Heavy-Flavor Theory for Heavy-Ion Collisions

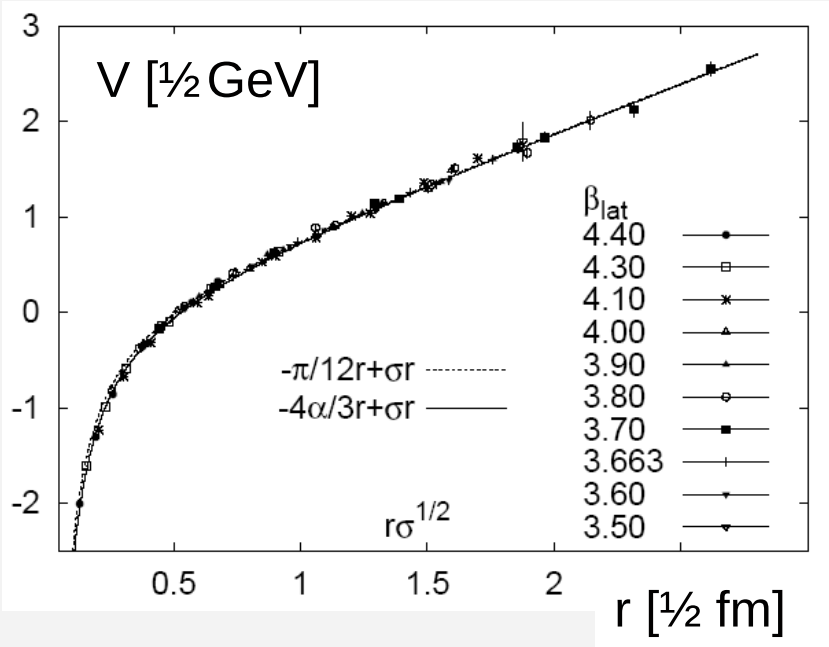
Ralf Rapp
Cyclotron Institute +
Dept. of Physics & Astronomy
Texas A&M University
College Station, TX
USA

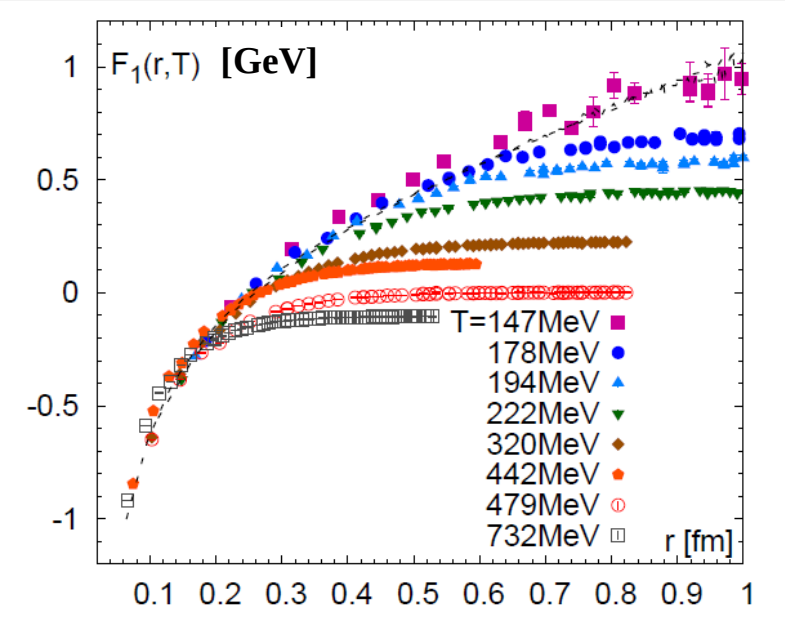
RHIC & AGS Users' Meeting 2017

"From Protons to Heavy Ions, and Back Again"

BNL, Upton (NY), June 20-23, 2017

1.) Introduction: A "Calibrated" QCD Force





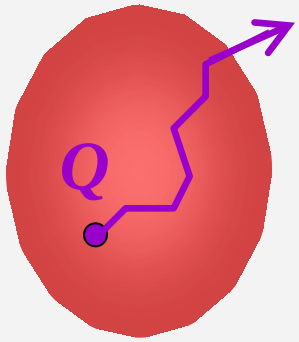
[Bazavov et al '13]

- Vacuum quarkonium spectroscopy well described
- Confinement ↔ **linear part of potential**

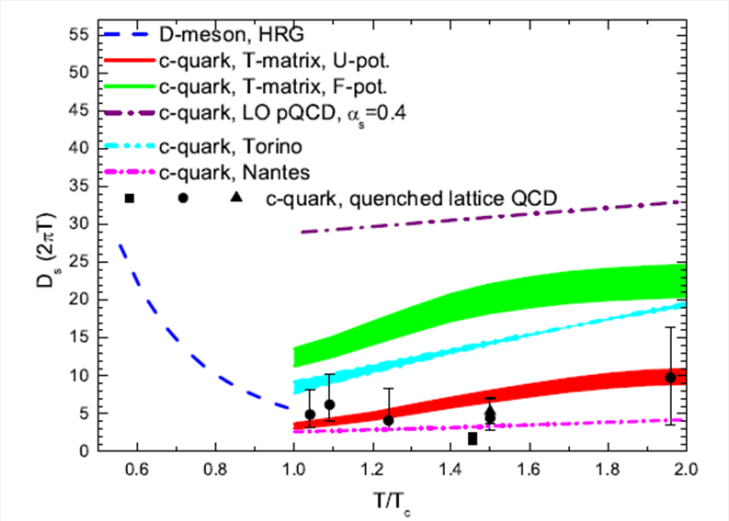
Objective: **Determine medium-modifications of QCD force and infer HF transport properties + spectral functions to probe QGP at varying resolution.**

Exploit $m_Q >> \Lambda_{QCD}$, T_c, $T_{RHIC,LHC}$

1.2 Heavy Quarks in Medium



- **Diffusion: "Brownian motion"**
- Thermalization delayed by $\mathbf{m_Q/T}$
  → memory in URHICs
- Direct access to transport coefficient $\mathcal{D}_s$ **(2πT)** **(~ η/s ~ σ_{EM}/T)**
- Elastic scattering rates (radiation suppressed, $\mathbf{q_0^2 \ll q^2}$)
  → widths; quasiparticles? ($\mathbf{m_Q \gg T}$)
- Probe of hadronization
- Non-perturbative effects until ~ $\mathbf{2T_{pc}}$
- Potential-type interactions ($\mathbf{q_0 \sim q^2/m_Q \ll q}$)
  → same as for heavy quarkonia!?
- Ample connections to lattice QCD



[He et al '12]

Outline

1.) Introduction

2.) Heavy-Flavor Interactions in QCD Matter

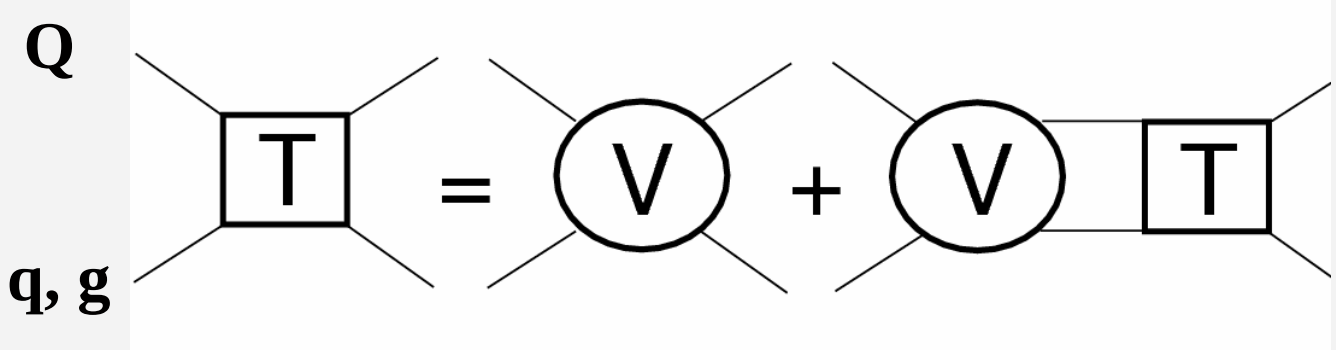
3.) Heavy-Flavor Transport

4.) Phenomenology

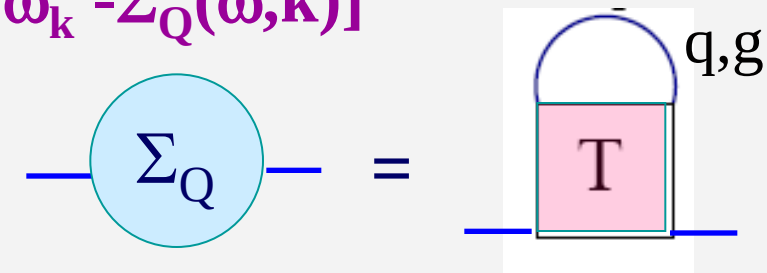
5.) Conclusions

2.1 Heavy-Light Scattering Amplitude in QGP

- **Scattering equation**



- Perturbative approximation (**weak** coupling): $\mathbf{T_{qQ} \approx V_{qQ}}$
- **Strong** coupling → resummation: $\mathbf{T_{qQ} = V_{qQ} + \int V_{qQ}\, D_q\, D_Q\, T_{qQ}}$
- Thermal quark propagator: $\mathbf{D_Q = 1 / [\omega - \omega_k - \Sigma_Q(\omega,k)]}$
- Quark self-energies → self-consistency:

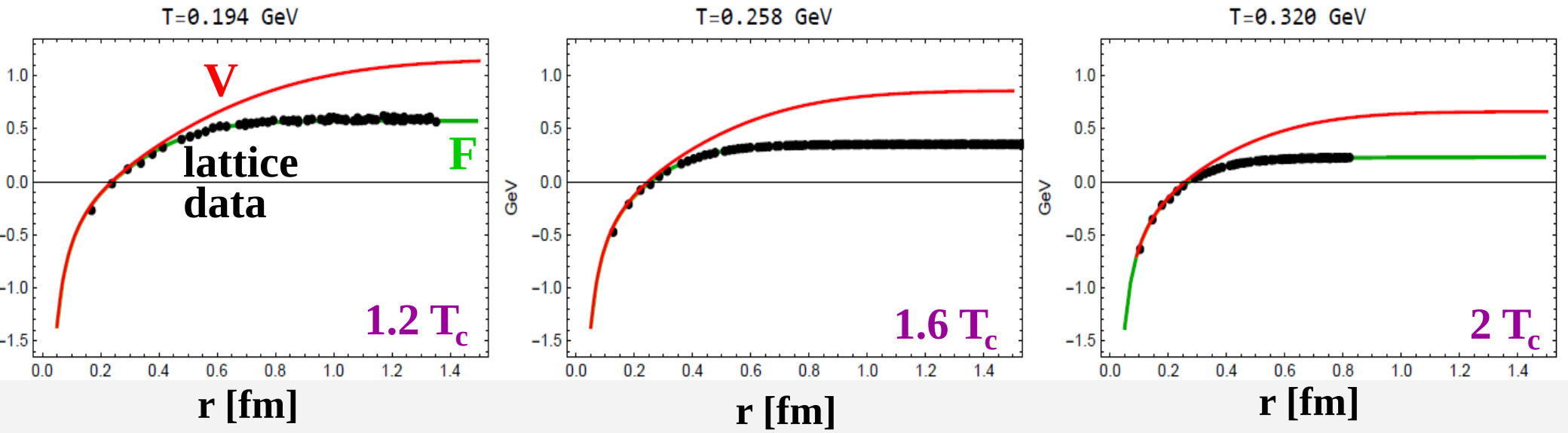


- **In-medium potential V?**

[Mannarelli+RR '05, Cabrera+RR '06, Riek+RR '10, Liu+RR '15]

2.1.2 Free Enegy + Potential from T-Matrix

- Free Energy $$F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\,\sigma(\omega, r_1 - r_2)\,e^{-\beta\omega}\right)$$
- Spectral Function $$\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$$
- Potential ansatz: $$V_R(E, r) = -\frac{4}{3}\alpha_s\frac{e^{-m_D r}}{r} - \sigma\frac{e^{-m_s r}}{m_s} \quad -\frac{4}{3}\alpha_s m_D + \sigma\frac{1}{m_s}$$

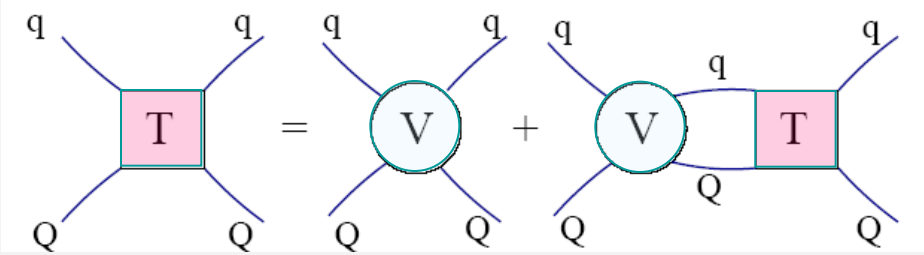


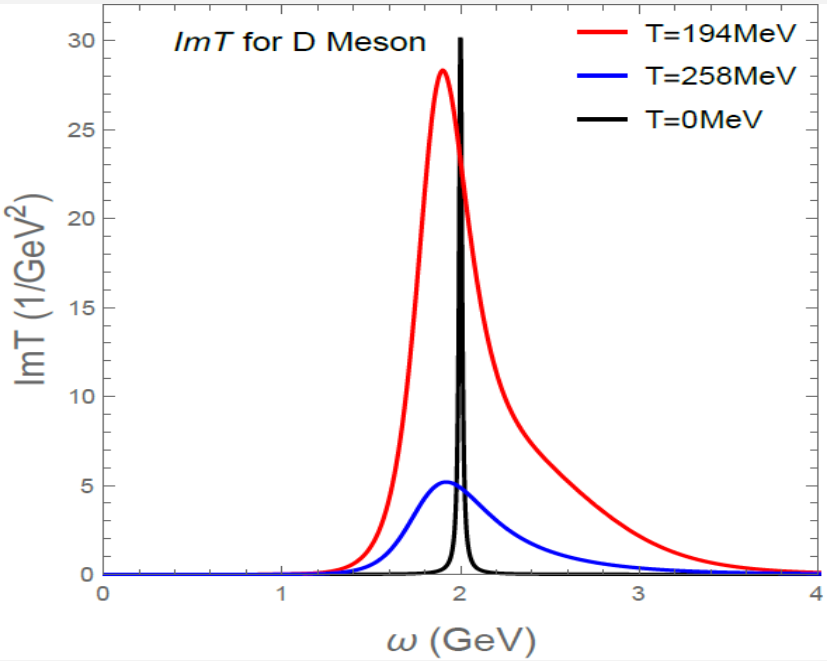
- **remnant of long-range "confining" force in QGP**

[S.Liu+RR '15]

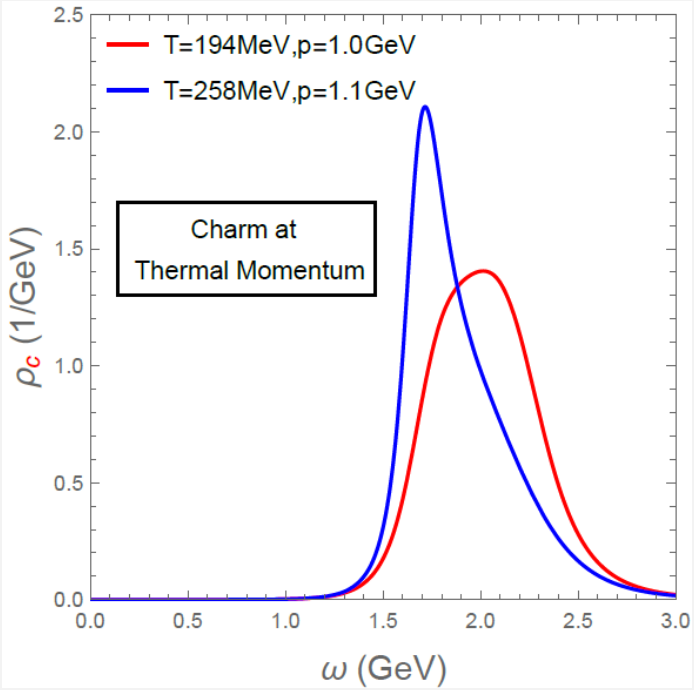
2.2 D-Meson + c-Quark Spectral Functions in QGP

T-matrix w/ "lattice potential" V





In-Medium c-Quark Selfenergy



D-meson resonances near T_{pc} → **c**-quark quasi-particles at high **T**

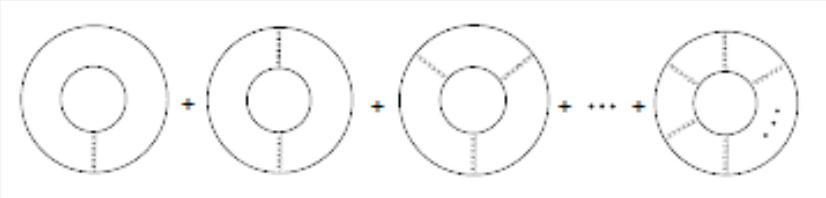
2.3 Self-Consistent Equation of State for QGP

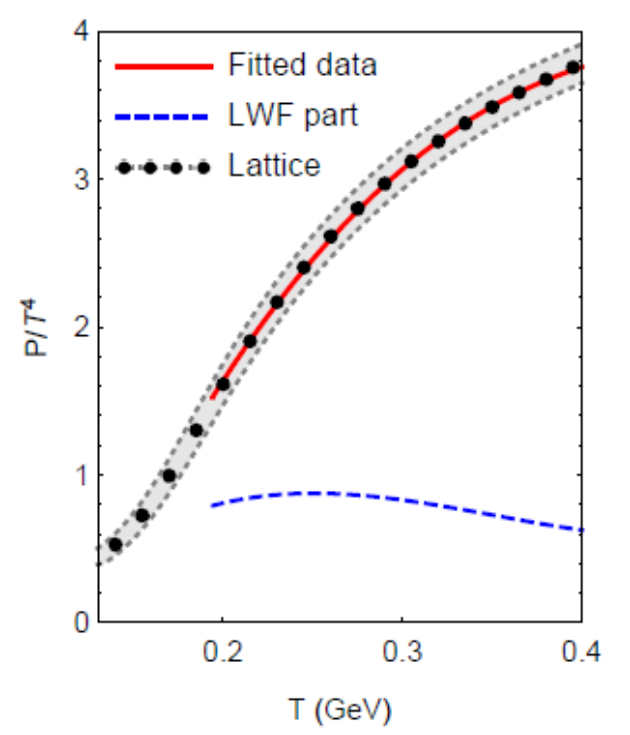
Thermodynamic Potential

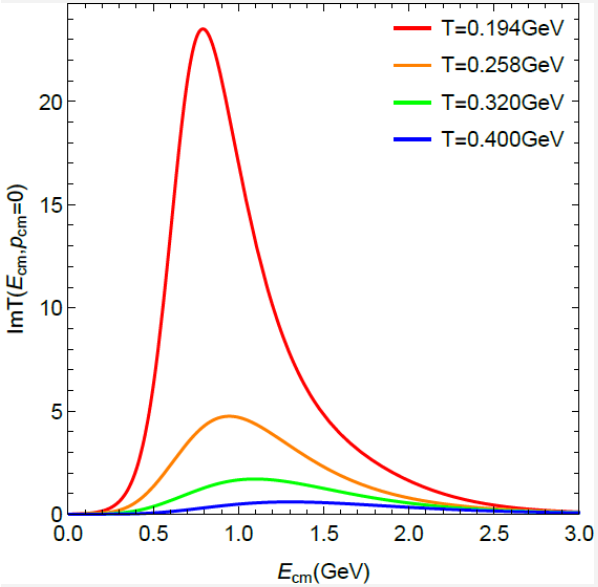
$$\Omega = \mp \frac{-1}{\beta} \sum_n \mathrm{Tr}\{\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi$$

"Skeleton" Diagrams

$$\Phi = \frac{-1}{\beta} \sum_{n,\nu} \mathrm{Tr}\{\frac{1}{2\nu}(\frac{-1}{\beta})^{\nu}[(-\beta)^{\nu}\Sigma_{\nu}(G)]G\}$$





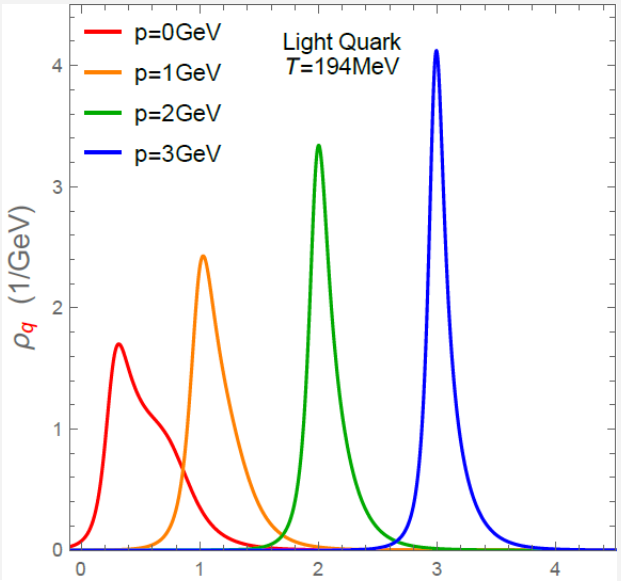


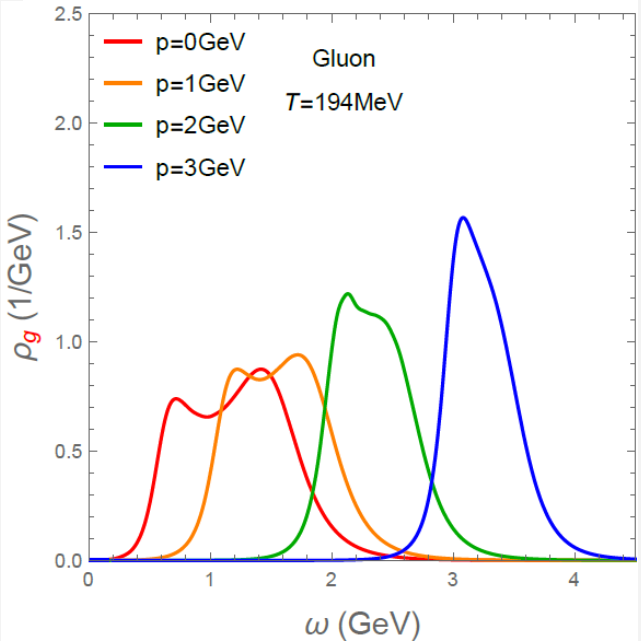
[S.Liu+RR '16]

- **Broad hadronic resonances emerge near T_{pc}, dominate EoS**

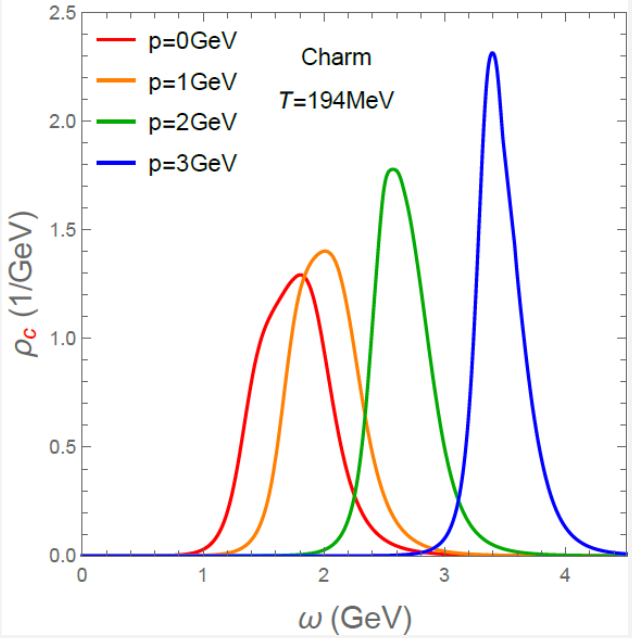
2.3.2 Parton Spectral Functions in QGP

Up/Down + Gluon

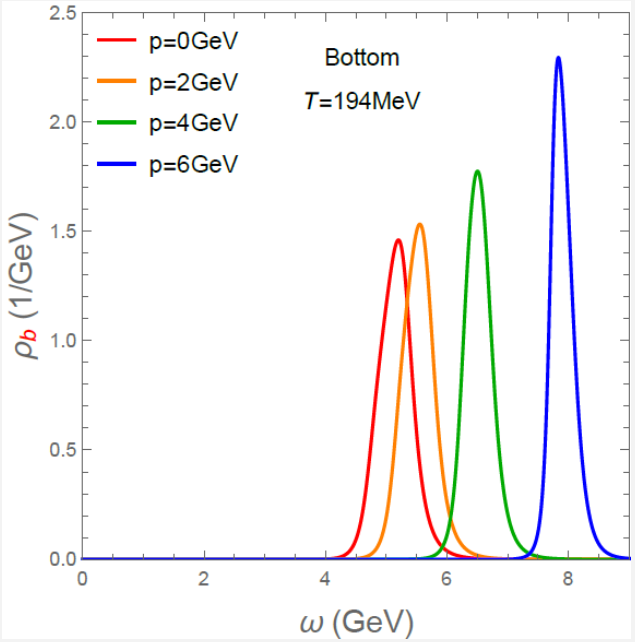




Charm



Bottom



- QGP structure changes with resolution scale
- Quasi-particles re-emerge when increasing $\mathbf{M_q}$, **p**, **T**

Outline

1.) Introduction

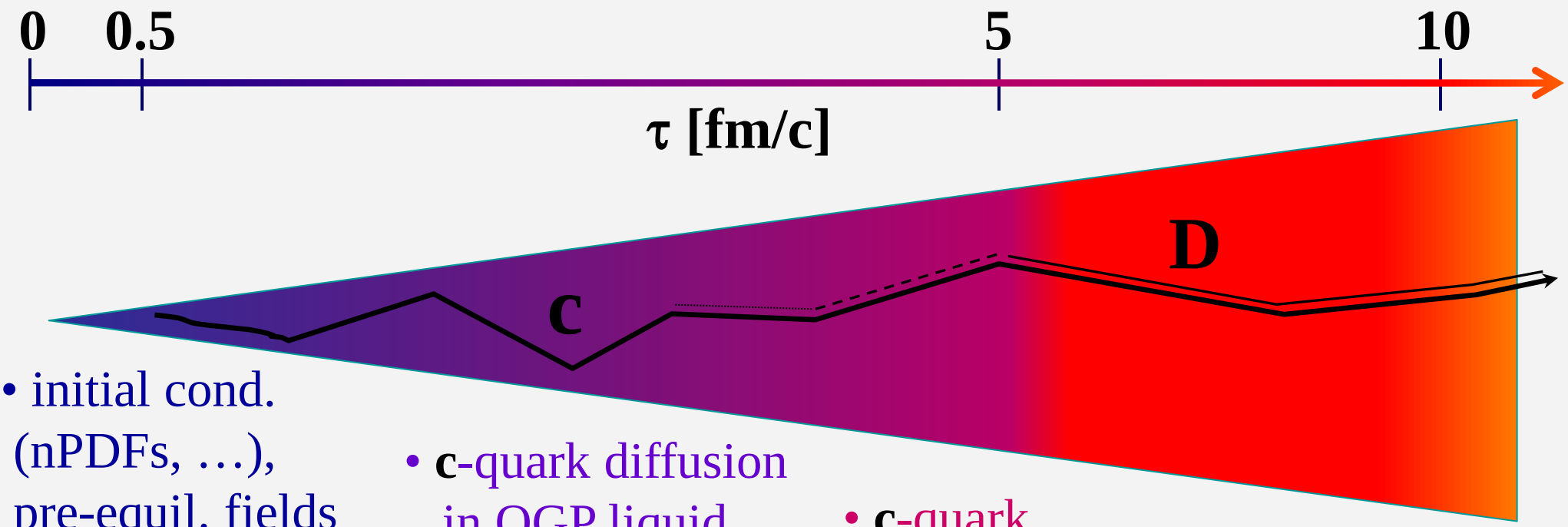
2.) Heavy-Flavor Interactions in QCD Matter

3.) Heavy-Flavor Transport

4.) Phenomenology

5.) Conclusions

3.1 Heavy-Flavor Transport in URHICs



- initial cond. (nPDFs, …), pre-equil. fields
- **c**-quark diffusion in QGP liquid
- **c**-quark hadronization
- **D**-meson diffusion in hadron liquid

- no "discontinuities" in interaction

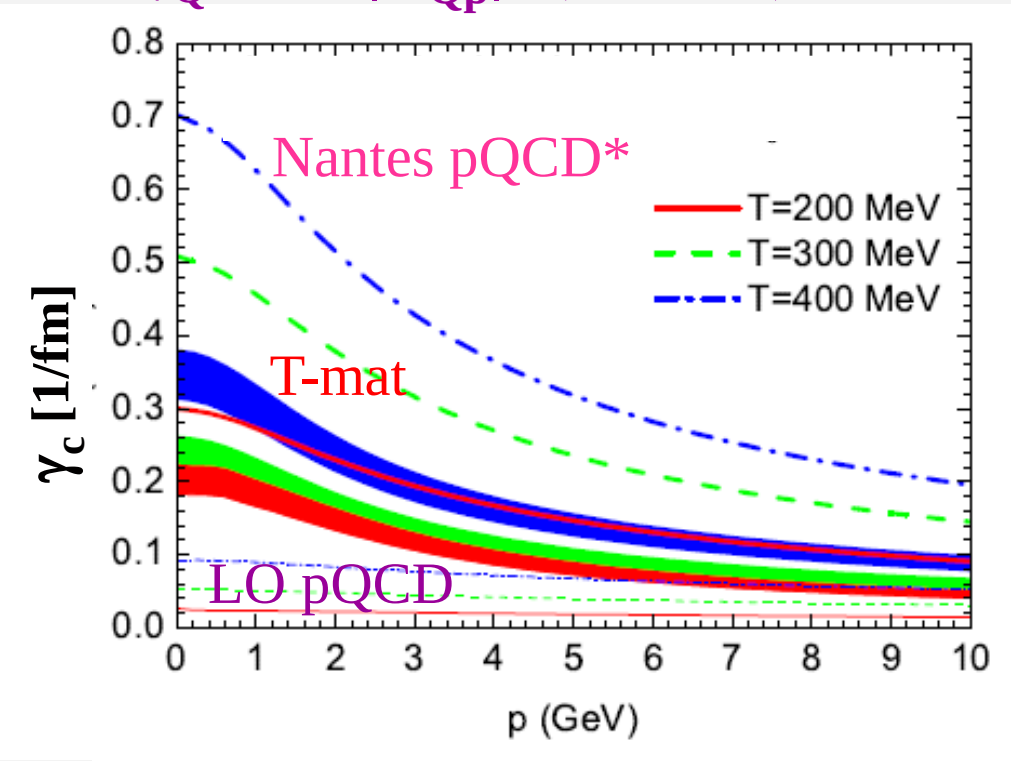
⇒ **diffusion toward T_{pc}** and **hadronization** same interaction (**confining**!)

[Moore+Teaney '05, van Hees et al '05, Gossiaux et al '08, Vitev et al '08, Das et al '09, Uphoff et al '10, M.He et al '11, Beraudo et al '11, Cao et al '13, Bratkovskaya et al '14, …]

3.2 Charm Transport Coefficients

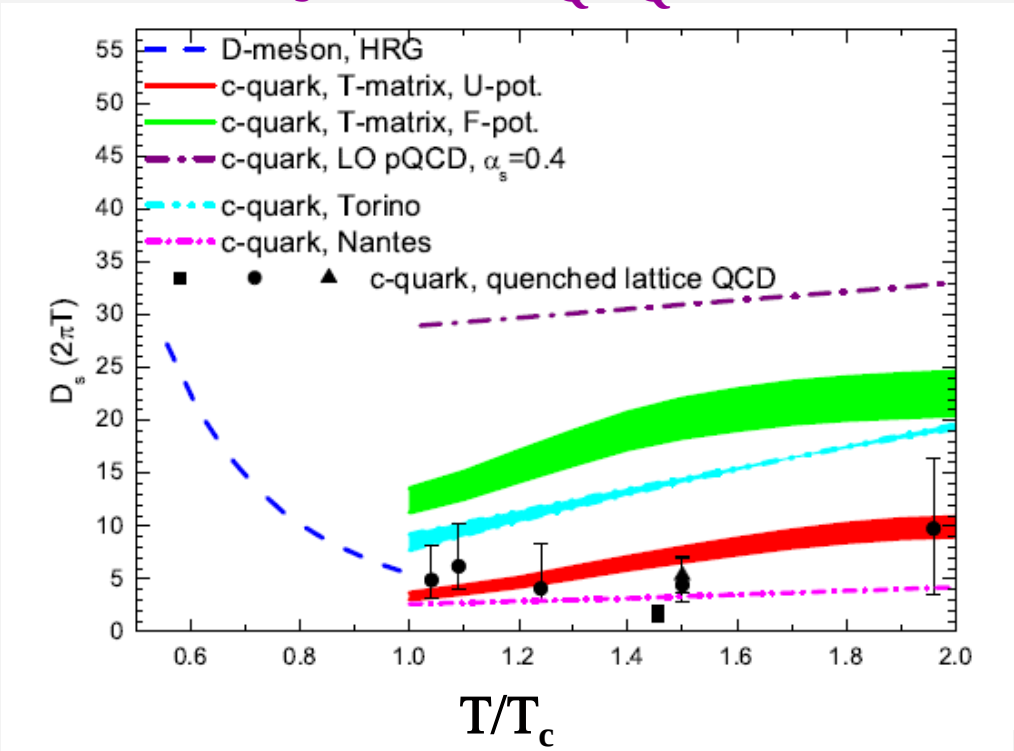
Thermalization Rate

$$\gamma_Q\, p = \int |\mathcal{T}_{Qp}|^2\, (1-\cos\Theta)\, f^p$$



Spatial Diffusion Coefficient

$$\mathcal{D}_s = T/\,(m_Q\, \gamma_Q(p=0))$$



- **p**- and **T**-dependence reflect core properties of QCD
- suggests minimum of $\mathcal{D}_s(2\pi T) \sim 2\text{-}4$ near T_{pc}
- width: $\Gamma_{coll} \sim 3/\mathcal{D}_s \sim 0.5\text{-}1$ GeV – **no light quasi-particles!**

Outline

1.) Introduction

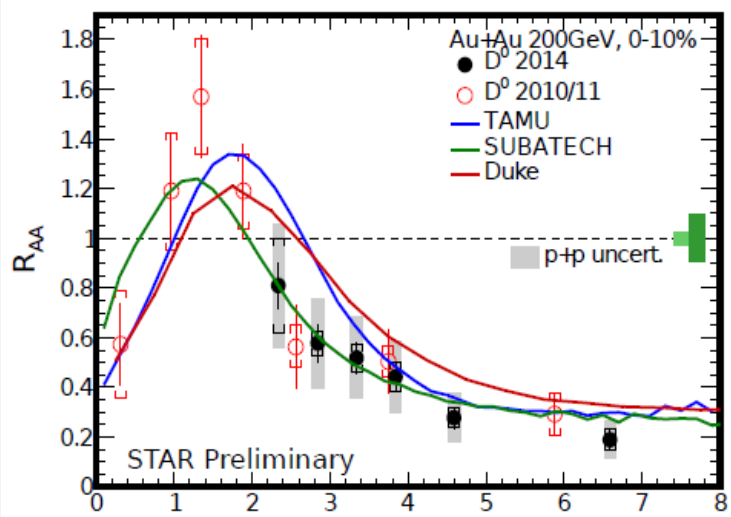
2.) Heavy-Flavor Interactions in QCD Matter

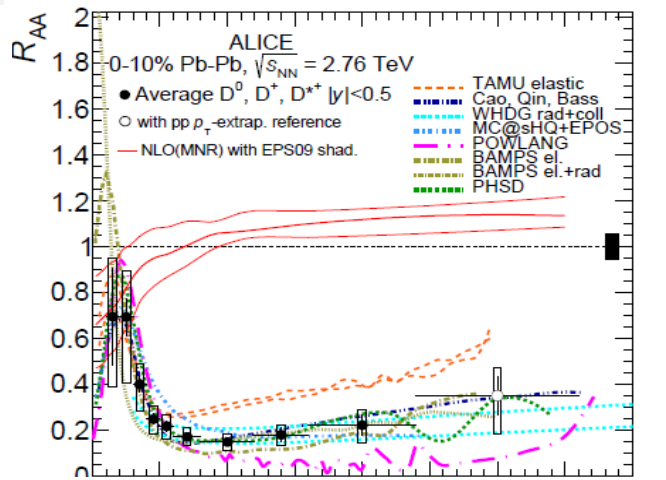
3.) Heavy-Flavor Transport

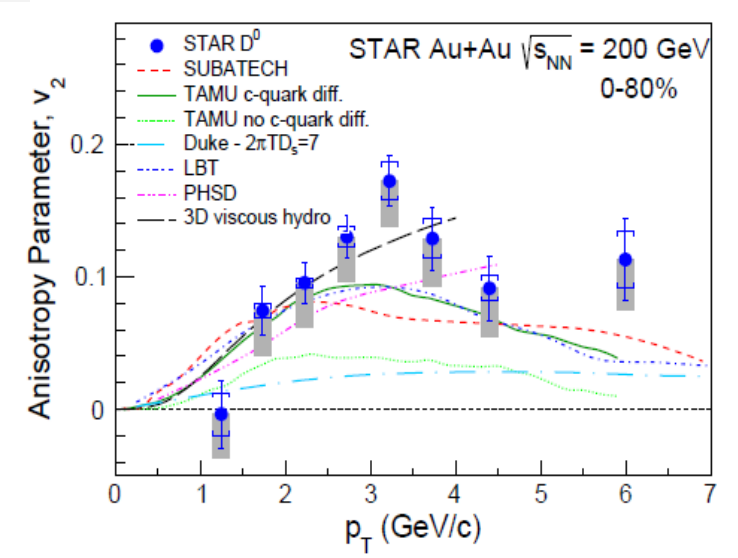
4.) Phenomenology

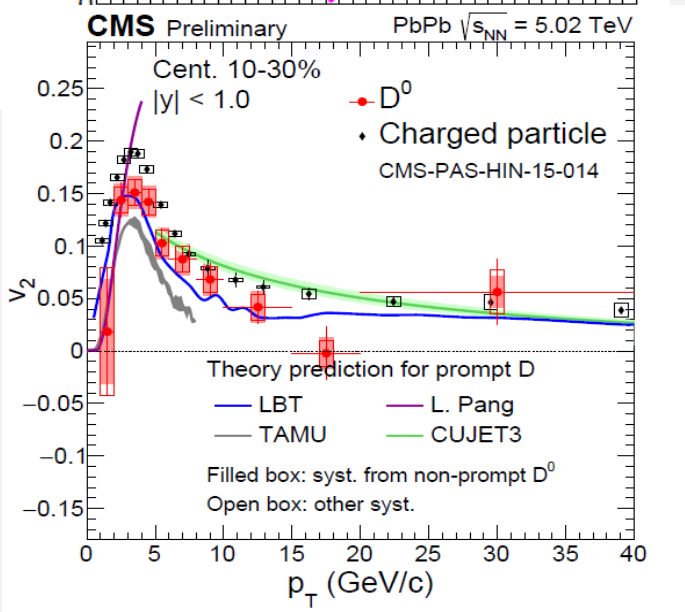
5.) Conclusions

4.1 Heavy-Flavor Transport at RHIC + LHC





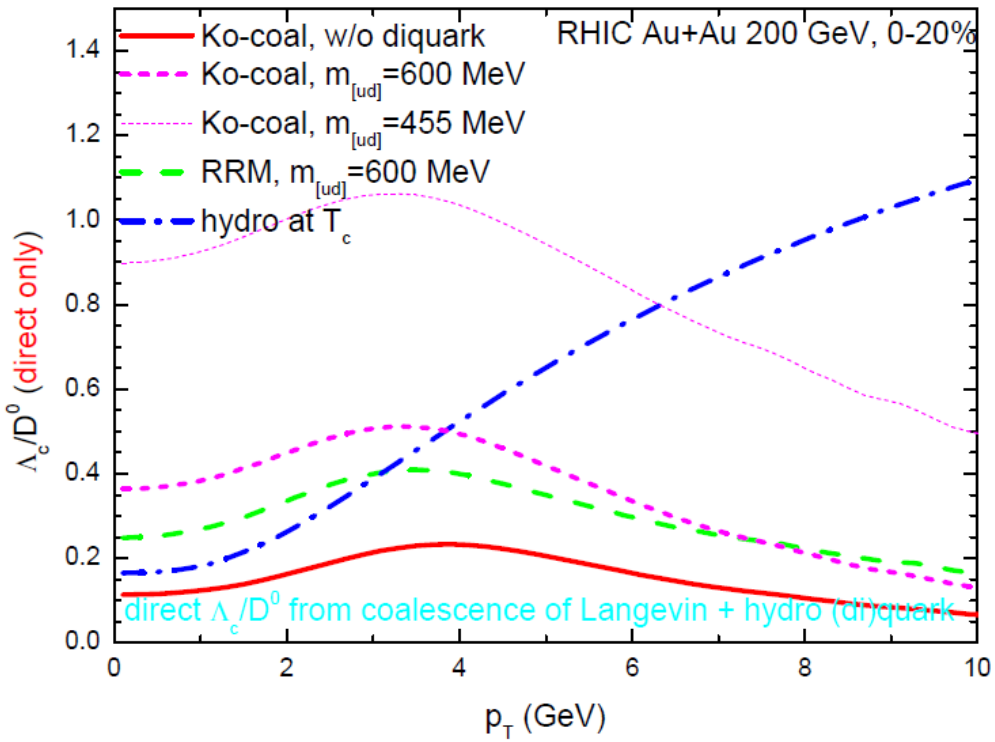




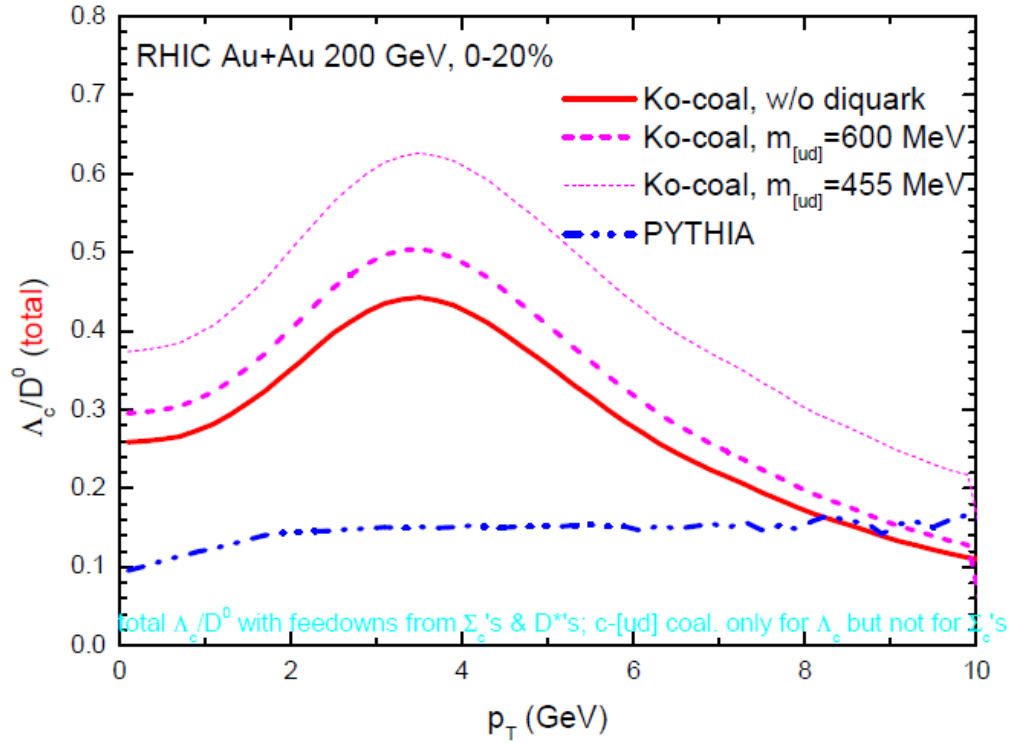
- flow bump in $\mathbf{R_{AA}}$ + large $\mathbf{v_2}$ ↔ strong coupling near $\mathbf{T_{pc}}$ (**recombination**)
- high-precision $\mathbf{v_2}$: transition from elastic to radiative regime?

4.2 The Λ_c Puzzle at RHIC

Ratio of Direct Production



Ratio including Feeddowns



- recent studies with instantaneous coalescence ("Ko-coal") vs. resonance recombination model (**RRM**) **[Min He '17]**
- **STAR** datum of **~1** remains a puzzle at this point…

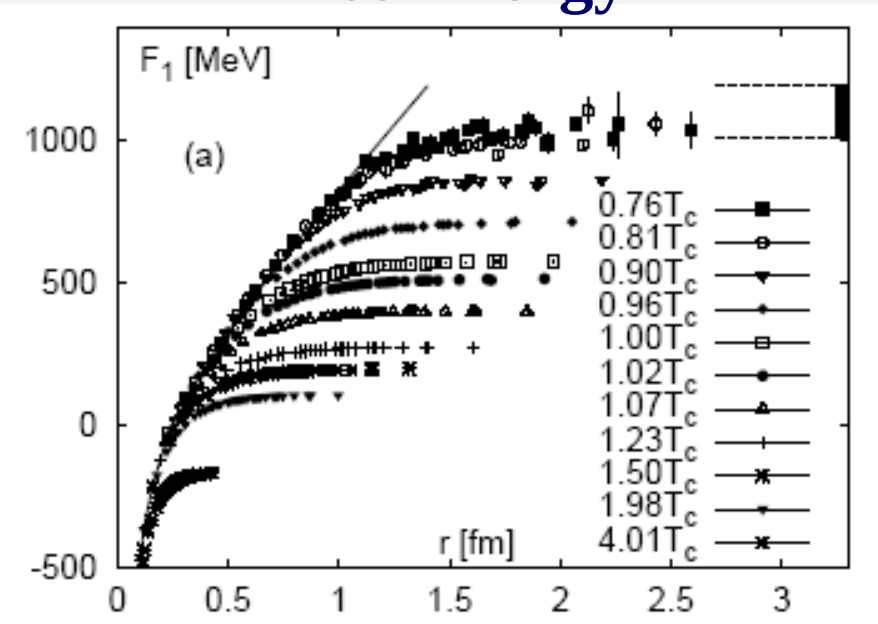
5.) Summary

- Many-body approach to extract heavy-quark potential in QGP from lQCD
- Remnants of long-range confining force survive well above $\mathbf{T_{pc}}$
  → large scattering rates, strong-coupling diffusion (small $\mathcal{D}_s$)
  → extended transition from heavy-quark to -hadron dofs
- Same approach to describe QGP equation of state
  → no long-wavelength quasi-partons near $\mathbf{T_{pc}}$ (re-emerge at high $\mathbf{p,T,m_Q}$)
  → broad hadronic bound states emerge near $\mathbf{T_{pc}}$
- **In-medium heavy flavor firmly rooted in sQGP / lat-QCD properties**
- Heavy-flavor phenomenology at **RHIC** + **LHC**
  - drag force imprinted on **D**-meson $\mathbf{R_{AA}}$ + $\mathbf{v_2}$, transition to radiation
  - test hadronization: $\mathbf{\Lambda_c}$ ”puzzle”
- Ongoing theory working groups (**EMMI RRTF**, **HQ-jet**)

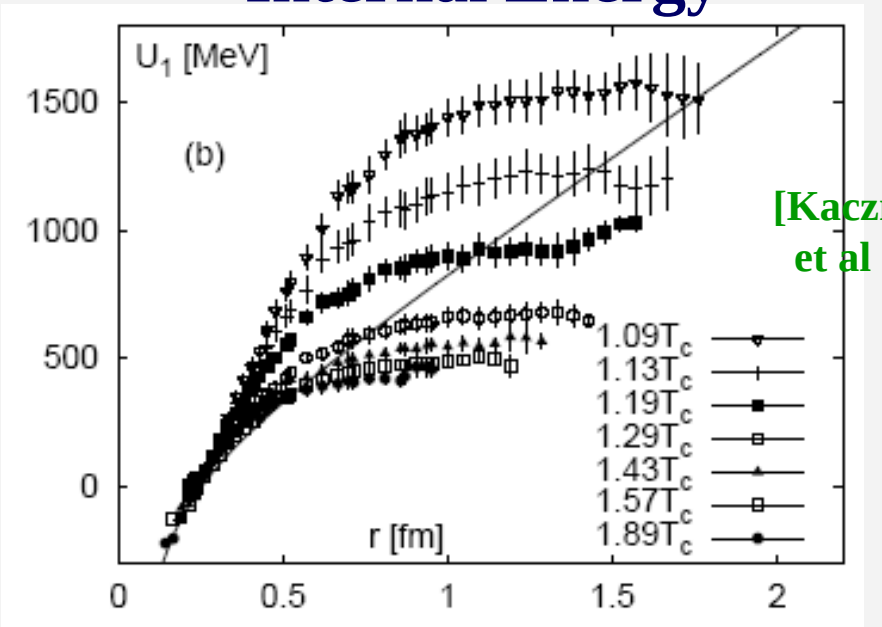
2.3 Heavy-Quark Free and Internal Energies in Lattice QCD

$$F_1(r,T) = U_1(r,T) - T\,S_1(r,T)$$

Free Energy



Internal Energy



[Kaczmarek et al '05]

- "**weak**" $Q\bar{Q}$ potential
- "**strong**" $Q\bar{Q}$ potential, $U = \langle H_{int} \rangle$

- **F, U, S thermodynamic quantities**
- **Entropy**: many-body effects

2.9 D-Mesons in Hadronic Matter

- effective ***D-h*** scattering amplitudes

  [He,Fries+RR '11, Tolos+Torres-Ricon '13]

Hadrons	$L_{I,2J}$	γ_D [fm^{-1}]
π	$S_{1/2,0},\ P_{1/2,2},\ D_{1/2,4},\ S_{3/2,0}$	0.0371
$K+\eta$	$S_{0,0},\ S_{1,0}$	0.0236
$\rho+\omega+K^*$	$S_{1/2,2},\ S_{0,2},\ S_{1,2}$	0.0129
$N+\bar{N}$	$S_{0,1},\ S_{1,1}$	0.0128
$\Delta+\bar{\Delta}$	$S_{1,3}$	0.0144

- **D**-meson in pion gas:

  - consistent with unitarized HQET **[Cabrera et al '11]**

  - factor **~10** larger in heavy-meson χPT **[Laine '11]**

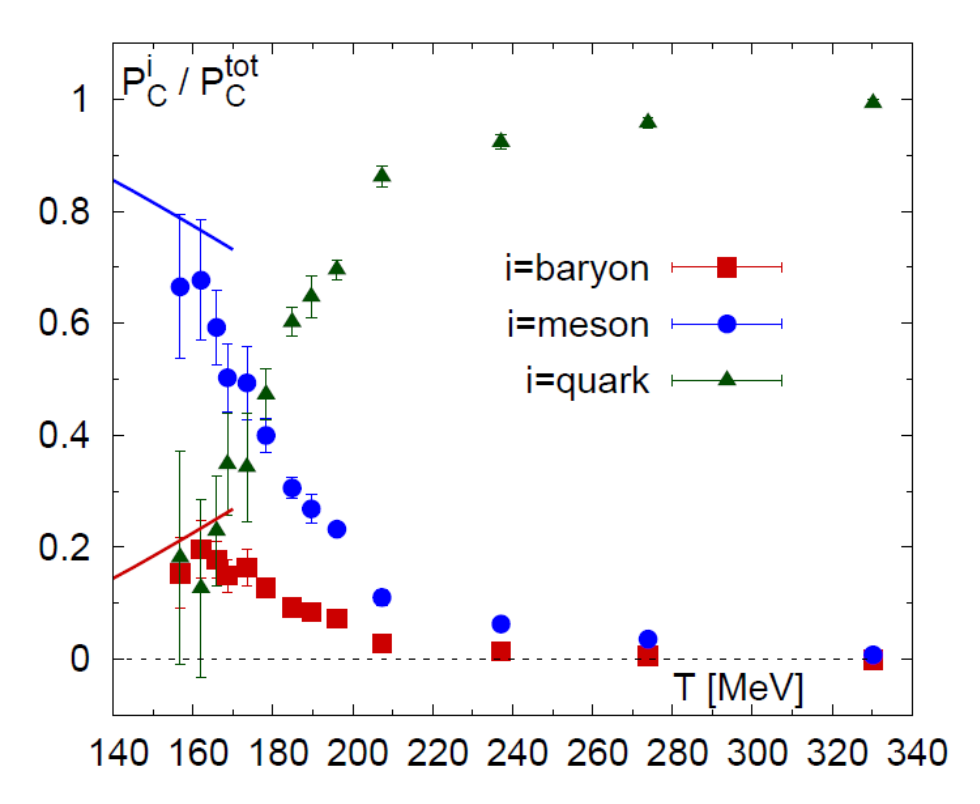
- hadron gas at **~T_c**: **$\tau_D \approx$ 10fm/c**

2.5 Charm Susceptibilities from Lattice QCD

- Extract partial charm pressures from susceptibilities

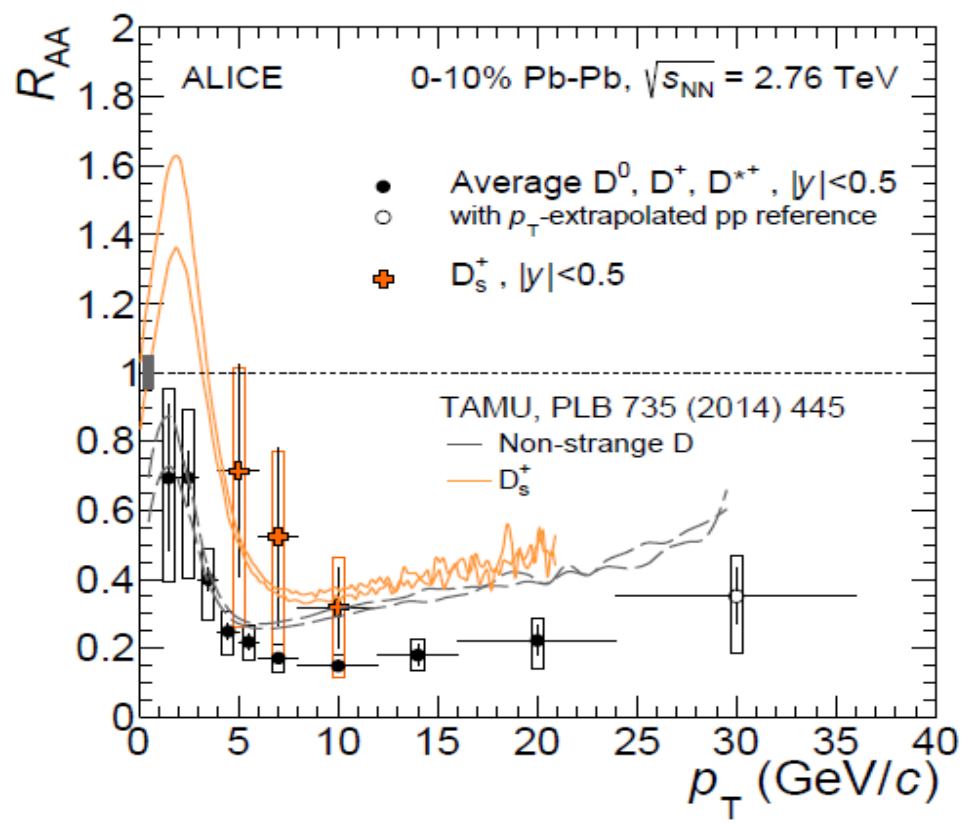
[Mukherjee,Petreczky +Sharma '15]

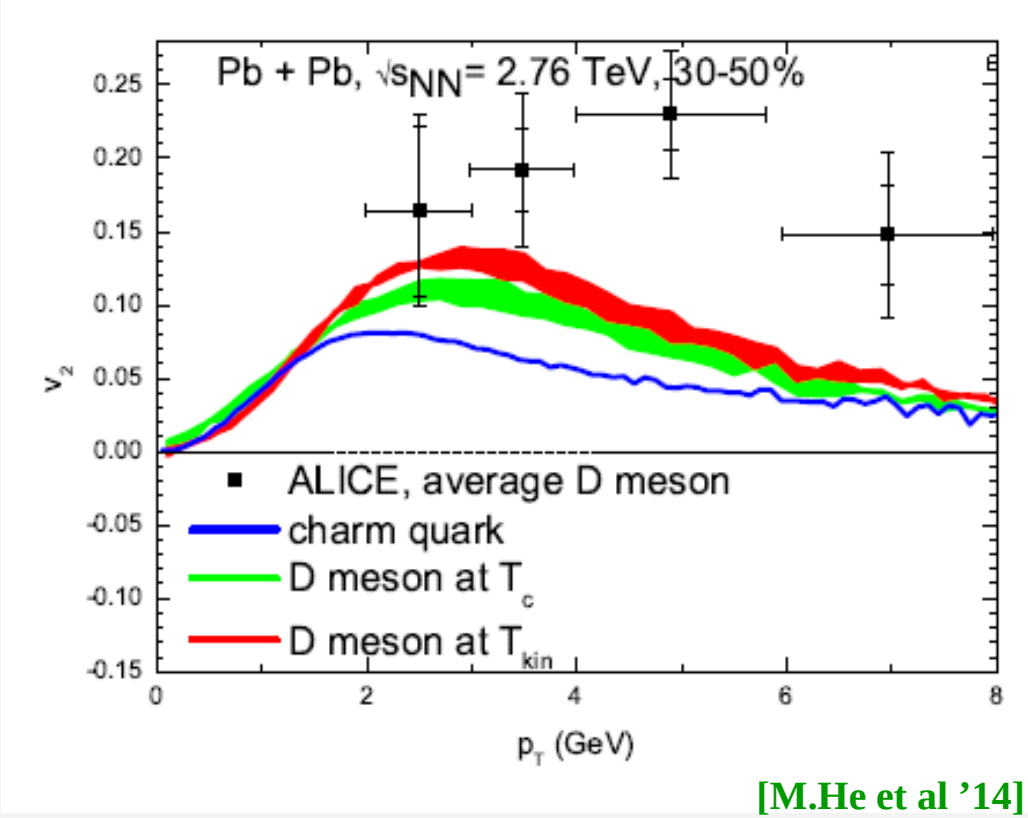
$$p_c(T,\mu_B,\mu_c) = p_M(T)\cosh\left(\frac{\mu_c}{T}\right) + p_B(T)\cosh\left(\frac{\mu_c+\mu_B}{T}\right) + p_Q(T)\cosh\left(\frac{\mu_c+\mu_B/3}{T}\right)$$



- Hadronic (**D**-meson) correlations contribute until **~1.4 T_c**
- Realistic models of heavy-quark diffusion should account for this (induces HQ-**v_2** + hadronization)

4.2 Charm Transport at LHC: D-Meson Spectra

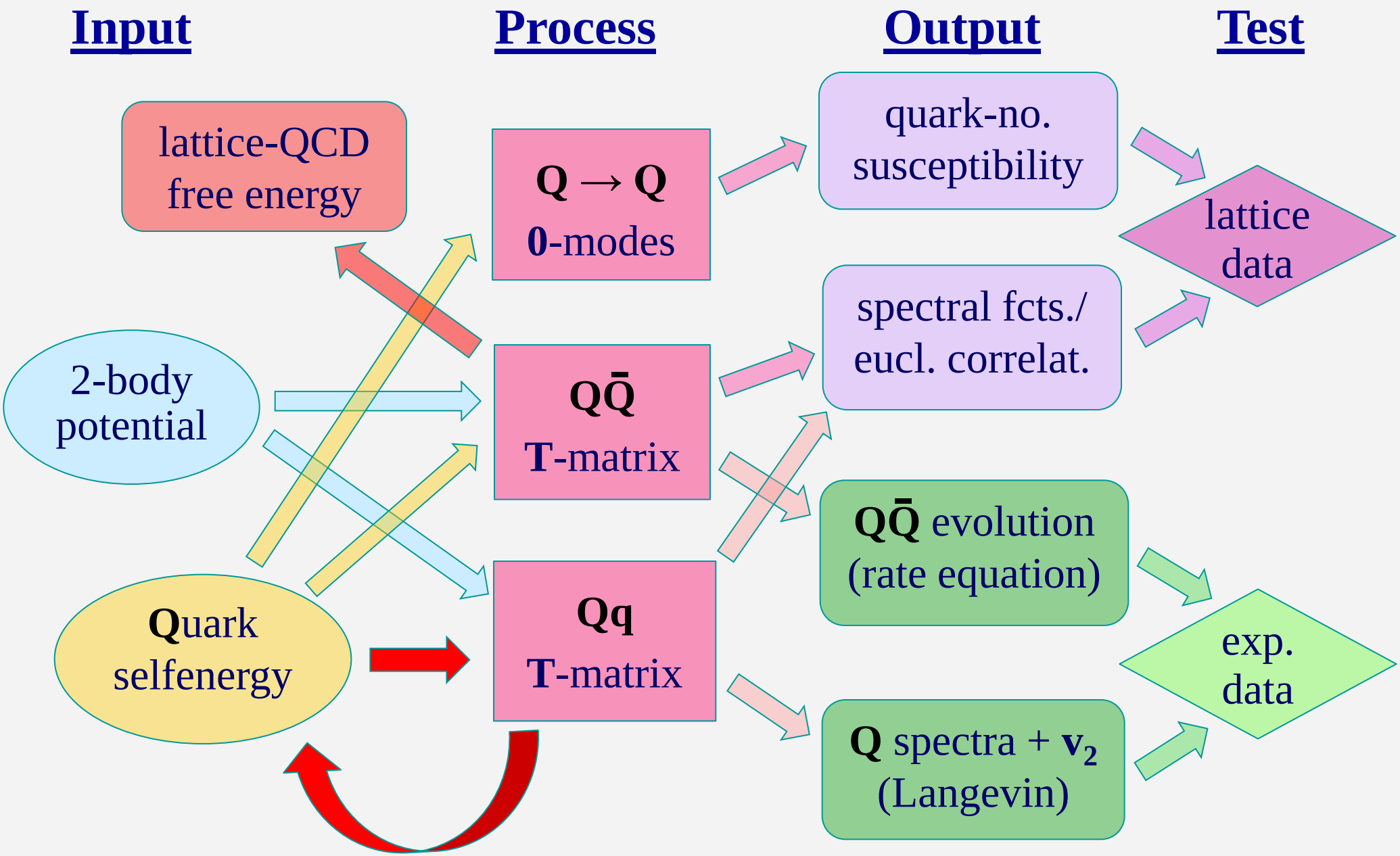




[M.He et al '14]

- R_{AA} "bump" from radial flow
- D_s meson (**cs**) enhanced from coalescence with strange quarks
- Coalescence + hadronic diffusion increase v_2
- similar features at **RHIC**

2.5 Brueckner Theory of Heavy Flavor in QGP



2.4.2 Free Energy from T-Matrix

- **Free Energy** $F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$

[Beraudo et al '08]

- Euclidean **T-matrix** in static limit

$$\tilde{T}(z_t|r) = V(z_t, r) + V(z_t, r)\,\tilde{G}_0^{(2)}(z_t - v^a, v^a)\;\tilde{T}(z_t|r) = \frac{V(z_t, r)}{1 - V(z_t, r)\,\tilde{G}_0^{(2)}(z_t)}$$

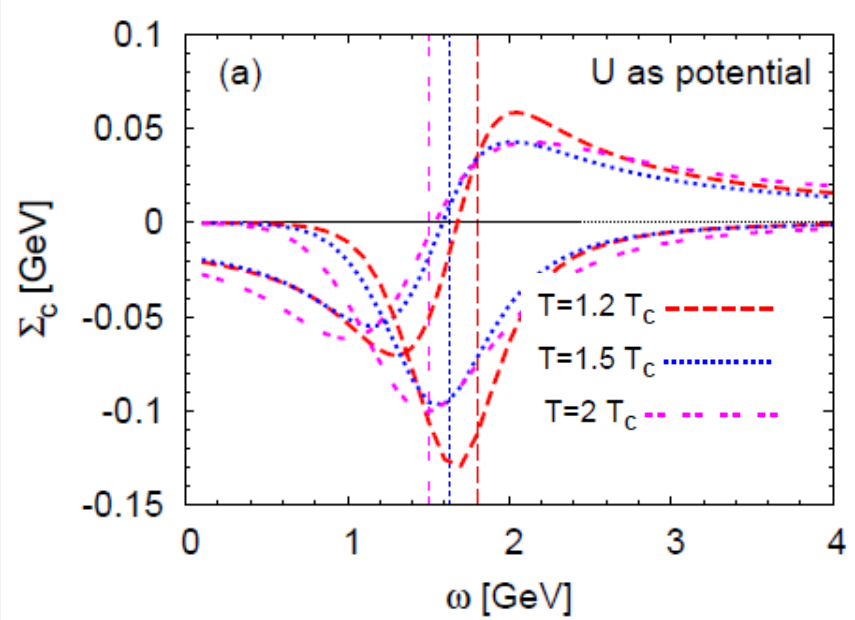
- **Spectral Function** $\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$

[S.Liu+RR '15]

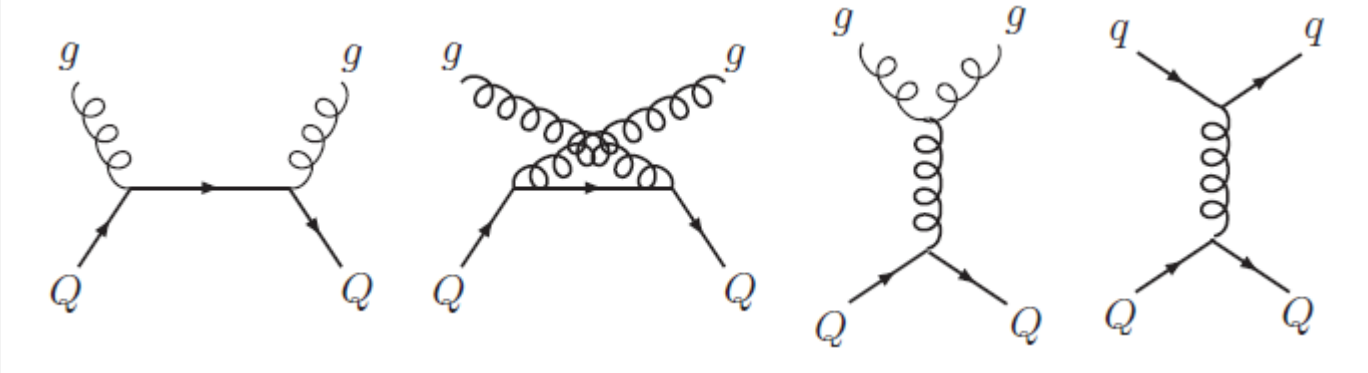
- Key ingredients:

  imaginary parts + their ω dependence

- heavy-quark self-energies calculated self-consistently from **Qq T-matrix**



2.1 Leading-Order Perturbative QCD



- gluon exchange regularized by Debye mass**:**

$$G(t) = \frac{1}{t} \rightarrow \frac{1}{t - \mu_D^2}\ , \quad \mu_D = gT$$

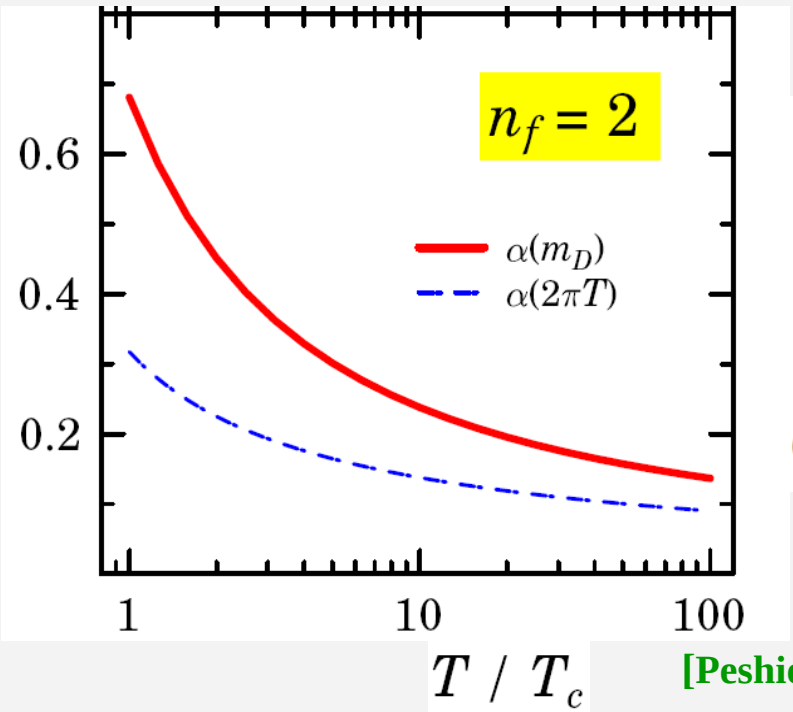
[Svetitsky '88, Mustafa et al '98, Molnar et al '04, Zhang et al '04, Hees+RR '04, Teaney+Moore'04]

- dominated by forward scattering
- long thermalization time **$\gamma^{-1} = \tau_c \geq 20$ fm/c (T≤ 300MeV, α_s=0.4)**

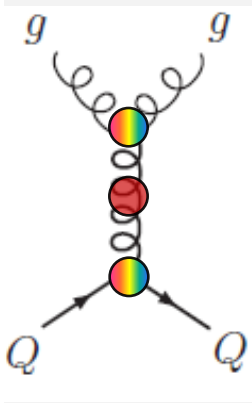
2.2 Perturbative QCD with Running Coupling

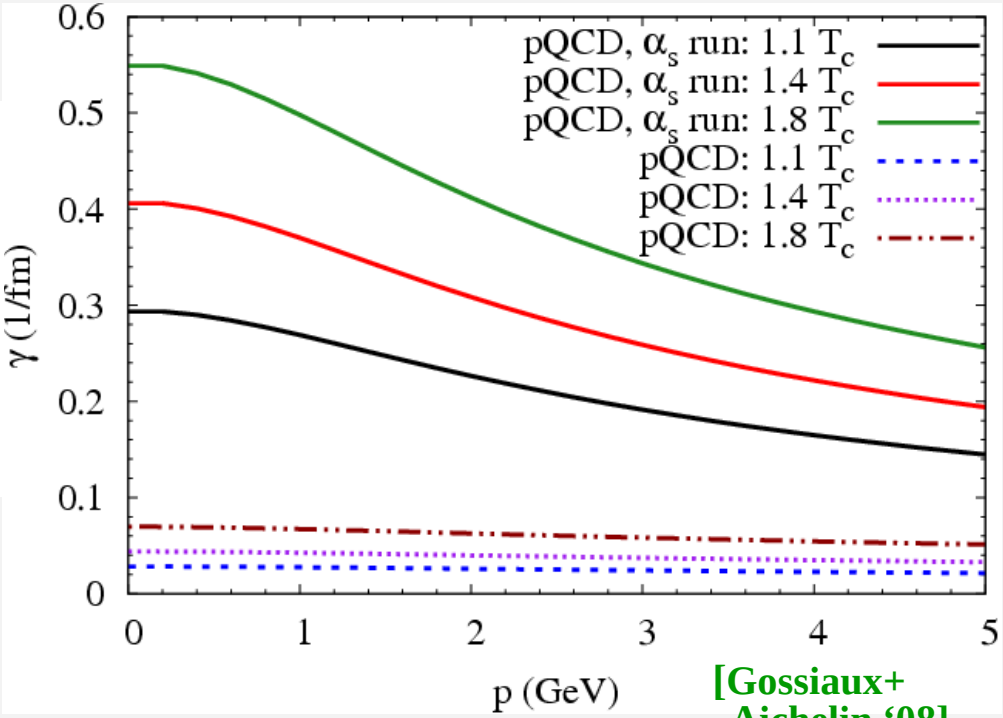
- run α_s to $m_D \sim gT$, rather than $2\pi T$
- reduced Debye mass $0.2\, m_D^2$

$$G(t) = \frac{\alpha}{t} \rightarrow \frac{\alpha_{\rm eff}(t)}{t - \tilde{\mu}^2}$$



[Peshier '07]





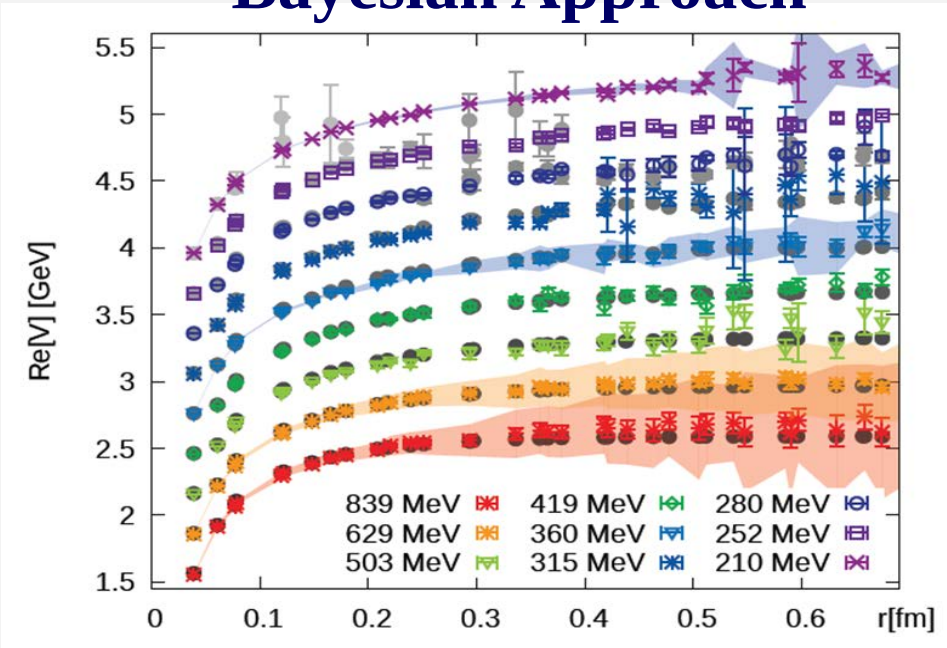
[Gossiaux+ Aichelin '08]

- factor ~10 faster thermalization: $\tau_c \approx$ **2-3 fm/c**
- perturbative regime? Need to resum large diagrams…

2.4 Potential Extraction from Lattice Data

- **Free Energy** $F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$
- **$Q\bar{Q}$ Spectral Function** $\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$

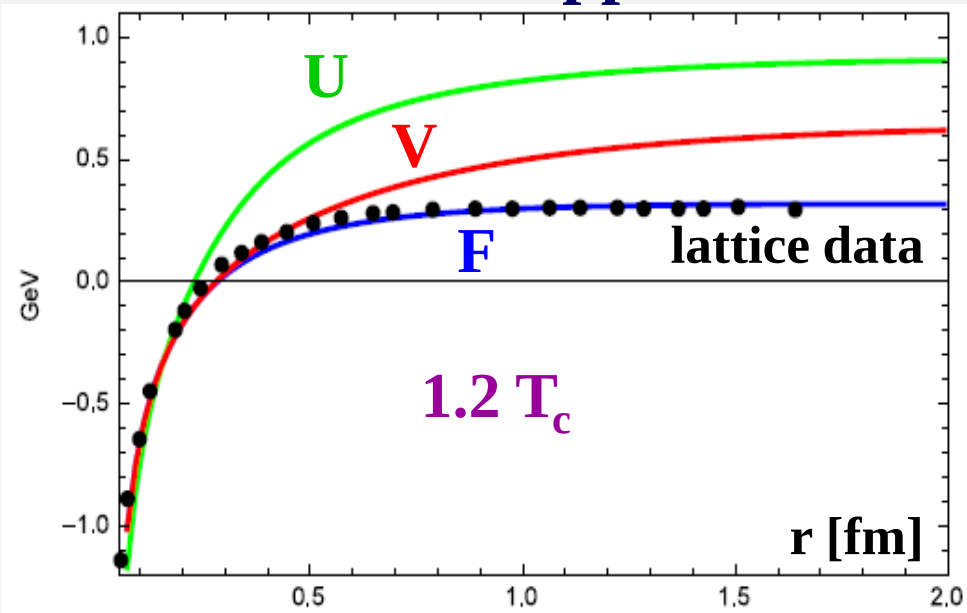
Bayesian Approach



- **Potential close to free energy**

[Burnier et al '14]

T-Matrix Approach



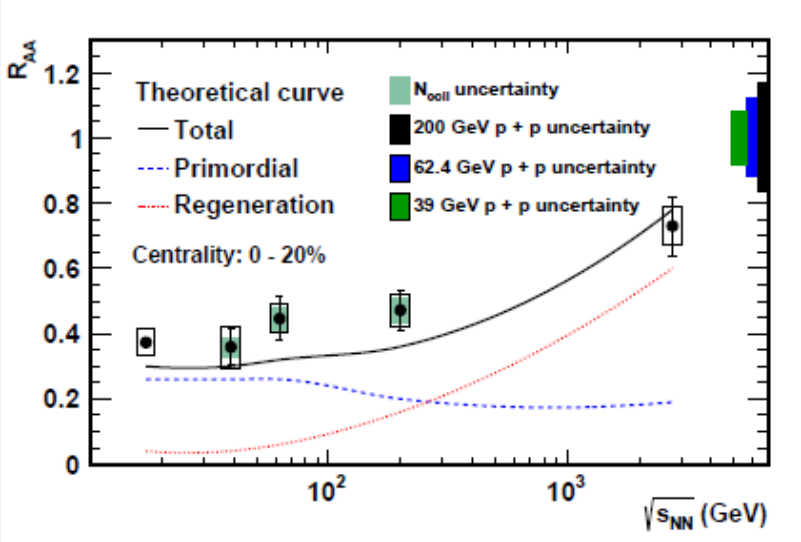
- Account for large imaginary parts
- **Remnant of confining force!**

[S.Liu+RR '15]

4.1 Upshot of Quarkonium Phenomenology

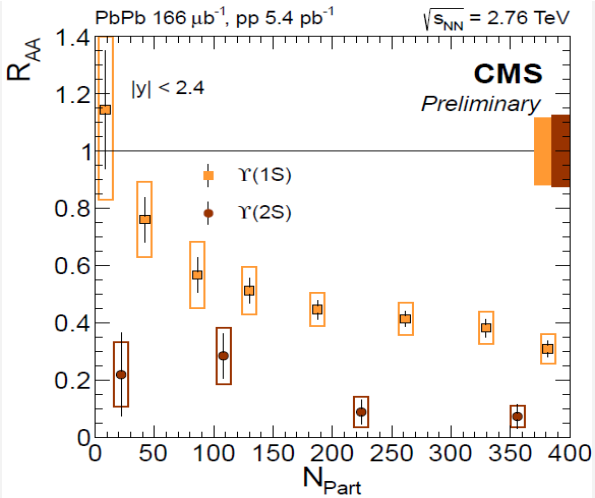
A. Charmonia (J/ψ)

- **SPS**: large cold-nucl.-abs., $R_{AA}^{hot} \sim 0.8$
- **RHIC**: $R_{AA}^{hot} \sim 0.6$
- **LHC**: $R_{AA}^{hot} \sim 0.7$,
  low-p_T excess, sizable v_2



B. Bottomonia

- **RHIC**: $R_{AA}[\Upsilon(1S)] \sim 0.7$
- **LHC**: $R_{AA}[\Upsilon(1S)] \sim 0.4$, $R_{AA}[\Upsilon(2S)] \sim 0.1$

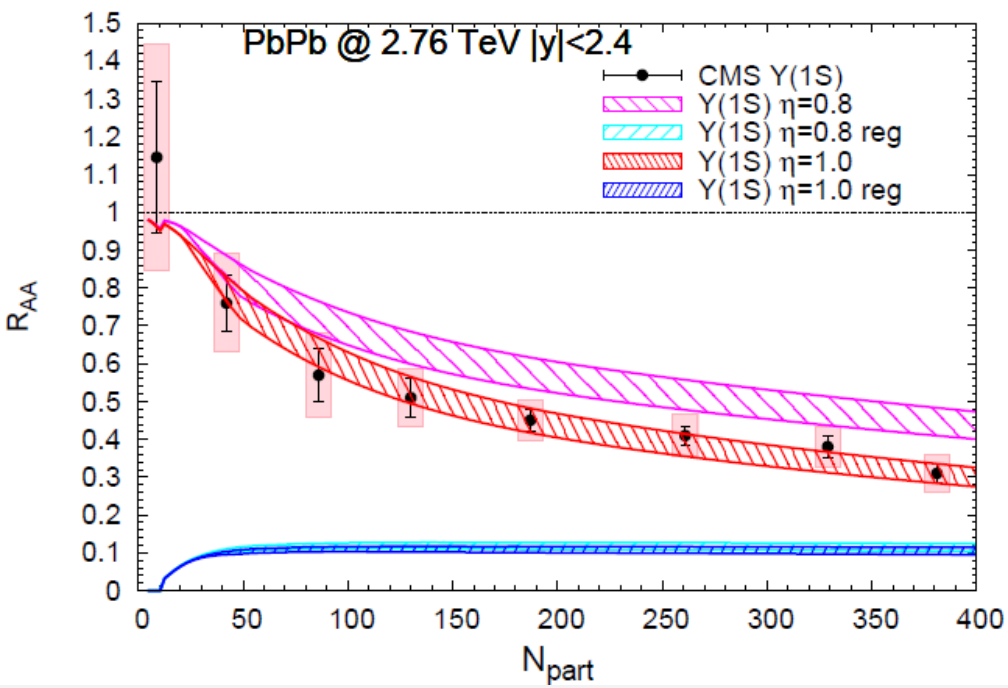


C. Implications

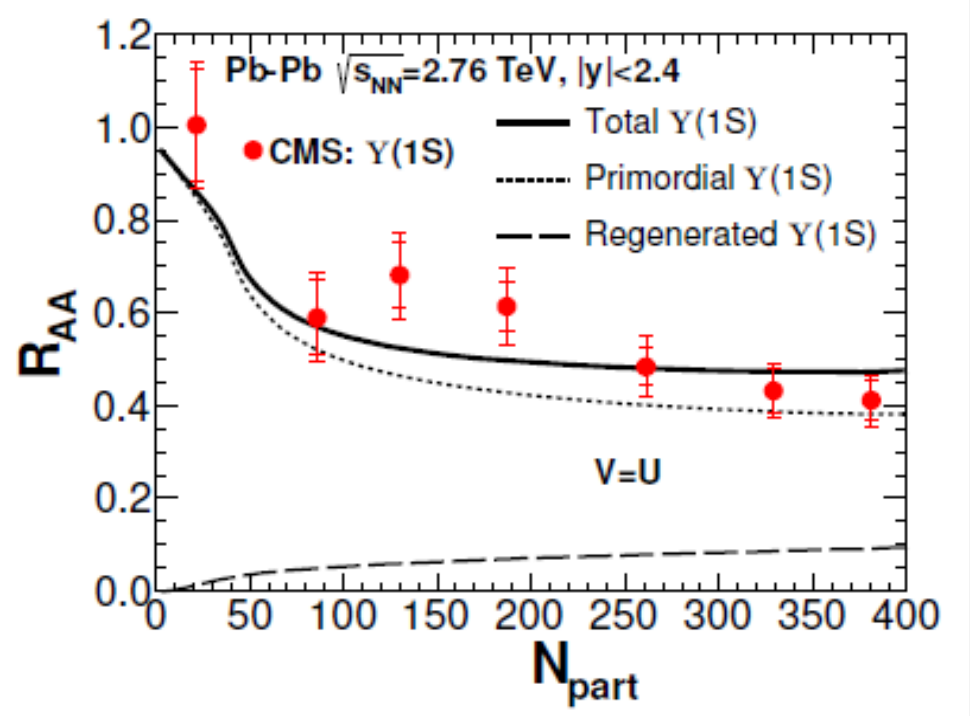
- T_0^{SPS} (~230) < $T_{diss}(J/\psi, \Upsilon')$ < T_0^{RHIC} (~350) < T_0^{LHC}(~550) ≤ $T_{diss}(\Upsilon)$
- **confining force screened at RHIC+LHC**
- thermalizing(!) charm quarks recombine at **LHC**

4.2 ϒ(1S): Screening and Regeneration?

… as implemented in current transport approaches



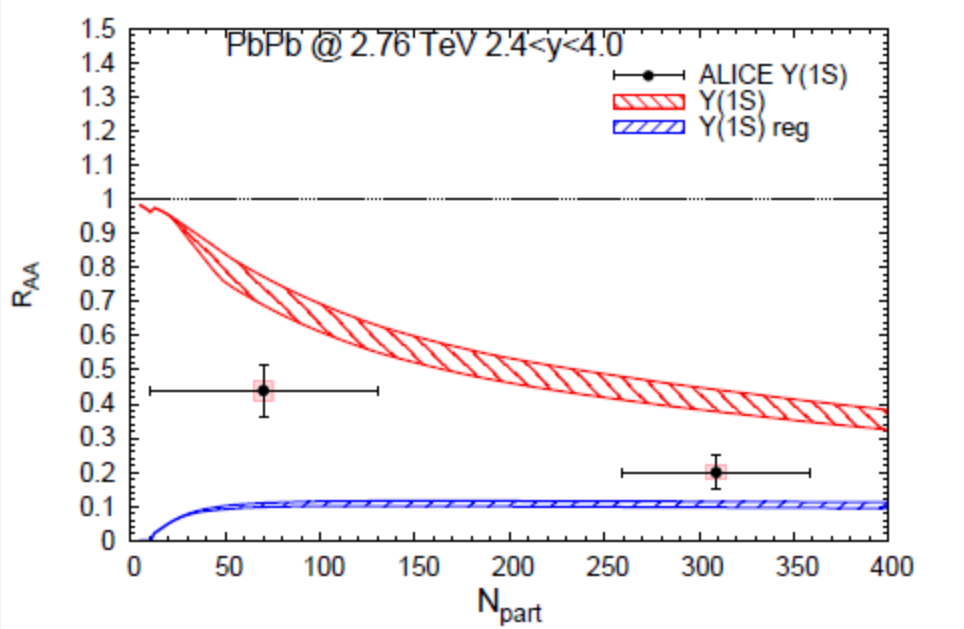
[TAMU]

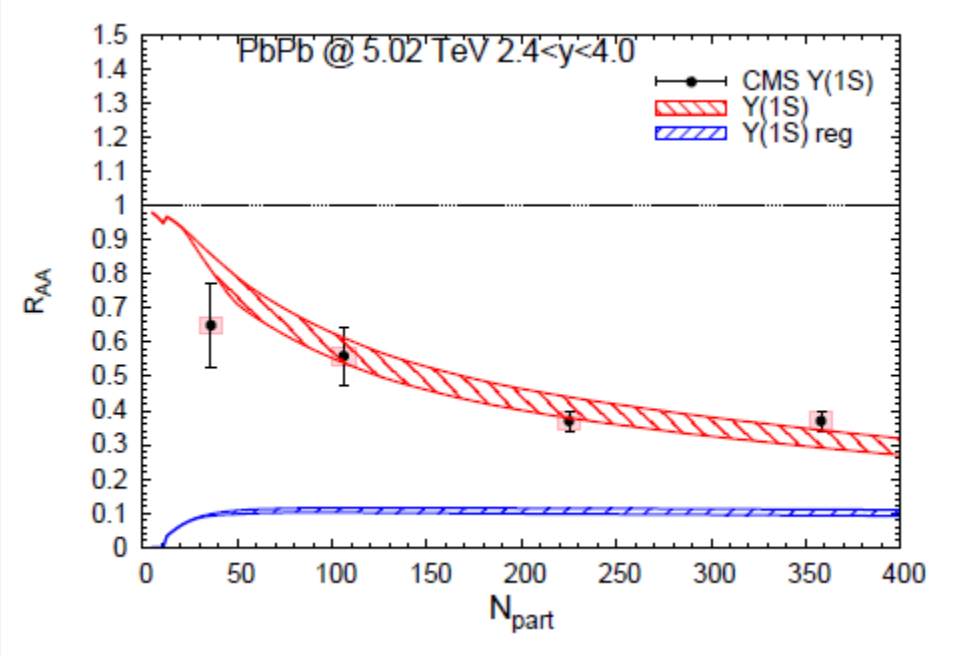


[Tsinghua]

- sensitive to color-screening; prefer strong binding (**U**-pot)
- role of regeneration for **ϒ(1S)**? (larger for **ϒ(2S)**)

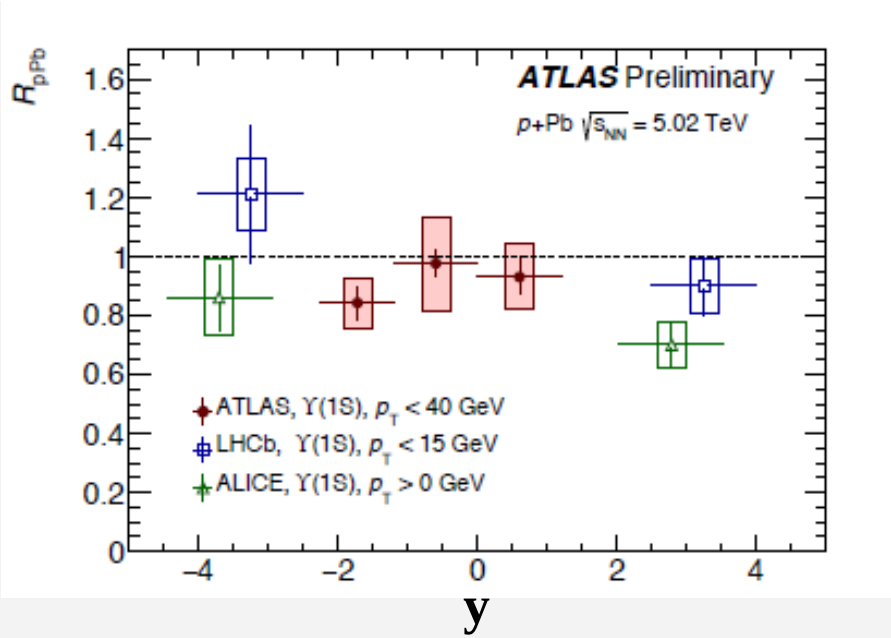
4.3 ϒ(1S): Rapidity Puzzle



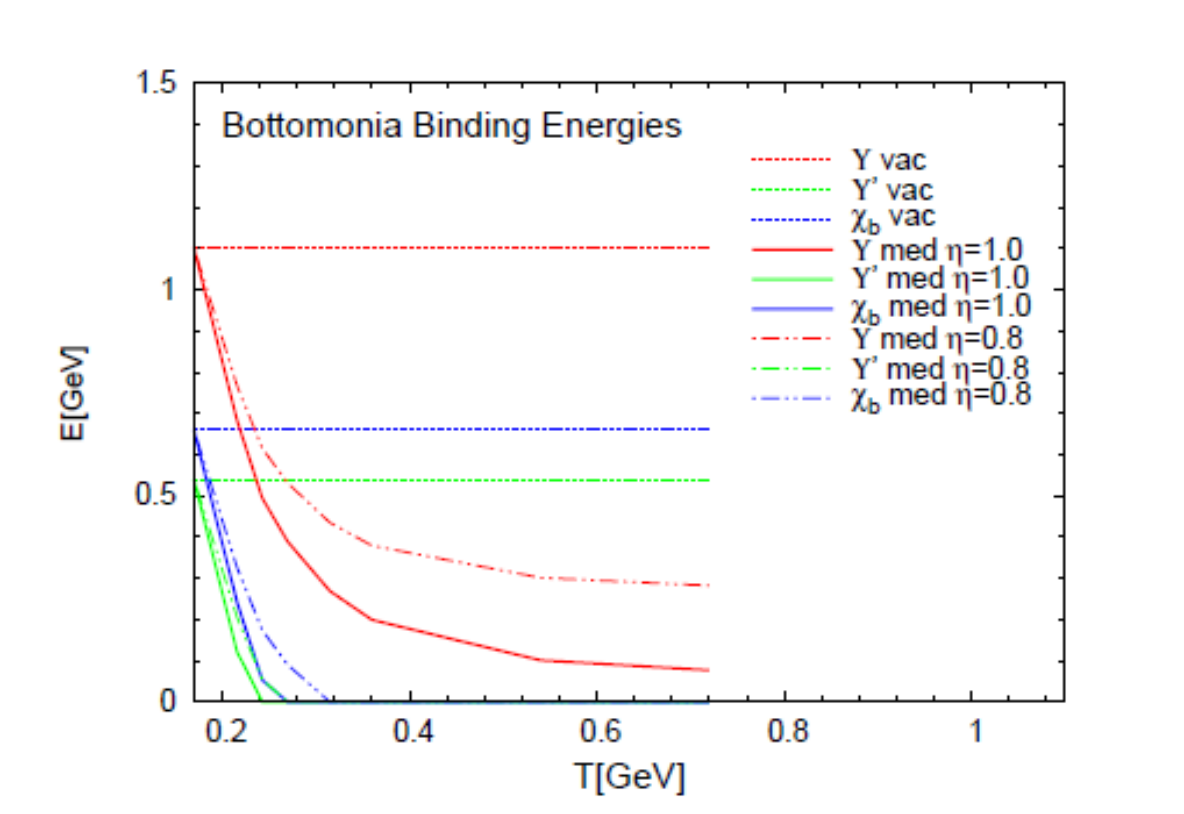


- problem of large(r) suppression in 2.76 TeV **ALICE** data
- beware of cold nuclear matter effects
- Regeneration: $N_{bb} \sim 1$ for central PbPb

  $\Rightarrow$ canonical limit $N_{\Upsilon} \sim (N_{bb})^1$

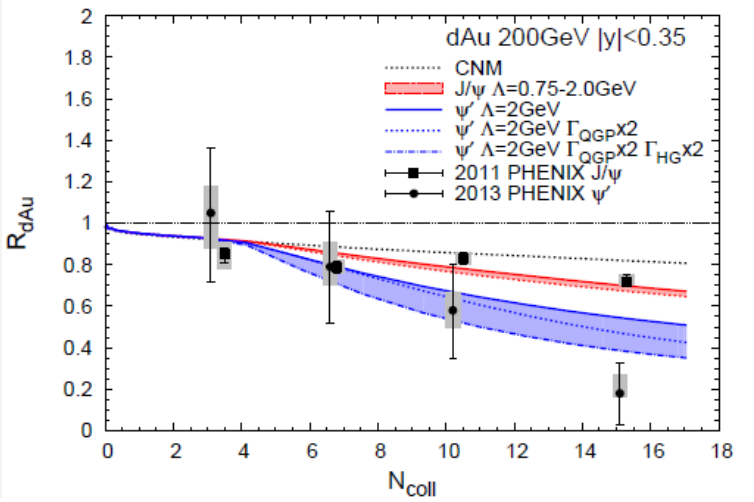


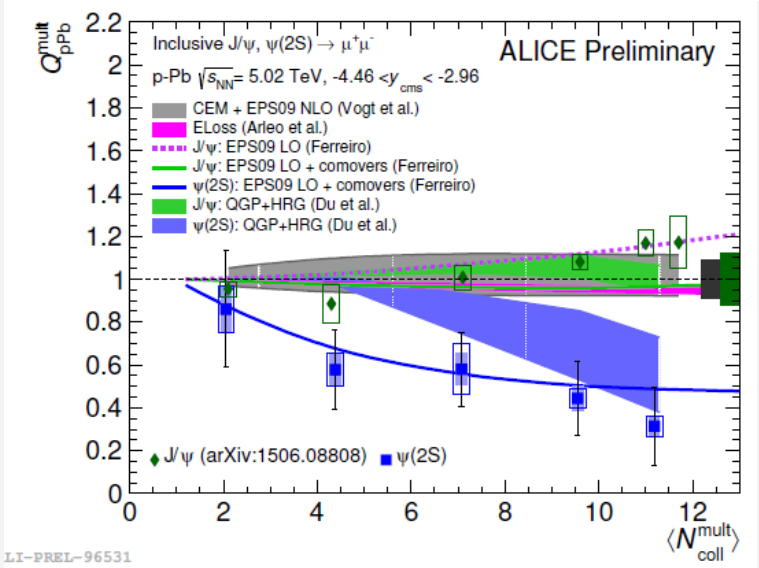
4.5 ϒ Binding Energies



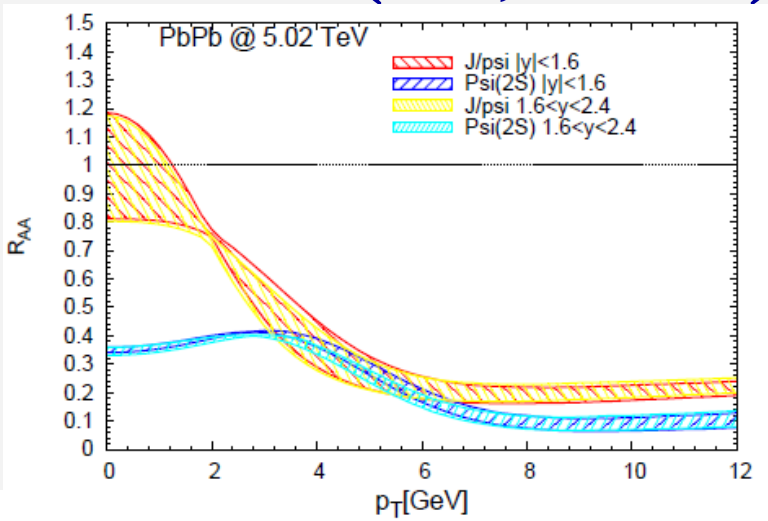
4.6 ψ(2S): Sequential Regeneration?

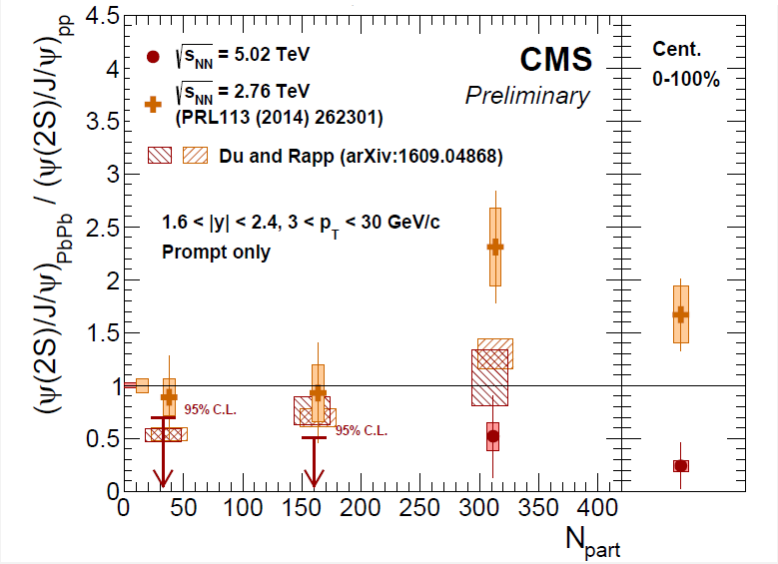
d-Au(0.2TeV) + p-Pb(5TeV)





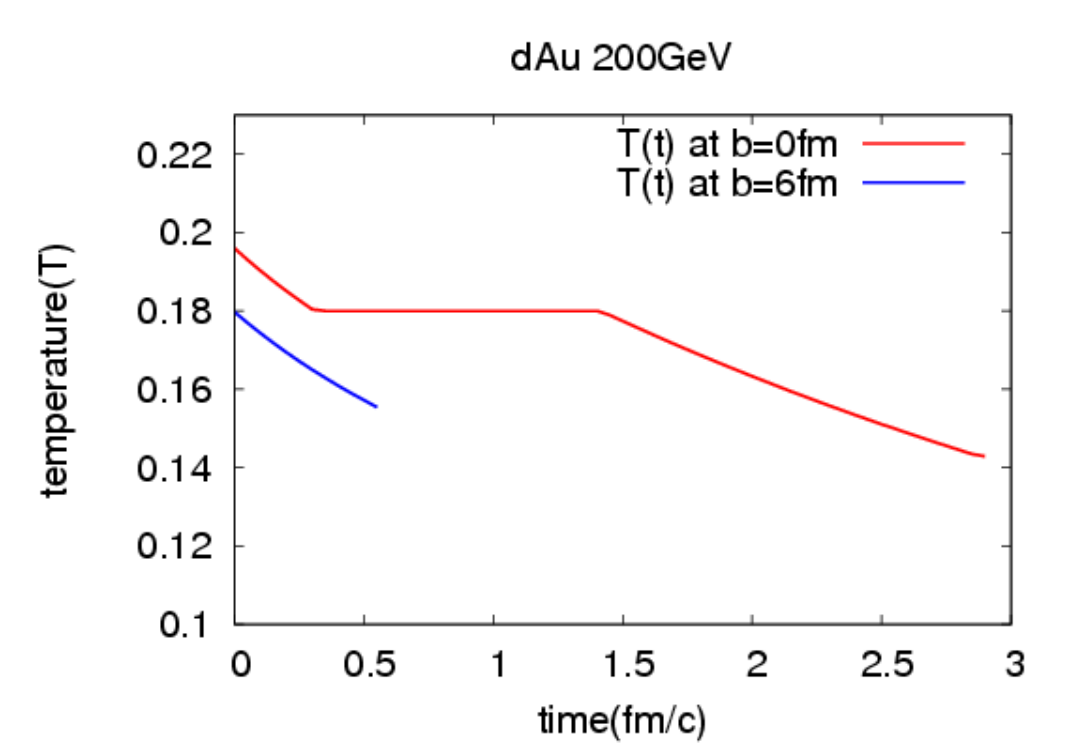
Pb-Pb(2.76,5.02TeV)

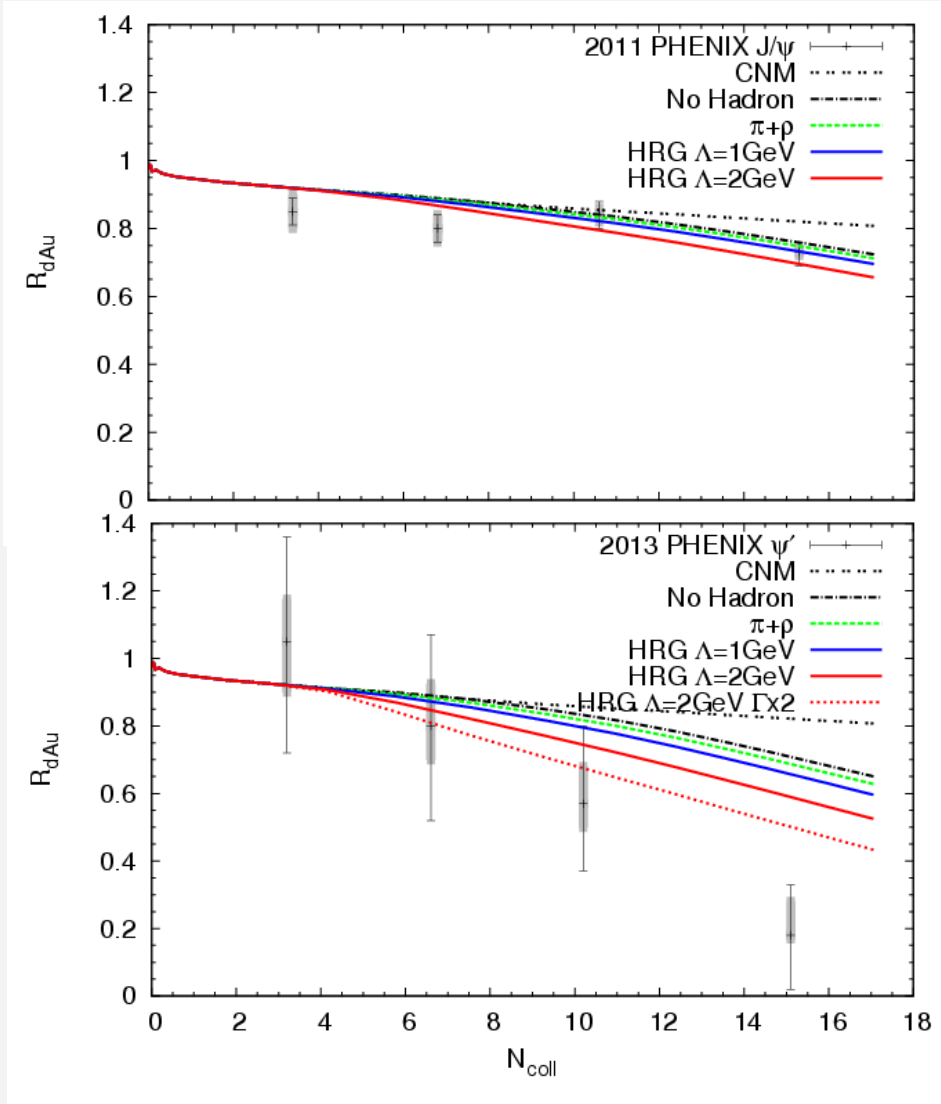




- **ψ'** suppression dAu, pPb → regenerated later in AA (larger flow than **J/ψ**)

4.2 Charmonia in d+Au Fireball





- construct fireball + evolve rate equat.
  → ψ′ suppression from hot medium
- similar in spirit to comover approach
  [Ferreiro '14]
- formation time effects?!

[X.Du+RR, in prep]

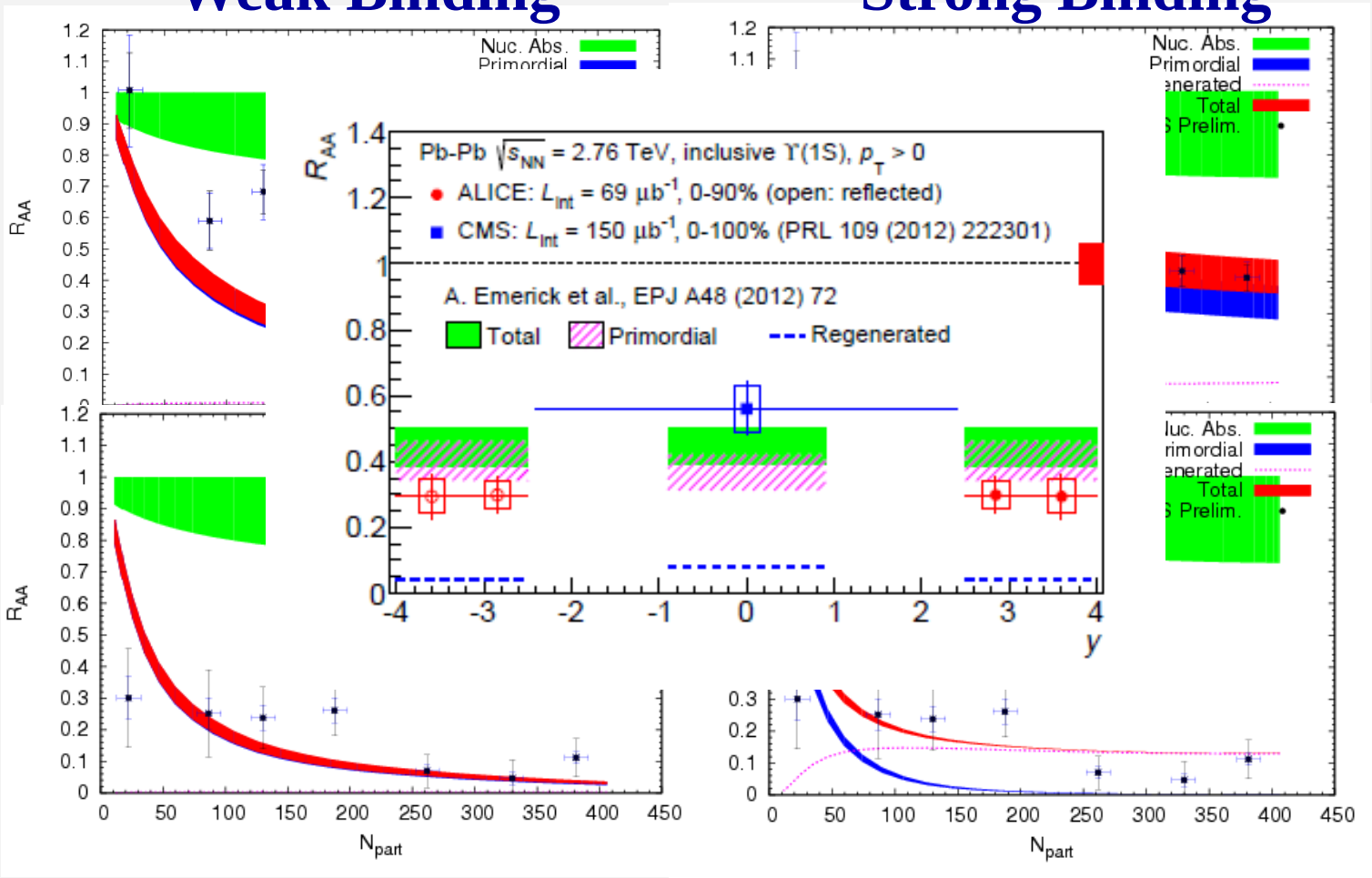
[Y.Liu, Ko et al '14]

3.6 ϒ(1S) and ϒ(2S) at LHC

Weak Binding **Strong Binding**

ϒ(1S) →

ϒ(2S) →



[Grandchamp et al '06, Emerick et al '11]

- sensitive to color-screening + early evolution times
- clear preference for strong binding (**U** potential)
- similar results by [Strickland '12]
- possible problem in rapidity dependence

3.2 Transport Approaches

- **Boltzmann equation for HQ phase-space distribution f_Q**

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{\omega_{\boldsymbol{p}}}\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_{\mathrm{Q}}(t,\boldsymbol{x},\boldsymbol{p}) = C[f_{\mathrm{Q}}]$$

  - explicit simulation of medium (quasi-) particles in collision term
  - semi-classical approximation

- **Fokker-Planck equation**

$$\frac{\partial}{\partial t} f_Q(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{A_i(\boldsymbol{p}) f_Q(t,\boldsymbol{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(t,\boldsymbol{p})]\right\}$$

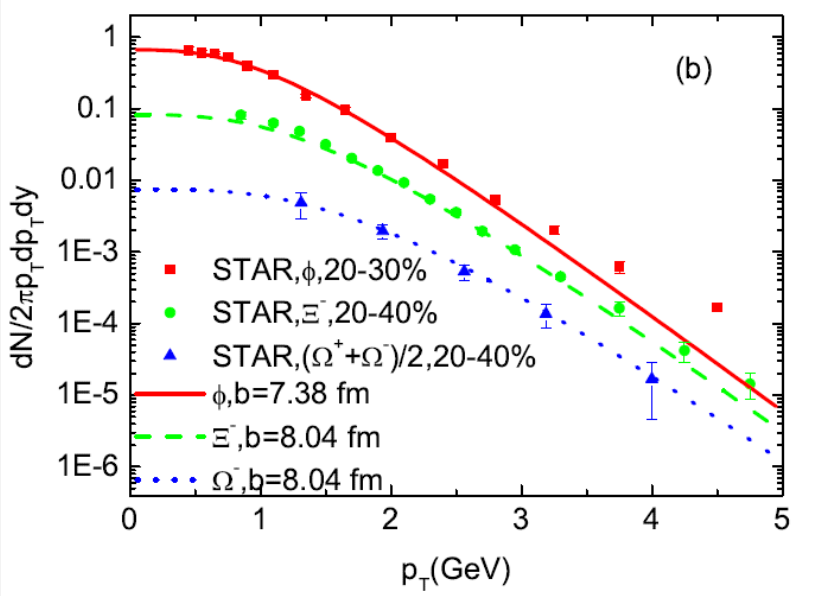
  - follows from Boltzmann with $\mathbf{q^2 \ll p^2}$ ($\mathbf{T^2 \ll 2m_Q T}$); ok for $\mathbf{m_Q/T \geq 5}$
  - does not require quasi-particle medium
  - well suited for strongly coupled medium where $\mathbf{E_{th} \leq \Gamma_{q,Q} < m_Q}$

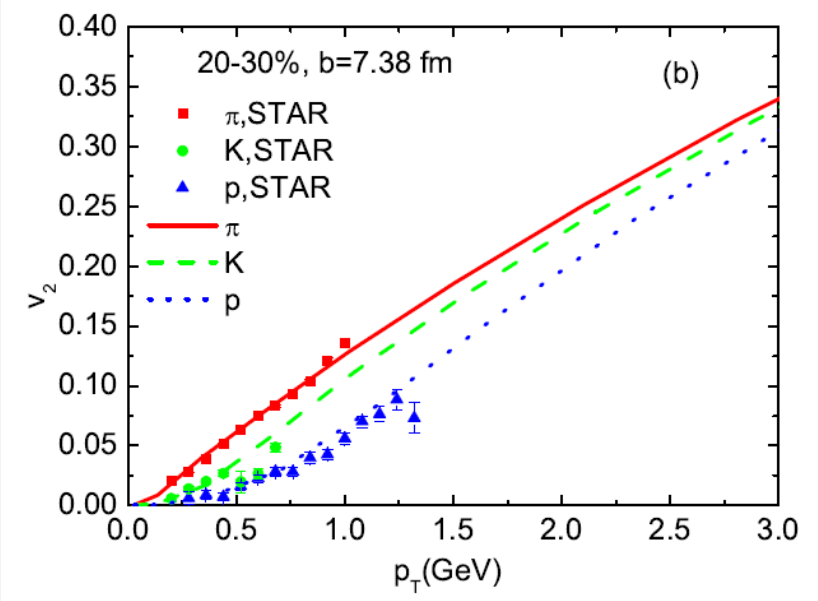
3.3 Quantitative Bulk-Medium Evolution

- initial conditions (compact, initial flow?)
- EoS: lattice (QGP, T_c~170MeV) + chemically frozen hadronic phase
- spectra + elliptic flow: multistrange at T_{ch} ~ 160MeV

[He et al '11]

π, K, p, Λ, … at T_{fo} ~ 110MeV





- v_2 saturates at T_{ch}, good light-/strange-hadron phenomenology