QCD Phase Diagram and Beam Energy Scan

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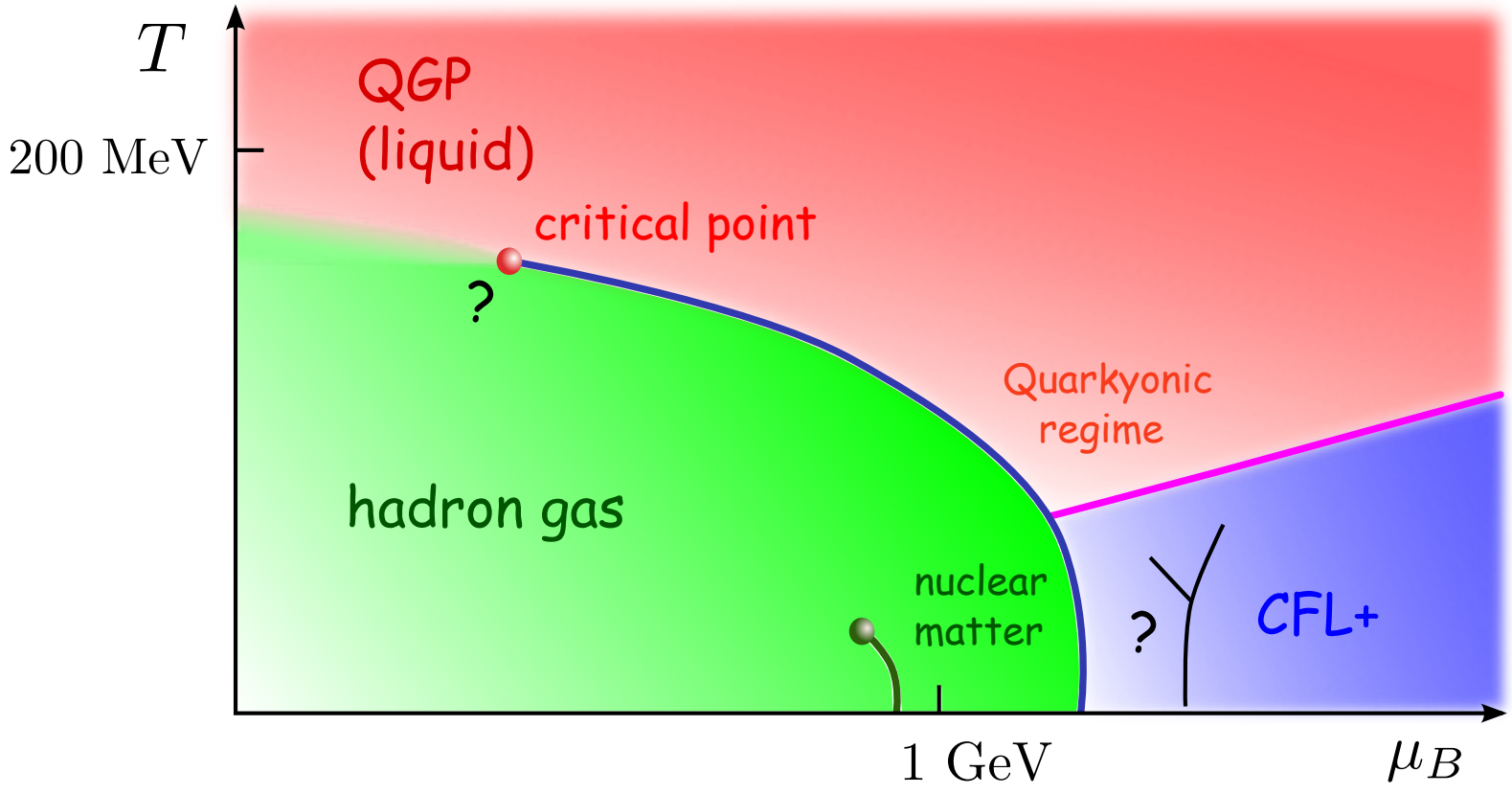
Outline

1. QCD phase diagram, critical point and fluctuations
   - Higher moments
2. RHIC beam energy scan
   - Mapping QCD phase diagram

QCD Phase Diagram *(a theorist's view)*



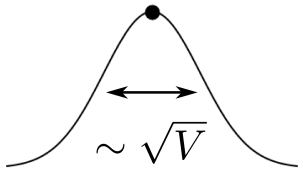
Lattice at $\mu_B \lesssim 2T$

Critical point – a singularity of EOS, anchors the 1st order transition.

Critical point and fluctuations

- The key equation:

$$P(X) \sim e^{S(X)} \qquad \textit{(Einstein 1910)}$$



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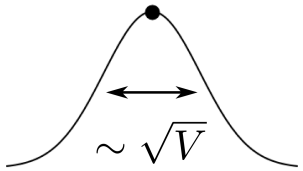

- An extensive fluctuation measure:

$$\langle X^2 \rangle = -\left(\frac{\partial^2 S}{\partial X^2}\right)^{-1} = VT\chi$$

Susceptibility χ is finite in thermodynamic limit $V \to \infty$ — CLT.

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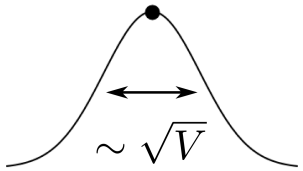

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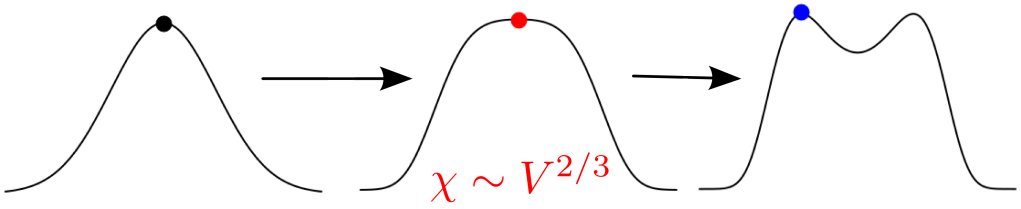

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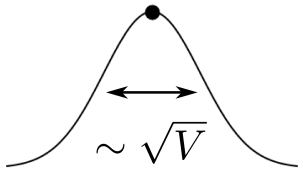

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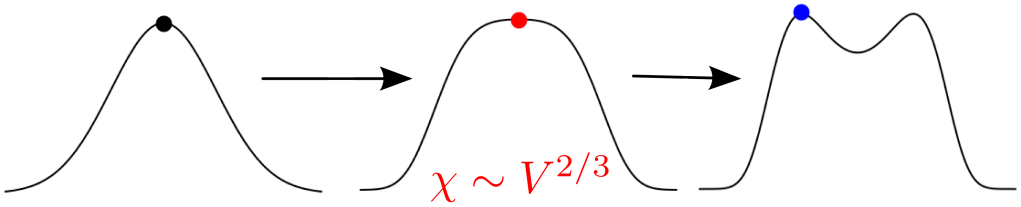
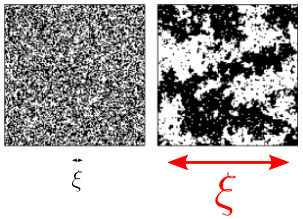




CLT? Fluctuations are not averaging out, but add coherently: $\xi \to \infty$

Fluctuations of order parameter and ξ

- Fluctuations at CP – conformal field theory.
  Parameter-free $\to$ universality. Near CP $\xi = m_\sigma^{-1} < \infty$,

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right].$$

- Moments of order parameter $\sigma_V \equiv \int d^3x\, \sigma(x)$:

  - Each propagator gives ξ^2. Thus $\langle \sigma_V^2 \rangle = VT\,\xi^2$.
  - As a result higher moments grow faster with ξ with universal exponents: $\langle \sigma_V^n \rangle \sim V\xi^k$, $k = n(3-[\sigma]) - 3$, $[\sigma] = \beta/\nu \approx 1/2$.

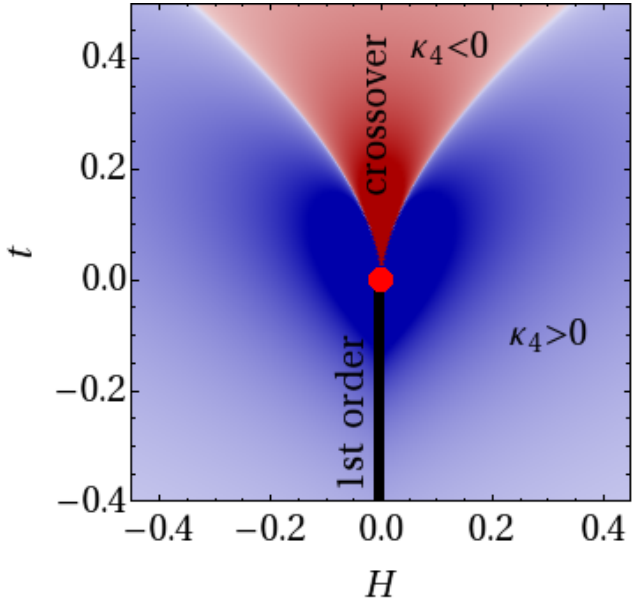
Sign

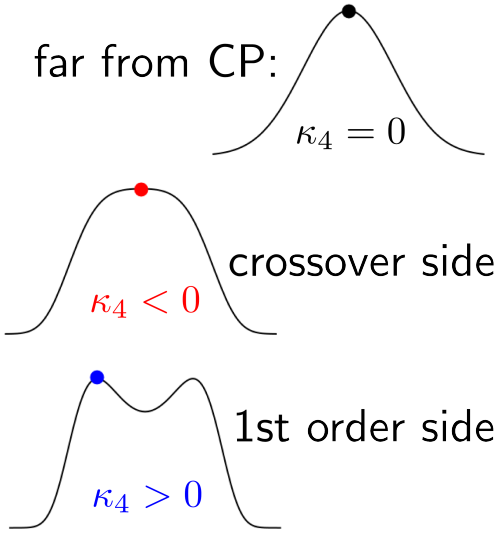
- Higher moments also depend on which side of the CP we are

$$\kappa_3[\sigma_V] = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}\,; \quad \kappa_4[\sigma_V] = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7\,.$$

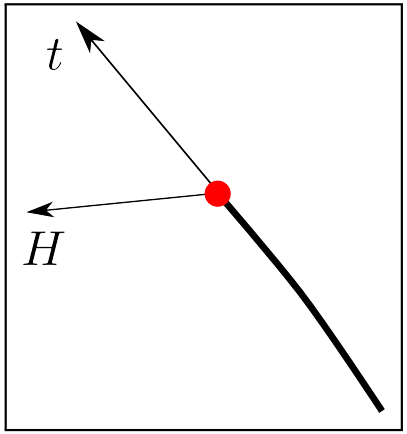
E.g., if symmetry ($\pm\sigma$) constrains $\lambda_3 = 0$ then $\kappa_3 = 0$ and $\kappa_4 < 0$.

- 2 relevant directions. Using universal Ising model variables:



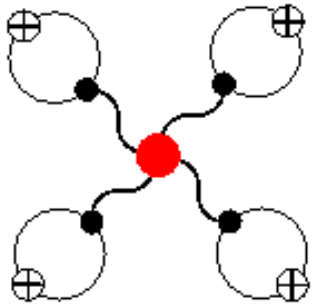


Mapping to QCD



- In QCD $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$

- $\kappa_4[N] = \langle N \rangle + \kappa_4[\sigma_V] \times g^4 \Big($ $\Big)^4 + \ldots,$

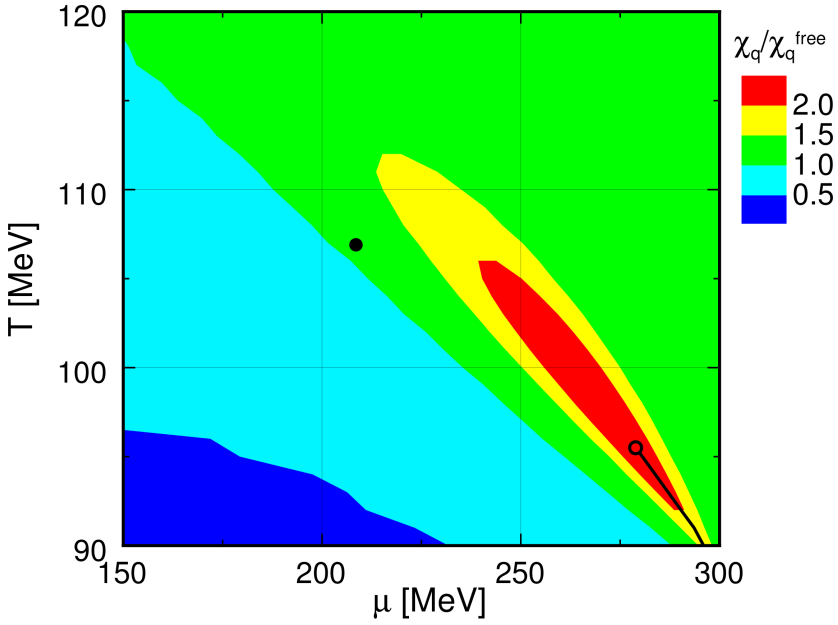


  $\kappa_4[\sigma_V] < 0$ means $\dfrac{\kappa_4[N]}{\langle N \rangle} < 1$

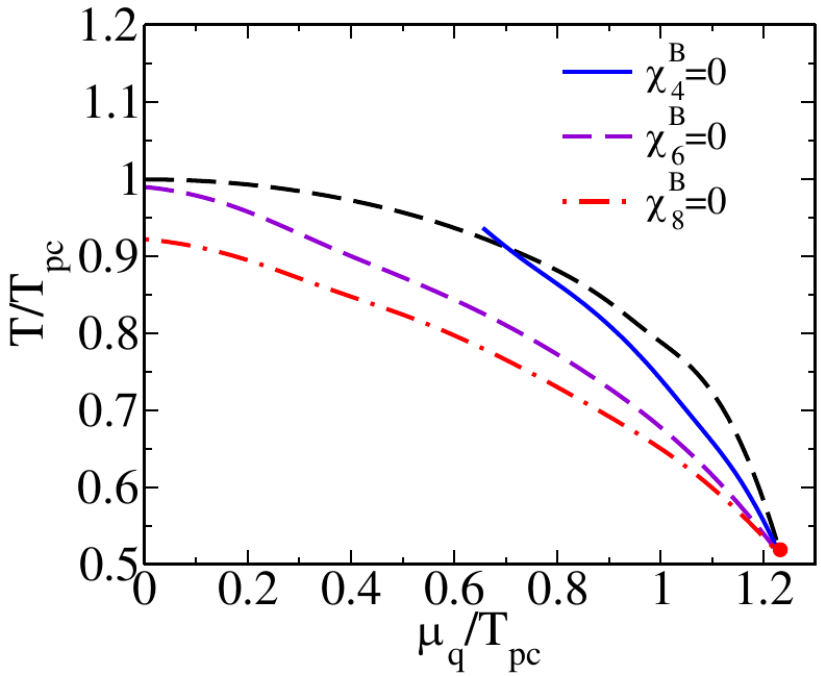
- Lessons:
  - Sensitivity to μ_B[CP] (exponential $e^{4\mu_B/T}$).
  - Sensitivity to $N_{\rm accepted}$: $(\kappa_4)_{\rm CP} \sim N^4$ (number of 4-tets).

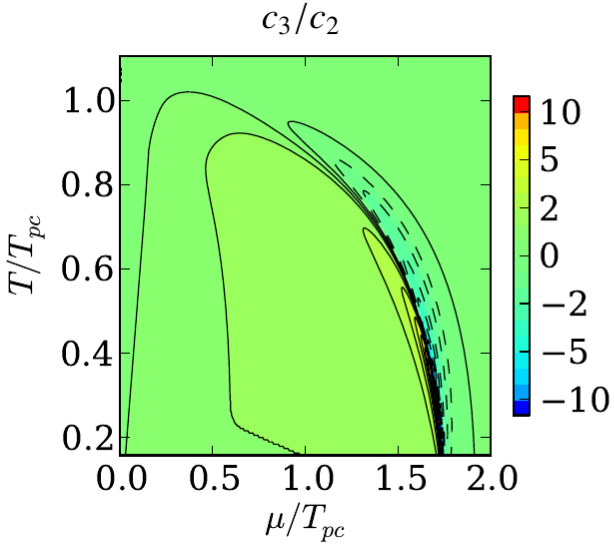
Critical region in models

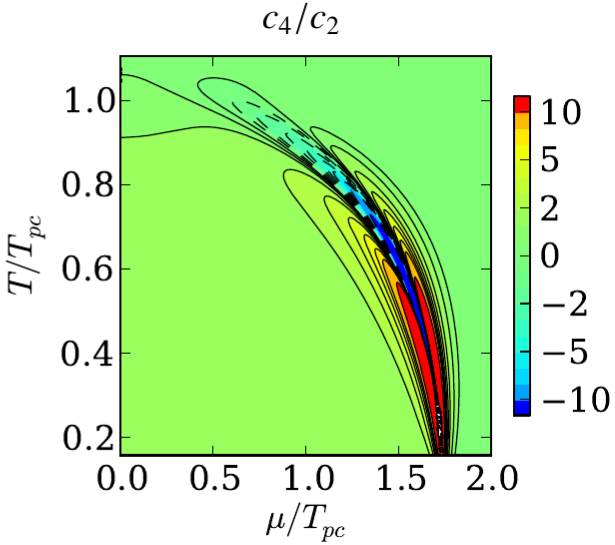
Hatta-Ikeda (2003)



Skokov-Friman-Karsch-Redlich (2011)



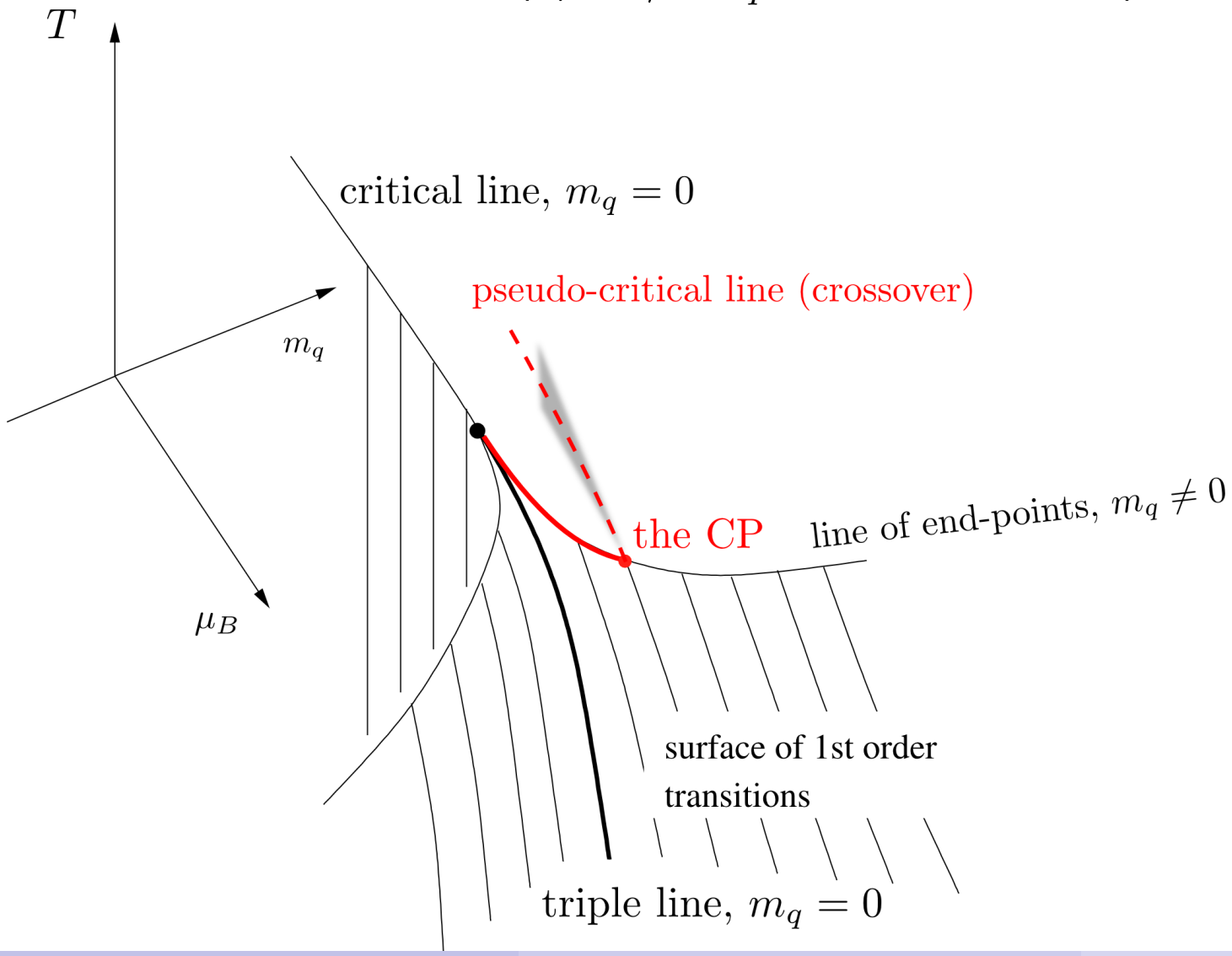




Skokov (2014)

Why the critical region is stretched

Critical region includes the "shadow" of the line of end-points.
($d\mu_{\rm CP}/dm_q \to \infty$ as $m \to 0$).



Why ξ is finite

System expands and is out of equilibrium

Critical slowing down. Universal scaling law:

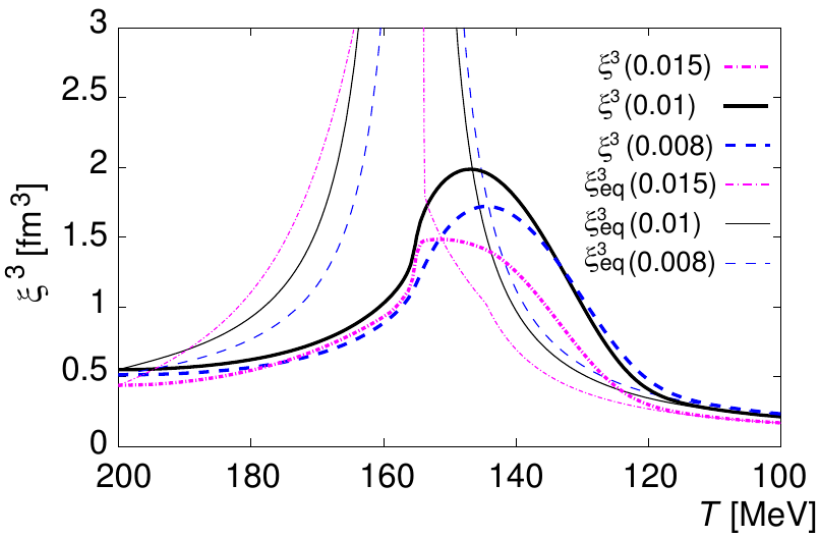
$$\xi \sim \tau^{1/z}, \qquad \text{where } 1/\tau \text{ is expansion rate}$$

and $z \approx 3$ (Son-MS).

Berdnikov-Rajagopal estimate $\xi \sim 2 - 3$ fm.

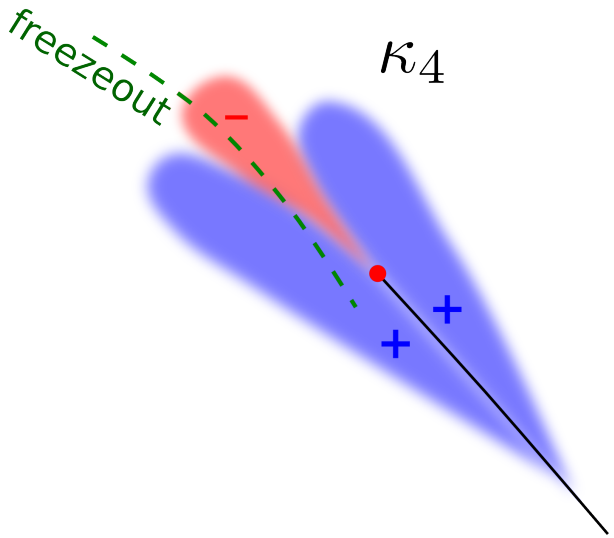
Hydrodynamics with a model EOS by Asakawa-Nonaka:

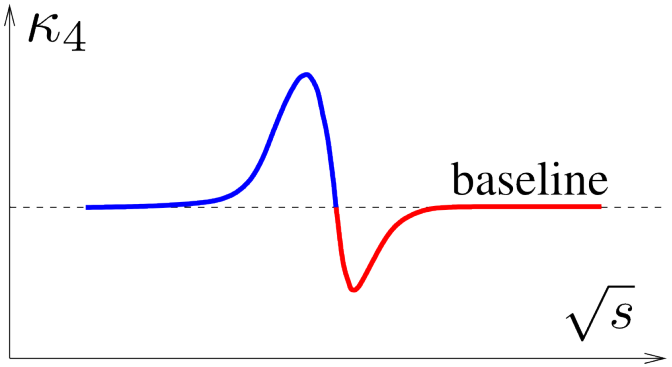
Significant for higher powers of ξ.



- Need full critical dynamics to take non-equilibrium into account

What should we see in the BES?

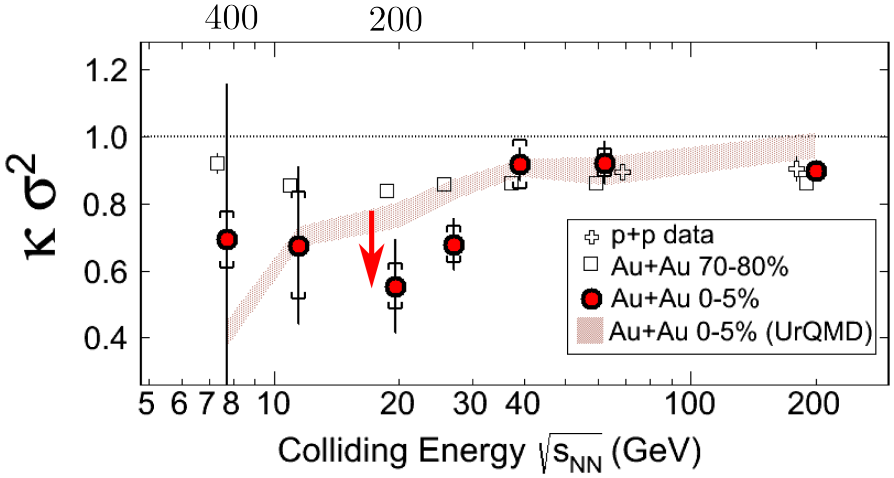


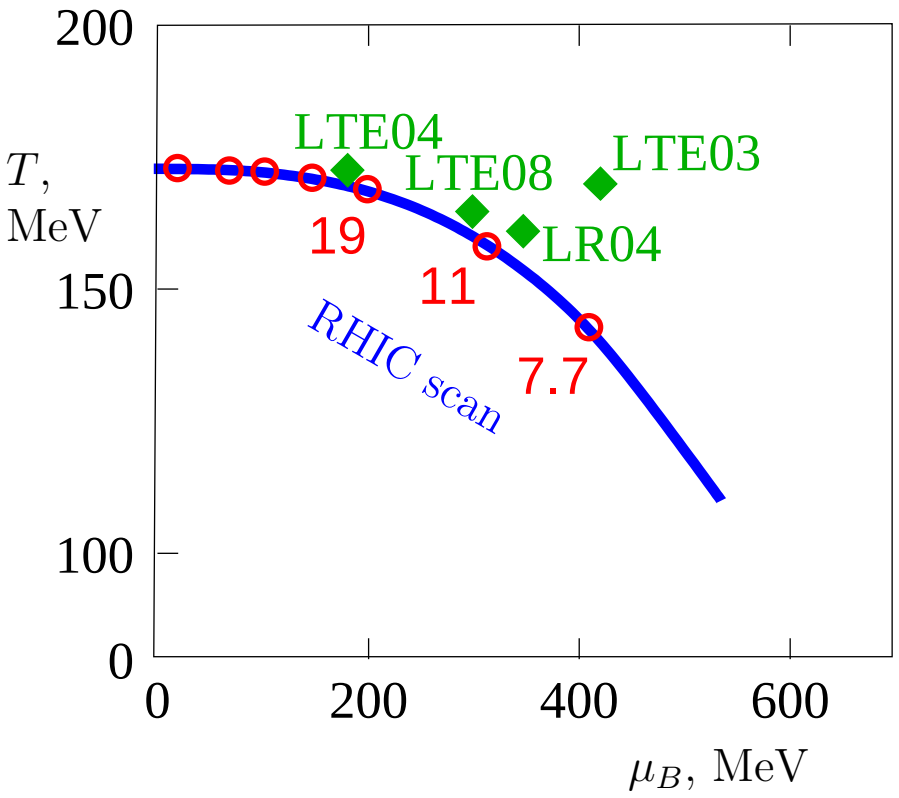


RHIC beam energy scan

Before CPOD 2014

STAR, PRL 112, 032302 (2014)



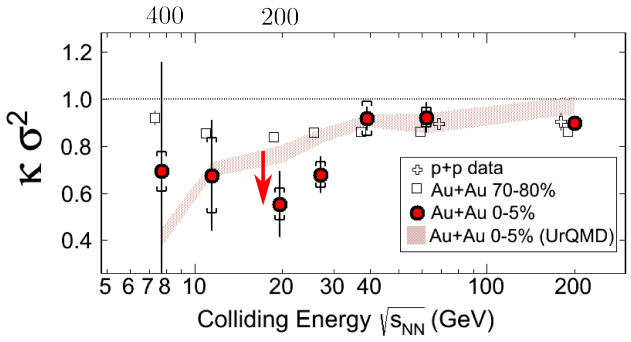


- Negative contribution to κ_4 around 19 GeV ($\mu_B \sim 200$ MeV).
- Large uncertainties at 7.7 and 11 GeV.

What do we learn?

- Before CPOD 2014
  - Consistent with lattice – no signals of the CP at $\mu_B < 200$ MeV.
  - Possible scenarios:
    μ_B[CP] ~ 250 MeV (11-19 gap)
    or μ_B[CP] > 400 MeV.

    Inconclusive without κ_4 rising above the baseline.



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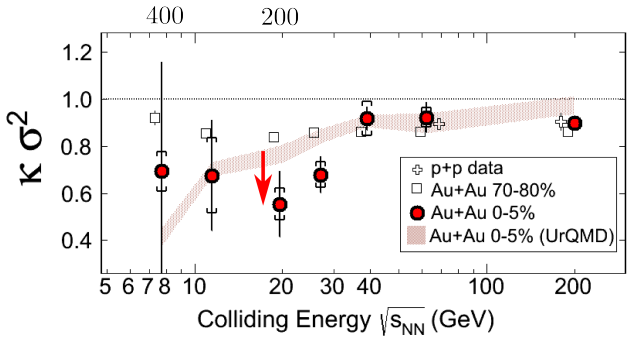
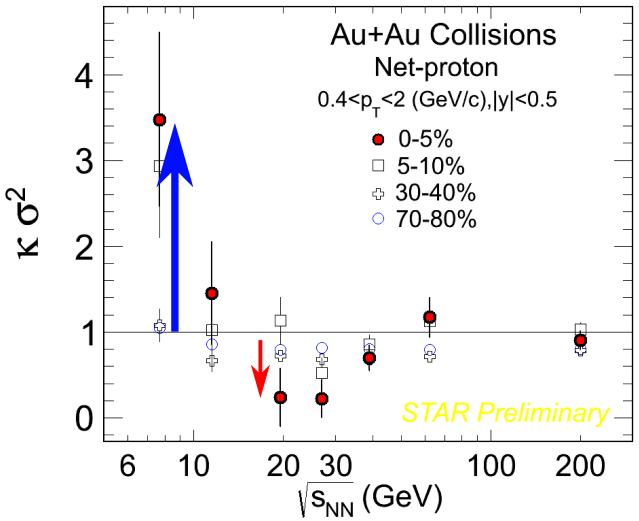
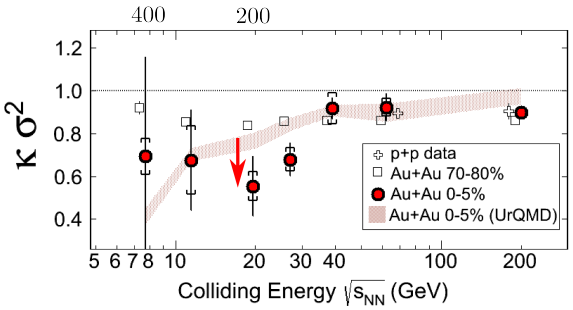
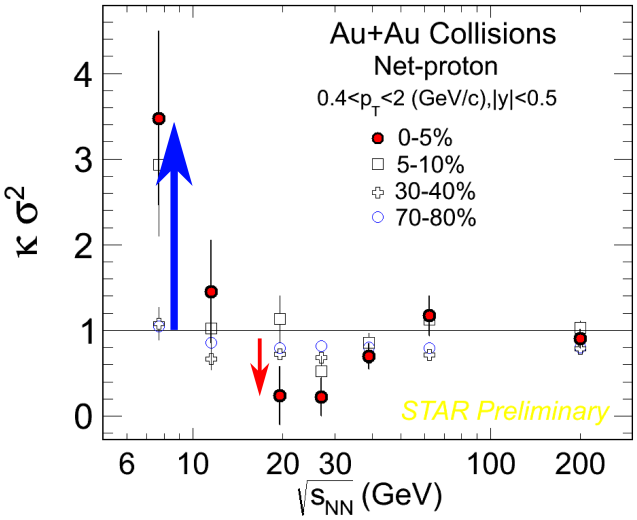


- Since CPOD 2014
  - New data (X.Luo) suggest a scenario with CP at $\mu_B \gtrsim 400$ MeV.



Questions and Thoughts





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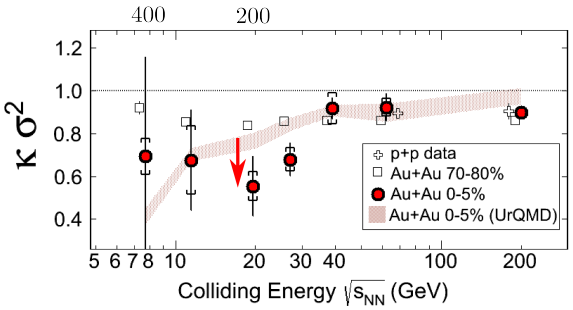

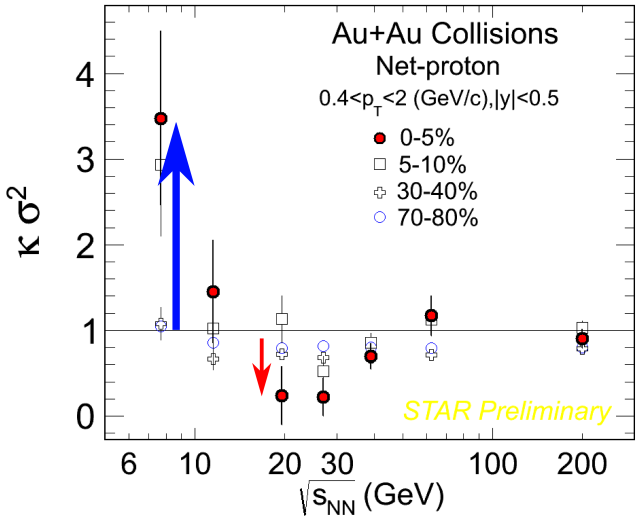

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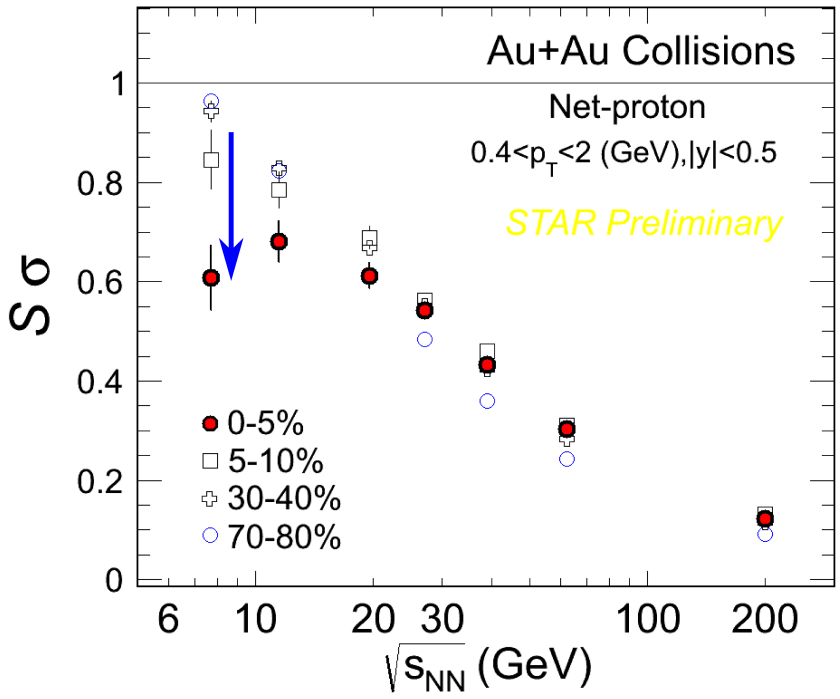
  - The $N_{\text{accepted}} \sim$ doubled and $\kappa_4 \sim N^4$.
- Why in 0-5% but not in 70-80%?
  - Bigger system. Cools *slower*.
    Larger ξ (Berdnikov-Rajagopal) and $\kappa_4 \sim \xi^7$.
  - Important to study dynamical evolution of fluctuations.

The sign of skewness



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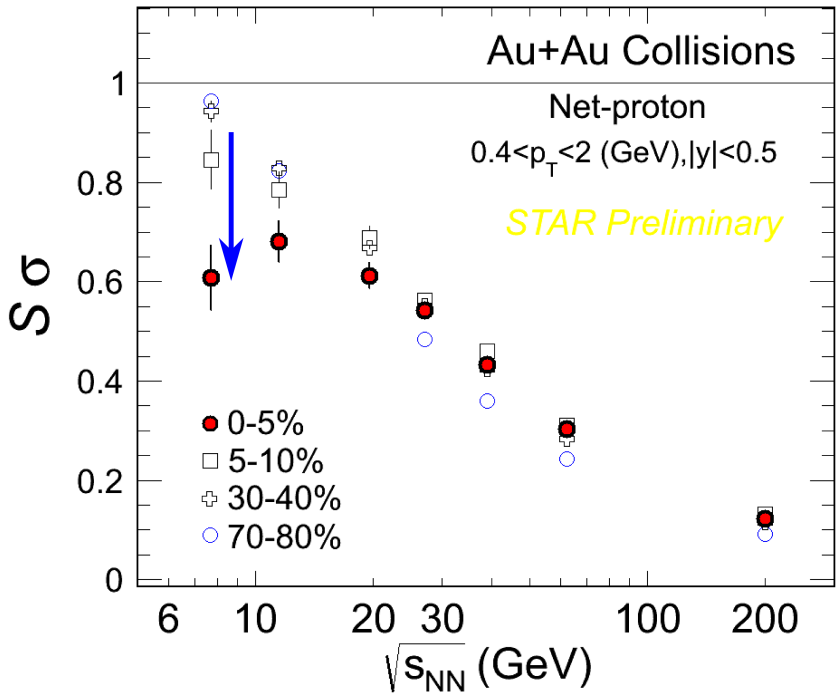
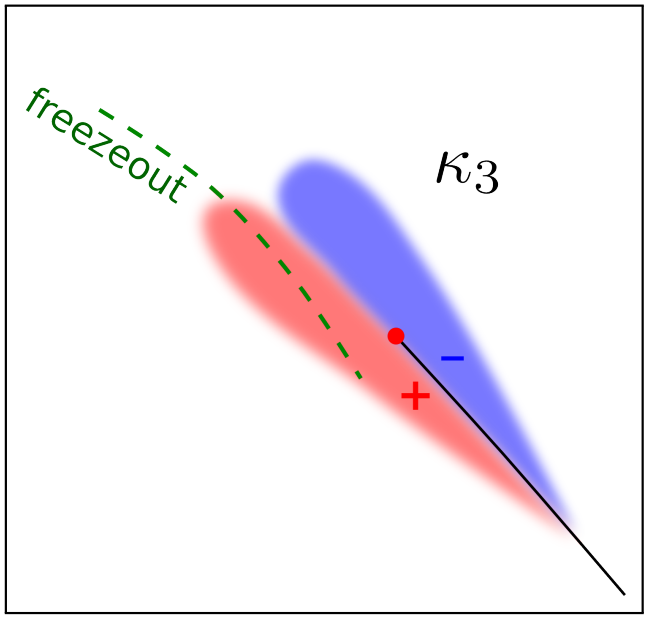


The critical point contribution is negative *above* CP.



Time evolution of cumulants (memory)

Critical slowing down.

Mukherjee-Venugopalan-Yin

$$\frac{dP}{d\tau} = F[P]$$

$$\Downarrow$$

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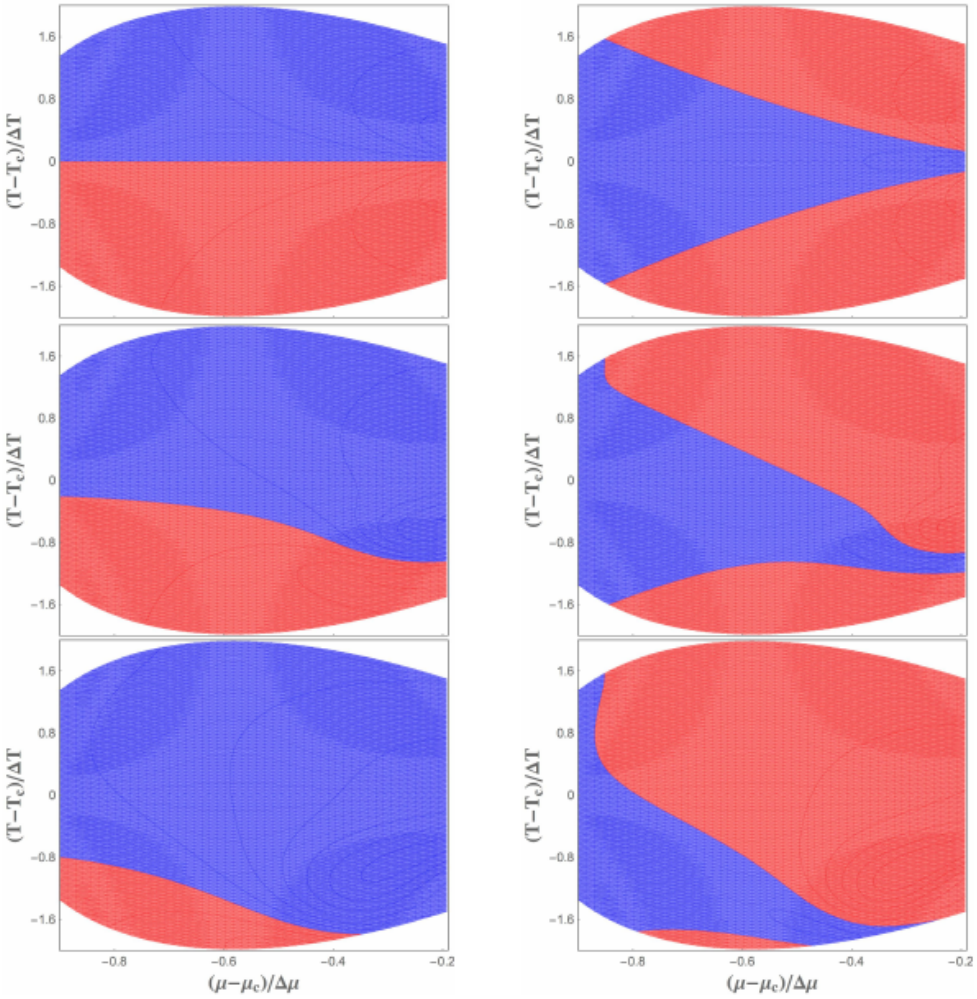

κ_3 κ_4

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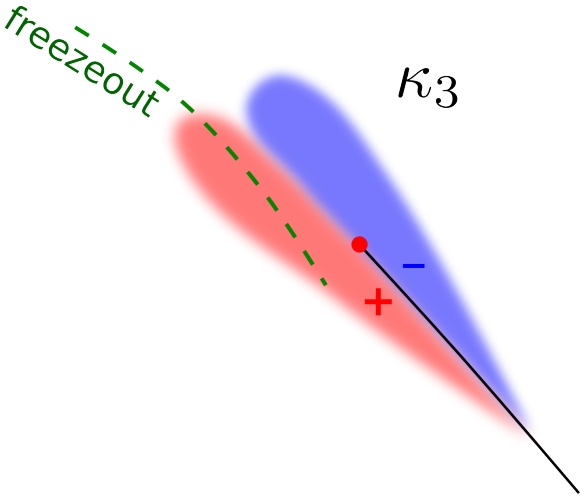

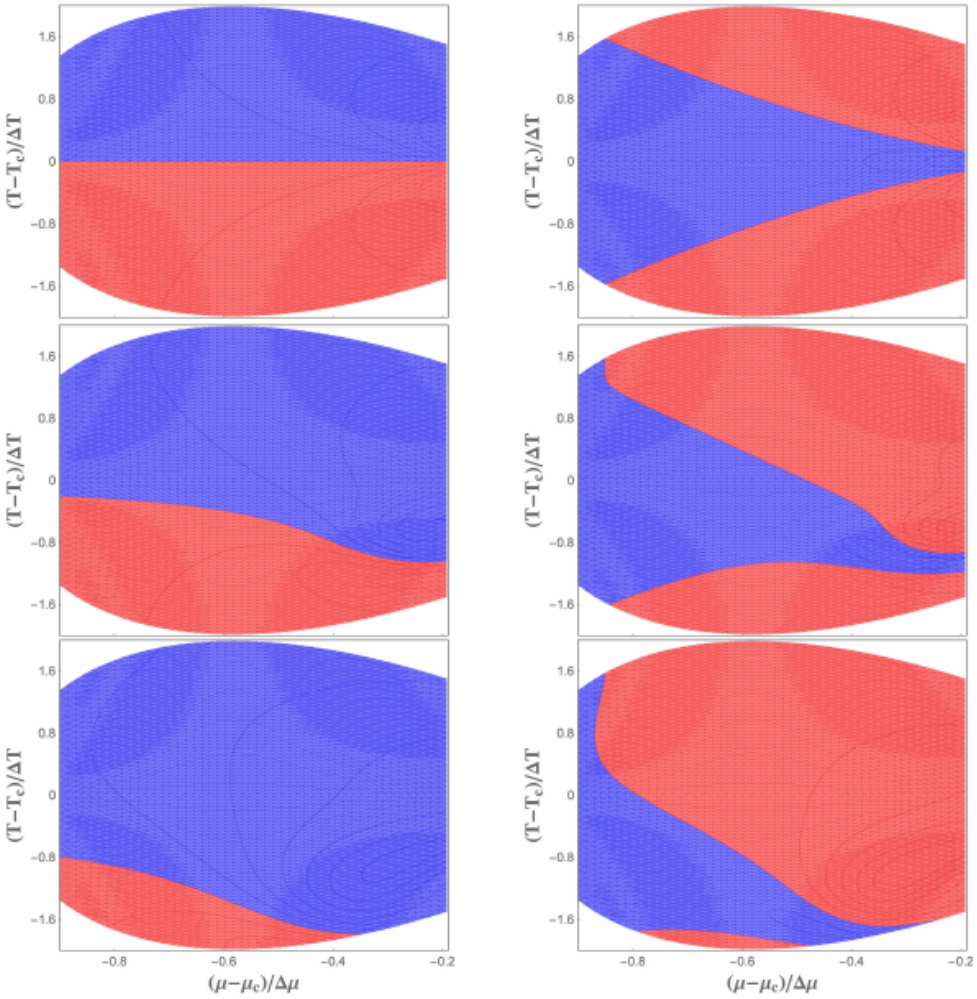

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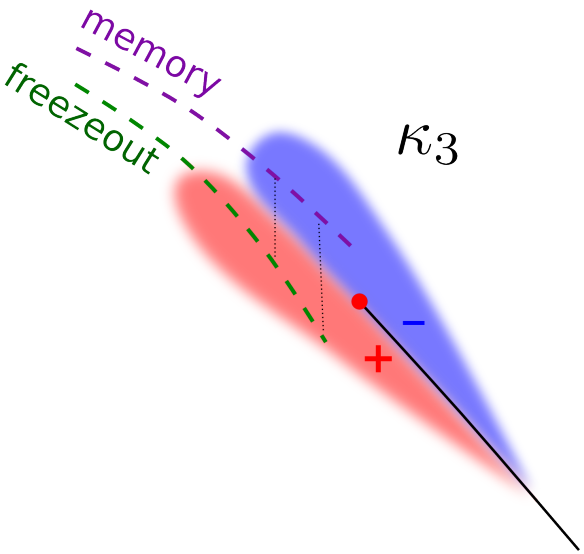

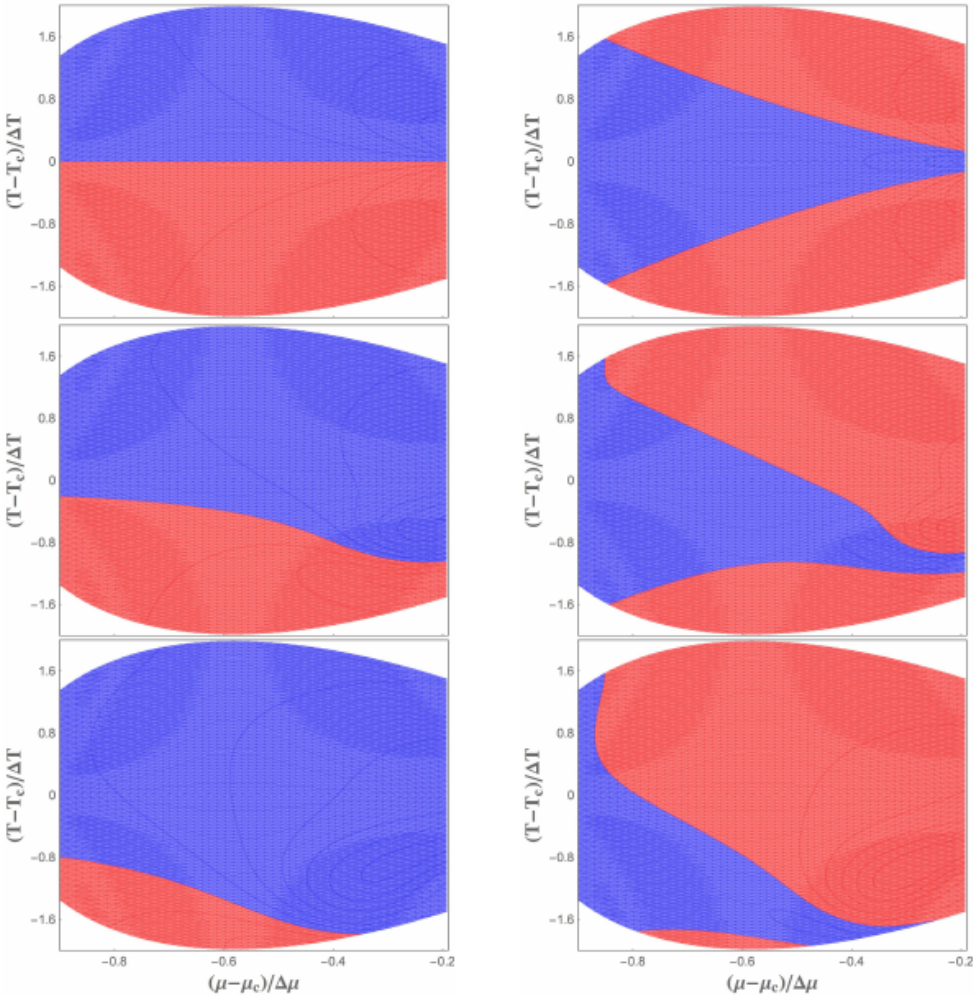

κ_3 κ_4

Summary and conclusions

- Fluctuations reflect universal features of the CP and could be used to discover it via the BES.
  - 4-th moment of (net-)proton multiplicity: sensitive to ξ while measurable with available statistics.
- Interesting data from recent analysis (wider p_T acceptance). Needs better understanding and *quantitative* comparison.
  - Needed better knowledge of QCD EoS. Lattice. (Karsch)
  - Dynamical description of fluctuations is essential. (talks by Nahrgang, Schenke, Yin)
- Concerted theory effort is needed: lattice + critical EoS + dynamics modelling (hydro, cascade) + MHD for chiral effects $\Rightarrow$ BES Theory collaboration.