Heavy-Flavor Theory for Heavy-Ion Collisions

Ralf Rapp
Cyclotron Institute +
Dept. of Physics & Astronomy
Texas A&M University
College Station, TX
USA

RHIC & AGS Users’ Meeting 2017
“From Protons to Heavy Ions, and Back Again”
BNL, Upton (NY), June 20-23, 2017
1.) Introduction: A “Calibrated” QCD Force

- Vacuum quarkonium spectroscopy well described
- Confinement ↔ linear part of potential

**Objective:** Determine medium-modifications of QCD force and infer HF transport properties + spectral functions to probe QGP at varying resolution.

Exploit $m_Q \gg \Lambda_{QCD}, T_c, T_{RHIC,LHC}$
1.2 Heavy Quarks in Medium

- **Diffusion:** “Brownian motion”
- Thermalization delayed by $m_Q/T$
  \[ \rightarrow \text{memory in URHICs} \]
- Direct access to transport coefficient $D_s (2\pi T)$ \[ \sim \frac{\eta}{s} \sim \frac{\sigma_{EM}}{T} \]
- Elastic scattering rates (radiation suppressed, $q_0^2 \ll q^2$)
  \[ \rightarrow \text{widths; quasiparticles? ($m_Q \gg T$)} \]
- Probe of hadronization
- Non-perturbative effects until $\sim 2T_{pc}$
- Potential-type interactions ($q_0 \sim q^2/m_Q \ll q$)
  \[ \rightarrow \text{same as for heavy quarkonia!} \]
- Ample connections to lattice QCD

[He et al ‘12]
1.) Introduction

2.) Heavy-Flavor Interactions in QCD Matter

3.) Heavy-Flavor Transport

4.) Phenomenology

5.) Conclusions
2.1 Heavy-Light Scattering Amplitude in QGP

- Scattering equation

\[ T_{qQ} = V_{qQ} + \int V_{qQ} \, D_{q} \, D_{Q} \, T_{qQ} \]

- Perturbative approximation (weak coupling): \( T_{qQ} \approx V_{qQ} \)

- Strong coupling \( \rightarrow \) resummation:

- Thermal quark propagator:

\[ D_{Q} = \frac{1}{[\omega - \omega_k - \Sigma_{Q}(\omega,k)]} \]

- Quark self-energies \( \rightarrow \) self-consistency:

- In-medium potential \( V_{\text{in}} \)

\[ \Sigma_{Q} = \]

\[ [\text{Mannarelli+RR '05, Cabrera+RR '06, Riek+RR '10, Liu+RR '15}] \]
2.1.2 Free Energy + Potential from T-Matrix

- Free Energy

\[ F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta} \ln (G^> (-i\beta, r_1 - r_2)) = -\frac{1}{\beta} \ln \left( \int_{-\infty}^{\infty} d\omega \sigma(\omega, r_1 - r_2) e^{-\beta\omega} \right) \]

- Spectral Function

\[ \sigma(\omega, r) = \frac{1}{\pi} \frac{(V + \Sigma_I(\omega))}{(\omega - (V + \Sigma)_R)^2 + (V + \Sigma)^2(\omega)} \]

- Potential ansatz:

\[ V_R(E, r) = -\frac{4}{3} \alpha_s \frac{e^{-m_D r}}{r} - \sigma \frac{e^{-m_s r}}{m_s} - \frac{4}{3} \alpha_s m_D + \sigma \frac{1}{m_s} \]

- remnant of long-range “confining” force in QGP

[S.Liu+RR ’15]
2.2 D-Meson + c-Quark Spectral Functions in QGP

**T-matrix w/ “lattice potential” V**

\[ T = V + VT \]

**In-Medium c-Quark Selfenergy**

\[ \Sigma_Q = T \]

D-meson resonances near \( T_{pc} \) \[\rightarrow\] c-quark quasi-particles at high \( T \)
2.3 Self-Consistent Equation of State for QGP

Thermodynamic Potential

“Skeleton” Diagrams

\[
\Omega = \mp \frac{1}{\beta} \sum_n \text{Tr}\{\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi
\]

\[
\Phi = \frac{-1}{\beta} \sum_{n, \nu} \text{Tr}\left\{\frac{1}{2\nu} \left(\frac{-1}{\beta}\right)^\nu \left[(-\beta)^\nu \Sigma_{\nu}(G)\right]G\right\}
\]

- Broad hadronic resonances emerge near $T_{pc}$, dominate EoS

[S.Liu+RR ’16]
2.3.2 Parton Spectral Functions in QGP

- QGP structure changes with resolution scale
- Quasi-particles re-emerge when increasing $M_q$, $p$, $T$
Outline

1.) Introduction

2.) Heavy-Flavor Interactions in QCD Matter

3.) Heavy-Flavor Transport

4.) Phenomenology

5.) Conclusions
3.1 Heavy-Flavor Transport in URHICs

- no “discontinuities” in interaction
  ⇒ diffusion toward $T_{pc}$ and hadronization same interaction (confining!)

- initial cond. (nPDFs, …), pre-equil. fields
- $c$-quark diffusion in QGP liquid
- $c$-quark hadronization
- $D$-meson diffusion in hadron liquid

3.2 Charm Transport Coefficients

- **Thermalization Rate**
  \[ \gamma_Q(p) = \int |T_{Qp}|^2 \left(1 - \cos \Theta\right) f^p \]

- **Spatial Diffusion Coefficient**
  \[ D_s = \frac{T}{m_Q \gamma_Q(p=0)} \]

- \( p \) - and \( T \)-dependence reflect core properties of QCD
- Suggests minimum of \( D_s(2\pi T) \sim 2\) - 4 near \( T_{pc} \)
- Width: \( \Gamma_{coll} \sim \frac{3}{D_s} \sim 0.5\) - 1 GeV – no light quasi-particles!
Outline

1.) Introduction

2.) Heavy-Flavor Interactions in QCD Matter

3.) Heavy-Flavor Transport

4.) Phenomenology

5.) Conclusions
4.1 Heavy-Flavor Transport at RHIC + LHC

- Flow bump in $R_{AA}$ + large $v_2$ $\leftrightarrow$ strong coupling near $T_{pc}$ (recombination)
- High-precision $v_2$: transition from elastic to radiative regime?
4.2 The $\Lambda_c$ Puzzle at RHIC

- recent studies with instantaneous coalescence (“Ko-coal”) vs. resonance recombination model (RRM) [Min He ’17]
- STAR datum of $\sim 1$ remains a puzzle at this point…
5.) Summary

- Many-body approach to extract heavy-quark potential in QGP from lQCD
- Remnants of long-range confining force survive well above $T_{pc}$
  $\rightarrow$ large scattering rates, strong-coupling diffusion (small $\mathcal{D}_s$)
  $\rightarrow$ extended transition from heavy-quark to -hadron dofs
- Same approach to describe QGP equation of state
  $\rightarrow$ no long-wavelength quasi-partons near $T_{pc}$ (re-emerge at high $p, T, m_Q$)
  $\rightarrow$ broad hadronic bound states emerge near $T_{pc}$
- In-medium heavy flavor firmly rooted in sQGP / lat-QCD properties
- Heavy-flavor phenomenology at RHIC + LHC
  - drag force imprinted on D-meson $R_{AA} + v_2$, transition to radiation
  - test hadronization: $\Lambda_c$ “puzzle”
- Ongoing theory working groups (EMMI RRTF, HQ-jet)
2.3 Heavy-Quark Free and Internal Energies in Lattice QCD

- **Free Energy**

  \[ F_1(r,T) = U_1(r,T) - T S_1(r,T) \]

- **Internal Energy**

  - “weak” $Q\bar{Q}$ potential
  - “strong” $Q\bar{Q}$ potential, $U = \langle H_{\text{int}} \rangle$

- **Thermodynamic Quantities**
  - $F$, $U$, $S$
  - **Entropy**: many-body effects

[Kaczmarek et al. '05]
2.9 D-Mesons in Hadronic Matter

- effective $D$-$h$ scattering amplitudes
  [He, Fries+RR ‘11, Tolos+Torres-Ricon ‘13]

- D-meson in pion gas:
  - consistent with unitarized HQET [Cabrera et al ‘11]
  - factor $\sim 10$ larger in heavy-meson $\chi$PT [Laine ‘11]

- hadron gas at $\sim T_c$: $\tau_D \approx 10\text{fm/c}$
2.5 Charm Susceptibilities from Lattice QCD

- Extract partial charm pressures from susceptibilities

\[ p_c(T, \mu_B, \mu_c) = p_M(T) \cosh\left( \frac{\mu_c}{T} \right) + p_B(T) \cosh\left( \frac{\mu_c + \mu_B}{T} \right) + p_Q(T) \cosh\left( \frac{\mu_c + \mu_B/3}{T} \right) \]

- Hadronic (D-meson) correlations contribute until \( \sim 1.4 T_c \)

- Realistic models of heavy-quark diffusion should account for this (induces HQ-\( v_2 \) + hadronization)
4.2 Charm Transport at LHC: D-Meson Spectra

- **$R_{AA}$** “bump” from radial flow
- **$D_s$** meson ($cs$) enhanced from coalescence with strange quarks
- Coalescence + hadronic diffusion increase $v_2$
- similar features at RHIC

[M.He et al ’14]
2.5 Brueckner Theory of Heavy Flavor in QGP

**Input**
- lattice-QCD free energy
- 2-body potential
- Quark selfenergy

**Process**
- \( Q \rightarrow Q \)
- 0-modes
- \( Q\bar{Q} \)
- T-matrix
- \( Qq \)
- T-matrix

**Output**
- quark-no. susceptibility
- spectral fcts./ eucl. correlat.
- \( Q\bar{Q} \) evolution (rate equation)
- \( Q \) spectra + \( v_2 \) (Langevin)

**Test**
- lattice data
- exp. data
2.4.2 Free Energy from T-Matrix

- **Free Energy**
  
  $$ F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta} \ln \left( G^> (-i\beta, r_1 - r_2) \right) = -\frac{1}{\beta} \ln \left( \int_{-\infty}^{\infty} d\omega \sigma(\omega, r_1 - r_2) e^{-\beta \omega} \right) $$

- **Euclidean T-matrix** in static limit

  $$ \tilde{T}(z_t|r) = V(z_t, r) + V(z_t, r) \tilde{G}^{(2)}_0(z_t - v^a, v^a) \tilde{T}(z_t|r) = \frac{V(z_t, r)}{1 - V(z_t, r) \tilde{G}^{(2)}_0(z_t)} $$

- **Spectral Function**

  $$ \sigma(\omega, r) = \frac{1}{\pi} \frac{(V + \Sigma)_I(\omega)}{(\omega - (V + \Sigma)_R)^2 + (V + \Sigma)_I^2(\omega)} $$

- **Key ingredients:**
  - imaginary parts + their $\omega$ dependence
  - heavy-quark self-energies calculated self-consistently from Qq T-matrix

[Beraudo et al '08]
[S.Liu+RR '15]
2.1 Leading-Order Perturbative QCD

- gluon exchange regularized by Debye mass:
  \[ G(t) = \frac{1}{t} \to \frac{1}{t - \mu_D^2}, \quad \mu_D = gT \]

- dominated by forward scattering

- long thermalization time \( \gamma^{-1} = \tau_c \geq 20 \text{ fm/c} \) (\( T \leq 300 \text{MeV}, \alpha_s=0.4 \))

[Svetitsky '88, Mustafa et al '98, Molnar et al '04, Zhang et al '04, Hees+RR '04, Teaney+Moore'04]
2.2 Perturbative QCD with Running Coupling

- run $\alpha_s$ to $m_D \sim gT$, rather than $2\pi T$
- reduced Debye mass $0.2m_D^2$

\[ G(t) = \frac{\alpha}{t} \rightarrow \frac{\alpha_{\text{eff}}(t)}{t - \tilde{\mu}^2} \]

- factor $\sim 10$ faster thermalization: $\tau_c \approx 2-3$ fm/c
- perturbative regime? Need to resum large diagrams…
2.4 Potential Extraction from Lattice Data

- **Free Energy**
  \[ F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta} \ln (G^\gamma (-i\beta, r_1 - r_2)) = -\frac{1}{\beta} \ln \left( \int_{-\infty}^{\infty} d\omega \sigma(\omega, r_1 - r_2) e^{-\beta\omega} \right) \]

- **Q\bar{Q} Spectral Function**
  \[ \sigma(\omega, r) = \frac{1}{\pi} \frac{(V + \Sigma)_I(\omega)}{(\omega - (V + \Sigma)_R)^2 + (V + \Sigma)_I^2(\omega)} \]

**Bayesian Approach**

- Potential close to free energy
  [Burnier et al ’14]

**T-Matrix Approach**

- Account for large imaginary parts
- Remnant of confining force!
  [S.Liu+RR ’15]
4.1 Upshot of Quarkonium Phenomenology

A. Charmonia (J/ψ)
- **SPS**: large cold-nucl.-abs., $R_{AA}^{hot} \sim 0.8$
- **RHIC**: $R_{AA}^{hot} \sim 0.6$
- **LHC**: $R_{AA}^{hot} \sim 0.7$, low-\(p_T\) excess, sizable \(v_2\)

B. Bottomonia
- **RHIC**: $R_{AA}[\Upsilon(1S)] \sim 0.7$
- **LHC**: $R_{AA}[\Upsilon(1S)] \sim 0.4$, $R_{AA}[\Upsilon(2S)] \sim 0.1$

C. Implications
- $T_0^{SPS} (\sim 230) < T_{diss}(J/\psi, \Upsilon') < T_0^{RHIC} (\sim 350) < T_0^{LHC} (\sim 550) \leq T_{diss}(\Upsilon)$
- confining force screened at RHIC+LHC
- thermalizing(!) charm quarks recombine at LHC
4.2 $\Upsilon(1S)$: Screening and Regeneration?

... as implemented in current transport approaches

- sensitive to color-screening; prefer strong binding ($U$-pot)
- role of regeneration for $\Upsilon(1S)$? (larger for $\Upsilon(2S)$)
4.3 $\Upsilon(1S)$: Rapidity Puzzle

- problem of large(r) suppression in 2.76 TeV ALICE data
- beware of cold nuclear matter effects
- Regeneration: $N_{bb} \sim 1$ for central PbPb
  \[ N_Y \sim (N_{bb})^1 \]
4.5 $\Upsilon$ Binding Energies
4.6 $\psi(2S)$: Sequential Regeneration?

$d$-Au($0.2\text{TeV}$) + $p$-Pb($5\text{TeV}$) vs. Pb-Pb($2.76, 5.02\text{TeV}$)

- $\psi'$ suppression $d$Au, $p$Pb → regenerated later in AA (larger flow than $J/\psi$)

- $J/\psi$, $\psi(2S)$: ALICE Preliminary
  - Inclusive $J/\psi$, $\psi(2S)$ → $\mu^+\mu^-$
  - $p$-Pb ($S_N = 5.02\text{ TeV}$, $-4.46 < y_{CM} < -2.96$

- $J/\psi$, $\psi(2S)$: CMS Preliminary
  - $1.6 < |y| < 2.3$, $p_T < 30\text{ GeV}$
  - Prompt only

Du and Rapp (arXiv:1609.04868)

- $J/\psi$, $\psi(2S)$: Regeneration
  - Large flow in AA
  - Fitted with PHENIX $\rho^0$ (argin)
4.2 Charmonia in d+Au Fireball

• construct fireball + evolve rate equat. → $\psi'$ suppression from hot medium

• similar in spirit to comover approach [Ferreiro ‘14]

• formation time effects?! [X.Du+RR, in prep] [Y.Liu, Ko et al ‘14]
3.6 $\Upsilon(1S)$ and $\Upsilon(2S)$ at LHC

- sensitive to color-screening + early evolution times
- clear preference for strong binding (U potential)
- similar results by [Strickland '12]
- possible problem in rapidity dependence

$\Upsilon(1S) \rightarrow \Upsilon(2S) \rightarrow \ldots$

[Grandchamp et al '06, Emerick et al '11]
3.2 Transport Approaches

- **Boltzmann equation for HQ phase-space distribution** $f_Q$

\[
\left[ \frac{\partial}{\partial t} + \frac{p}{\omega_p} \frac{\partial}{\partial x} + F \frac{\partial}{\partial p} \right] f_Q(t, x, p) = C[f_Q]
\]

- explicit simulation of medium (quasi-) particles in collision term
- semi-classical approximation

- **Fokker-Planck equation**

\[
\frac{\partial}{\partial t} f_Q(t, p) = \frac{\partial}{\partial p_i} \left\{ A_i(p) f_Q(t, p) + \frac{\partial}{\partial p_j} [B_{ij}(p) f_Q(t, p)] \right\}
\]

- follows from Boltzmann with $q^2 \ll p^2$ ($T^2 \ll 2m_Q T$); ok for $m_Q/T \geq 5$
- does not require quasi-particle medium
- well suited for strongly coupled medium where $E_{th} \leq \Gamma_{q,Q} < m_Q$
3.3 Quantitative Bulk-Medium Evolution

- initial conditions (compact, initial flow?)
- EoS: lattice (QGP, $T_c \sim 170\text{MeV}$) + chemically frozen hadronic phase
- spectra + elliptic flow: multistrange at $T_{ch} \sim 160\text{MeV}$
  \[ \pi, K, p, \Lambda, \ldots \text{ at } T_{fo} \sim 110\text{MeV} \]

$\nu_2$ saturates at $T_{ch}$, good light-/strange-hadron phenomenology

[He et al. '11]