(Some) Theory perspective on correlations and fluctuations in small systems

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Why small systems?

p+p and p(d,..)+A:

used to be baselines for studying novel A+A physics ideas

$\rightarrow$ **now became testing grounds for how far A+A models apply**

heavy ion theory:

- **calculations for various theoretical limits, applicability not clear** *a priori*

  e.g., hydrodynamics, kinetic theory, classical field theory

- **doesn't get simpler for p+A (in most cases)**

  still many degrees of freedom, and larger fluctuations

small systems mean a new knob to vary (esp. geometry) $\rightarrow$ extra constraints

Hydrodynamics

Long-wavelength, long-timescale limit, (in general*) near local equilibrium.

Conservation laws: $\partial_\mu T^{\mu\nu}(x) = 0$, $\partial_\mu N_c^\mu(x) = 0$ **(**$c = B, S, Q, ...$**)**

+ **equation of state** $p(e, \{n_c\})$, $T(e, \{n_c\})$

+ **transport coefficients** $\eta(T)$, $\zeta(T)$, $\kappa(T)$, $D_{c,b}(T)$, ...

+ **for 2nd-order theories: relaxation times** $\tau_\eta(T)$, $\tau_\zeta(T)$, $\tau_\kappa(T)$, ...

that enter in dynamical eqns for shear stress, bulk pressure, heat flow...

* isotropized conformal systems can have $e = 3p$ out of equilibrium

In heavy-ion physics applications also need:

- **initial conditions**
- **stopping condition (typically a** $T = const$ **hypersurface)**
- **model for converting hydro fields to particles (usually Cooper-Frye)**
- **optional: late stage hadron kinetic theory ("hybrid" hydro+transport)**

Only combinations of all these can be tested against data.

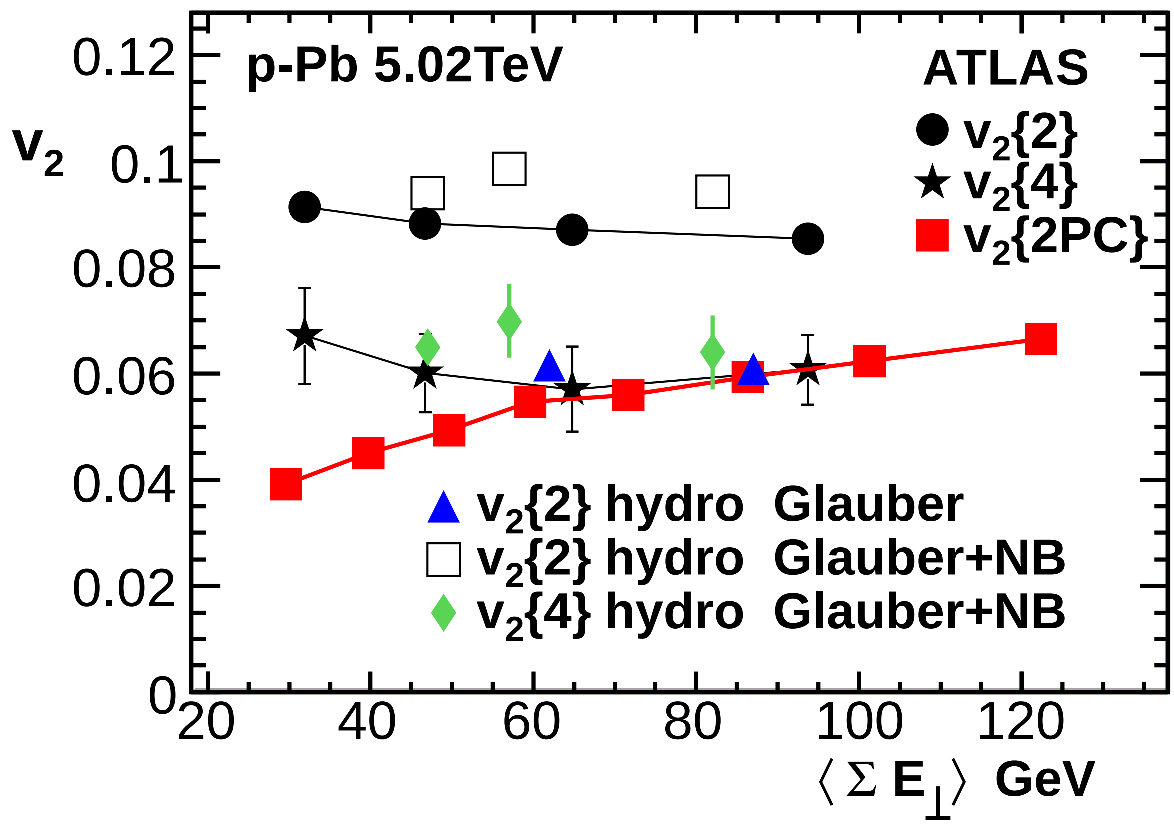
SIGNS OF COLLECTIVITY IN SMALL SYSTEMS

Bozek @ QM2015

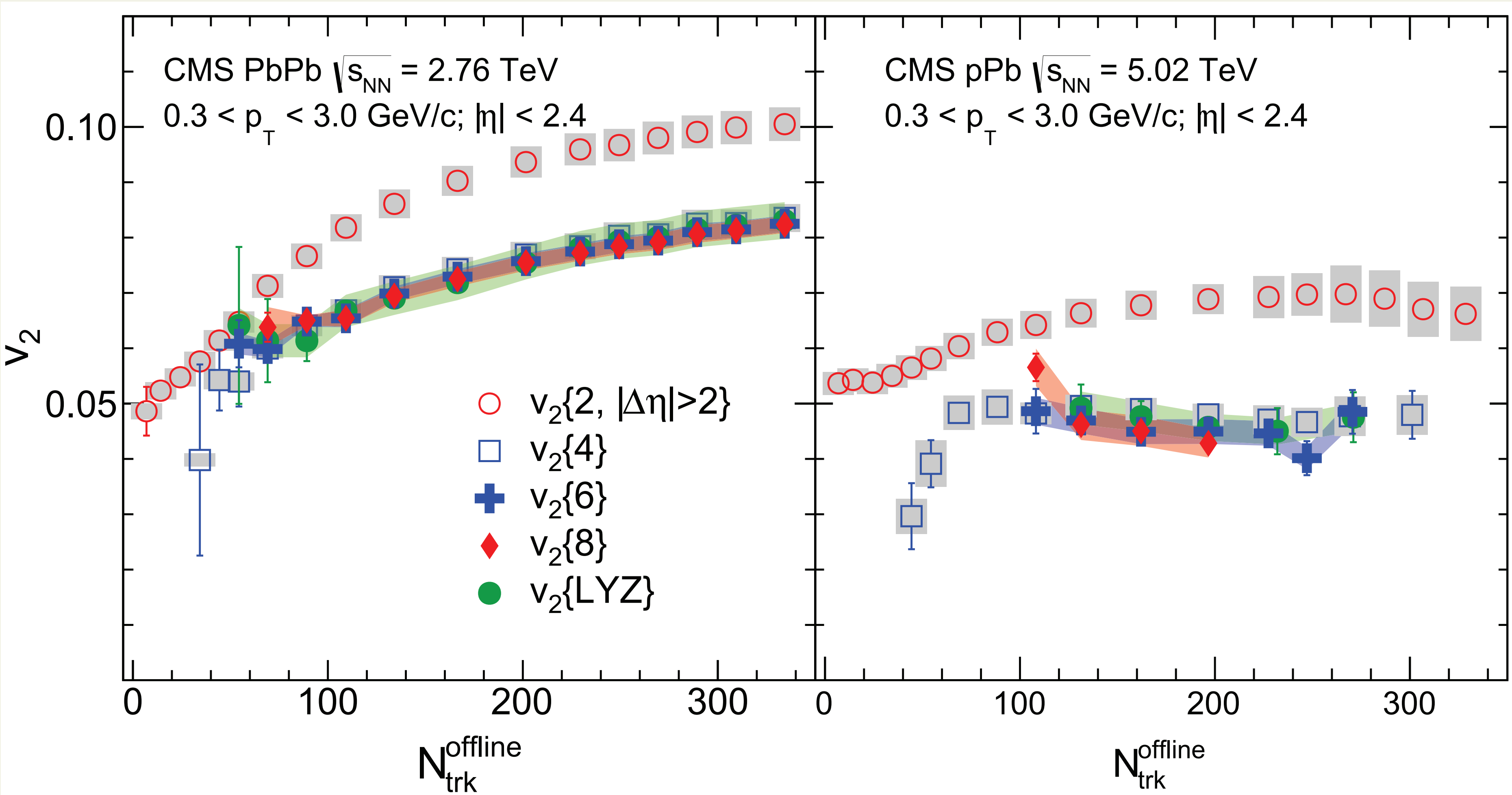
1. **Elliptic and triangular flow** $v_n(p_T, ...) = \langle \cos n(\phi - \psi_n) \rangle_{p_T,...}$, $n=2,3$
2. **Flow from higher cumulants** $v_n\{2\} > v_n\{4\} \approx v_n\{6\} \approx v_n\{8\}$
   all particles flow
3. **Hierarchy of v_2 and v_3 in p-A, d-A, He-A**
   collective response to geometry (final state effect)
4. **$k_\perp$ dependence of HBT radii**
   $V_{n\Delta}(a,b) \approx \sqrt{V_{n\Delta}(a,a)V_{n\Delta}(b,b)}$
5. **Factorization at intermediate $p_\perp$ and large $\Delta\eta$**
   particles at intermediate $p_\perp$, large η, correlated to geometry
6. **Mass splitting of v_2**
   $\approx$ **boosted nearly thermal sources**
7. Mass hierarchy of spectra $(< p_\perp >)$

- **Density driven collective expansion**
- **Hydrodynamics describes data for $p_\perp < 1.5$GeV**

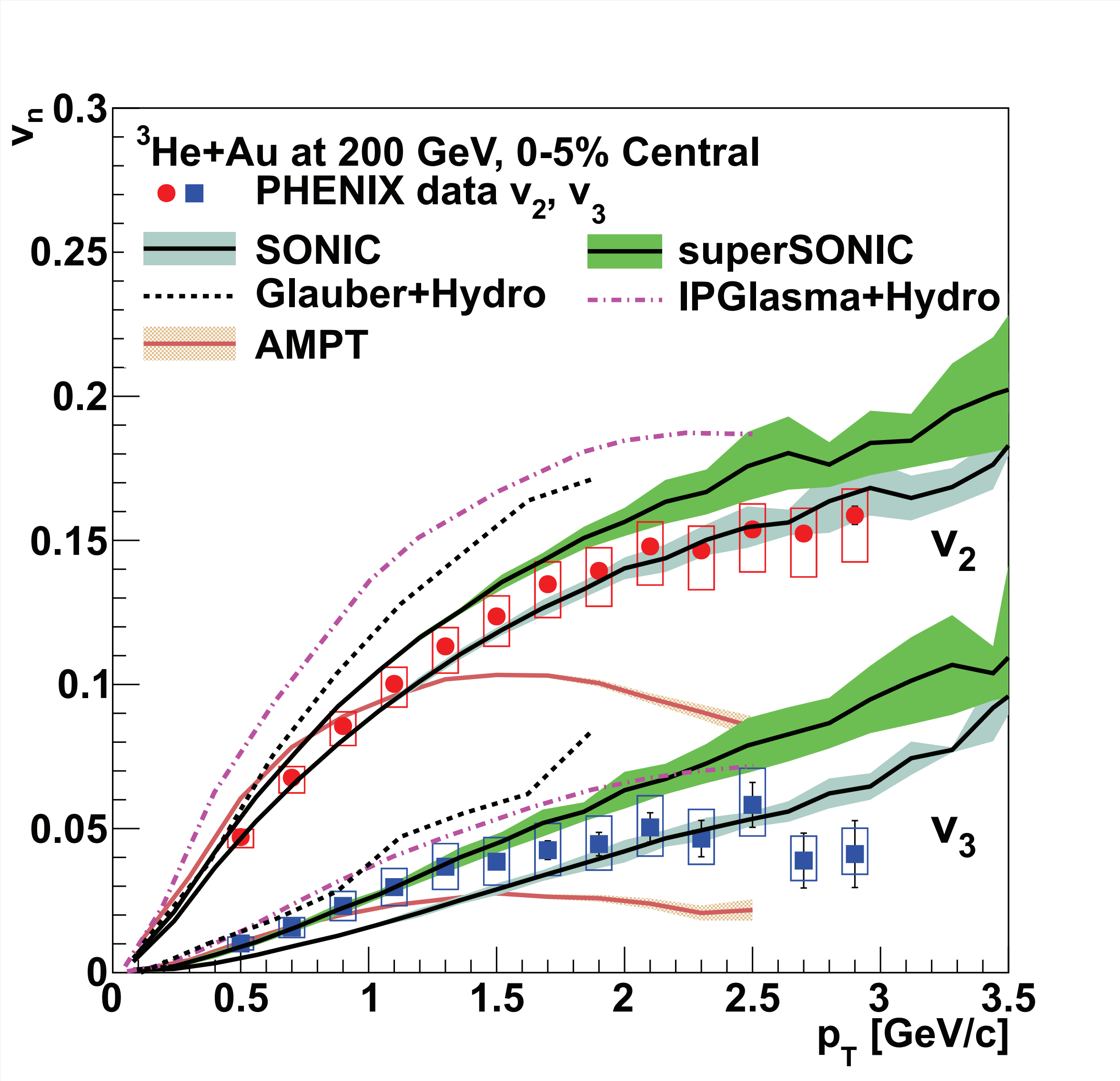
Bozek & Broniowski, PRC88 ('13): **shear & bulk viscous hydro, GLISSANDO initconds**



CMS, PRL115 ('15): **multiparticle azimuthal correlations in pPb**



PHENIX, PRL ('15): ^{3}**He+Au**



rather different models do reasonably well up to $p_T = 1 - 1.5$ **GeV**

Where is the energy loss??

On high-pT physics in small systems, see Chanwook Park's talk this afternoon...

If medium is indeed hydrodynamic (opaque), should there be jet energy loss?

central Pb+Pb | **vs** | **p+Pb**

$\frac{dN_h}{d\eta} \sim 2400 \;\; \propto \;\; 30 \times \text{p+Pb}$ $\qquad \sim 80$

RMS radius ~ 3 **fm** $\;\; \propto \;\; 3 \times \text{p+Pb}$ $\qquad \sim 1$ **fm**

"density" $\sim \langle s \rangle \sim \frac{dN_h}{d\eta}\frac{1}{R^2} \;\; \propto \;\; 3 \times \text{p+Pb}$

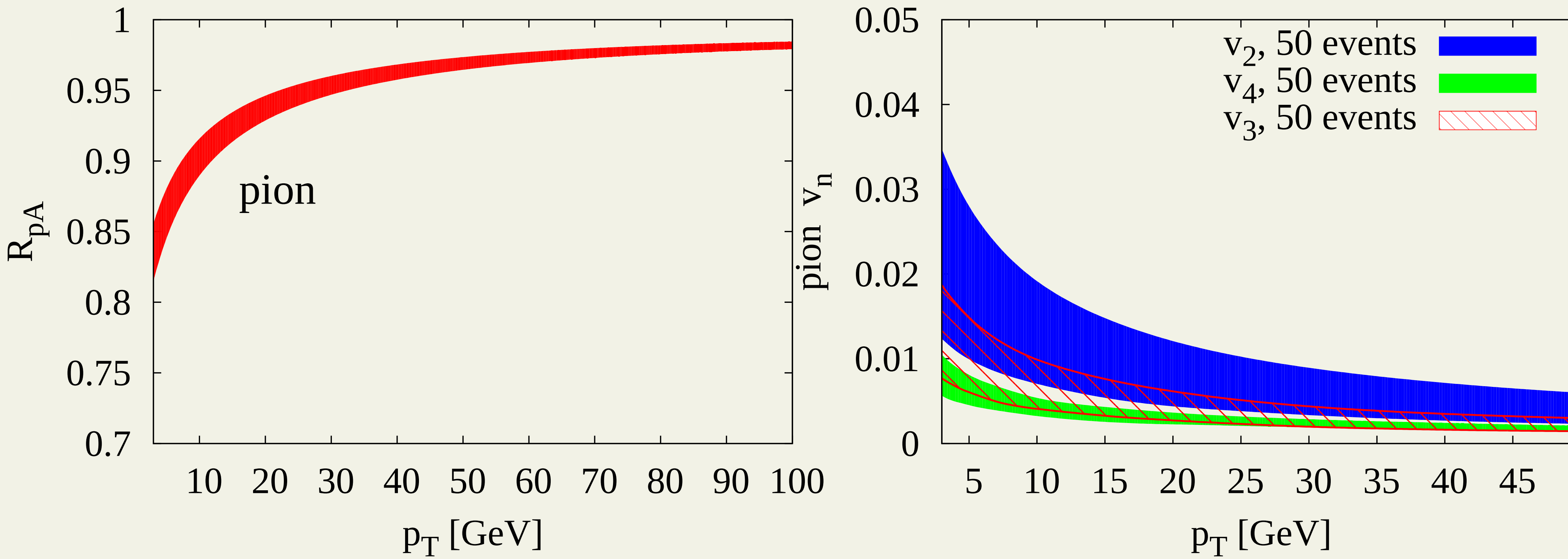
opacity $\sim < s > \cdot R \;\; \propto \;\; 10 \times \text{p+Pb}$

but this ignores density fluctuations, which are larger in the smaller system

e.g., GLV energy loss for 3.4% most central p+Pb at $\sqrt{s_{NN}} = 5.02$ TeV

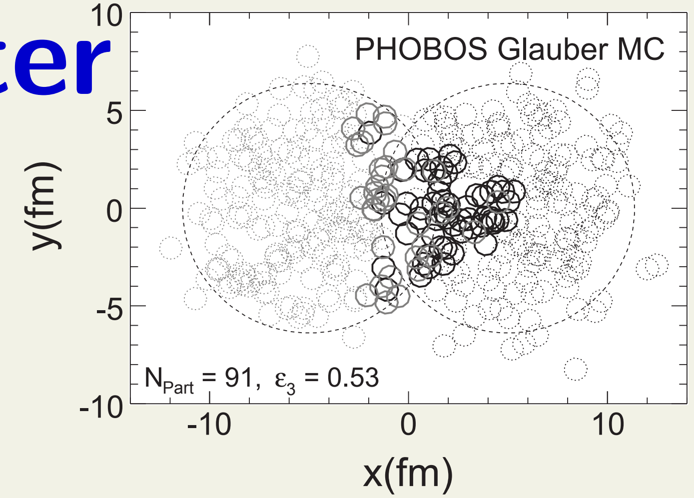
- medium: 2+1D ideal hydro with fluctuating GLISSANDO initconds
- covariant E-loss, with $(1 - \vec{v}_F \vec{v}_{jet})$ jet-flow coupling DM & Sun, NPA932 ('14)

DM & Sun @ QM2015:



expect ballpark $\sim 5\%$ suppression and $\sim 1\%$ v_2 for pions, on average

Initial conditions matter



common theme: density/shape fluctuations
due to random nucleon positions in nucleus

Alver & Roland, PRC81 ('10)

MC-Glauber, GLISSANDO: postulate transverse profile based on number of wounded nucleons/binary collisions

AMPT: color flux tubes based on random nucleon position, dissolved later to quarks ("string melting")

EPOS: based on "pomerons" produced in initial parton scatterings

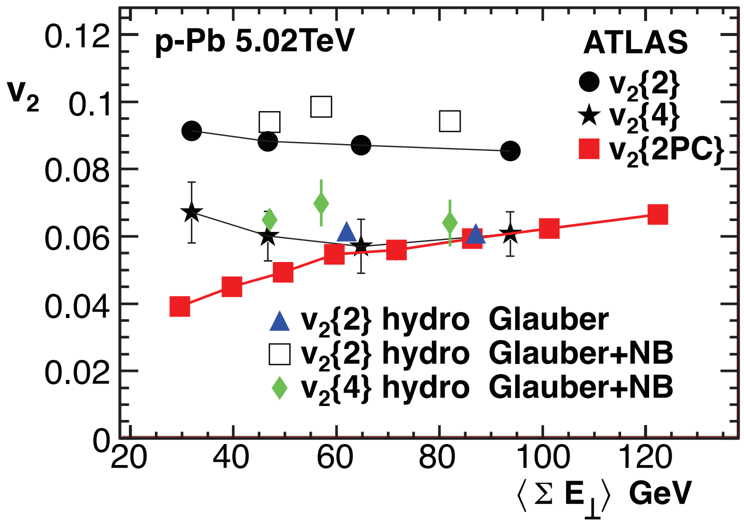
MC-KLN, MCrcBK: gluon saturation in low-x limit of QCD with $Q^2_{s(0)} \sim N_{part}$ from nucleons and k_T factorization

IP-Glasma: classical Yang-Mills evolution for low-x gluons with random color sources based on nucleon positions $\langle \rho^a(x_T)\rho^b(y_T)\rangle \sim Q_s^2(x_T)\delta^{ab}\delta^2(x_T - y_T)$

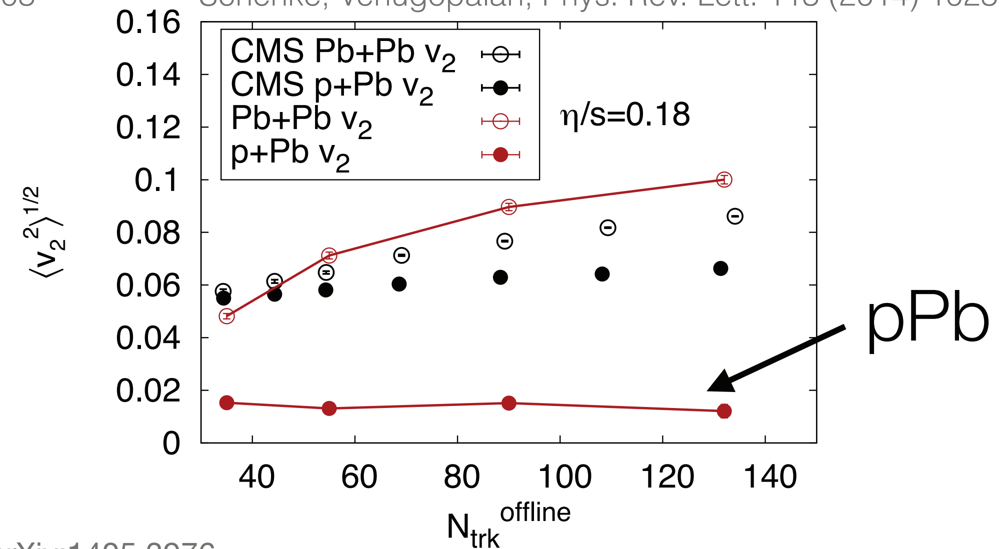
Trento: phenomenological model that interpolates between Glauber and saturation profiles

Different initial states: very different results

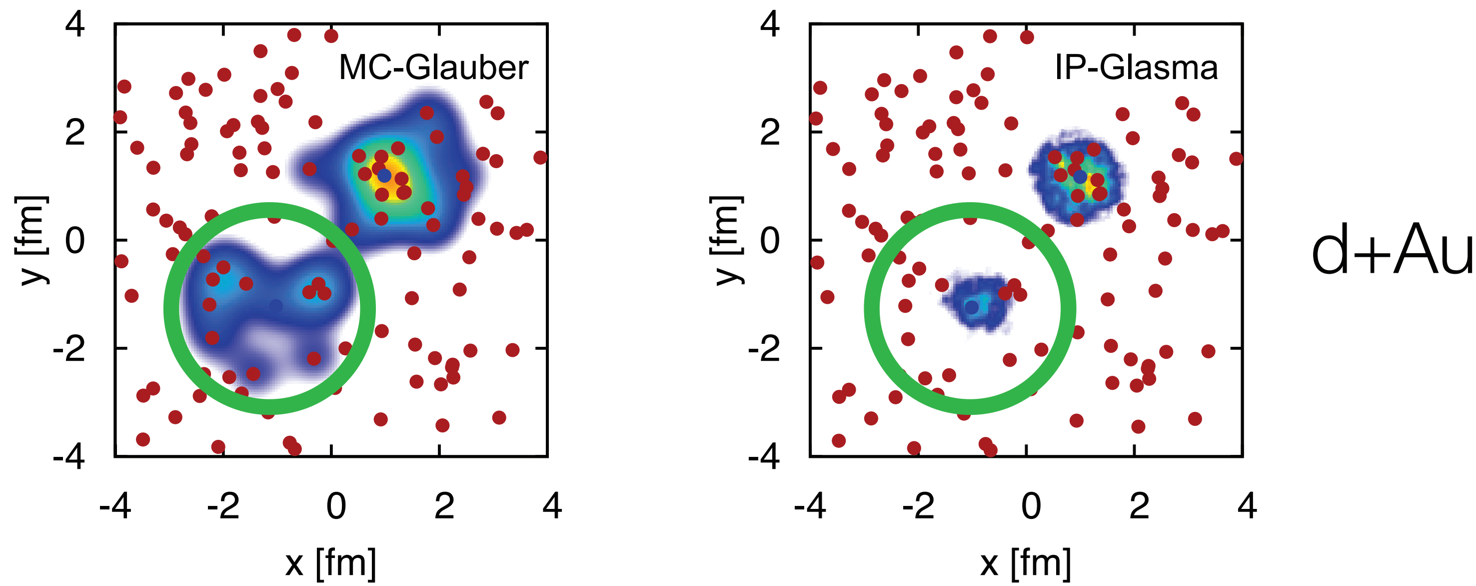
Bozek, Broniowski, Phys.Rev. C88 (2013) 014903



Schenke, Venugopalan, Phys. Rev. Lett. 113 (2014) 102301



see also: Kozlov, Luzum, Denicol, Jeon, Gale, arXiv:1405.3976



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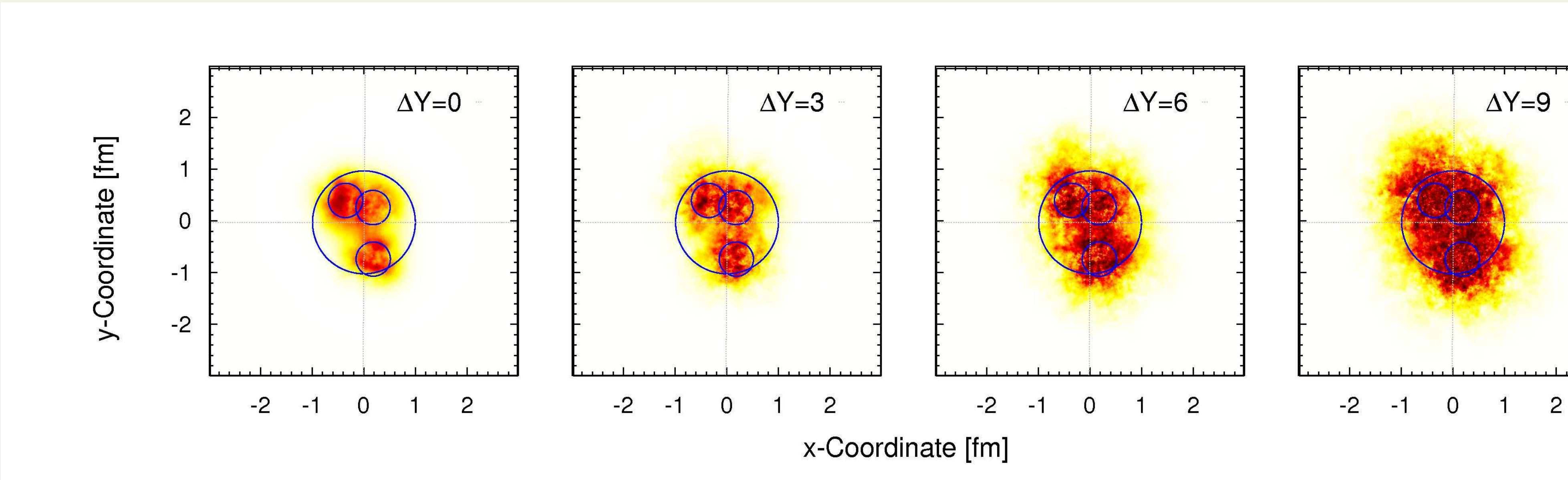
one idea: proton shape from low-x evolution (JIMWLK), with 3 constituent quark seeds at high x Schlichtig & Schenke, PLB739 ('14)



$\rightarrow$ "fatter" and anisotropic transverse profile, for single proton

interesting if such sub-fermi structures matter in hydro, a supposedly long-wavelength theory

Kinetic theory

Incoherent, particle limit of QCD (not very soft momenta).

(typically on-shell) phase-space density $f(x, \vec{p}) \equiv \frac{dN(\vec{x},\vec{p},t)}{d^3x d^3p}$

transport equation:

$$p^\mu \partial_\mu f_i(x,p) = C^i_{2\to 2}[\{f_j\}](x,p) + C^i_{2\leftrightarrow 3}[\{f_j\}](x,p) + \cdots$$

with, e.g.,

$$C^i_{2\to 2} = \frac{1}{2}\sum_{jkl}\int_{234}(f_3^k f_4^l - f_1^i f_2^j) W^{ij\to kl}_{12\to 34} \qquad \left(\int_j \equiv \int \frac{d^3p_j}{2E_j}\,, \quad f_a^k \equiv f^k(x,p_a)\right)$$

collision terms can be obtained from perturbative QCD matrix elements

fully causal and stable, thermalizes (in box)
$\to ggg \leftrightarrow gg$ **has a very important role in thermalization** Greiner & Xu, PRC71 ('05)

near hydro limit, transport coeffs & relaxation times: $\eta \approx 1.2T/\sigma$, $\tau_\pi \approx 1.2\lambda_{tr}$

one full-fledged example: A Multi-Phase Transport (AMPT)

Lin, Ko et al, PRC72 ('05)

also quite successful with observables

$$\begin{aligned} \textbf{AMPT} \approx & \textbf{ Lund string model (HIJING)} \to \textbf{geometry fluctuations} \\ & + 2 \to 2 \textbf{ parton cascade (ZPC)} \\ & + \textbf{hadron transport (ART)} \end{aligned}$$

version with "string melting":

- **strings converted to quarks/antiquarks**
- **hadronization via coalescence**

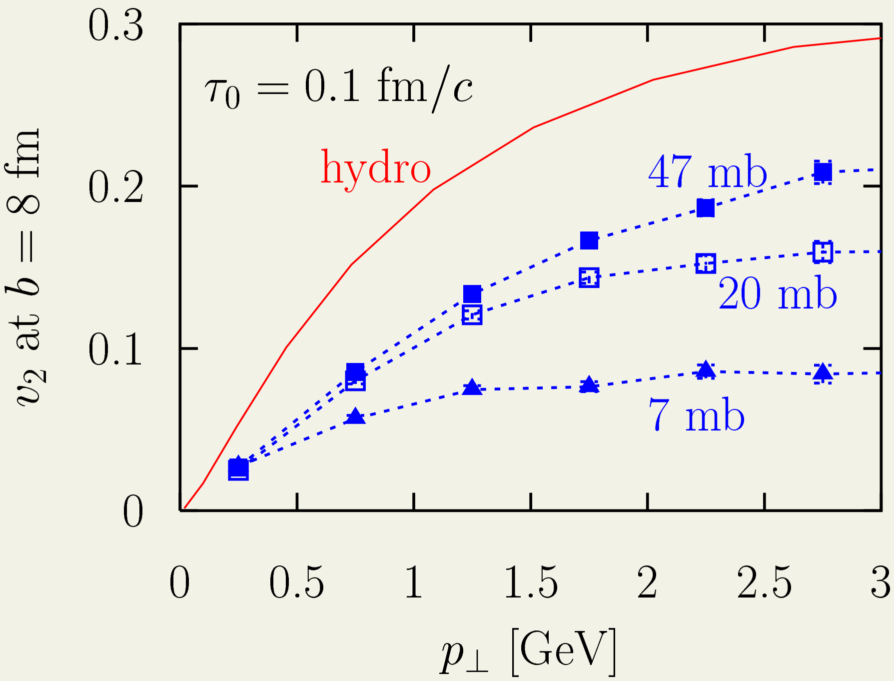
$\to$ **higher parton density, enhanced collectivity**

HERE: especially d+Au, p+Pb

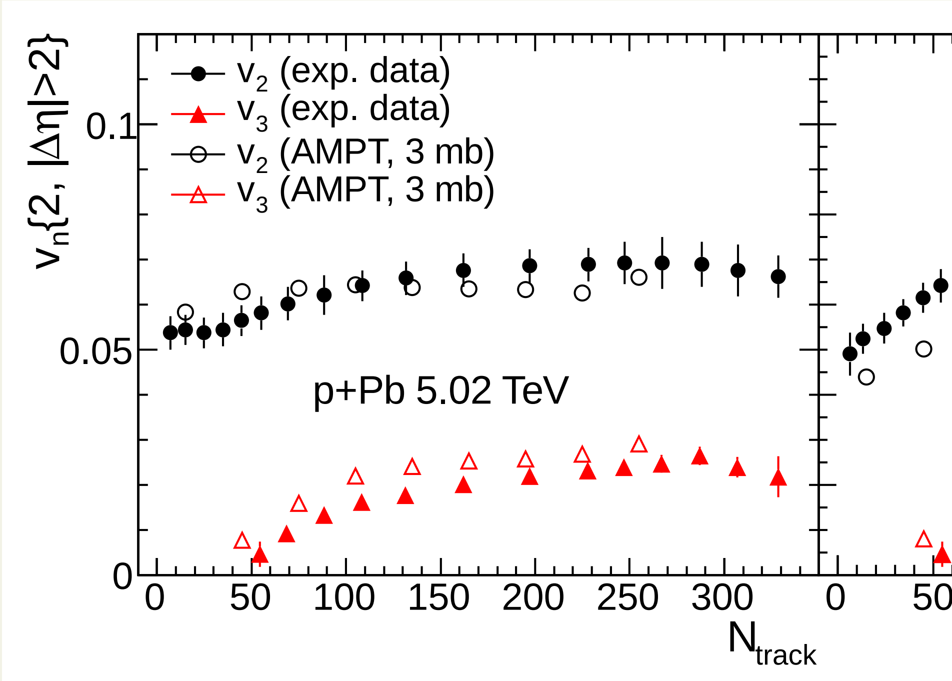
Anisotropic escape - AMPT$\neq$hydro

- **high opacity limit of transport is indeed hydro (can be hard to reach)**

DM & Huovinen, PRL94 ('05)



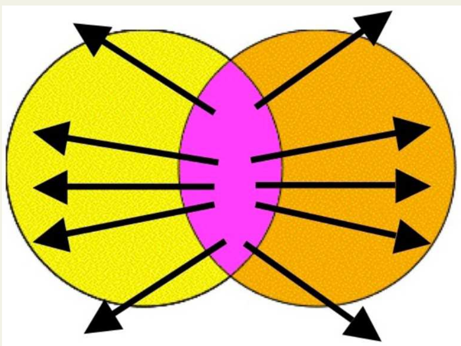
Bzdak & Ma, PRL 113 ('14): **3 mb**



- **but at more modest opacities $\langle N_{coll}\rangle \sim 5$**

  $\rightarrow$ **finite, anisotropic chance to escape collision zone**

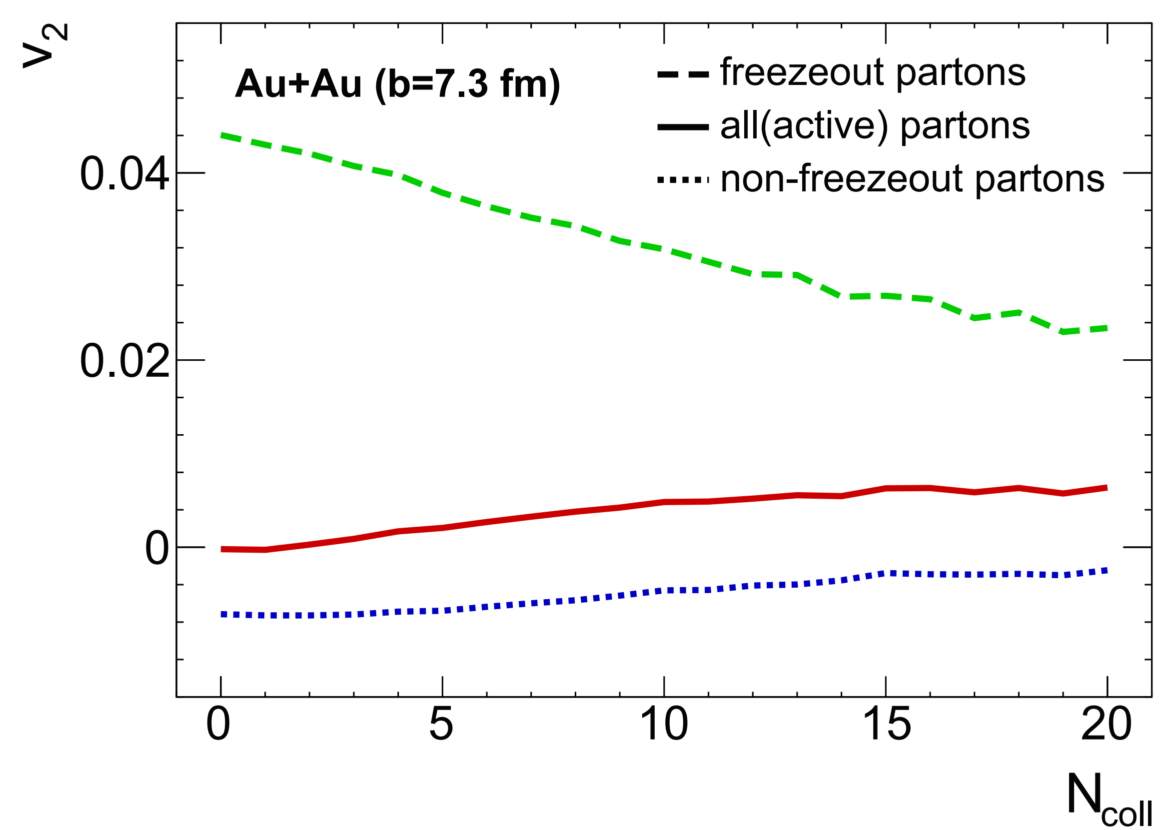
He et al, PLB 753 ('16)



AMPT = string melting + parton cascade + coalescence + hadron cascade

In AMPT, still interacting part of system carries surprisingly small v_n

He et al, PLB753 ('16):



$$\frac{d\sigma_{qq}}{dt} \propto \frac{1}{(t - \mu_D^2)^2}$$

$$\mu_D^2 = 0.2 \text{ GeV}^2$$

$$\sigma_{TOT} = \frac{9\pi\alpha_s^2}{2\mu_D^2} = 3 \text{ mb}$$

$$\Rightarrow \langle N_{coll} \rangle \approx 4.5$$

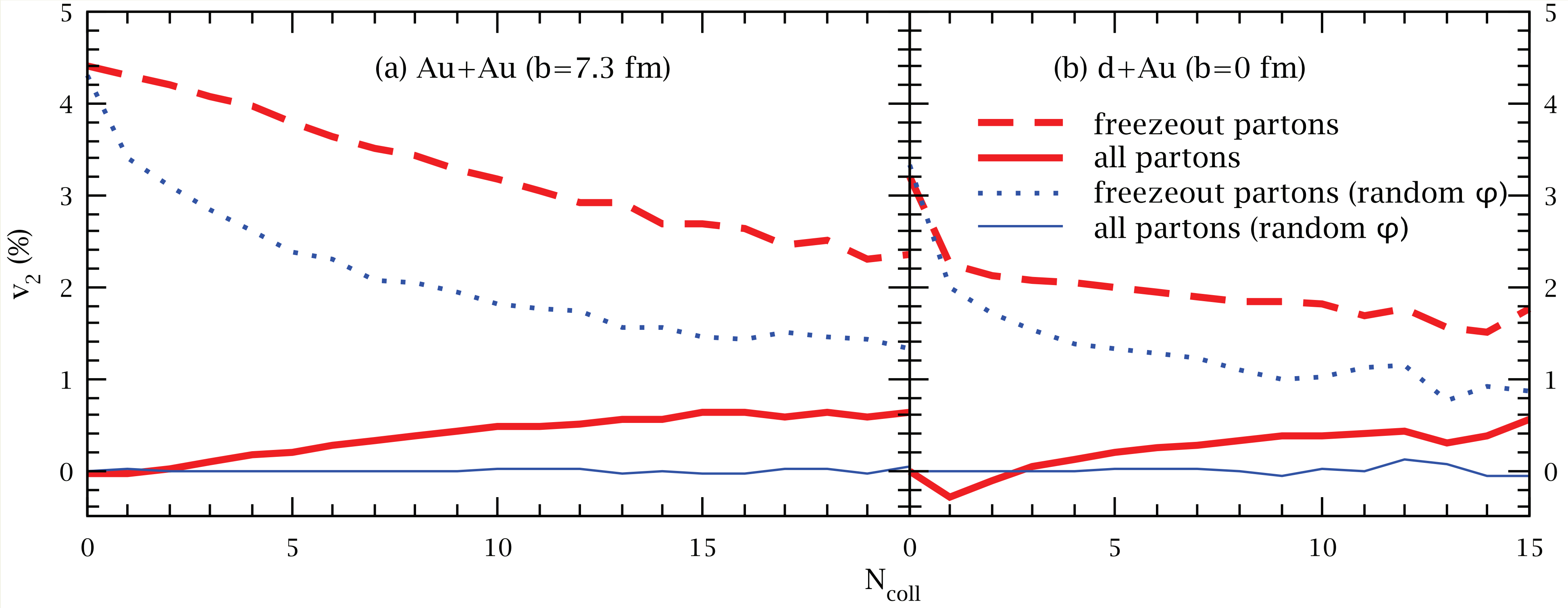
solid: all partons right after their N_{coll}-th collision

dashed: frozen out partons that do not interact after the N_{coll}-th collision

dotted: still interacting part - will have more than N_{coll} collisions in future

This is due to the low opacities in AMPT, and also holds for d+Au.

He et al, PLB753 ('16): **Au+Au, $\langle N_{coll} \rangle = 4.5$** **central d+Au, $\langle N_{coll} \rangle = 1.2$**



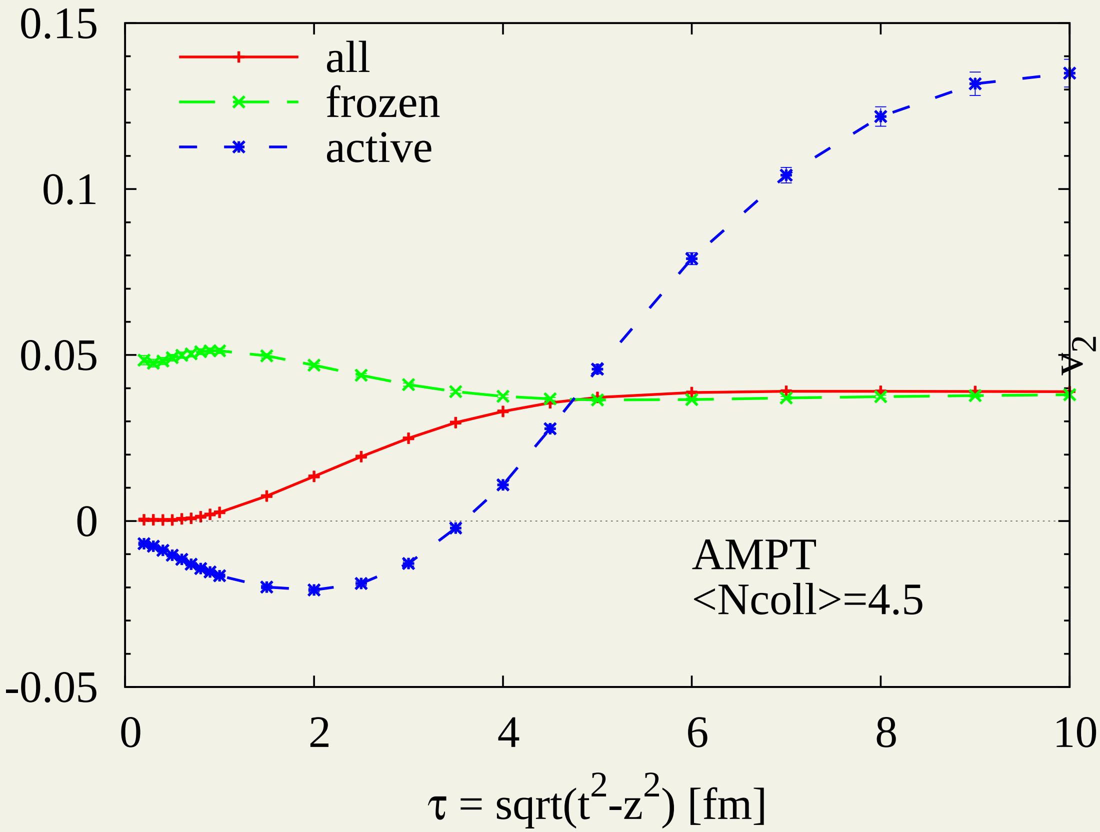
→ **large $\mathcal{O}(1)$ change in v_2 near the very last collision**

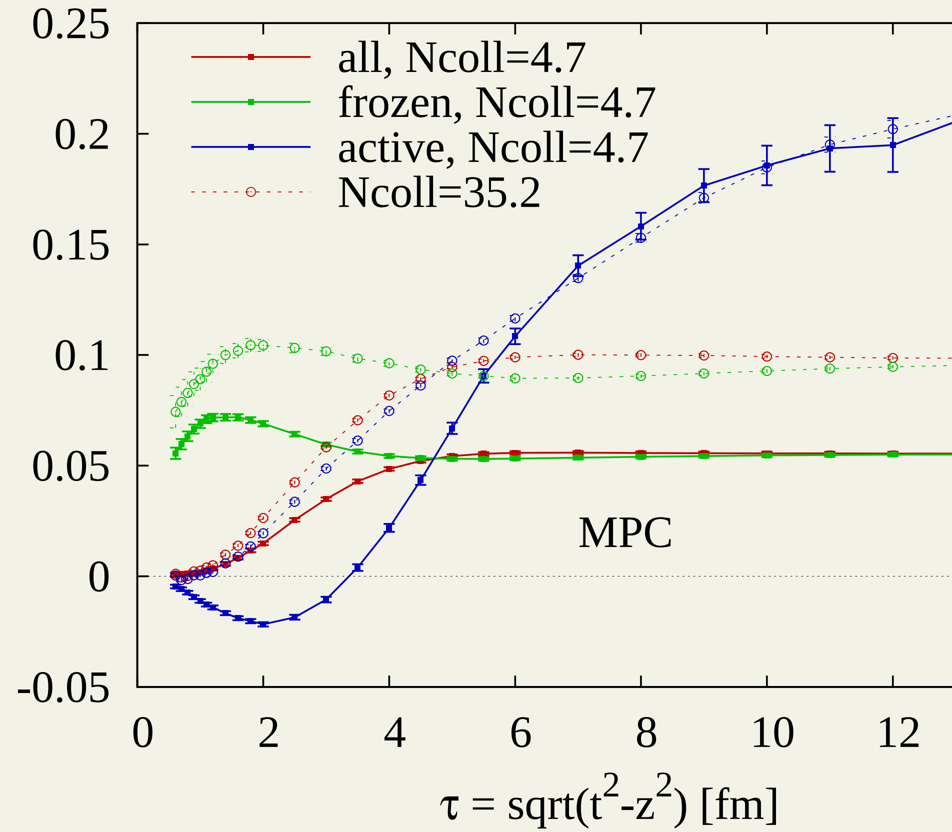
→ **effect does not come in the microscopic collision itself because it survives randomization of outgoing azimuths in each collision**

- anisotropic escape is generic, and also manifest vs time (instead of Ncoll)

e.g., MPC transport code, with smooth Au+Au profile, isotropic $2 \to 2$

DM ('15): **AMPT $v_2(\tau)$** **MPC $v_2(\tau)$**

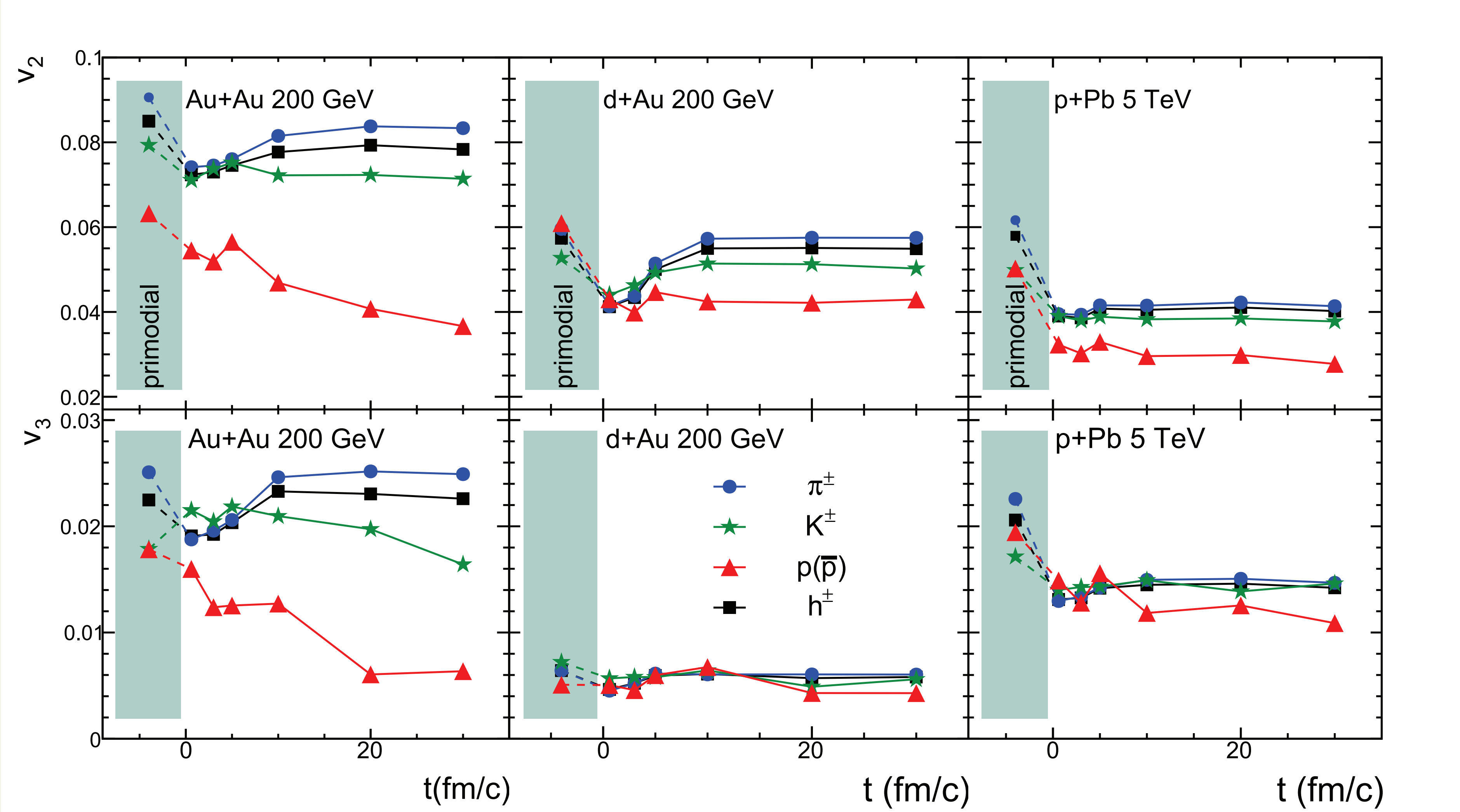




interacting component dominates v_2 only at very high opacity ~ 35

The mass effect in $v_n(p_T)$ also comes out, due to coalescence and hadronic rescatterings. (Rescatterings mattered already in RQMD Teaney et al, nucl-th/0110037)

Li et al, 1604.07387v2: **Au+Au** **d+Au** **p+Pb**



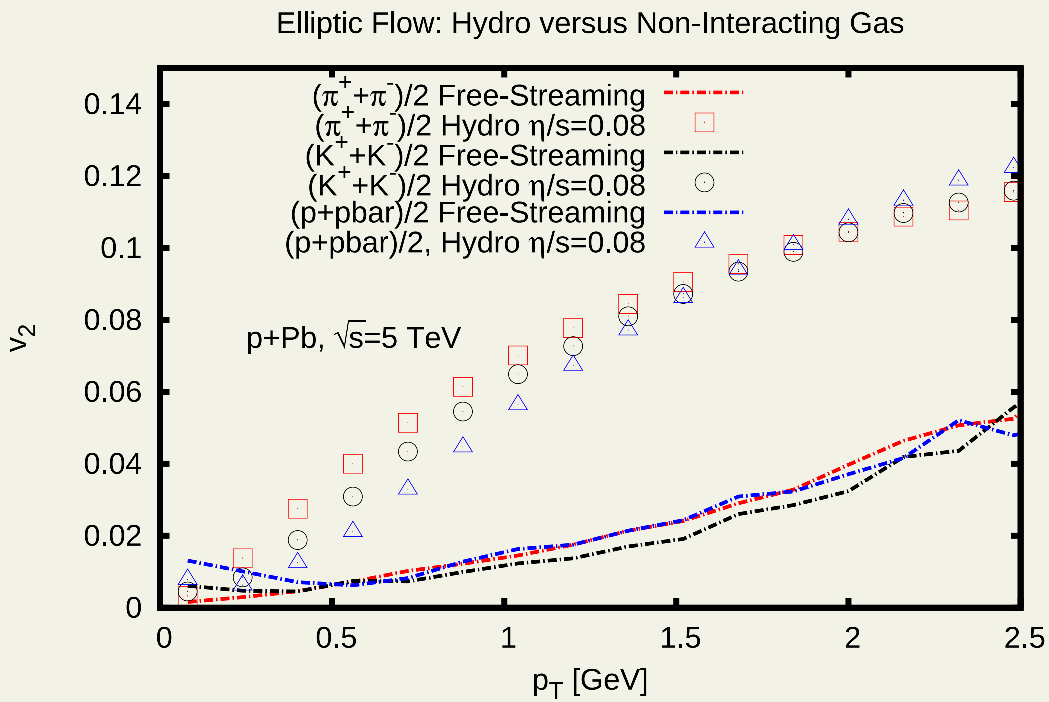
Coalescence is also how AMPT gets large hadronic v_n with small σ. But rescaling v_2/n_q vs p_T/n_q does <u>not</u> give you the parton flow Chen & Ko, PRC73 ('06)

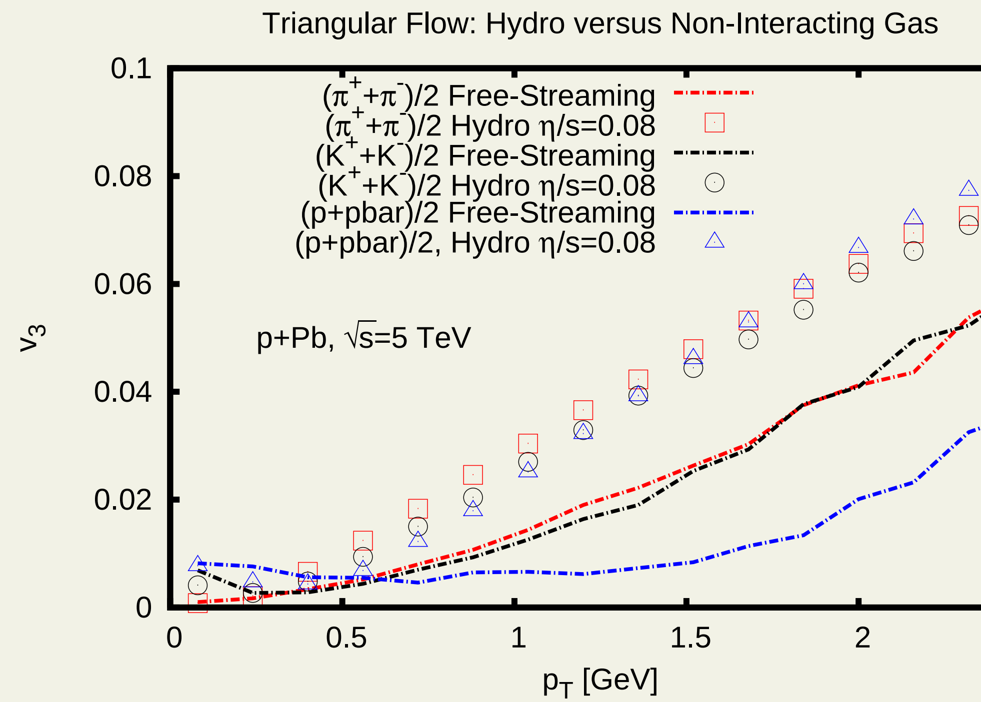
Flow without hydro(?)

observation of flow may not inevitably mean hydrodynamics

Romatschke, EPJC75 ('15): **significant flow from "free streaming"**

v_2 in p+Pb **v_3 in p+Pb**





how come v_n is nonzero for free streaming?

"free streaming" = free streaming + sudden momentum rearrangement on a spacetime hypersurface

- for any transport solution $f(x,t)$, can construct $T^{\mu\nu}(x) = \int \frac{d^3p}{E} p^\mu p^\nu f$

  and find energy & momentum conserved: $\partial_\mu T^{\mu\nu}(x) = 0$

- also can define $T = const$ Cooper-Frye hypersurfaces, and get particles:

$$E\frac{dN}{d^3p} = p_\mu d\sigma^\mu(x) f(x,\vec{p})$$

- but guessing the correct f from hydro fields is nontrivial $(f = f_{eq} + \delta f)$

  ALL hydro simulations face this 'δf problem' when comparing to data

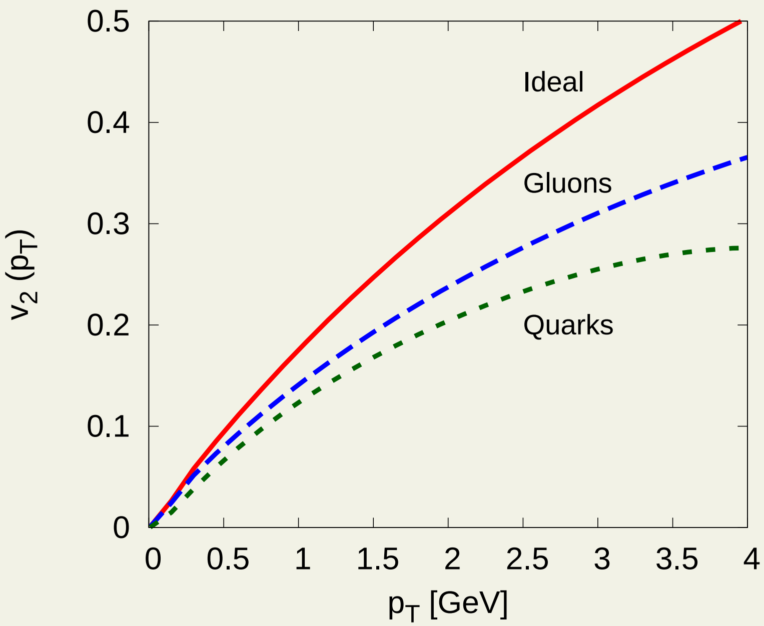
  unless the system is quickly driven towards a fixed point, such as local thermal equilibrium, the answer depends on the evolution history

  δf still depends on the microscopic dynamics even near equilibrium

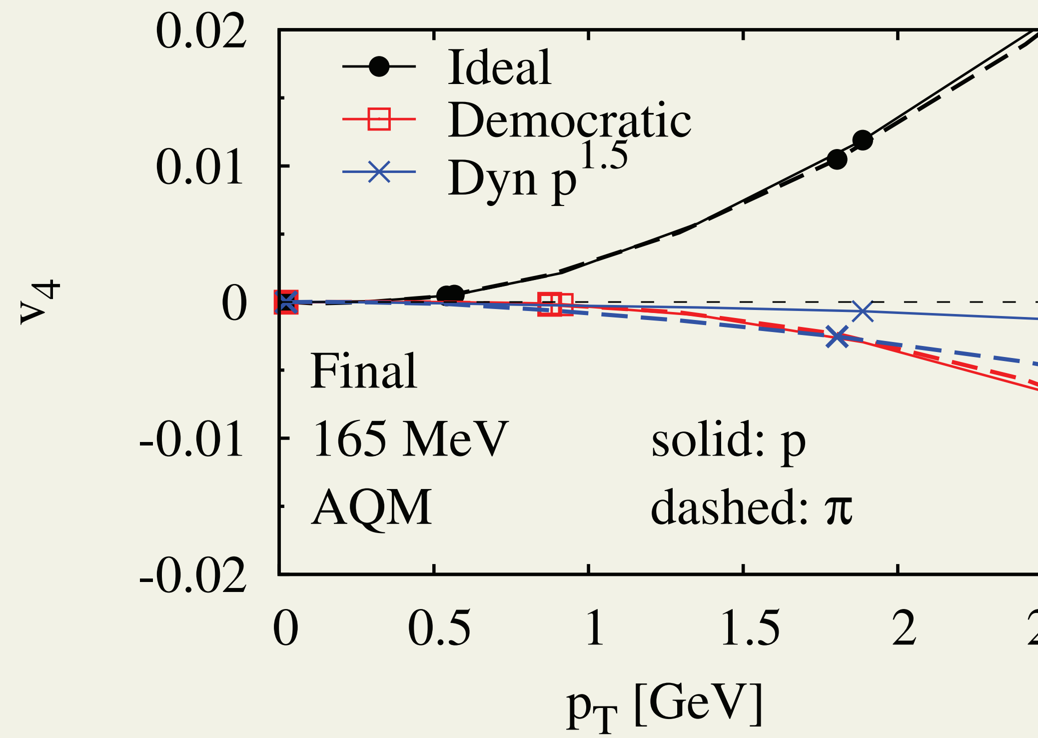
DM & Wolff, 1405:7850; Teaney, Moore, Dusling, PRC81 ('10); El et al, EPJA48 ('12) ...

self-consistently determined viscous δf corrections are smaller for species that interact more frequently (closer to equilibrium), and tend to have weaker $\delta f / f_{eq} \sim p^{\sim 1.5}$ dependence than the commonly used quadratic (Grad) form

Dusling et al, PRC81 ('10): **quark/gluon $v_2(p_T)$**



DM & Wolff, JPCS 535 ('14): **π/p $v_4(p_T)$**



matters a lot for identified particle $v_6(p_T)$ as well

v_n from wave physics

hydro used at sub-fermi wavelengths - should we worry about quantum mechanics/wave effects?

Back of the envelope: QM + uncertainty relation ($\hbar = 1$)

$$v_2 \sim \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}\,, \quad \langle p_x^2 \rangle \sim \frac{1}{R_x^2}\,, \quad \langle p_y^2 \rangle \sim \frac{1}{R_y^2} \qquad \Rightarrow \qquad v_2 \sim \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2} = \varepsilon \quad (!)$$

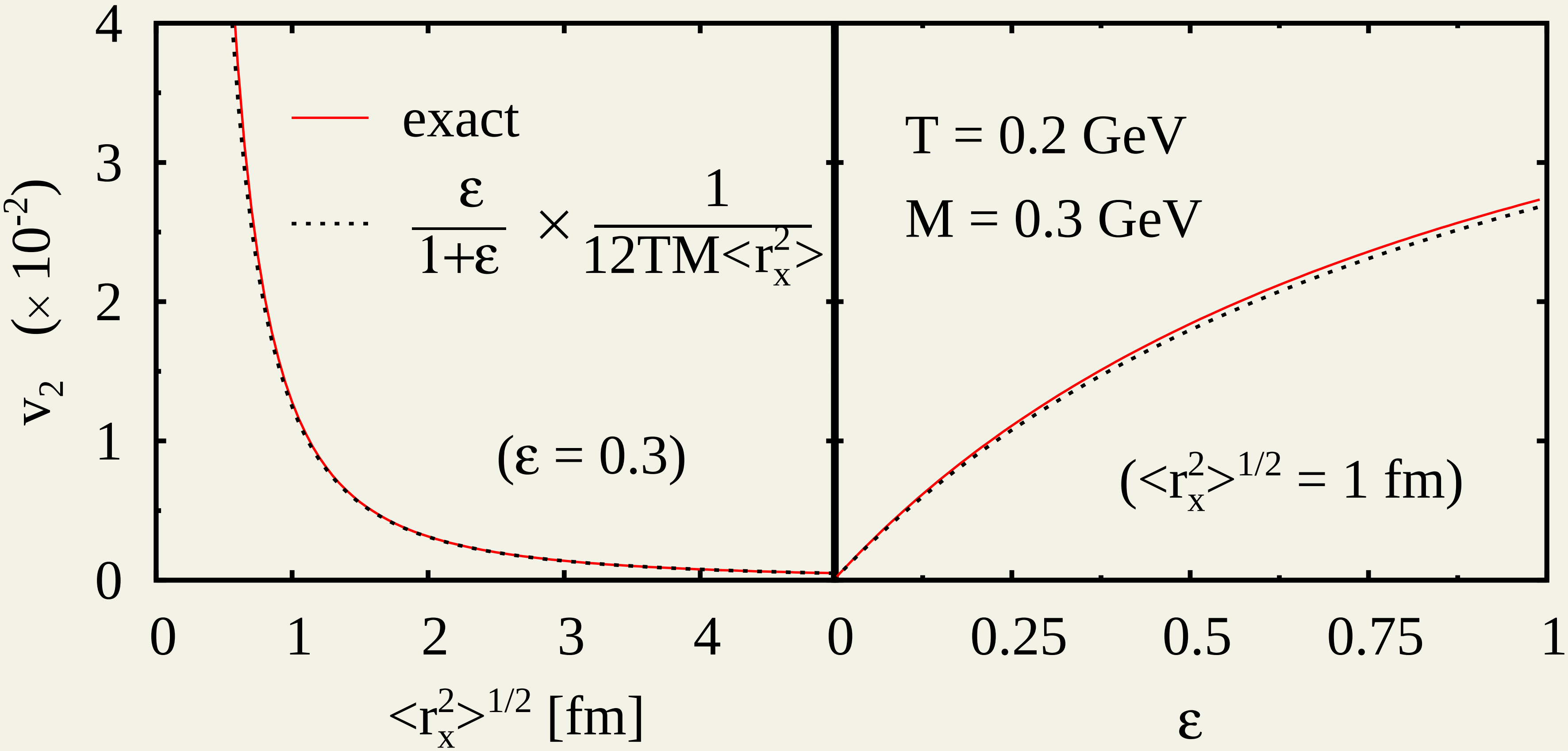
At finite T, excited states also constribute. In classical stat phys $(H = K+V)$:

$$\frac{dN}{d\mathbf{p}} = N \frac{\int d\mathbf{r}\, e^{-H_1(\mathbf{p},\mathbf{r})/T}}{\int d\mathbf{r}\, d\mathbf{p}\, e^{-H_1(\mathbf{p},\mathbf{r})/T}} = N \frac{e^{-K(\mathbf{p})/T}}{\int d\mathbf{p}\, e^{-K(\mathbf{p})/T}} = isotropic \qquad \Rightarrow \qquad v_n \equiv 0$$

But in QM, level spacing matters $\int d^3r e^{-H/T} \to \sum\limits_n |\psi_n(\mathbf{p})|^2 e^{-E_n/T}$**, and for Gaussian potential** DM, Wang & Greene, 1404.4119

$$v_2 \approx \frac{\hbar^2}{12 k_B T M \langle R_x^2 \rangle} \cdot \frac{\varepsilon}{1+\varepsilon} = \frac{\hbar^2}{12 p_{th}^2 \langle R_x^2 \rangle} \cdot \frac{\varepsilon}{1+\varepsilon} \neq 0$$

anisotropy vanishes for “hot” systems $v_2 \sim 1/T$



but expect on the order of 1% v_2 **in small systems (p+A)**

in comparision, for cold atomic gases $v_2 \sim \mathcal{O}(10^{-5})$ [^{6}Li, $\sim \mu$K, $r_x \sim 20\ \mu$m]

Main question: how to combine this with dynamics?

This is just a simple wave effect, so classical Yang-Mills may show it too.

Interestingly, $v_{2n} \neq 0$**, but there is no hydrodynamic flow anywhere:**

$$\mathcal{L} = \frac{i\hbar}{2}\left(\psi^*\dot{\psi} - \dot{\psi}^*\psi\right) - \frac{\hbar^2}{2M}(\nabla\psi^*)(\nabla\psi) - V(\mathbf{r},t)\psi^*\psi$$

so from Noether's theorem

$$\begin{aligned}
T^{00} &= \frac{\hbar^2}{2M}(\nabla\psi^*)(\nabla\psi) + V(\mathbf{r},t)\psi^*\psi && (1)\\
T^{0i} &= \frac{i\hbar}{2}\left(\psi\nabla_i\psi^* - \psi^*\nabla_i\psi\right) && (2)\\
T^{i0} &= \frac{i\hbar}{2M}\left(\frac{\hbar^2}{2M}\Delta\psi - V\psi\right)(\nabla_i\psi^*) + c.c. && (3)\\
T^{ij} &= \frac{\hbar^2}{2M}\left\{(\nabla_i\psi^*)(\nabla_j\psi) - \frac{1}{2}\delta_{ij}\left[\psi^*\Delta\psi + (\nabla\psi^*)(\nabla\psi)\right]\right\} + c.c. && (4)
\end{aligned}$$

The HO wave functions are real $\Rightarrow T^{0i} \equiv 0 \equiv T^{i0}$

Correlated multi-particle production

Intuitive picture:

- Color electric fields inside the projectile and target fluctuate from event-to-event and are locally organized in domains of size ~1/Q_s

  $\rightarrow$ **breaks axial symmetry E-by-E**

- Each parton receives a kick in the direction of the chromo-electric field which leads to a correlation in azimuthal angle

  can get $v_n \neq 0$ already from initial state

(c.f. talk by T. Lappi)

(Kovner,Lublinsky PRD 83 (2011) 034017; Dumitru, Giannini NPA 933 (2014) 212-228; Dumitru, Skokov PRD 91 (2015) 7, 074006; Lappi, Schenke, Schlichting, Venugopalan 1509.03499)

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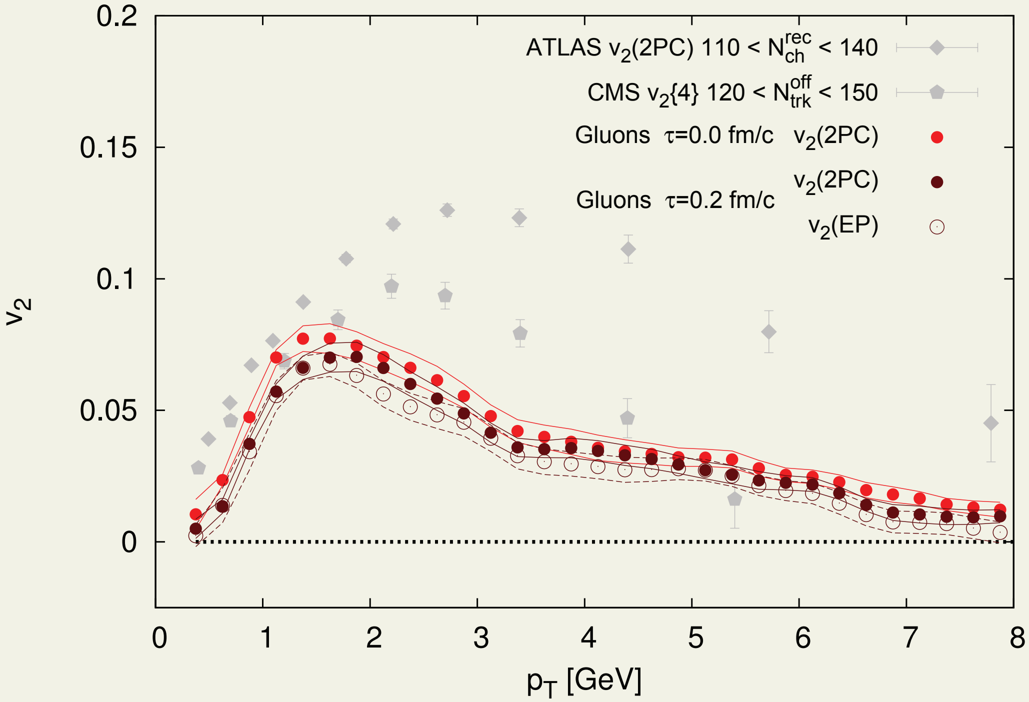
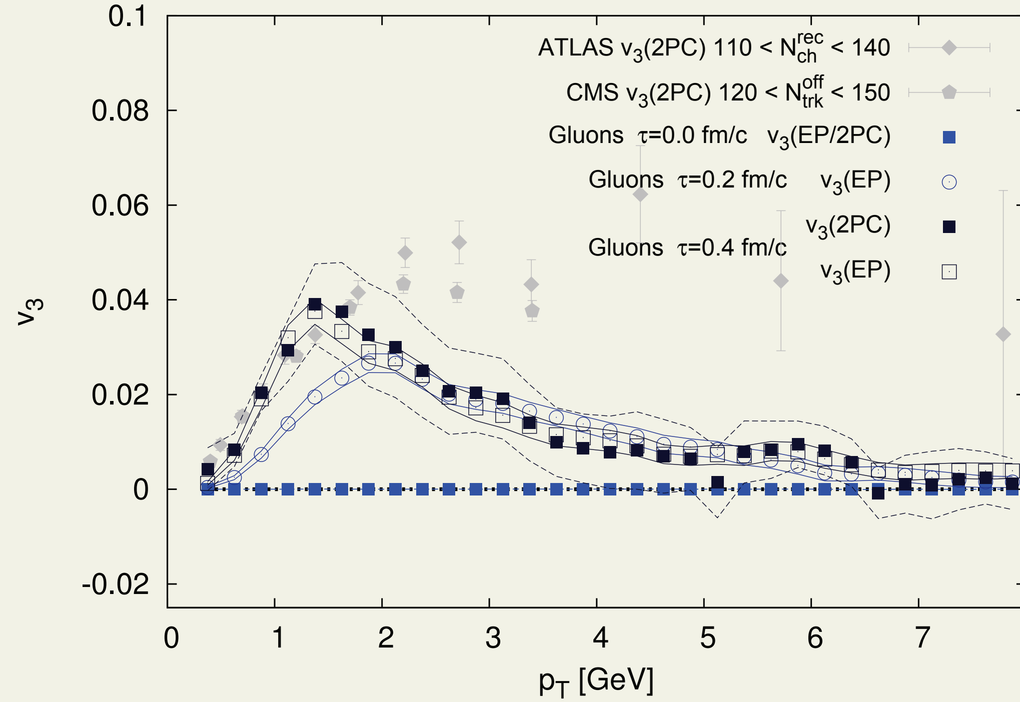
Schnenke, Schlichting, Venugopalan, PLB747 ('15)

gluon $v_2(p_T)$ in p+Pb



$v_3(p_T)$



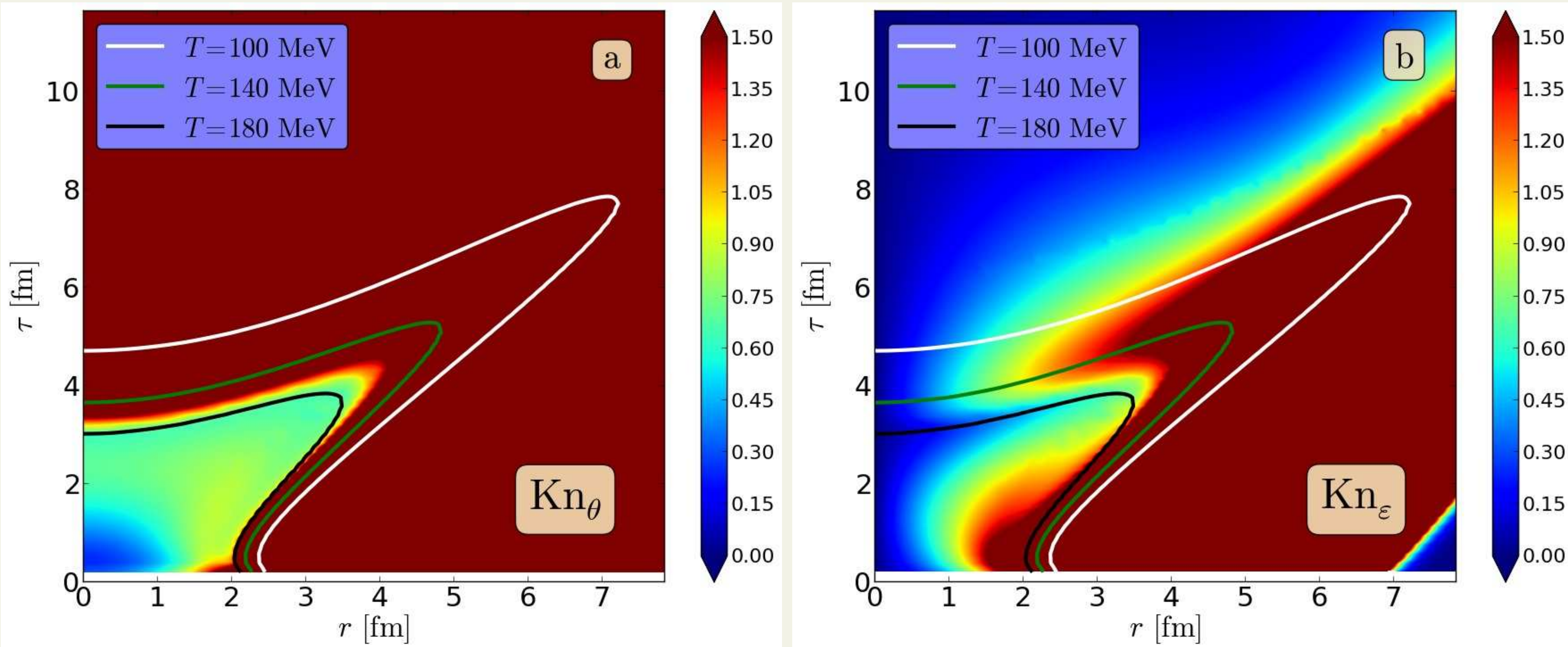
- **large <u>initial</u>** $v_2 > 0$**, <u>not</u> from collective flow** $(T^{0i} \propto \vec{E}_a \times \vec{B}_a = 0)$

- **classical Yang-Mills evolution also builds up** $v_3 > 0$ **for** $t > 0$

Validity of second-order hydro

Hydrodynamics is an expansion around equilibrium in small gradients compared to microscopic scales. **How large are gradients in practice?**

Denicol & Niemi, 1404.7327: **trouble with shear in p+Pb, even from smooth initconds**



Two large Knudsen numbers: $Kn_\theta = \tau_\pi(\partial_\mu u^\mu) > 1$ **and** $Kn_\epsilon = \tau_\pi|\partial_\mu \epsilon|/\epsilon > 1$

$\rightarrow$ **need to check higher orders in hydro?**

Smallest droplets from AdS/CFT

- **lots of insight from $\mathcal{N} = 4$ SYM using gauge-gravity duality**

e.g., the "minimal" shear viscosity $\eta/s = 1/4\pi$ Policastro, Son, Starinets, PRL87 ('02)
Kovtun, Son, Starinets, PRL94 ('05)

main advantage: - solvable at strong coupling (Einstein equations in 5D)
- QCD-like (but no confinement, conformal)

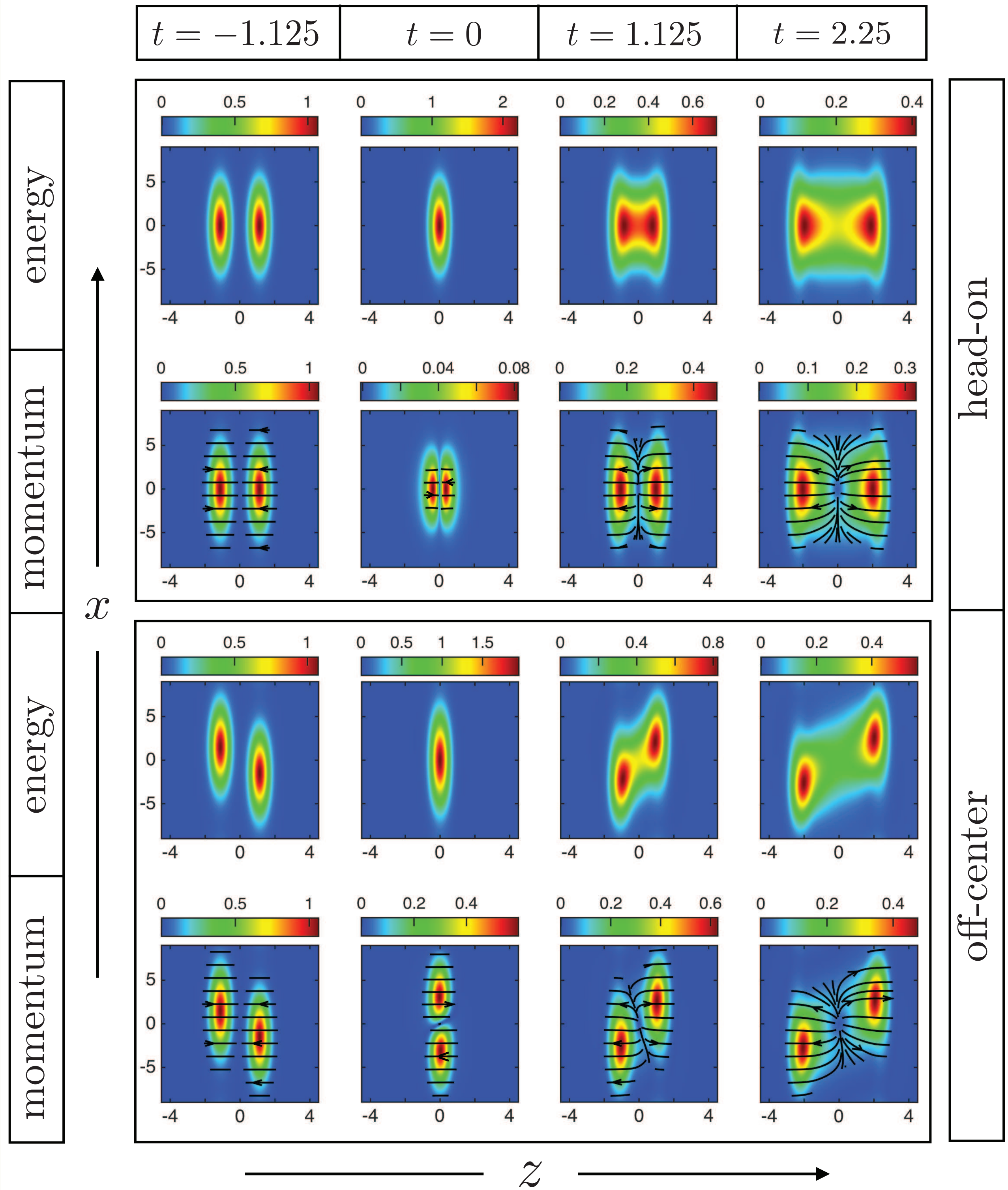
- **can be used to test applicability of hydrodynamics**

e.g., Casalderrey-Solana et al, 1101.0618; ... Chesler, PRL 115 ('15)

or even generate initconds for hydro: (super)SONIC van der Schnee et al, PRL111 ('13)

one recent work: collide two Gaussian droplets to test hydro in p+p

Chesler, JHEP03 ('16)



Chesler, JHEP03 ('16)

$$T^{00}_{ini} \propto N_c^2 \mu^3 e^{-\frac{(\mathrm{x}_T \pm \mathrm{b}/2)^2}{2\sigma^2}} \delta_w(z \pm t)$$

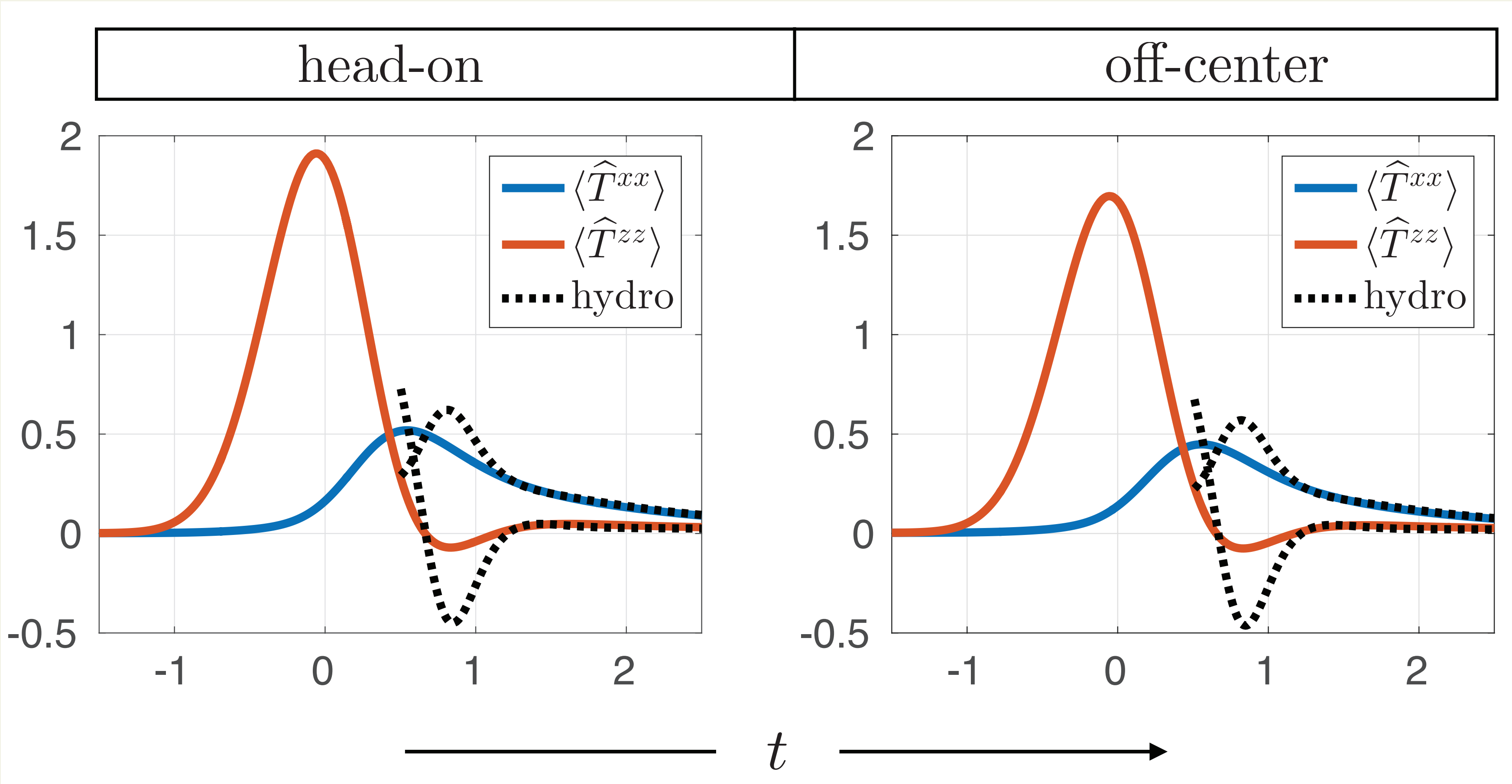
$$E_{TOT} \propto N_c^2 \mu^3 \sigma^2$$

with $\sigma = 3/\mu$,

and $b = 0$ (central),
or $b = \sigma$ (peripheral)

$$T_{eff} \sim (\epsilon/N_c^2)^{1/4}$$

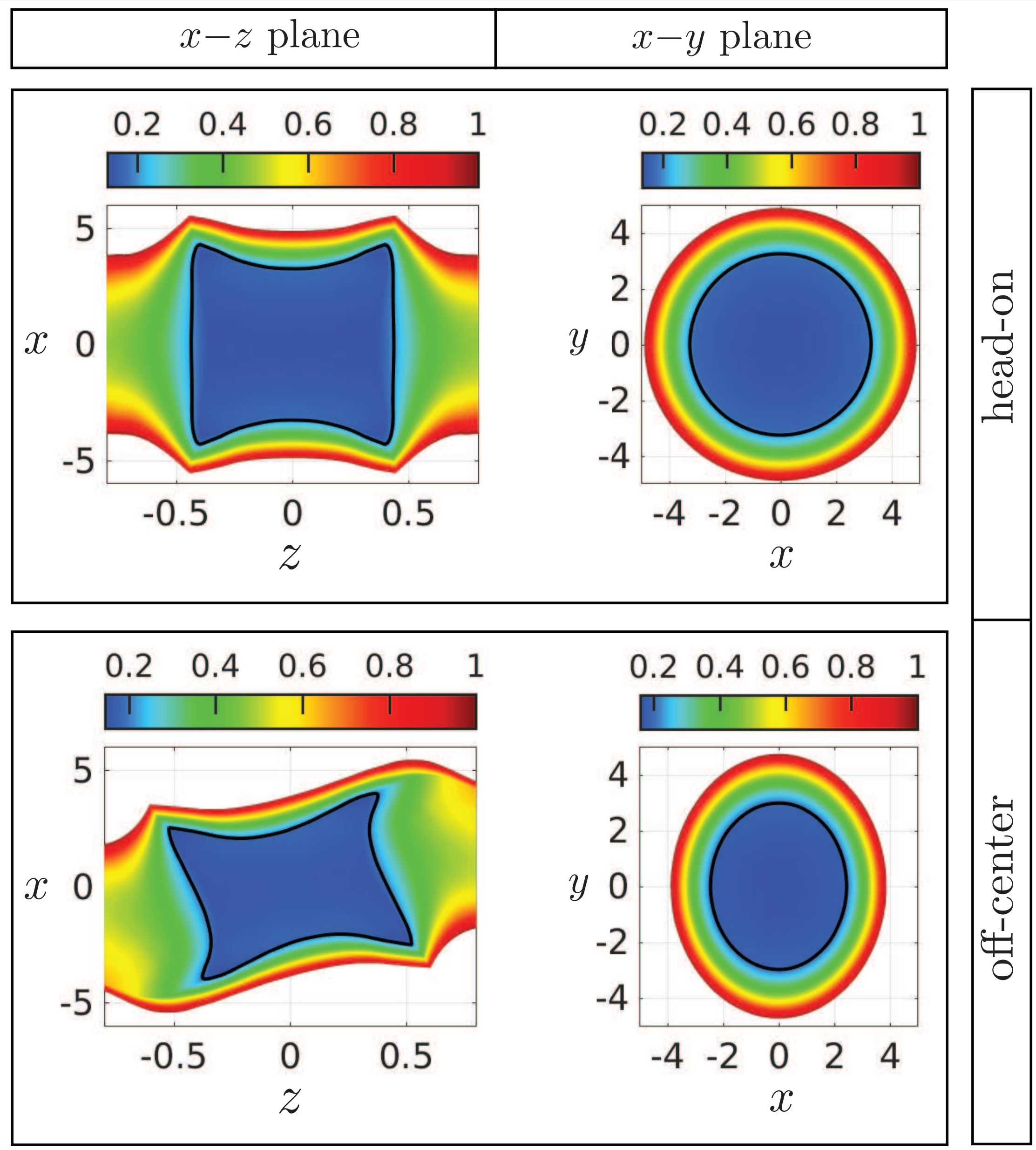
Chesler, JHEP03 ('16): **pressure evolution at center $x = y = z = 0$**



first term in gradient expansion $T^{\mu\nu} = T^{\mu\nu}_{ideal} + T^{\mu\nu}_{linear\ grad.} + \ldots$ **good enough for** $t \gtrsim 1.2/\mu$

despite $\mathcal{O}(1)$ **relative correction to ideal isotropic pressure (**$p_T/p_L > 10$**(!))**

maximum future error in $T^{\mu\nu}(\vec{x}, t)$, relative to pressure, if we only keep first-order term in gradient expansion. Good: $< 20\%$, bad: $> 80\%$



$$\Delta(t) \equiv \max_{t'>t} \frac{T^{\mu\nu}(t') - T^{\mu\nu}_{hydro}(t')}{p(t')}$$

viscous hydro stays valid near center: $R \lesssim 1/T_{eff}$

even though gradients and the pressure anisotropy remain large

if true for QCD, hydro for (smooth) small systems may not be unreasonable

Conclusions

Small system collisions are very interesting, and many theoretical developments are in progress. Most intriguing possibility (to me): quark-gluon plasma in p+A or perhaps even p+p.

One lesson is that "flow observables" do not necessarily have a (solely) hydrodynamic origin. Anisotropic escape, initial production from the glasma/classical Yang-Mills, or even wave packet spatial anisotropy can play a role.

Applicability of hydrodynamics may be questionable (too large gradients in p+A) but an encouraging AdS/CFT work suggests that even p+p might be accommodated by 2nd-order viscous hydro.

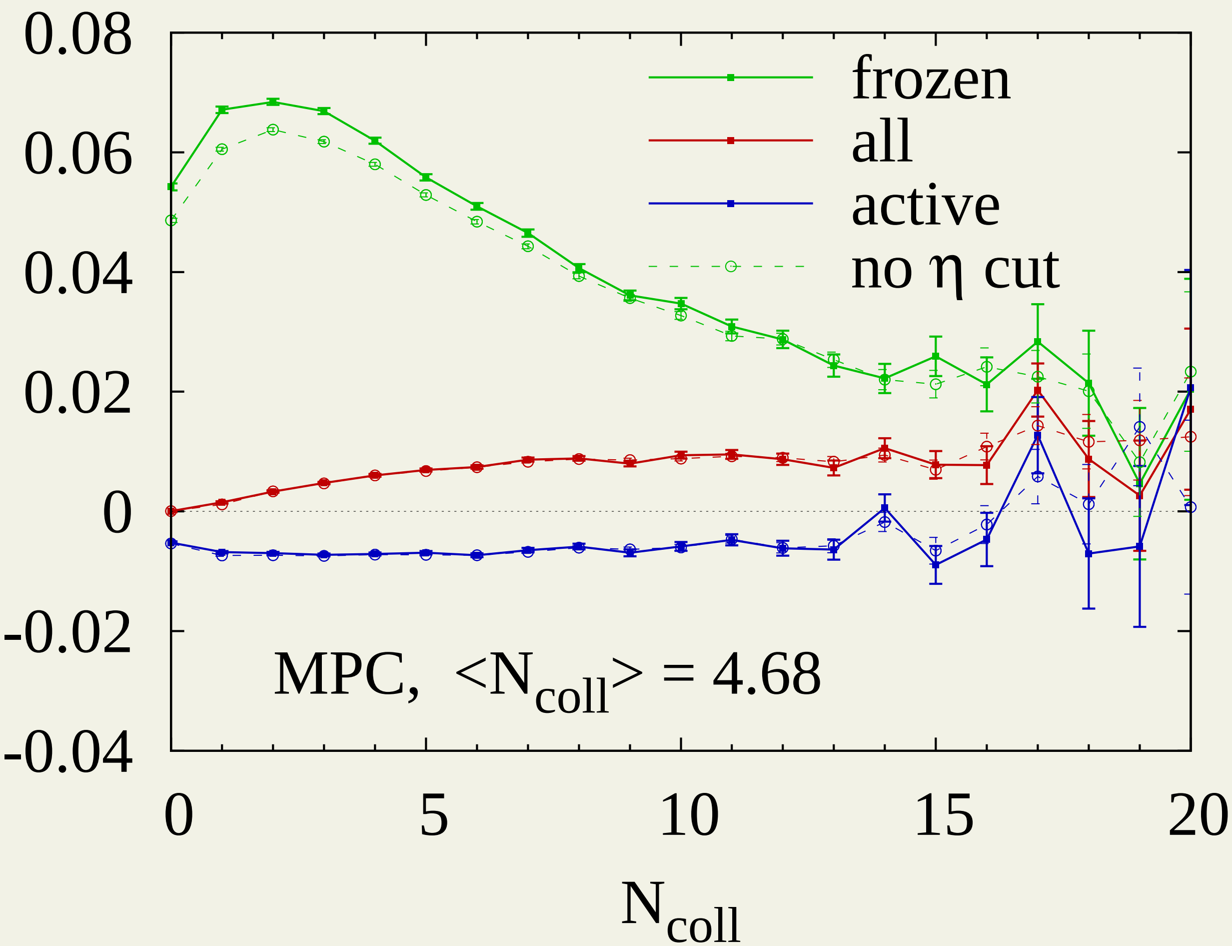
There is quite some more work to be done on the theory side especially. Relentlessly keeping models' feet to the fire should help sort things out. High-order identified particle v_n up to $p_T \sim 3-4$ GeV would be welcome.

—

I left out many exciting areas, e.g.: dynamical fluctuations in hydro, longitudinal fluctuations, rigorous parameter extraction from data, ultra-central A+A collisions, thermalization, anisotropic hydro ...

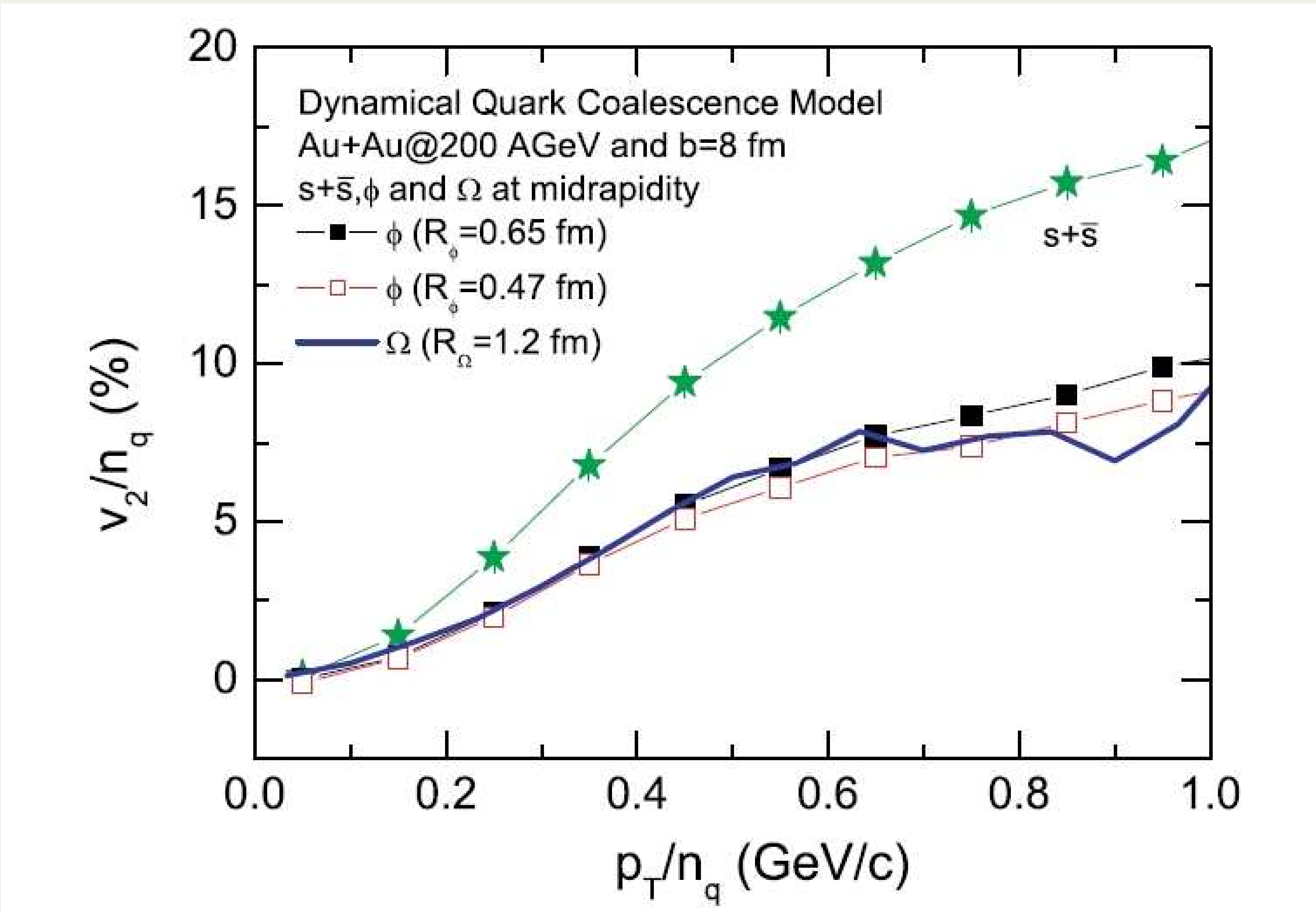
v2 vs Ncoll from MPC in Au+Au at RHIC

He et al, PLB753 ('16):



Quark number scaling from AMPT - v_2 for ϕ, Ω, and s-quarks

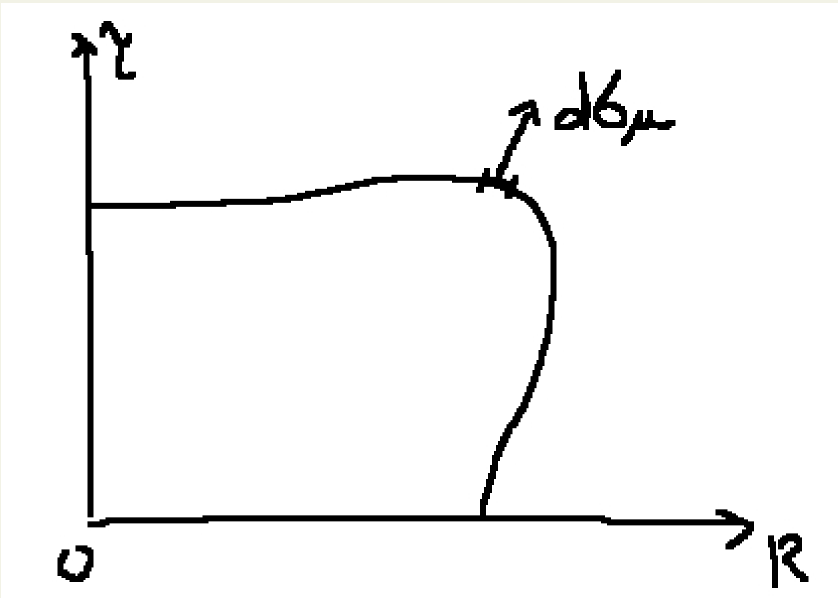
Chen & Ko, PRC73 ('06):



Cooper-Frye freezeout

Not a very satisfactory solution (still open problem)

<u>Assume</u> sudden transition to a gas on a 3D hypersurface (typically $T = const$ or $\varepsilon = const$)



$$E\, dN = p^\mu d\sigma_\mu(x)\, d^3p\, f_{gas}(x, \vec{p})$$

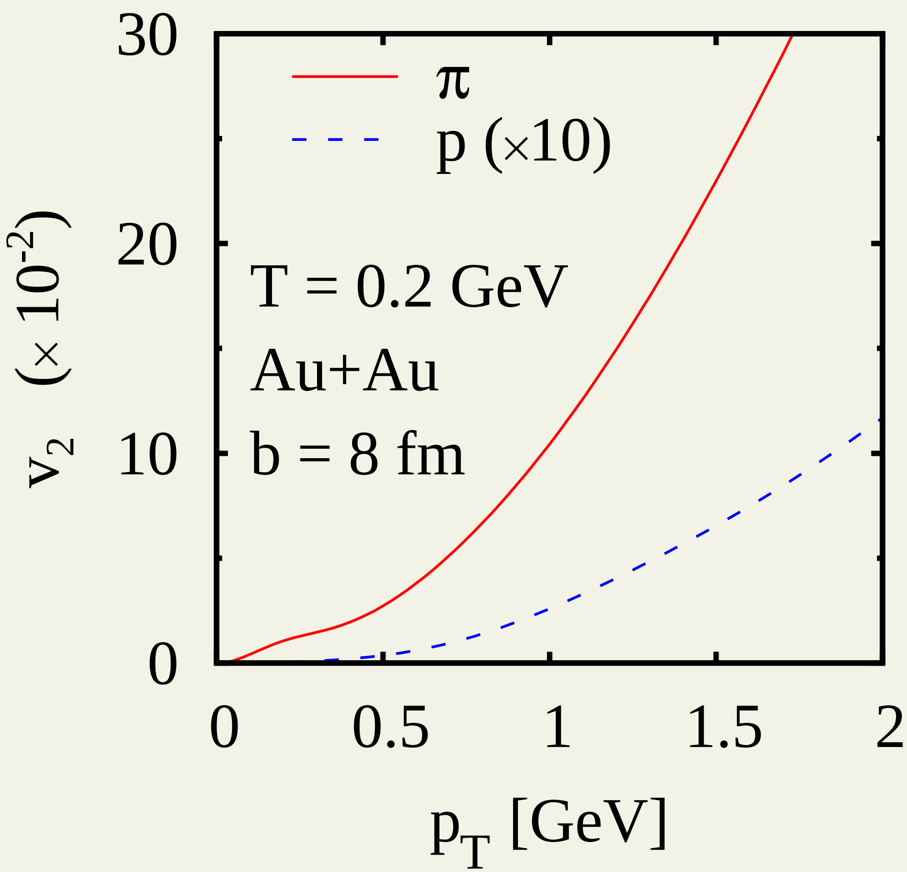
(covariant analog of $t = const$ freezeout $dN/d^3x d^3p = f(\vec{x}, \vec{p}, t_{fo})$)

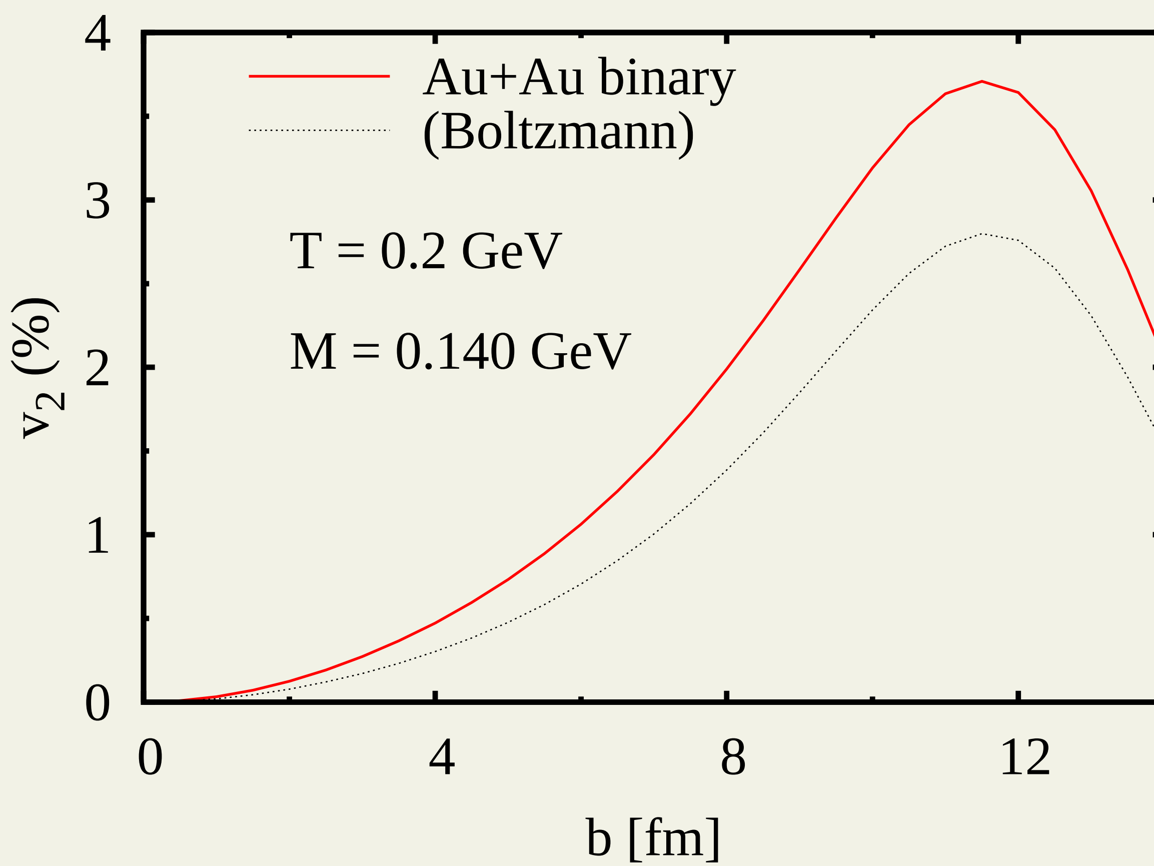
Good: - conserves energy-momentum and charges locally

Bad: - negative contributions possible $p \cdot d\sigma < 0$
- arbitrariness in choice of HS & self-consistency problem

v_2 from QM DM, Wang & Greene, 1404.4119

Au+Au at RHIC:





with notable caveats (nonrelativistic treatment, no expansion dynamics)