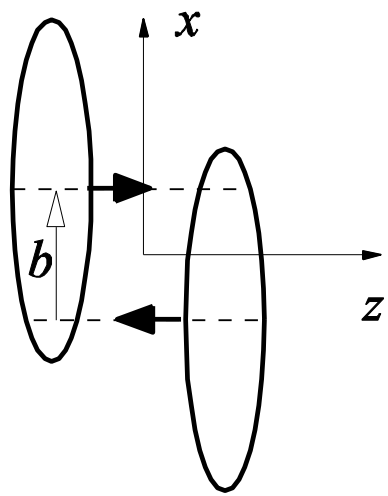
Polarization of fermions in a vortiular fluid

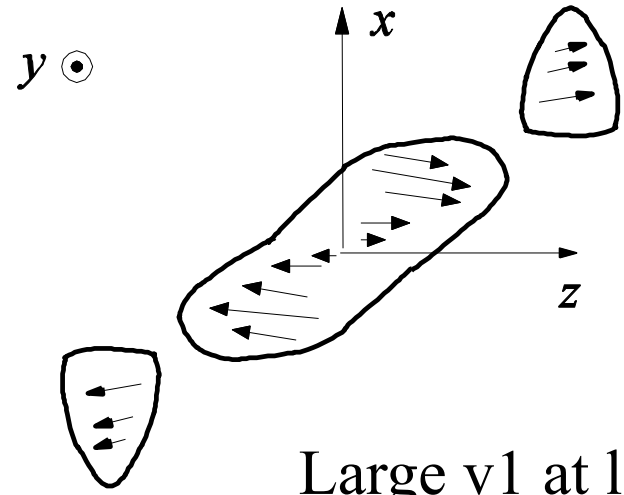
AGS/RHIC Users meeting
June 7, 2016

Xin-Nian Wang
CCNU/LBNL

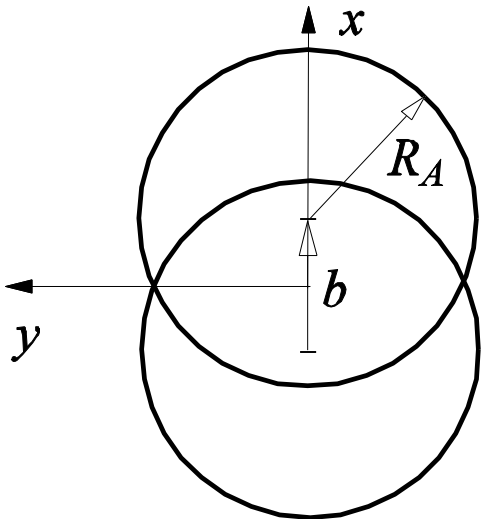
Fang, Pang, Wang & XNW arXiv:1604.04036
Pang, Petersen, Wang & XNW arXiv:1605.04024

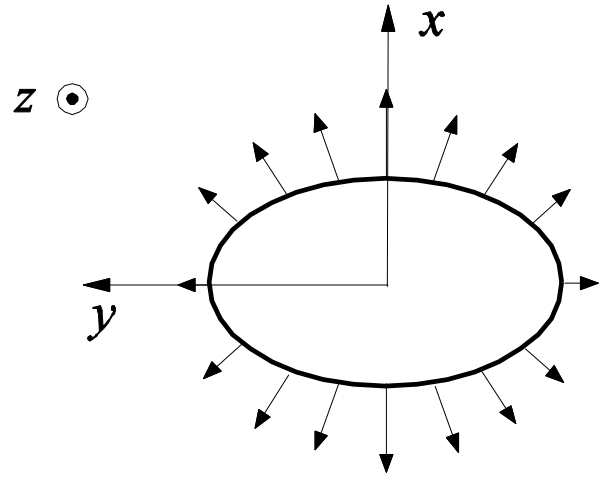
Global Orbital Angular Momentum



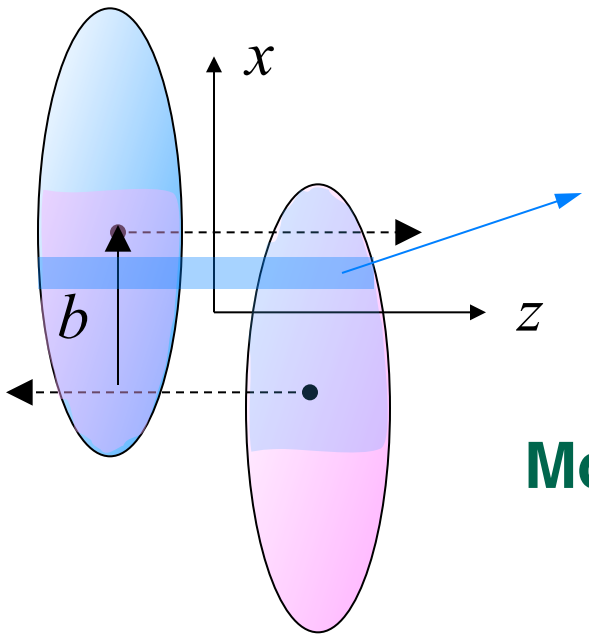


Large v1 at large rapidity





Transverse gradient of longitudinal fluid velocity in a Landau fireball



$$\frac{dN_{part}^{P,T}}{dx}$$

Distribution of number of participant projectile or target nucleons

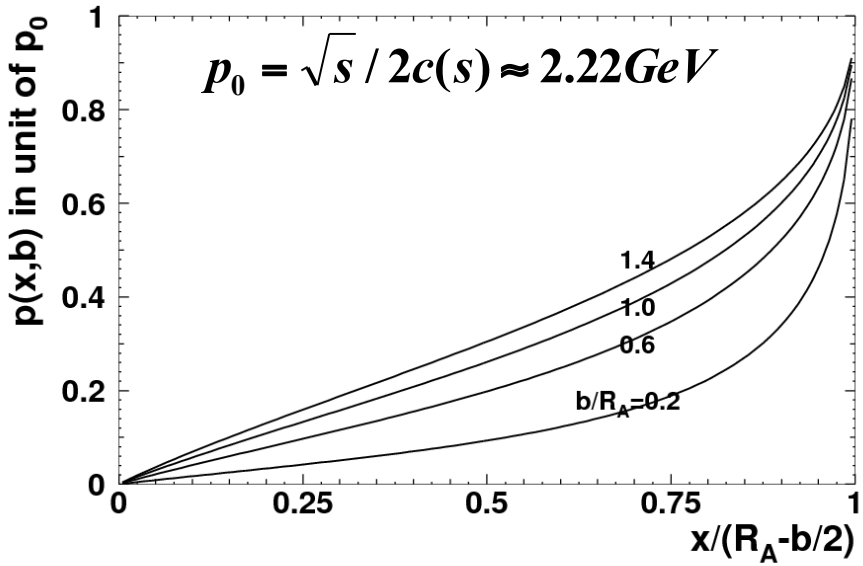
Momentum conservation

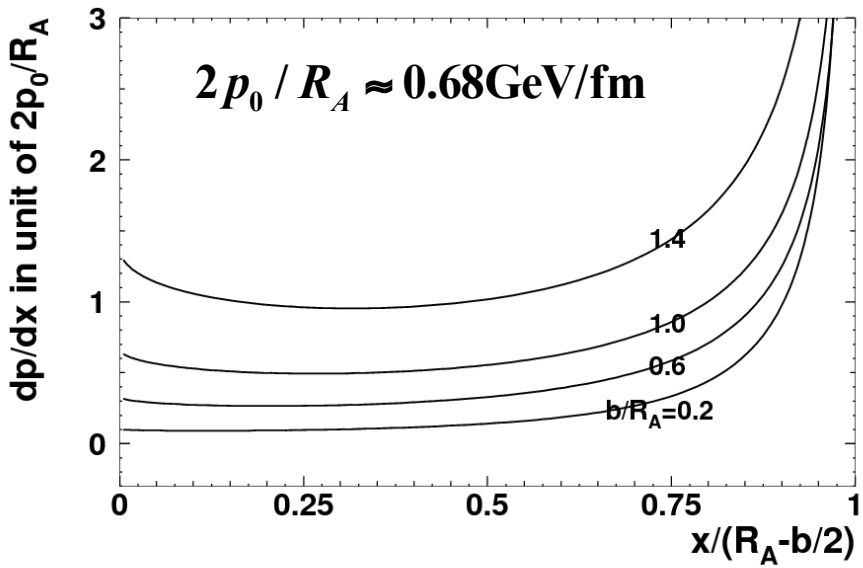
→ BJ scaling violation

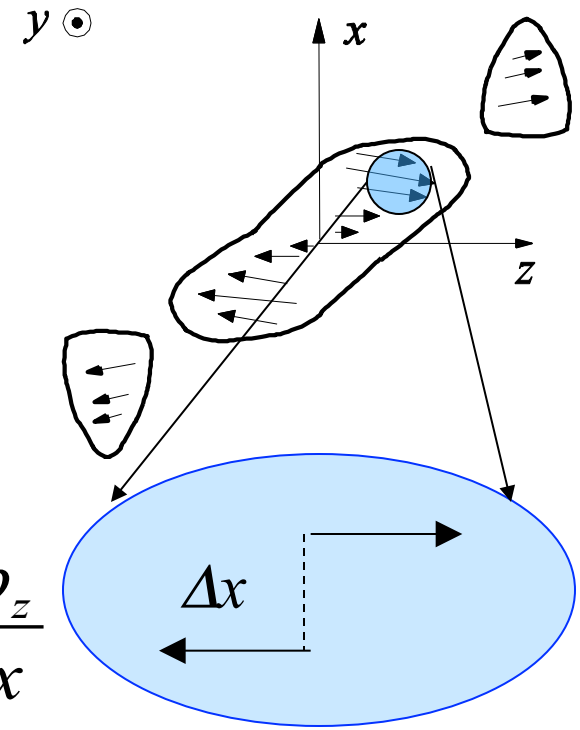
Collective longitudinal momentum per produced parton

$$p_z(x,b) = \frac{\sqrt{s}}{2} \frac{\frac{dN_{part}^{P}}{dx} - \frac{dN_{part}^{T}}{dx}}{c(s)\left(\frac{dN_{part}^{P}}{dx} + \frac{dN_{part}^{T}}{dx}\right)}$$

Local Orbital Angular Momentum







$$\Delta p_z = \Delta x \frac{dp_z}{dx}$$

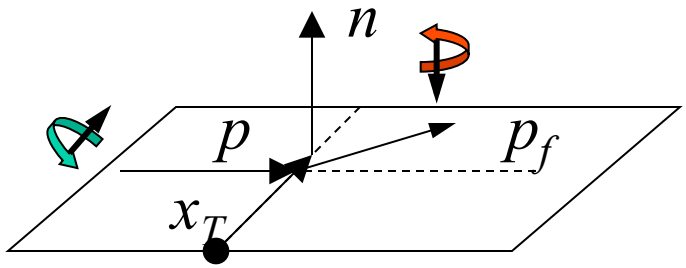
$$L_y = -\Delta x \Delta p_z = -\Delta x^2 \frac{dp_z}{dx}$$

$$\vec{\omega} = \vec{\nabla} \times \vec{v} \sim -\hat{y}\langle \frac{1}{E}\frac{dp_z}{dx}\rangle$$

Quark Polarization

Unpolarized cross section:

$$\frac{d\sigma}{d^2x_T} = \frac{d\sigma_+}{d^2x_T} + \frac{d\sigma_-}{d^2x_T} = 4C_T\alpha_s^2 K_0^2(\mu x_T)$$



$$\frac{d^2\sigma}{d^2q_T} = \frac{1}{(2\pi)^2}\frac{1}{2}\sum_{\lambda,\lambda_i}|M(q_T)|^2 = C\frac{g^2}{(q_T^2+\mu^2)^2}$$

Polarized cross section:

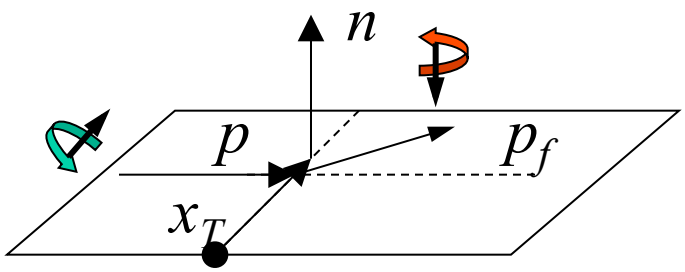
$$\frac{d\Delta\sigma}{d^2x_T} = \frac{d\sigma_+}{d^2x_T} - \frac{d\sigma_-}{d^2x_T} = -\mu\frac{\vec{p}\cdot(\hat{x}_T\times\vec{n})}{E(E+m_q)}4C_T\alpha_s^2 K_0(\mu x_T)K_1(\mu x_T)$$

$$P_q \equiv \frac{\Delta\sigma}{\sigma} = -\pi\frac{\mu p}{2E(E+m_q)}$$

Liang & XNW, PRL 94 (2005) 102301

Spin-orbital coupling

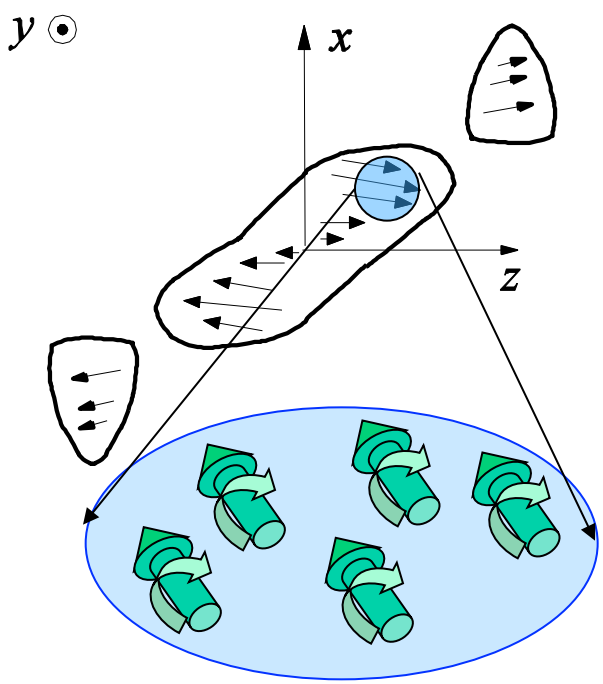
$$P_q \equiv \frac{\Delta\sigma}{\sigma} = -\pi \frac{\mu p}{4E(E+m_q)}$$



spin polarization in A+A

nonrelativistic limit: $(m_q \gg p, \mu)$

$$P_q \approx -\pi \frac{\mu p}{4m_q^2} \sim -\omega/m$$



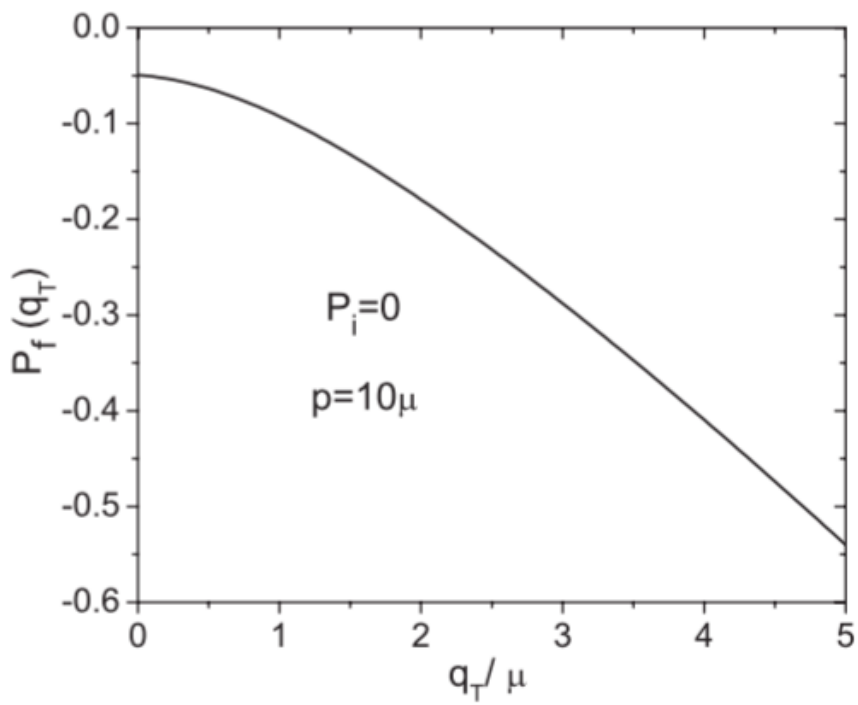
Liang & XNW, PRL 94 (2005) 102301

Transverse momentum dependence

$$P_f = P_i - \frac{(1-P_i^2)\pi\mu p}{2E(E+m) - P_i\pi\mu p}$$

Polarization rate Eq.

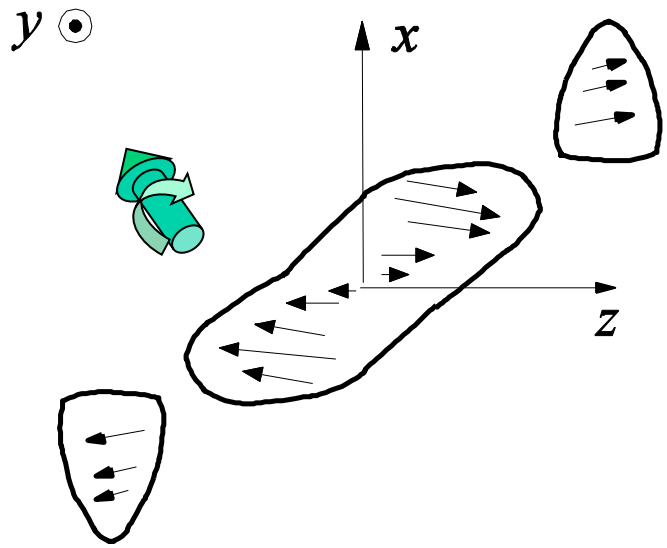
$$P_f(q_T) = \frac{\pi E(E+m)P_i - p\sqrt{q_T^2+\mu^2}K(q_T/\sqrt{q_T^2+\mu^2})}{\pi E(E+m) - P_i p\sqrt{q_T^2+\mu^2}K(q_T/\sqrt{q_T^2+\mu^2})}$$



Huang, Pasi & XNW
PRC 84 (2011) 054910

Consequences in A+A collisions

Globally Polarized thermal dilepton, J/Ψ,
Hyperons and vector mesons



Liang & XNW, PRL 94 (05) 102301
Liang & XNW, PLB 629(05)20
Gao et al, PRC 77 (08) 044902

Spin polarization in equilibrium

Dirac Eq. $$\left[\gamma^\mu(i\partial_\mu + e_q A_\mu) - m\right]\psi(x) = 0$$

Spin: vorticity coupling Magnetic coupling

$$\delta E_s = \frac{\hbar}{2}\mathbf{n}\cdot\omega + e_q\hbar\frac{\mathbf{n}\cdot\mathbf{B}}{E_p}$$

$$\Pi = \frac{1}{2}\int\frac{d^3p}{(2\pi)^3}\left[f(E_p+\delta E_s) - f(E_p+\delta E_s)\right]$$

$$\approx \int\frac{d^3p}{(2\pi)^3}\delta E_s\frac{\partial f(E_p)}{\partial E_p}$$ gradient expansion

Becattini & Ferroni, EJPC 52 (2007) 597, Betz, Gyulassy & Torrieri, PRC 76 (2007) 044901, Becattini, Piccinini & Rizzo, PRC 77 (2008) 024906, Beccatini, Csernai & Wang, PRC 87 (2013) 034905, Xie, Glastad & Csernai, PRC 92 (2015) 064901, Deng & Huang, arXiv 1603.06117

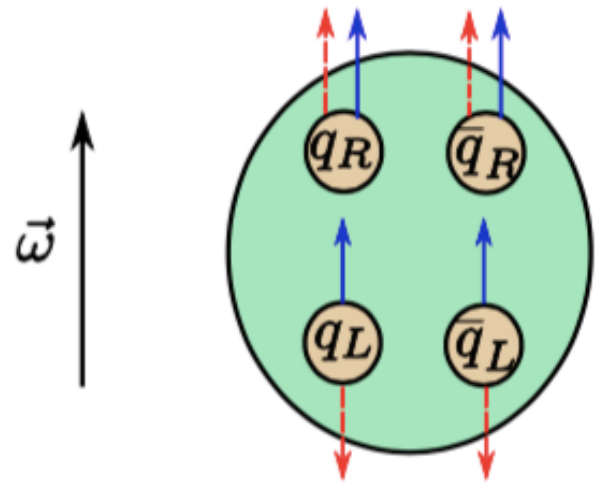
CME/CVE and spin polarization

Chiral Magnetic Effect — Kharzeev, McLerran, Warringa (2008), Fukushima, Kharzeev, Warringa (2008)

Chiral Vorticity Effect — Son, Surowka (2009), Kharzeev, Son (2011)

$$j^\mu = \int d^4p\, \mathcal{V}^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu,$$

$$j_5^\mu = \int d^4p\, \mathcal{A}^\mu = n_5 u^\mu + \xi_5 \omega^\mu + \xi_{B5} B^\mu.$$



R (L) fermion's spin parallel (opposite) to vorticity & magnetic field

Pu, Gao, Liang, Wang & XNW, PRL 109 (2012) 232301

Quantum Kinetic Theory

Fang, Pang, Wang, XNW arXiv:1604.04036

$$\gamma_\mu \left(p^\mu + \frac{1}{2} i \nabla^\mu \right) W(x,p) = 0,$$

Polarization density

$$\Pi^\alpha(x) = \frac{1}{2}\hbar\widetilde{\omega}^\alpha \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{e^{\beta(E_p-\mu)}}{[e^{\beta(E_p-\mu)}+1]^2} + \frac{e^{\beta(E_p+\mu)}}{[e^{\beta(E_p+\mu)}+1]^2} \right\}$$

$$+ \frac{1}{2}\hbar Q \beta B^\alpha \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \left\{ \frac{e^{\beta(E_p-\mu)}}{[e^{\beta(E_p-\mu)}+1]^2} - \frac{e^{\beta(E_p+\mu)}}{[e^{\beta(E_p+\mu)}+1]^2} \right\}$$

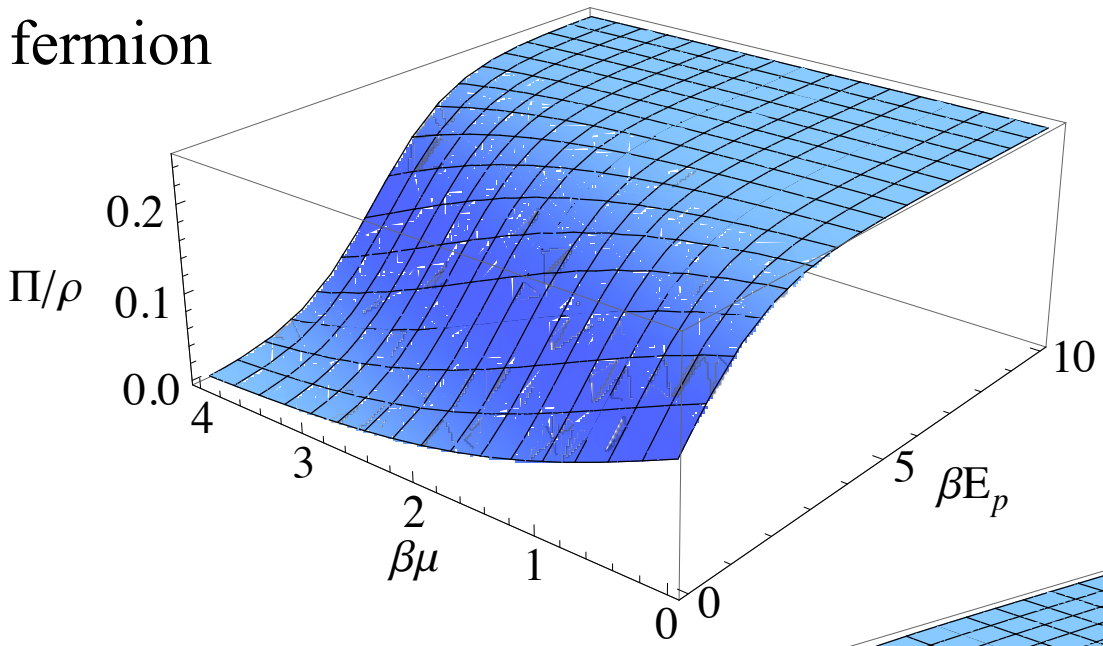
Polarization on the freeze-out surface:

μ chemical potential

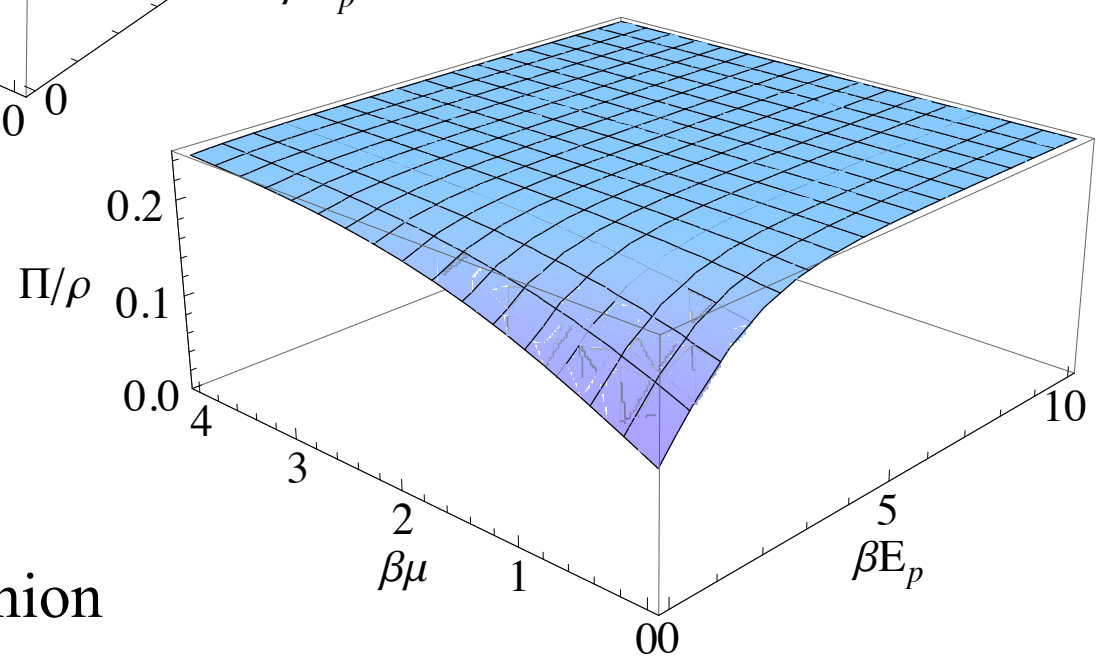
$$\frac{d\Pi^\alpha(p)/d^3p}{d\rho(p)/d^3p} = \frac{\hbar}{4m} \frac{\int d\Sigma_\lambda p^\lambda \tilde{\Omega}^{\alpha\sigma} p_\sigma f_{\rm FD}(x,p)[1 - f_{\rm FD}(x,p)]}{\int d\Sigma_\lambda p^\lambda f_{\rm FD}(x,p)}.$$

Vorticity induced polarization per fermion

fermion



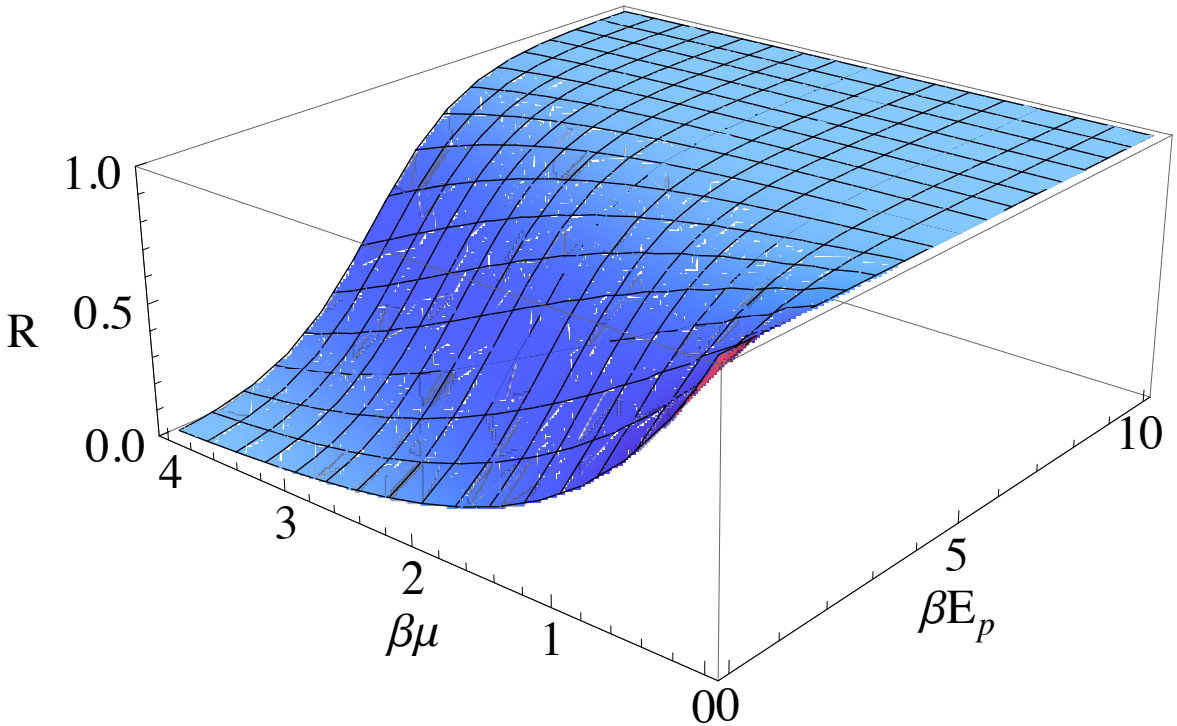
In the unit of ω



anti-fermion

polarization fermion/anti-fermion

$$R = \frac{[\Pi/\rho]_{\text{fermion}}}{[\Pi/\rho]_{\text{anti-fermion}}}$$

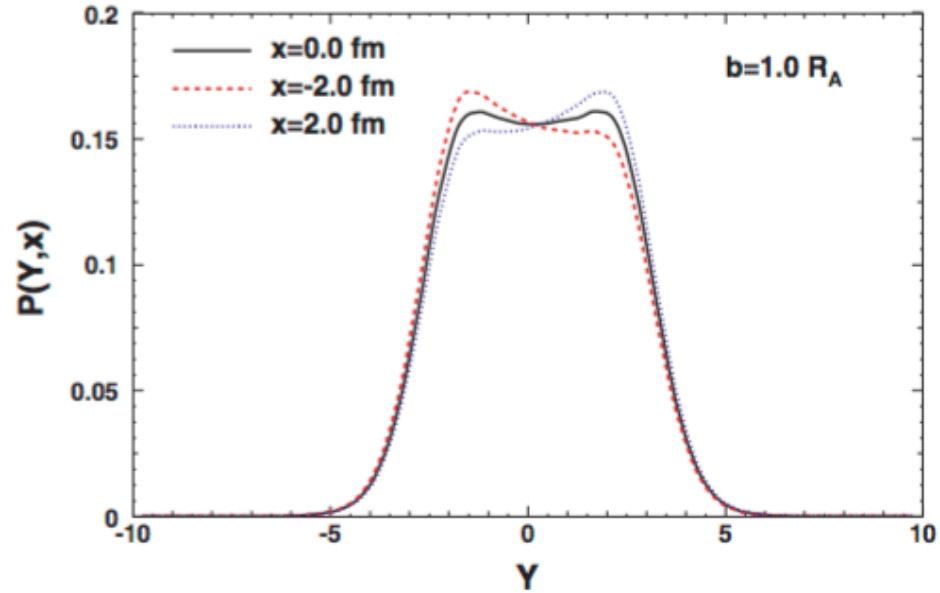


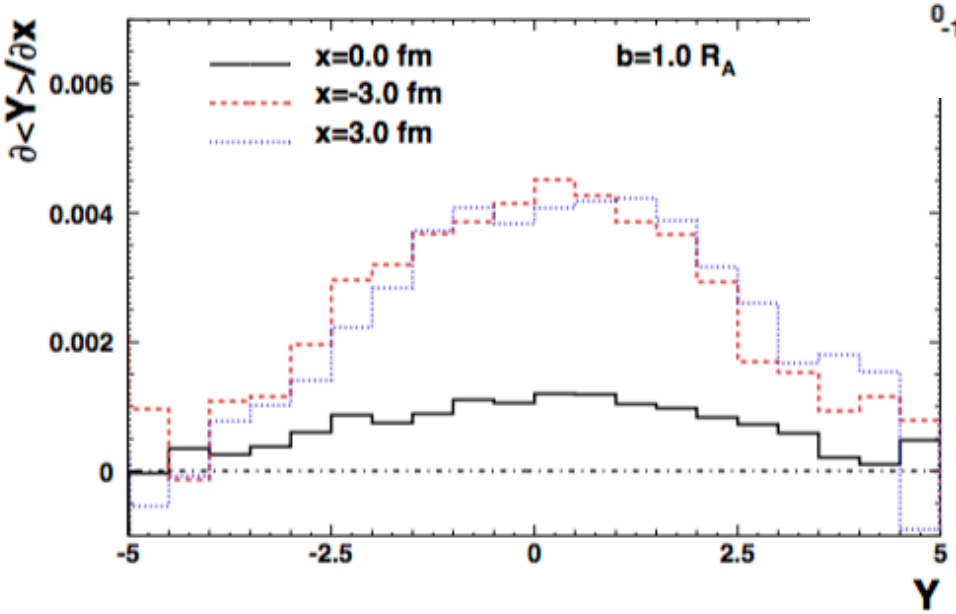
Fang, Pang, Wang, XNW arXiv:1604.04036

Transverse gradient of longitudinal fluid velocity in the HIJING model

Approximate BJ scaling at y=0, no transverse gradient in longitudinal fluid velocity,

BJ scaling violation at large y or lower collision energy

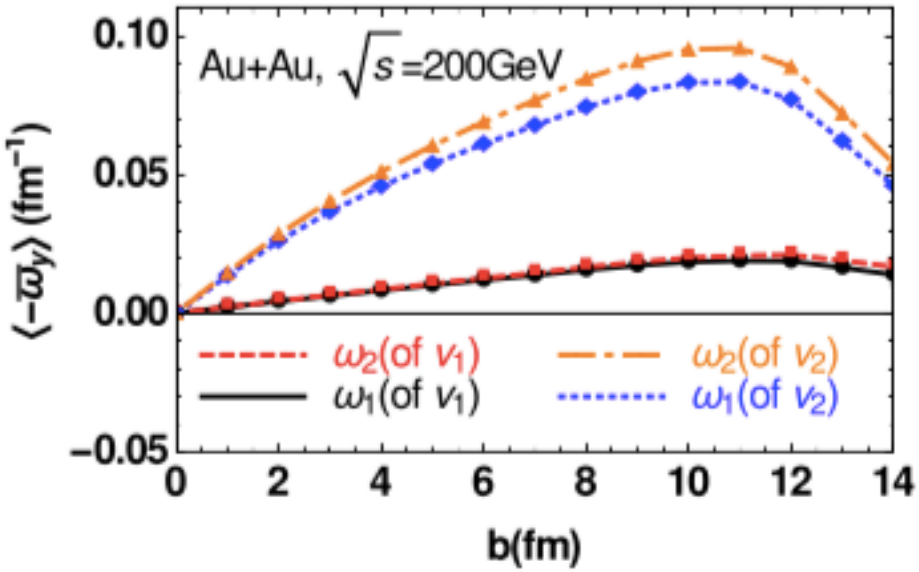


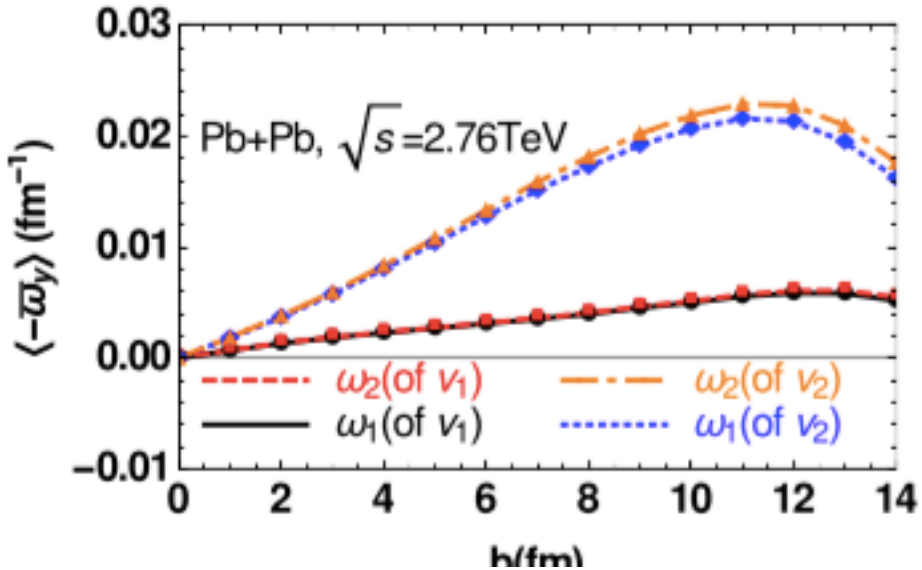


$$\omega_y \sim \sinh(Y)\frac{\partial Y}{\partial x}$$

Gao, Chen, Deng, Liang, Q. Wang, XNW, PRC 77 (2008)044902

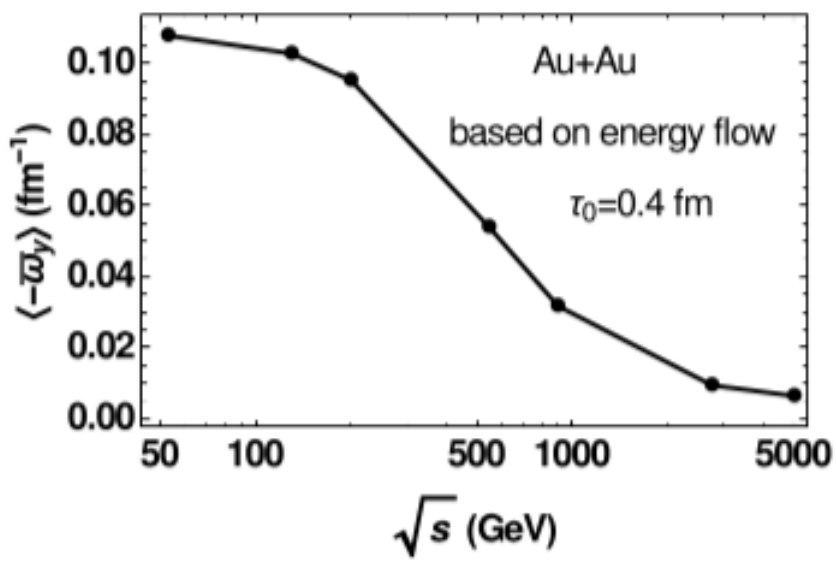
Vorticity from HIJING model





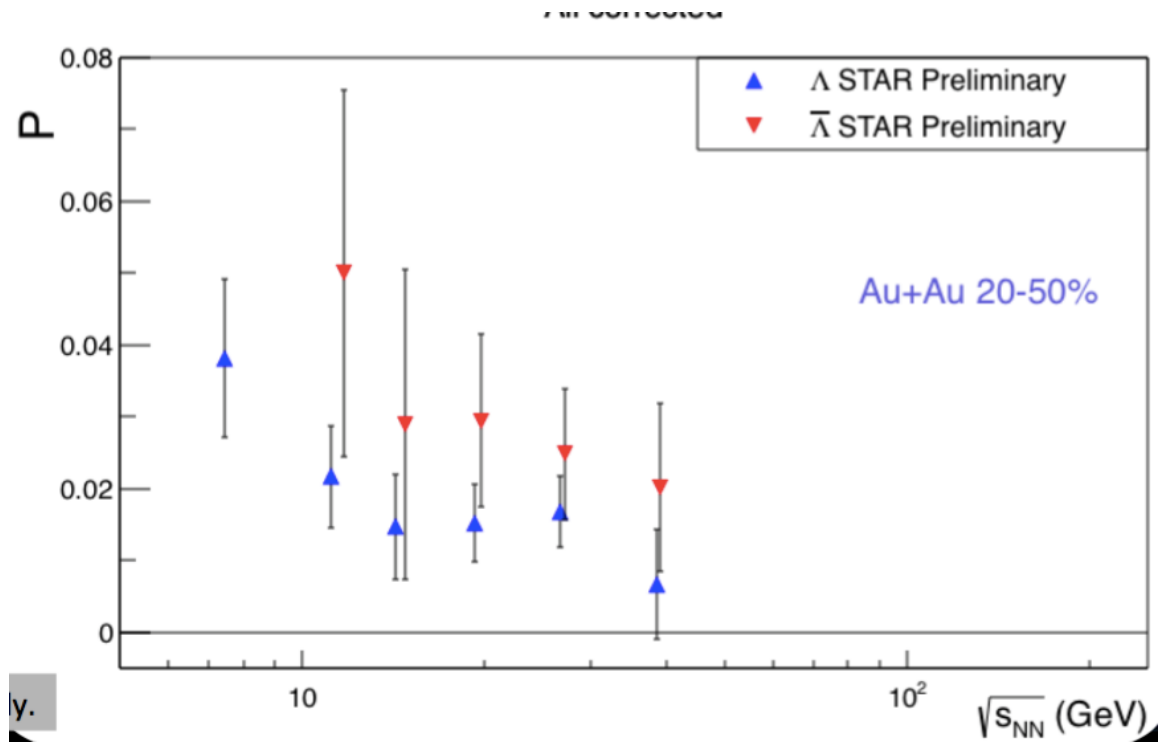
Global vorticity increases with impact-parameter and decreasing colliding energy

Deng & Huang, arXiv:1603.06117



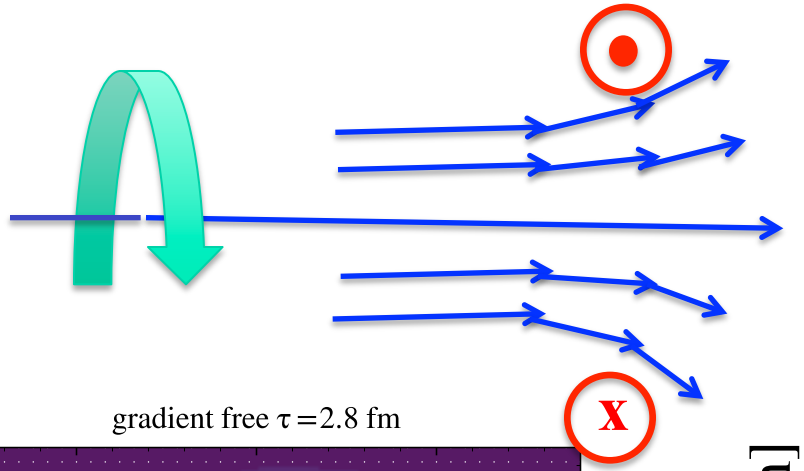
Features of Global Hyperon Polarization

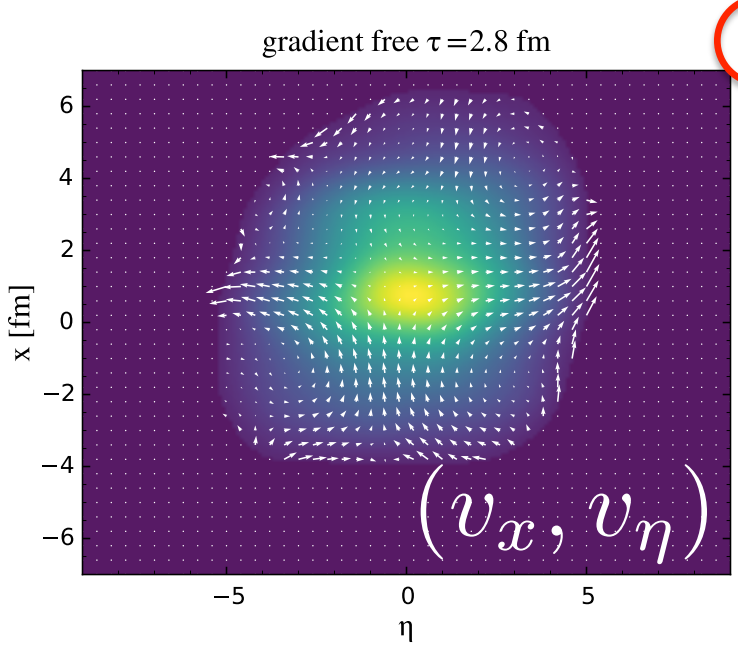
- Hyperons & anti-hyperons are similarly polarized
- P_H=0 in central and increases with b
- Increases with y at fixed collision energy
- Increase at lower energy at fixed y

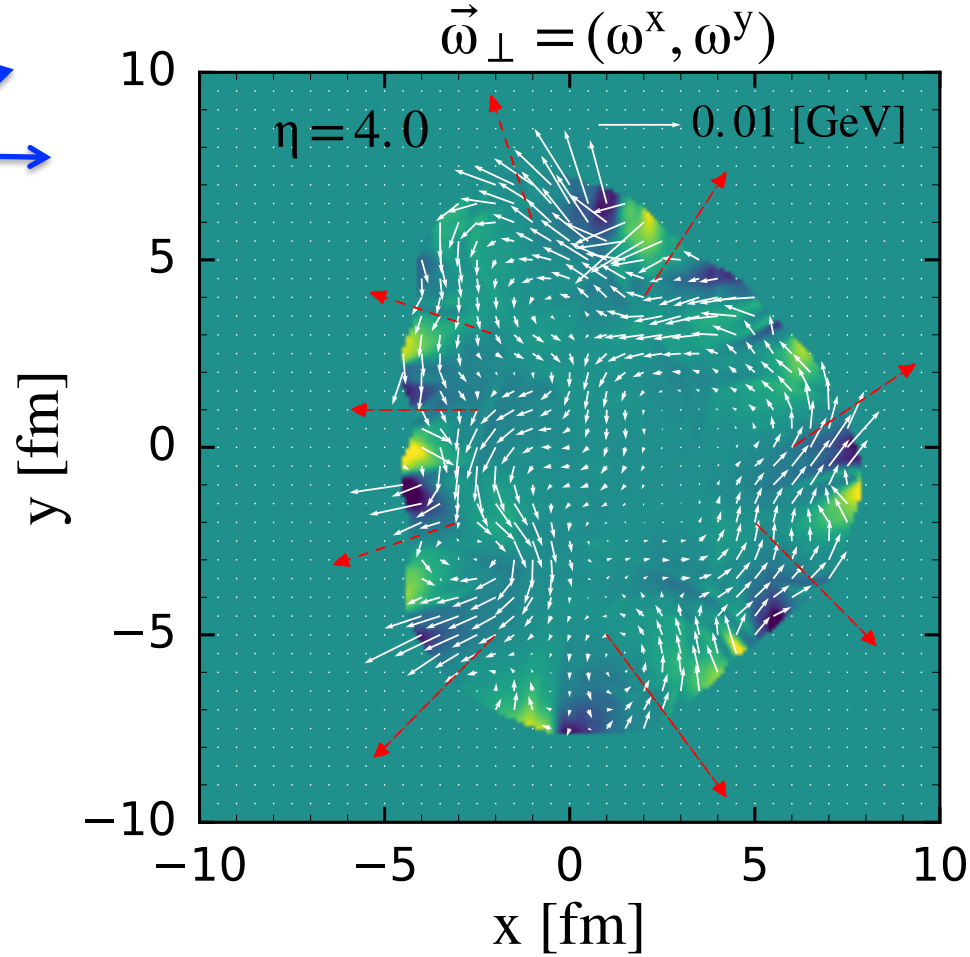


Talk by M. Lisa
Workshop on Vorticity
& magnetic field in
heavy-ion collisions
@UCLA, 2016

Transverse vorticity: Toroidal structure



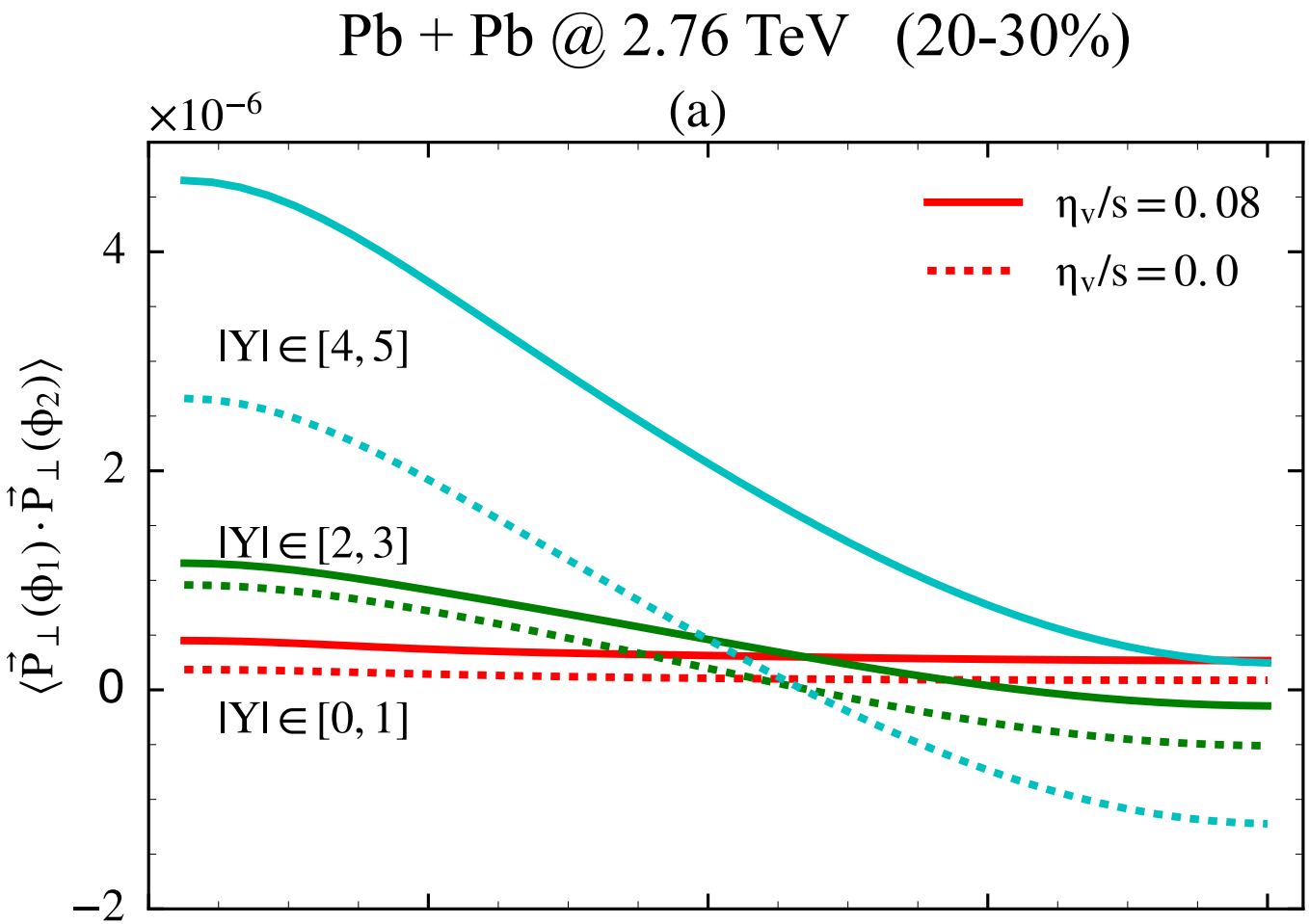




Toroidal structure of transverse fluid vorticity, in addition to the global net vorticity

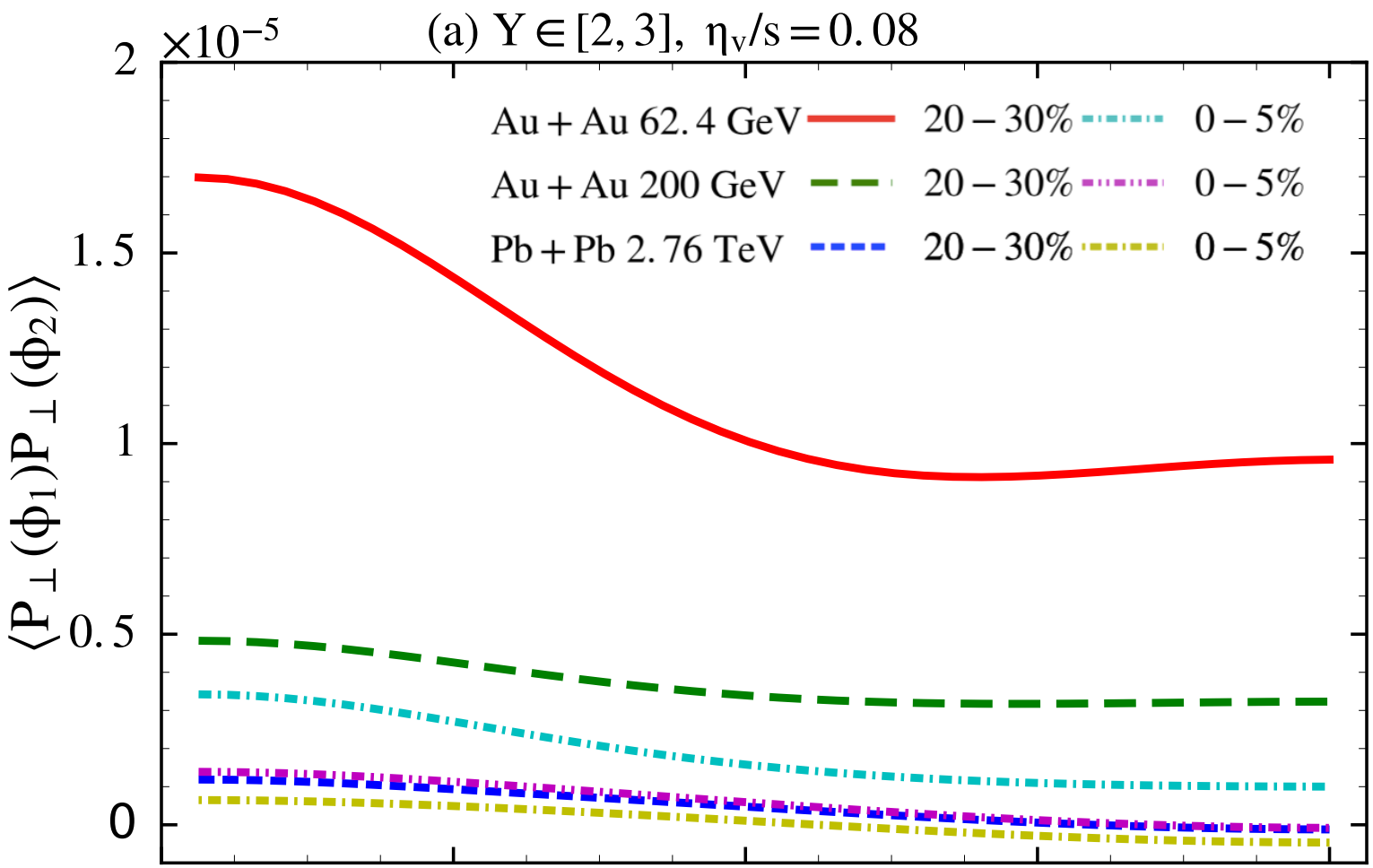
Pang, Petersen, Wang & XNW, arXiv:1605.04024

Transverse spin correlation



Pang, Petersen, Wang & XNW, arXiv:1605.04024

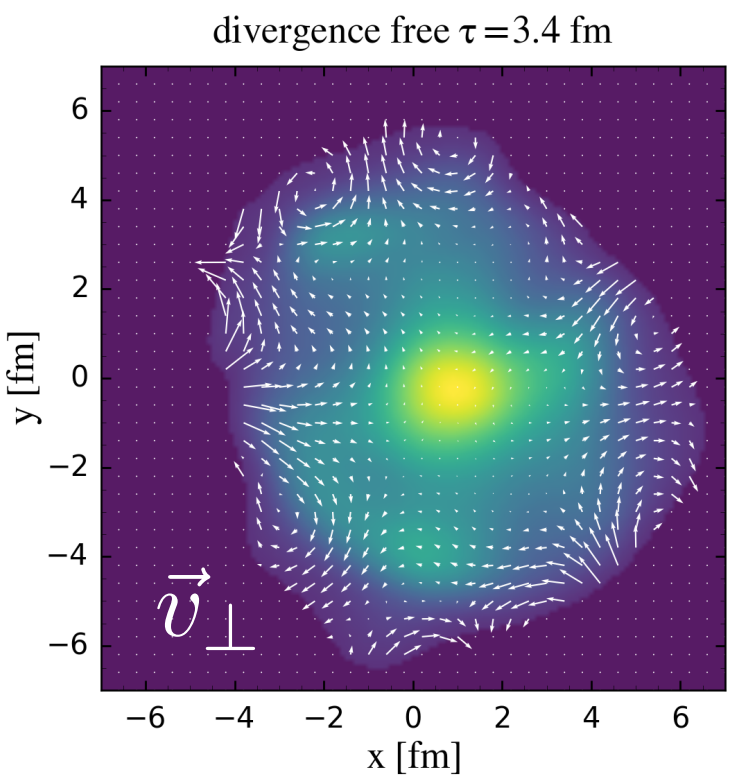
Transverse spin correlation



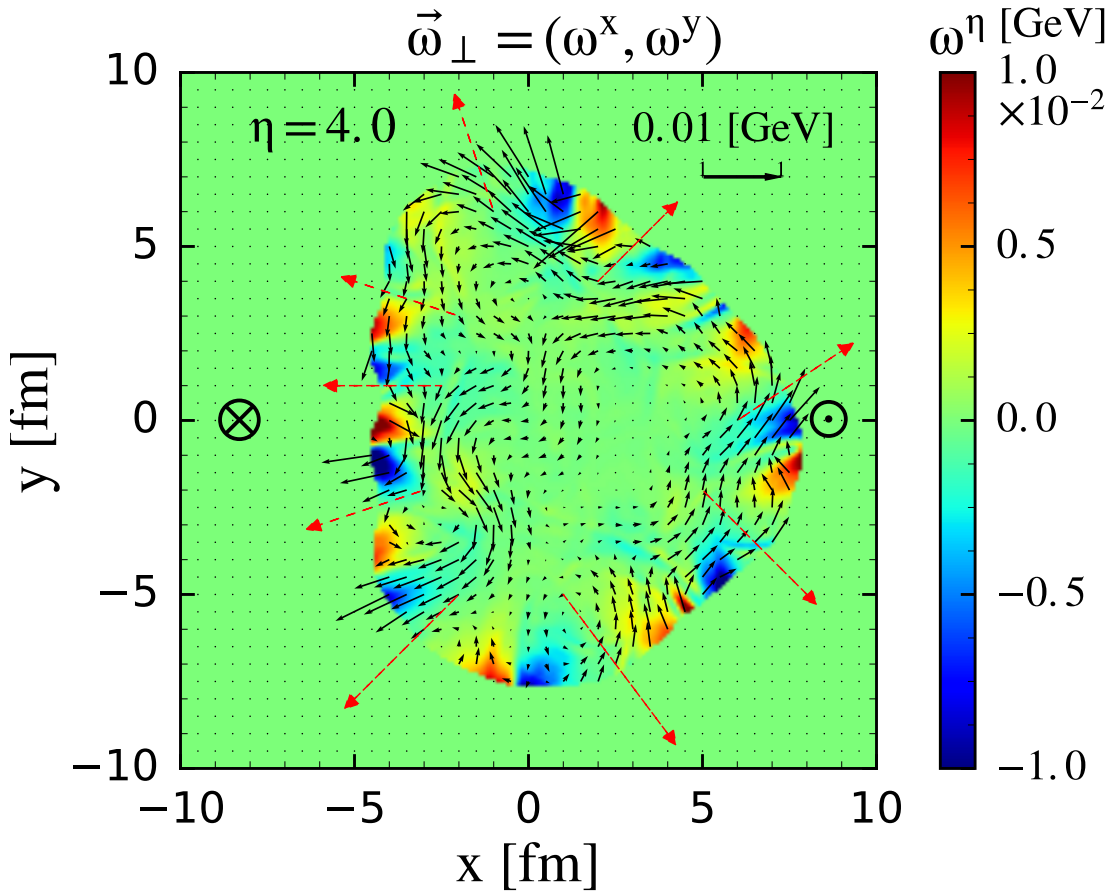
Pang, Petersen, Wang & XNW, arXiv:1605.04024

Longitudinal vorticity: vortex-pairing

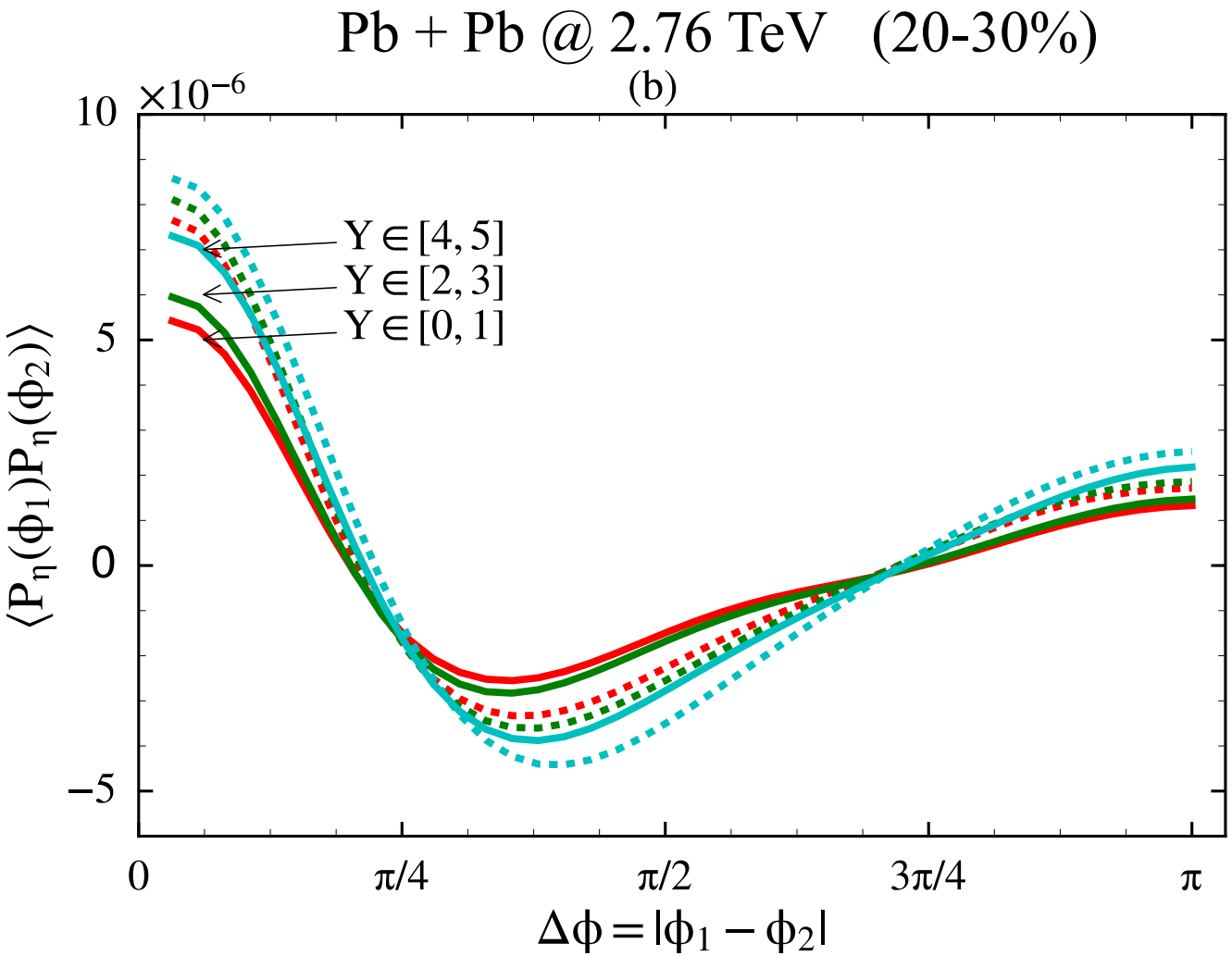
Pang, Petersen, Wang & XNW, arXiv:1605.04024



Vortex Pairing

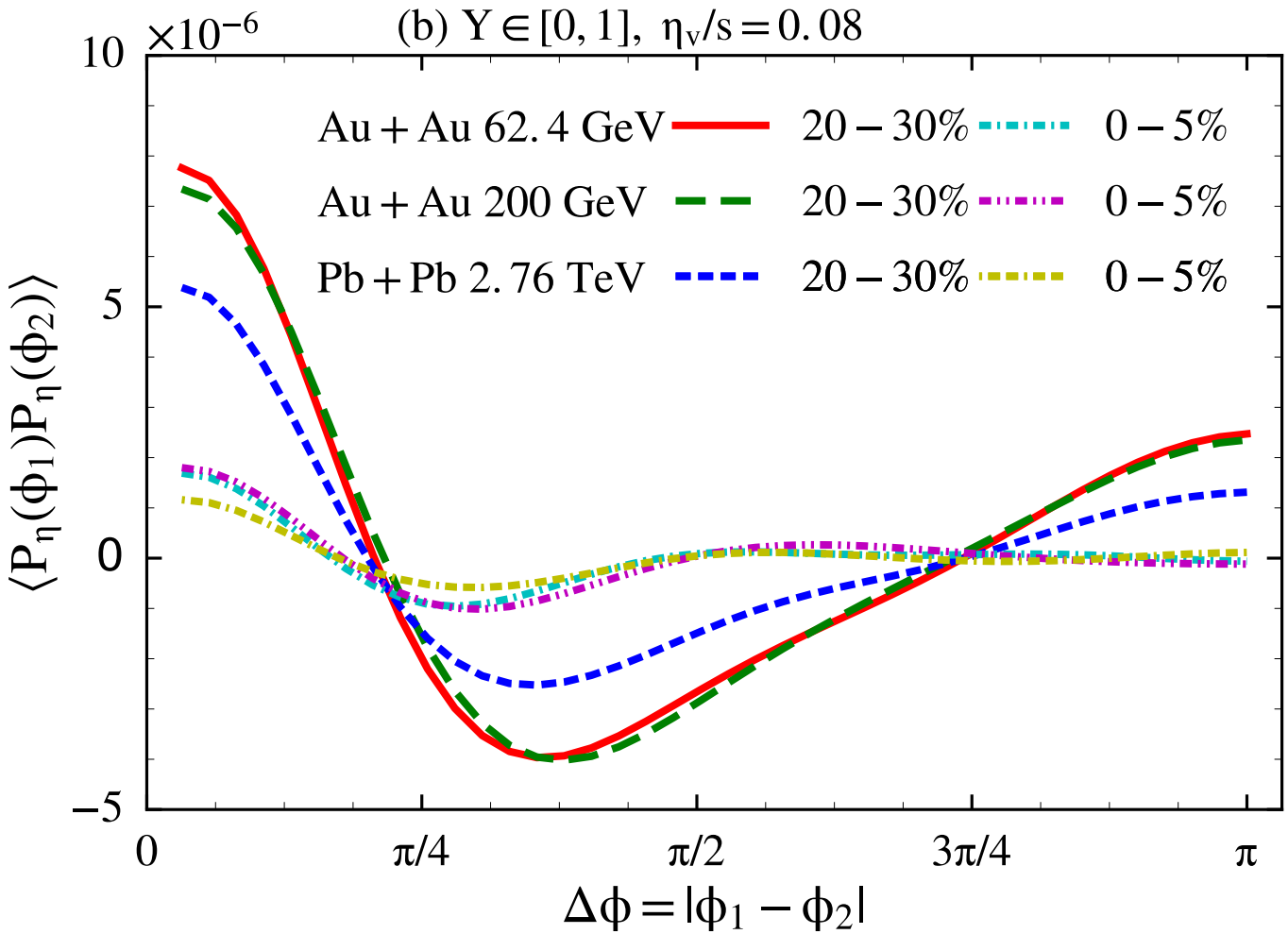


Longitudinal spin correlation



Pang, Petersen, Wang & XNW arXiv:1605.04024 ,

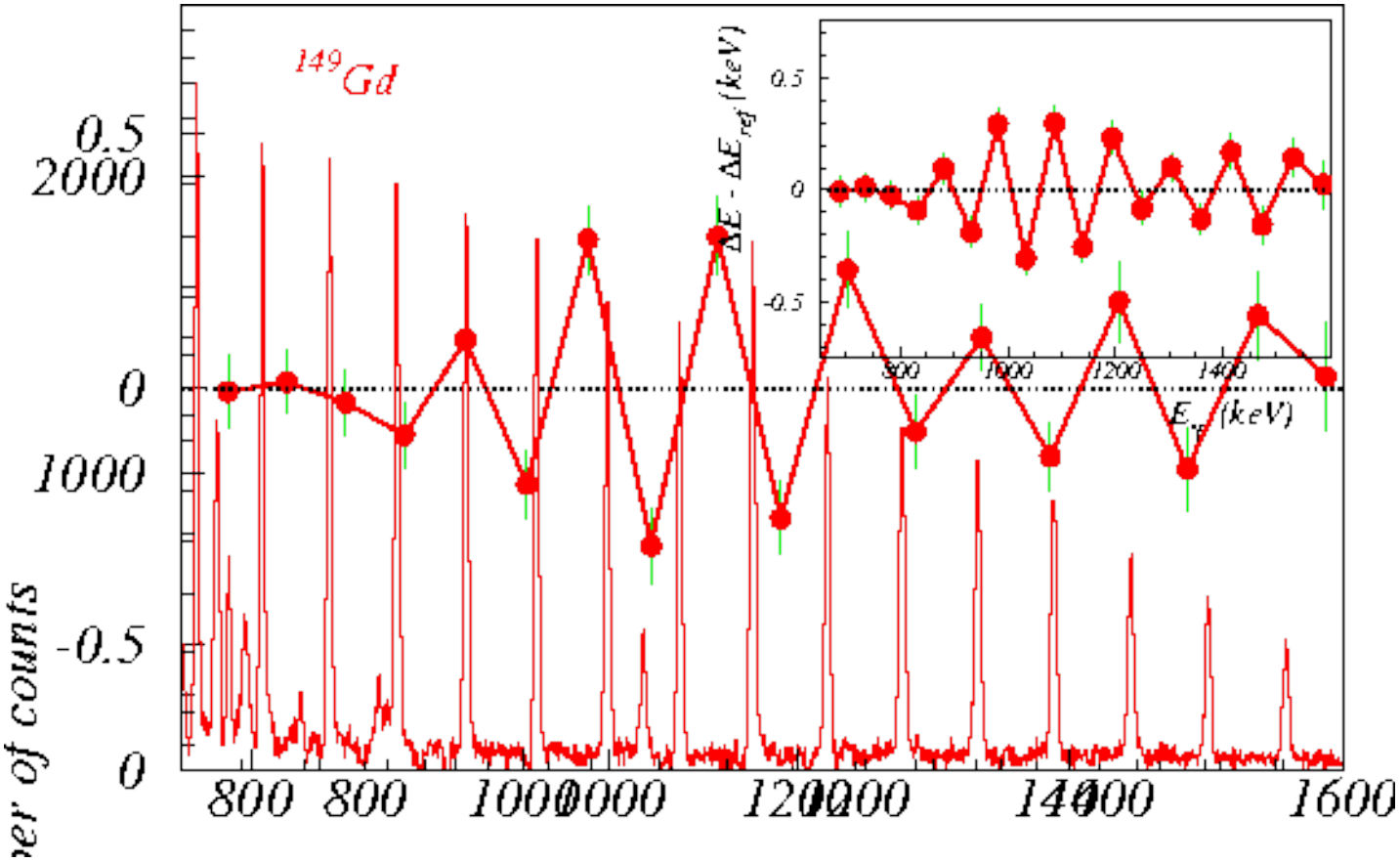
Longitudinal spin correlation



Pang, Petersen, Wang & XNW, arXiv:1605.04024

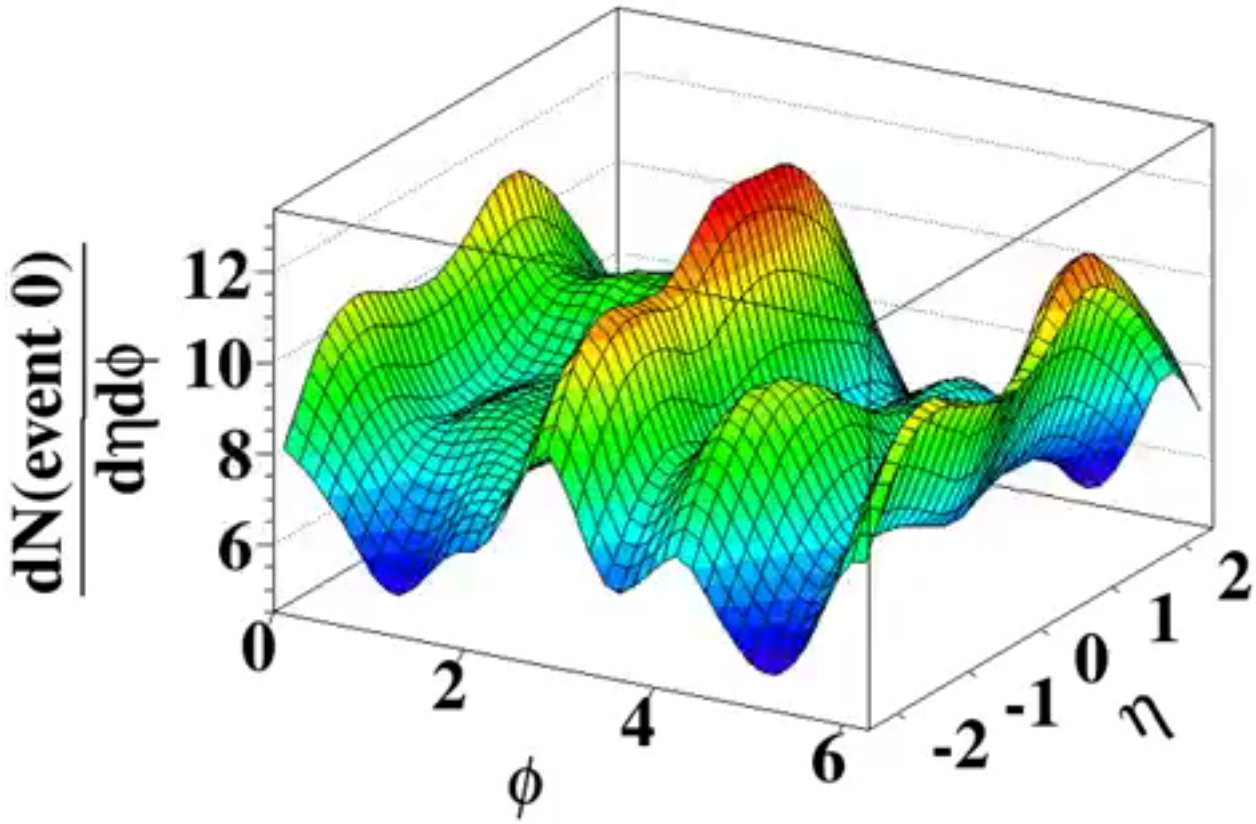
Summary

- fluid velocity gradient or fluid vorticity leads to global spin polarization
- Spin structure in QGP in A+A collisions
  - Hyperon polarization, vector meson spin alignment, contribution to vn (small)
- Convective longitudinal flow induces toroidal structure of transverse vorticity → transverse spin-spin correlation
- Convective radial flow of hot spots induces longitudinal vortex-pairing → longitudinal spin-spin correlation
- Study of spin polarization can shed light on transport properties of QGP and hadronization mechanism


149Gd
0.5
2000
0
1000
-0.5
0
800
1000
1200
1400
1600
ber of counts
ΔE - ΔE_ref (keV)
0.5
0
-0.5
800
1000
1200
1400
E_γ (keV)

Longitudinal structure in A+A

E-by-E 3+1D ideal hydro + HIJING initial condition

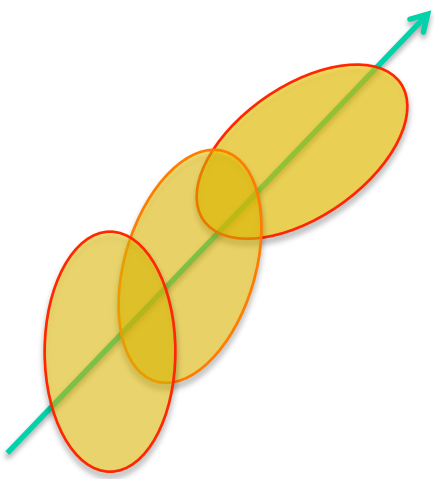


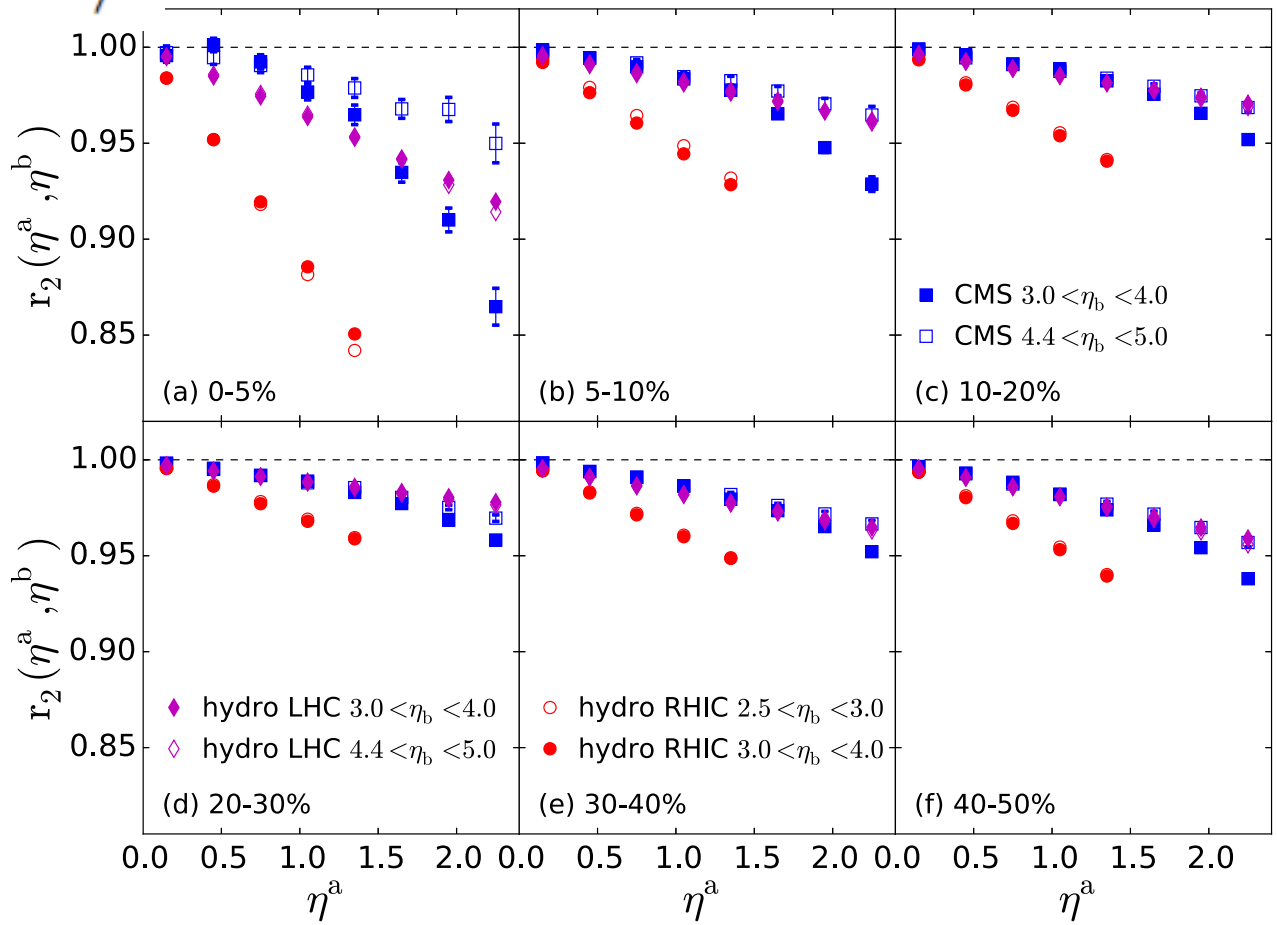
Pang & XNW (2013)

Longitudinal decorrelation of vn

$$r_n(\eta_a,\eta_b) = \frac{\left\langle \vec{Q}_n(-\eta_a)\vec{Q}_n^*(\eta_b)\right\rangle}{\left\langle \vec{Q}_n(\eta_a)\vec{Q}_n^*(\eta_b)\right\rangle}$$

Fluctuation and twist of the event plane





Pang, Petersen, Qin, Roy & XNW, EPJA 52(06) 97

Vorticity in convective fluid

FLOW

KTH

ROYAL INSTITUTE OF TECHNOLOGY

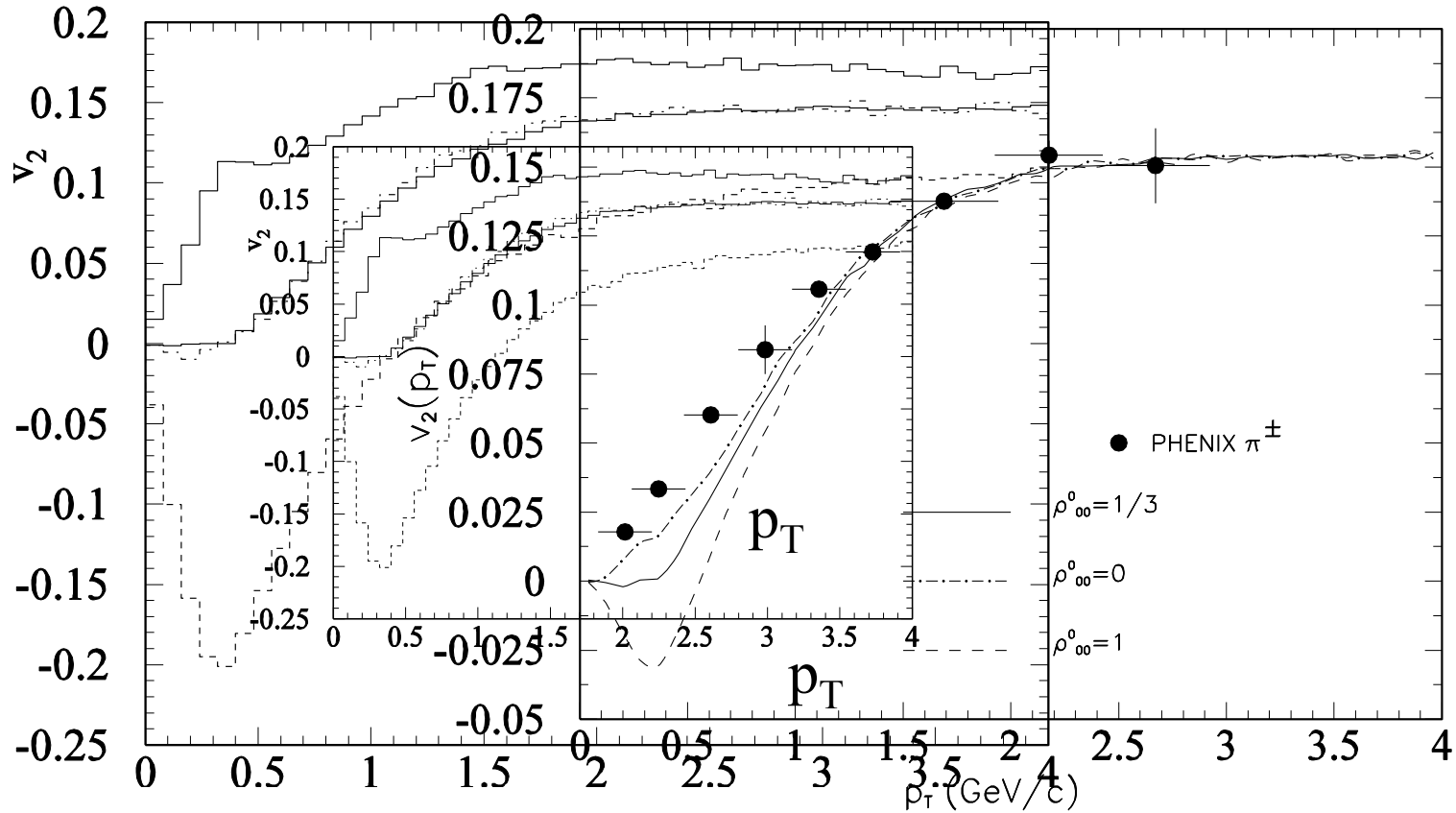
Round Jet, Re_D=5000

Ramis Örlü, 2009, Linné Flow Centre, KTH Mechanics

Constraints via v2

$$\frac{dN}{d\cos\theta} = 1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta$$

$v_2 > 0 \quad (\rho_{00} < 1/3)$ $\qquad$ $v_2 < 0 \quad (\rho_{00} > 1/3)$



Spin Alignment of Vector Mesons

Recombination model

$$\rho_{-1-1} = \frac{(1-P_q)(1-P_{\bar{q}})}{3+P_q P_{\bar{q}}} \qquad \rho_{00} = \frac{1-P_q P_{\bar{q}}}{3+P_q P_{\bar{q}}} \le \frac{1}{3} \qquad \rho_{11} = \frac{(1+P_q)(1+P_{\bar{q}})}{3+P_q P_{\bar{q}}}$$

Fragmentation

First rank: q-bar anti-parallel to leading q

$$e^+e^- \to Z^0 \to q\bar{q} \to \rho^{\rightarrow} + X$$

$$\rho_{00} = \frac{1+\alpha P_q^2}{3-\alpha P_q^2} \ge \frac{1}{3}$$

Other effects

Hadronic scattering: $\Lambda+\pi \rightarrow \Lambda\,\pi$

$P_\Lambda>0$ $\qquad$ $P_\Lambda<0$

$$M_{\Lambda\pi} = \bar{u}(p_f)\left[S(q) + \frac{1}{2}(\not{k} + \not{k}_f)V(q)\right]u(p)$$

Quark polarization due to Chromo-magnetic field? (Kharzeev' 04)

No global chromo-magnetic field ← color correlation length

$L \sim r_N$ inside nuclei, $L \sim 1/\mu$ in QGP

$L \sim 1/Q_s$ in CGC

Towards a numeric estimation at RHIC

Semi-central: $b=R_A$

$c(s) = 45$ at RHIC (assume parton/hadron=2)

$$\frac{dp_0}{dx} = \frac{\sqrt{s}}{2c(s)R_A} \approx 0.34 \text{ GeV/fm}$$

$$\frac{1}{\Delta x} : \mu : 0.5 \text{ GeV}$$

$$\Delta p_z : 0.1 \text{ GeV} = \mu$$

"Small angle approximation" is NOT valid at RHIC.

We need to go beyond small angle approximation

QK equation for massive fermions

Fang, Pang, Wang, XNW arXiv:1604.04036

$$(\gamma_\mu K^\mu - m)W(x,p) = 0,$$

Spin quantized in a given direction **n** in the fermion's rest frame

$$A^\mu_{(0)} = m\left[\theta(p_0)n^\mu(\mathbf{p},\mathbf{n}) - \theta(-p_0)n^\mu(-\mathbf{p},-\mathbf{n})\right]\delta(p^2-m^2)A,$$

$$A^\alpha_{(1)}(x,p) = -\frac{1}{2}\hbar\tilde{\Omega}^{\alpha\sigma}p_\sigma\frac{dV}{d(\beta p_0)}\delta(p^2-m^2) - Q\hbar\tilde{F}^{\alpha\lambda}p_\lambda V\frac{\delta(p^2-m^2)}{p^2-m^2},$$

$$A \equiv \frac{2}{(2\pi)^3}\sum_s s\left[\theta(p^0)f_{\rm FD}(p_0-\mu_s) + \theta(-p^0)f_{\rm FD}(-p_0+\mu_s)\right]$$

$$V \equiv \frac{2}{(2\pi)^3}\sum_s \left[\theta(p^0)f_{\rm FD}(p_0-\mu_s) + \theta(-p^0)f_{\rm FD}(-p_0+\mu_s)\right].$$

Quantum Kinetic approach to CME/ CVE and spin polarization

Wignar function:

$$U(x_+, x_-) \equiv e^{-iQ\int_{x_-}^{x_+} dz^\mu A_\mu(z)},$$

$$\hat{W}_{\alpha\beta} = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta(x_+) U(x_+, x_-) \psi_\alpha(x_-),$$

$$x_\pm \equiv x \pm \tfrac{1}{2}y$$

Quantum Kinetic equation

$$\gamma_\mu \left(p^\mu + \frac{1}{2} i \nabla^\mu \right) W(x,p) = 0,$$

$$\nabla^\mu \equiv \partial_x^\mu - Q F^\mu{}_\nu \partial_p^\nu$$

$$W = \frac{1}{4}\left[F + i\gamma^5 P + \gamma^\mu V_\mu + \gamma^5\gamma^\mu A_\mu + \frac{1}{2}\sigma^{\mu\nu} S_{\mu\nu} \right]$$

Gradient expansion:

$$j^\mu = \int d^4p \mathscr{V}^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu,$$

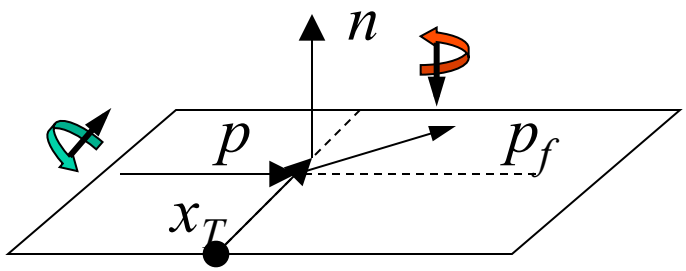
$$j_5^\mu = \int d^4p \mathscr{A}^\mu = n_5 u^\mu + \xi_5 \omega^\mu + \xi_{B5} B^\mu.$$

Pu, Gao, Liang, Wang & XNW, PRL 109 (2012) 232301

Quark Scattering with fixed L_y

Screened static potential model
With small angle approximation: $q_T \ll p$

λ: final quark spin



$$M_{\lambda,\lambda_i}(\vec{q}_T) = \frac{-ig}{2E}\bar{u}_\lambda(p_f)\not{A}(q_T)u_{\lambda_i}(p)$$

$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

$$\frac{d\sigma_\lambda}{d^2x_T} = C_T \int \frac{d^2q_T}{(2\pi)^2}\frac{d^2k_T}{(2\pi)^2} e^{i(\vec{k}_T - \vec{q}_T)\cdot\vec{x}_T}\,\frac{1}{2}\sum_{\lambda_i} M_{\lambda,\lambda_i}(\vec{q}_T)M^\dagger_{\lambda,\lambda_i}(\vec{k}_T)$$

$$\downarrow$$

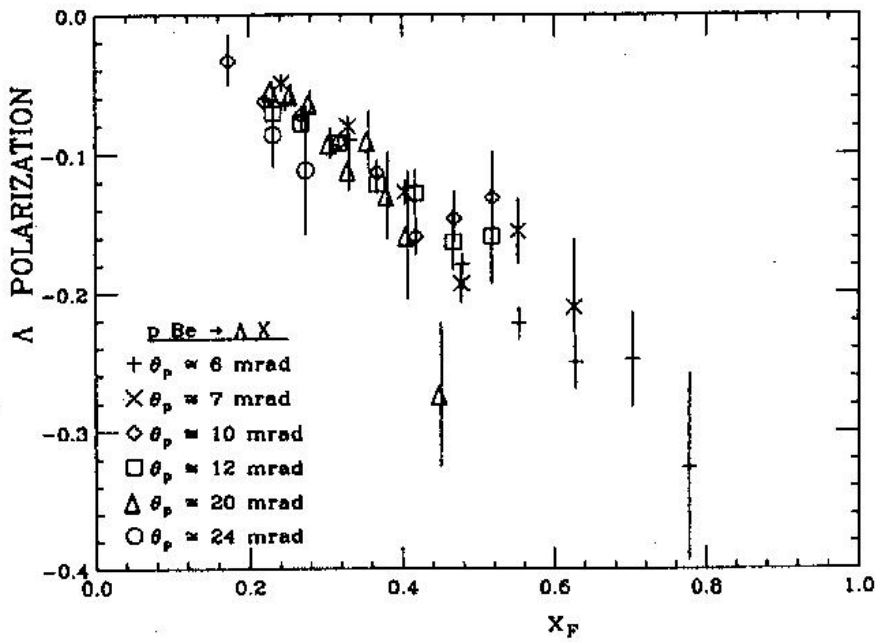
$$I_\lambda \simeq \frac{g^2}{2}A^0(q_T)A^0(k_T)\left[1 - i\lambda\frac{(\vec{q}_T - \vec{k}_T)\cdot(\vec{n}\times\vec{p})}{2E(E+m_q)}\right]$$

Polarization in unpolarized collisions

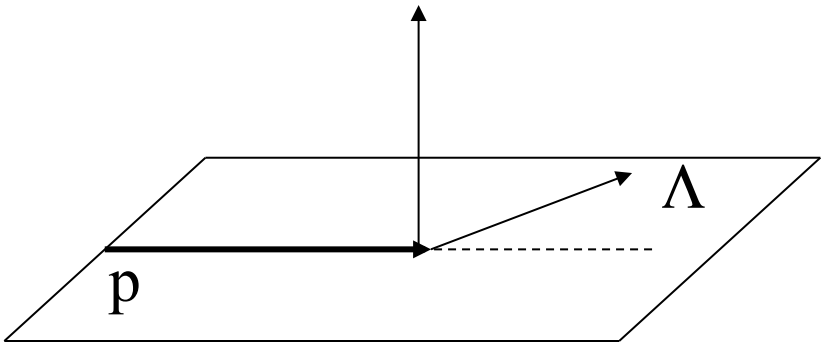
$$\frac{dN}{d\cos\theta} = \frac{1}{2}(1 + \alpha P_\Lambda \cos\theta)$$

$\alpha_{p\pi^-}$=0.642

θ : angle of decay products in rest fame relative to the polarization direction



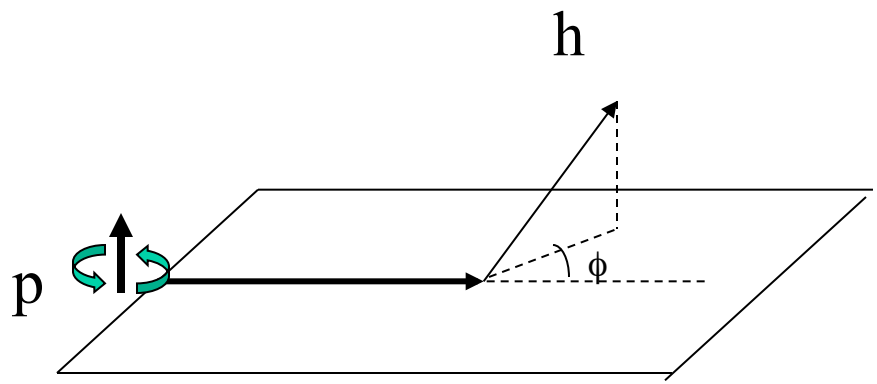
Transverse polarized hyperons in unpolarized p+p, p+A collisions.



Hyperon Polarization in p+p, p+A

- P_{Λ},P_{Ξ}, $P_{anti\text{-}\Xi}$, <0, P_{Σ} , $P_{anti\text{-}\Sigma}$ >0.
- P=0 for anti-Λ, Ω and anti-Ω.
- increase with x_f and p_T
- Weak A-dependence in p+A collisions.
- Measured in A+A at AGS: vanishes

Connection to single-spin asymmetry and spin-orbital angular momentum coupling



Constituent Quark Recombination

Polarization of quarks

Polarization of hyperons:

Constituent quark model:

(l=0, color anti-symmetric)

$$\left|\Lambda^{\uparrow}\right\rangle = \sqrt{1/2}\left(u^{\uparrow}d^{\downarrow} - u^{\downarrow}d^{\uparrow}\right)s^{\uparrow}$$

$$\left|\Xi^{0\uparrow}\right\rangle = \sqrt{1/6}\left[2s^{\uparrow}s^{\uparrow}u^{\downarrow} - \left(s^{\uparrow}s^{\downarrow} + s^{\downarrow}s^{\uparrow}\right)u^{\uparrow}\right]$$

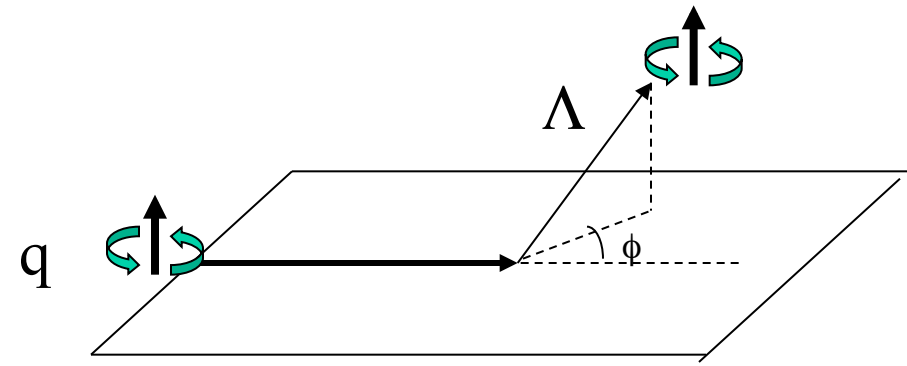
$$\left|\Sigma^{0\uparrow}\right\rangle = \sqrt{1/6}\left[2u^{\uparrow}d^{\uparrow}s^{\downarrow} - \left(u^{\uparrow}d^{\downarrow} + u^{\downarrow}d^{\uparrow}\right)s^{\uparrow}\right]$$

$$P_{\Lambda} = P_s \qquad P_{\Sigma} = \frac{4P_q - P_s - 3P_sP_q^2}{3 - 4P_qP_s + P_q^2} \qquad P_{\Xi} = \frac{4P_s - P_q - 3P_qP_s^2}{3 - 4P_qP_s + P_s^2}$$

$$P_{\Omega} = P_s\frac{5 + P_s^2}{3(1 + P_s^2)}$$

Fragmentation Scenario

Model:
Only first rank hadron (containing q with the same initial polarization) is polarized



$$e^+e^- \rightarrow Z^0 \rightarrow q\bar{q} \rightarrow \Lambda^{\rightarrow} + X$$

$$P_\Lambda = P_s \frac{n_s}{n_s + 2f_s}$$

$$P_\Sigma = \left(4 f_s P_q - n_s P_s\right) \frac{1}{3(n_s + 2f_s)}$$

$$P_\Omega = P_s / 3$$

$$P_\Xi = \left(4 n_s P_s - f_q P_q\right) \frac{1}{3(2n_s + f_s)}$$

n_s, f_s strange quark abundance relative to u, d in QGP and fragmentation