Universität Bielefeld

Transport and spectral properties of heavy quarks

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RHIC&AGS meeting
BNL
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Outline

1) Color electric field correlation function

[A.Francis, OK, M.Laine, T.Neuhaus, H.Ohno PRD92(2015)116003]

Heavy quark momentum diffusion coefficient κ

estimate for the time scale for kinetic equilibration of heavy quarks

2) Charmonium and Bottomonium dissociation patterns

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]

using a complex valued heavy quark potential

we determine the spectra in the bound state region

and their thermal modifications

and give an estimate for Ψ' to J/Ψ ratio due to regenration at T_c

3) Vector meson correlation functions for heavy quarks

[with H-T.Ding, A.Francis, M.Laine and H.Ohno]

relativistic treatment of charm and bottom on the lattice

Motivation - Transport Coefficients

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

Need to be determined from QCD using first principle lattice calculations!

for heavy flavour:

Heavy Quark Diffusion Constant D

[H.T.Ding, OK et al., PRD86(2012)014509]

Heavy Quark Momentum Diffusion κ

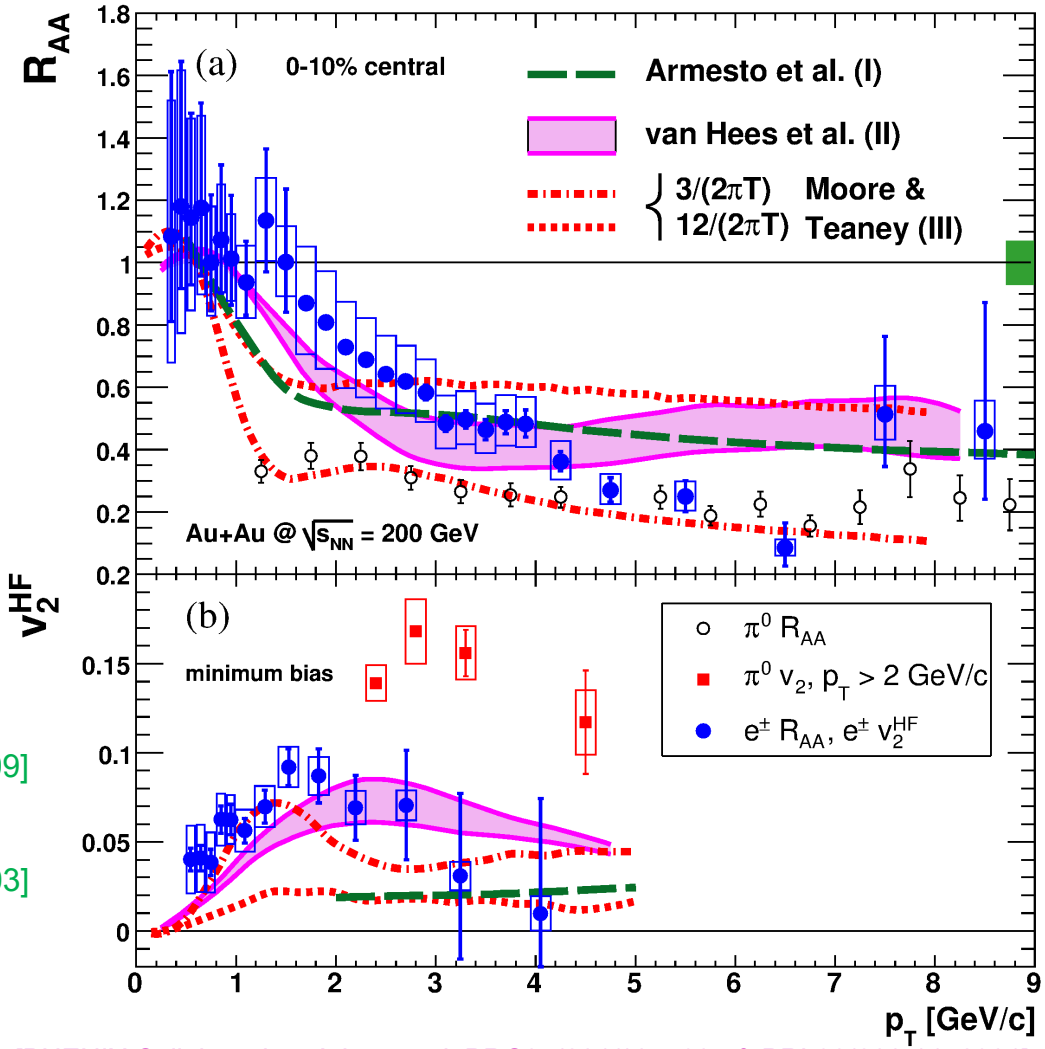
[A.Francis, OK, et al., PRD92(2015)116003]

for light quarks:

Light quark flavour diffusion

Electrical conductivity

[A.Francis, OK et al., PRD83(2011)034504
H-T.Ding, F.Meyer, OK, arXiv:1604.06712]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

Vector meson spectral function – hard to separate different scales

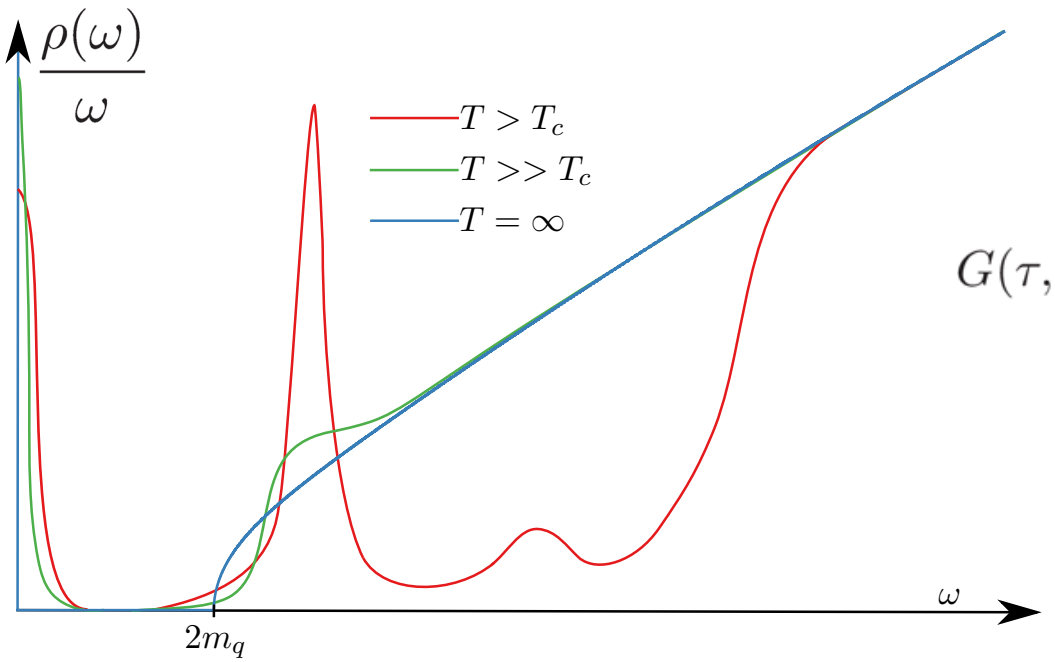
Different contributions and scales enter in the spectral function

Quarkonium

Spectral functions in the QGP

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- **in addition cut-off effects on the lattice**

notoriously difficult to extract from correlation functions



$$G(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)$$

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}, \qquad \eta = \frac{T}{MD}$

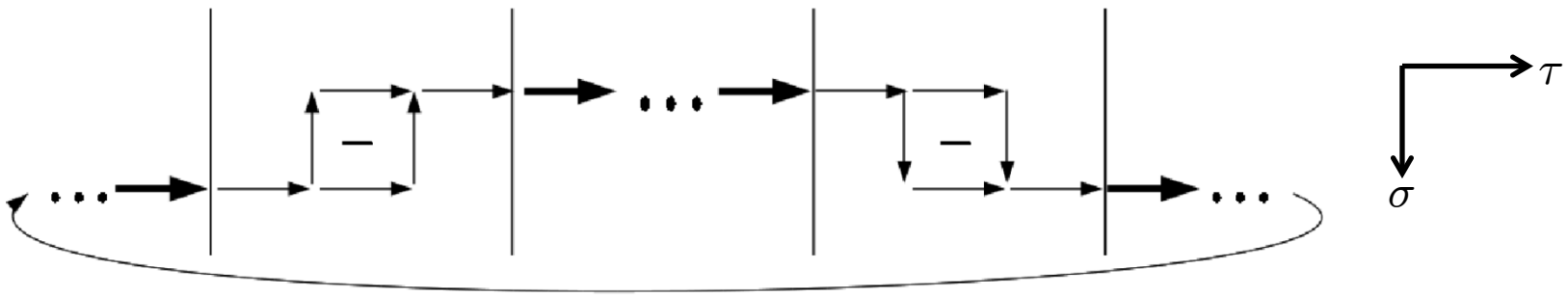
Heavy Quark Momentum Diffusion Constant – Single Quark in the Medium

Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\frac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\frac{1}{T};0)]\right\rangle}$$

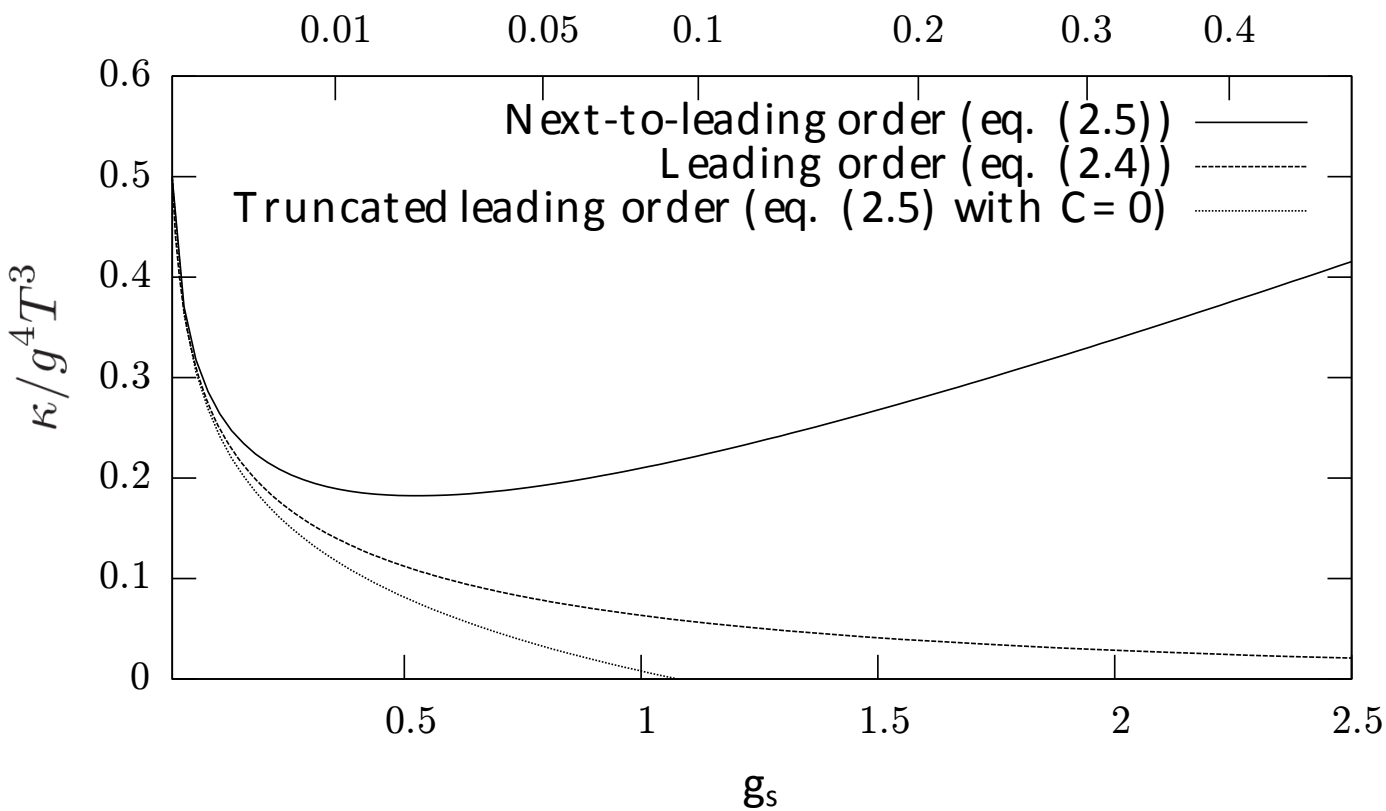
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

can be related to the thermalization rate: $$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

NLO in perturbation theory: [Caron-Huot, G.Moore, JHEP 0802 (2008) 081]

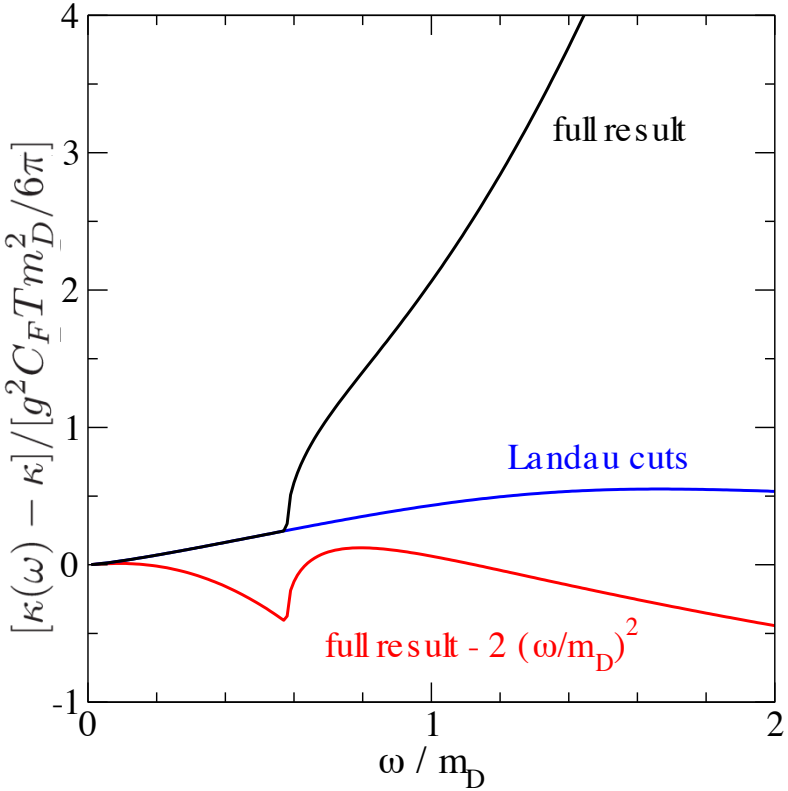


very poor convergence

→ **Lattice QCD study required in the relevant temperature region**

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



in contrast to a narrow transport peak, from this a smooth limit

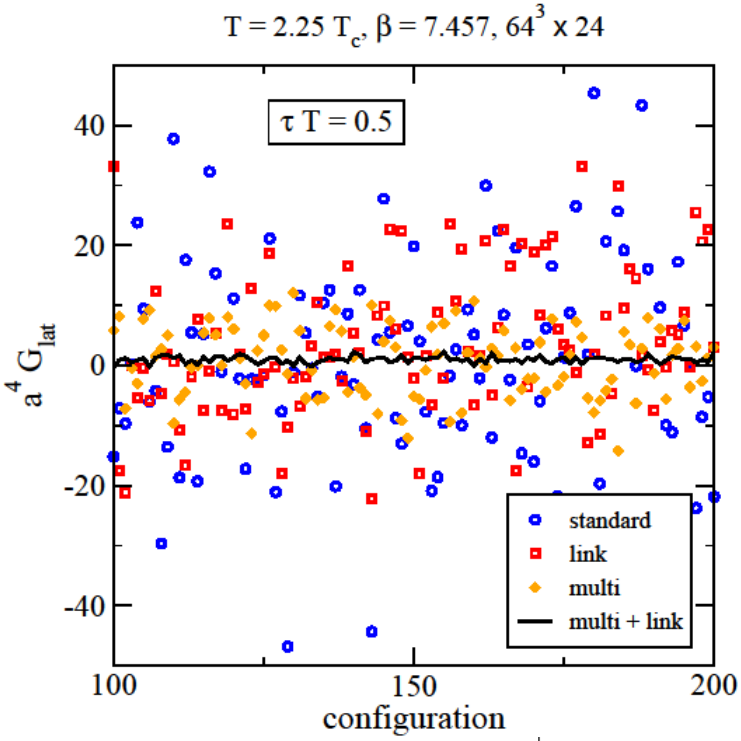
$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

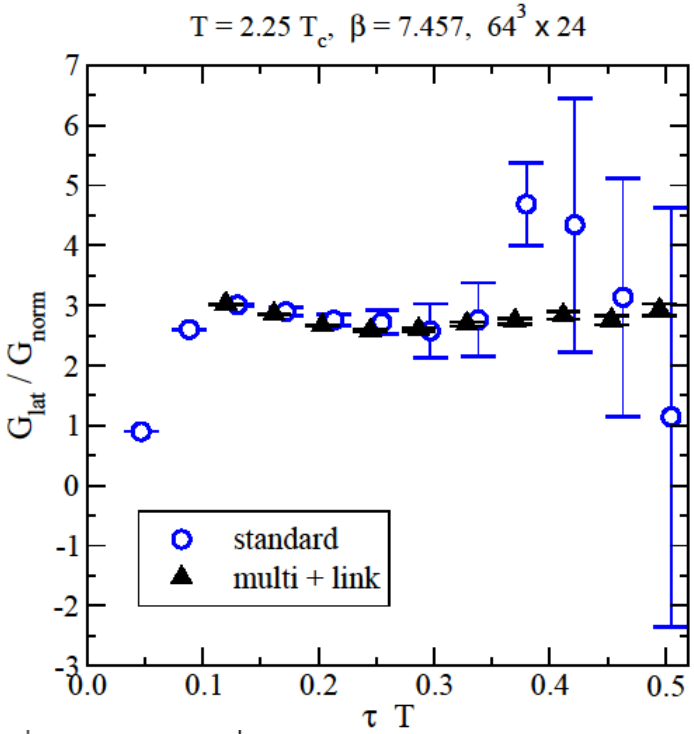
is expected

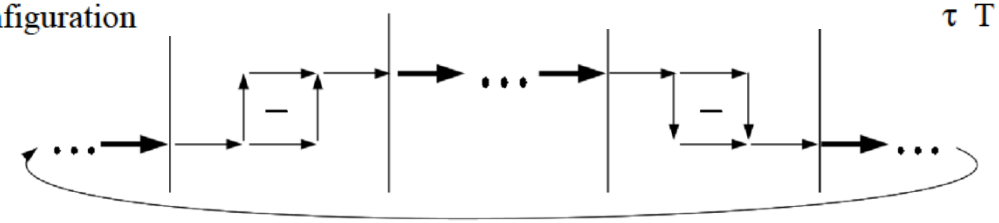
qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

Heavy Quark Momentum Diffusion Constant – Lattice algorithms

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]







due to the gluonic nature of the operator, signal is extremely noisy

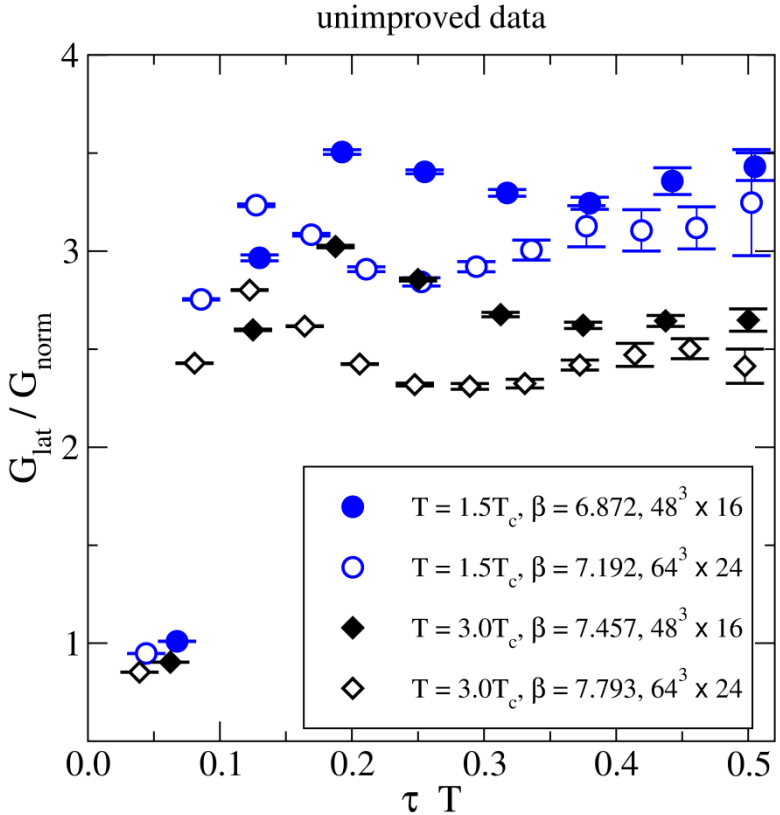
→ **multilevel combined with link-integration techniques to improve the signal**

[Lüscher,Weisz JHEP 0109 (2001)010 and H.B.Meyer PRD (2007) 101701]

[Parisi,Petronzio,Rapuano PLB 128 (1983) 418, and de Forcrand PLB 151 (1985) 77]

Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



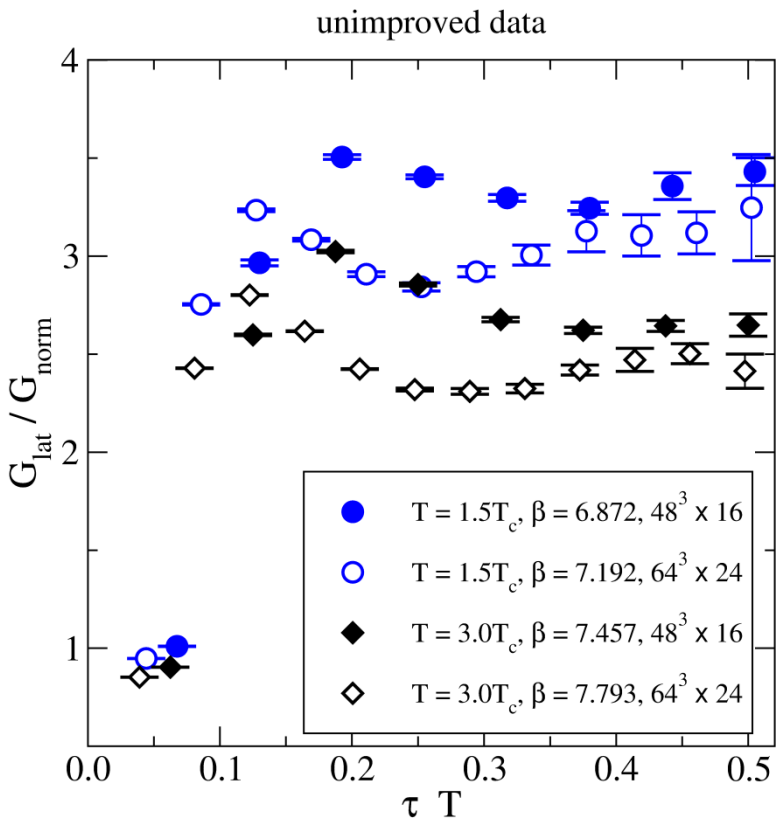
normalized by the LO-perturbative correlation function:

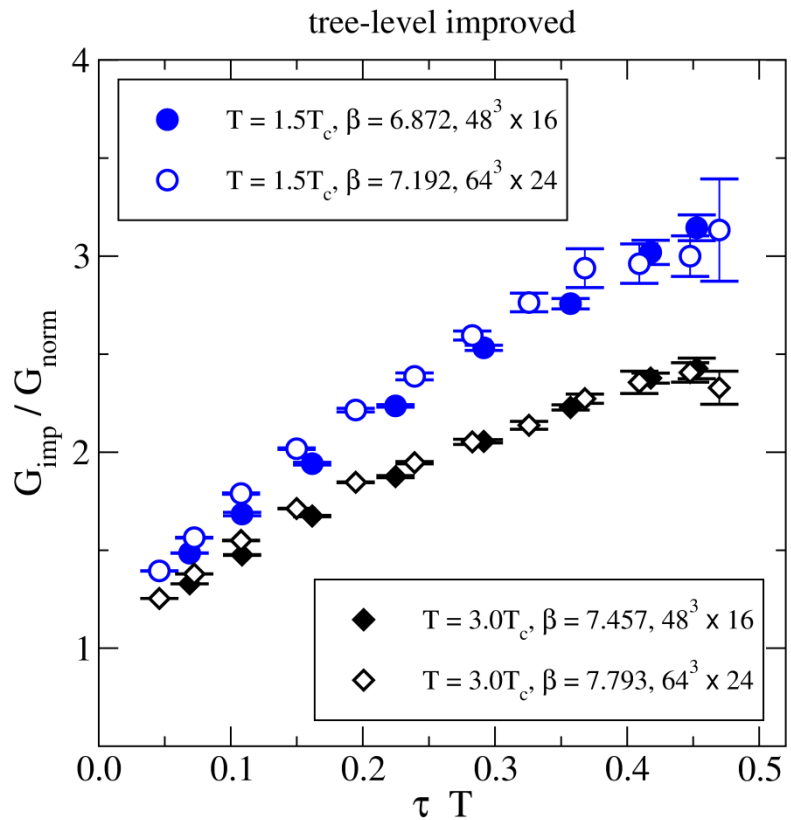
$$G_{\rm norm}(\tau T) \equiv \frac{G^{\rm LO}_{\rm cont}(\tau T)}{g^2 C_F} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right] \qquad C_F \equiv \frac{N_c^2 - 1}{2N_c}$$

and renormalized using NLO renormalization constants $Z(g^2)$

Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]





lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G^{\mathrm{LO}}_{\mathrm{cont}}(\overline{\tau T}) = G^{\mathrm{LO}}_{\mathrm{lat}}(\tau T)$$

leads to an effective reduction of cut-off effect for all τT

Heavy Quark Momentum Diffusion Constant – Lattice results

[A.Francis, OK, M.Laine, T.Neuhaus, H.Ohno, PRD92(2015)116003]

Quenched Lattice QCD on large and fine isotropic lattices at $T \simeq 1.5\ T_c$

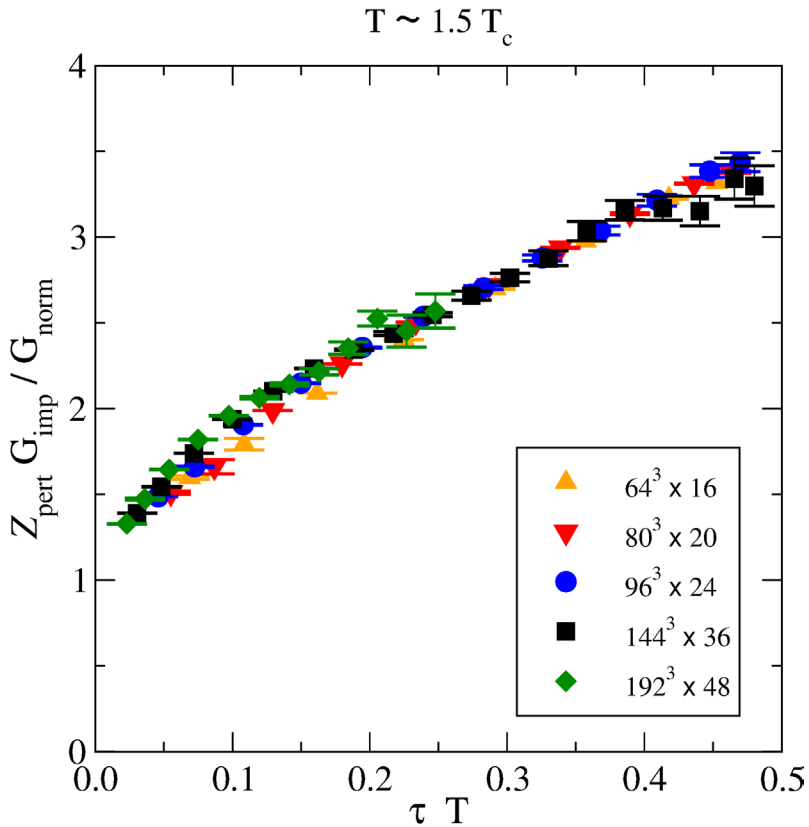
- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration $N_s/N_t = 4$, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, $a \to 0 \ \leftrightarrow\ N_t \to \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$
- scale setting using r_0 and t_0 scale [A.Francis,OK,M.Laine, T.Neuhaus, H.Ohno, PRD91(2015)096002]

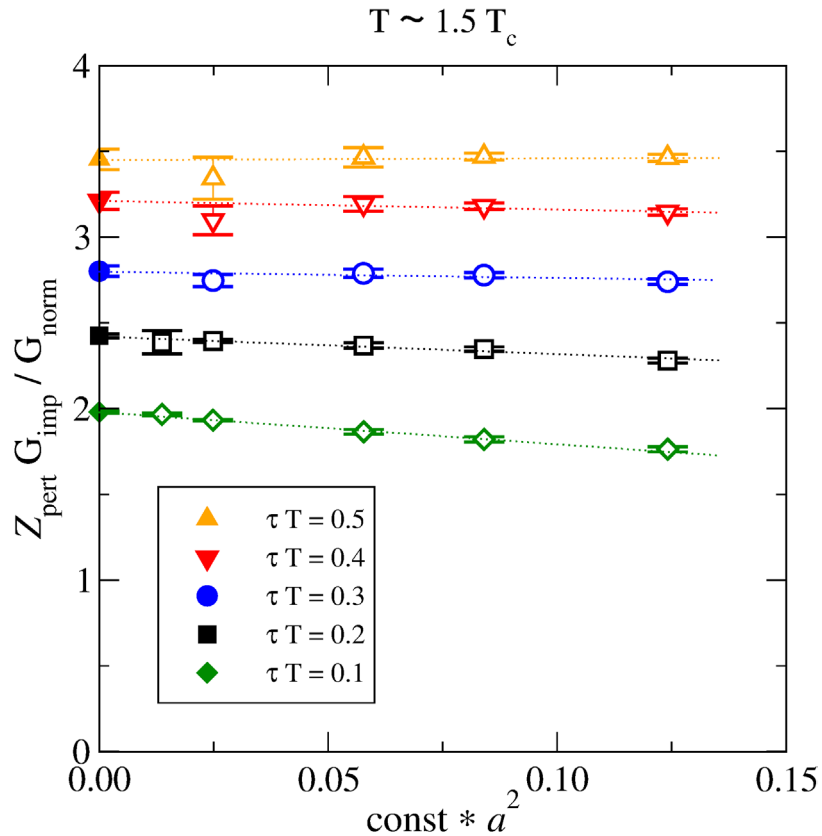
β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}^{(\mathrm{imp})}$	$T/T_c\|_{t_0}^{(\mathrm{imp})}$	$T\sqrt{t_0}^{(\mathrm{clov})}$	$T/T_c\|_{t_0}^{(\mathrm{clov})}$	Tr_0	$T/T_c\|_{r_0}$
6.872	$64^3 \times 16$	172	0.3770	1.52	0.3805	1.53	1.116	1.50
7.035	$80^3 \times 20$	180	0.3693	1.48	0.3739	1.50	1.086	1.46
7.192	$96^3 \times 24$	160	0.3728	1.50	0.3790	1.52	1.089	1.46
7.544	$144^3 \times 36$	693	0.3791	1.52	0.3896	1.57	1.089	1.46
7.793	$192^3 \times 48$	223	0.3816	1.53	0.3955	1.59	1.084	1.45

similar studies by [Banerjee,Datta,Gavai,Majumdar, PRD85(2012)014510]
and [H.B.Meyer, New J.Phys.13(2011)035008]

Heavy Quark Momentum Diffusion Constant – Lattice results

we performed a continuum extrapolation, $a \to 0 \leftrightarrow N_t \to \infty$ **, at fixed** $T=1/a\,N_t$





well behaved continuum extrapolation for $0.05 \leq \tau T \leq 0.5$

finest lattice already close to the continuum

coarser lattices at larger τT show almost no cut-off effects

how to extract the spectral function from the correlator?

Heavy Quark Momentum Diffusion Constant – IR and UV asymptotics

$\omega \ll$T: linear behavior motivated at small frequencies

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$\omega \gg$T: vacuum perturbative results and leading order thermal correction:

$$\rho_{\mathrm{UV}}(\omega) = \left[\rho_{\mathrm{UV}}(\omega)\right]_{T=0} + \mathcal{O}\left(\frac{g^4T^4}{\omega}\right)$$

using a renormalization scale $\bar{\mu}_\omega = \omega$ for $\omega \gg \Lambda_{\overline{MS}}$ leading order becomes

$$\rho_{\mathrm{UV}}(\omega) = \Phi_{UV}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{MS})}\right)\right]$$

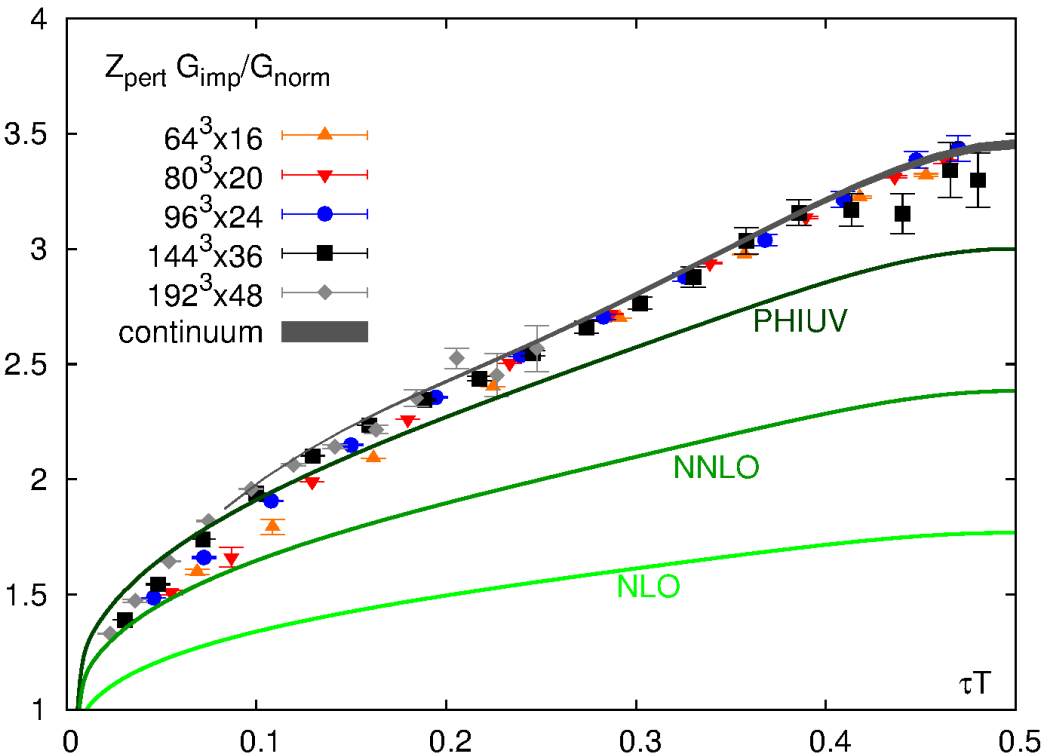
$$\Phi_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega)C_F\omega^3}{6\pi} \quad , \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

model the spectral function using these asymptotics with two free parameters

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{A\Phi_{\mathrm{UV}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



$$\rho_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

already closer to the data

Model spectral function: transport contribution + UV-asymptotics

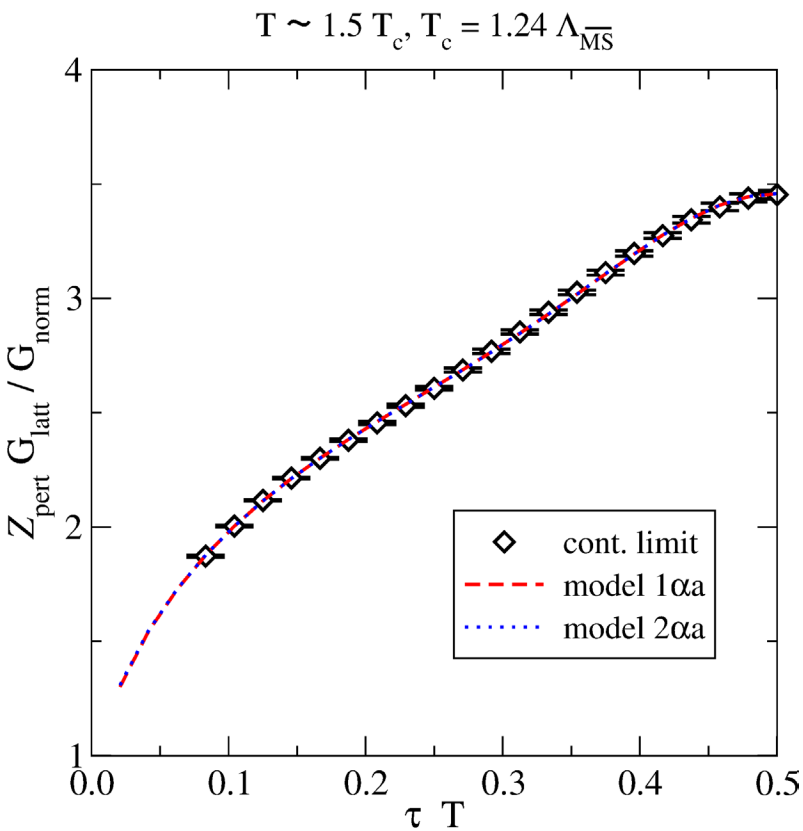
$$\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm UV}(\omega), \frac{\omega\kappa}{2T} \right\}$$

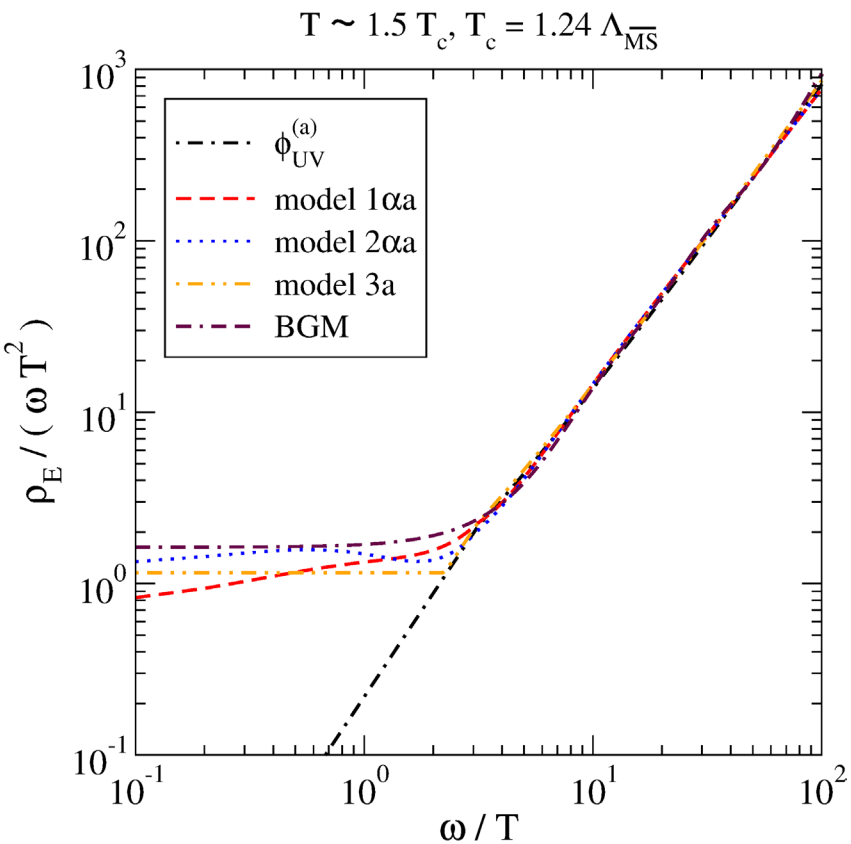
$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

analysis of the systematic uncertainties by

modeling corrections to ρ_{IR} by a power series in ω





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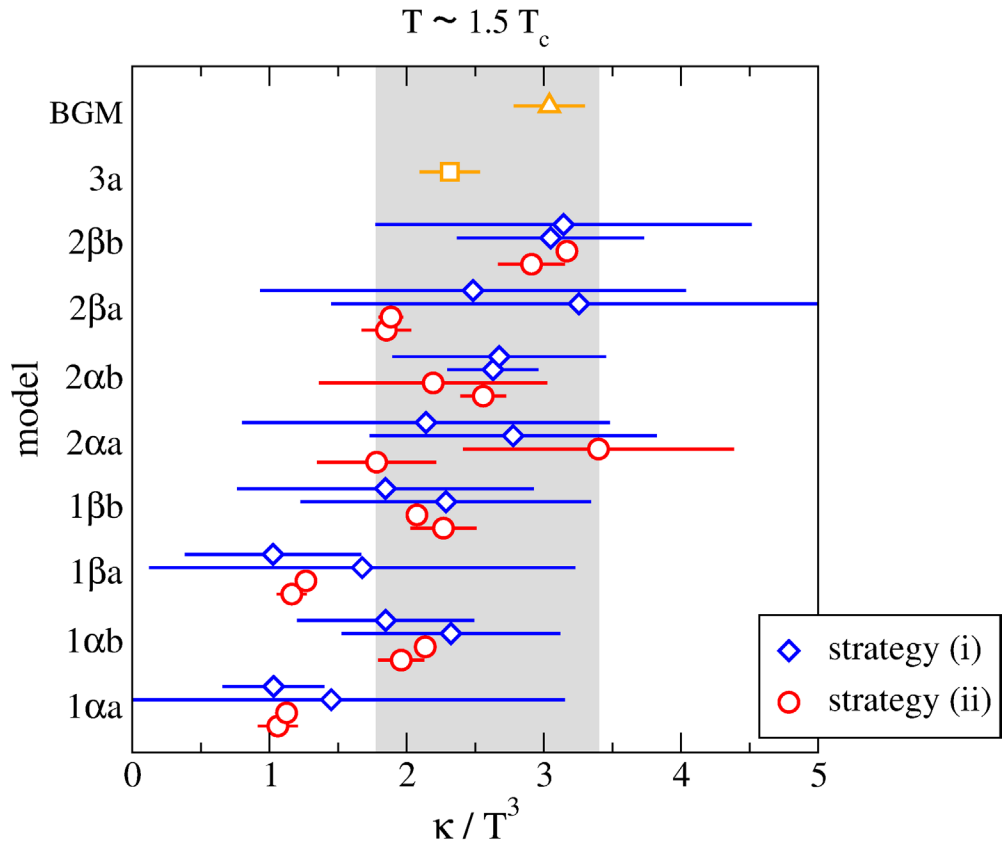
$$\kappa/T^3 = \lim_{\omega \to 0} \frac{2T \rho_{\rm E}(\omega)}{\omega}$$

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

model corrections to ρ_{IR} by a power series in ω



analysis of the systematic uncertainties

→ **continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} = 1.8...3.4$$

Equilibration time scale for heavy quarks

in the non-relativistic limit, κ is related to the diffusion coefficient D:

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7...7.0$$

and to the drag coefficient η_D:

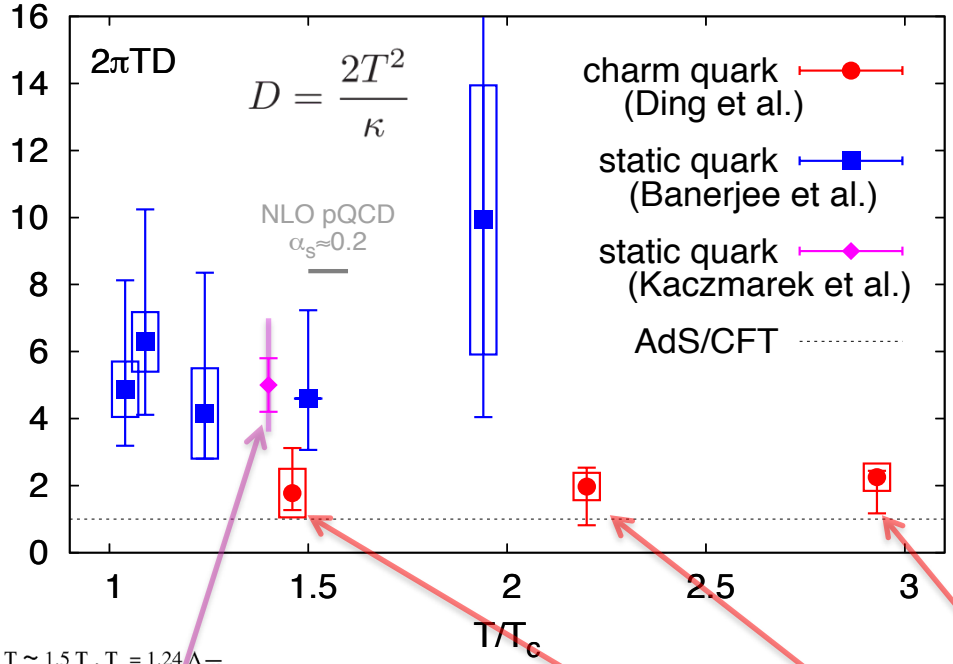
$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

used to estimate the time scale associated with kinetic equilibration of heavy quarks:

$$\tau_{\rm kin} = \frac{1}{\eta_D} = (1.8\ldots3.4)\left(\frac{T_{\rm c}}{T}\right)^2\left(\frac{M}{1.5\ {\rm GeV}}\right){\rm fm/c}$$

close to T_c, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.

Lattice QCD results on heavy quark diffusion coefficients

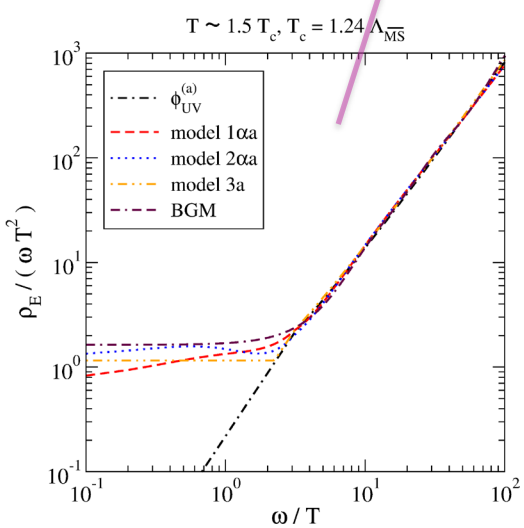


[H.T.Ding, OK, H.Satz et al., PRD86(2012)014509]

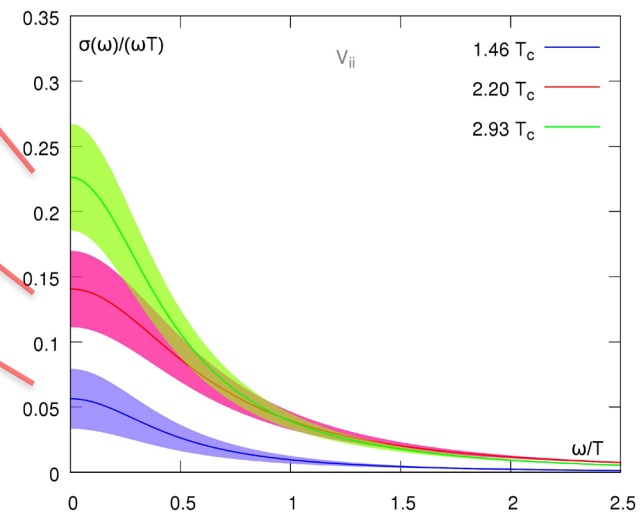
[Banerjee et al., PRD85(2012) 014510]

[A.Francis, OK et al., PRD92(2015)116003]

charm quark mass
quenched approximation
no continuum yet



heavy quark mass limit
quenched approximation
continuum extrapolated



next goals: continuum extrapolation for charm and bottom + including dynamical quarks

→ quark mass dependence of diffusion coefficients + sequential melting of quarkonia

Vector meson spectral function – hard to separate different scales

Charmonium and Bottomonium

Spectral functions in the QGP

Different contributions and scales enter in the spectral function

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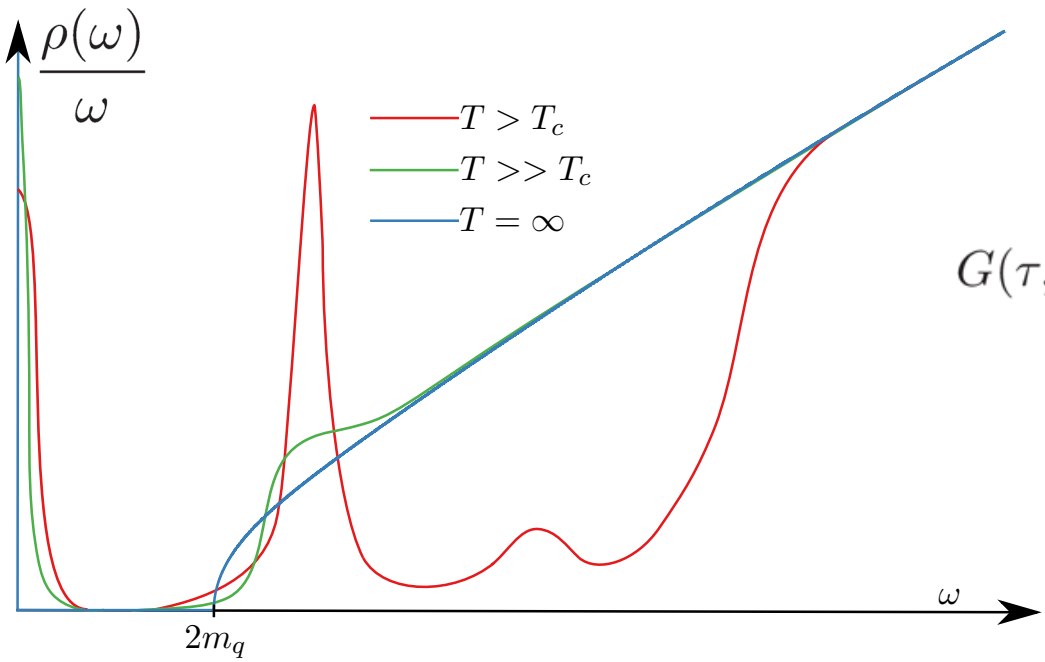
notoriously difficult to extract from correlation functions



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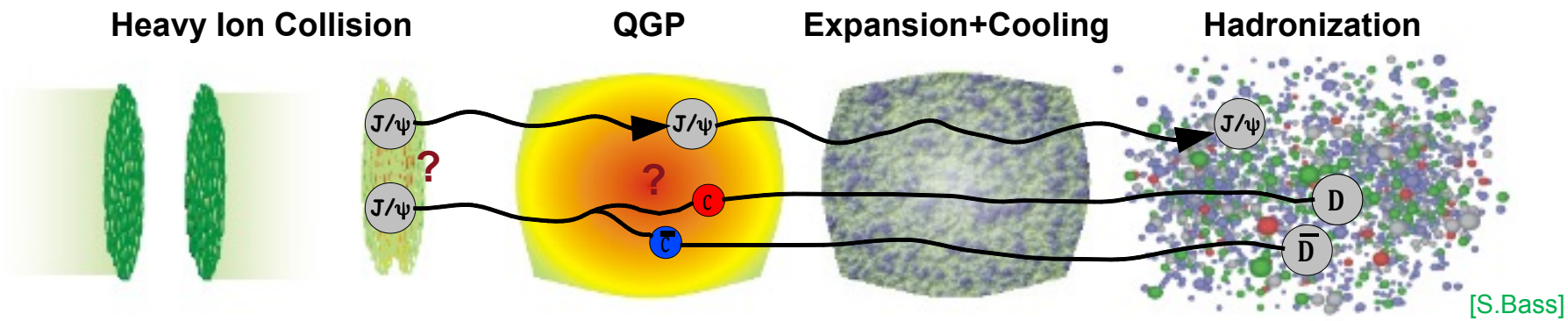
(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}$, $\eta = \frac{T}{MD}$

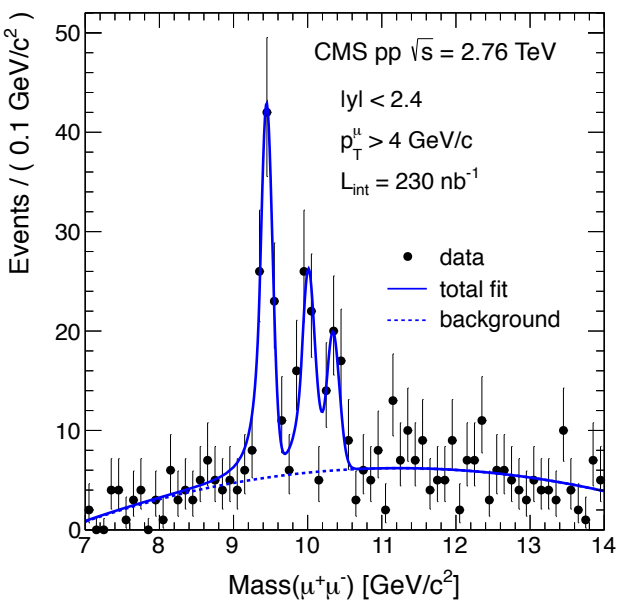
Motivation - Quarkonium in Heavy Ion Collisions



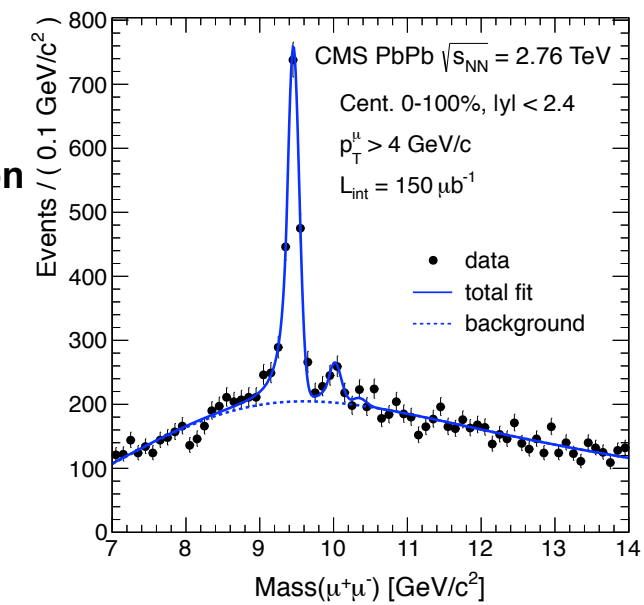
Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

Depending on the Dissociation Temperature

- **remain as bound states in the whole evolution**
- **release their constituents in the plasma**



Sequential suppression for bottomonium observed at CMS



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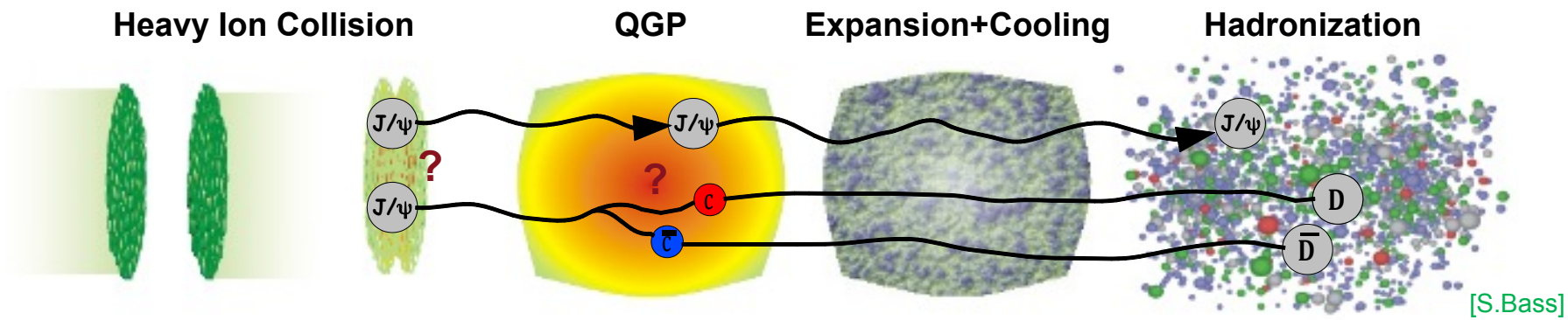

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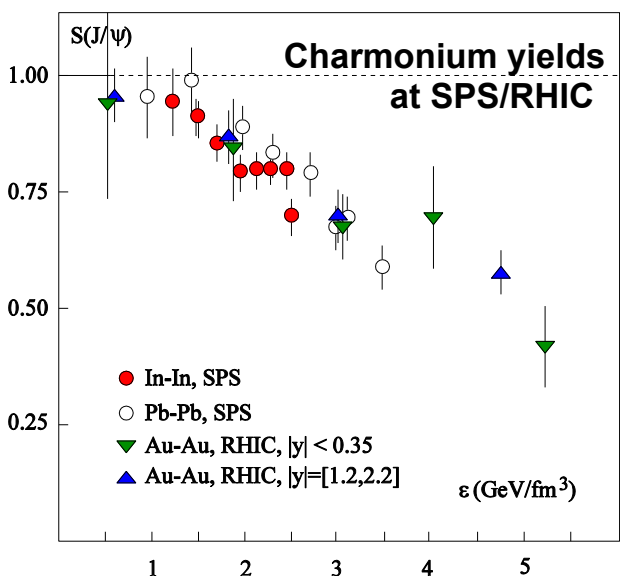
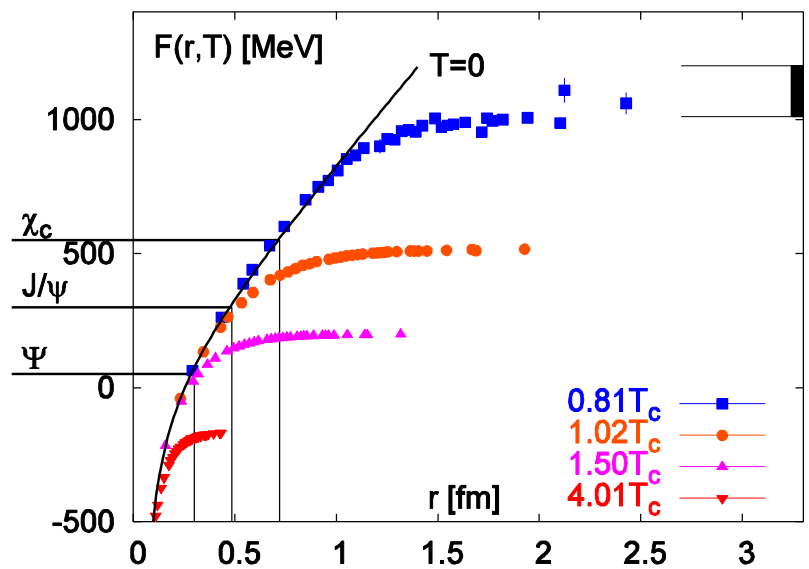


First estimates on Dissociation Temperatures

from detailed knowledge of Heavy Quark Free Energies and Potential Models

[Kaczmarek, Zantow, 2005]



Complex valued heavy quark potential

Large distance behavior of the real-time Wilson loop

$$W(r,t) = \left\langle \mathrm{Tr}\left(\exp\left[-ig\int dx^\mu A^a_\mu T^a\right]\right)\right\rangle$$

can be used to determine the heavy quark potential

$$V(r) = \lim_{t\to\infty} \frac{i\partial_t W(t,r)}{W(t,r)}$$

which can be evaluated in perturbation theory: [Laine et al., JHEP 0703 (2007) 054]

$$V_{\mathrm{HTL}}(r) = -\tilde{\alpha}_s\left[m_D + \frac{e^{-m_D r}}{r} + iT\phi(m_D r)\right] + \mathcal{O}(g^4)$$

$$\phi(x) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left(1 - \frac{\sin(xz)}{xz}\right)$$

and contains a real and an imaginary contribution.

Heavy quark potential from lattice QCD – real and imaginary part

Both, the real and imaginary time Wilson loop are related to the same spectral function

$$W(\tau, r) = \int d\omega e^{-\omega\tau} \rho(\omega, r) \leftrightarrow \int d\omega e^{-i\omega t} \rho(\omega, r) = W(t, r)$$

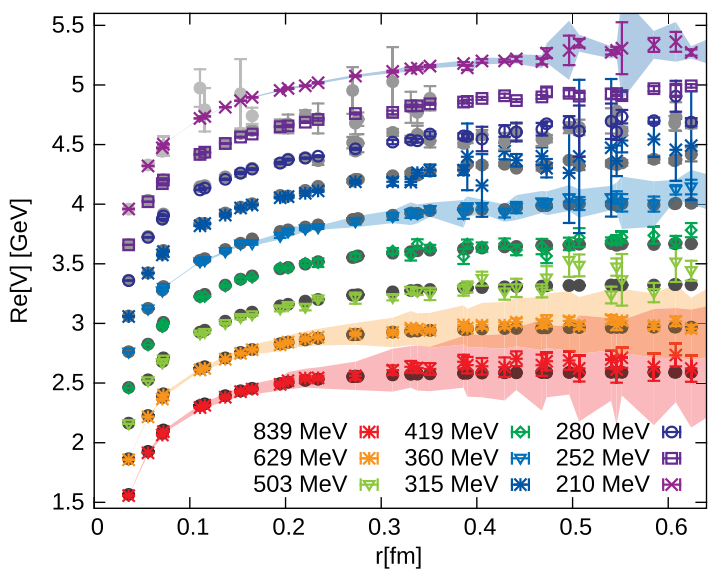
which allows to calculate the heavy quark potential using Euclidean lattice QCD

[Rothkopf,Hatsuda,Sasaki, PRL108(2012)162001]

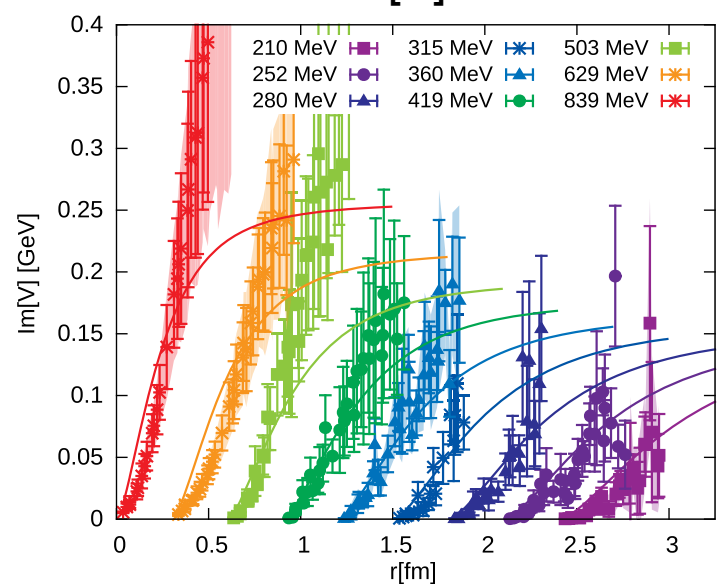
$$V(r) = \lim_{t\to\infty} \int d\omega\, \omega e^{-i\omega t} \rho(\omega, r) / \int d\omega\, e^{-i\omega t} \rho(\omega, r)$$

Quenched results: [Y.Burnier, OK, A.Rothkopf, PRL114(2015)082001]

Re[V]



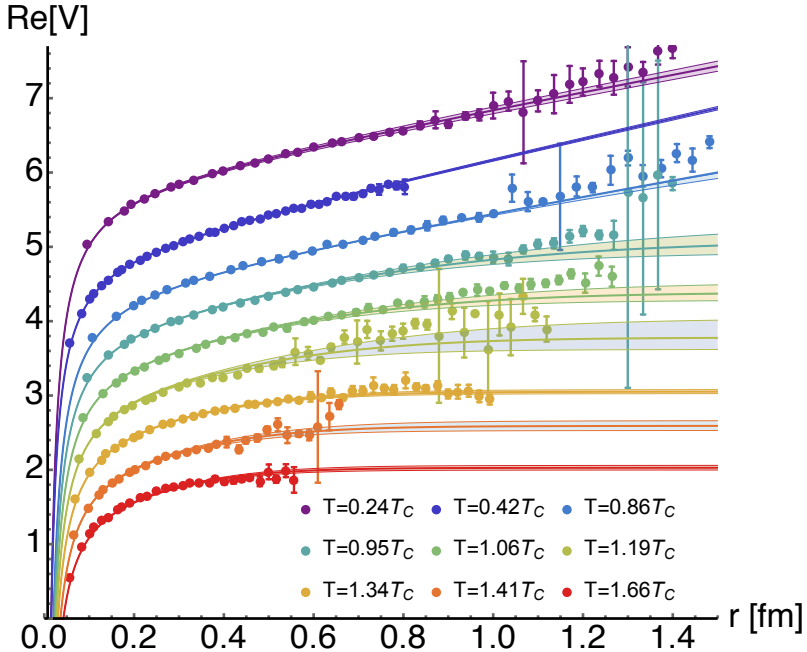
Im[V]

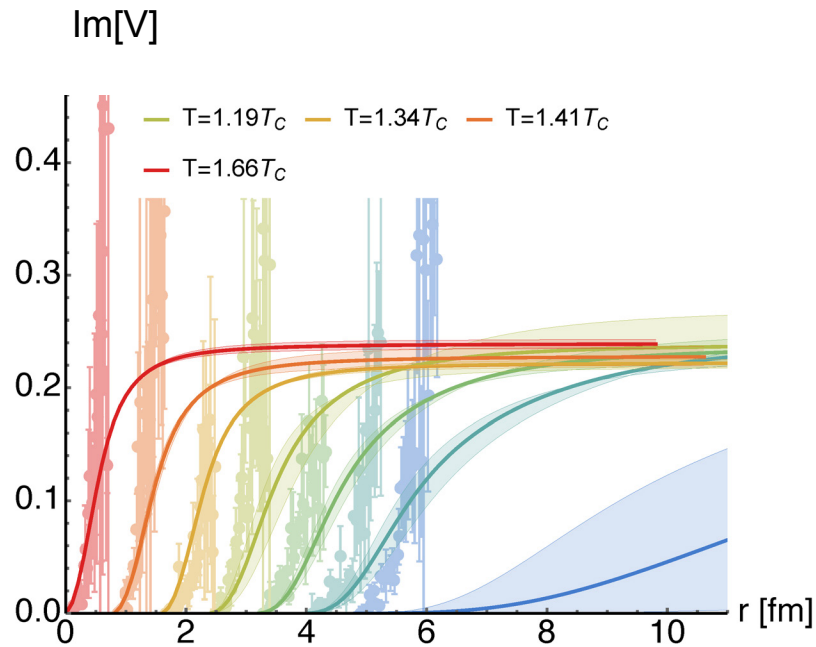


Heavy quark potential from lattice QCD – real and imaginary part

Using asqtad improved staggered quarks with physical strange and $m_l=m_s/20$ on $48^3\times48$ (zero temperature) and $48^3\times12$ (finite temperature) lattices (generated by Bielefeld-Brookhaven-Wuhan and HotQCD PRD85(2012)054503)

[Burnier ,OK,Rothkopf,JHEP12(2015)101]





We use an Ansatz to fit Re[V] and Im[V] [Burnier, Rothkopf, arXiv:1506.08684, Dixit, MPLA5(1990)227] with two T-independent parameters α and σ and one T-dependent parameter $m_D(T)$

Schroedinger equation → spectral function

Schroedinger fct for correlator using complexed valued potential can be written as

[Burnier, Laine, Vepsalainen JHEP0801 (2008) 043]

$$\left[\tilde{H} - i|ImV(r)|\right] D^{>}(t, \mathbf{r}, \mathbf{r}') = i\partial_t D^{>}(t, \mathbf{r}, \mathbf{r}'), \quad t > 0$$

$$\left[\tilde{H} + i|ImV(r)|\right] D^{>}(t, \mathbf{r}, \mathbf{r}') = i\partial_t D^{>}(t, \mathbf{r}, \mathbf{r}'), \quad t < 0$$

using the Hamiltonian

$$\tilde{H} = 2m_q - \frac{\nabla^2}{2m_q} + \mathrm{Re}[V](r) + \frac{l(l+1)}{m_q r^2}$$

and as an initial condition

$$D^{>}(0, \mathbf{r}, \mathbf{r}') = -6N_c \delta^{(3)}(r - r')$$

taking the Fourier transform

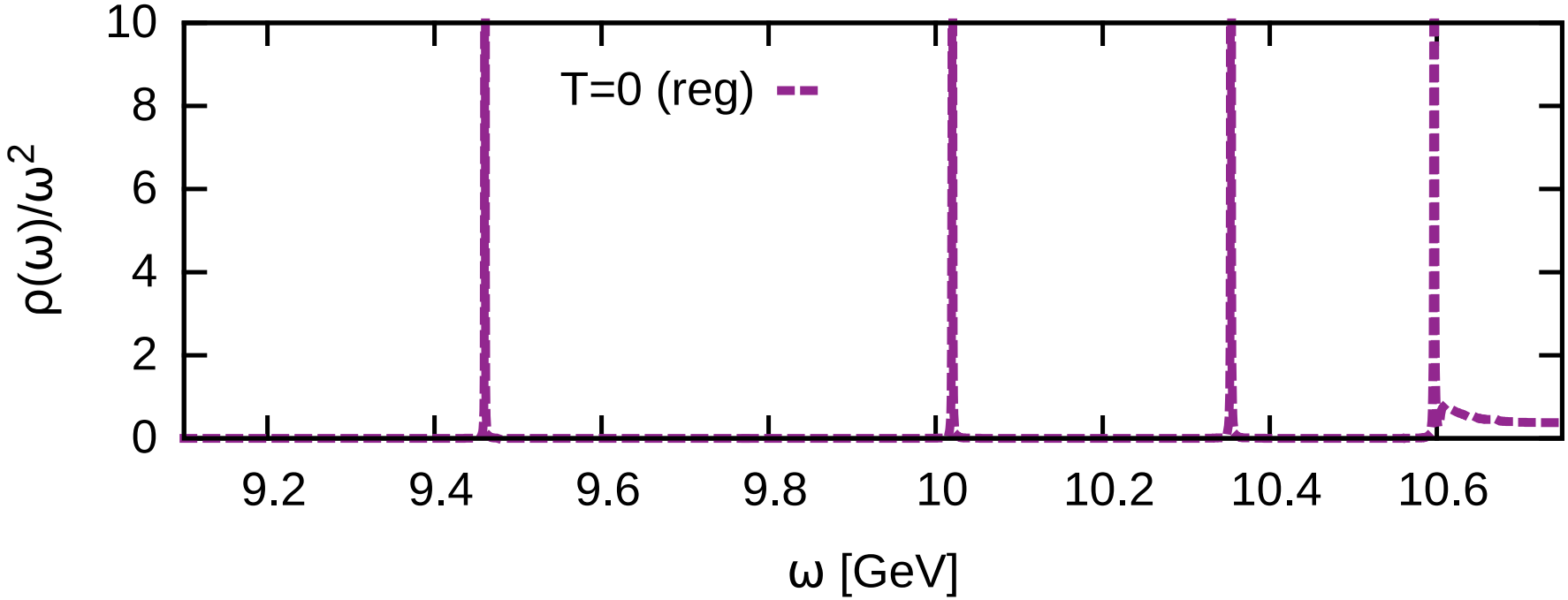
$$\tilde{D}(\omega, \mathbf{r}, \mathbf{r}') \equiv \int_{-\infty}^{\infty} dt s^{i\omega t} D^{>}(t, \mathbf{r}, \mathbf{r}')$$

one can obtain the spectral function

$$\rho^{\mathrm{V}}(\omega) = \lim_{\mathbf{r}, \mathbf{r}' \to \mathbf{0}} \frac{1}{2} \tilde{D}(\omega, \mathbf{r}, \mathbf{r}')$$

Bottomonium spectral function – zero temperature

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



we use a renormalon subtracted bottom mass m_b^{RS} = 4.882 GeV motivated by pNRQCD

[Pineda, JHEP106(2001)022, Brambilla et al., Rev.Mod.Phys.77(2005)1423]

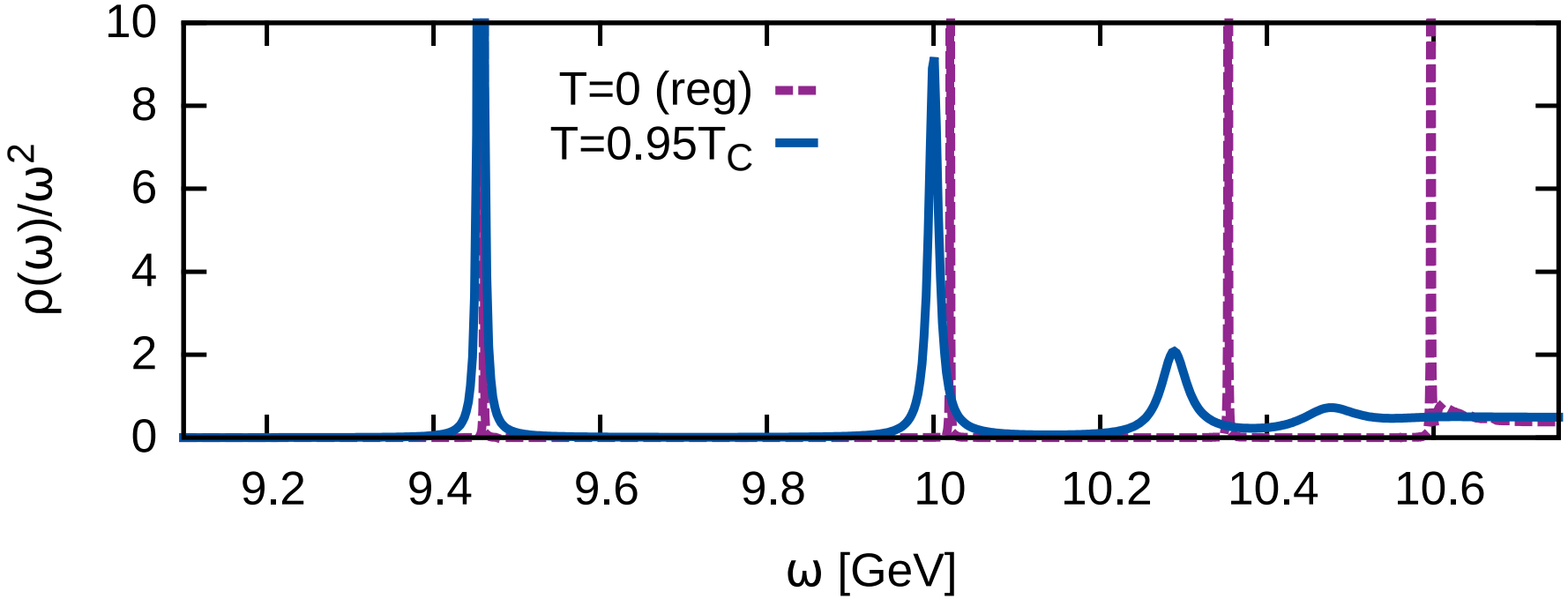
and use a Cornell potential with string breaking at r_{sb}=1.25fm

fitted to the experimental bottomonium states

states	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$	$\chi_b(1P)$	$\chi_b(2P)$	$\chi_b(3P)$
m [GeV]	9.4603	10.020	10.353	10.597	9.92597	10.269	10.538
$m^{\rm PDG}$ [GeV]	9.4603	10.023	10.355	10.579	9.88814	10.252	10.534
r [$\rm GeV^{-1}$]	1.489	2.985	4.385	10.17	2.435	3.898	5.586
r [fm]	0.2934	0.5881	0.8639	2.004	0.4797	0.7679	1.100

Bottomonium spectral function

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



We use the functional form for Re[V] and Im[V] at finite temperature derived in

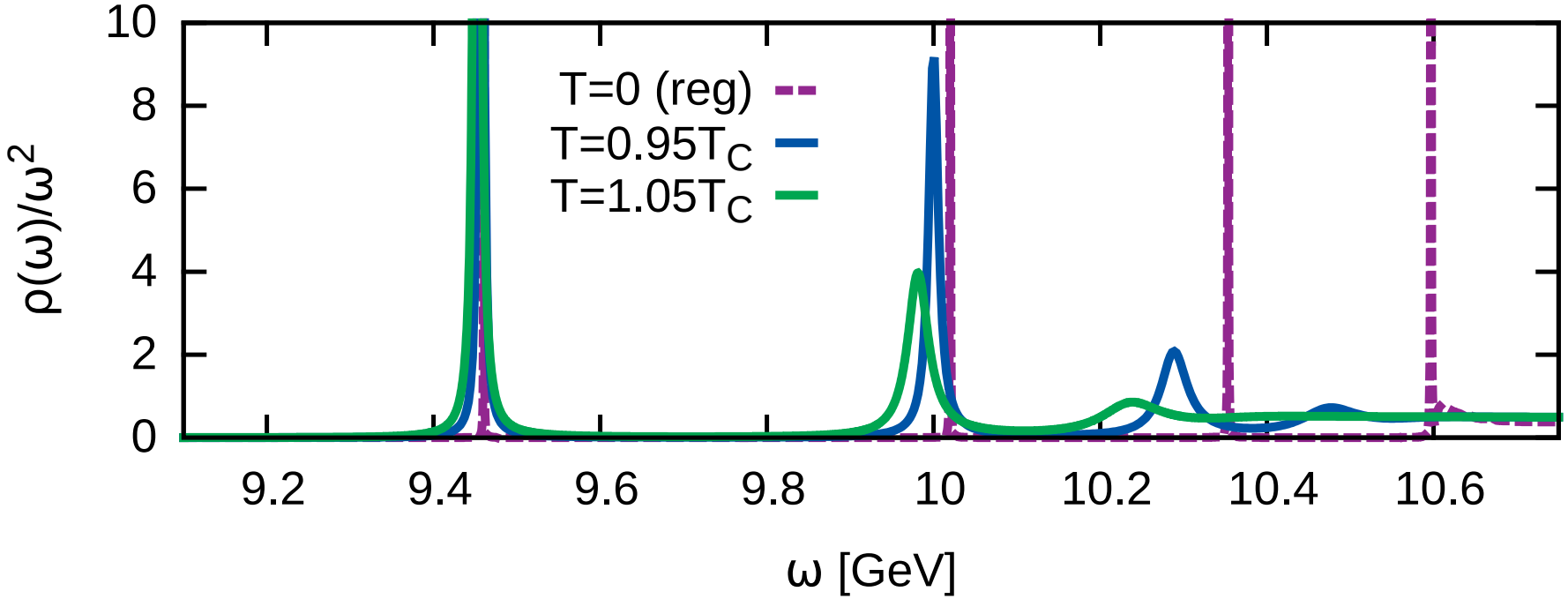
[Burnier, Rothkopf, PLB753(2016)232, Dixit, MPLA5(1990)227]

with only one T-dependent parameter $m_D(T)$

- decrease of bound state masses (larger than possible thermal mass corrections)
- thermal broadening of the states due to Im[V]
- decrease of the continuum towards smaller energies
- sequential dissociation of excited states

Bottomonium spectral function

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



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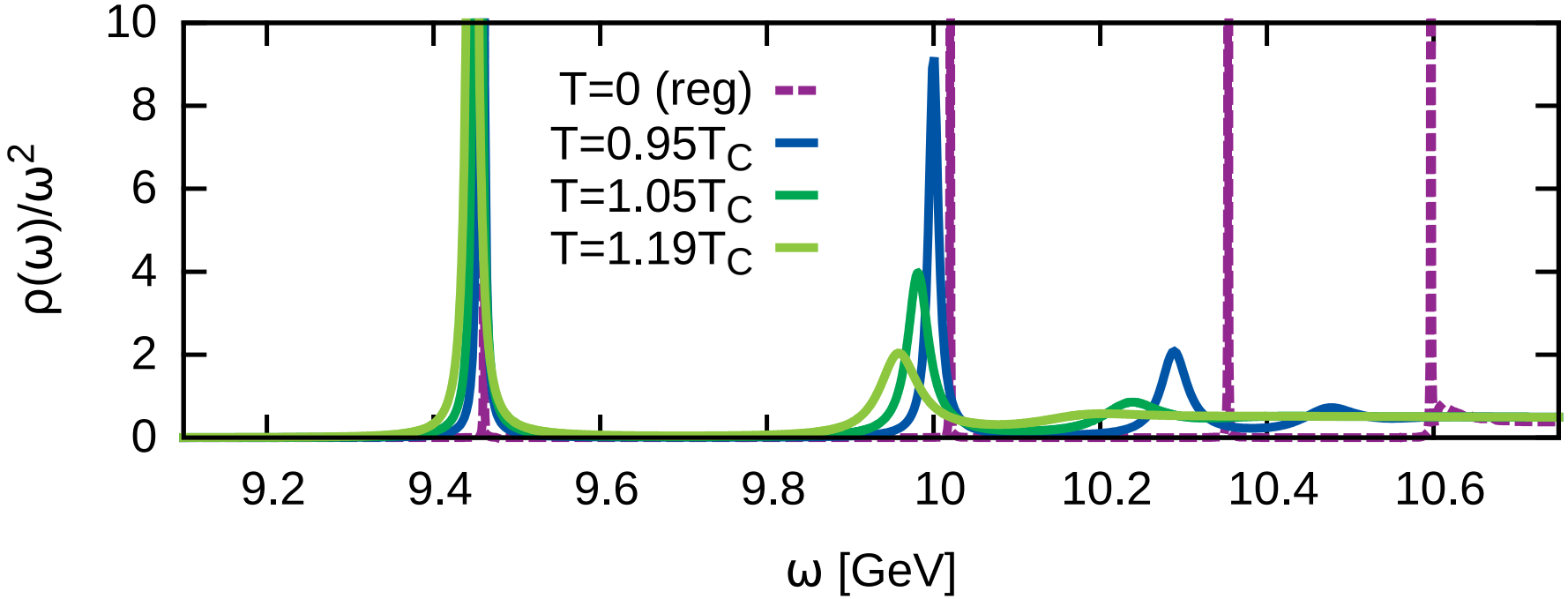

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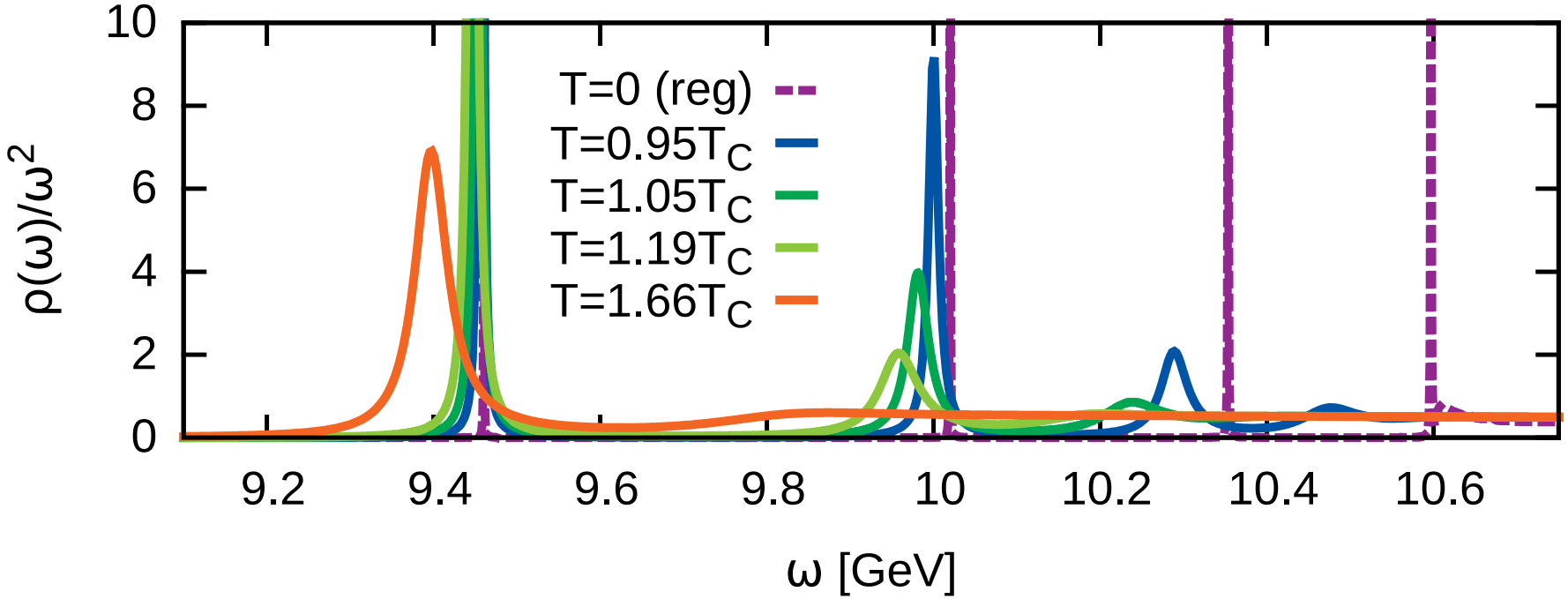

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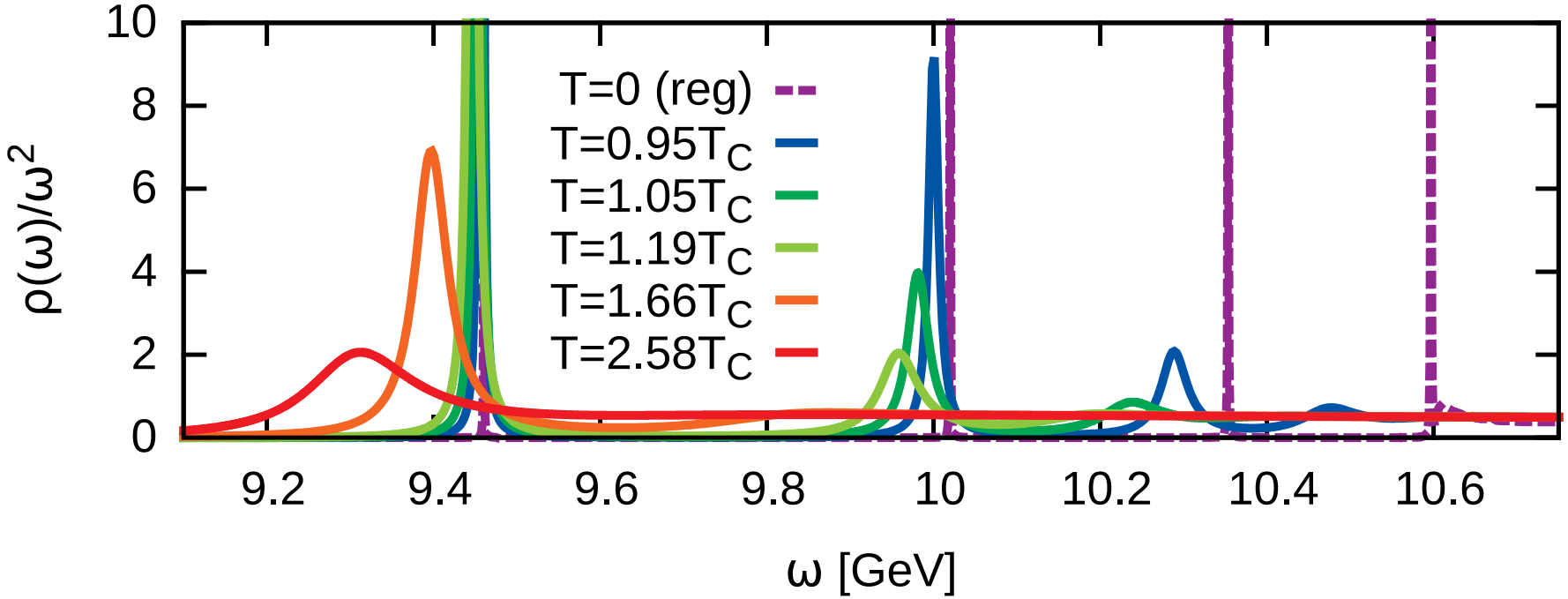

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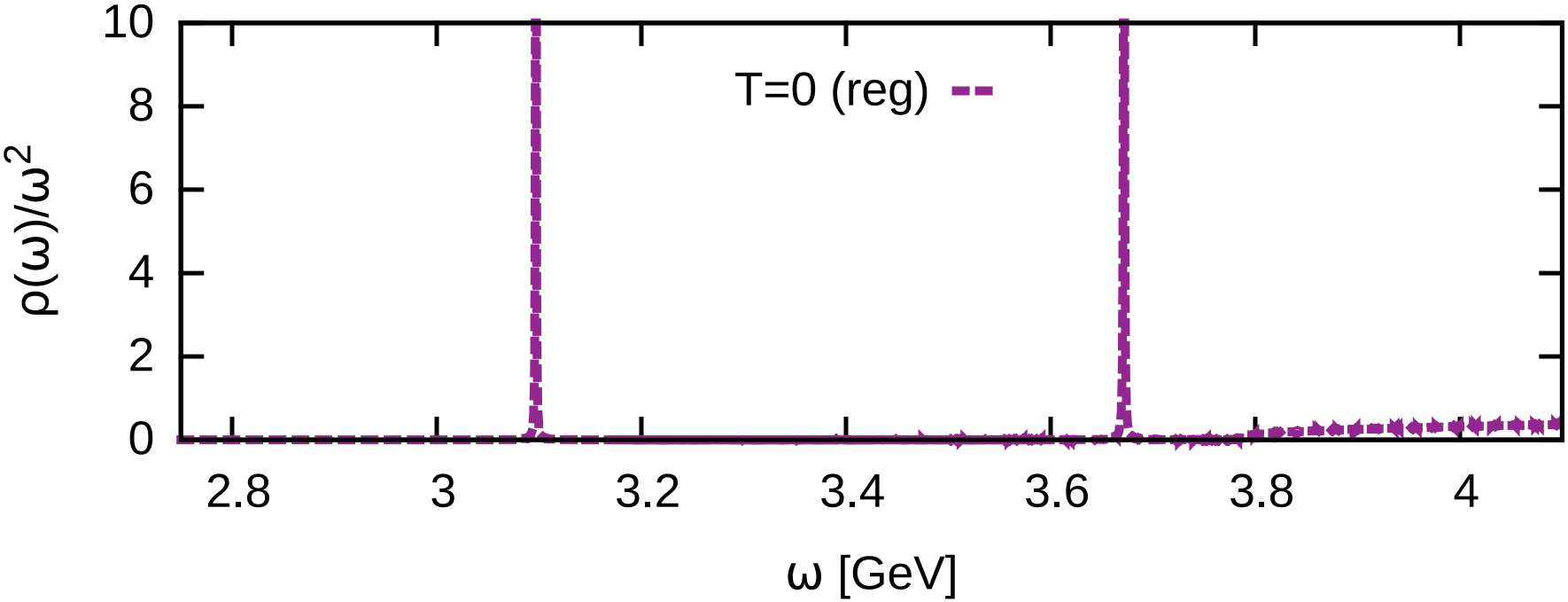
Charmonium spectral function – zero temperature

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



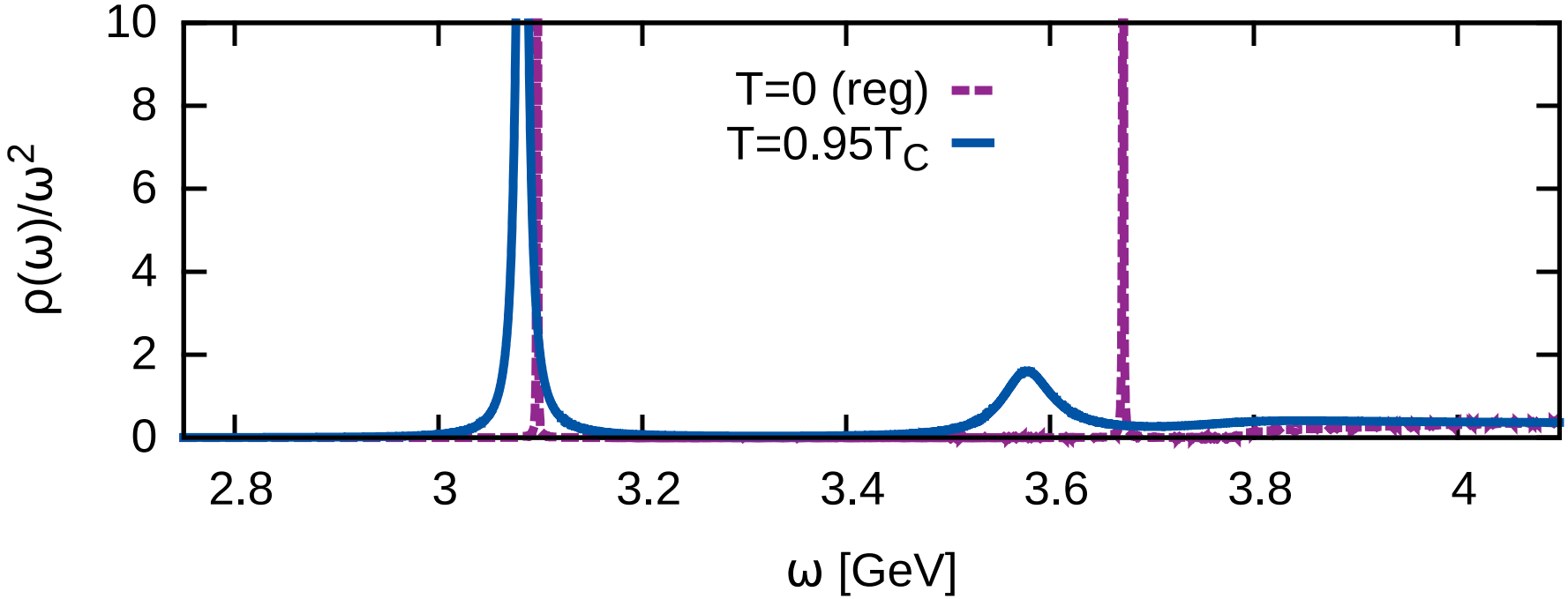
Cornell potential determined from bottomonium spectrum

charm quark mass fitted to lowest charmonium states, m_c=1.472GeV

states	$J/\Psi(1S)$	$\Psi'(2S)$	$\chi_c(1P)$	$\chi_c(2P)$
m [GeV]	3.0969	3.6717	3.5089	3.7918
m^{PDG} [GeV]	3.0969	3.6861	3.4939	3.9228
r [GeV^{-1}]	2.861	5.839	4.136	25.42
r [fm]	0.5635	1.150	0.814824	5.00813

Charmonium spectral function

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



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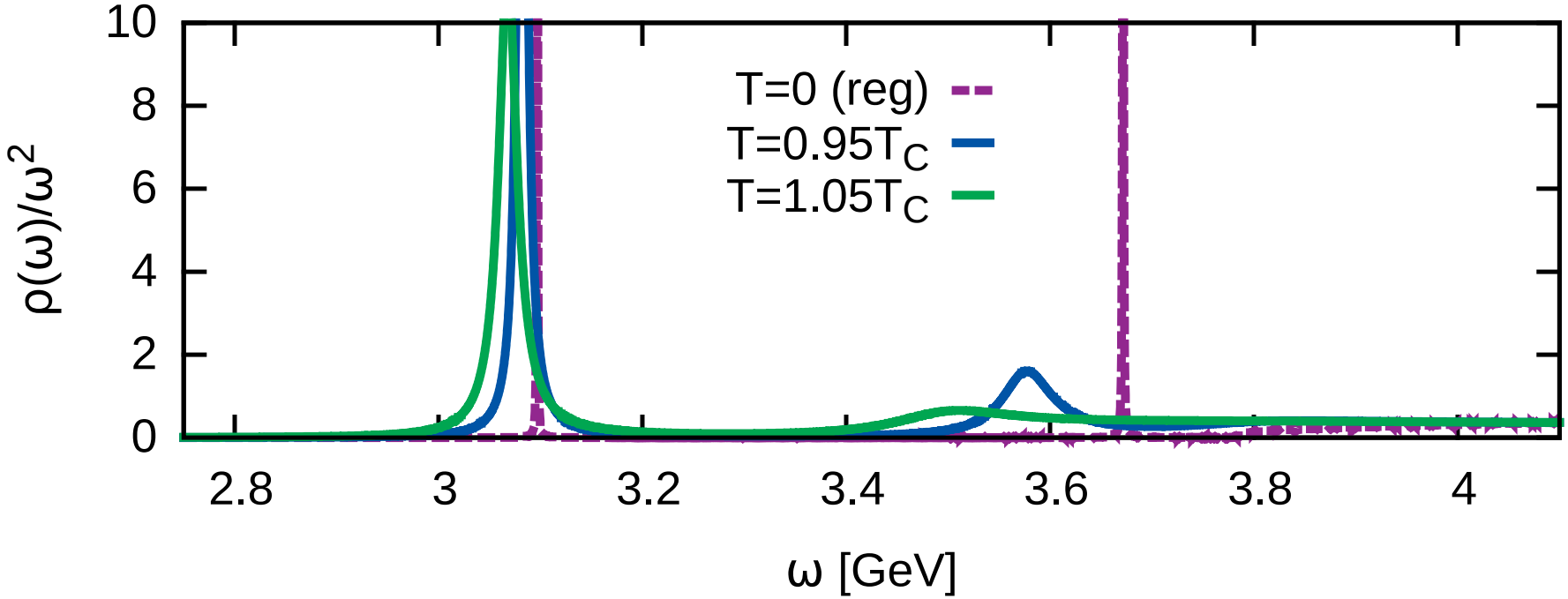

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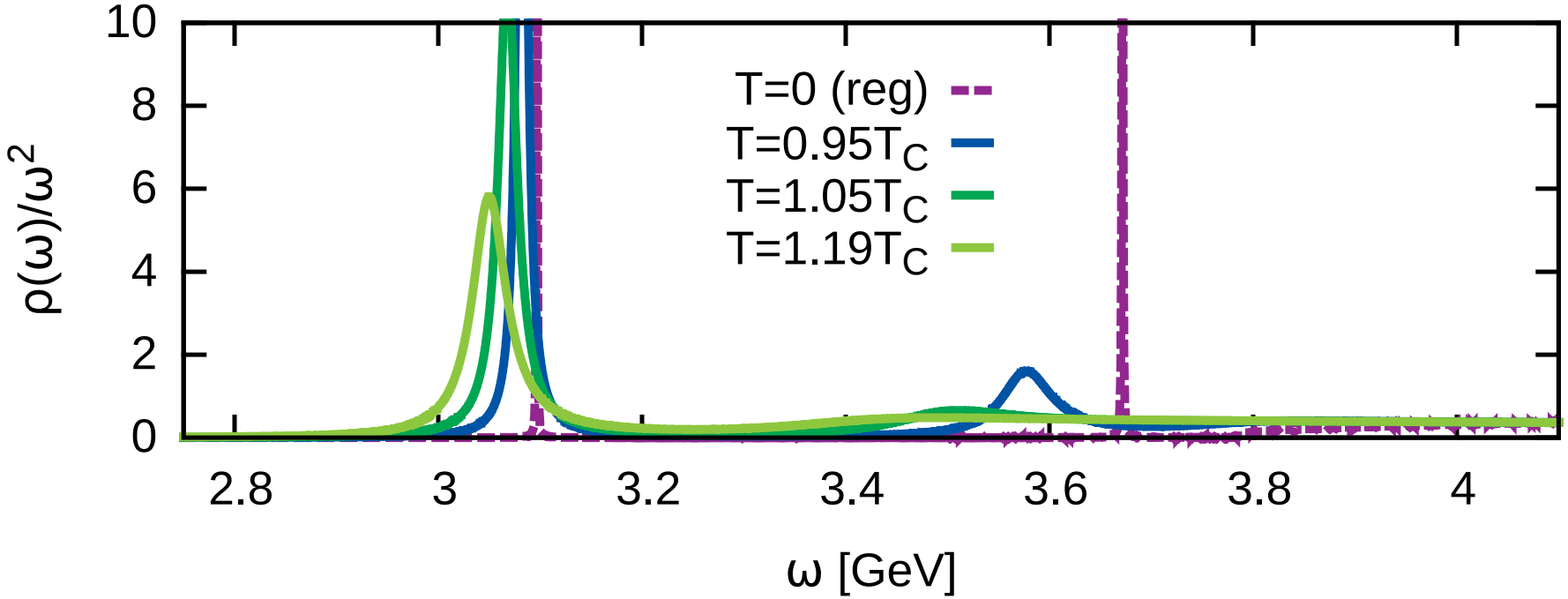

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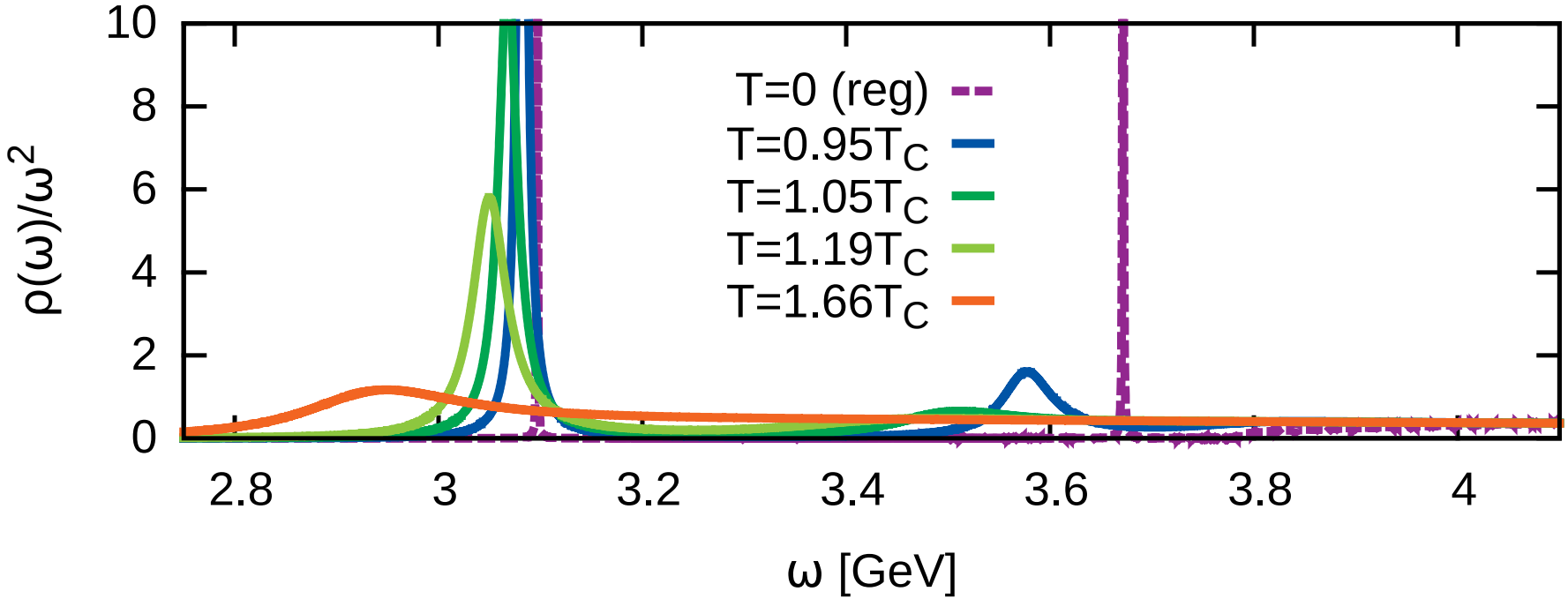

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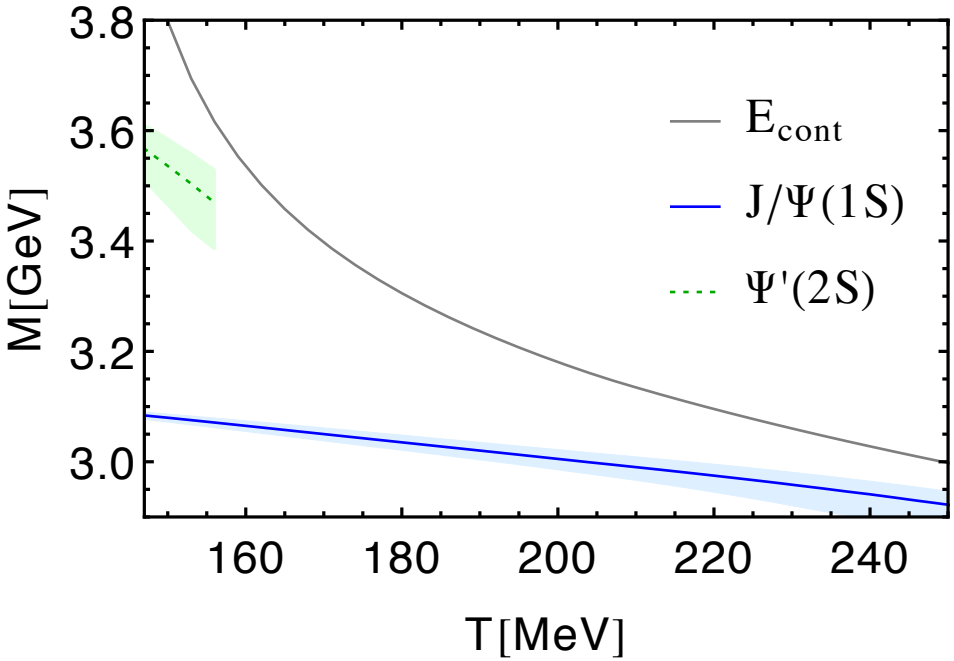
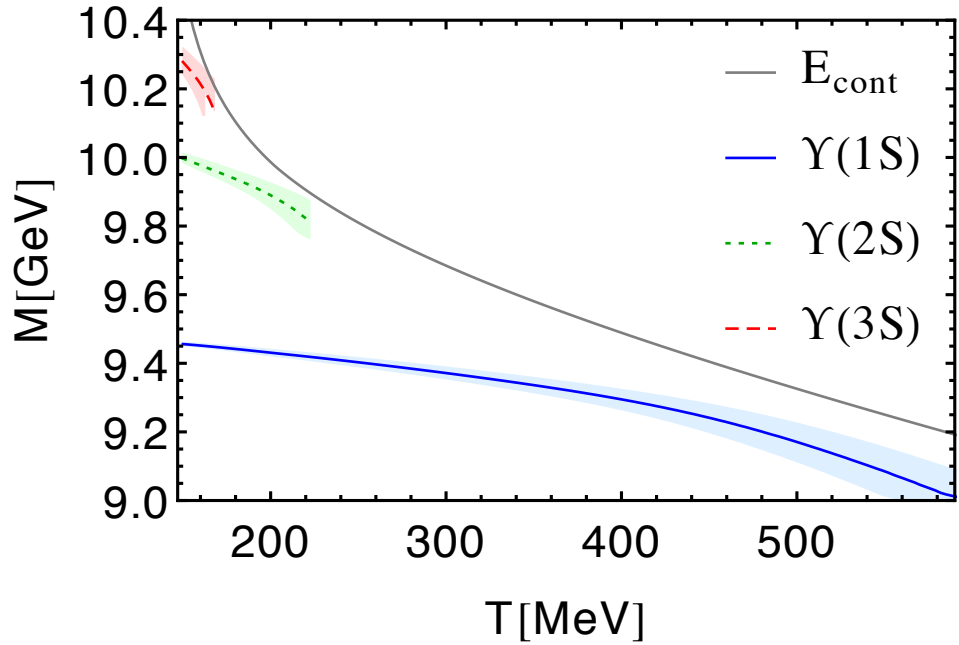
Shift of bound state masses

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]





thermal mass shift up to 100-300 MeV before the states are melted

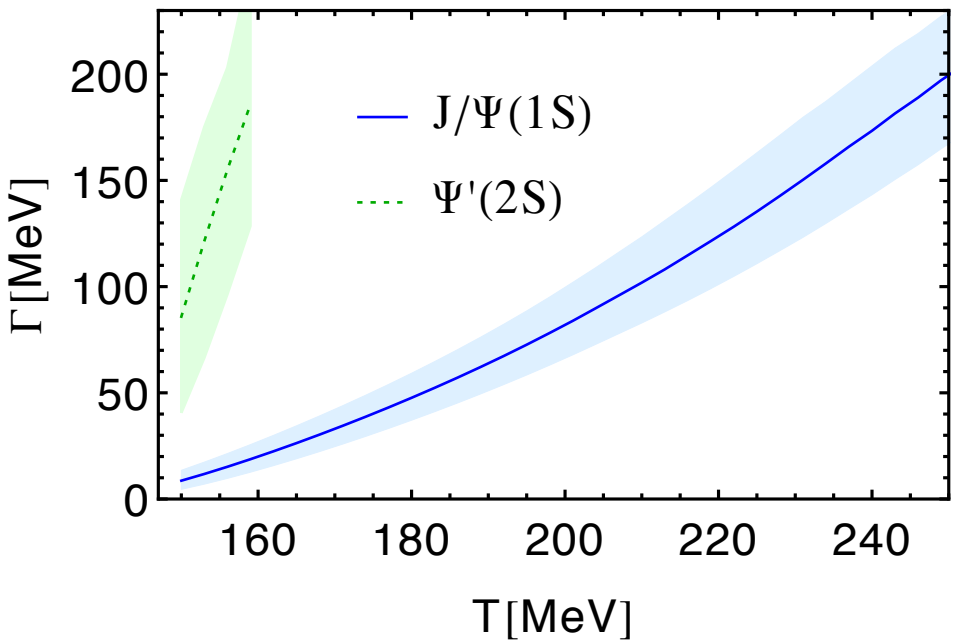
note that thermal quark mass corrections due to the medium

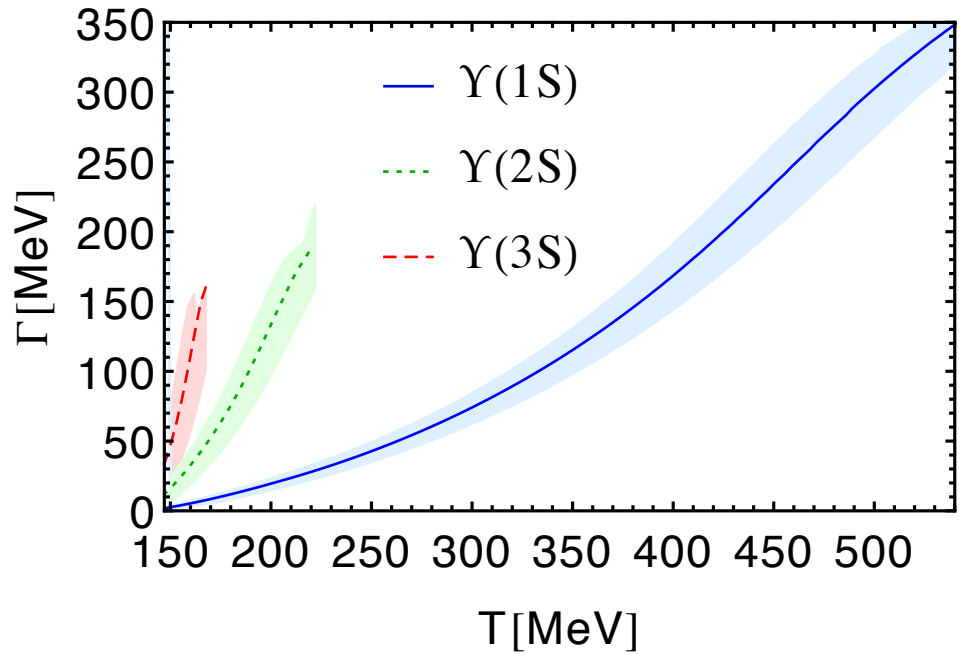
$$m_Q(T) = m_Q + \frac{g^2 T^2 C_F}{12 m_Q}$$

would only contribute a few MeV and are of higher order in the potential

Melting temperatures

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]





estimates of the melting temperatures defined by the point where

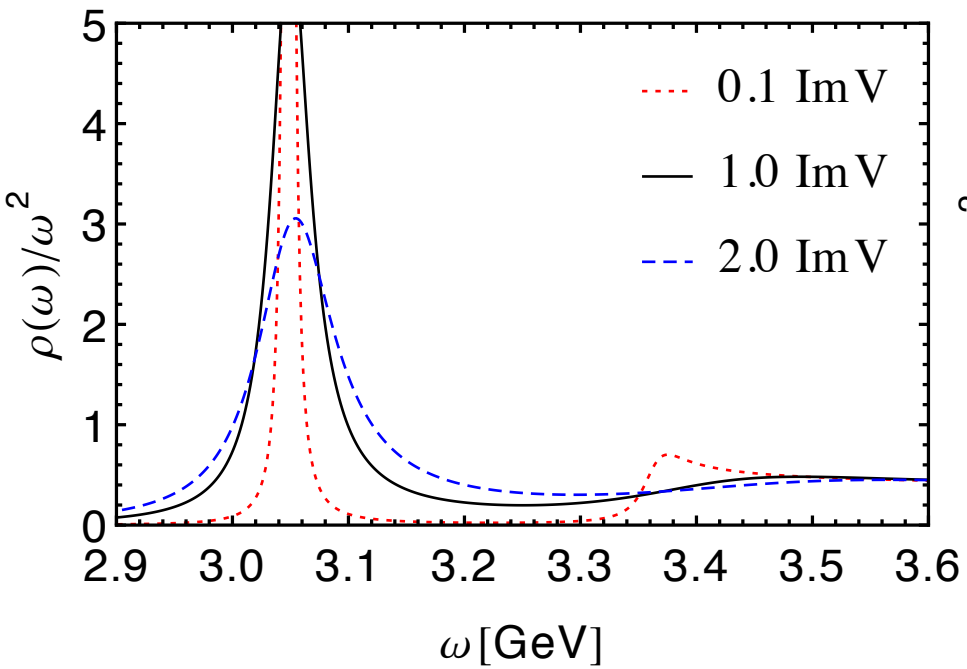
the width of the state becomes as large as the binding energy :

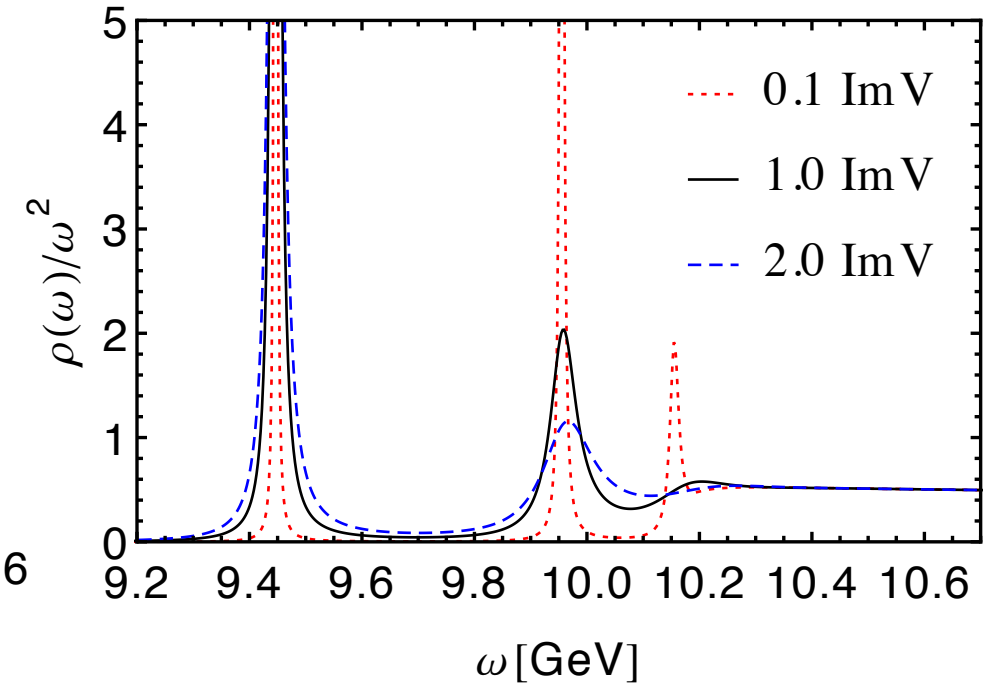
states	$J/\Psi(1S)$	$\Psi'(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$
$T_{\rm melt}^{\Gamma=E_{\rm bind}}/T_C$	$1.37^{+0.08}_{-0.07}$	< 0.95	$2.66^{+0.49}_{-0.14}$	$1.25^{+0.17}_{-0.05}$	$1.01^{+0.03}_{-0.03}$	< 0.95

Effect of the imaginary part

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]

to see the effect of the imaginary part of the potential, one can vary the strength of Im V:





mainly results in a broadening of the peaks without changing the position and area

→ larger imaginary part leads to an earlier melting of the bound states

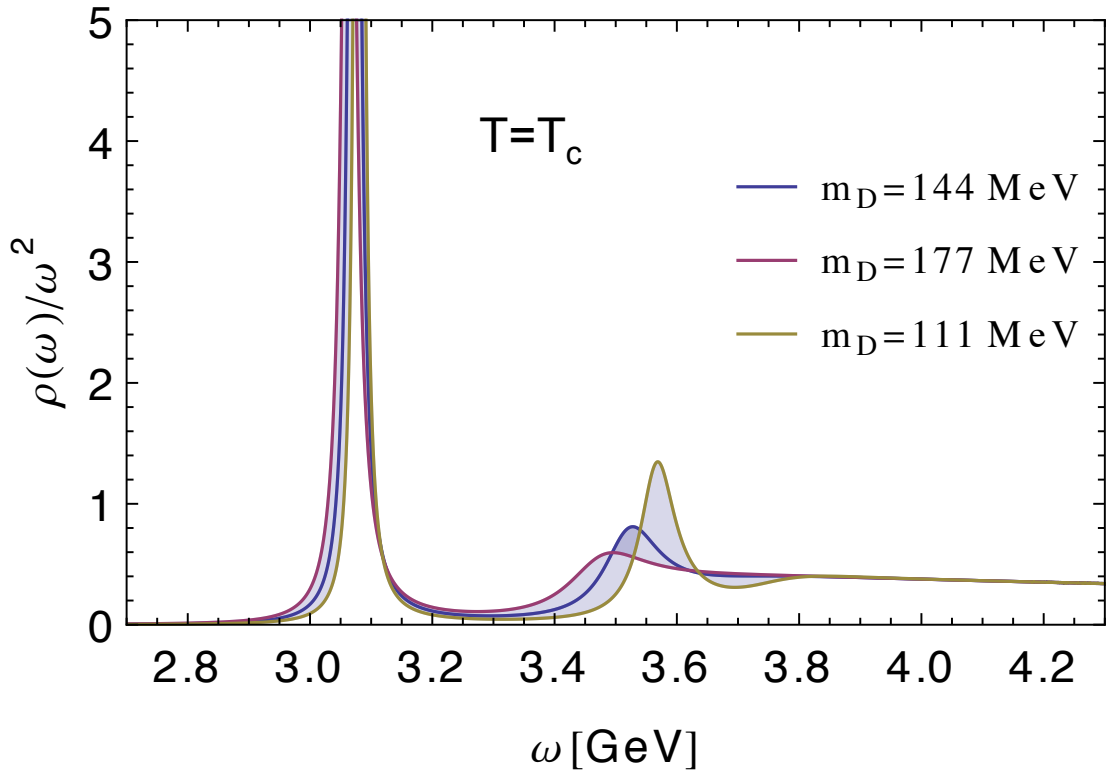
Regeneration of charmonium at freeze-out

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]

At high energies at RHIC and LHC all charmonium states may be melted in the QGP

→ observed dilepton yields mainly due to recombined charmonia at freeze-out

Ratio of Ψ' to J/Ψ yields can be estimated by the spectral weights at T_c



ratio of dilepton yields at T_c:

$$\frac{R^{\Psi'}_{\ell\bar{\ell}}}{R^{J/\Psi}_{\ell\bar{\ell}}} = 0.023 \pm 0.004$$

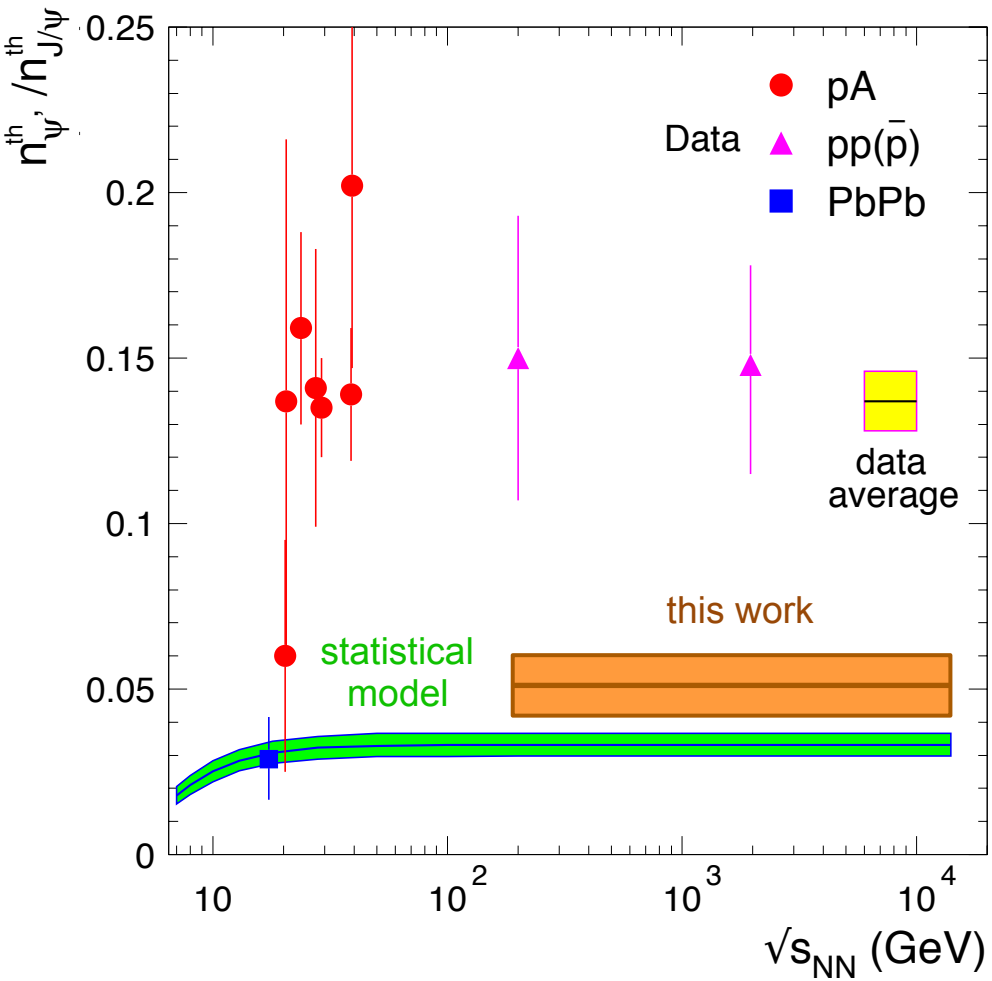
Regeneration of charmonium at freeze-out

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]

Decay to dileptons will happen in the vacuum

→ Need to divide by the electromagnetic decay rate of the vacuum states

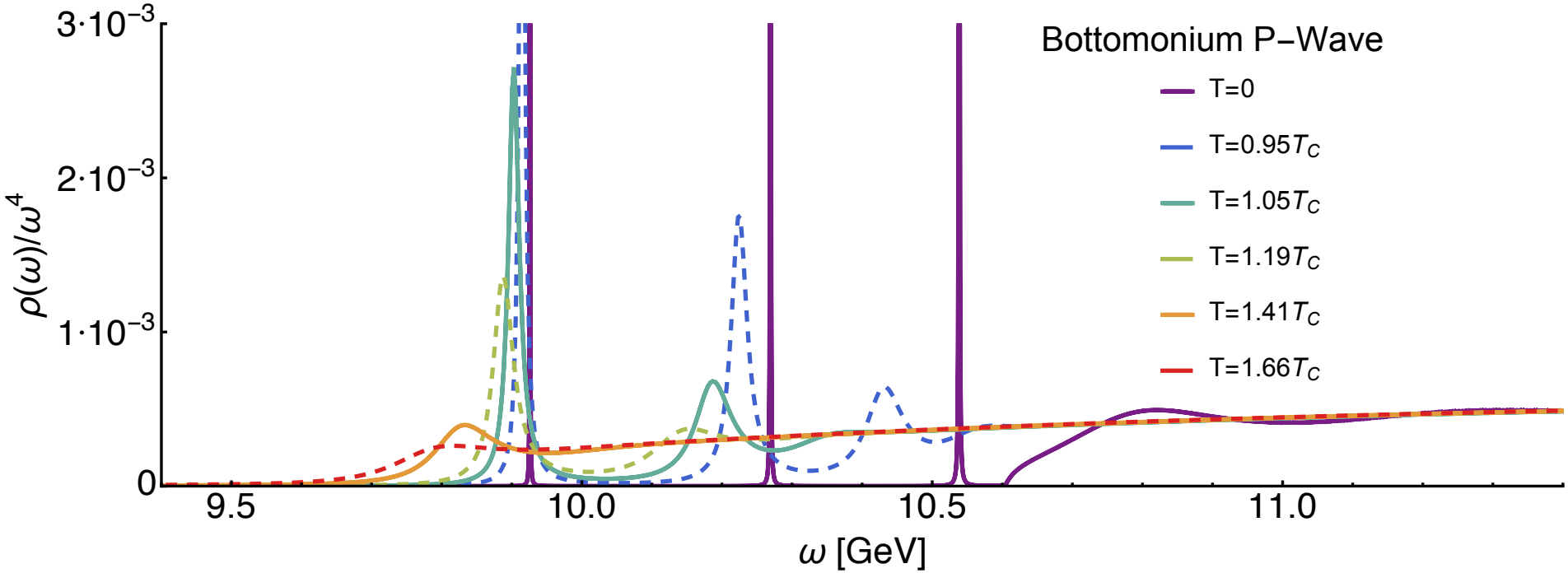
$$\frac{N_{\Psi'}}{N_{J/\Psi}} = \frac{R^{\Psi'}_{\ell\bar{\ell}}}{R^{J/\Psi}_{\ell\bar{\ell}}} \frac{M^2_{\Psi'}|\Phi_{J/\Psi}(0)|^2}{M^2_{J/\Psi}|\Phi_{\Psi'}(0)|^2} = 0.052 \pm 0.009$$



compared to statistical model
[A. Andronic et al.,PLB678(2009) 350]

Preliminary results for P-wave states

[Y. Burnier ,OK, A.Rothkopf, in preparation]



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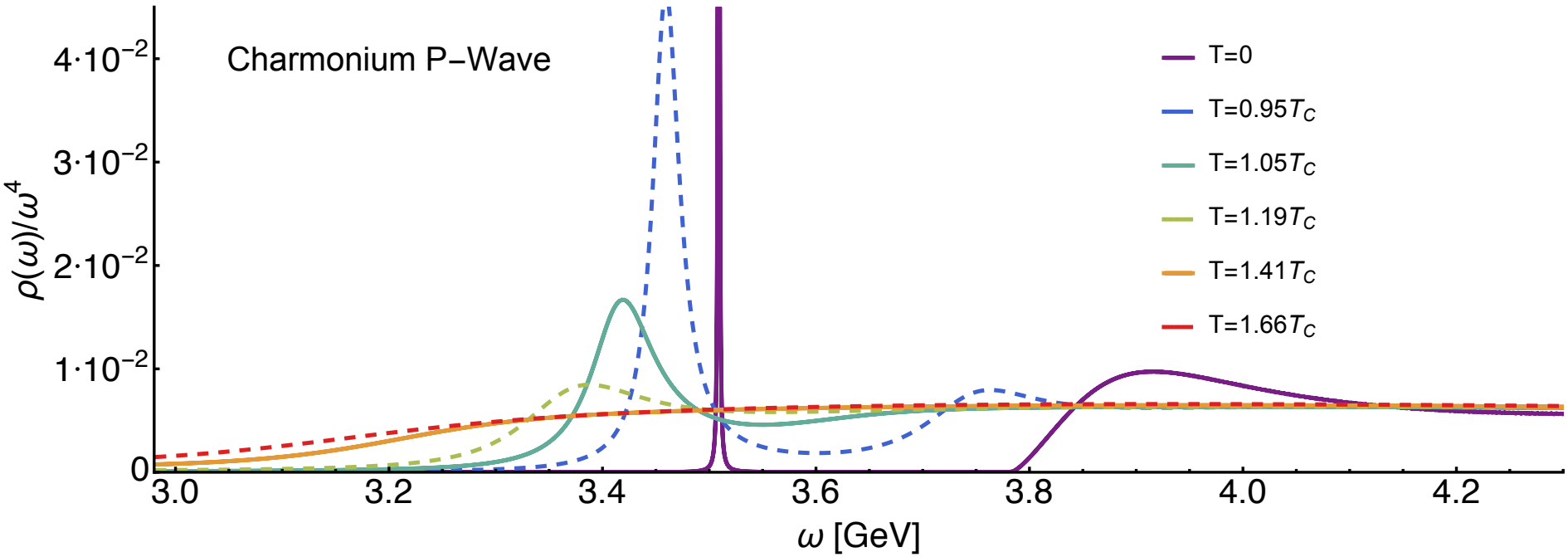

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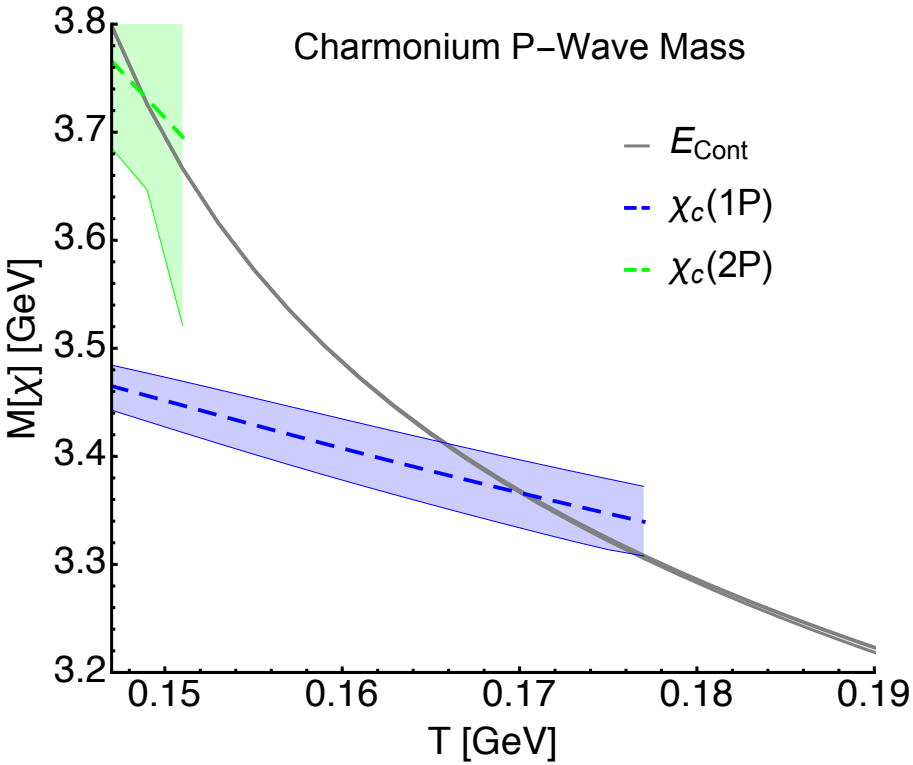

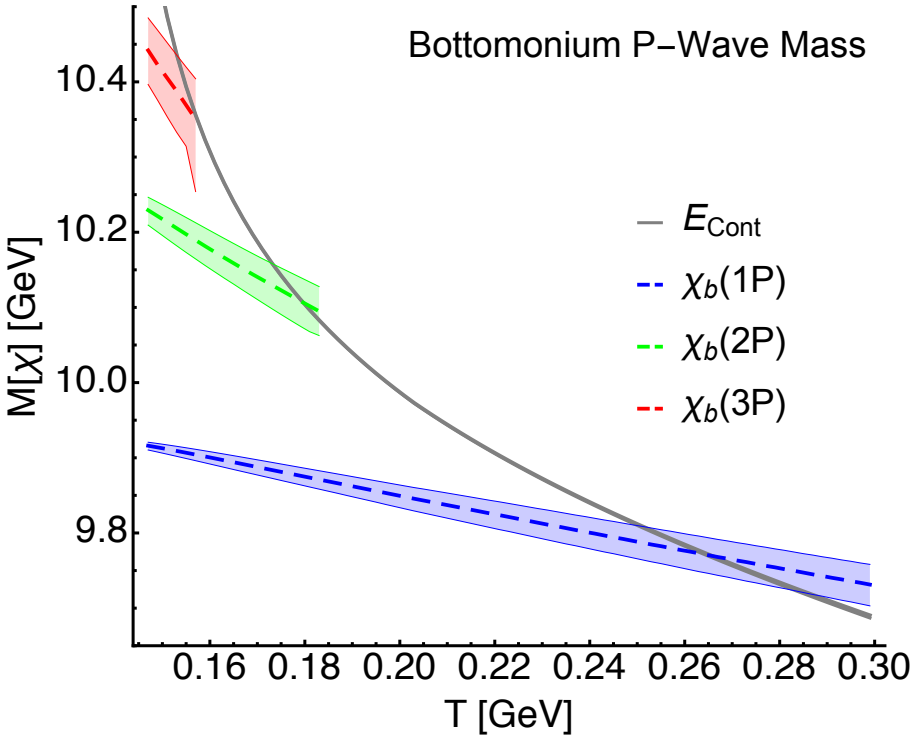

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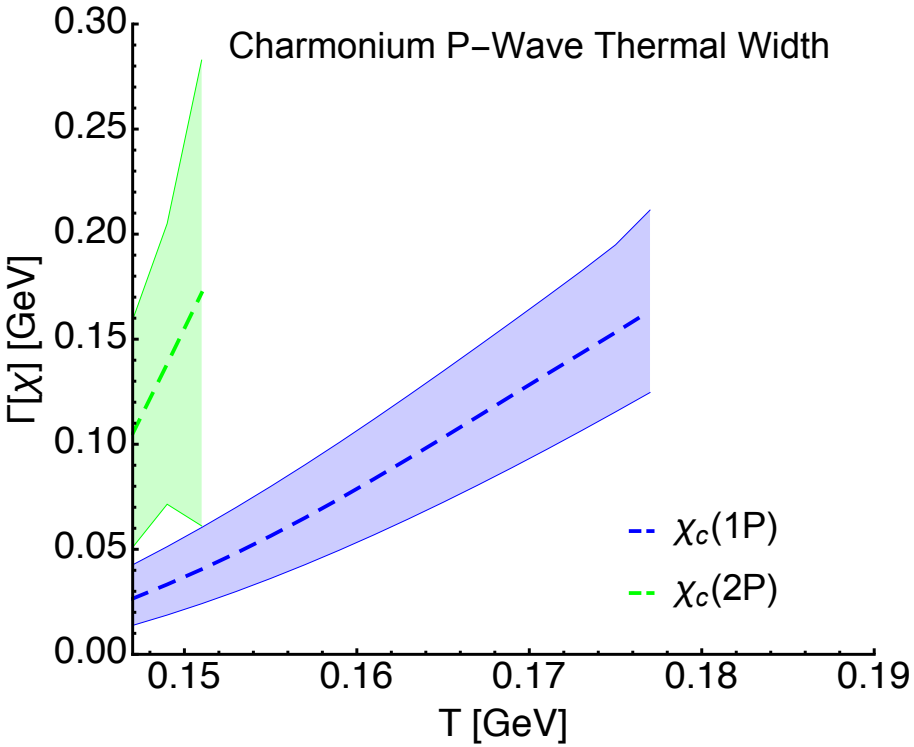
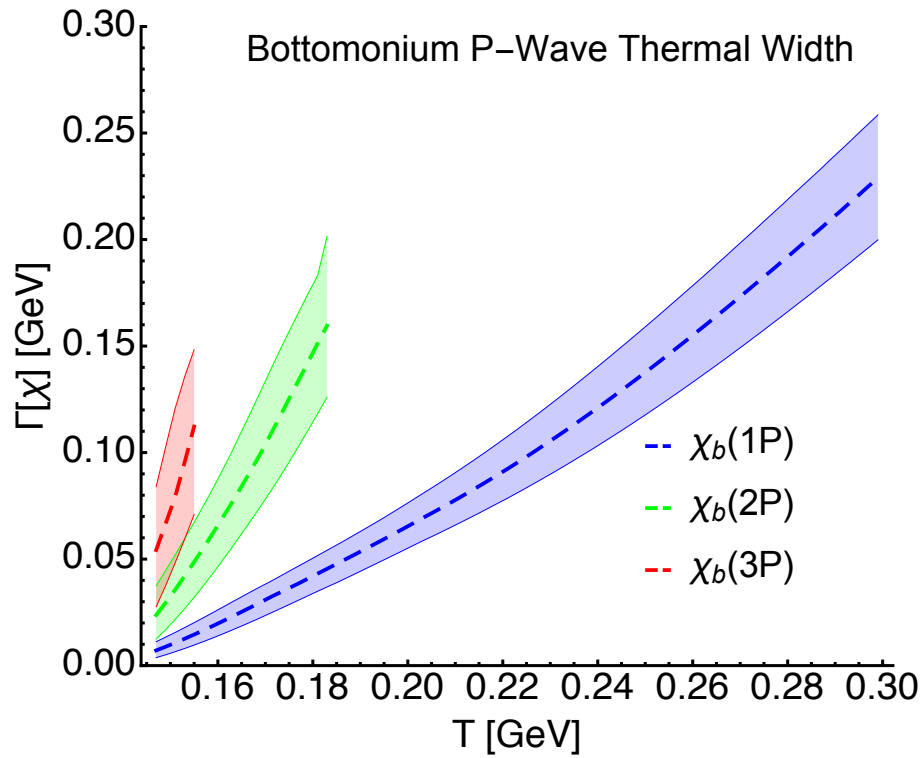
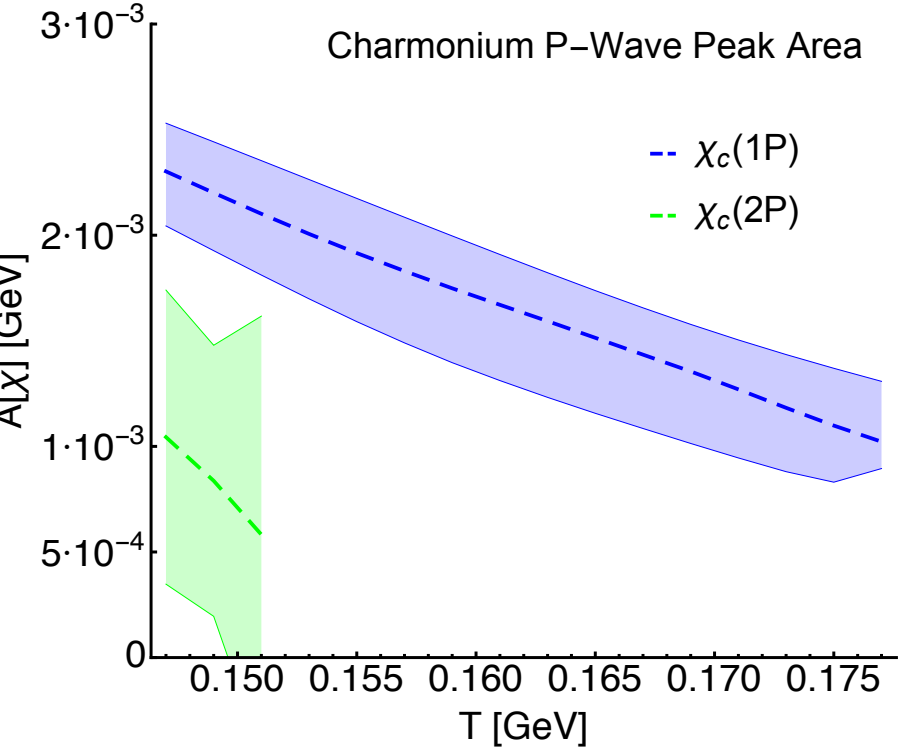


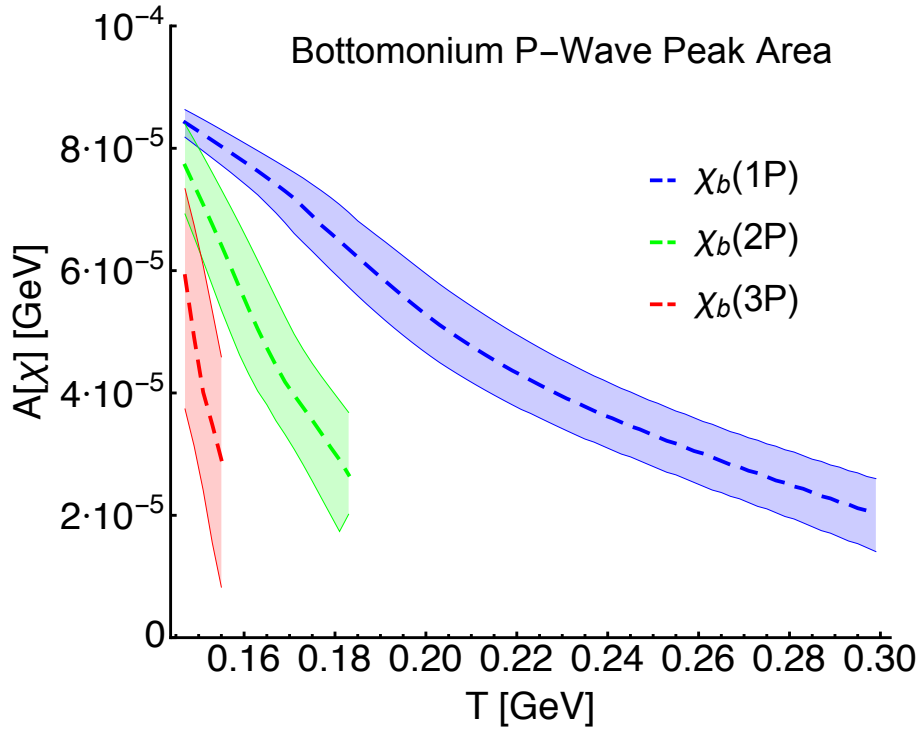


states	$\chi_c(1P)$	$\chi_c(2P)$	$\chi_b(1P)$	$\chi_b(2P)$	$\chi_b(3P)$
$T_{\rm melt}^{\Gamma=E_{\rm bind}}/T_C$	$1.03^{+0.006}_{-0.006}$	< 0.95	$1.41^{+0.006}_{-0.006}$	$1.06^{+0.006}_{-0.006}$	$0.98^{+0.006}_{-0.006}$
$T_{\rm melt}^{A=\frac{1}{2}A_0}/T_C$	$1.2^{+0.04}_{-0.04}$	< 0.95	$1.72^{+0.006}_{-0.006}$	$1.08^{+0.006}_{-0.006}$	$0.97^{+0.012}_{-0.012}$

Preliminary results for P-wave states

[Y. Burnier ,OK, A.Rothkopf, in preparation]





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Quarkonium Spectral Functions – full relativistically on the lattice

In the following: Meson Correlation Functions

$$G(\tau, \mathbf{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \mathbf{p}, T) K(\tau, \omega, T)$$

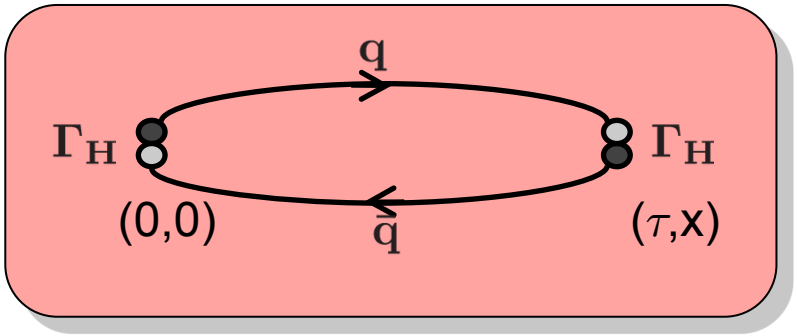
$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J_\nu^\dagger(0, \vec{0}) \rangle$$

$$J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x})$$

$$G_{\mu\nu}(\tau, \vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau, \vec{x}) e^{i\vec{p}\vec{x}}$$



related to a conserved current in the vector channel

Channel	Γ_H	$^{2S+1}L_J$	J^{PC}	Quarkonia
Pseudoscalar (PS)	γ_5	$^1S_0$	0^{-+}	η_c, η_b
Vector (V)	γ_i	$^3S_1$	1^{--}	J/ψ, Υ
Scalar (S)	$\mathbf{1}$	$^1P_0$	0^{++}	χ_{c0}, χ_{b0}
Axialvector (AV)	$\gamma_i\gamma_5$	$^3P_1$	1^{++}	χ_{c1}, χ_{b1}

Quarkonium Spectral Functions – HQ Diffusion and Dissociation

In the following: Meson Correlation Functions

$$G(\tau, \mathbf{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \mathbf{p}, T) K(\tau, \omega, T)$$

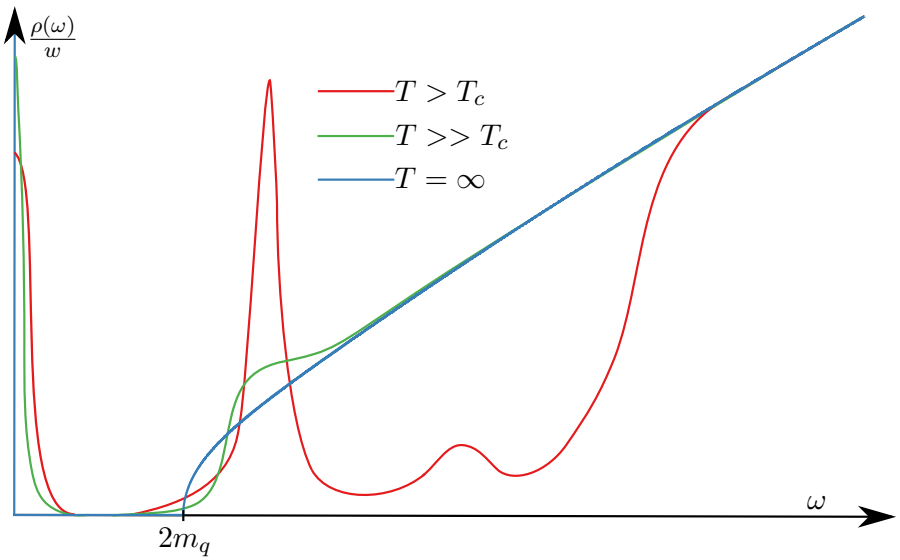
$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J_\nu^\dagger(0, \vec{0}) \rangle$$

$$J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x})$$

$$G_{\mu\nu}(\tau, \vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau, \vec{x}) e^{i\vec{p}\vec{x}}$$



only correlation functions calculable on lattice but

Transport coefficient determined by slope of spectral function at ω=0 (Kubo formula)

$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$

Charmonium and Bottomonium correlators

[H.T.Ding, H.Ohno, OK, M.Laine, work in progress]

- standard plaquette gauge & O(a)-improved Wilson quarks
- quenched gauge field configurations
- on fine and large isotropic lattices
- $T = 0.75 - 2.25 T_c$
- 3 different lattice spacing

  analysis of cut-off effects

  continuum limit (in the future)
- both charm & bottom

  tuned close to their physical masses

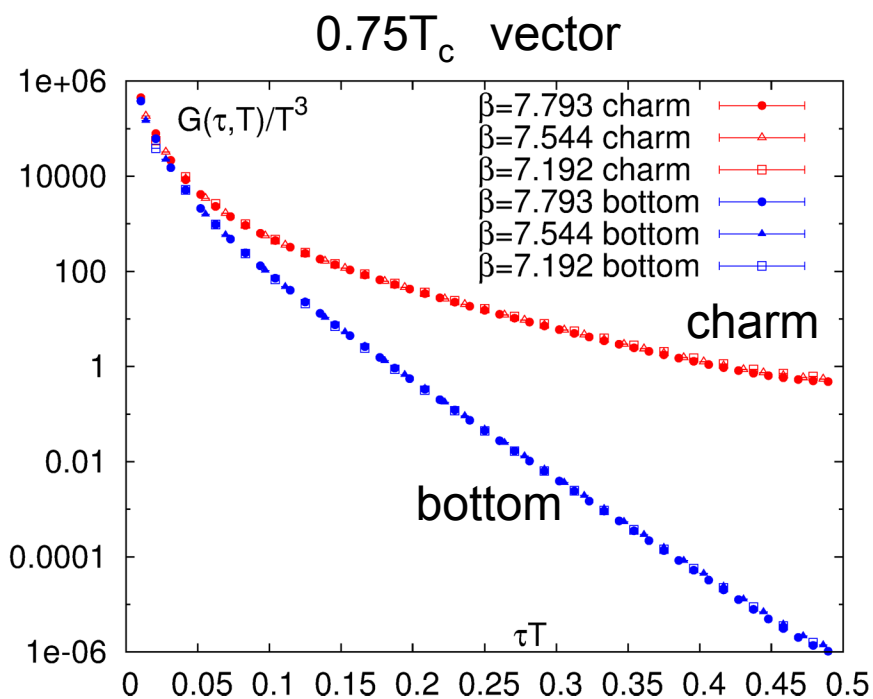
β	a [fm]	a^{-1} [GeV]	$\kappa_{\rm charm}$	$\kappa_{\rm bottom}$	$m_{J/\Psi}$ [GeV]	m_Υ [GeV]
7.192	0.0190	10.4	0.13194	0.12257	3.105(3)	9.468(3)
7.793	0.00968	20.4	0.13221	0.12798	3.092(5)	9.431(5)

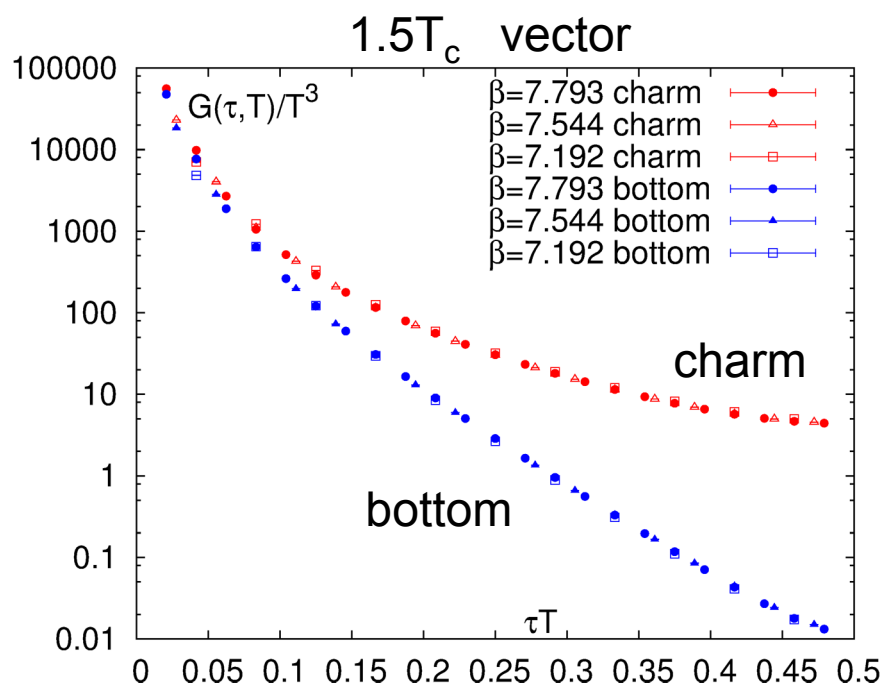
Experimental values: $m_{J/\Psi}$ = 3.096.916(11) GeV, m_Υ = 9.46030(26) GeV

Charmonium and Bottomonium correlators

Lattice setup

β	c_{SW}	r_0/a	a [fm] (a^{-1} [GeV])	N_σ	N_τ	T/T_c	# confs (measured/generated)
7.192	1.367261	26.1	0.0184 (10.7)	96	48	0.75	237/237
					32	1.1	476/476
					28	1.3	336/336
					24	1.5	336/336
					16	2.25	237/237
7.544	1.330868	39.2	0.0121 (16.3)	144	72	0.75	221/221
					48	1.1	462/462
					42	1.3	660/660
					36	1.5	288/288
					24	2.25	237/237
7.793	1.310381	52.0	0.00905 (21.8)	192	96	0.75	**183/224**
					64	1.1	**178/291**
					56	1.3	**190/291**
					48	1.5	**210/348**
					32	2.25	**109/121**





Reconstructed correlation function

$$G_{\rm rec}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$

$$\frac{G(\tau, T)}{G_{\rm rec}(\tau, T; T')}$$ equals to unity at all τ

if the spectral function doesn't vary with temperature

S. Datta *et al.*, PRD 69 (2004) 094507

can be calculated directly from correlation function
for suitable ratios of N'_τ / N_τ without knowledge of spectral function:

$$\frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh[\omega N_\tau/2]} = \sum_{\tau'=\tau;\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh[\omega N'_\tau/2]}$$

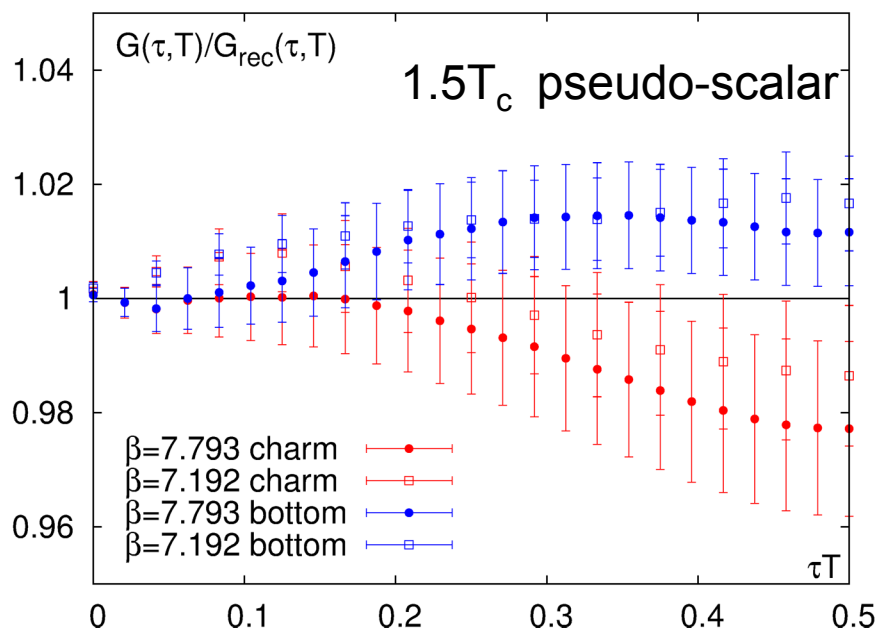
$$T = 1/(N_\tau a) \qquad N'_\tau = m N_\tau \qquad m = 1, 2, 3, \cdots$$

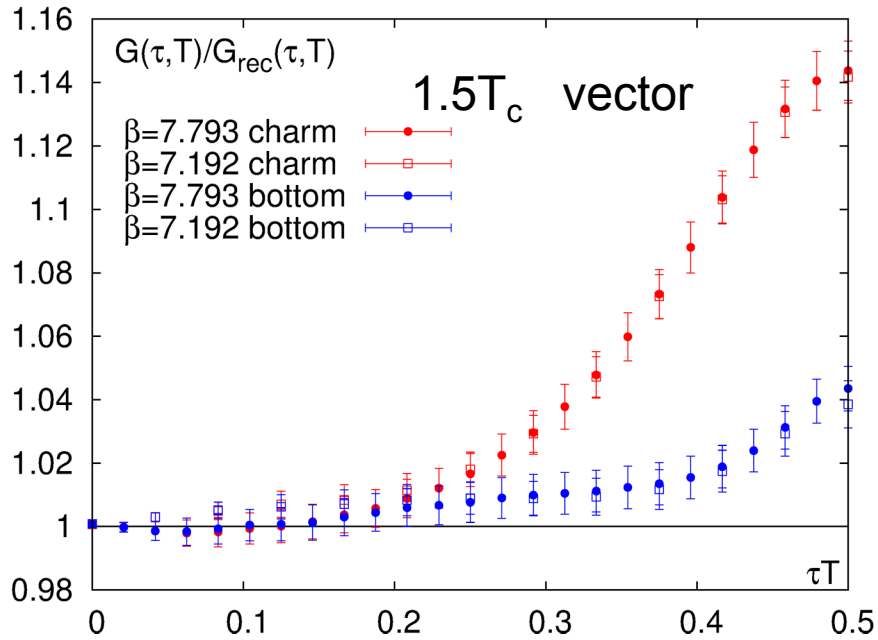
$$\boxed{G_{\rm rec}(\tau, T; T') = \sum_{\tau'=\tilde{\tau};\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau', T')}$$

H.-T. Ding *et al.*, PRD 86 (2012) 014509

Charmonium and Bottomonium correlators – S-wave channels

$$G_{\rm rec}(\tau,T;T') \equiv \int_0^\infty d\omega\,\rho(\omega,T')K(\omega,\tau,T)$$





different behavior in pseudo-scalar (left) and vector (right) channel

strong modification at large τ in vector channel

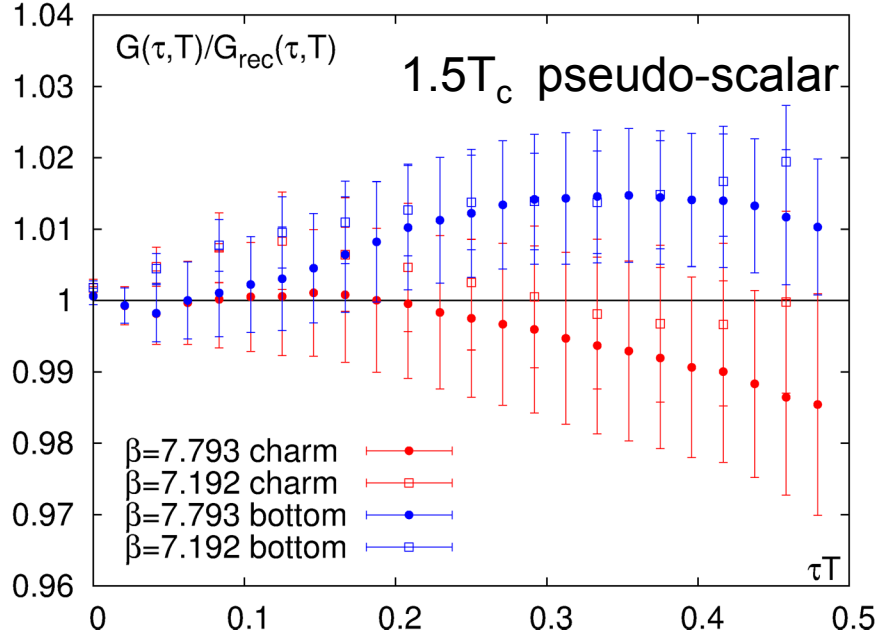
stronger for charm compared to bottom

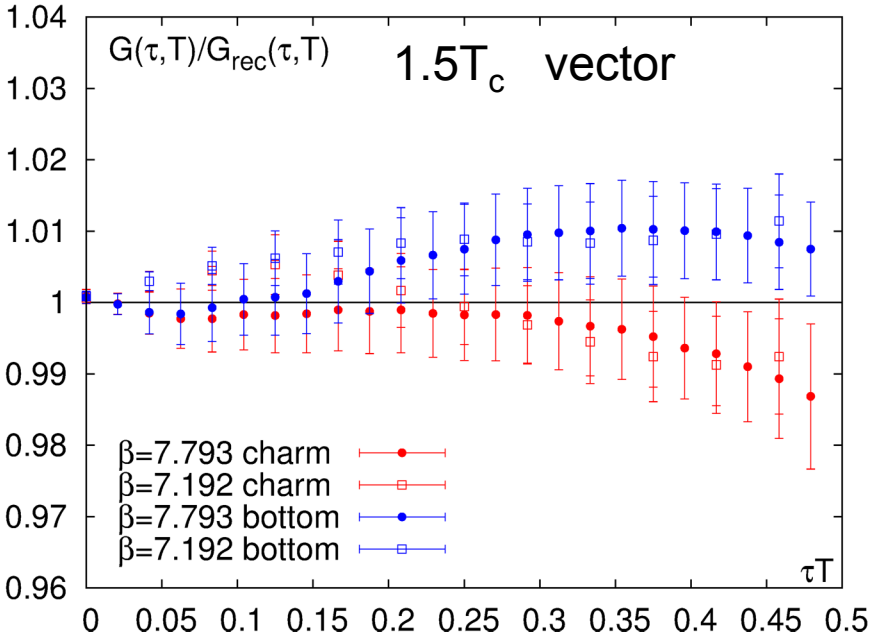
related to transport contribution

Mid-point subtracted correlators – S-wave channels

$$\bar{G}(\tau) \equiv G(\tau) - G(1/2T)$$

mid-point subtracted correlator





small ω region gives (almost) constant contribution to correlators

effectively removed by mid-point subtraction

pseudo-scalar (left) and vector (right) very comparable

need to understand cut-off effects and quark-mass effects

next goals: continuum extrapolation and determination of the spectral fct

Conclusions

1) Heavy quark momentum diffusion coefficient κ

continuum extrapolation of the color electric field correlation function

estimate for the heavy quark momentum diffusion coefficient κ at $1.5T_c$

estimate for the time scale for kinetic equilibration of heavy quarks

2) Charmonium and Bottomonium dissociation patterns

using a complex valued heavy quark potential

we determined the sequential melting patterns for quarkonium states

and give an estimate for Ψ' to J/Ψ ratio due to regenration at T_c

Outlook – full relativistic treatment of charm and bottom on the lattice

work is still in progress

continuum extrapolation for the quarkonium correlators still needed

detailed analysis of the systematic uncertainties

extract spectral properties (on continuum extrapolated correlators) by

- comparing to perturbation theory

- Fits using Ansätze for the spectral function

- Bayesian techniques to extract the spectral function

final goal:

understand the temperature and quark mass dependence of

heavy quark diffusion coefficient

dissociation temperatures for different states