Resummation in High Energy Scattering
-- BFKL vs Sudakov

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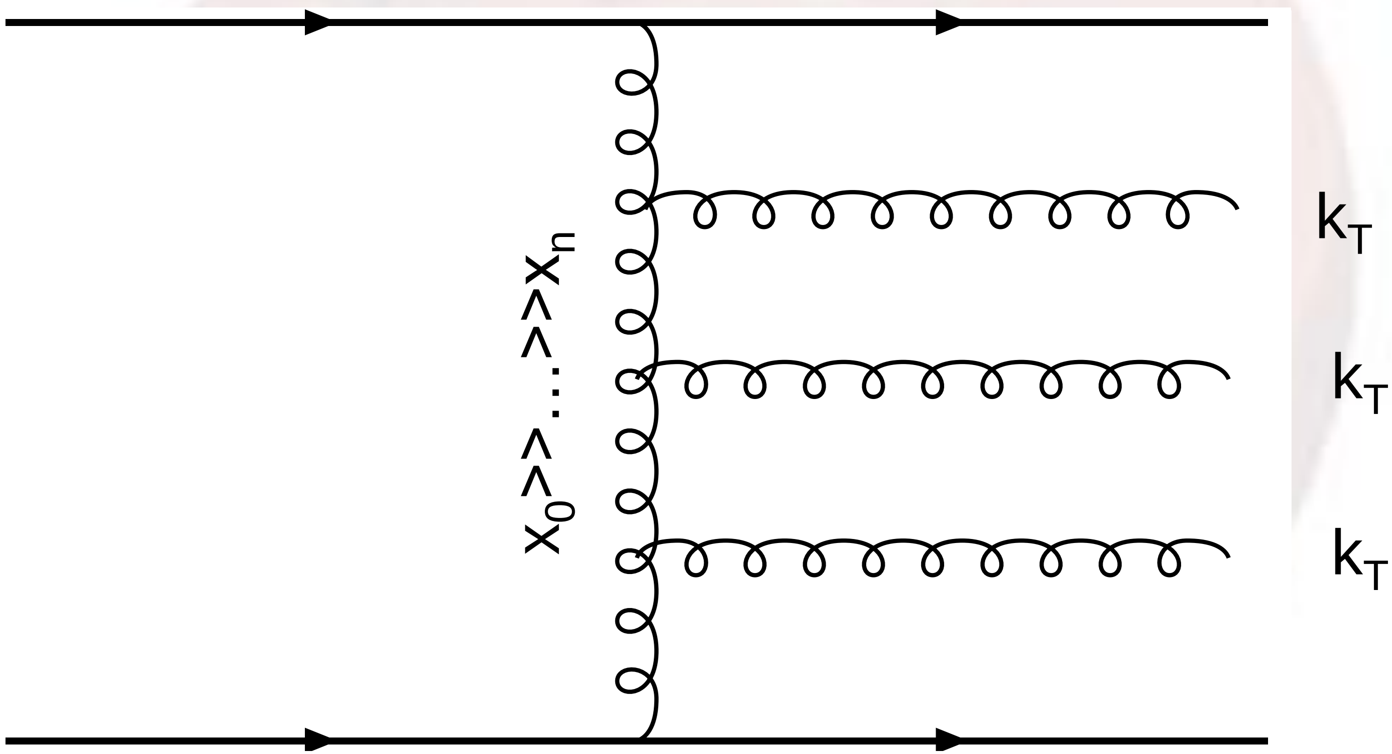
Refs: Mueller, Xiao, Yuan, PRL110, 082301 (2013); Phys.Rev. D88 (2013) 114010.

High energy scattering



BFKL: $$\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \mathcal{K} \otimes \mathcal{F}$$ Un-integrated gluon distribution

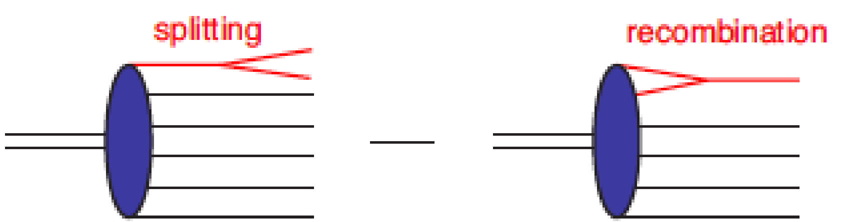
Non-linear term at high density

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

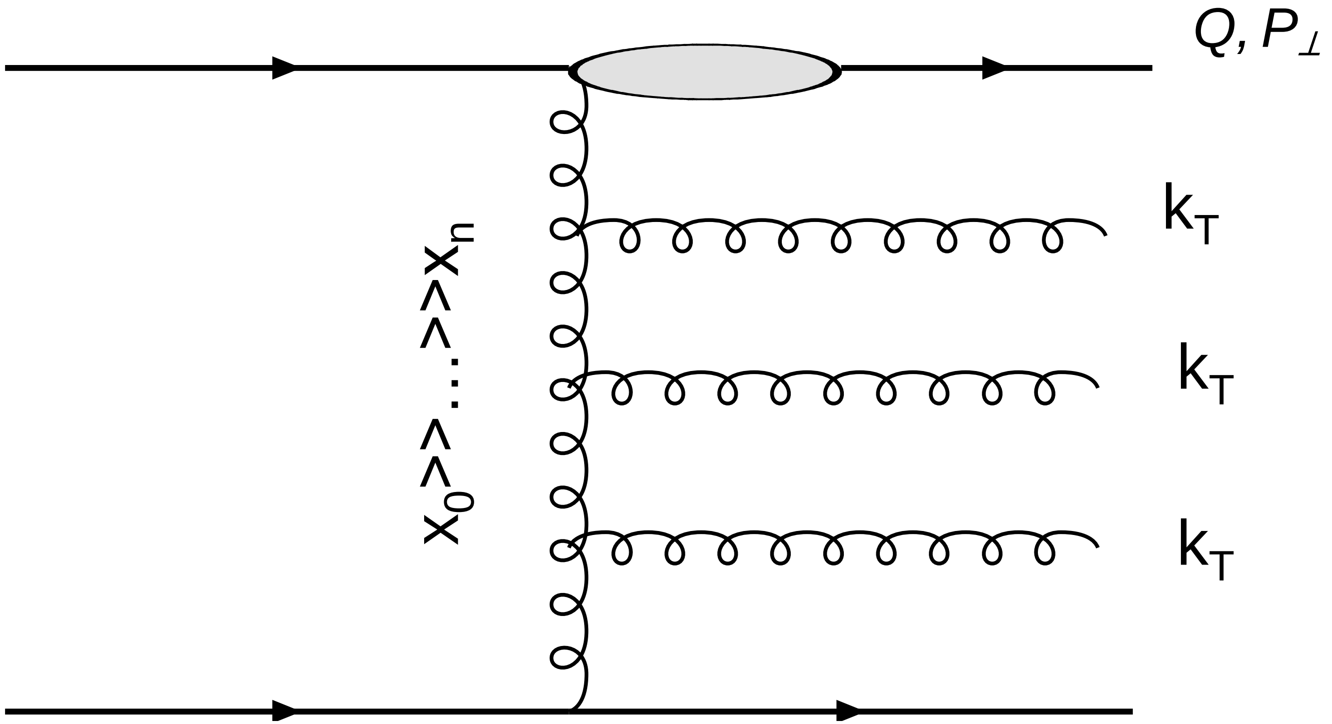
$$\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x, r_T)$$

- Balitsky-Kovchegov: Non-linear term, 98

$$\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x, r_T) - \alpha_s [N(x, r_T)]^2.$$

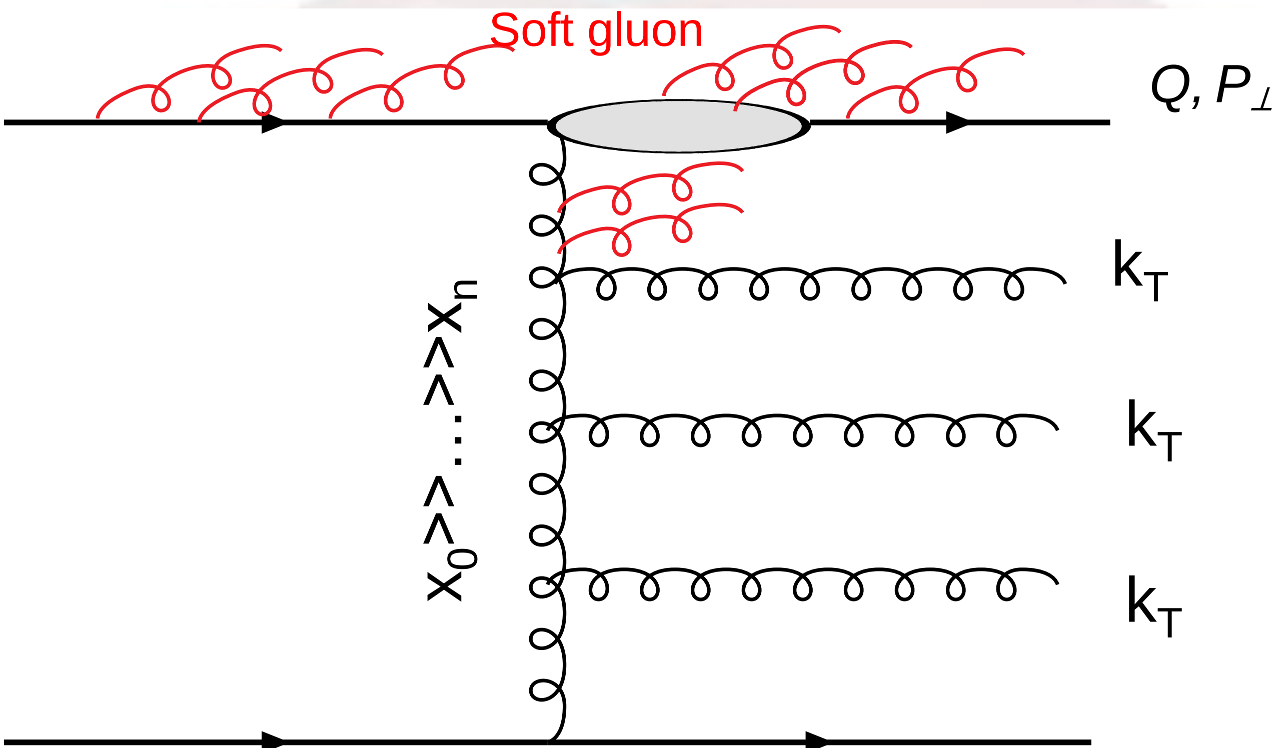


Hard processes at small-x



- Manifest dependence on un-integrated gluon distributions
  - Dominguiz-Marquet-Xiao-Yuan, 2010

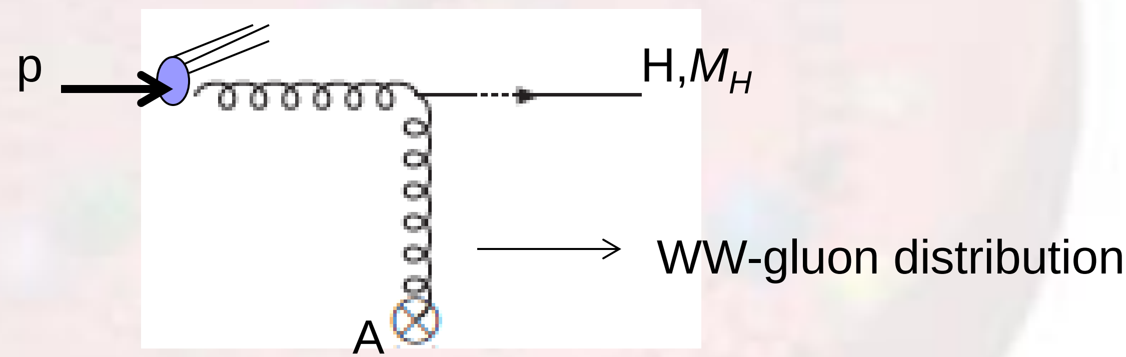
Additional dynamics comes in



- BFKL vs Sudakov resummations (LL)

Sudakov resummation at small-x

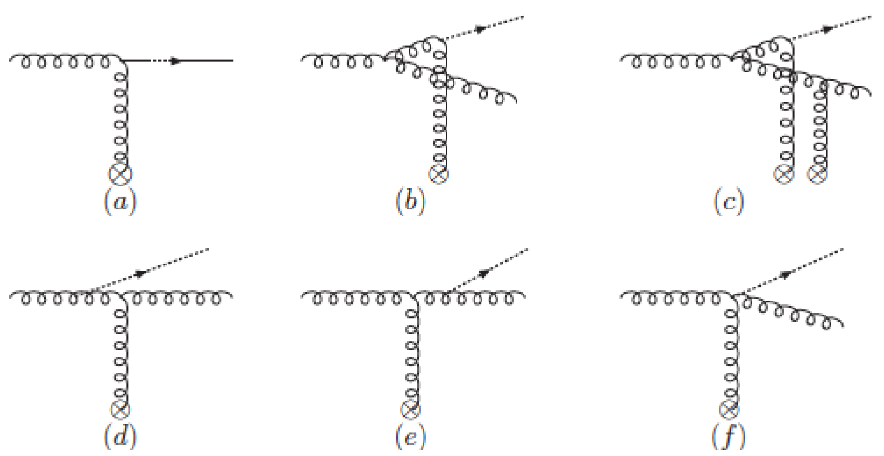
- Take massive scalar particle production p+A->H+X as an example to demonstrate the double logarithms, and resummation

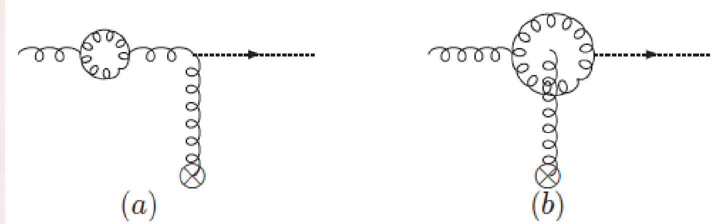


$$\frac{d\sigma^{(\mathrm{LO})}}{dy\,d^2k_\perp} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp\cdot r_\perp} x_0 g_p(x_0) S^{(WW)}(x_\perp, x'_\perp)$$

$$S_Y^{WW}(x_\perp,y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp)U^\dagger(y_\perp)\partial_\perp^\beta U(y_\perp)U^\dagger(x_\perp)\right]\right\rangle_Y$$

Explicit one-loop calculations





$$x_0 g_p(x_0) \int \frac{d\xi}{\xi} \mathbf{K}_{DMMX} \otimes S^{WW}(x_\perp, y_\perp) + \left(-\frac{1}{\epsilon}\right) S^{WW}(x_\perp, y_\perp) \mathcal{P}_{g/g} \otimes x_0 g(x_0) \ ,$$

- Collinear divergence ➔ DGLAP evolution
- Small-x divergence ➔ BK-type evolution

Dominguiz-Mueller-Munier-Xiao, 2011

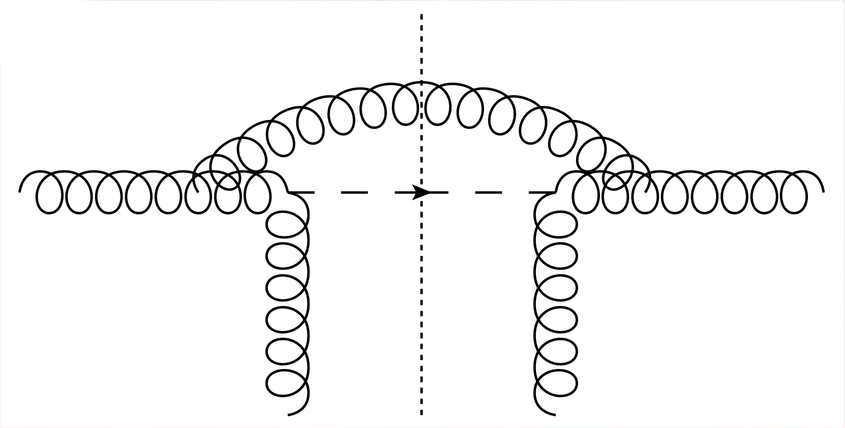
Soft vs Collinear gluons

- Radiated gluon momentum

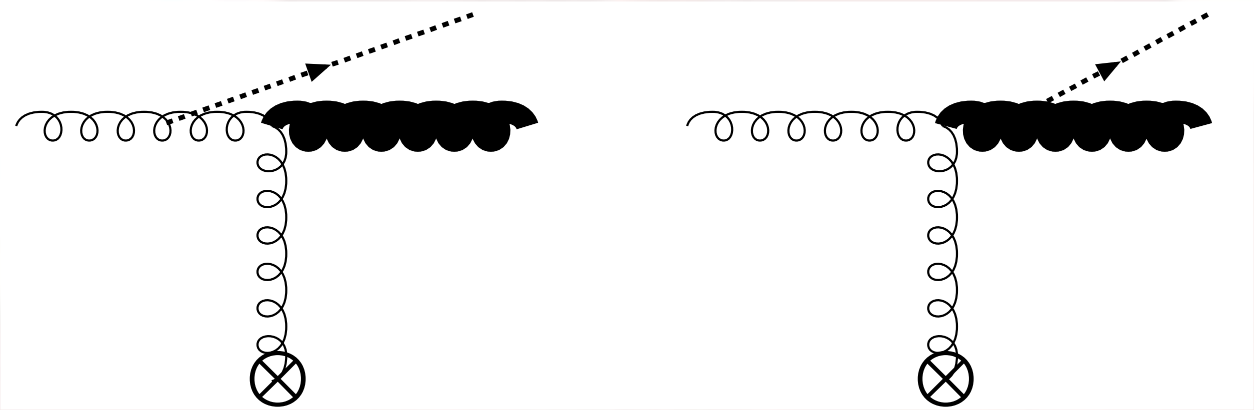
$$k_g = \alpha_g p_1 + \beta_g p_2 + k_{g\perp} \ ,$$

- Soft gluon, $\alpha \sim \beta \ll 1$
- Collinear gluon, $\alpha \sim 1$, $\beta \ll 1$
- Small-x collinear gluon, $1-\beta \ll 1$, $\alpha \to 0$
  - Rapidity divergence

Examples



- Contributes to
  - Collinear gluon from the proton
  - Collinear gluon from nucleus
  - Soft gluon to Sudakov double logs



- Only contributes to small-x collinear gluon

Final result

- Double logs at one-loop order

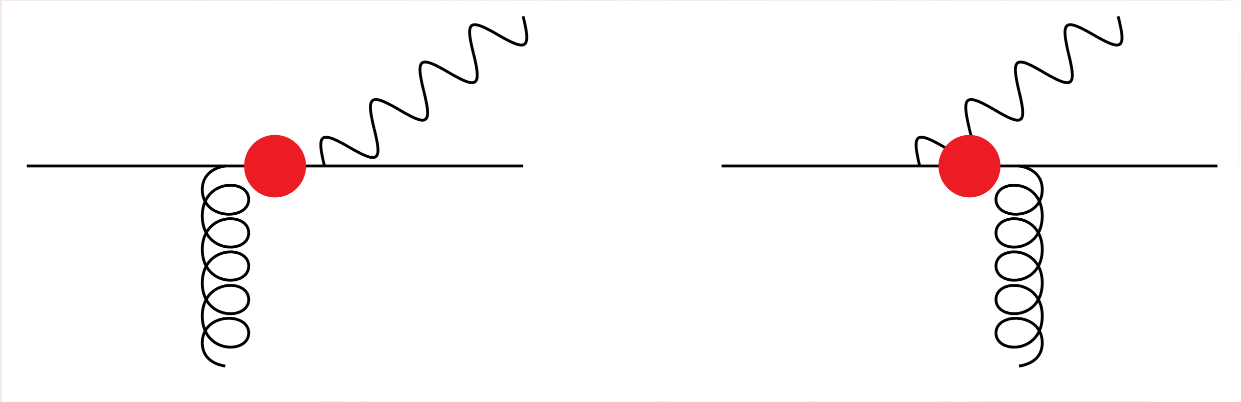
$$\frac{d\sigma^{(\mathrm{LO+NLO})}}{dyd^2k_\perp}|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} S^{WW}_{Y=\ln 1/x_a}(x_\perp, x'_\perp) x g_p(x, \mu^2 = \frac{c_0^2}{r_\perp^2}) \left\{1 + \frac{\alpha_s}{\pi} C_A \left[\beta_0 \ln \frac{Q^2 r_\perp^2}{c_0^2} - \frac{1}{2}\left(\ln \frac{Q^2 r_\perp^2}{c_0^2}\right)^2 + \frac{\pi^2}{2}\right]\right\} ,$$

- Include both BFKL (BK) and Sudakov

$$\frac{d\sigma^{(\mathrm{resum})}}{dyd^2k_\perp}|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} e^{-S_{sud}(Q^2, r_\perp^2)} S^{WW}_{Y=\ln 1/x_a}(x_\perp, x'_\perp) \times x g_p(x, \mu^2 = c_0^2/r_\perp^2) \left[1 + \frac{\alpha_s}{\pi} \frac{\pi^2}{2} N_c\right] ,$$

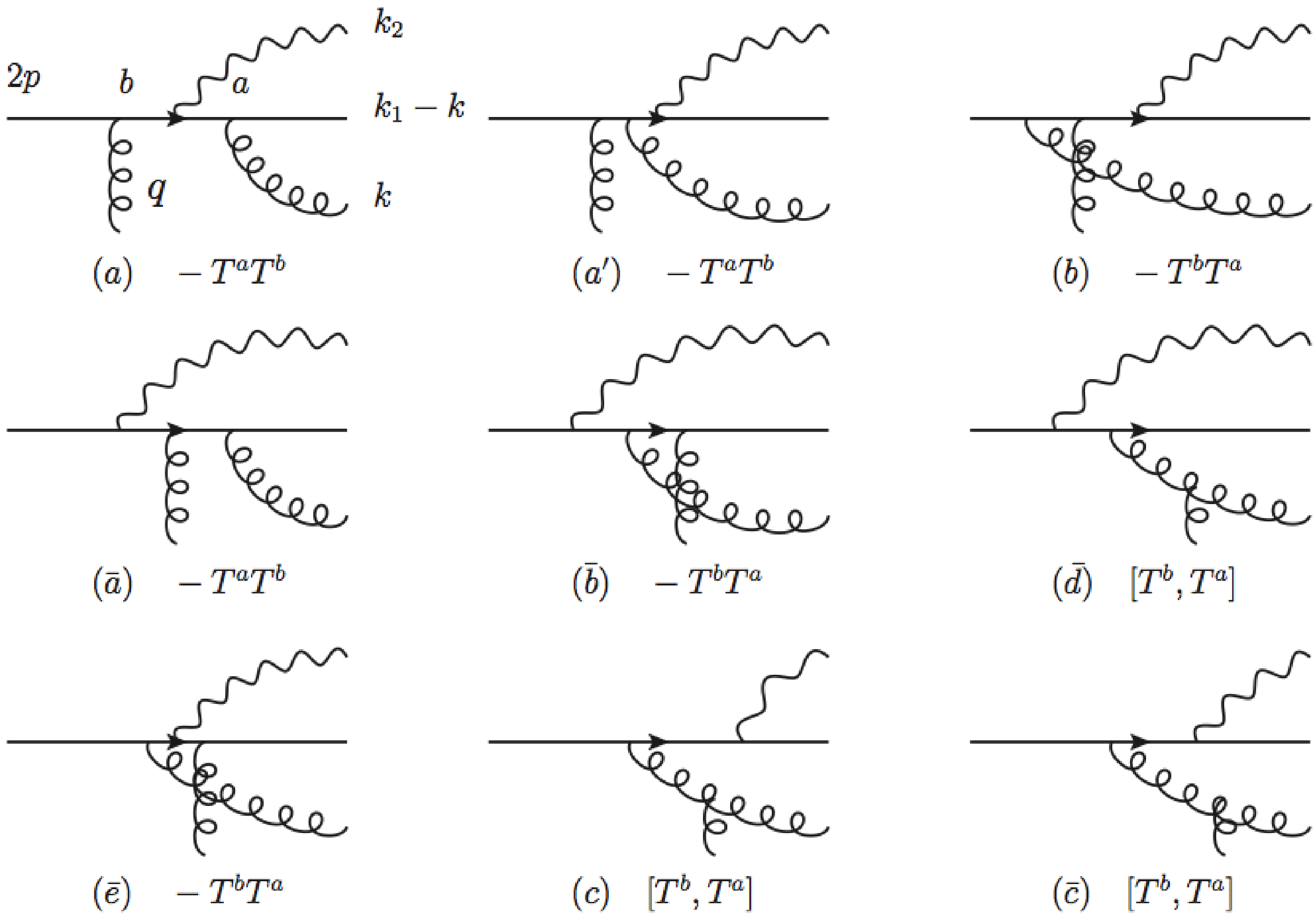
Photon-Jet correlation

- Leading order

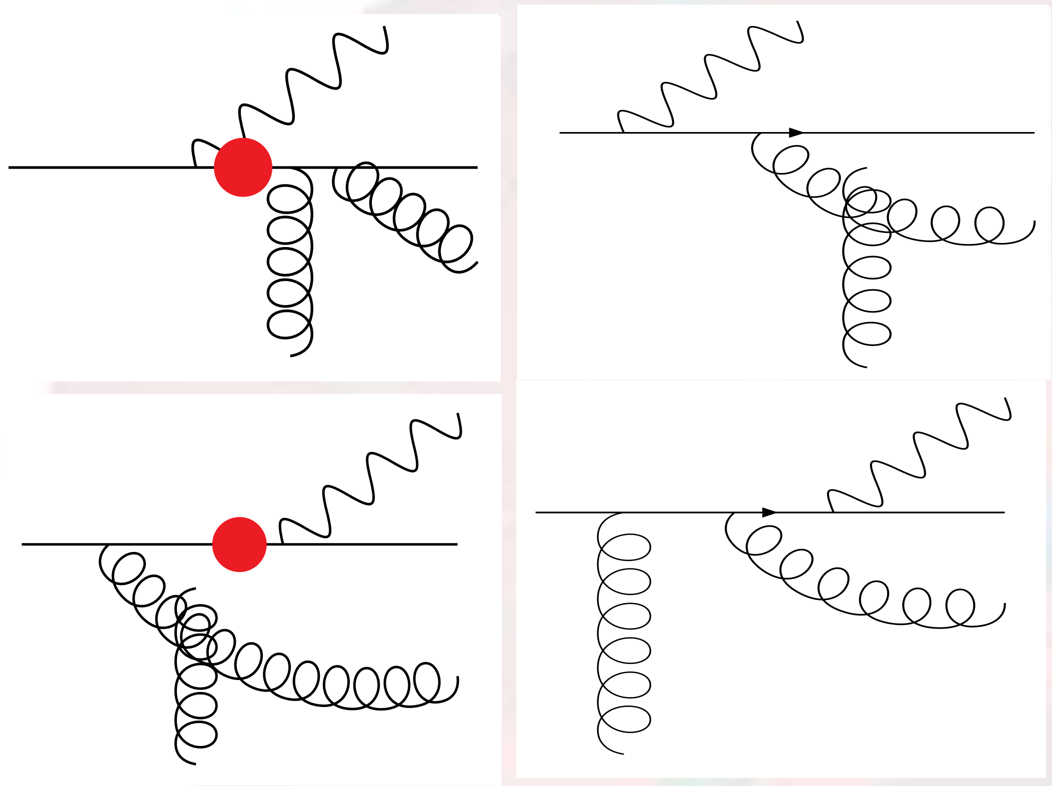


Dipole gluon distribution

One gluon radiation (real)



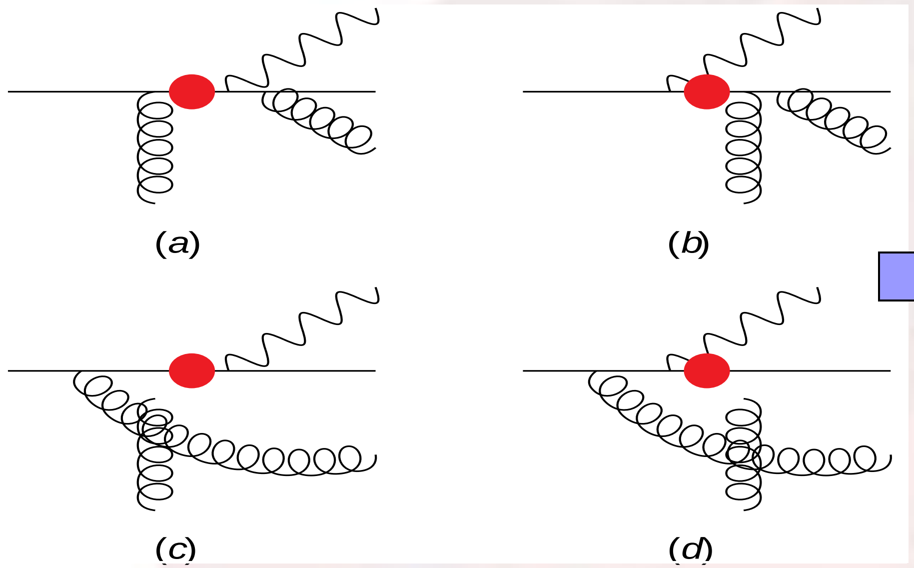
BK-evolution



These two do not Contribute to soft Gluon radiation

$$\frac{\partial}{\partial Y} S_Y^{(2)}(x_\perp, y_\perp) = -\frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2 b_\perp (x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2 (y_\perp - b_\perp)^2} \left[S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp) \right]$$

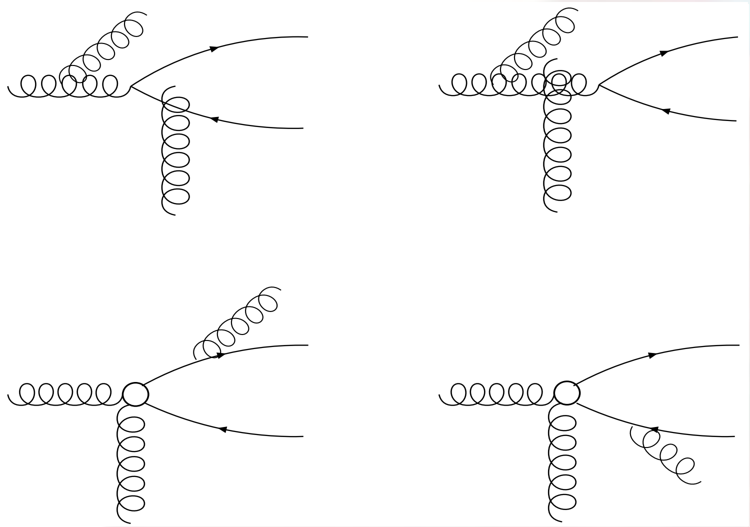
Soft gluon radiation



$$\left(-\frac{\alpha_s}{2\pi}\frac{C_F+C_A}{2}\right)\ln^2\left(\frac{Q^2(x_\perp-y_\perp)^2}{c_0^2}\right)$$

- A^2 from (a,b) contribute to $C_F/2$ (jet)
- A^2 from (c,d) contribute to C_F
- Interference contribute to 1/2Nc

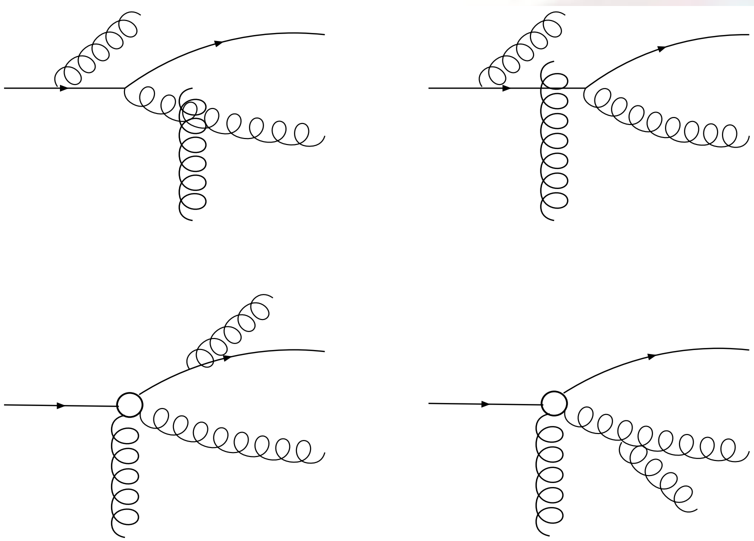
$gg \rightarrow qq$



$$\Rightarrow \left(-\frac{\alpha_s}{2\pi}N_c\right)\ln^2\left(\frac{Q^2(x_\perp - y_\perp)^2}{c_0^2}\right)$$

- $|A_1|^2 \rightarrow C_A$, $|A_2|^2 \rightarrow C_F/2$, $|A_3|^2 \rightarrow C_F/2$
- $2A_1^*(A_2+A_3) \rightarrow -Nc/2$
- $2A_2^*A_3$, 1/Nc suppressed

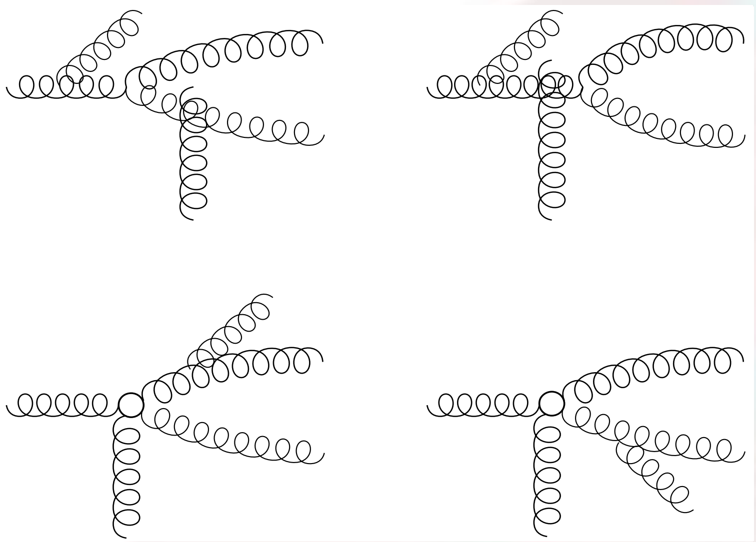
$qg \to qg$



$$\Rightarrow \left(-\frac{\alpha_s}{2\pi}\frac{C_F+C_A}{2}\right)\ln^2\left(\frac{Q^2(x_\perp-y_\perp)^2}{c_0^2}\right)$$

- $|A_1|^2 \to C_F$, $|A_2|^2 \to C_F/2$, $|A_3|^2 \to C_A/2$
- $2A_3^*(A_1+A_2) \to -N_c/2$
- $2A_1^*A_2$, large Nc suppressed

$gg \rightarrow gg$



$$\Rightarrow \left(-\frac{\alpha_s}{2\pi}N_c\right)\ln^2\left(\frac{Q^2(x_\perp - y_\perp)^2}{c_0^2}\right)$$

- $|A_1|^2 \rightarrow C_A$, $|A_2|^2 \rightarrow C_A/2$, $|A_3|^2 \rightarrow C_A/2$
- $2A_1^*(A_2+A_3)+2A_2^*A_3 \rightarrow -N_c$

Sudakov leading double logs

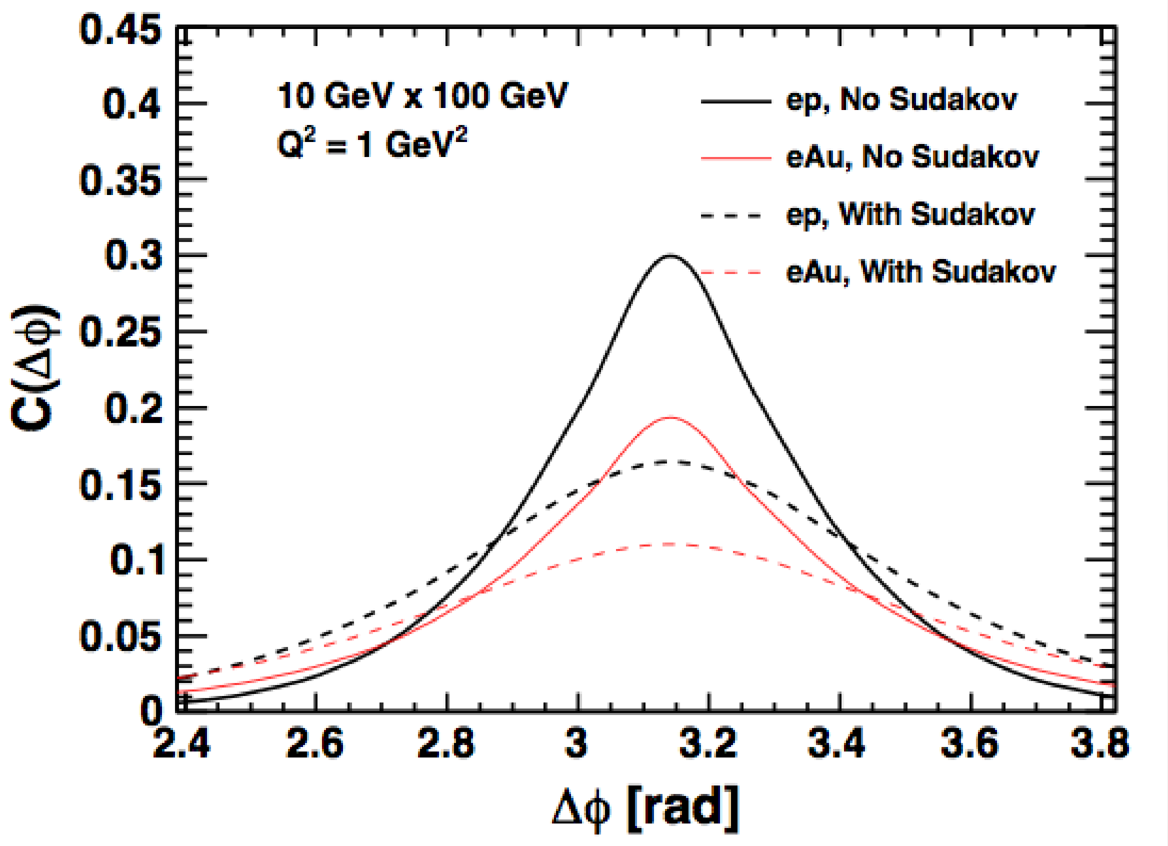
- Each incoming parton contributes to a half of the associated color factor
  - Initial gluon radiation, aka, TMDs

$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2k_\perp} \propto H(P_\perp^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot(x_\perp-y_\perp)}\widetilde{W}_{x_A}(x_\perp,y_\perp)$$

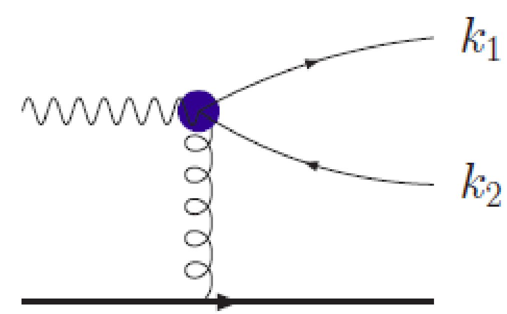
Sudakov

$$\Longrightarrow \quad H(P_\perp^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot R_\perp}e^{-S_{sud}(P_\perp,R_\perp)}\widetilde{W}_{x_A}(x_\perp,y_\perp)$$

Phenomenological applications



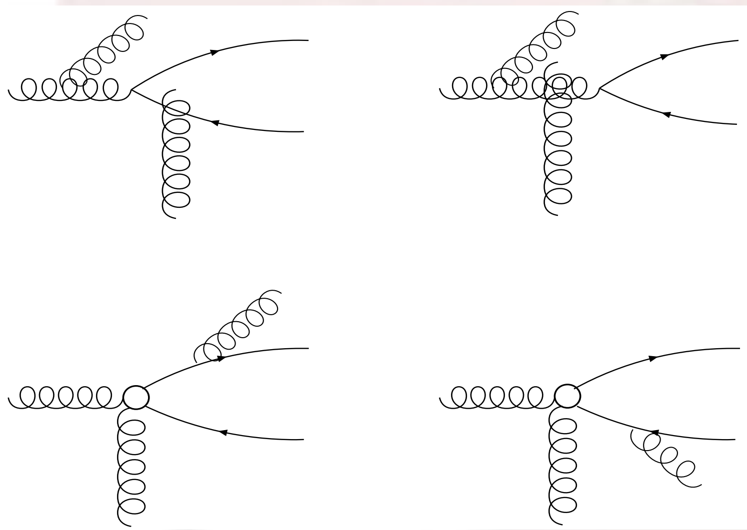
Di-hadron azimuthal Correlations at the Electron-ion Collider



Zheng, Aschenauer,Lee,Xiao, Phys.Rev. D89 (2014) 074037

Beyond leading logs

- Additional nuclear effects, such as energy loss will come in
  - Liou-Mueller, 1402.1647



Interference between
Them, leads to energy
Loss calculated in
Liou-Mueller

Next-to-leading-logarithms (NLL)

- Matrix form in the Sudakov resummation

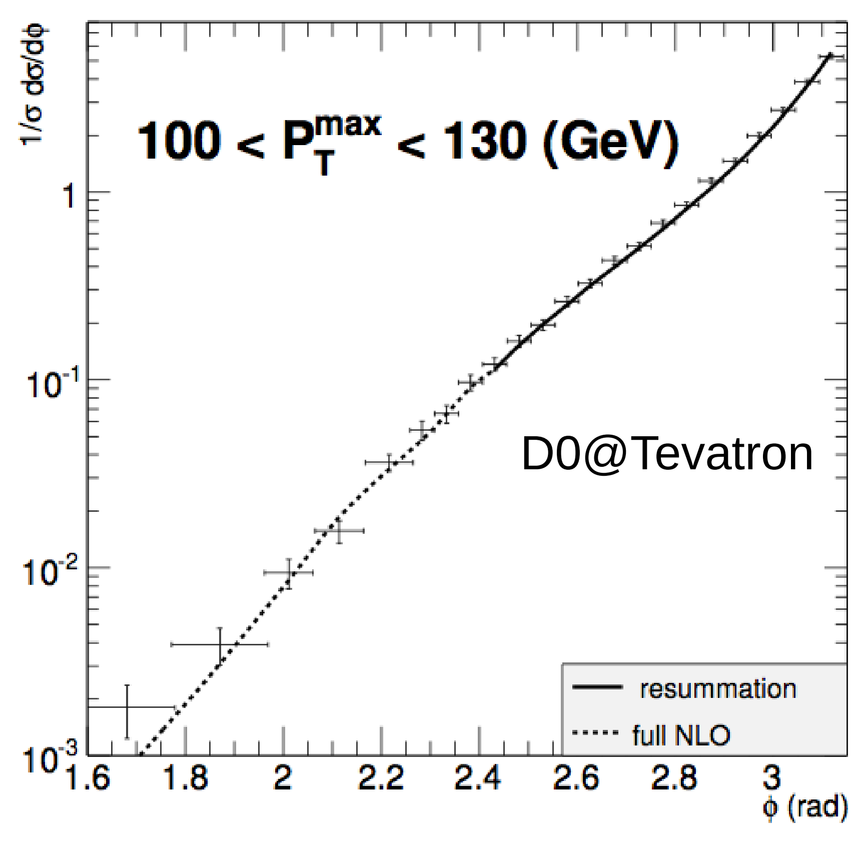
$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\rm Sud}(Q^2, b_\perp)}$$

$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}]\right]$$

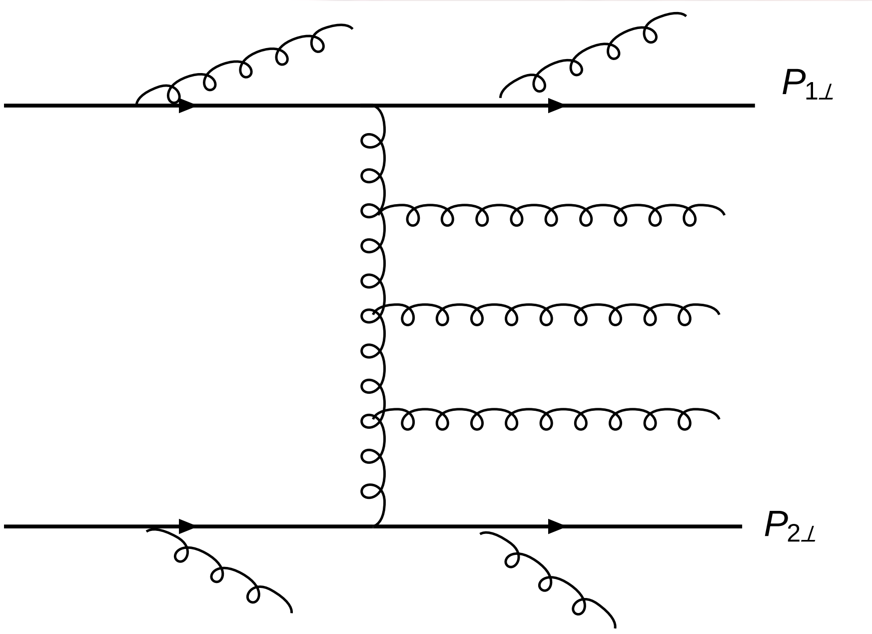
  - Sun,Yuan,Yuan,1405.1105

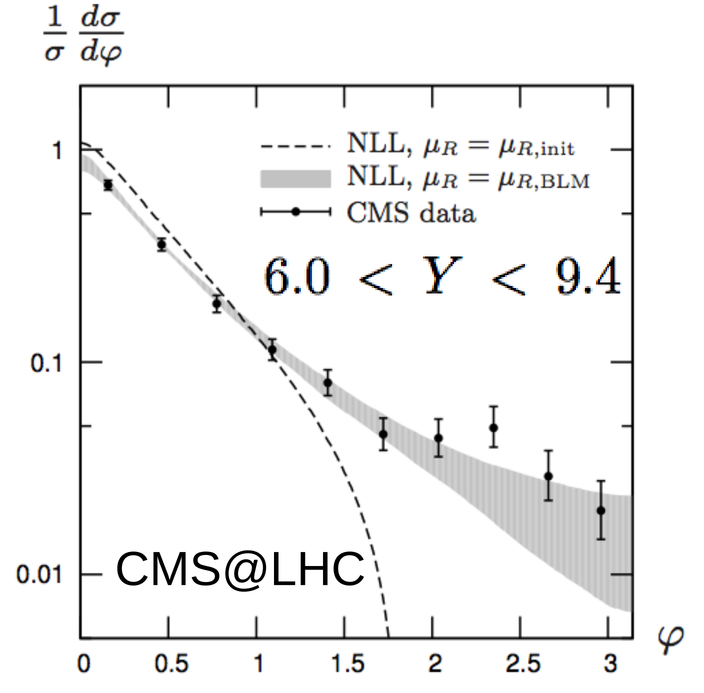
$$\int_{b_0^2/b_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right) A + B + D_1 \ln\frac{Q^2}{P_J^2 R_1^2} + D_2 \ln\frac{Q^2}{P_J^2 R_2^2}\right]$$

  - Kidonakis-Sterman, NPB 1997
  - Banfi-Dasgupta-Delenda, PLB2008



Dijet with large rapidity gap





Sudakov resummation will dominate
Small angle distribution

Ducloue,Szymanowski,Wallon 1309.3229, only take into account BFKL

Conclusion

- Sudakov double logs can be re-summed in the small-x saturation formalism
- Soft gluon and collinear gluon radiation is well separated in phase space
- Shall provide arguments to apply the effective kt-factorization to describe dijet correlation in pA collisions