Open Quantum Systems and Quarkonium Production in Heavy Ion Collisions

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Outline of the talk

- Quarkonium as a probe of the medium.
- Quarkonium is a non-relativistic system. Efficiently studied using Effective Field Theories.
- Physical interpretation of the imaginary part of the potential.
- Understanding of creation and destruction of quarkonium within the formalism of heavy ion collisions.
- Computation of $R_{AA}$ combining EFTs with the open quantum system framework.
Introduction
The original idea of Matsui and Satz (1986)

- Quarkonia is quite stable in the vacuum.
- Phenomena of colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.
- Is it really how quarkonium is dissociated?

\[ V(r) = -\alpha_s \frac{e^{-m_D r}}{r} \]
Another mechanism, the decay width

- A singlet state absorbing a gluon from the medium will turn into an octet.
- Interaction is attractive in a singlet state but repulsive in an octet state.
- Thermal decay width. *Finite lifetime.*
The potential picture

Can we study quarkonium using a Schrödinger equation? With which potential?
To answer these questions we used an strategy based of Effective Field Theories that was successful to answer similar question $T = 0$.

- The movement of the heavy quarks around the center of mass is non-relativistic.
- This induces a set of well separated energy scales. The mass $m$, the inverse of the typical radius $\frac{1}{r}$ and the binding energy $E$.
- pNRQCD (Brambilla, Pineda, Soto and Vairo (200)) is an EFT valid at energy scales much smaller than $m$ and $\frac{1}{r}$. Its degrees of freedom are color singlets and color octets.
- At LO it is equivalent to a Schrödinger equation, there are NLO corrections impossible to reproduce with a potential model.
The potential picture at finite temperature


- LO thermal corrections can be studied by a potential redefinition if $T \gg E$.
- This potential is complex and in the regime $T \gg gT \sim \frac{1}{r}$ coincides with the one in (Laine et al. (2007)).
- In perturbation theory, the only case we can calculate in QCD, the potential does not coincide with the free or internal energy and dissociation is due to the thermal decay width, not screening.
- Complex potentials has also been found in Lattice QCD and in AdS/CFT.
Physical mechanism behind the decay width

**Gluodissociation**
- Singlet absorbs a gluon and decays into an octet.
- Dominant mechanism if $E \gg gT$.
- Large $N_c$ limit studied in (Bhanot and Peskin (1979)).
- Finite $N_c$ EFT study in (Brambilla, M.A.E, Ghiglieri and Vairo (2011)).

**Inelastic scattering**
- Singlet scatters with parton and as a result decays into an octet.
- Dominant mechanism if $gT \gg E$. Always understandable, at LO, as a modification of the potential.
- (Grandchamp and R. Rapp (2001)). Studied in EFT at different $T$ regimes in (Brambilla, M.A.E, Ghiglieri and Vairo (2013)).
Experimental observation

Indeed quarkonium disappears in heavy-ion collisions. What is the mechanism? How can we described realistically the quarkonium-medium interaction. How can we understand the decay width and the imaginary part in a non-equilibrium environment?

Plot taken from Z. Ye talk in QM 2017.
Some of the things that have been learnt by applying EFTs

- LO thermal effects can be described by a potential model only if $T \gg E$.
- At any temperature such that $T \sim E$ or bigger quarkonium will have a decay width.
- The radius $r$ of the heavy quark will determine how strong is the interaction with the medium.
Open quantum systems
Why open quantum systems?

- We want to study the non-equilibrium evolution of the density matrix of the heavy quarks.
- We don’t need to know all the details all the gluons and light quarks that interact with quarkonium.
- Situation in which open quantum system framework is useful. Describe the evolution of an open system (heavy quarks) in contact with an environment (the rest of particles) from which we only know average properties.
- To a good approximation we can assume that the environment is not modified by the interaction with the system.
What is an open quantum system?

**Isolated quantum system**
- Information encoded in a wave-function. Schrödinger-like equation.
- Suitable to describe isolated systems. At early times (before the creation of the plasma) we expect this to be valid.

**Open classical system**
- Information encoded in a probability distribution. Boltzmann-like equation.
- Suitable to describe systems that interact with an environment when there is no quantum coherence. Quantum coherence is lost with time.

**Open quantum system**
- Information encoded in a density matrix. Lindblad-like equation.
- Generalizes and interpolates between the previous cases.
The $\frac{1}{r} \gg T$ case

- $\Upsilon(1S)$ is a quite small state. $\frac{1}{r}$ is a perturbative scale.
- At $T = 0$ the Bohr radius of $\Upsilon(1S)$ is such that $\frac{1}{a_0} \sim 1200$ MeV.
- At the most central collisions of LHC the higher temperature is around 450 MeV, that makes in the highest point $\frac{1}{r} \sim \pi T$.
- In not so central collisions the temperature is smaller.
- As time passes the system cools down.
Evolution of the density matrix in pNRQCD. Singlet

\[ \partial_t \rho_s = -i [h_s, \rho_s] - i \Sigma \rho_s + i \rho_s \Sigma^\dagger + \mathcal{F}(\rho_o) \]

We use the non-equilibrium quantum field theory formalism to perform this computation. Tree level is equivalent to the \( T = 0 \) LO evolution.
Evolution of the density matrix in pNRQCD. Singlet

\[ \partial_t \rho_S = -i[h_s, \rho_s] - i\Sigma \rho_s + i\rho_s \Sigma^\dagger + \mathcal{F}(\rho_o) \]

Self-energy diagram contributes to screening and to the decay width.
Evolution of the density matrix in pNRQCD. Singlet

\[ \partial_t \rho_S = -i[h_s, \rho_s] - i\Sigma \rho_s + i\rho_s \Sigma^\dagger + F(\rho_o) \]

Hermitian conjugate of the previous diagram. We can reorganize this to diagrams in a redefinition of \( h_s \) and a term that represents the decay width.
Evolution of the density matrix in pNRQCD. Singlet

\[ \partial_t \rho_s = -i[h_s, \rho_s] - i\Sigma \rho_s + i \rho_s \Sigma^\dagger + \mathcal{F}(\rho_o) \]

The number of singlets is increased due to the octets in the medium that absorb a gluon.
Evolution of the octet

Very similar reasoning.

\[ \partial_t \rho_o = -i[H_o, \rho_o] - \frac{1}{2}\{\Gamma, \rho_o\} + \mathcal{F}_1(\rho_s) + \mathcal{F}_2(\rho_o) \]

- Computationally costly system of coupled equations. The density matrix contains many information, especially in the non-Abelian case (QCD).
- Total number of heavy particles is conserved. \( Tr(\rho_s) + Tr(\rho_o) \) is a constant of the evolution.
- Only if \( T \gg E \) the interaction with the medium can be considered local in time and the evolution is Markovian.
The $\frac{1}{r} \gg T \sim m_D \gg E$ regime

At strong coupling the screening length $\frac{1}{m_D}$ is of the order of the temperature. When all the thermal scales are smaller than $\frac{1}{r}$ but bigger than $E$ the evolution equation is Markovian and can be written in the Lindblad form.

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2} \{C_k^\dagger C_k, \rho\})$$

there is a transition singlet-octet

$$C_{i}^{so} = \sqrt{\frac{\kappa}{N_c^2 - 1}} r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix}$$

and octet to octet

$$C_{i}^{oo} = \sqrt{\frac{(N_c^2 - 4)\kappa}{2(N_c^2 - 1)}} r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
The parameter $\kappa$

$$\kappa = \frac{g^2}{6 N_c} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^a, i(s, 0) E^a, i(0, 0) \rangle$$

quantity also appearing in heavy particle diffusion, recent lattice QCD evaluation in Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)

$$1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4$$

Picture taken from O. Kaczmarek talk in ”30 years in $J/\Psi$ suppression”. 
The $\frac{1}{r} \gg T \gg m_D \gg E$ regime

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than $E$ (around 500 MeV for $\Upsilon(1S)$) the evolution equation is of the Lindblad form.

\[
\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2} \{C_k^\dagger C_k, \rho\})
\]

\[
H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2 - 2}{2(N_c^2 - 1)} \end{pmatrix}
\]

\[
\gamma = \frac{g^2}{6 N_c} \text{Im} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, 0) E^{a,i}(0, 0) \rangle
\]

No lattice QCD information on this but we observe that we reproduce data better if $\gamma$ is small. In pQCD

\[
\gamma = -2\zeta(3) C_F \left(\frac{4}{3} N_c + n_f\right) \alpha_s^2(\mu_T) T^3 \approx -6.3 \ T^3
\]
Initial conditions and hydrodynamics

In order to understand well the underlying mechanism we worked in the simplest possible conditions. However it is possible and straightforward to couple our theory to the full hydro evolution and we will do in the future.

**Initial conditions**

- To create a pair of heavy particles is a high energy process \( \rightarrow \) pair initially created in a Dirac delta state in the relative coordinate.
- Naively (without taking into account \( P_T \) dependence) it is \( \alpha_s \) suppresses to create a spin 1 singlet compared to an octet.

**Hydrodynamics**

- Bjorken expansion.
- Optical Glauber model to compute dependence of initial temperature with centrality.
- Quarkonium propagates in the vacuum from \( t = 0 \) to \( t_0 = 0.6 \text{ fm} \), then the plasma is created.
Bjorken expansion

\[ T = T_0 \left( \frac{t_0}{t} \right)^{\nu_s^2} \]

- We assume the medium to be infinite and the temperature equal at all points.
- In a high temperature deconfined plasma \( \nu_s^2 = \frac{1}{3} \).
- We take \( t_0 = 0.6 \text{ fm} \).
- \( T_0 \) is a function of collision centrality computed by using the average impact factor of each centrality window.

<table>
<thead>
<tr>
<th>centrality (%)</th>
<th>( \langle b \rangle ) (fm)</th>
<th>( T_0 ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 10</td>
<td>3.4</td>
<td>471</td>
</tr>
<tr>
<td>10 – 20</td>
<td>6.0</td>
<td>461</td>
</tr>
<tr>
<td>20 – 30</td>
<td>7.8</td>
<td>449</td>
</tr>
<tr>
<td>30 – 50</td>
<td>9.9</td>
<td>425</td>
</tr>
<tr>
<td>50 – 100</td>
<td>13.6</td>
<td>304</td>
</tr>
</tbody>
</table>

To be sure that \( \frac{1}{r} \gg T \) is fulfilled during all the evolution we take the two more peripheral windows of centrality.

We stop the computation of thermal effects when \( T = 250 \text{ MeV} \) as we need to be sure that \( T \gg T_c \).
Reducing the degrees of freedom of the density matrix

- The density matrix contains much more information than a wave-function $\rightarrow$ it requires a lot of memory to be simulated.
- By using pNRQCD Lagrangian we know that a s-wave state can only decay to a p-wave, a p-wave either to s-wave, p-wave or d-wave and so on.
- We can make an expansion in spherical harmonics. If we are interested s-wave states a good approximation is to consider in the simulation only s-wave and p-wave.
Results. 30 − 50% centrality

Error bars only take into account uncertainty in the determination of $\kappa$. $\gamma$ is set to zero.
Results. 50 – 100% centrality

Error bars only take into account uncertainty in the determination of $\kappa$. $\gamma$ is set to zero.
Comparison between the CMS data of 2012 (triangles) and our computation (circles). Upper (red) entries refer to the $\Upsilon(1S)$, lower (green) entries to the $\Upsilon(2S)$. 

\[ R_{AA} \]

\[ \langle N_{\text{part}} \rangle \]
Outlook and conclusions
Outlook

- We have first studied the simplest case in which we can obtain a lot of information of lattice QCD. Already quite expensive computationally. The density matrix contains much more information that the wave-function.

- Reduce computational cost by using Monte Carlo techniques and parallel computing (master equation unravelling).

- Improve initial conditions and hydrodynamics.

- Generalize the procedure to other temperature regimes.

- Understand better when classical approximations are reliable (QED work on this direction in Blaizot, De Boni, Faccioli and Garberoglio (2016) and De Boni (2017). In QCD Blaizot and MAE (in preparation)).
Relation with other recent studies

- The difference with solving the Schrödinger equation with an imaginary potential (Kroupa and Strickland (2016)) is that we take into account the octet to singlet transition, hence number of particles is conserved in the equations.

- The stochastic potential approach (Kajimoto, Akamatsu, Asakawa and Rothkopf (2017)) is a very similar approach. Monte Carlo technique that efficiently computes the evolution of the density matrix in the Abelian case. We considered the non-Abelian case.

- The Schrödinger-Langevin approach (Gossiaux and Katz (2016)) is very similar to the stochastic potential but adding a non-linear term to impose thermalization. Connection with QFT calculations not very clear. Abelian approximation.

- The model consisting of a Langevin equation plus a decay width (Petreczky and Young (2016)) describes similar physical phenomena. Probably related to our equations in the limit of small quantum coherence.
Conclusions

- It is useful to view quarkonium in heavy-ion collisions as an open quantum system.
- The $\frac{1}{r} \gg T$ case is relevant to study experimental situation and can be efficiently studied using pNRQCD.
- The case $\frac{1}{r} \gg T, m_D \gg E$ can be studied without assuming weak coupling and using lattice QCD information.
- Reasonable comparison with experimental data making few assumptions.