Modeling

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Hydrodynamic Modeling for the search of anomalous chiral effects

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Outline

Modeling of chiral effects

Effects of dynamical EM fields

Outlook for future efforts

Anomalous chiral effects

$$\boxed{\boldsymbol{j}_{\mathrm{anom}} = \kappa_B \boldsymbol{B} + \kappa_\omega \boldsymbol{\omega}}$$

$$\boxed{\boldsymbol{j}_{5,\mathrm{anom}} = \xi_B \boldsymbol{B} + \xi_\omega \boldsymbol{\omega}}$$

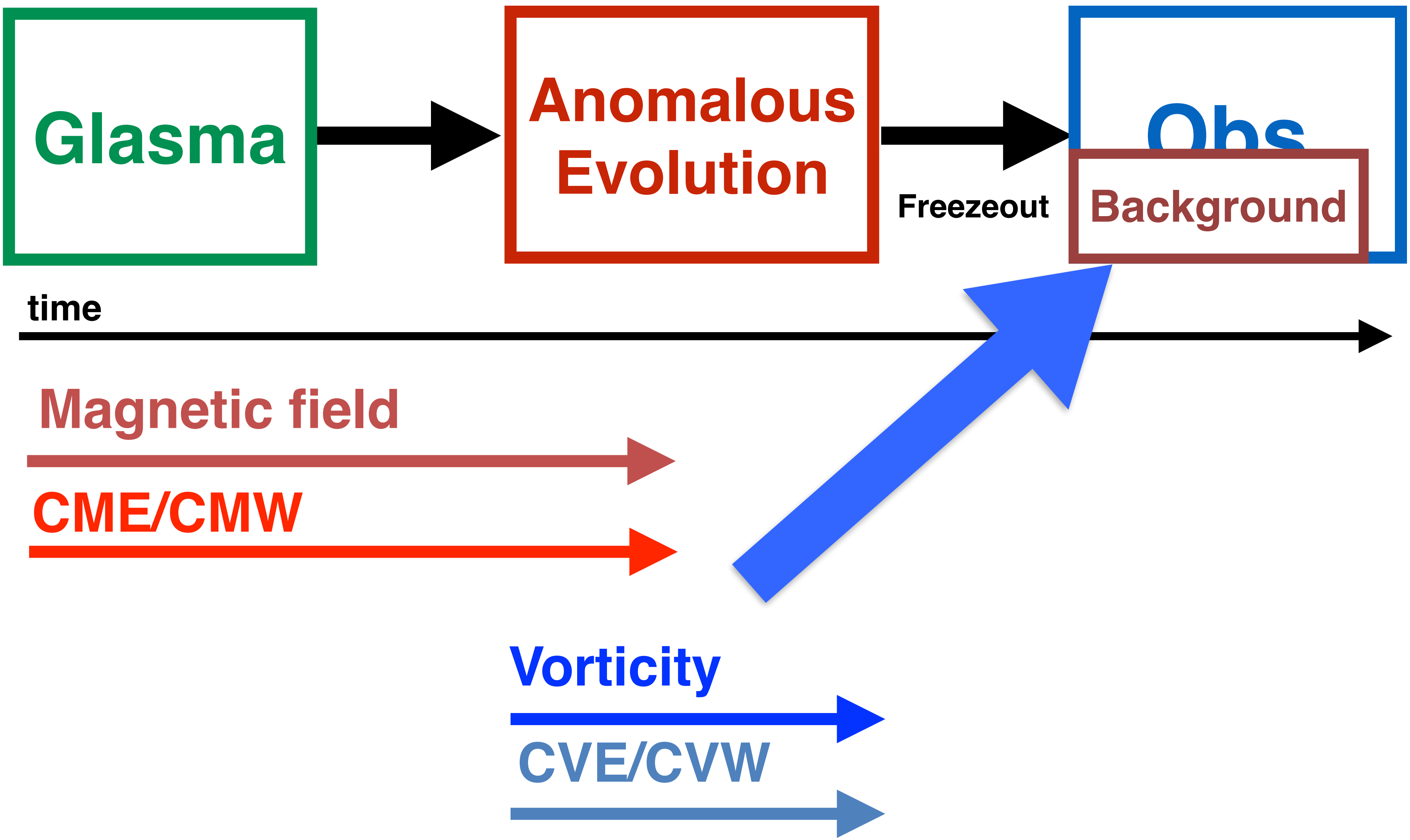
Theoretical frameworks

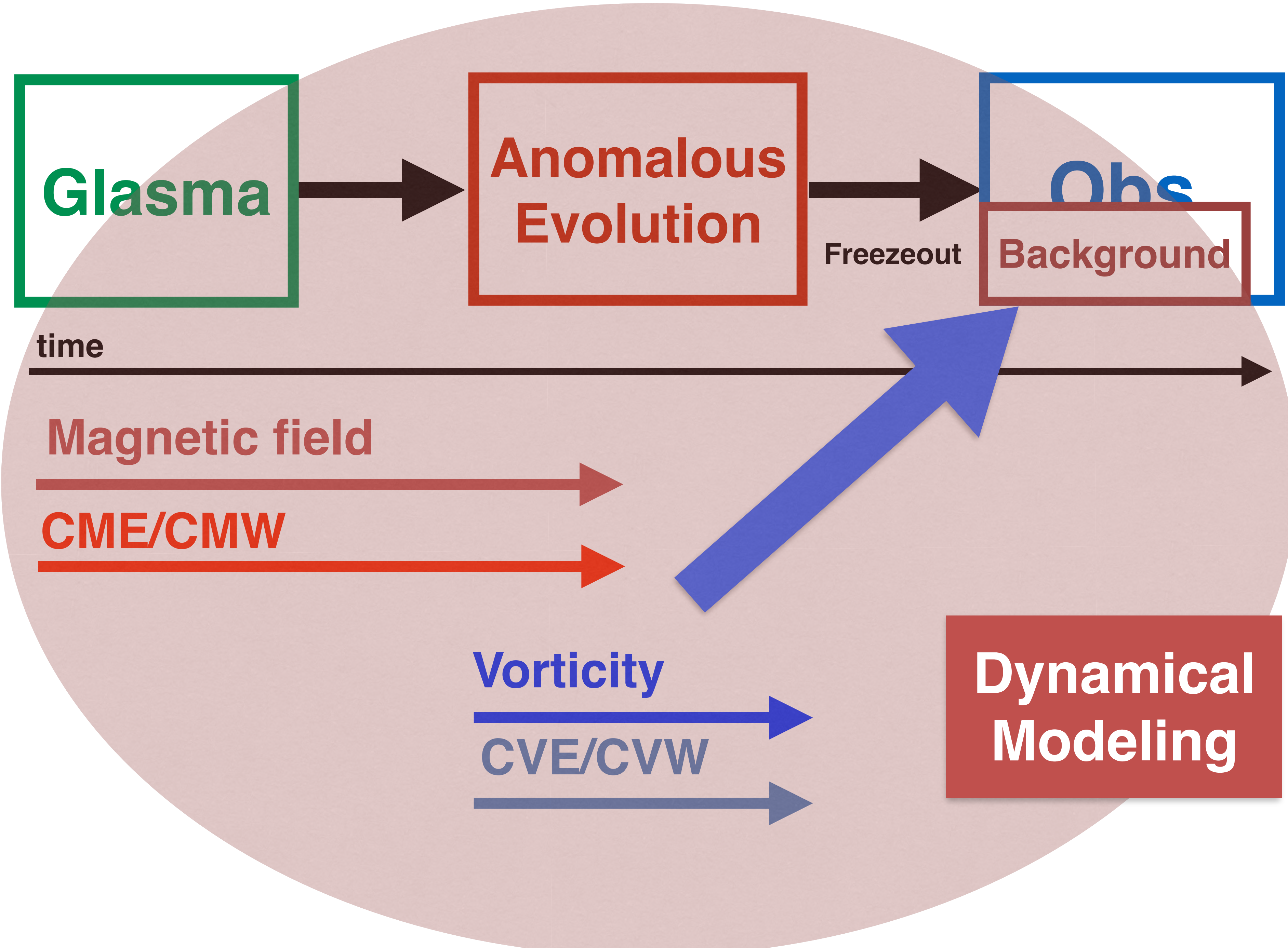
- Anomalous hydrodynamics

[Son-Surowka 2009; …]

- Chiral kinetic theory

[Son-Yamamoto; Stephanov-Yin; …]


Glasma
Anomalous Evolution
Obs
Background
Freezeout
time
Magnetic field
CME/CMW
Vorticity
CVE/CVW



Glasma
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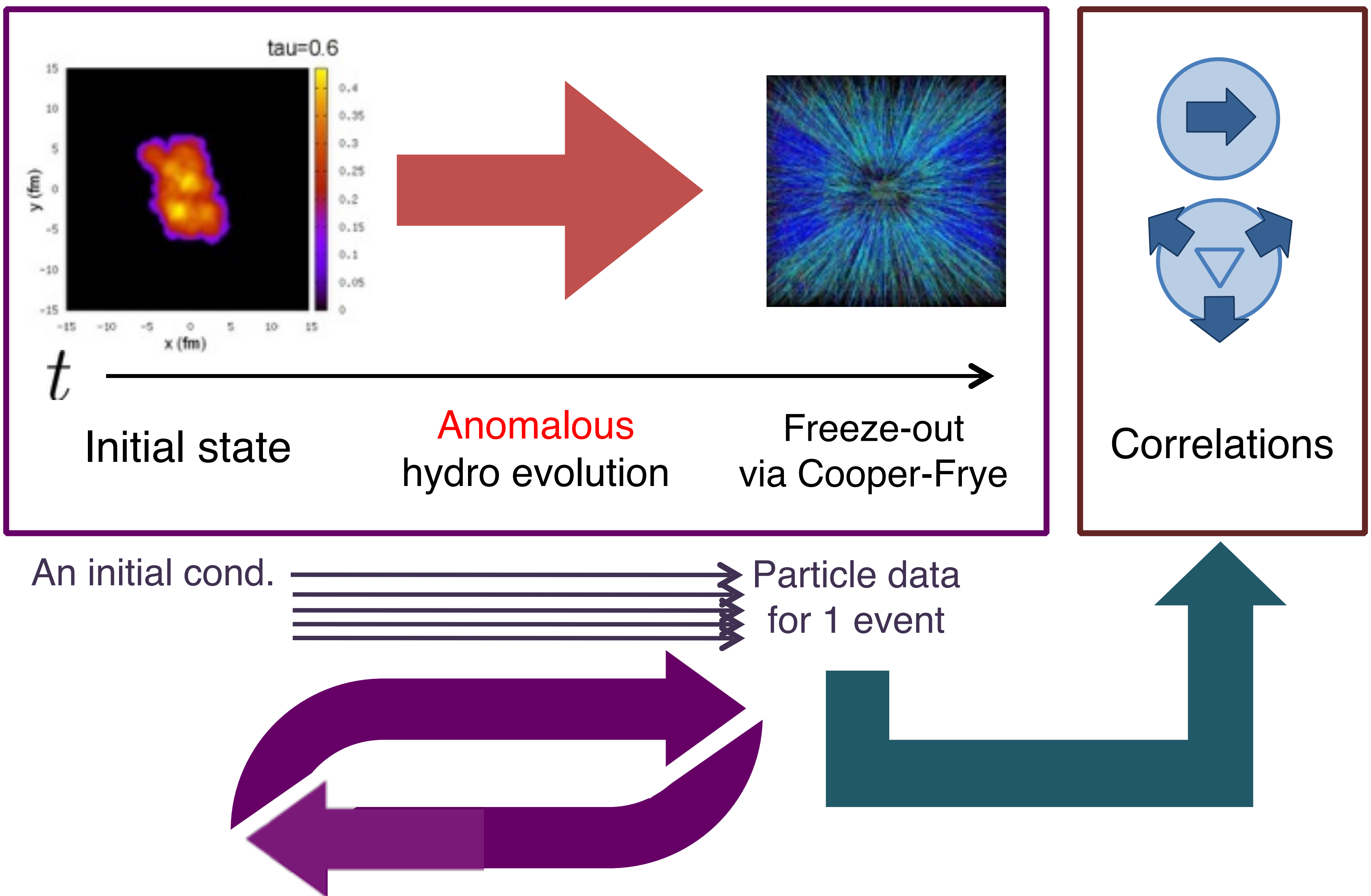
Dynamical Modeling

Event-by-event anomalous hydrodynamics for CME

[Hirono-Hirano-Khazeev 1412.0311]

Event-by-event anomalous hydrodynamic model



Anomalous hydrodynamic equations

- Non-dissipative anomalous fluid in 3+1D
  - no viscosity/Ohmic conductivity
- Background electromagnetic fields

$$\partial_\mu T^{\mu\nu} = \boxed{F^{\nu\rho} j_\rho}$$

$$\partial_\mu j^\mu = 0 \qquad \partial_\mu j_5^\mu = \boxed{C E_\mu B^\mu}$$

$$C = \frac{N_c N_f}{2\pi^2}$$

Anomalous hydrodynamic equations

- Constitutive equations

$$j^\mu = n u^\mu + \boxed{\kappa_B B^\mu} \quad j_5^\mu = n_5 u^\mu + \boxed{\xi_B B^\mu}$$

CME CSE

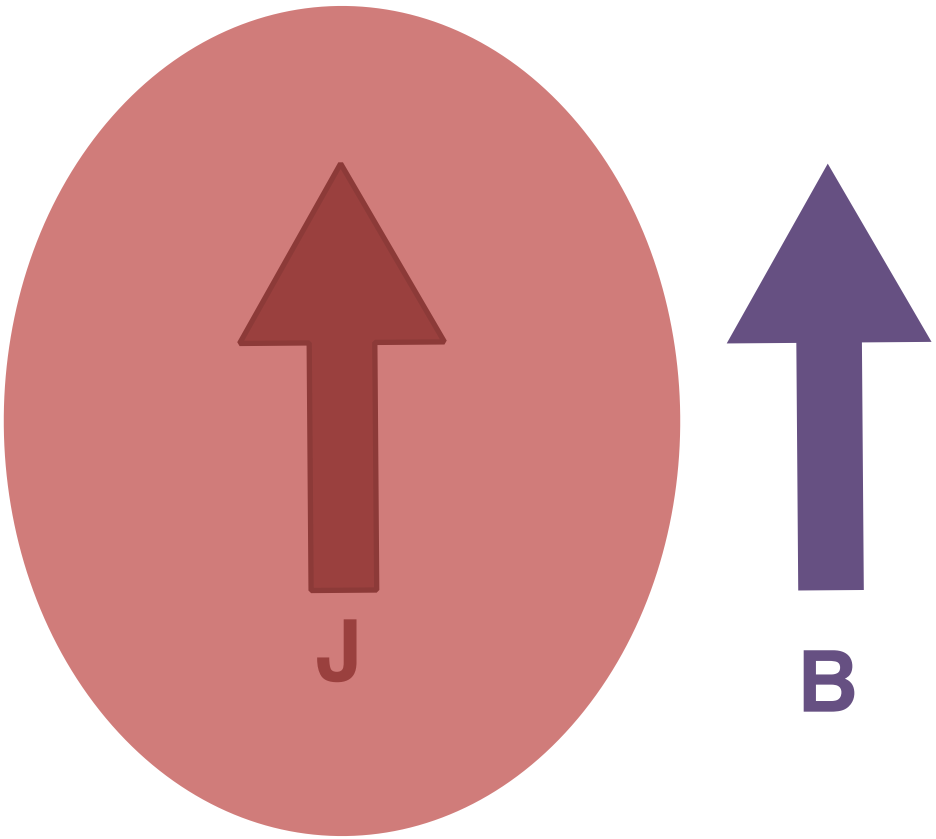
$$e\kappa_B = C\mu_5 \left(1 - \frac{\mu n}{e+p}\right) \quad e\xi_B = C\mu \left(1 - \frac{\mu_5 n_5}{e+p}\right)$$

[Son-Surowka 2009]
[Kalaydzhyan-Kirsch 2011]

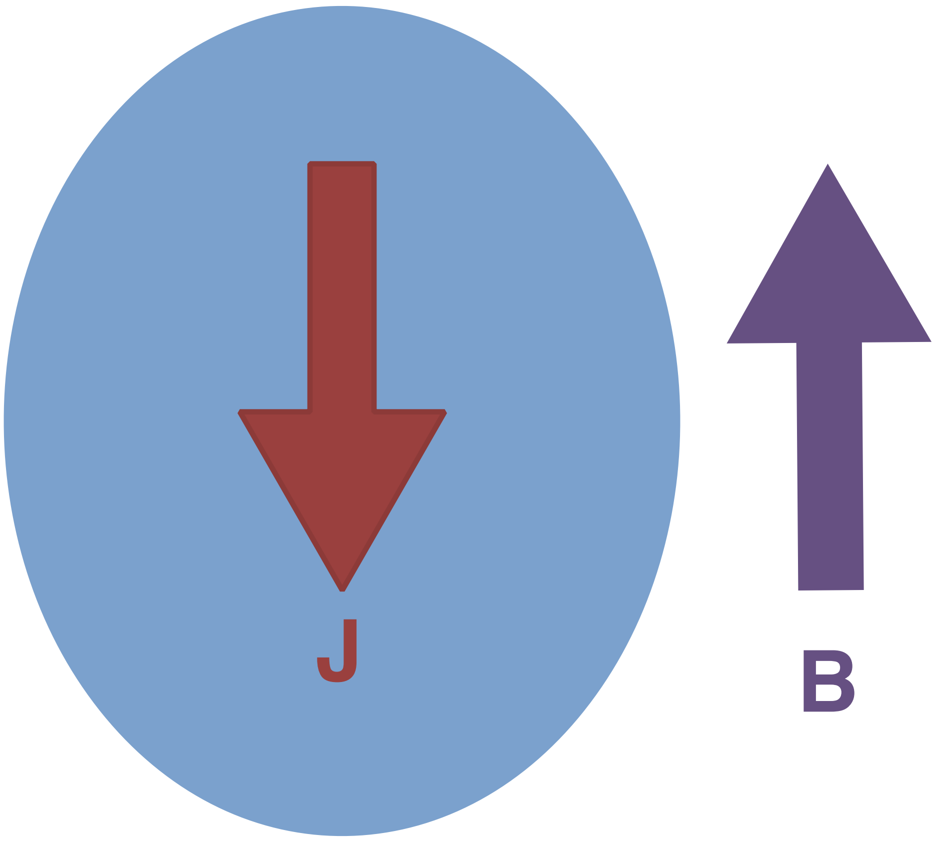
- Equation of state - conformal
  - massless quarks & gluons

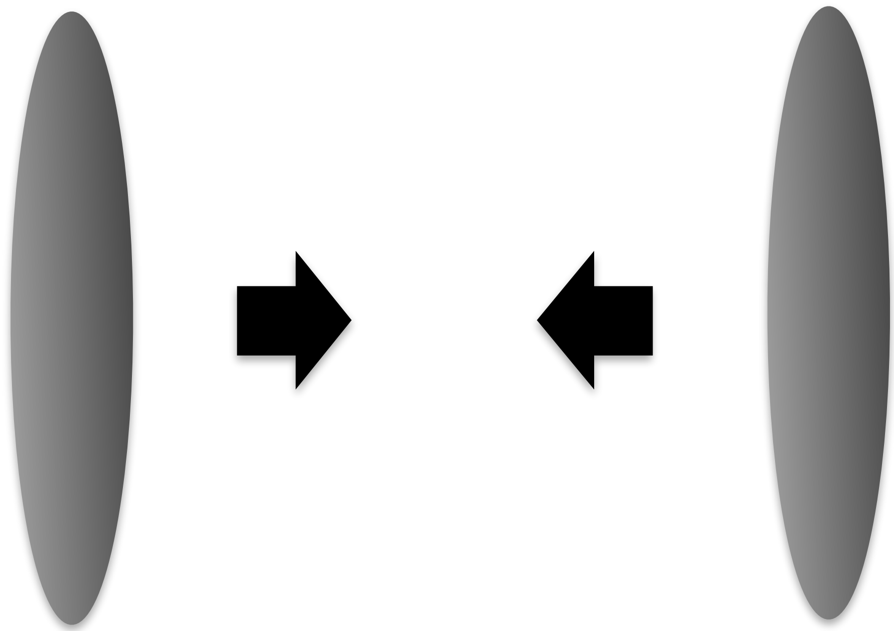
$$p\left(T, \mu, \mu_5\right) = \frac{g_{\mathrm{qgp}}\pi^2}{90} T^4 + \frac{N_c N_f}{6}\left(\mu^2 + \mu_5^2\right) T^2 + \frac{N_c N_f}{12\pi^2}\left(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4\right)$$

Initial axial charge?

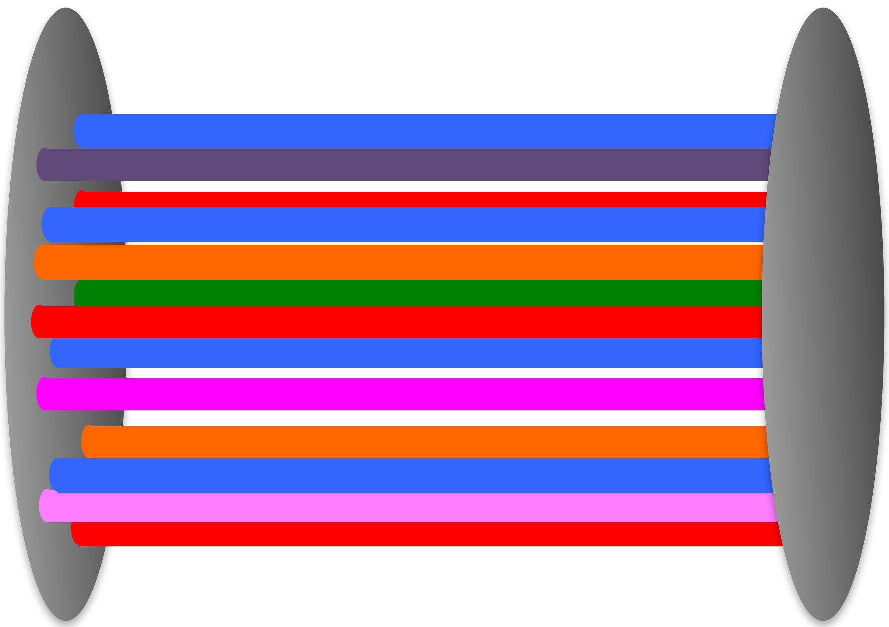


Initial axial charge?





$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2}\boldsymbol{E}^a \cdot \boldsymbol{B}^a$$



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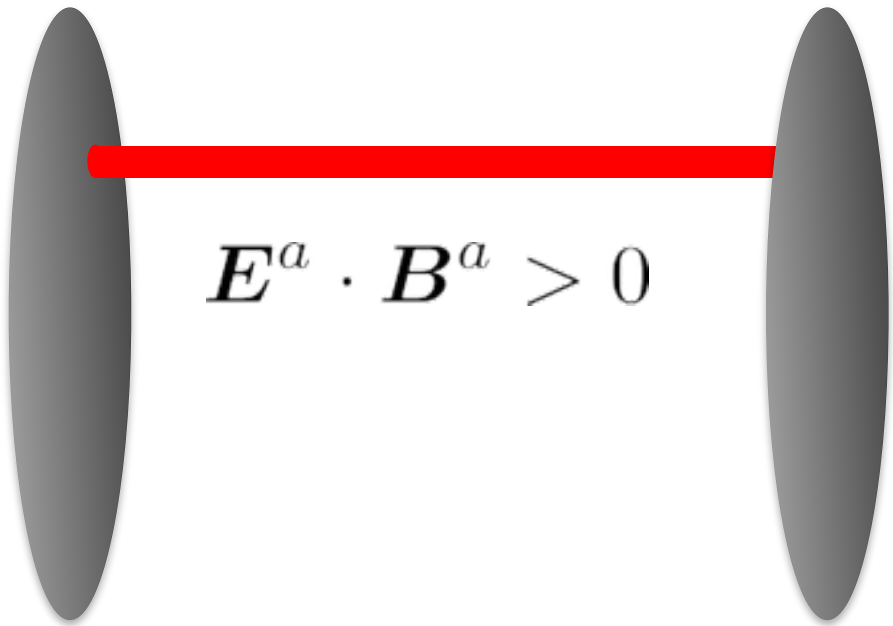

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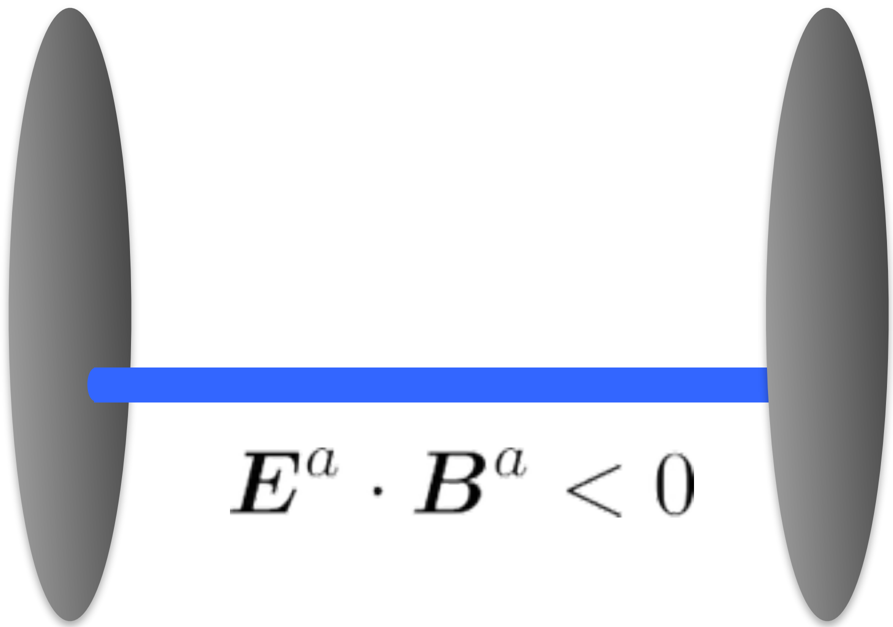
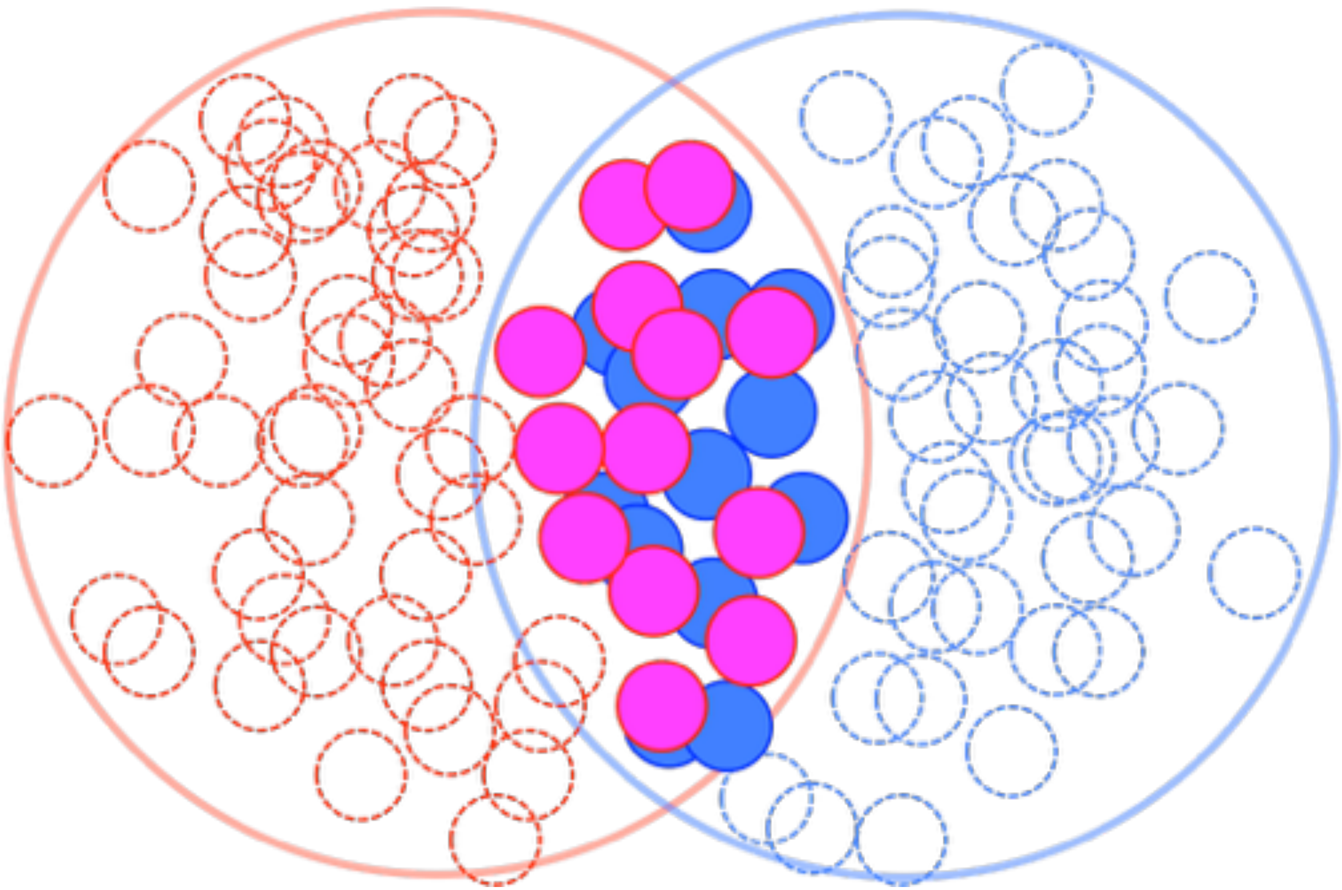


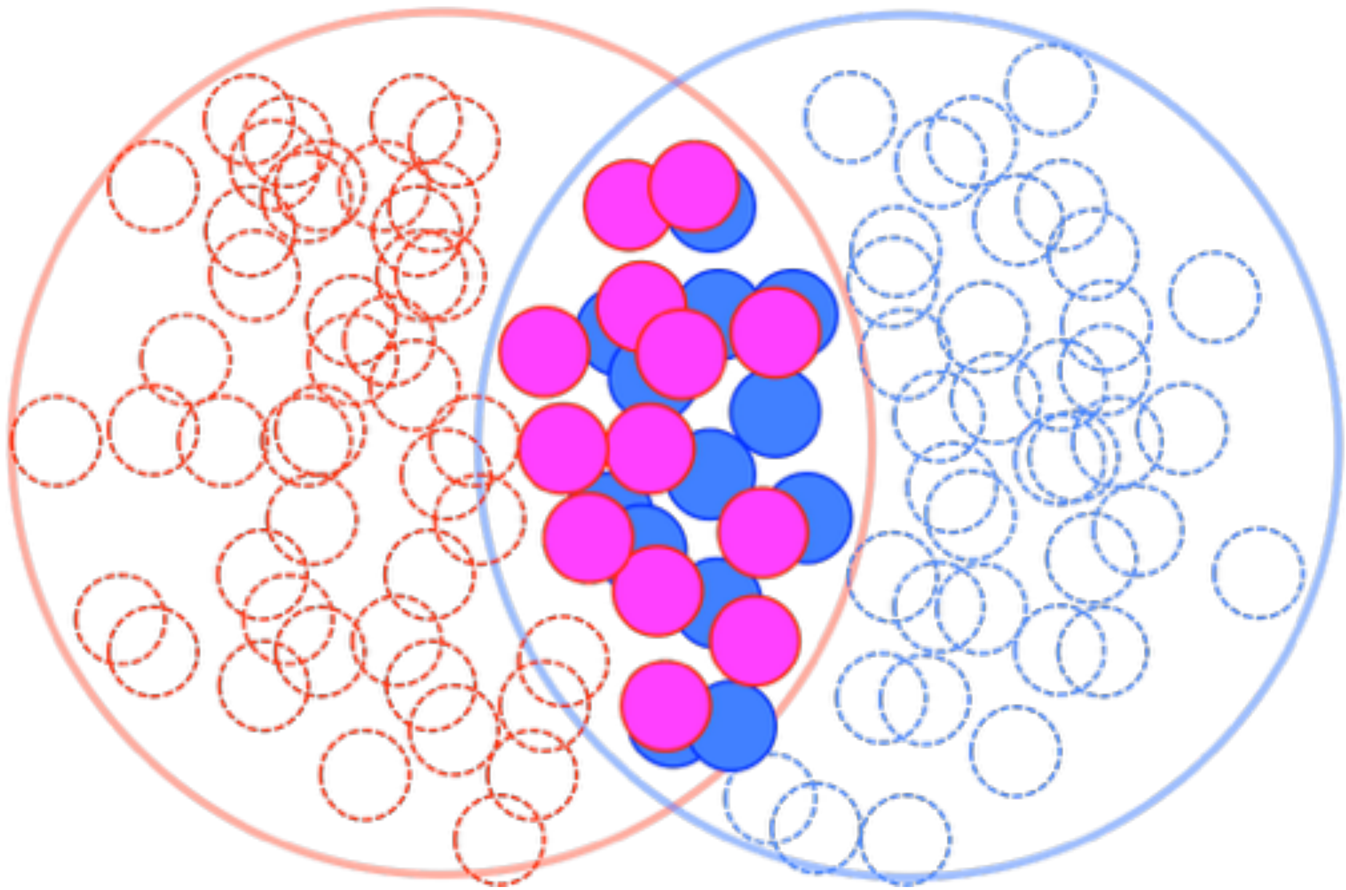
Axial charges from color flux tubes

$N_{\mathrm{part}}^{\mathrm{A(B)}}(\boldsymbol{x}_{\mathrm{T}})$: # of ( ) $N_{\mathrm{coll}}(\boldsymbol{x}_{\mathrm{T}})$: # of pairs



Axial charges from color flux tubes

$$s(\boldsymbol{x}_\mathrm{T}, \eta_s) = Af(\eta_s)\left[\frac{1-\alpha}{2}\frac{d^2N_\mathrm{part}}{dx_\mathrm{T}^2} + \alpha\frac{d^2N_\mathrm{coll}}{dx_\mathrm{T}^2}\right]$$

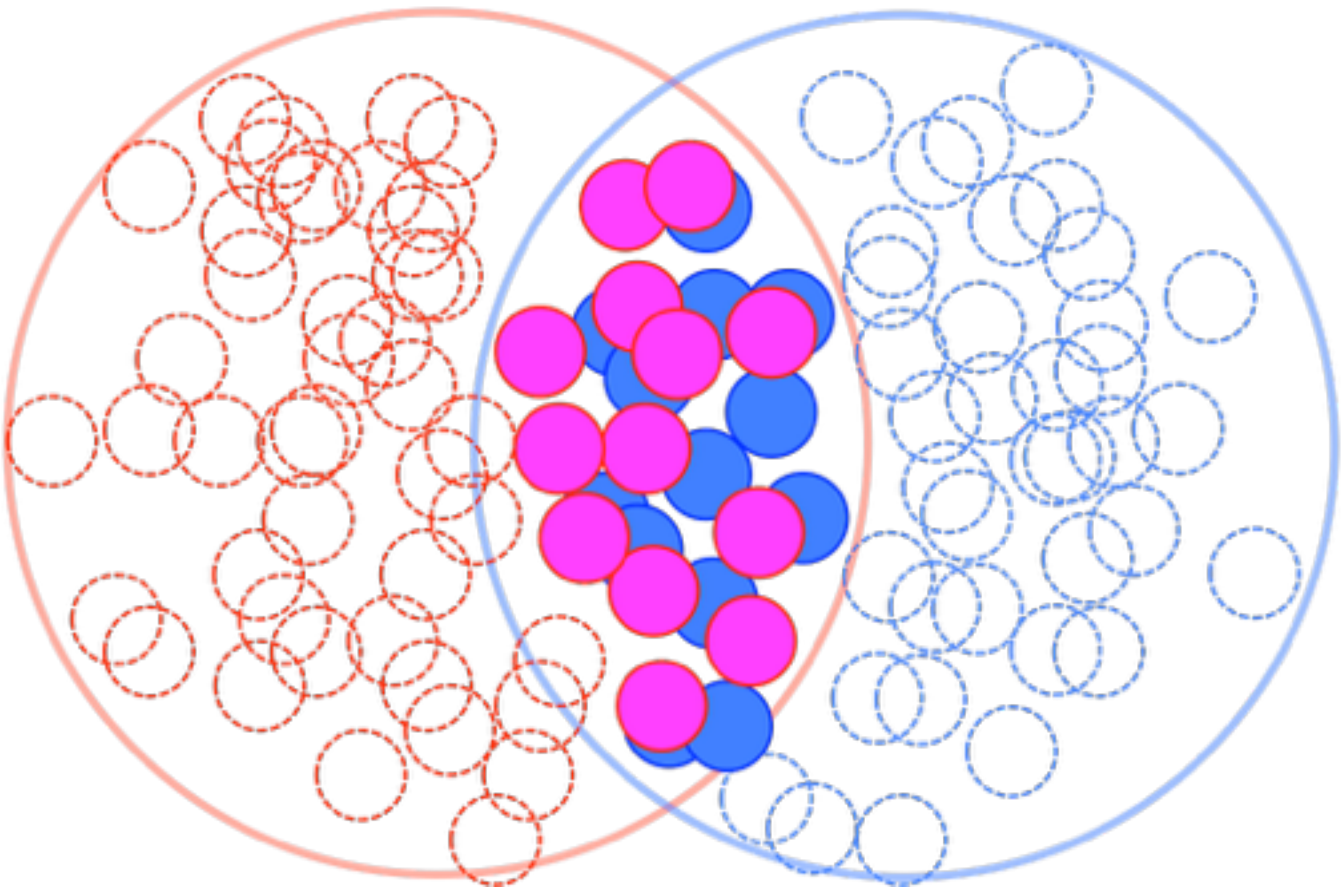


Axial charges from color flux tubes

$$X_j \in \{+1, -1\}$$

Sign of $\boldsymbol{E}^a \cdot \boldsymbol{B}^a$

$$\mu_5(\boldsymbol{x}_\mathrm{T}) = C_{\mu_5} \sum_{j=1}^{N_\mathrm{coll}(\boldsymbol{x}_\mathrm{T})} X_j$$



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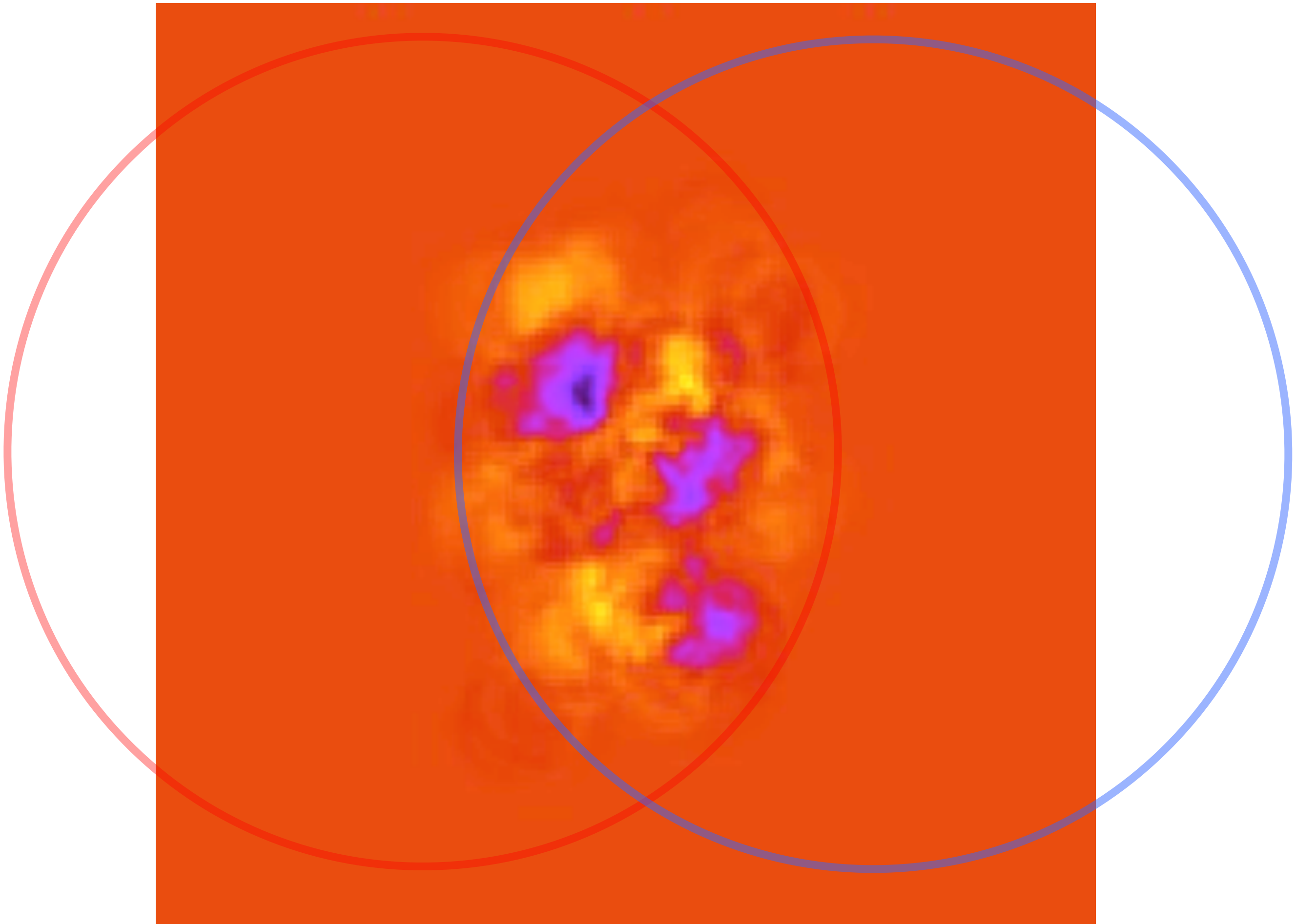
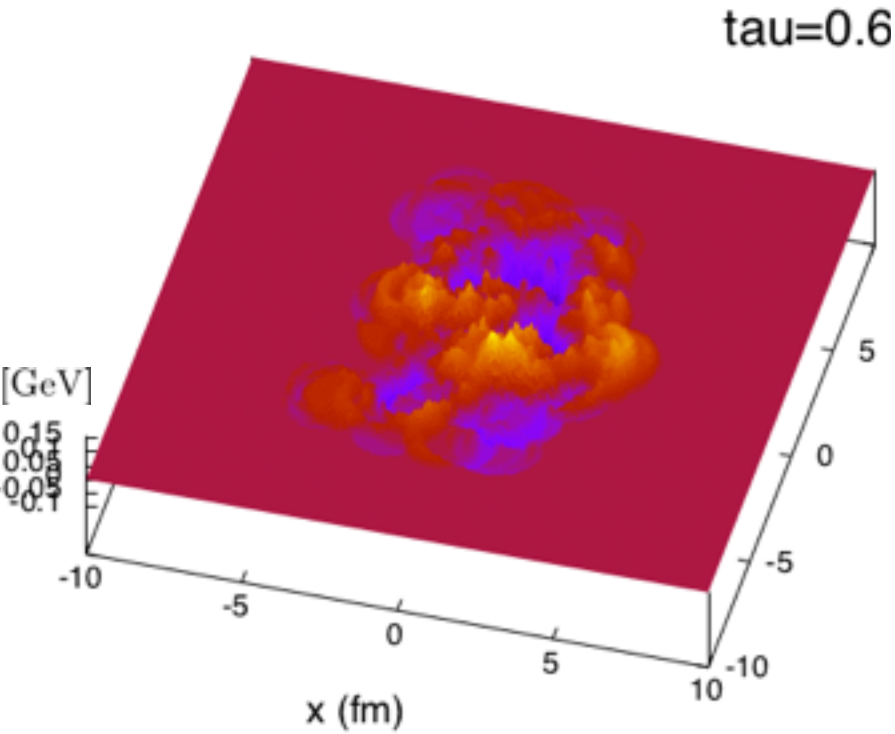
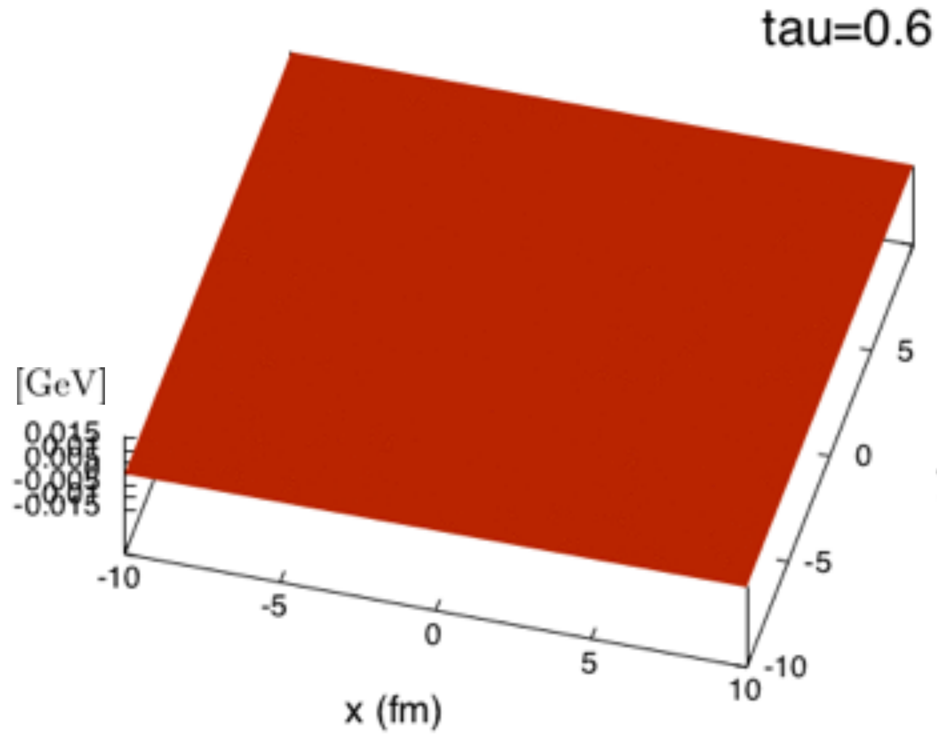


Time evolution of chemical potentials

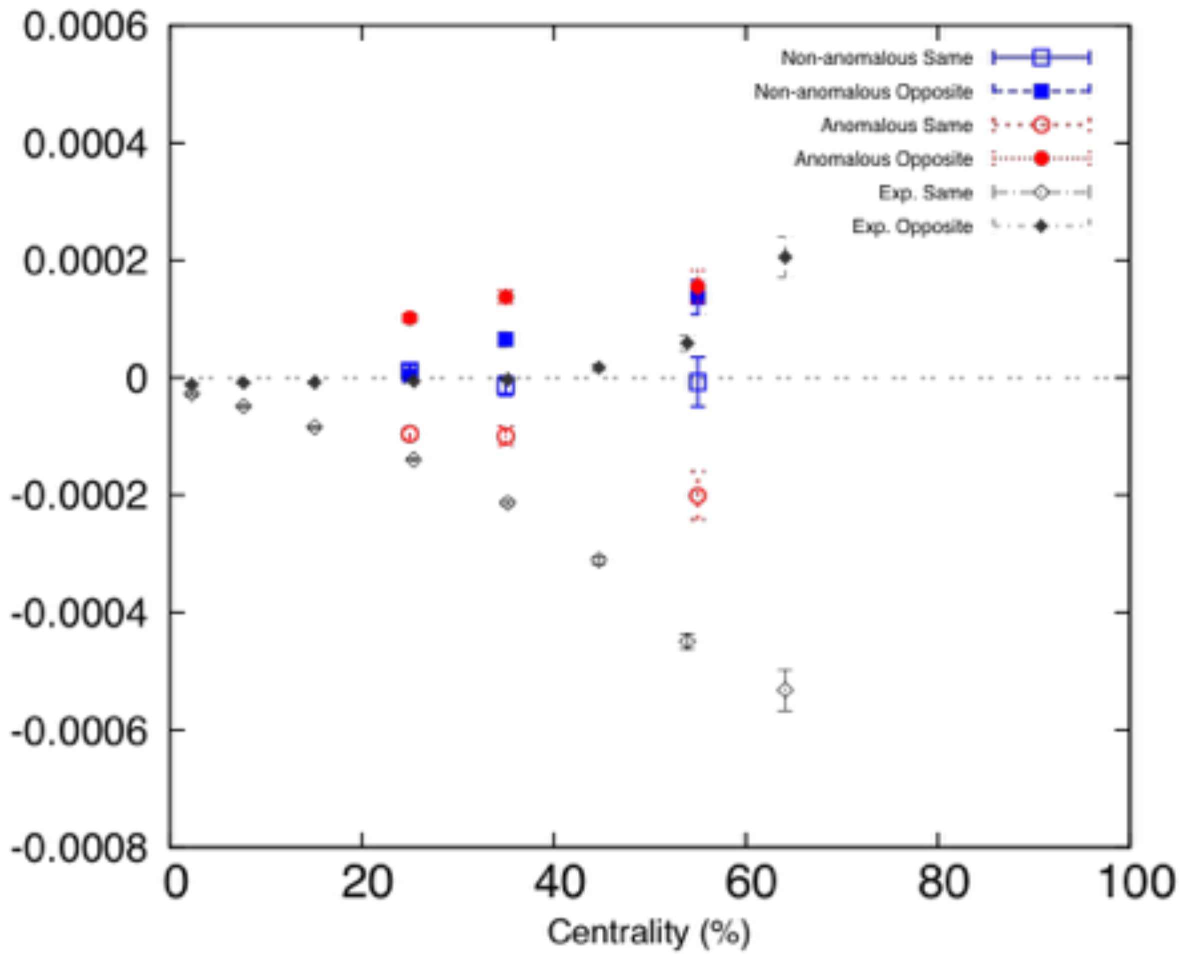


μ_5



μ

$$\gamma_{\alpha\beta} = <\cos(\phi_i + \phi_j - 2\psi_{\mathrm{RP}})>_{\alpha\beta}$$



Gamma correlations are indeed sensitive to CME/CSE

Values are similar to experimental ones

Hydrodynamics with chiral anomaly and charge separation in relativistic heavy ion collisions

Yi Yin[1,*] and Jinfeng Liao[2,3,†]

[1504.06906]

- Charge transport from CME/CSE is solved on top of the solution of 2+1D viscous hydro (VISH)
- Quantify the effects of transverse momentum conservation (TMC)
- Same-charge correlation is calculated

Hydrodynamics with chiral anomaly and charge separation in relativistic heavy ion collisions

Yi Yin[1,*] and Jinfeng Liao[2,3,†]

[1504.06906]

$$\gamma_{\alpha\beta} = < \cos(\phi_i + \phi_j - 2\psi_{\mathrm{RP}}) >_{\alpha\beta} \qquad \delta_{\alpha\beta} = < \cos(\phi_i - \phi_j) >_{\alpha\beta}$$

$$\gamma_{\alpha,\beta}^{\mathrm{data}} \simeq \boxed{\gamma_{\alpha,\beta}^{CME}} + \gamma_{\alpha,\beta}^{TMC} \;, \quad \delta_{\alpha,\beta}^{\mathrm{data}} \simeq \boxed{\delta_{\alpha,\beta}^{CME}} + \delta_{\alpha,\beta}^{TMC}$$

$$(\gamma + \delta)^{\mathrm{data}} \approx (\gamma + \delta)^{\mathrm{TMC}}$$

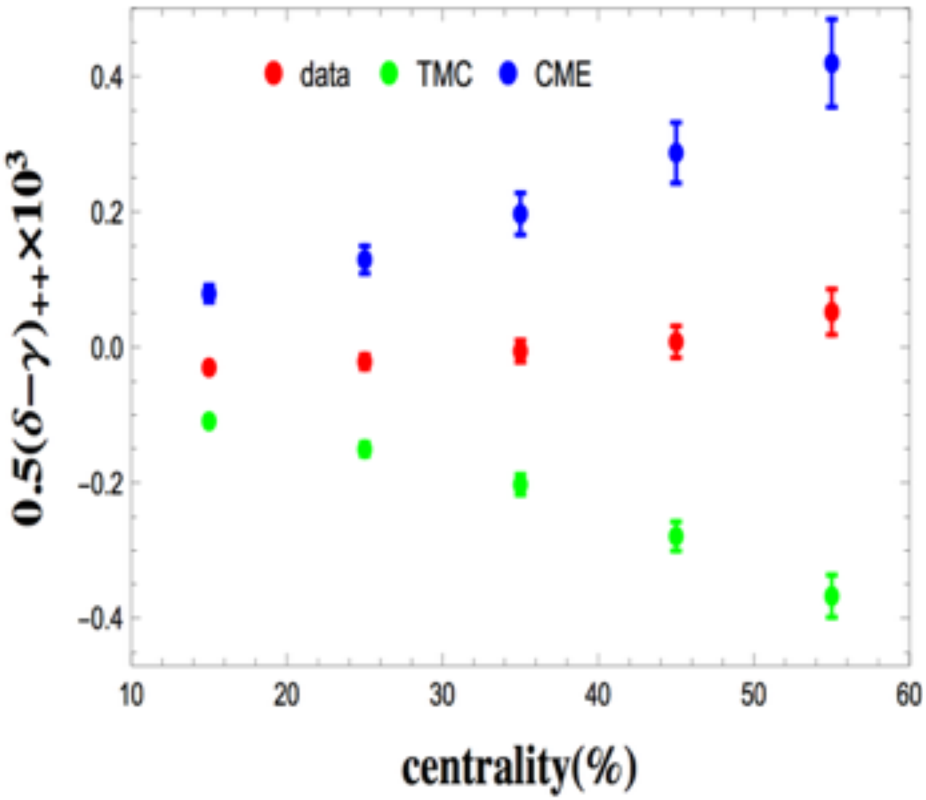
$$\delta_{\alpha\beta}^{\mathrm{TMC}} \pm \gamma_{\alpha\beta}^{\mathrm{TMC}} = \frac{[\langle p_\perp \rangle_\alpha (1 \pm \bar{v}_{2,\alpha})][\langle p_\perp \rangle_\beta (1 \pm \bar{v}_{2,\beta})]}{N_{TMC} \langle p_\perp^2 \rangle (1 \pm \bar{\bar{v}}_2)}$$

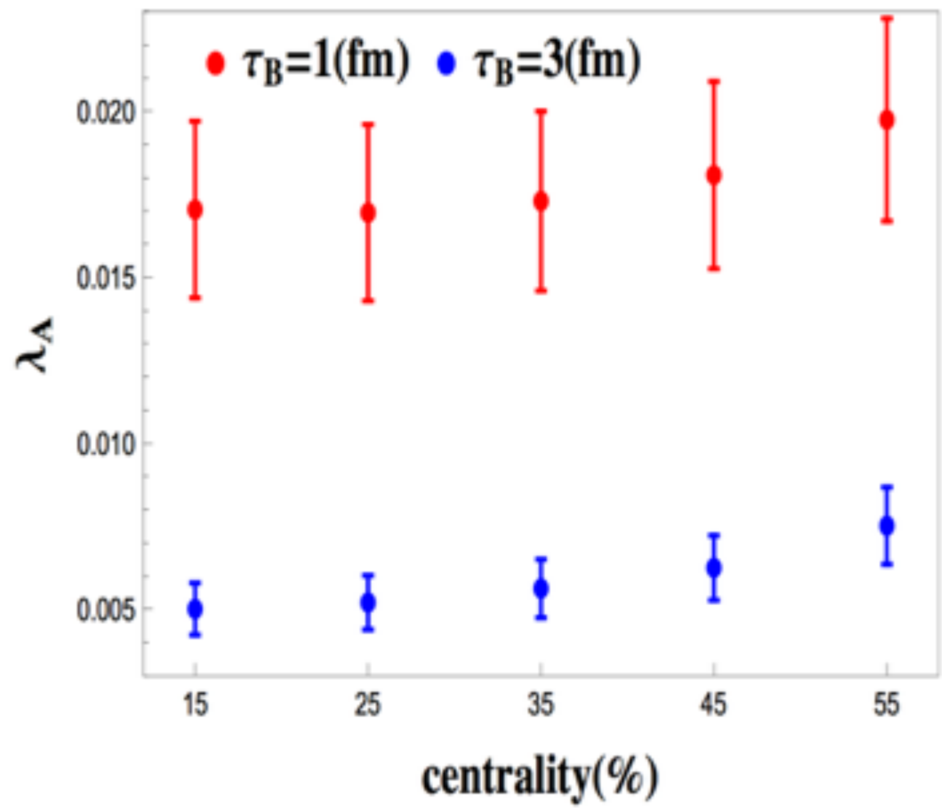
[Bzdak-Koch-Liao 1008.4919]

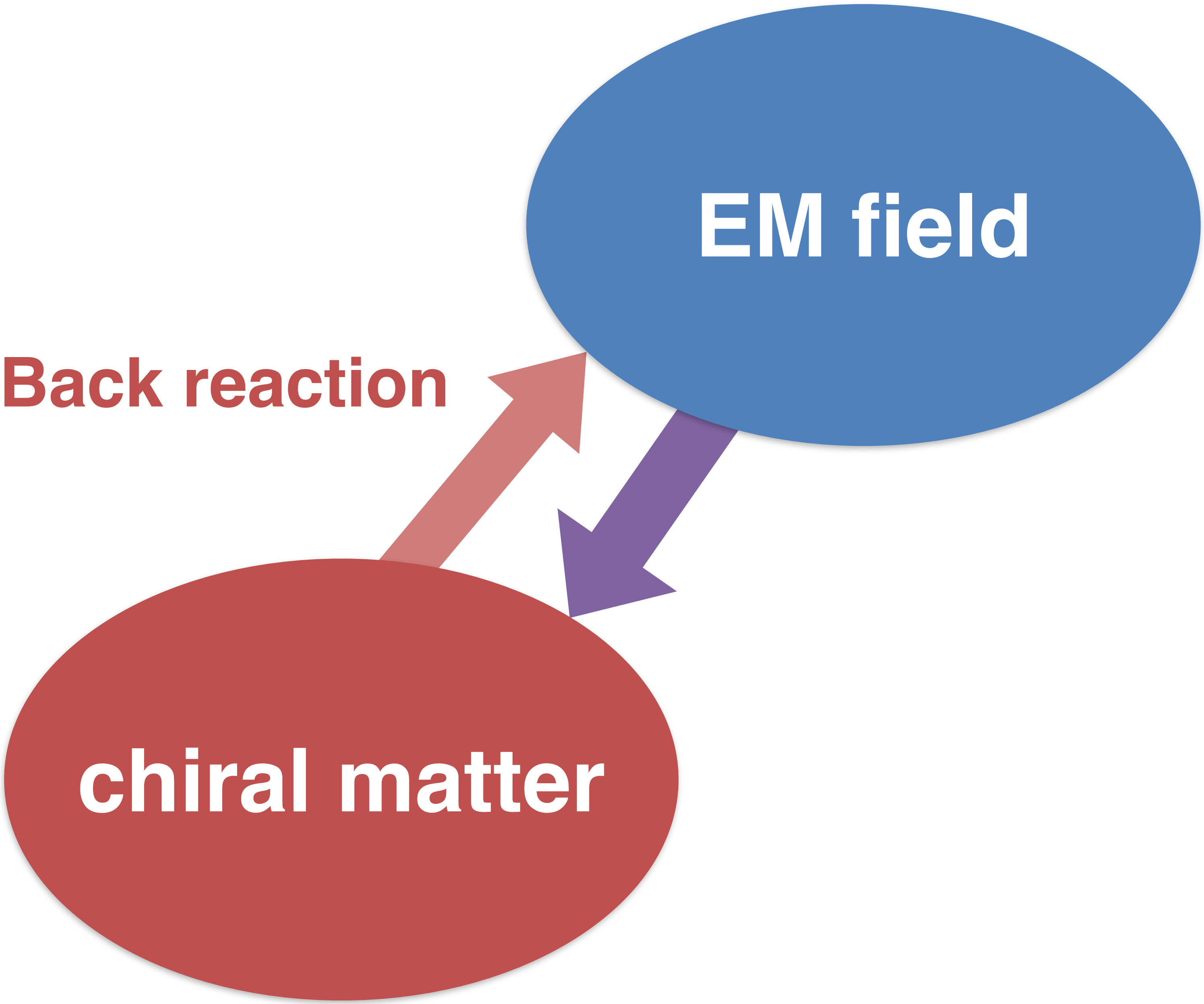
Hydrodynamics with chiral anomaly and charge separation in relativistic heavy ion collisions

Yi Yin[1,*] and Jinfeng Liao[2,3,†]

[1504.06906]






EM field
Back reaction
chiral matter

Magnetic & fermionic helicities

$$\partial_\mu j_A^\mu = C_A \boldsymbol{E} \cdot \boldsymbol{B}$$

$$\Rightarrow \boxed{\frac{d}{dt}\left[\mathcal{H} + \mathcal{H}_F\right] = 0}$$

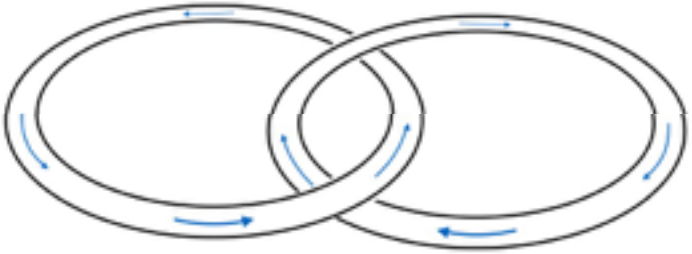
$$\mathcal{H} = \int d^3x \boldsymbol{A} \cdot \boldsymbol{B} \qquad \mathcal{H}_F = \frac{2}{C_A} \int d^3x \, n_A$$

Magnetic helicity

Fermionic helicity

Magnetic helicity knows topology

$$\mathcal{H} = \int d^3x \, \boldsymbol{A} \cdot \boldsymbol{B}$$

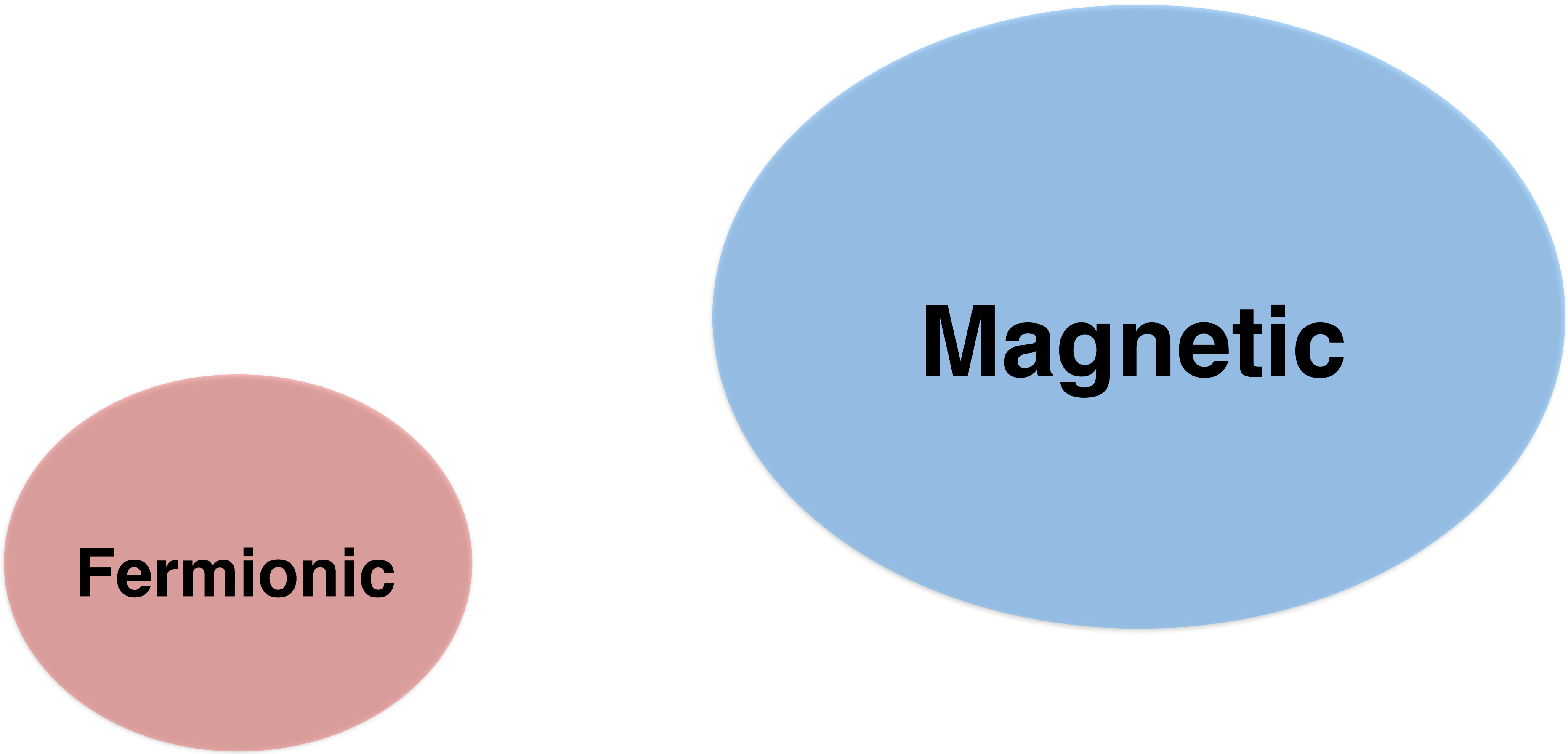


$$= \sum_i \mathcal{S}_i \varphi_i^2 + 2 \sum_{i,j} \mathcal{L}_{ij} \varphi_i \varphi_j$$

Self-linking number

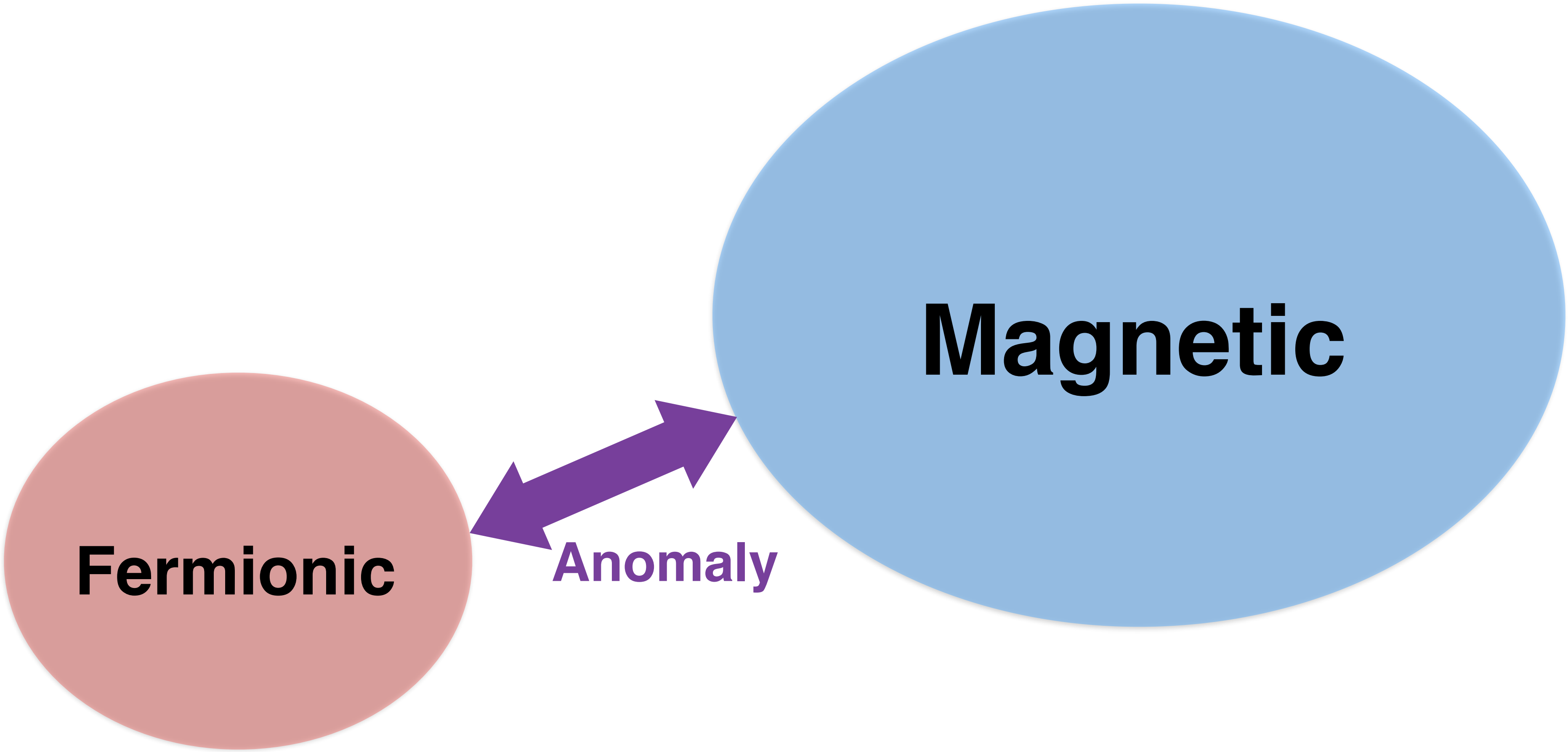
Linking number

Helicity transfer



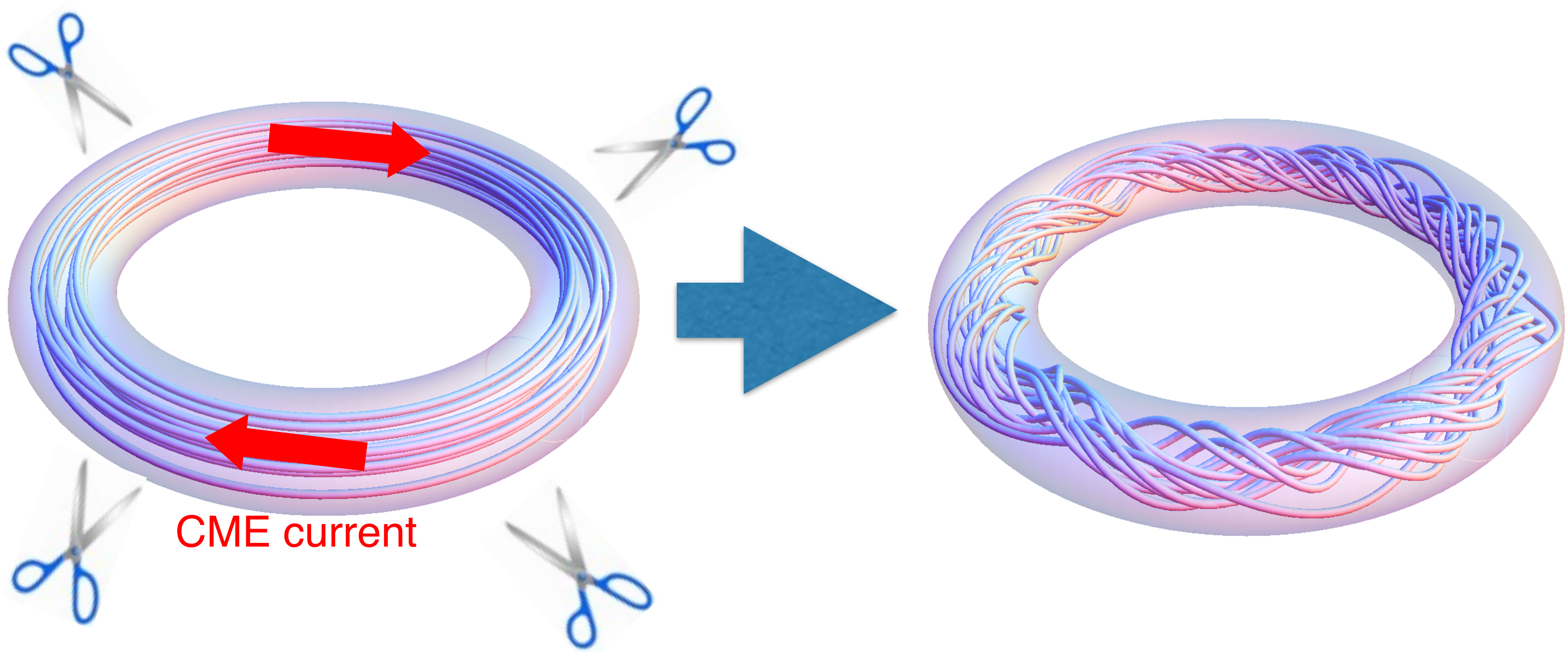
Without anomaly: two helicities are separately conserved

Helicity transfer



With anomaly: two helicities can be transferred
sum is conserved

Anomaly changes the topology of magnetic fields



What is the fate of the magnetic field?

Self-similar inverse cascade of magnetic fields

[Hirono-Khazeev-Yin 1509.07790]

Maxwell-Chern-Simons equations

$$\nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{j}_{\mathrm{EM}} \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = \nabla \cdot \boldsymbol{E} = 0$$

$$\boldsymbol{j}_{\mathrm{EM}} = \sigma \boldsymbol{E} + \boxed{\sigma_A \boldsymbol{B}}$$

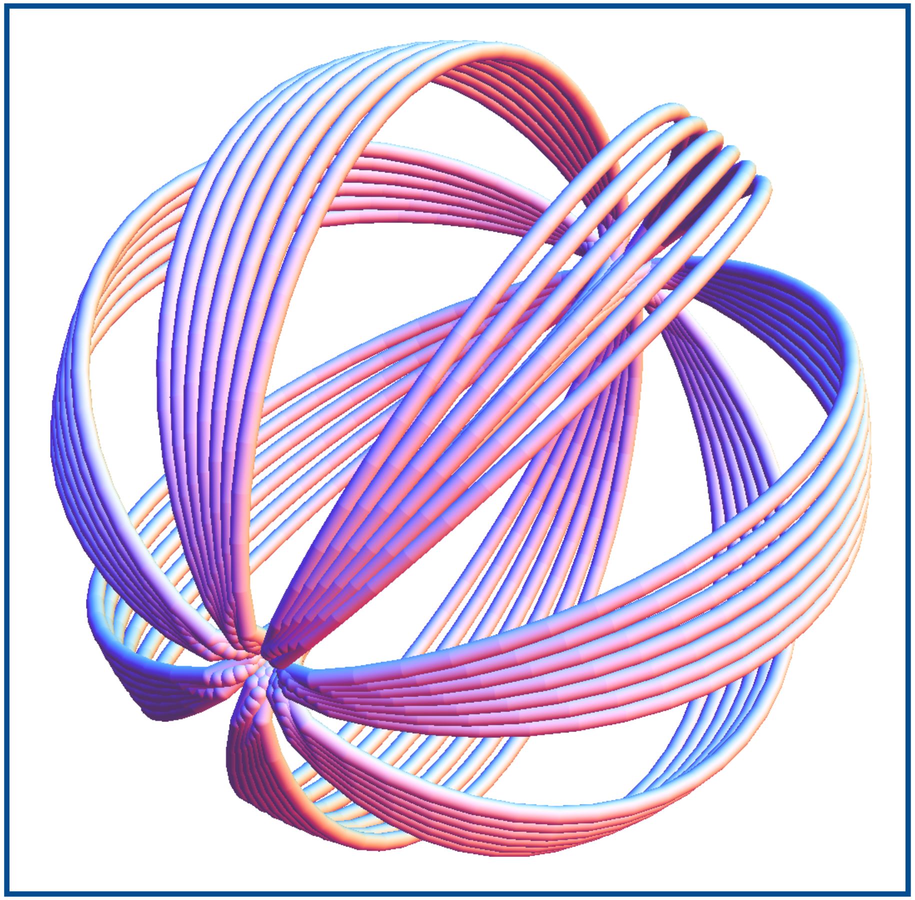
$$\sigma_A(t) = C_A \mu_A(t) = \frac{C_A n_A(t)}{\chi}$$

Axial charge is taken to be spatially constant

Chandrasekhar-Kendall states

$$\nabla \times \boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k) = \pm k \boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k)$$

Sign of helicity



Decomposition of the magnetic fields

$$\boldsymbol{B}(\boldsymbol{x},t) = \sum_{l,m} \int_0^\infty \frac{dk}{\pi} k^2 \left[\alpha_{lm}^+(k,t) \boldsymbol{W}_{lm}^+(\boldsymbol{x};k) + \alpha_{lm}^-(k,t) \boldsymbol{W}_{lm}^-(\boldsymbol{x};k) \right]$$

$$h_m(t) = \int_0^\infty \frac{dk}{\pi}\, k \boxed{\left[g_+(k,t) - g_-(k,t) \right]}$$

Helicity spectral function

$$\mathcal{E}_m(t) = \int_0^\infty \frac{dk}{\pi}\, k^2 \left[g_+(k,t) + g_-(k,t) \right]$$

$$g_\pm(k,t) \equiv \sum_{l,m} |\alpha_{lm}^\pm(k,t)|^2$$

Energy cost to store a unit helicity in $W^\pm(\boldsymbol{x},k)$ is k

EOM for Fourier coeff.

$$\boxed{\partial_t \alpha^{\pm}_{lm}(k,t) = \sigma^{-1}[-k^2 \pm \sigma_A(t) k]\alpha^{\pm}_{lm}(k,t)}$$

If σ_A is positive,

- Negative helicity modes decays exponentially
- Positive helicity modes

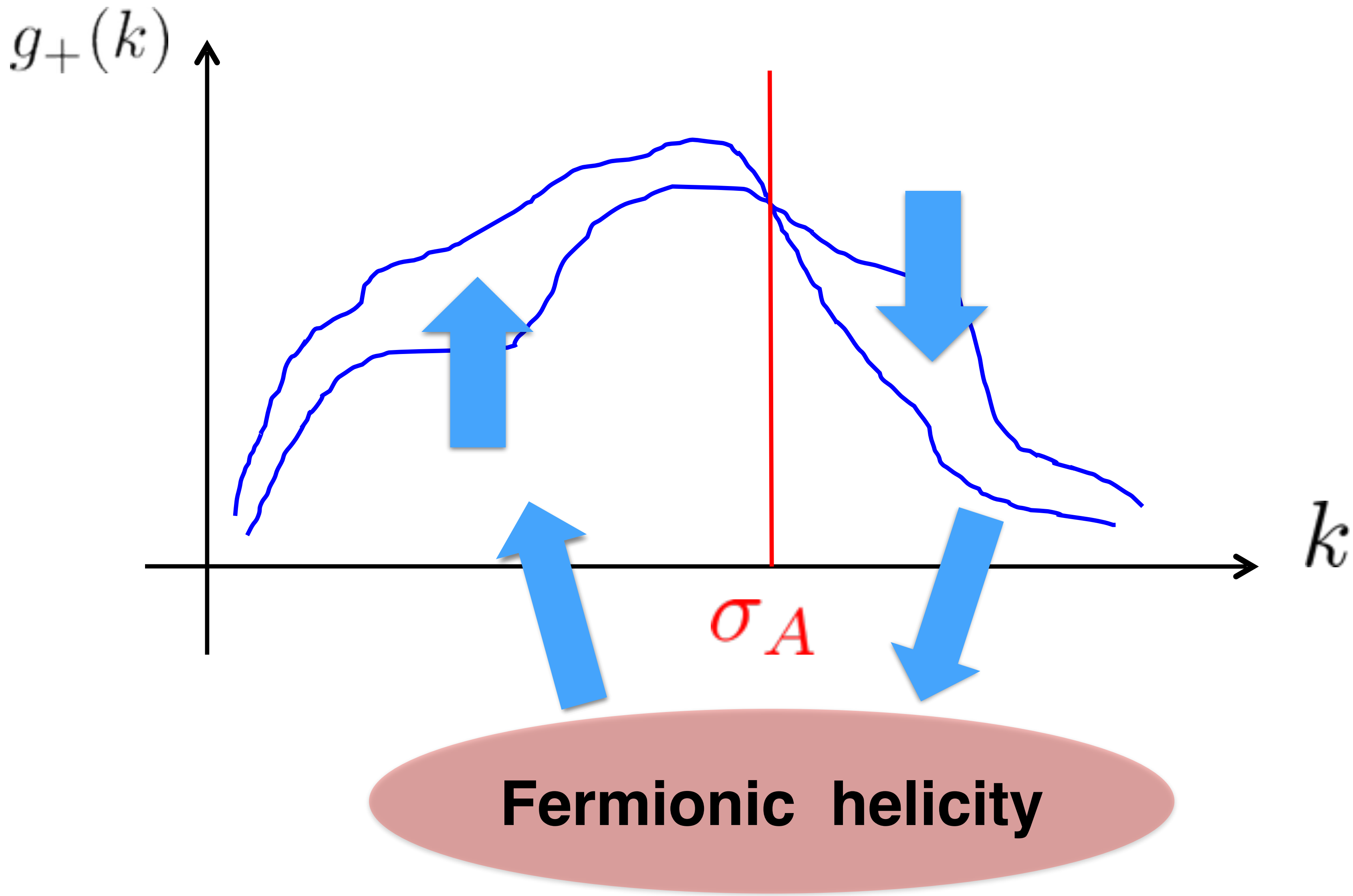
$k > \sigma_A$ decay

$k < \sigma_A$ grow (chiral plasma instability)

$\sigma_A(t)$: energy cost per unit fermionic helicity

$$h_F = \frac{1}{C_A}\int d^3x \, n_A \quad \sigma_A = C_A \mu_A = \frac{\mu_A}{1/C_A}$$

Magnetic helicity spectrum



We will see the fates of

$h_m(t)$: magnetic helicity

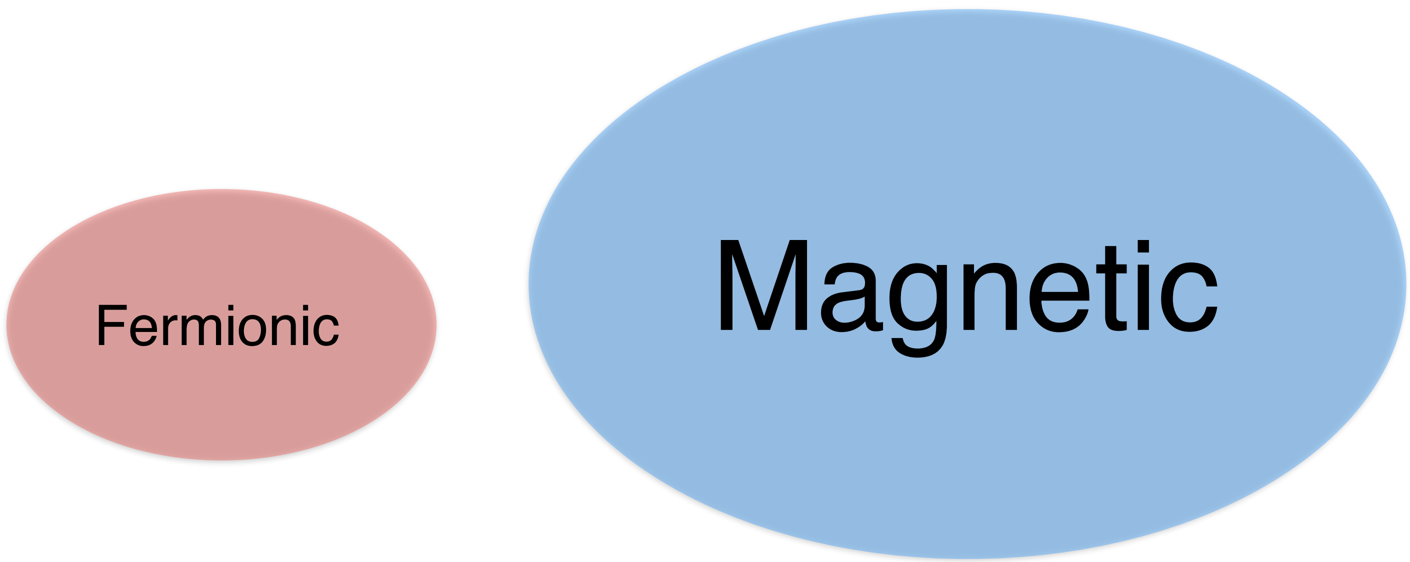
$\sigma_A(t)$: (prop. to) fermionic helicity

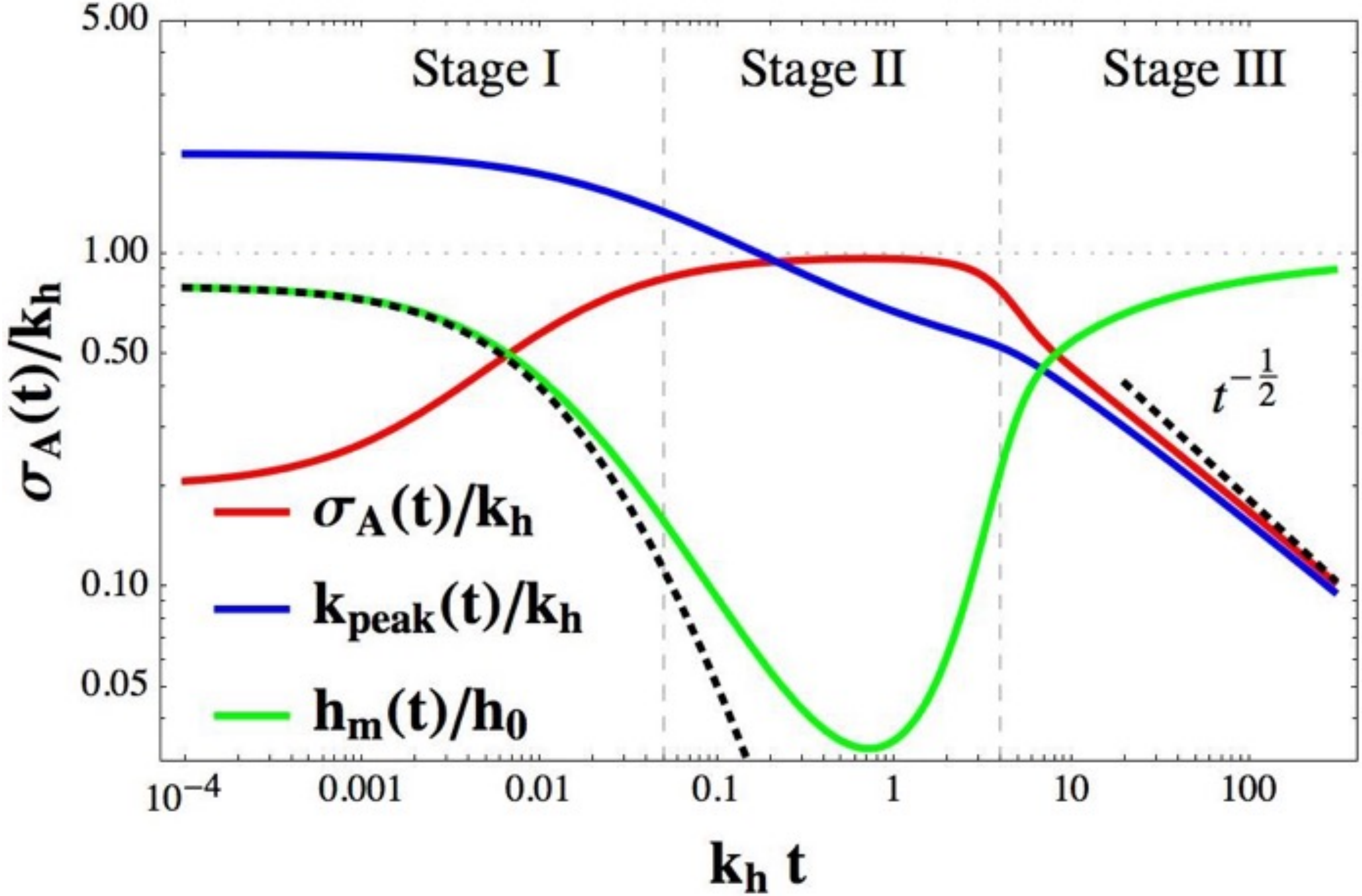
$k_{\rm peak}(t)$: peak momentum of magnetic helicity spectrum

$$h_m(t) = \int_0^\infty \frac{dk}{\pi}\, k \boxed{[g_+(k,t) - g_-(k,t)]}$$

Initial helicity is dominated by magnetic helicity

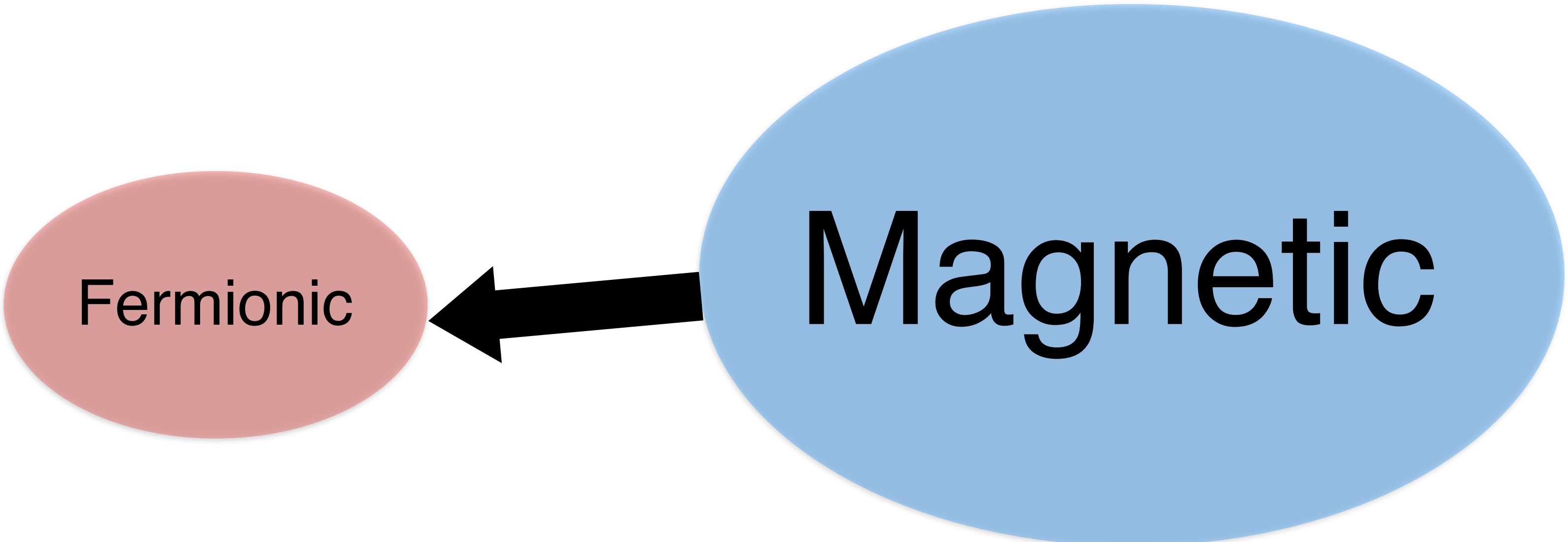
$$h_m(t=0) = 0.8h_0$$



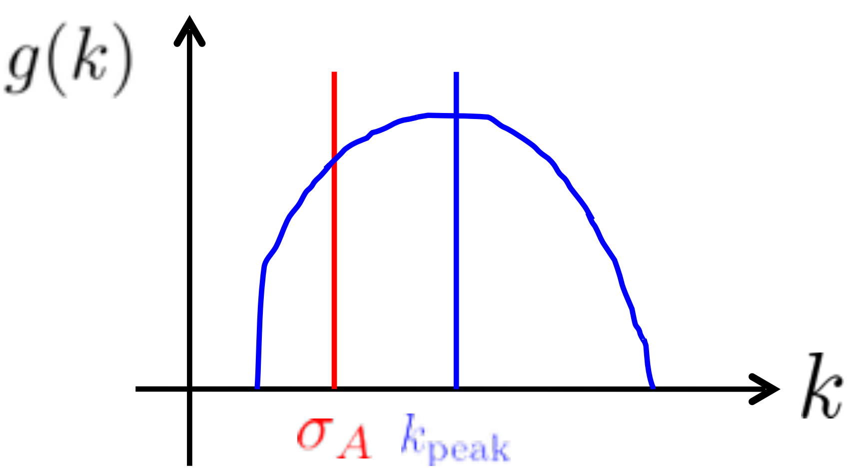


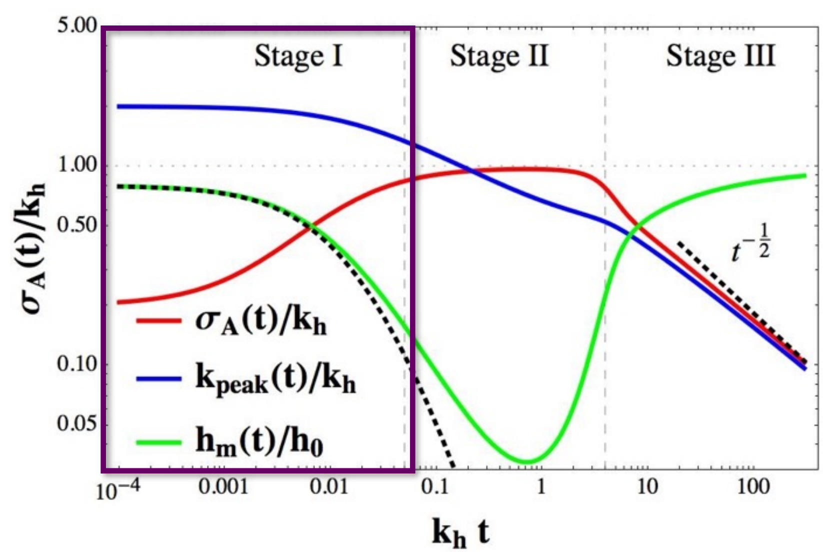
$k_h \equiv \frac{C_A^2 h_0}{\chi V}$ energy cost per unit fermionic helicity, when all the helicity is stored in fermions

Stage I: helicity transferred to fermions

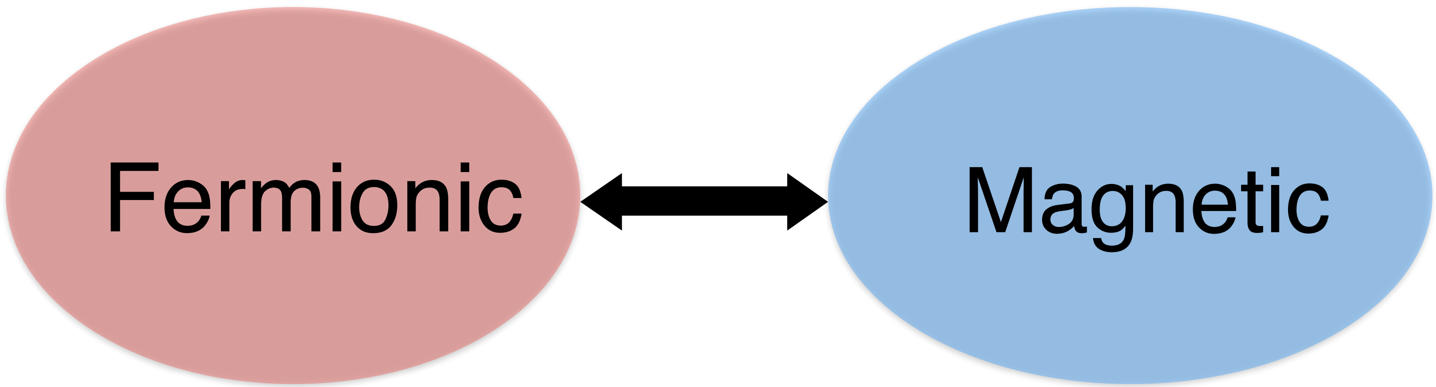


$$\sigma_A < k_{\text{peak}}$$

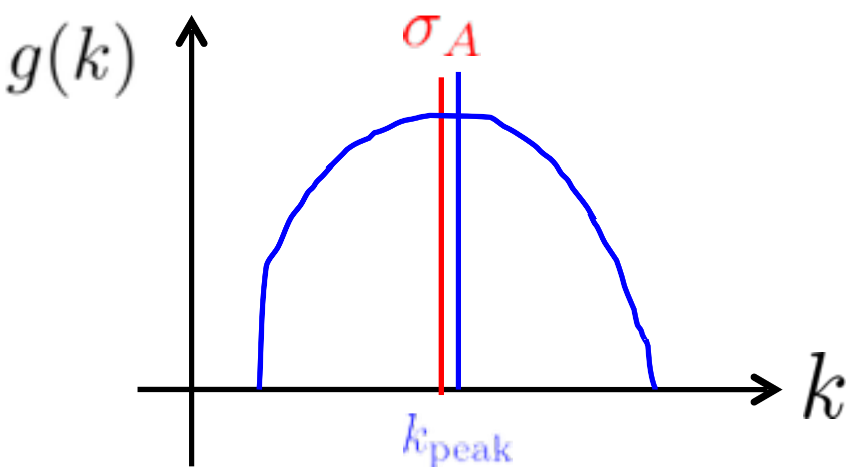


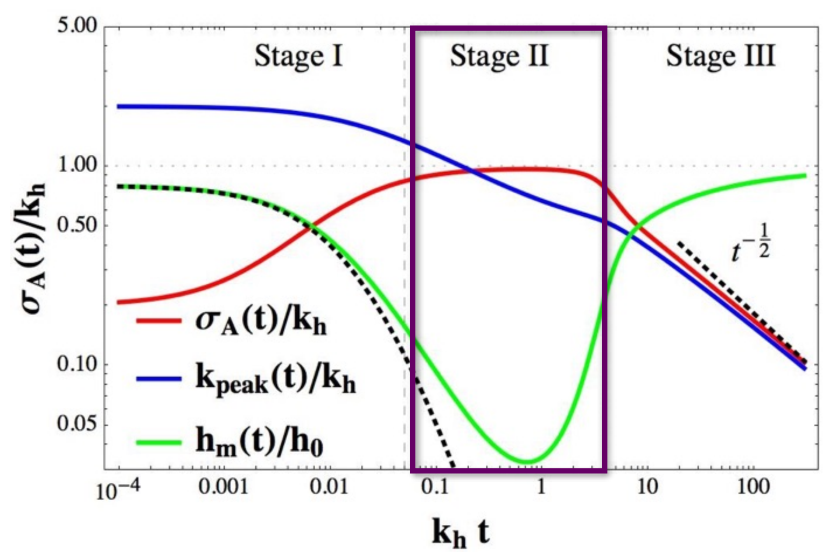


Stage II: intermediate stage

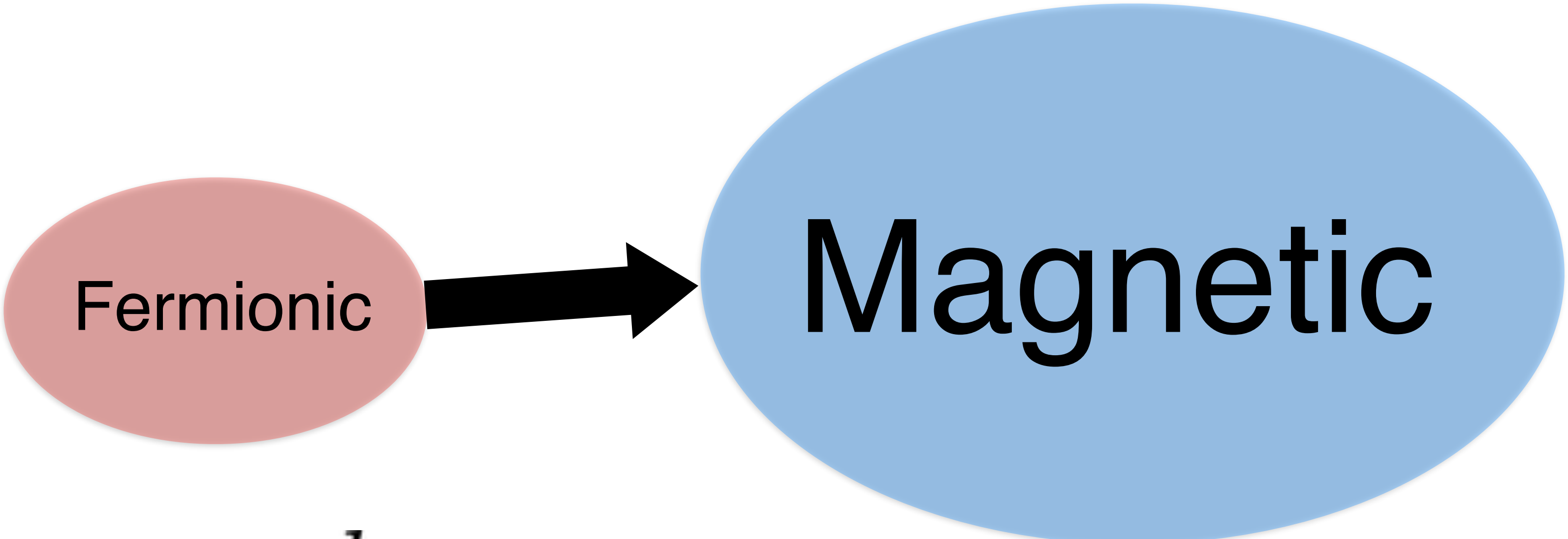


$$\sigma_A \approx k_{\text{peak}}$$

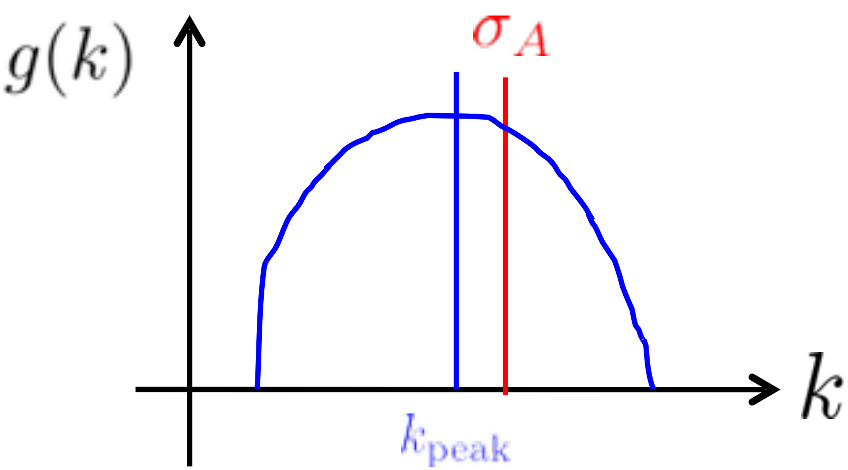


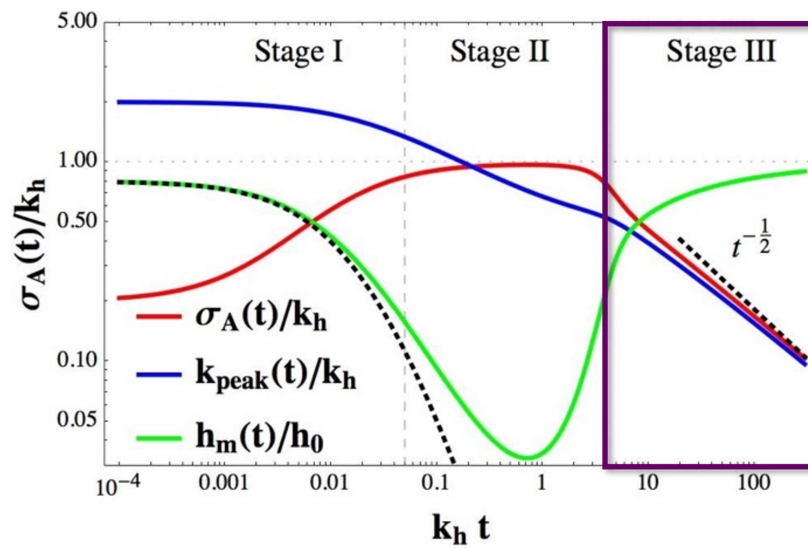


Stage III: self-similar evolution

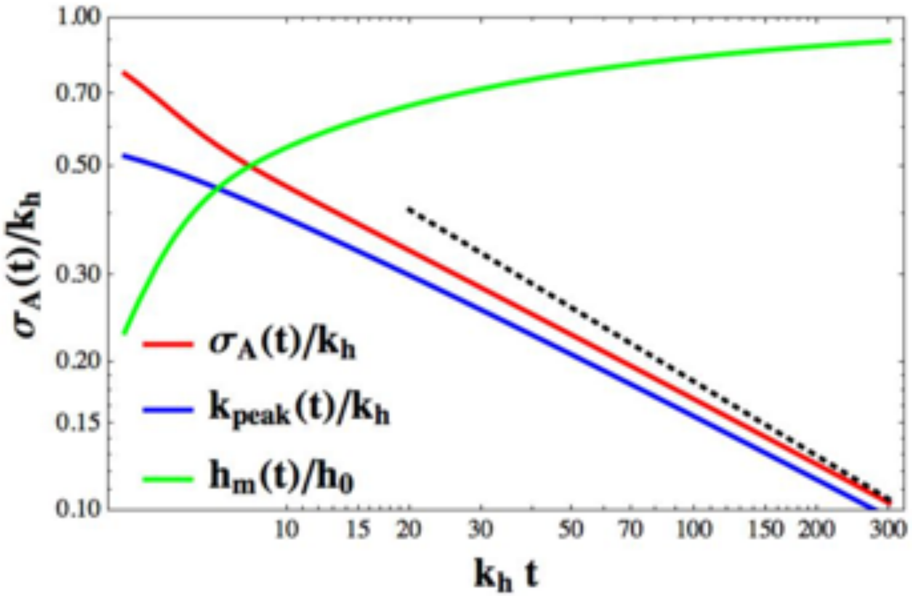


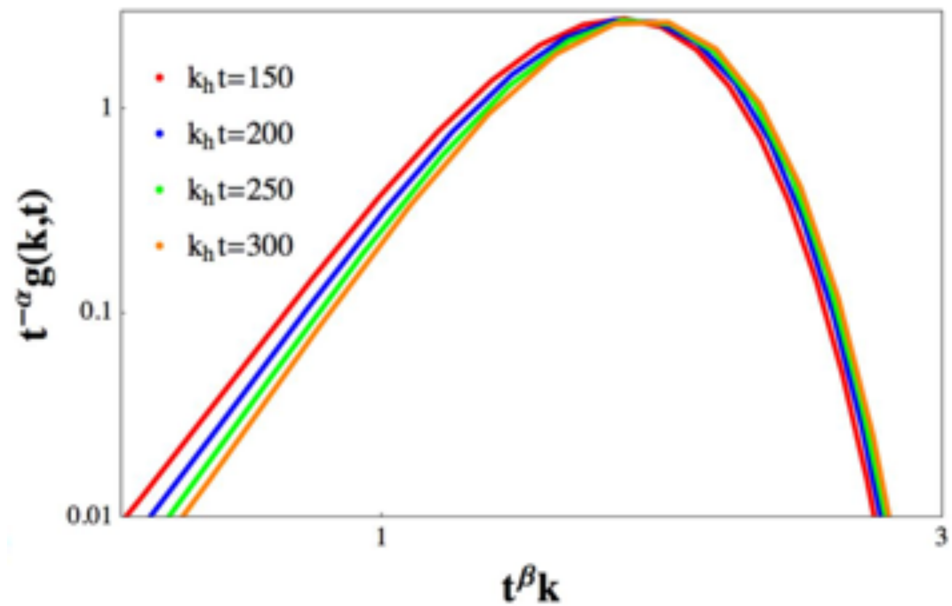
$$\sigma_A > k_{\text{peak}}$$





Stage III: self-similar evolution





$$\sigma_A(t) \sim k_{\mathrm{peak}}(t) \propto t^{-\beta}$$

$$g(k,t) \sim t^{\alpha}\tilde{g}(t^{\beta}k)$$

$$\alpha = 1, \quad \beta = 1/2$$

Stage III: self-similar evolution

$$g(k,t) \propto \exp\left[-2\sigma^{-1}(k - k_{\rm peak})^2 t\right]$$
$$\to \delta(k - k_{\rm peak}(t))$$

The magnetic field approaches a single CK state

Self-similarity extended to chiral MHD

Scaling laws in chiral hydrodynamic turbulence

Naoki Yamamoto [1603.08864]

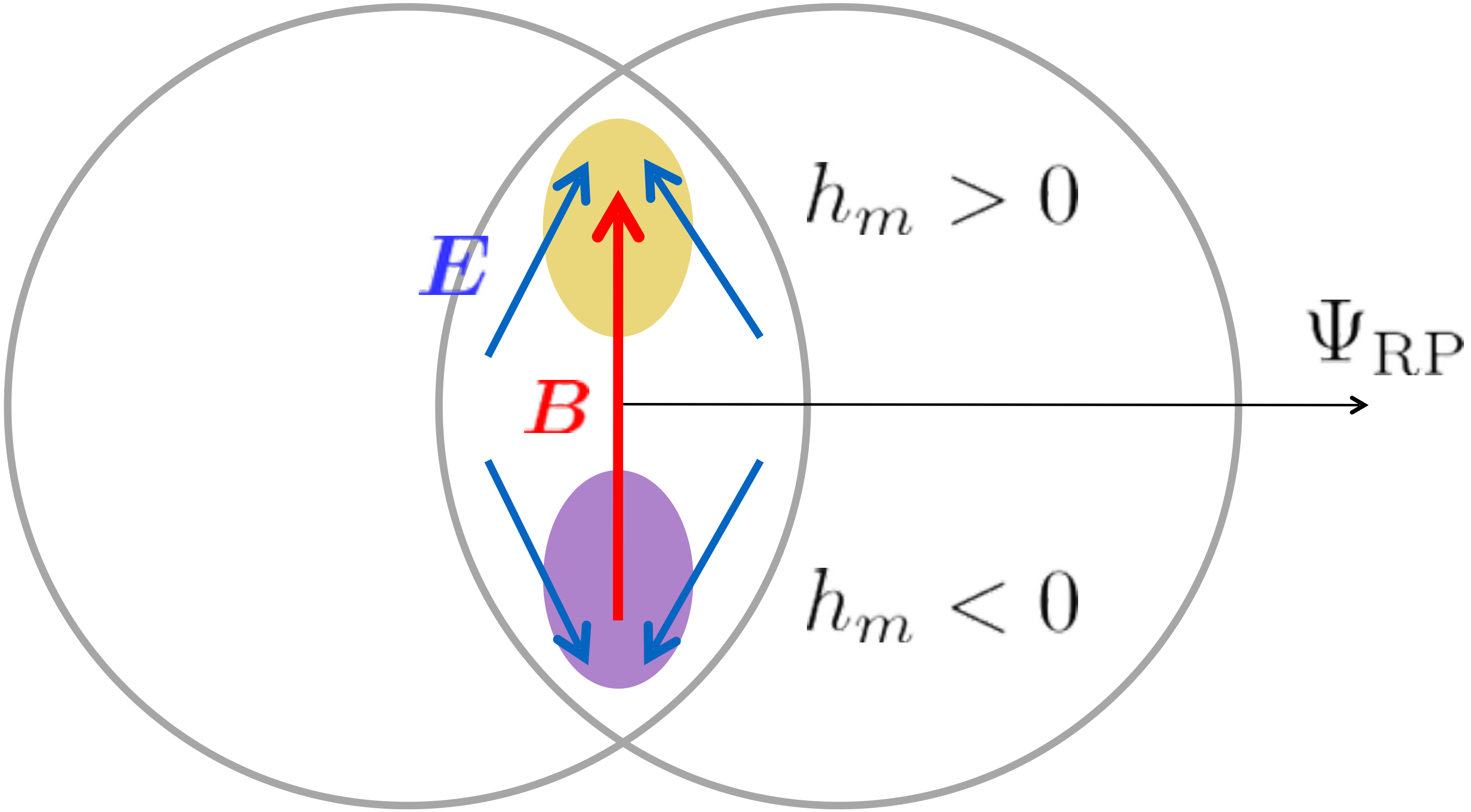
$$\bar{n}_5(t) = \bar{n}_5(t_s)\left(\frac{t_s}{t}\right)^{\frac{1}{2}}$$

$$\xi_B(t) = \xi_B(t_s)\left(\frac{t}{t_s}\right)^{\frac{1}{2}}$$ magnetic correlation length

$$\xi_v(t) = \xi_v(t_s)\left(\frac{t}{t_s}\right)^{\frac{1}{2}}$$ kinetic correlation length

Realized in heavy-ion collisions?

Helical magnetic field in HIC



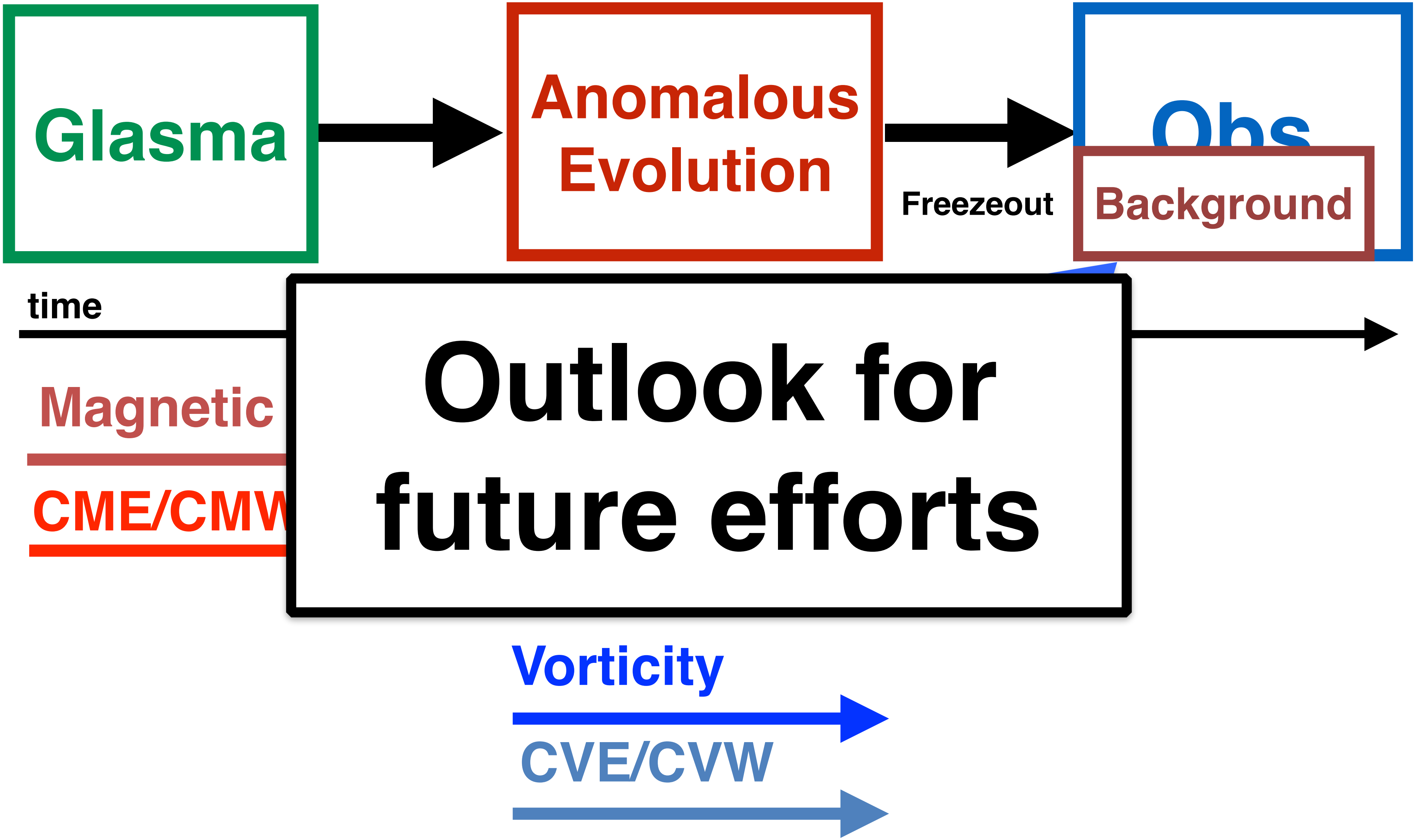
Condition for the inverse cascade to occur

$$eB = c_B m_\pi^2 \quad L \approx 10 \; [\mathrm{fm}]$$

$$c_B > 100$$ RHIC

$$c_B > 26$$ LHC

Outlook for future efforts

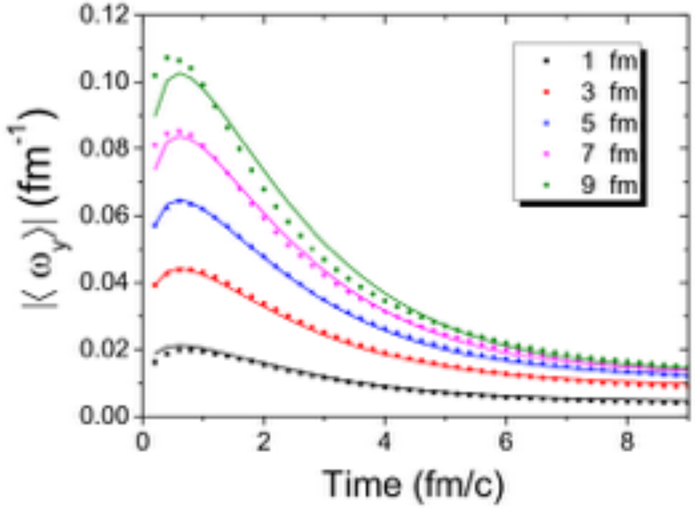


Toward the detection of anomalous effects

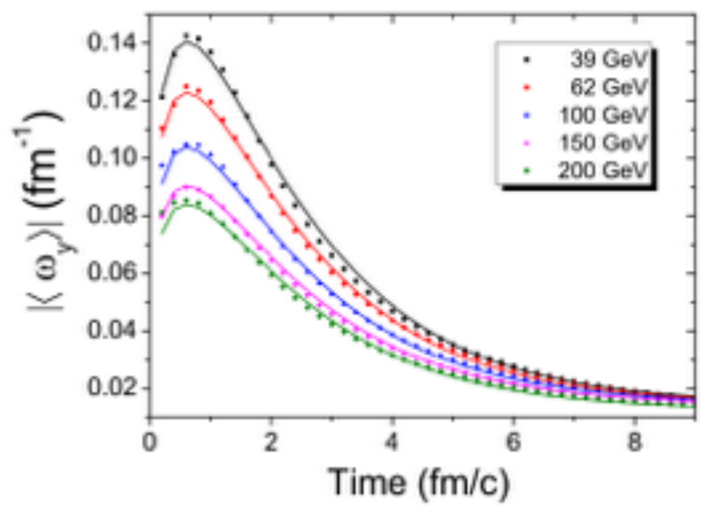
- **Challenges**
  - Lifetime of B
  - Initial axial charge density & CME current
  - Chiral vortical effect
  - Initial charge densities
  - Modeling of background effects like LCC

- **Chiral magnetohydrodynamics**
  - Solve anomalous hydro + Maxwell equations consistently

Vorticity from HIJING/AMPT

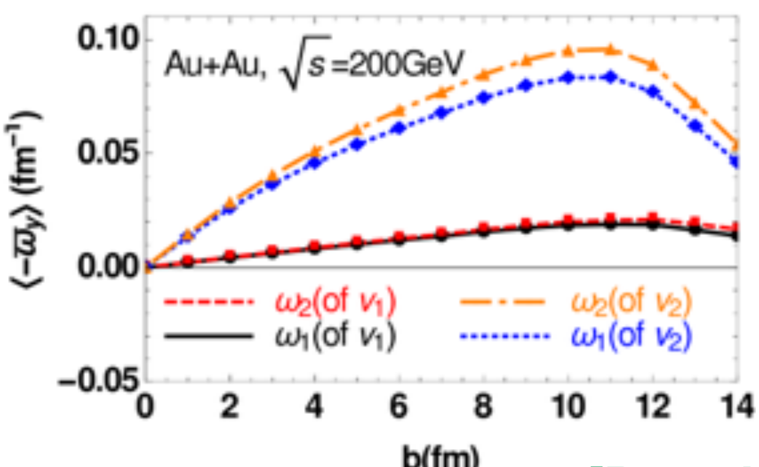


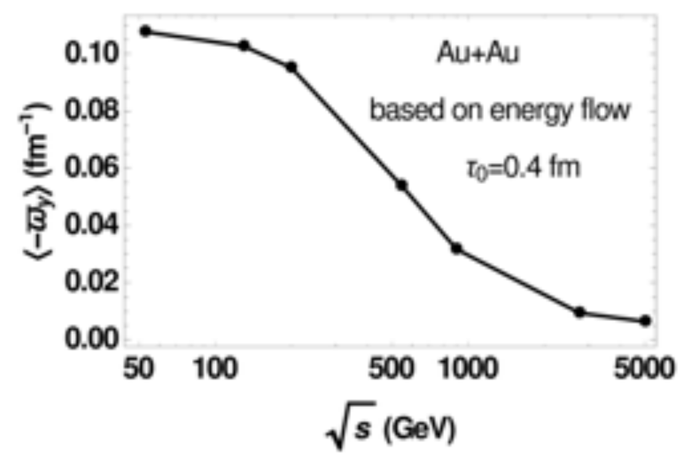
Impact parameter dep.



Collision energy dep.

[Jiang-Lin-Liao 1602.06580]





[Deng-Huang 1603.06117]

Chiral magnetic effect and anomalous transport from real-time lattice simulations

Niklas Müller,[1,*] Sören Schlichting,[2,†] and Sayantan Sharma[2,‡] [1606.00342]

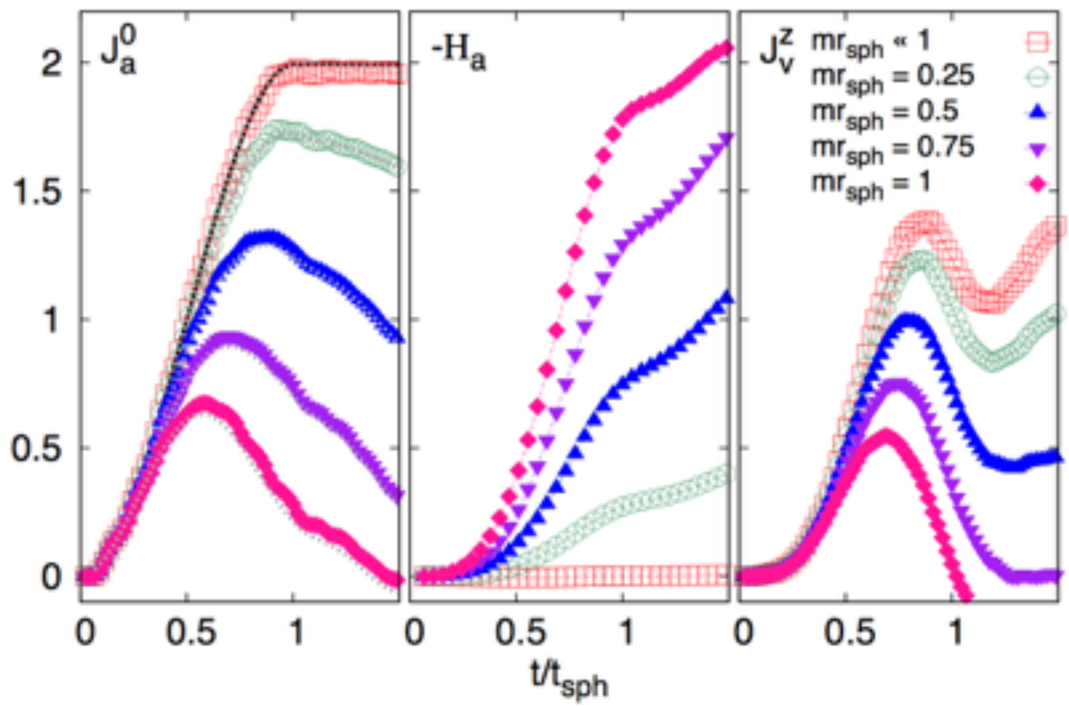


FIG. 3. Evolution of the axial charge J_a^0 (left), pseudo-scalar condensate H_a (center) and vector current J_v^z (right) for different values of the fermion mass $mr_{\rm sph} = 3 \cdot 10^{-3}, 0.25, 0.5, 0.75, 1.0$. Comparison with the gray lines in the left panel demonstrates that the axial anomaly relation (6) is satisfied in all cases.

Toward the detection of anomalous effects

- **Challenges**
  - Lifetime of B
  - Initial axial charge density & CME current
  - Chiral vortical effect
  - Initial charge densities
  - Modeling of background effects like LCC

- **Chiral magnetohydrodynamics**
  - Solve anomalous hydro + Maxwell equation consistently