U.S. DEPARTMENT OF ENERGY | Office of Science

BROOKHAVEN NATIONAL LABORATORY

BJÖRN SCHENKE, BROOKHAVEN NATIONAL LABORATORY

BEAM ENERGY SCAN THEORY

RECENT HIGHLIGHTS FROM THE BEST COLLABORATION

2018 RHIC & AGS Annual Users' Meeting

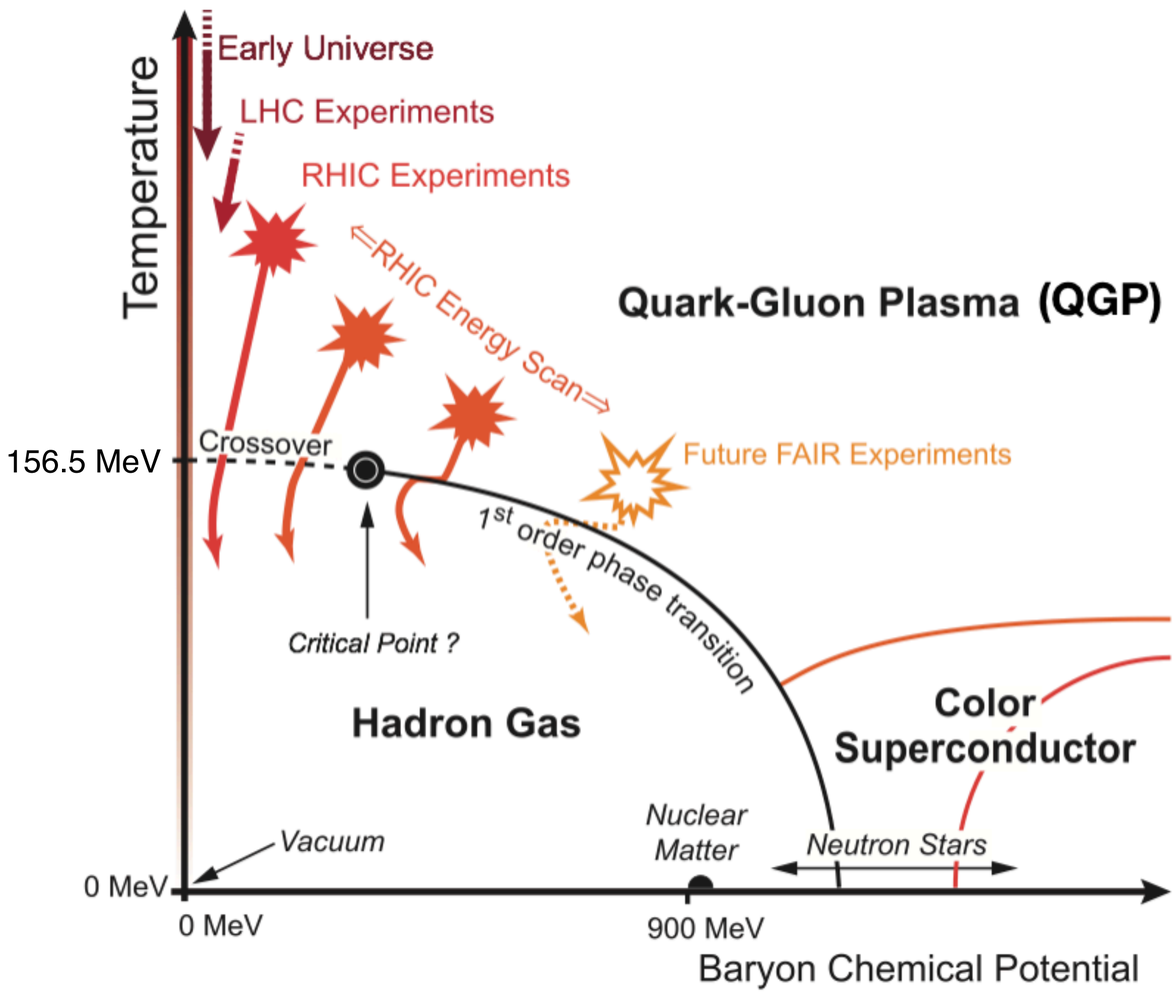
BEST COLLABORATION

BEAM ENERGY SCAN (BES) II AT RHIC

- QCD critical point & phase diagram
- Properties of baryon-rich QGP
- Onset of chiral symmetry restoration
- Unexpected new phenomena

Needs coordinated theory effort:

BEST COLLABORATION



THE BEAM ENERGY SCAN THEORY COLLABORATION

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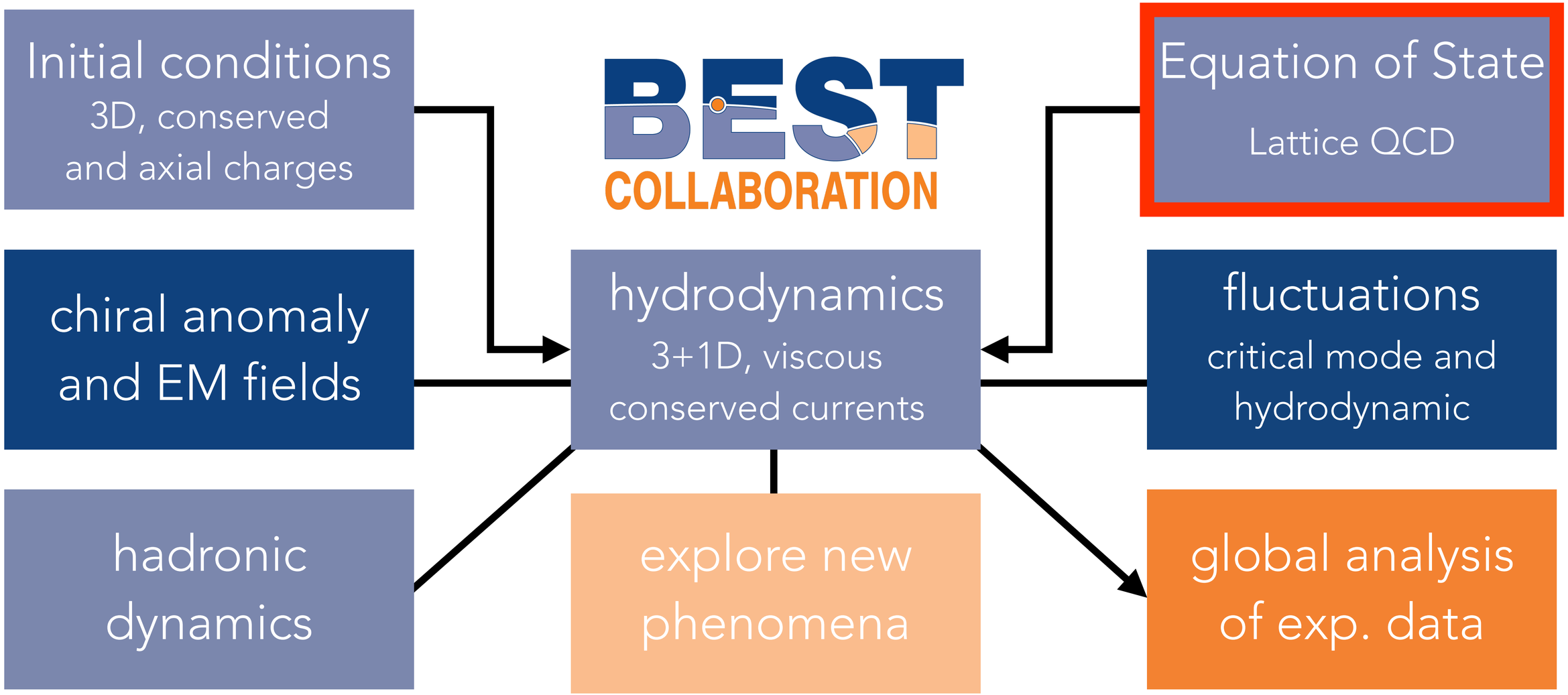
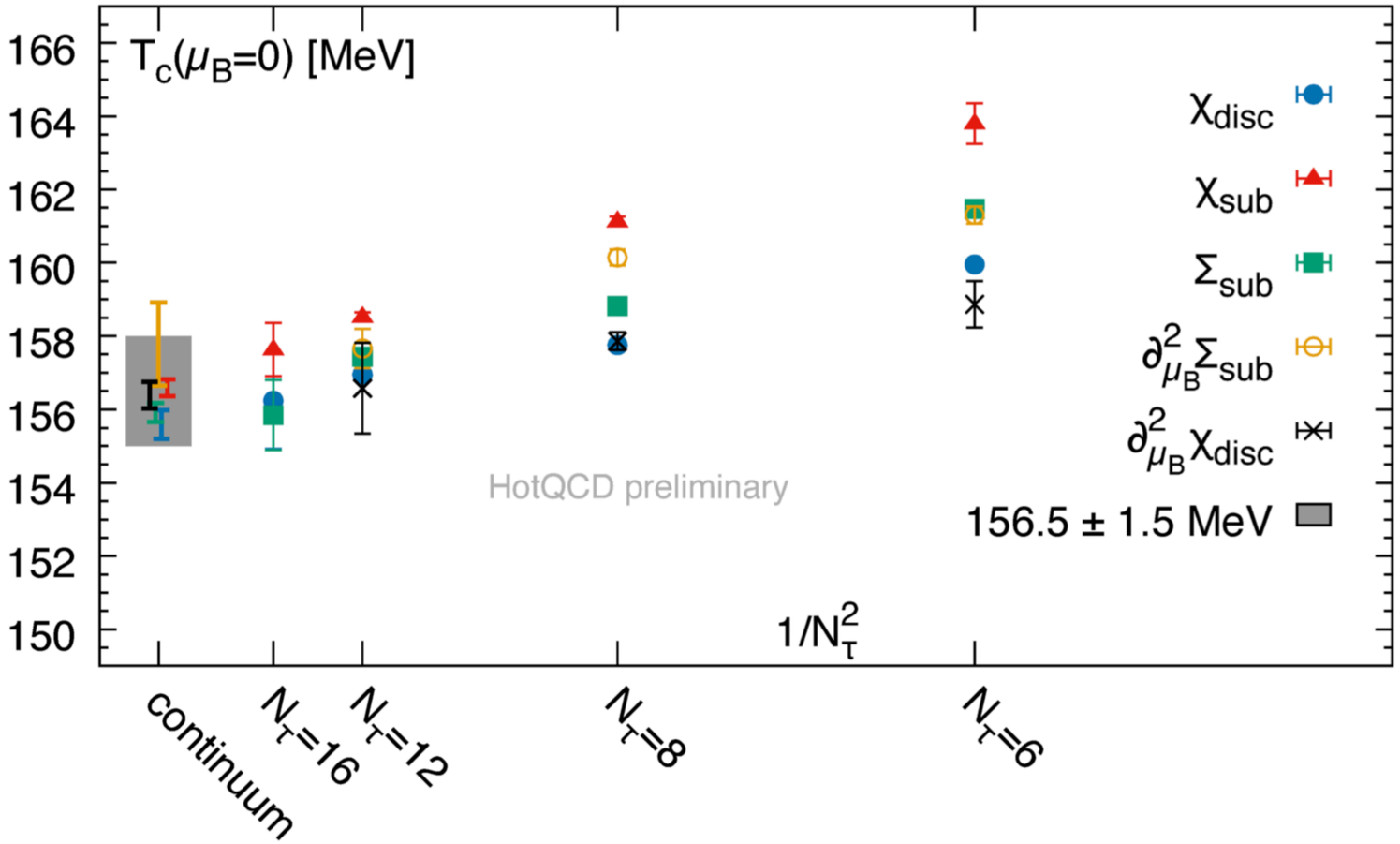


EQUATION OF STATE AT $\mu_B=0$



P. Steinbrecher @QM2018

Latest result for the QCD crossover temperature: Tc ≃ 156.5 ± 1.5 MeV

from the chiral condensate

EQUATION OF STATE AT $\mu_B \neq 0$

$$Z = \int \mathcal{D}U e^{-S_G[U]} \det M[\mu_B]$$

$\det M[\mu_B]$ is complex and Monte Carlo simulations are not feasible

→ sign problem

One way to cope: Taylor expansion around μ_B=0:

$$\frac{P(T,\mu_B)}{T^4} = \sum_n \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n \quad \text{where} \quad \chi_n^B = \left. \frac{\partial^n P(T,\mu_B/T)/T^4}{\partial(\mu_B/T)^n} \right|_{\mu_B=0}$$

are the susceptibilities

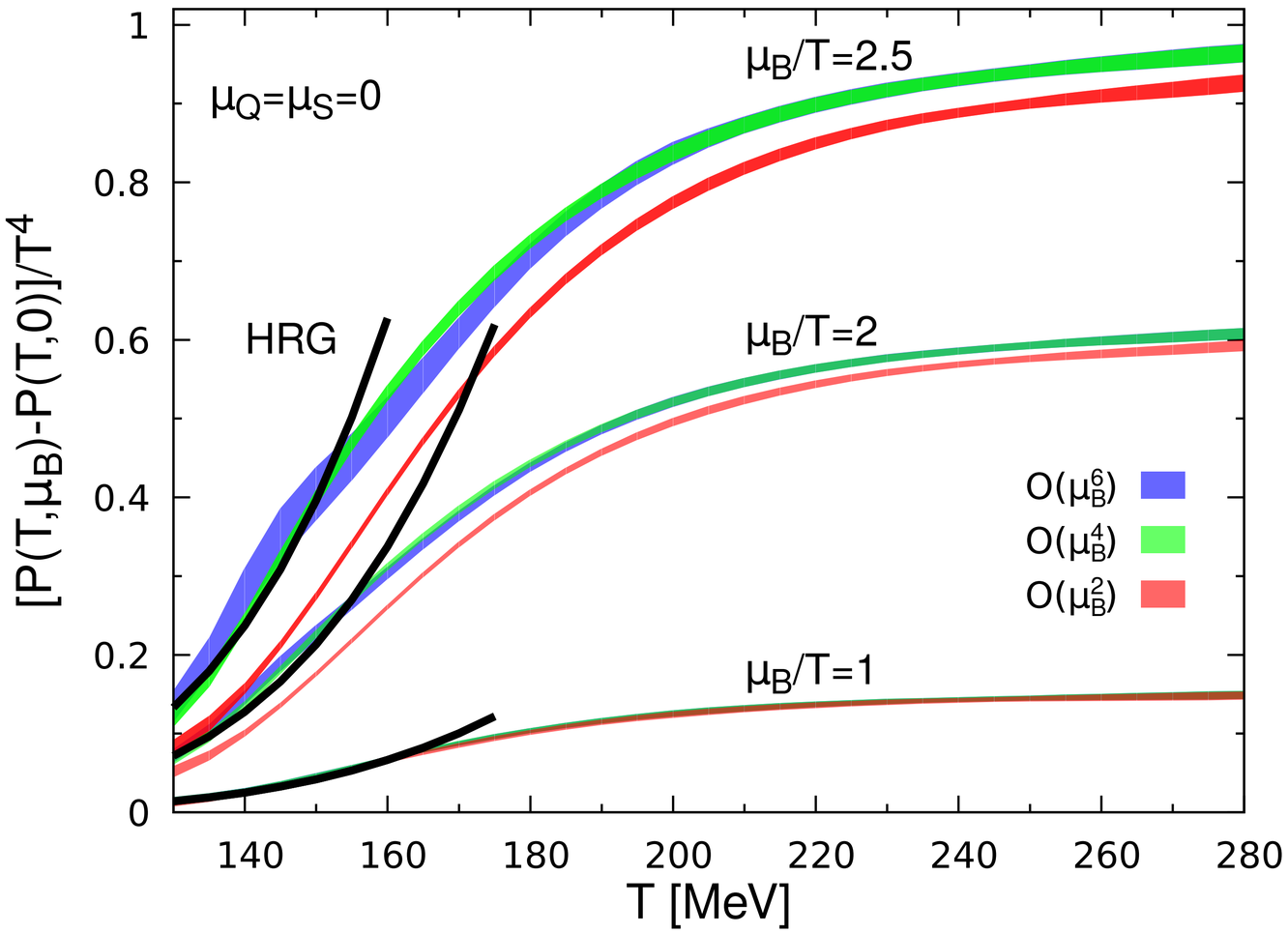
EQUATION OF STATE AT $\mu_B \neq 0$

Taylor expansion up to $\mathcal{O}(\mu_B^6)$

Present reach of the Lattice QCD EoS:

$$\mu_B/T \lesssim 2$$

HotQCD: Phys. Rev. D95, 054504 (2017)



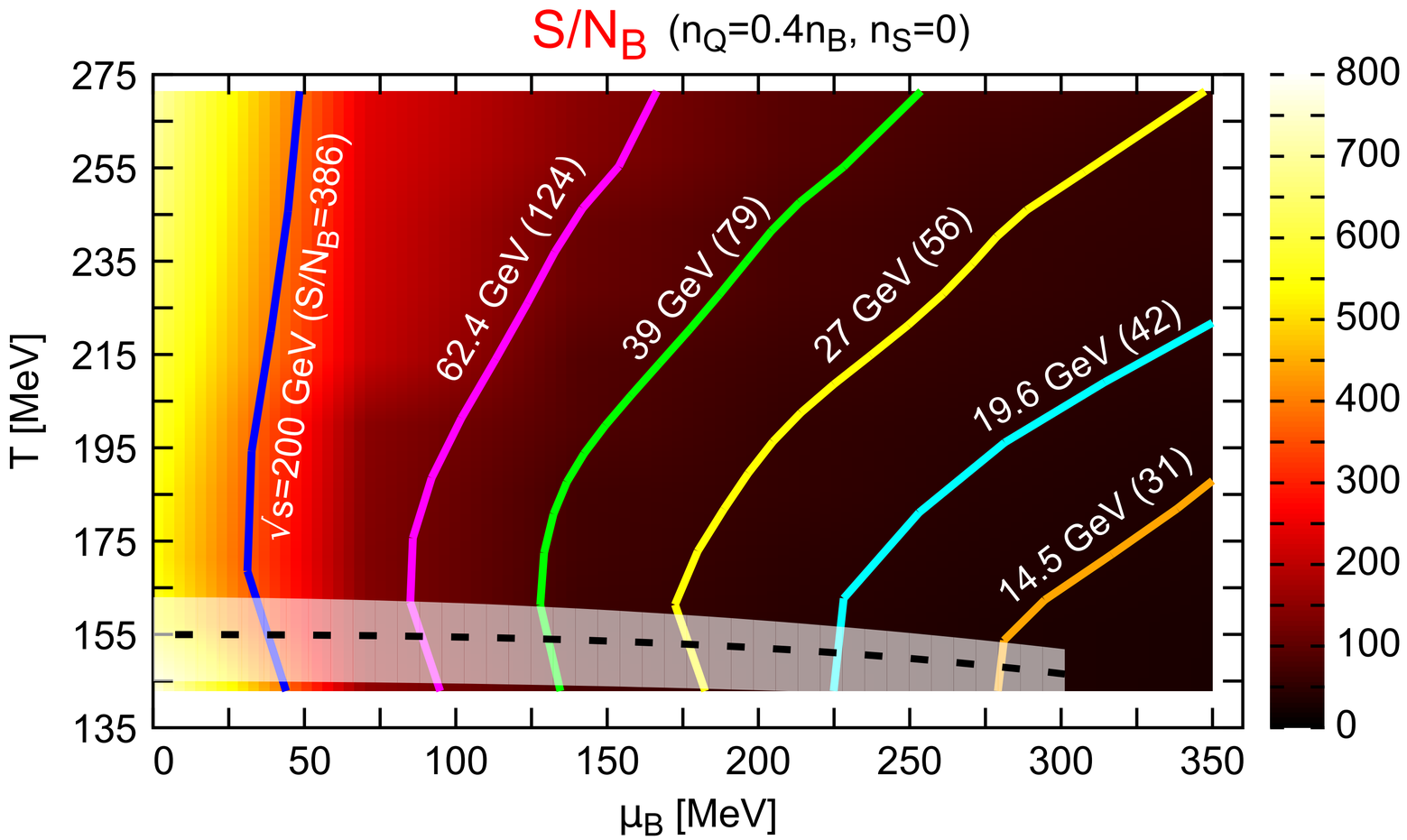
EQUATION OF STATE AT $\mu_B \neq 0$

Present reach of the Lattice QCD EoS:

$$\mu_B/T \lesssim 2$$

or

$$\sqrt{s} \gtrsim 14.5\,\mathrm{GeV}$$



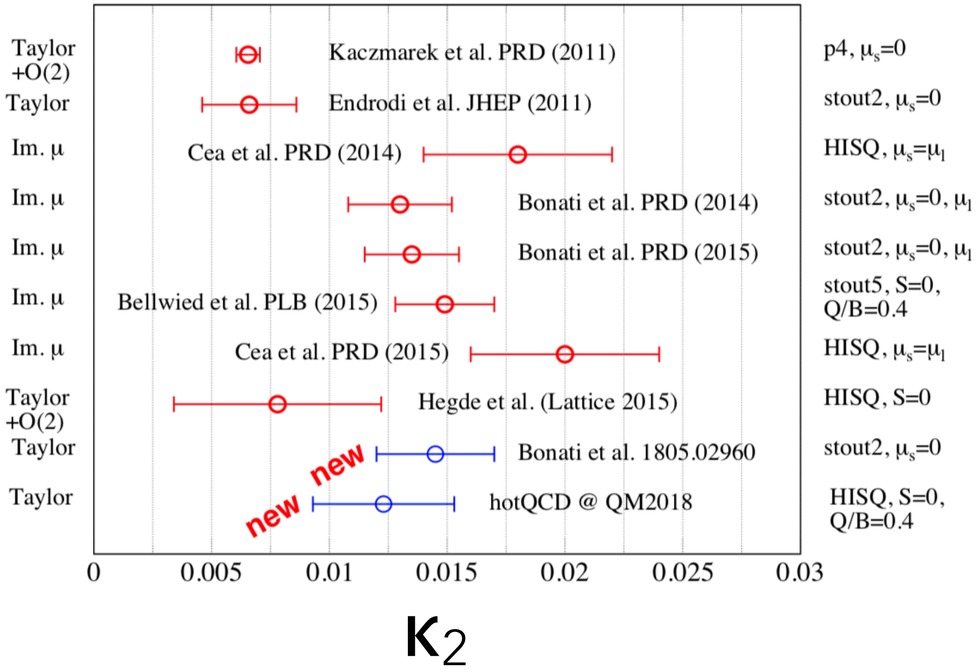
Lines: Constant entropy to net-baryon number
(Approximate evolution trajectories of inviscid QGP)

THE QCD CROSSOVER LINE

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

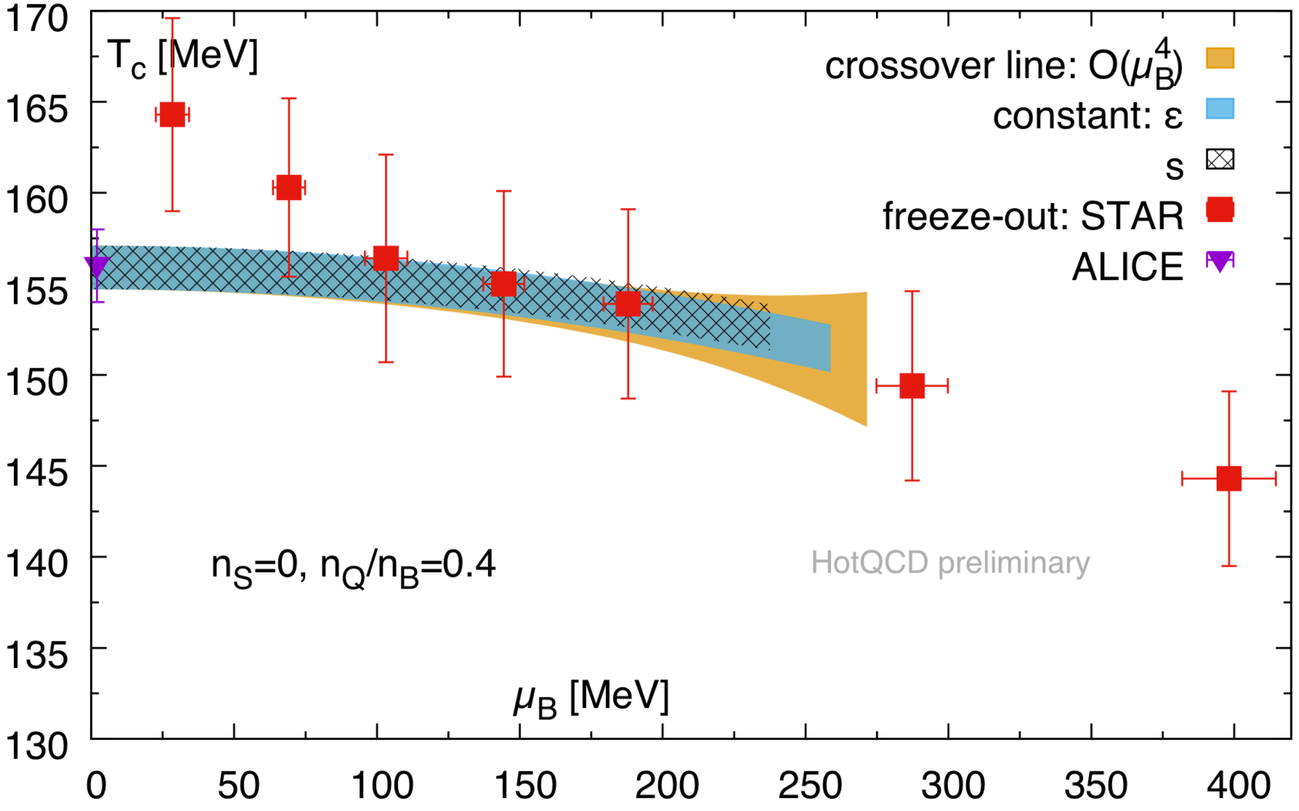
Imposing strangeness neutrality → $\kappa_2 = 0.0123(30)$

$\kappa_4 = 0.000131 \pm 0.0041$



STAR: arxiv:1701.07065

ALICE: arxiv:1408.6403



P. Steinbrecher @QM2018

CRITICAL POINT WITHIN REACH OF LQCD?

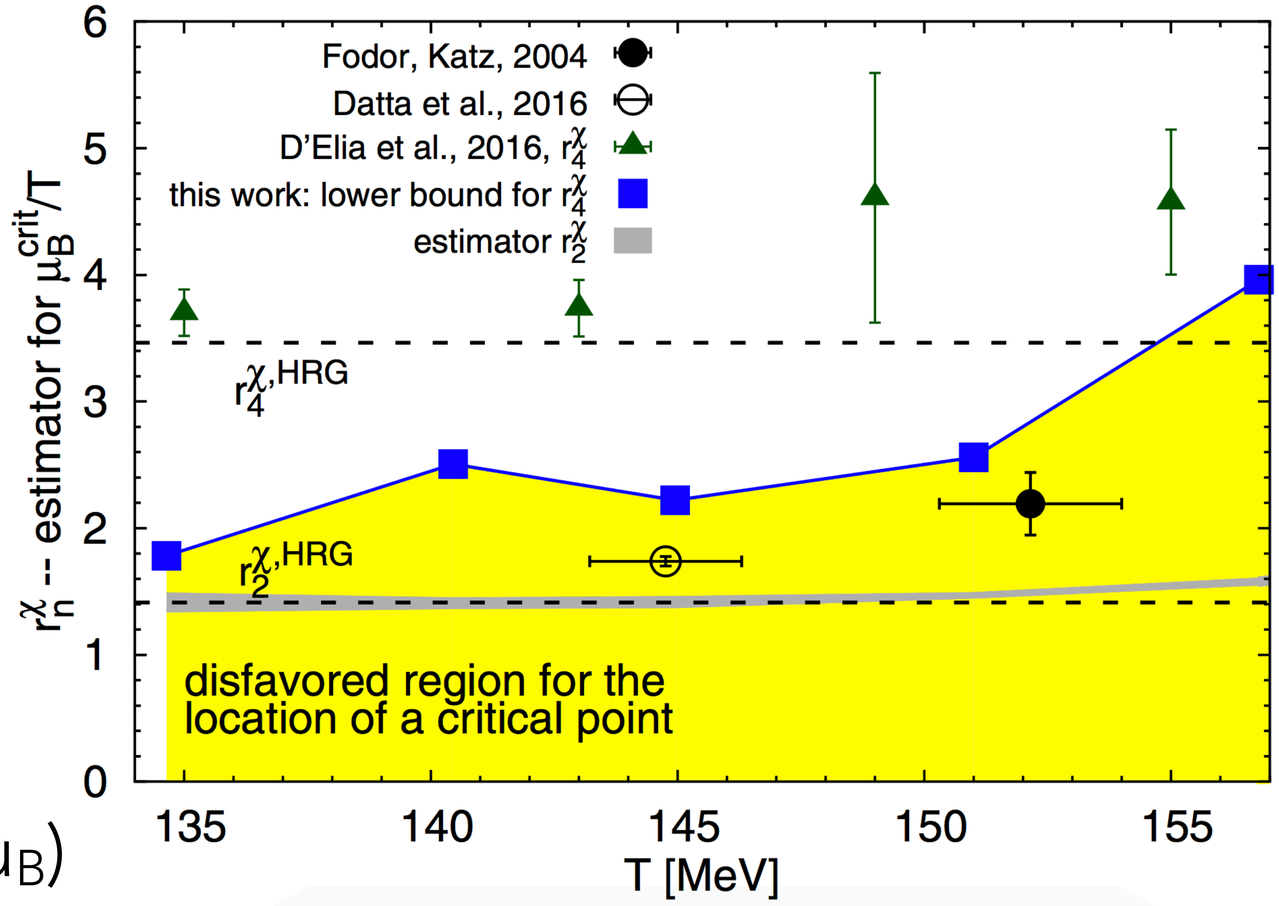
If there is a critical point close by, we expect radius of convergence of Taylor series of pressure or susceptibility, r_c to be $< \infty$:

$$\chi_2^B(T,\mu_B) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} \chi_{2n+2}^B \left(\frac{\mu_B}{T}\right)^{2n}$$

$$r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2}, \quad r_c = \lim_{n\to\infty} r_{2n}^{\chi}$$

To get r_c to be $< \infty$ we need $|\chi_{n+2}^B/\chi_n^B| \sim n^2$ which does not happen up to n=4

→Identify disfavored region (T,μ_B)



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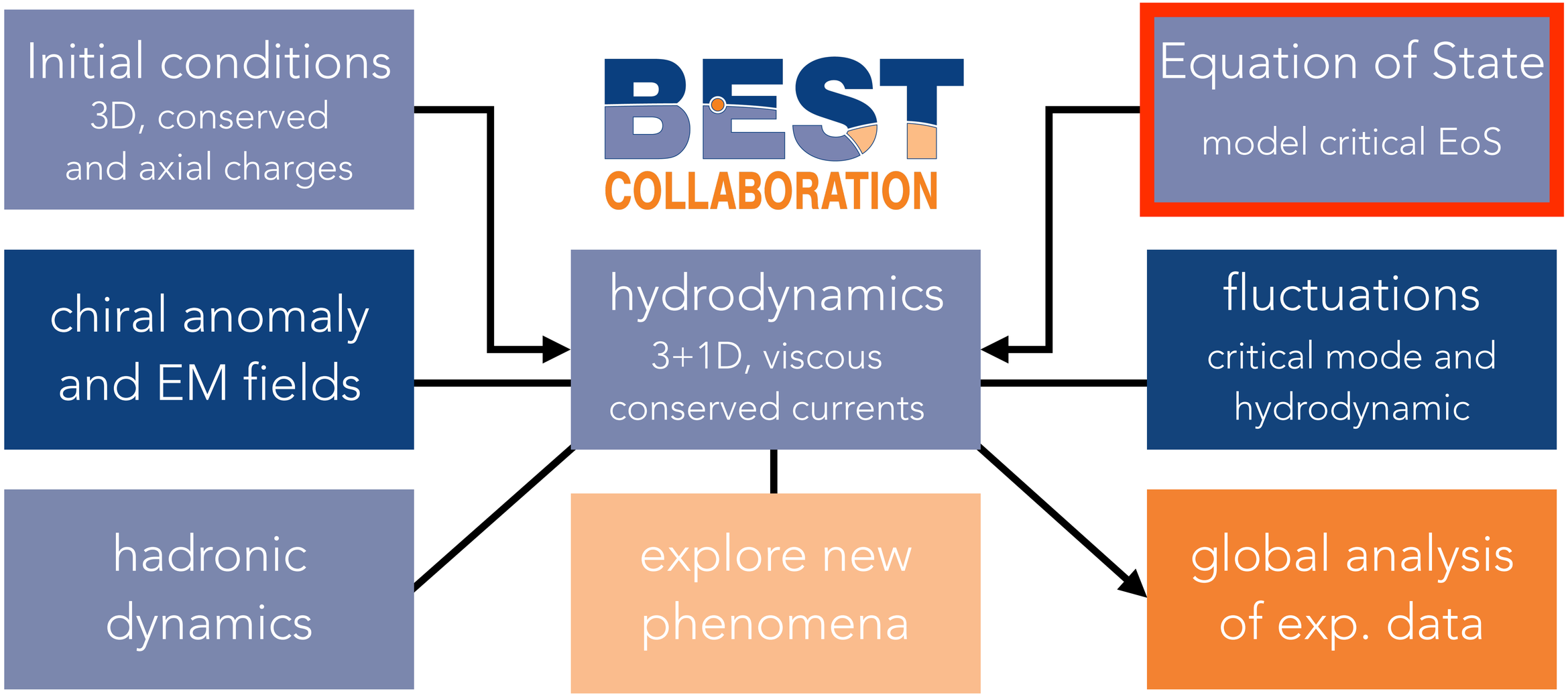
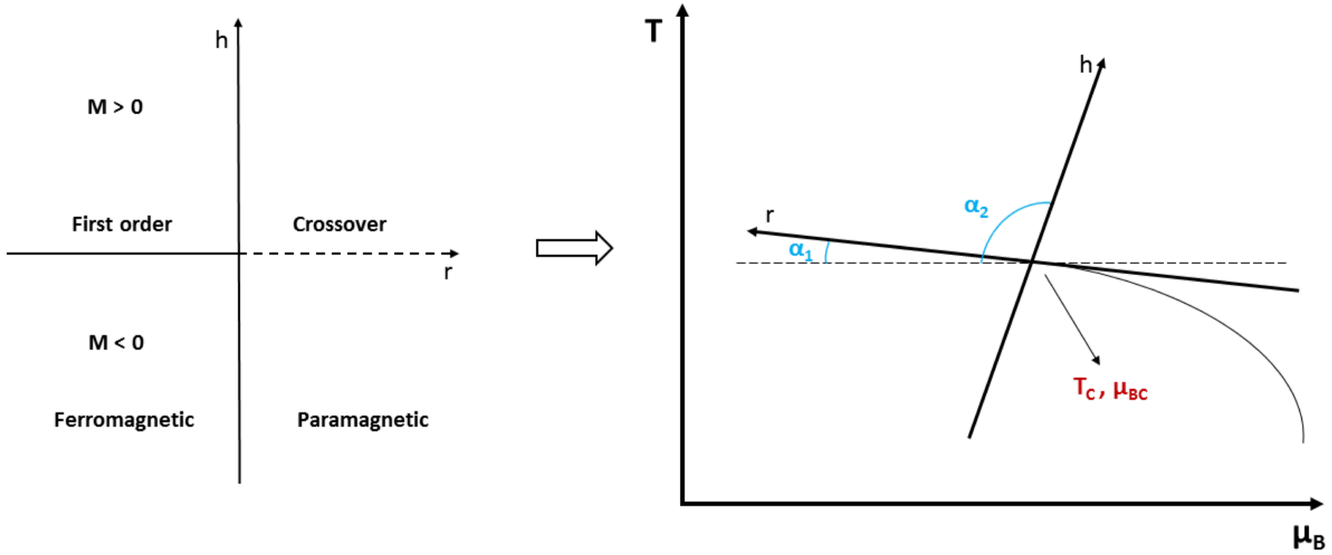
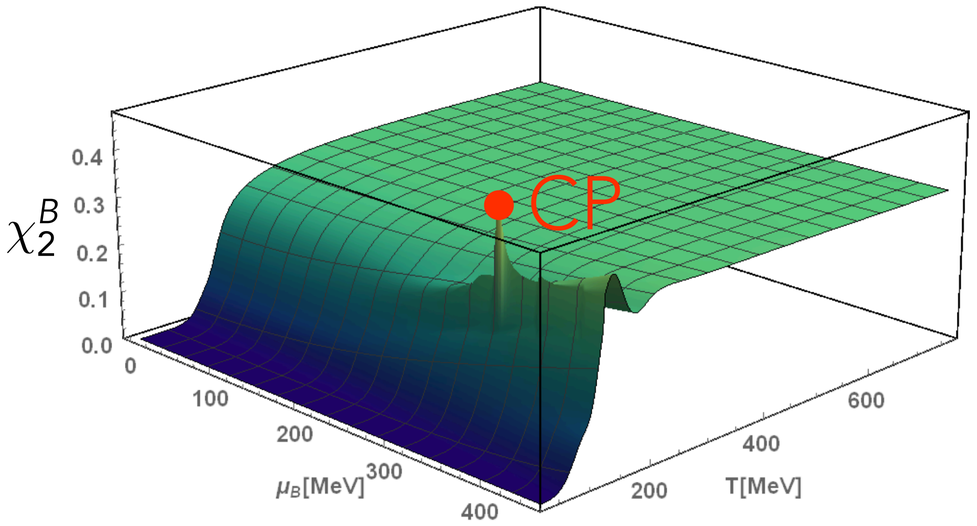


INCLUDE CRITICAL POINT IN LQCD EOS

P. Parotto et al, arXiv:1805.05249

Take LQCD EoS with Taylor expansion and place a critical point in the 3D-Ising model universality class: Add critical pressure to LQCD pressure





Map from Ising variables (r,h) to (T,μ_B) is not universal: 6 free parameters:

$$\frac{T - T_C}{T_C} = \mathbf{w}\,(r\rho \sin\alpha_1 + h \sin\alpha_2)$$

$$\frac{\mu_B - \mu_{BC}}{T_C} = \mathbf{w}\,(-r\rho \cos\alpha_1 - h \cos\alpha_2)$$

Available for download at
`https://www.bnl.gov/physics/best/resources.php`

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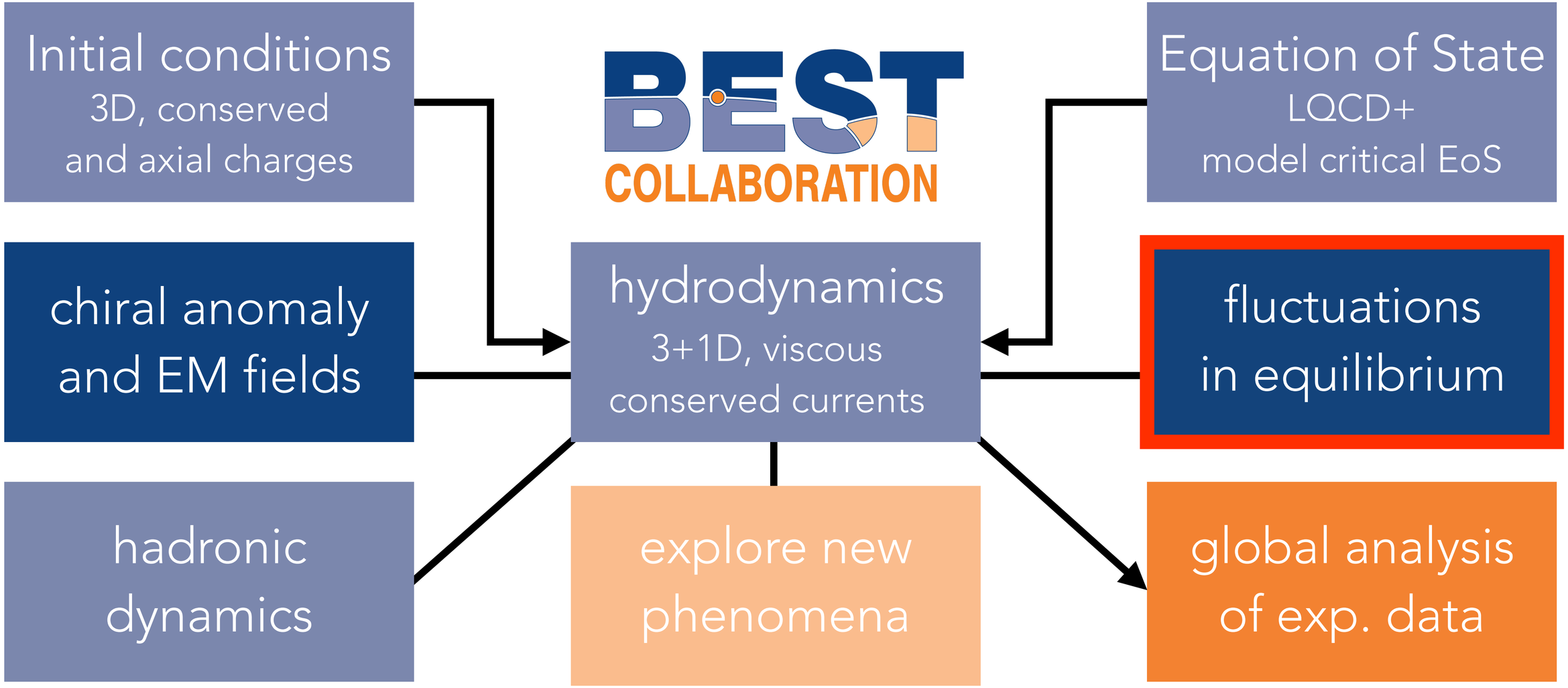


CUMULANTS OF NET BARYON NUMBER

- At the critical point the correlation length ξ of the order parameter diverges (for infinite volume)
- Growing correlation length means increasing fluctuations
- Order parameter for chiral critical point is chiral condensate
- Chiral condensate mixes with the baryon density
  → large baryon number fluctuations at the critical point
- Measure using cumulants

$$\kappa_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu_B/T)^{n-1}} \langle N_B \rangle$$

(note that $\chi_n^B = \kappa_n^B / V$)

$$\kappa_1^B = \langle N_B \rangle \quad , \quad \kappa_2^B = \langle N_B - \langle N_B \rangle \rangle^2 \quad , \quad \kappa_3^B = \langle N_B - \langle N_B \rangle \rangle^3$$

Sensitivity to crit. point increases with n: $\kappa_2^B \sim \xi^2$, $\kappa_3^B \sim \xi^{4.5}$, $\kappa_4^B \sim \xi^7$

CUMULANTS OF NET BARYON NUMBER

- Cumulants scale with volume, which is not well known in HIC
- So take ratios of cumulants

$$R^B_{12}(T,\mu_B) \equiv \frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)} \equiv \frac{M_B}{\sigma^2_B} ,$$

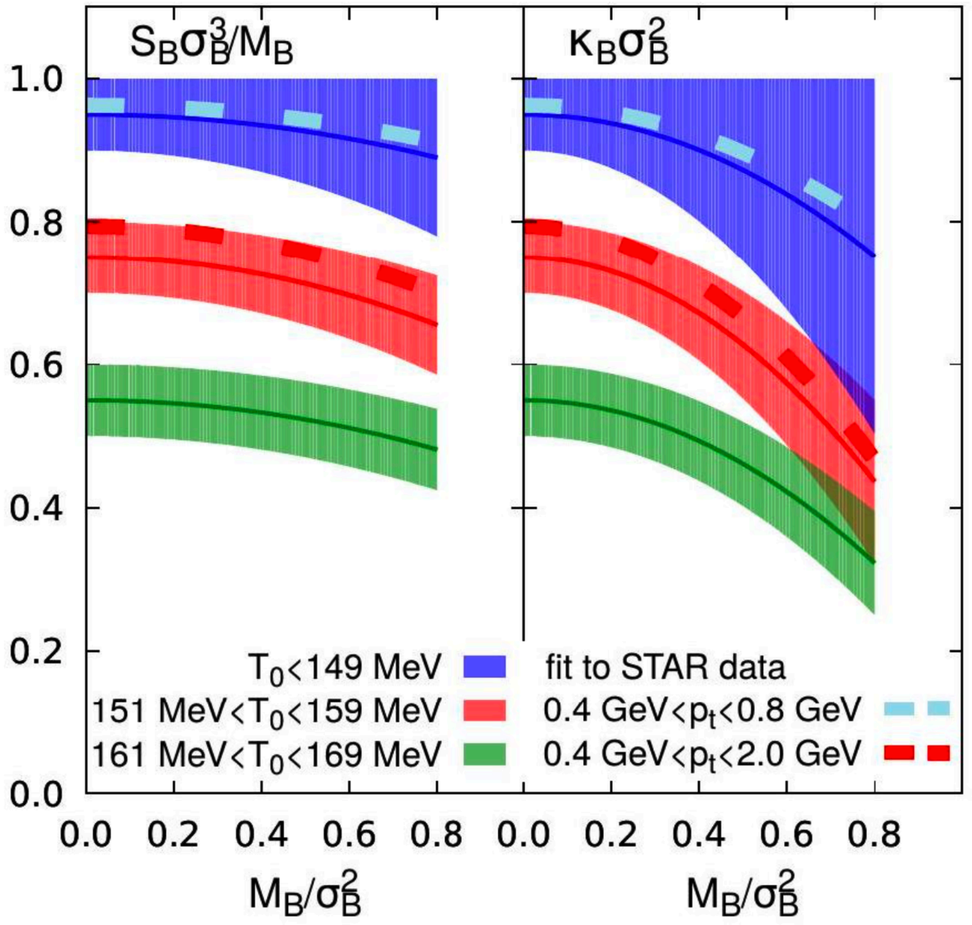
$$R^B_{31}(T,\mu_B) \equiv \frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)} \equiv \frac{S_B\sigma^3_B}{M_B} ,$$

$$R^B_{42}(T,\mu_B) \equiv \frac{\chi^B_4(T,\mu_B)}{\chi^B_2(T,\mu_B)} \equiv \kappa_B\sigma^2_B .$$

with mean (M_B), variance (σ_B^2), skewness (S_B), kurtosis (κ_B)

LQCD consistent with data but should not be compared directly

Compute on the lattice:



HotQCD, Phys.Rev. D96 (2017) 074510

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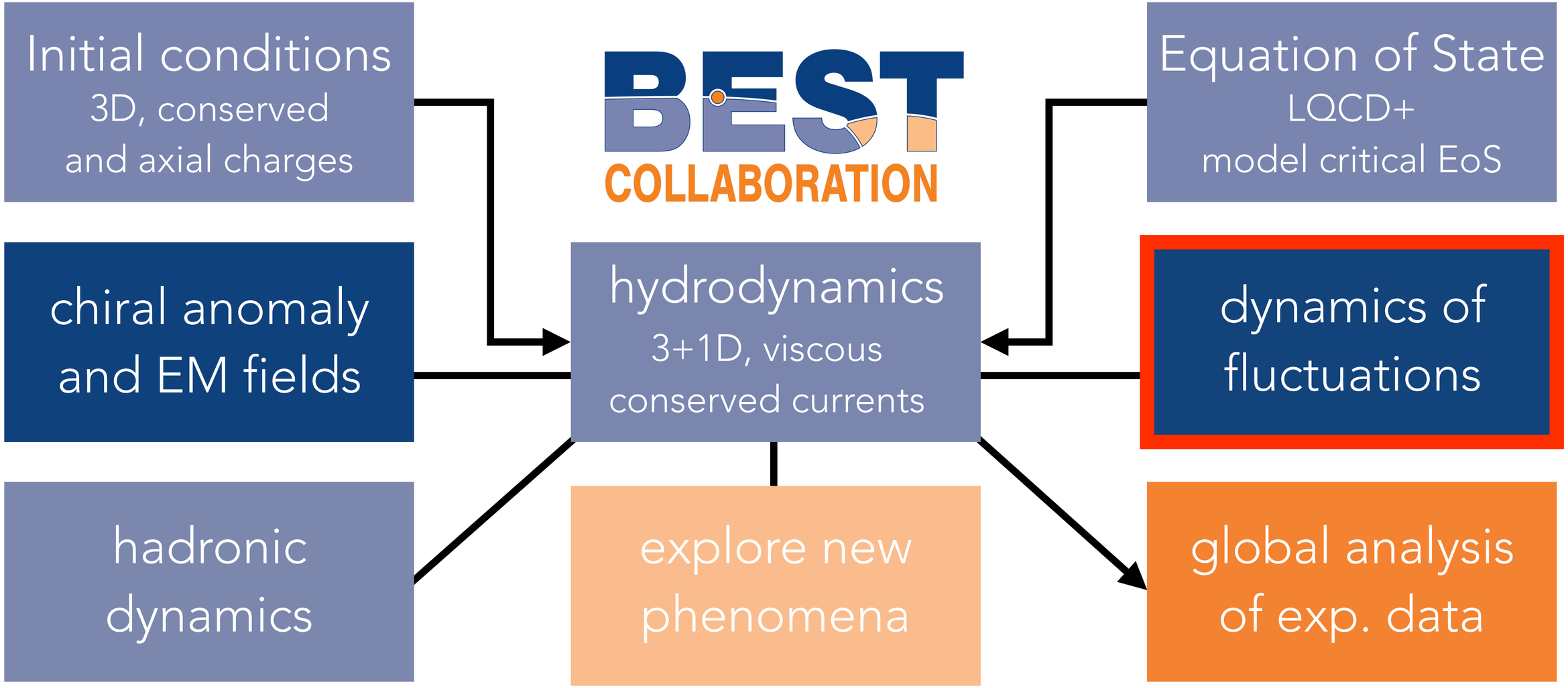
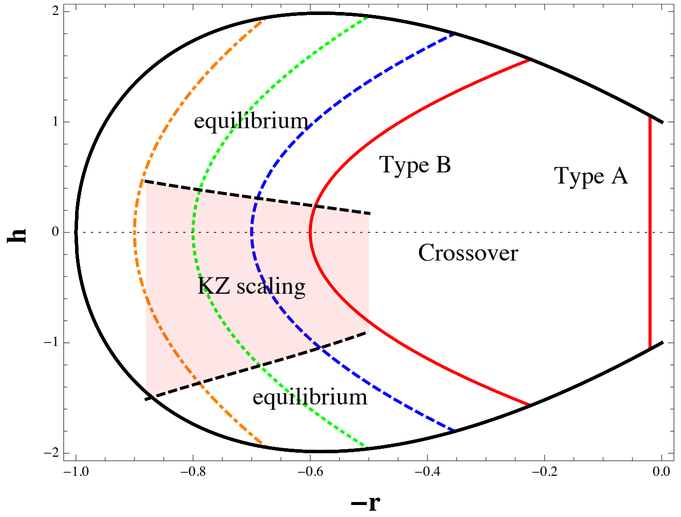
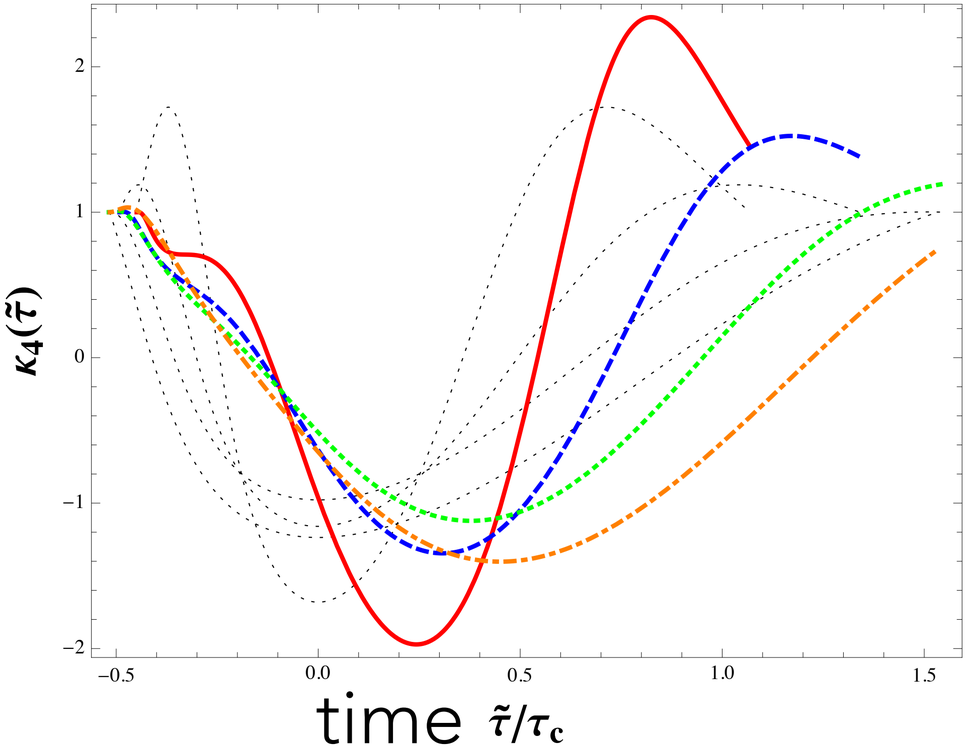


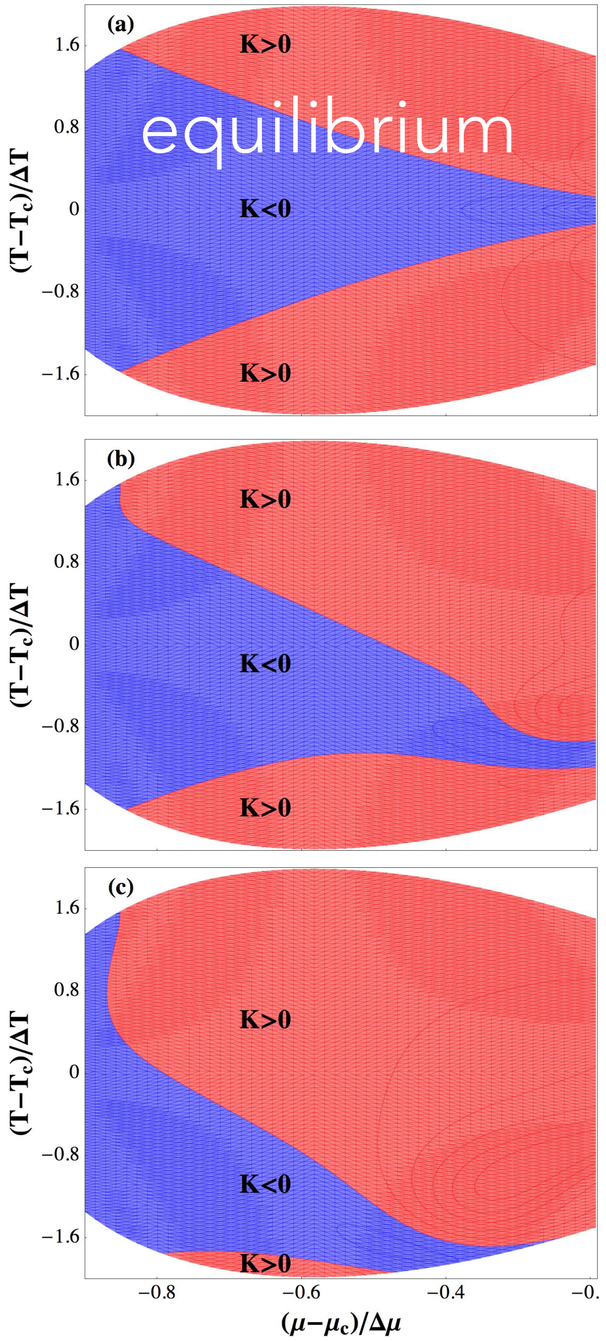
NON-EQUILIBRIUM EVOLUTION OF CRITICAL FLUCTUATIONS

Mukherjee, Venugopalan, Yin, Phys. Rev. C92, 034912 (2015)

Derive Fokker-Planck equations for the cumulants and study time evolution. Here we show kurtosis:



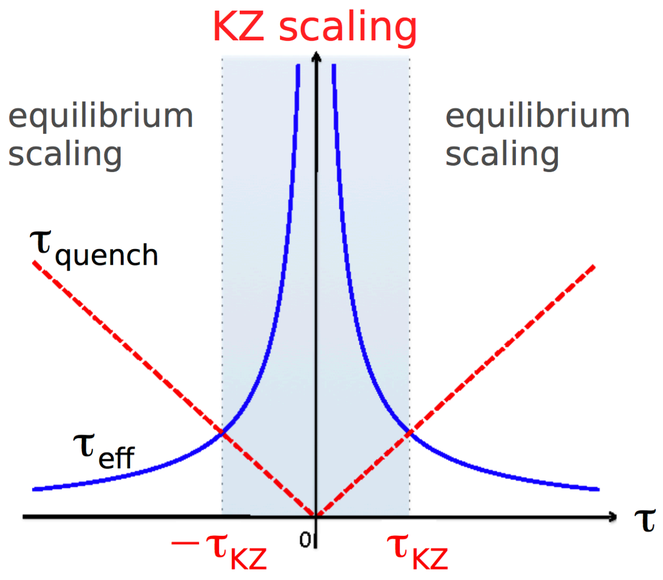




increasing relaxation time

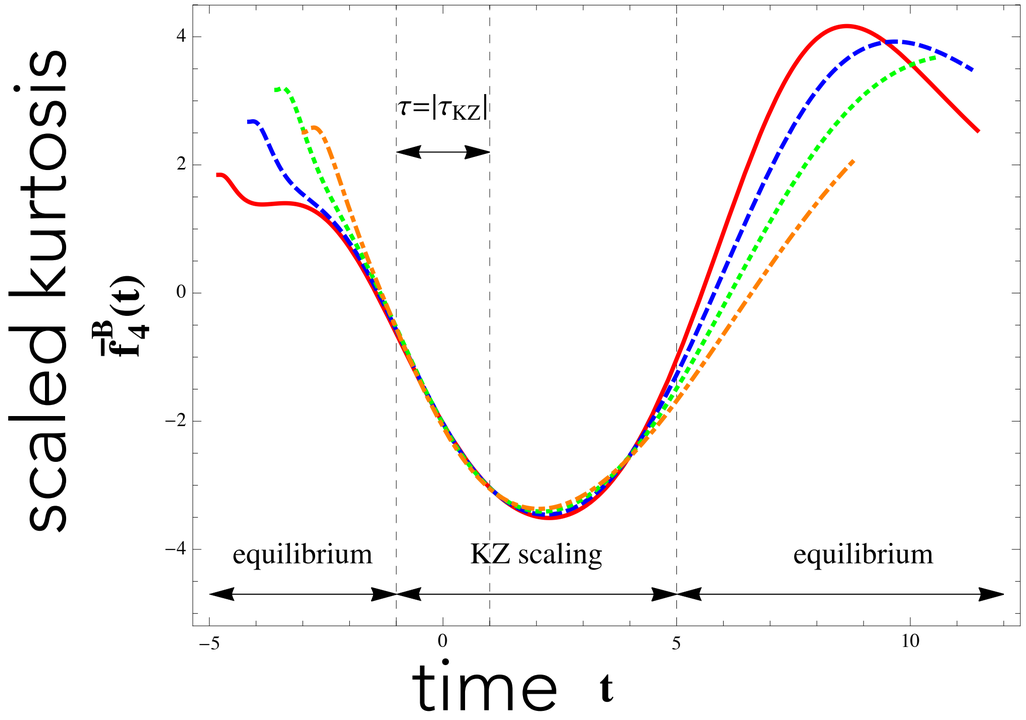
KIBBLE-ZUREK SCALING

Mukherjee, Venugopalan, Yin, Phys. Rev. Lett. 117, 222301 (2016)



Close to CP, relaxation time $\tau_{eff} \sim \xi^z$ gets larger than time in which system tries to change ξ, (τ_{quench})

Then relevant scales are $\tau_{KZ} = \tau_{eff}(\tau^*) = \tau_{quench}(\tau^*)$ and $l_{KZ} = \xi_{eq}(\tau^*)$



Cumulants show a certain scaling with ξ_{eq} in equilibrium. Using the same scaling form with the constant l_{KZ} and τ_{KZ} restores scaling in non-equilibrium

Universality is restored

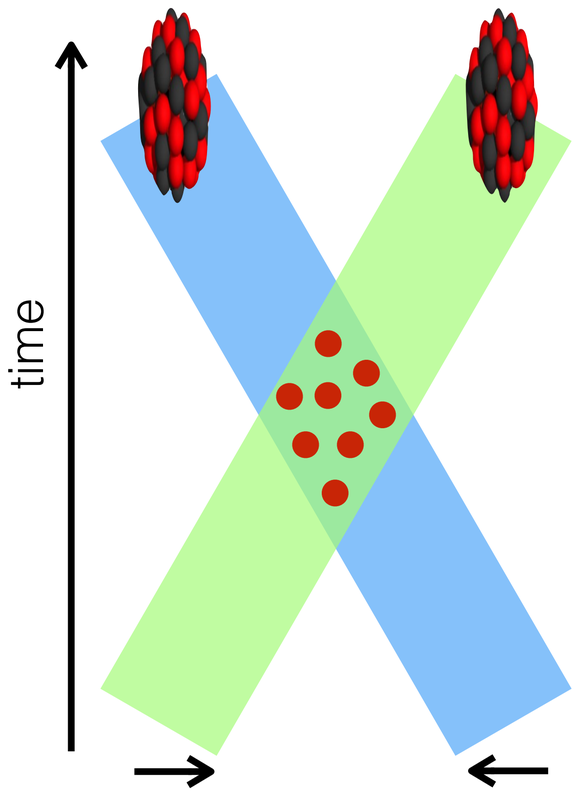
$$\tilde{\tau} = \tau - \tau_c \,, \quad t = \tilde{\tau}/\tau_{\rm KZ}$$

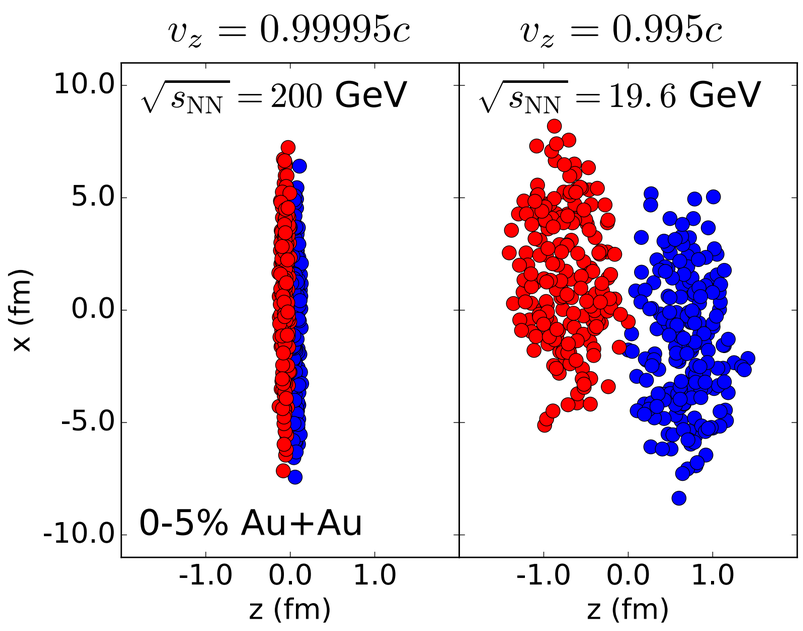
THE BEAM ENERGY SCAN THEORY COLLABORATION

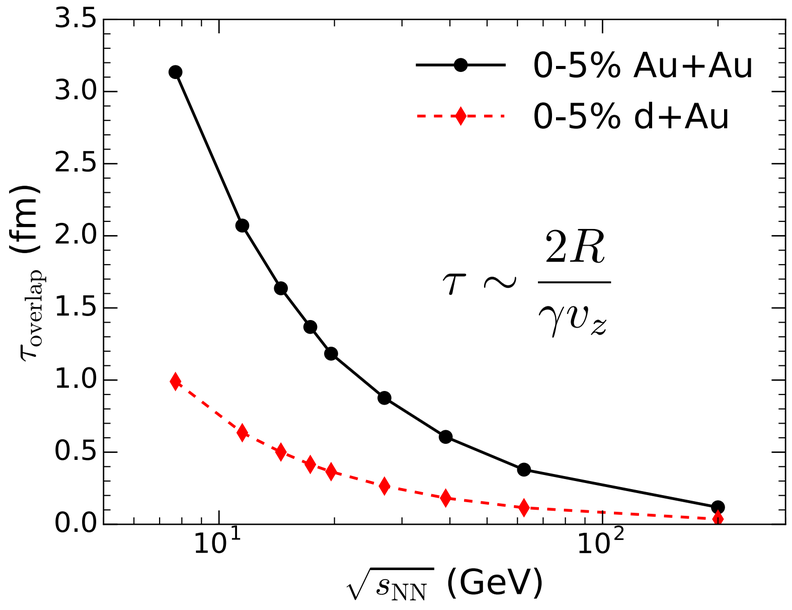
DYNAMICAL INITIAL STATE FOR 3D HYDRO

C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

Nuclear overlap time becomes large at lower energies:







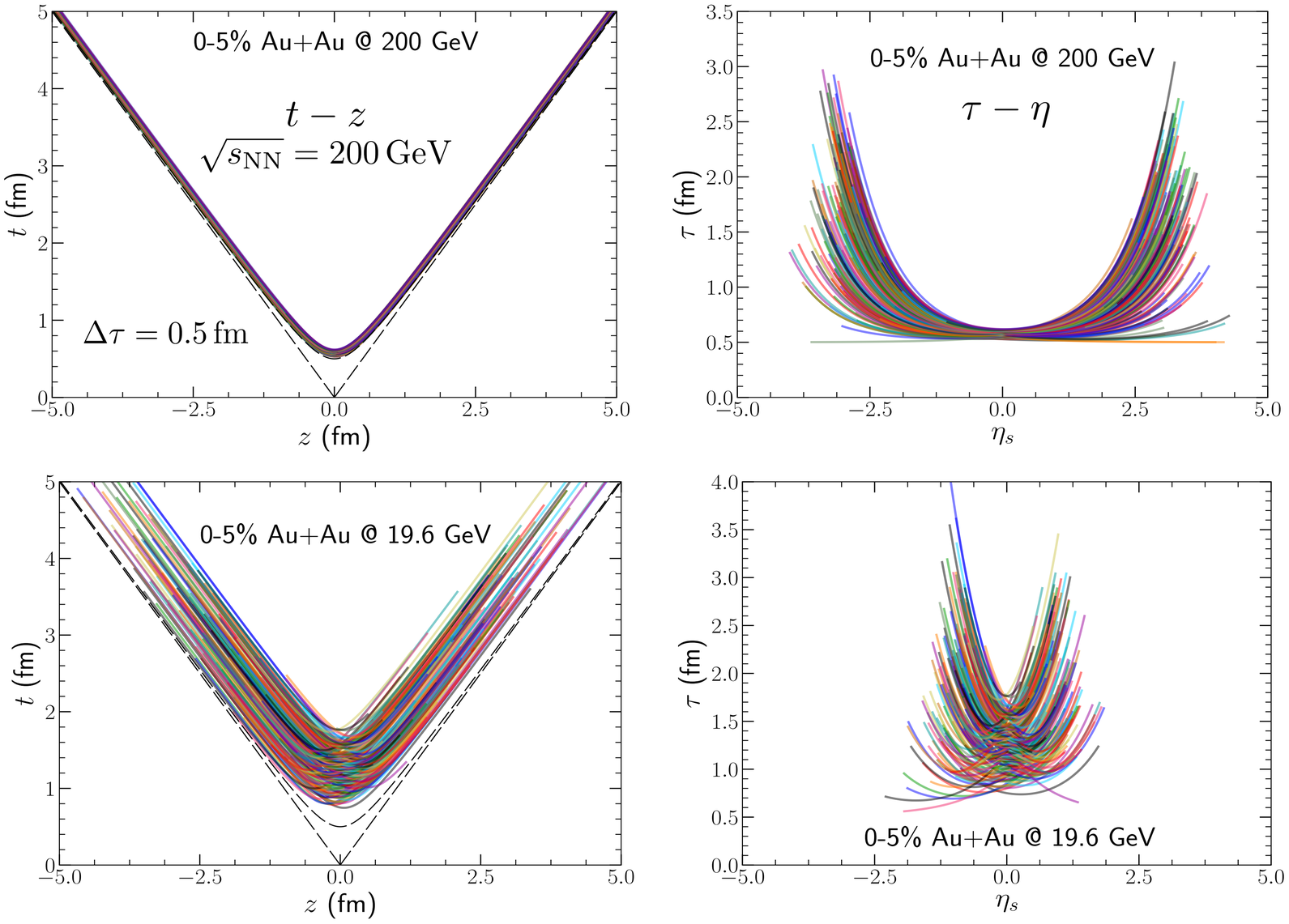
- The interaction zone is not point like
- Colliding nucleons are decelerated with a classical string model

  A. Bialas, A. Bzdak and V. Koch, arXiv:1608.07041
- The lost energy and momentum of the decelerated nucleons are fed into hydrodynamics as source terms

  C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907

ENERGY DEPOSITION IN SPACE-TIME

C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907



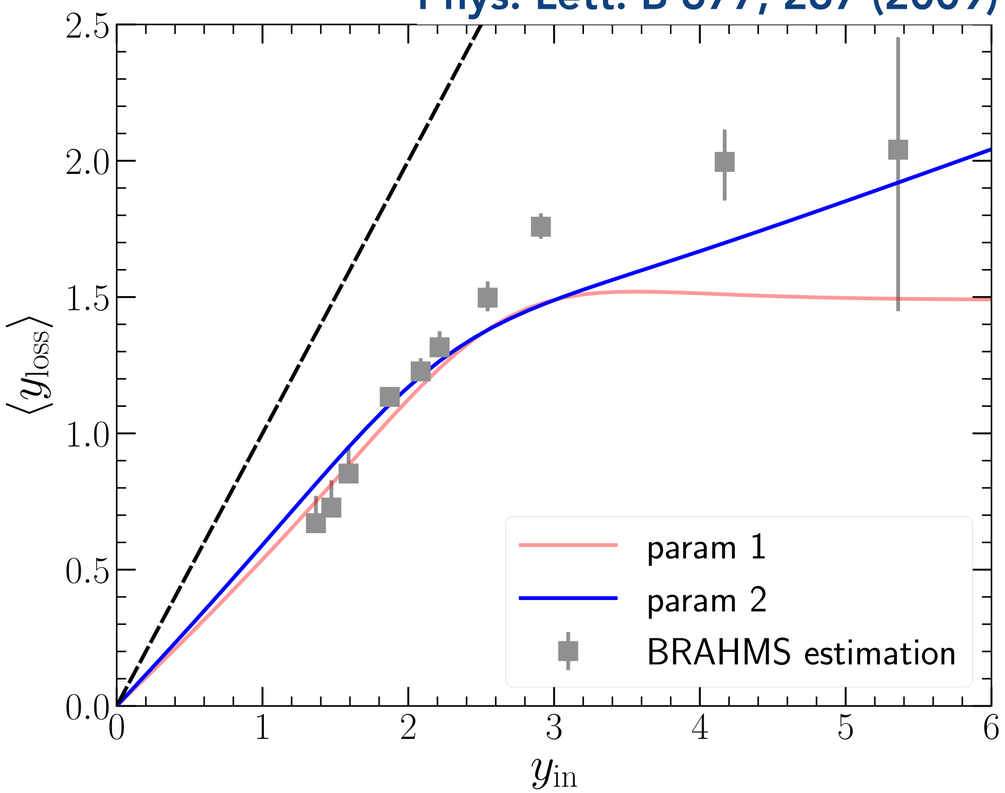
$\sqrt{s} = 200\,{\rm GeV}$

$\sqrt{s} = 19.6\,{\rm GeV}$

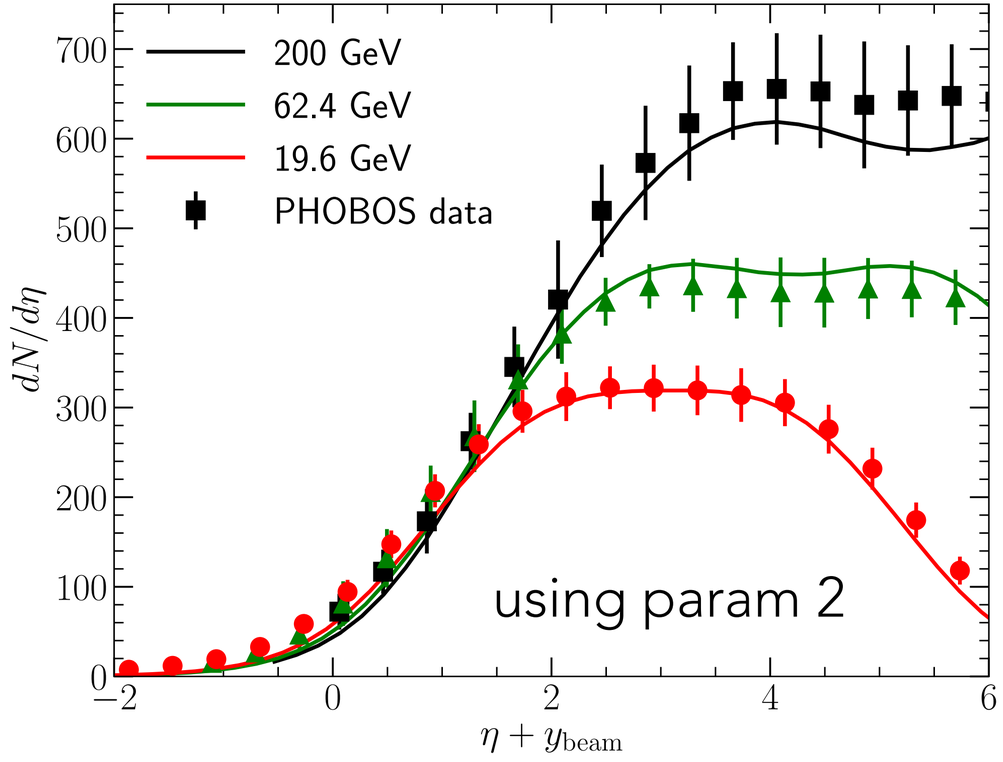
DESCRIBING RAPIDITY SPECTRA

C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

Phys. Lett. B 677, 267 (2009)



Dynamic initial state + hydro



Model baryon stopping - adjust string tension to get right amount of rapidity loss

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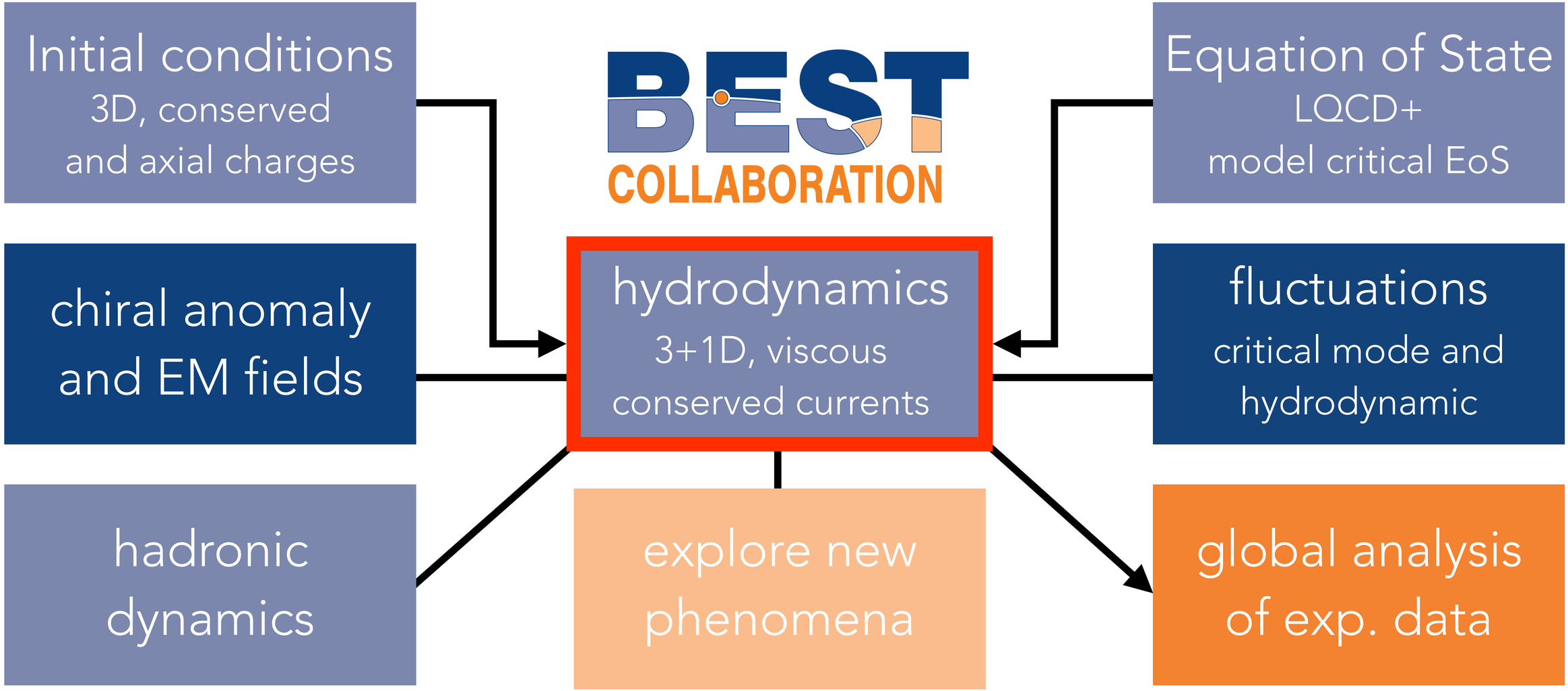
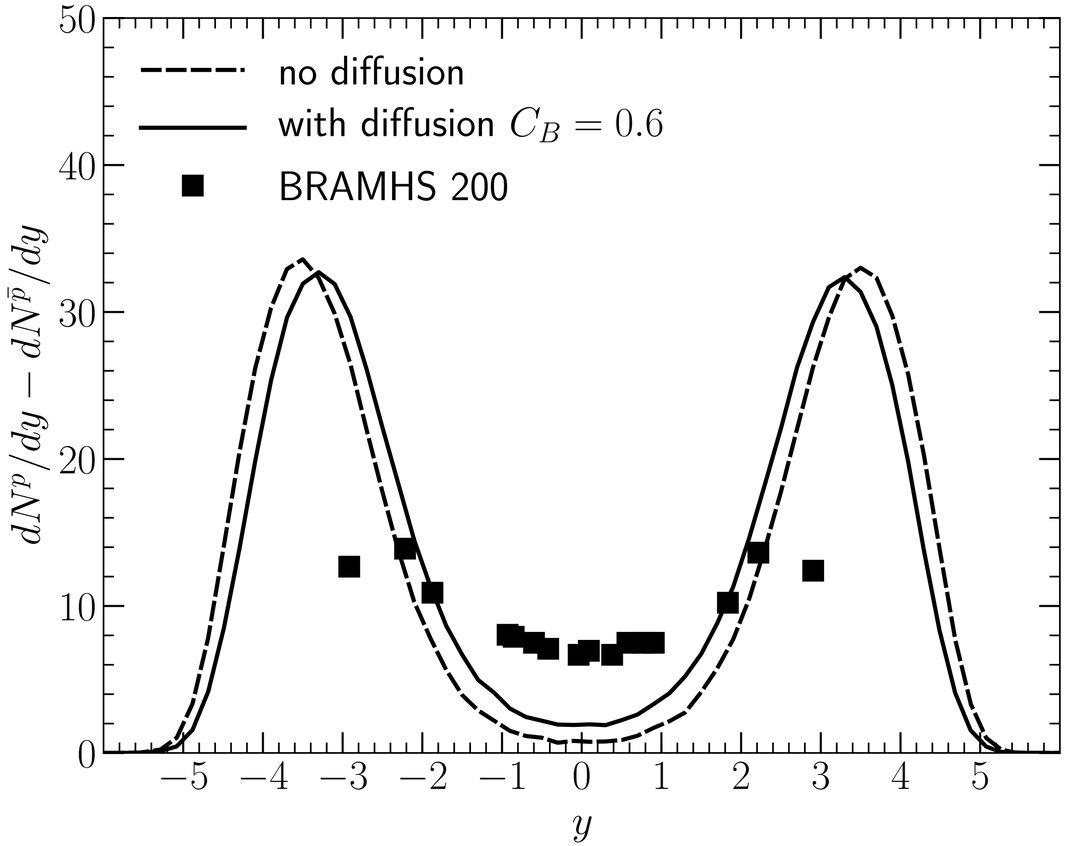
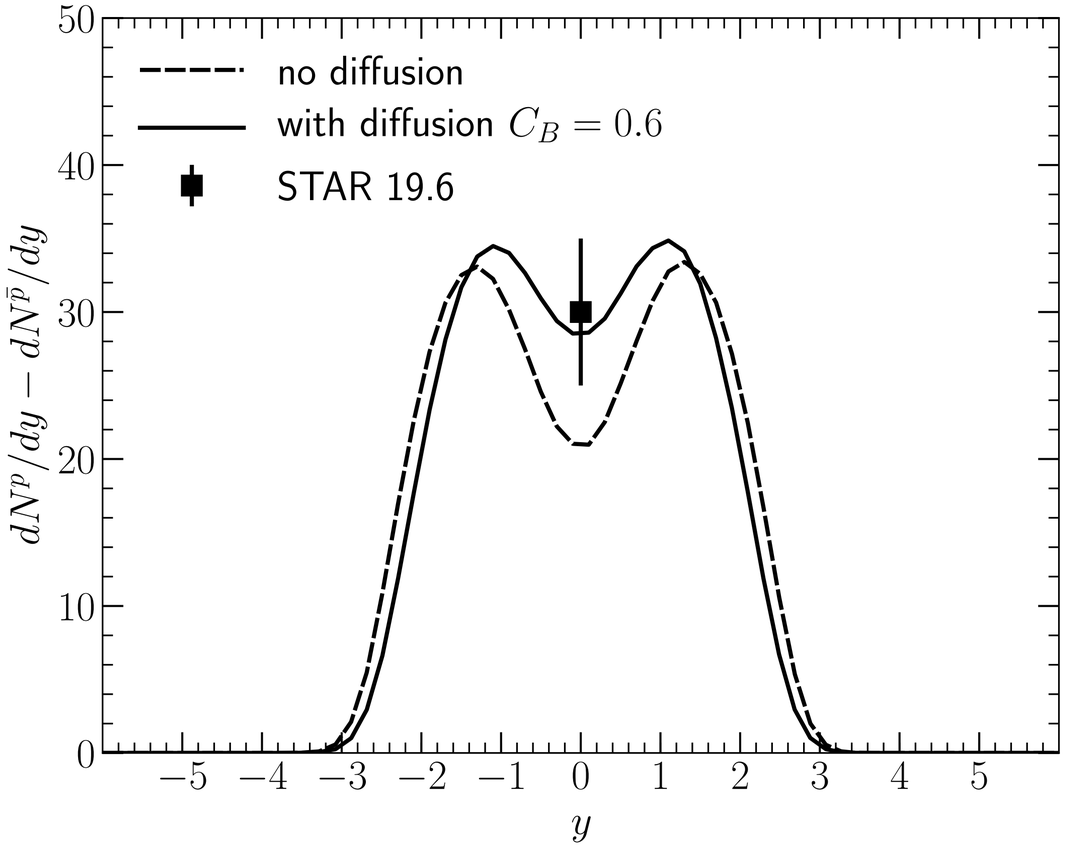


BARYON DIFFUSION

- Extended MUSIC 3+1D hydro to include baryon diffusion

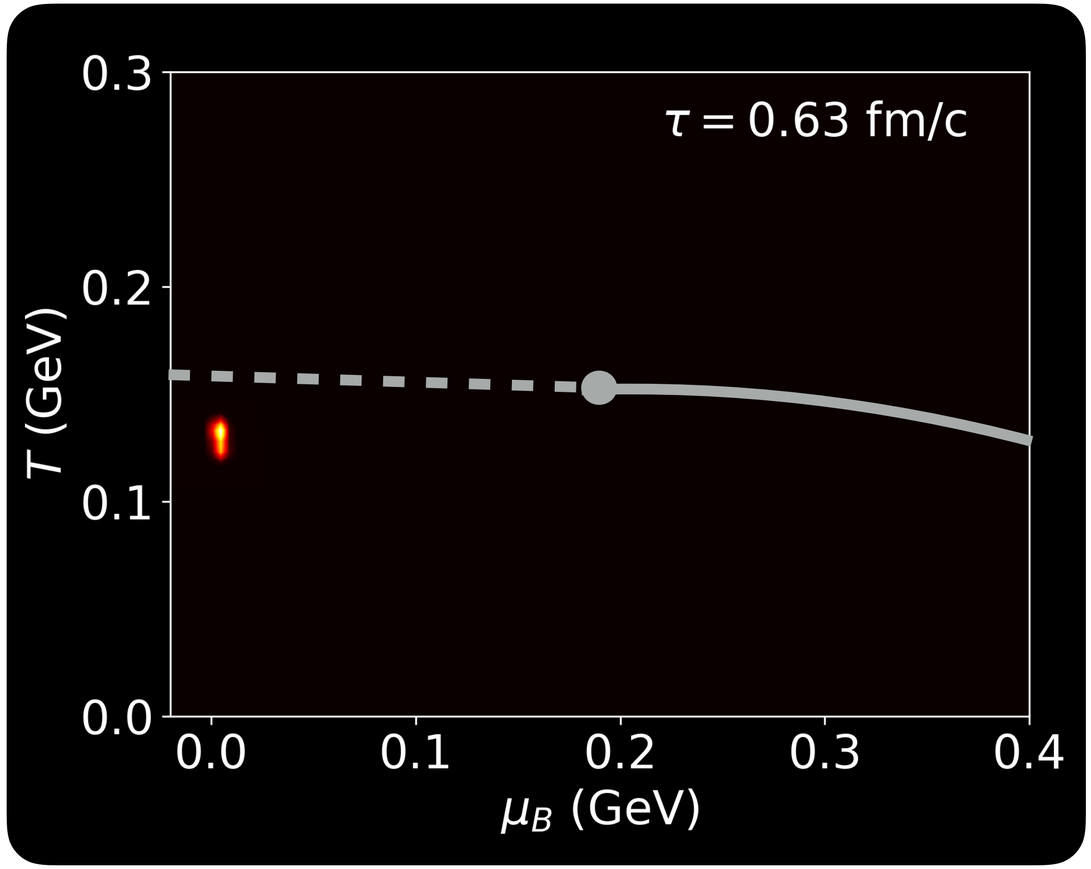




G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke and C. Shen, arXiv:1804.10557 [nucl-th]

EVOLUTION IN THE T-μ_B PLANE

- dynamic initial state + 3+1D hydrodynamic evolution



C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

0-5% AuAu@19.6 GeV

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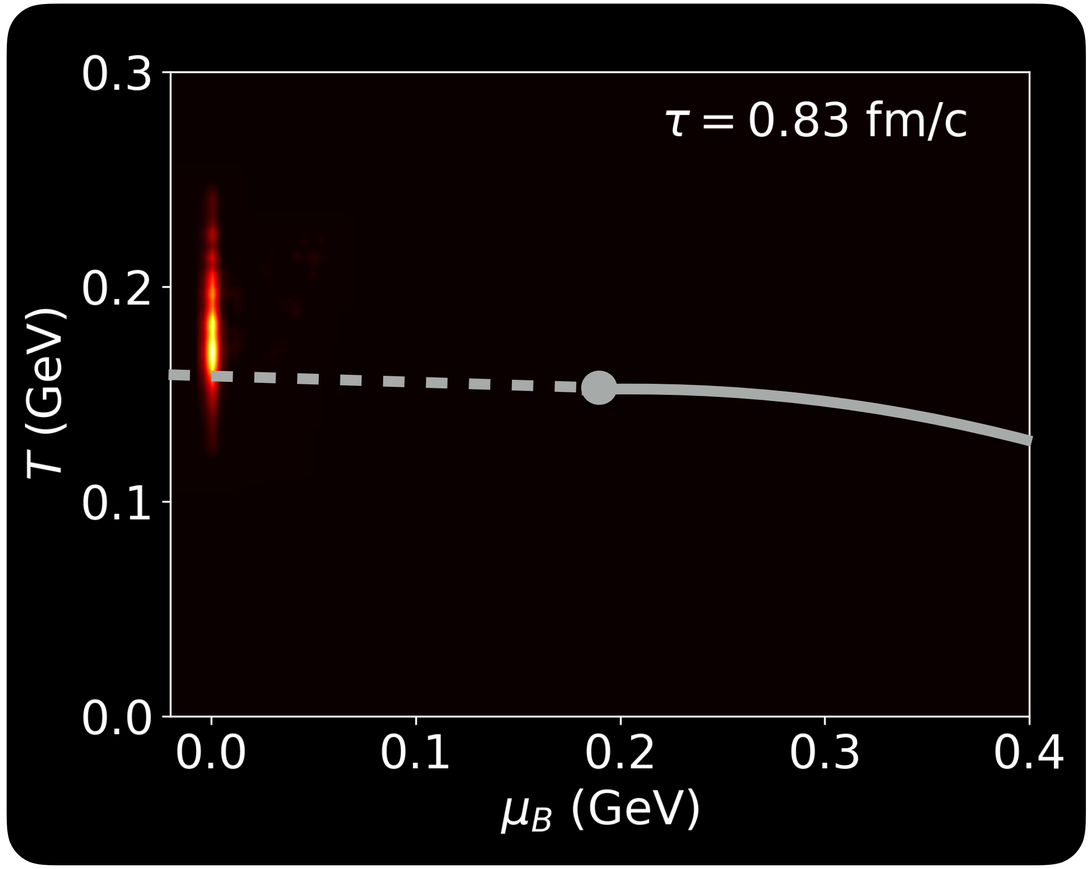

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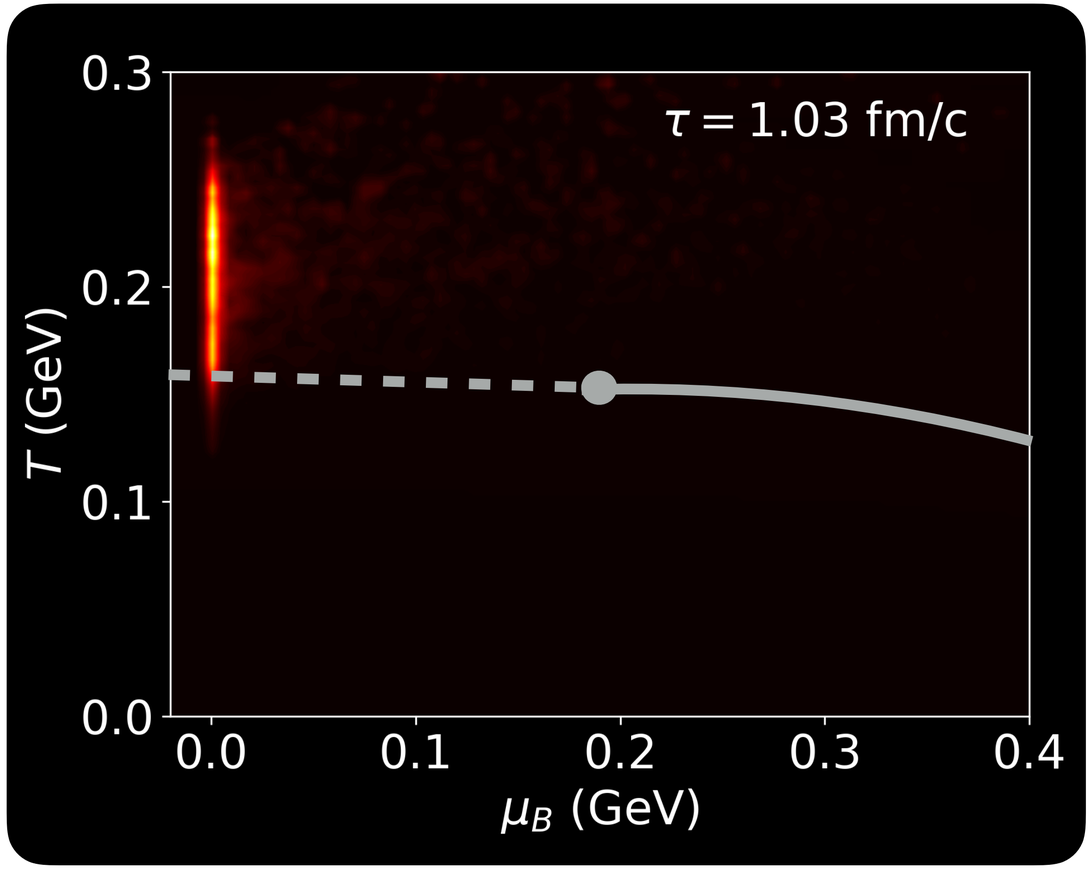

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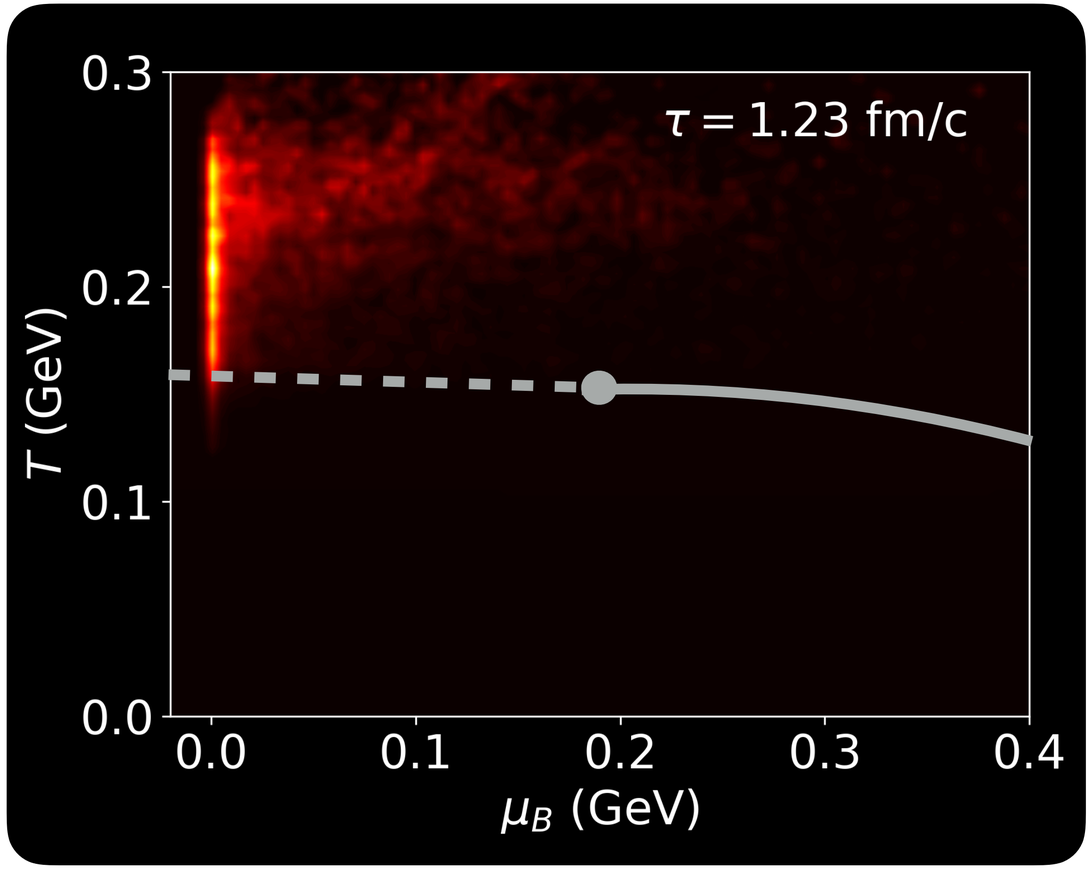

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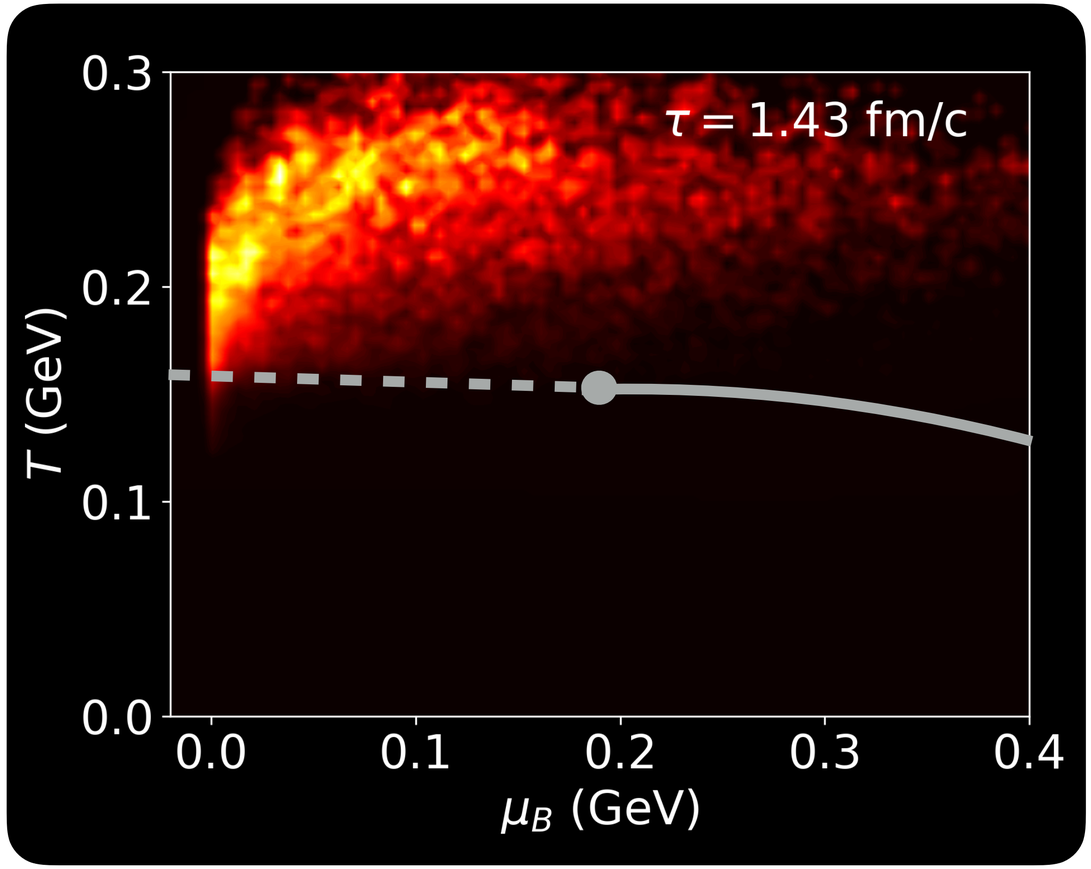

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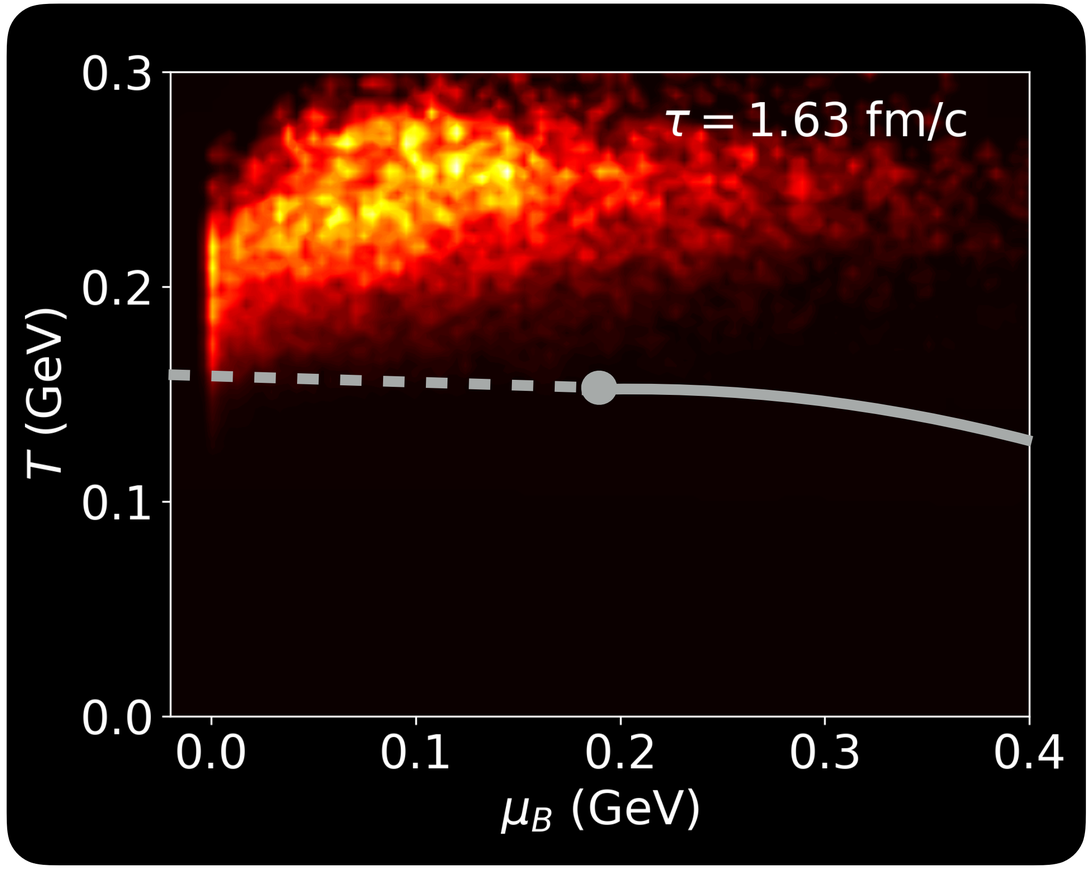

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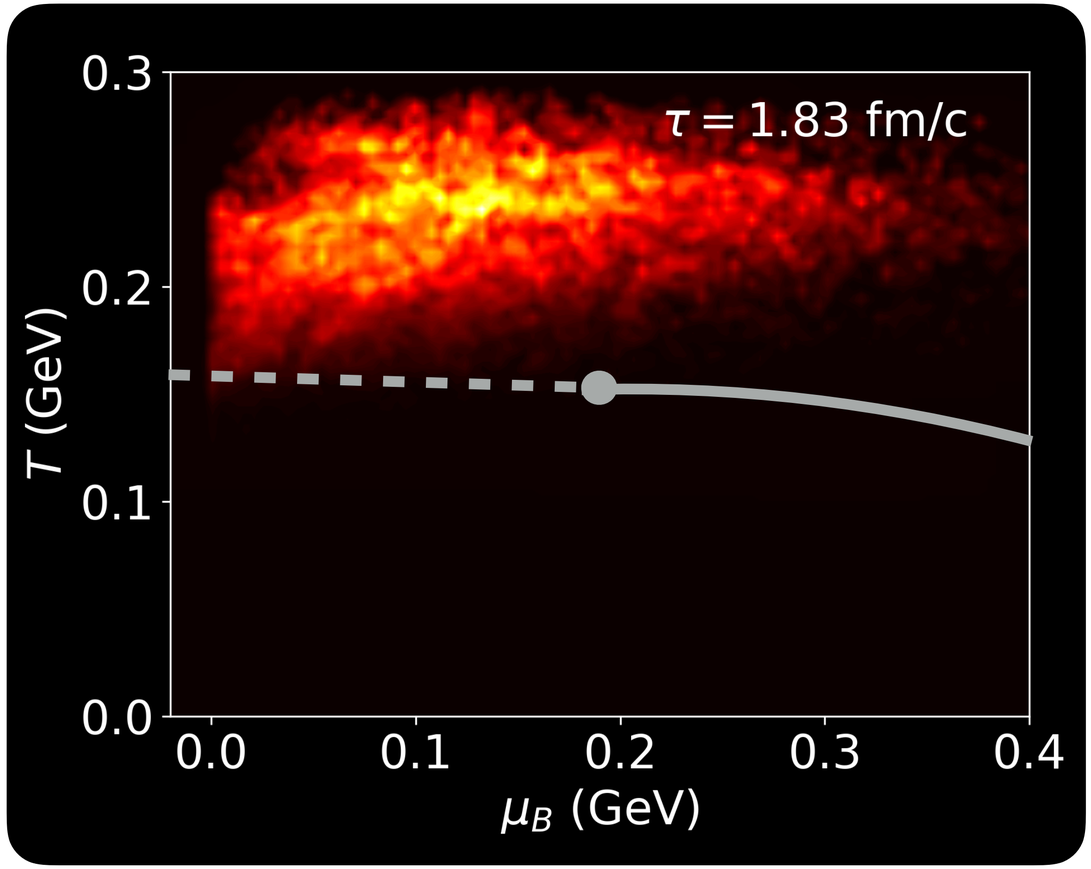

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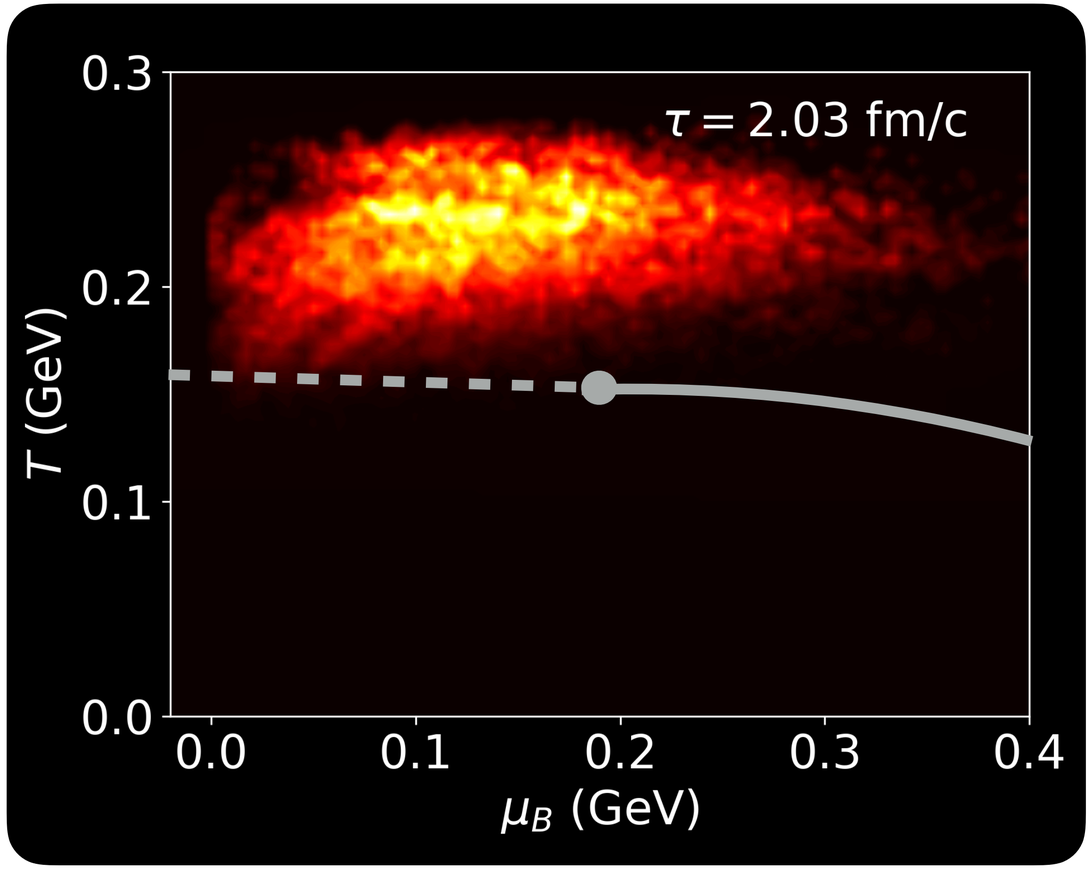

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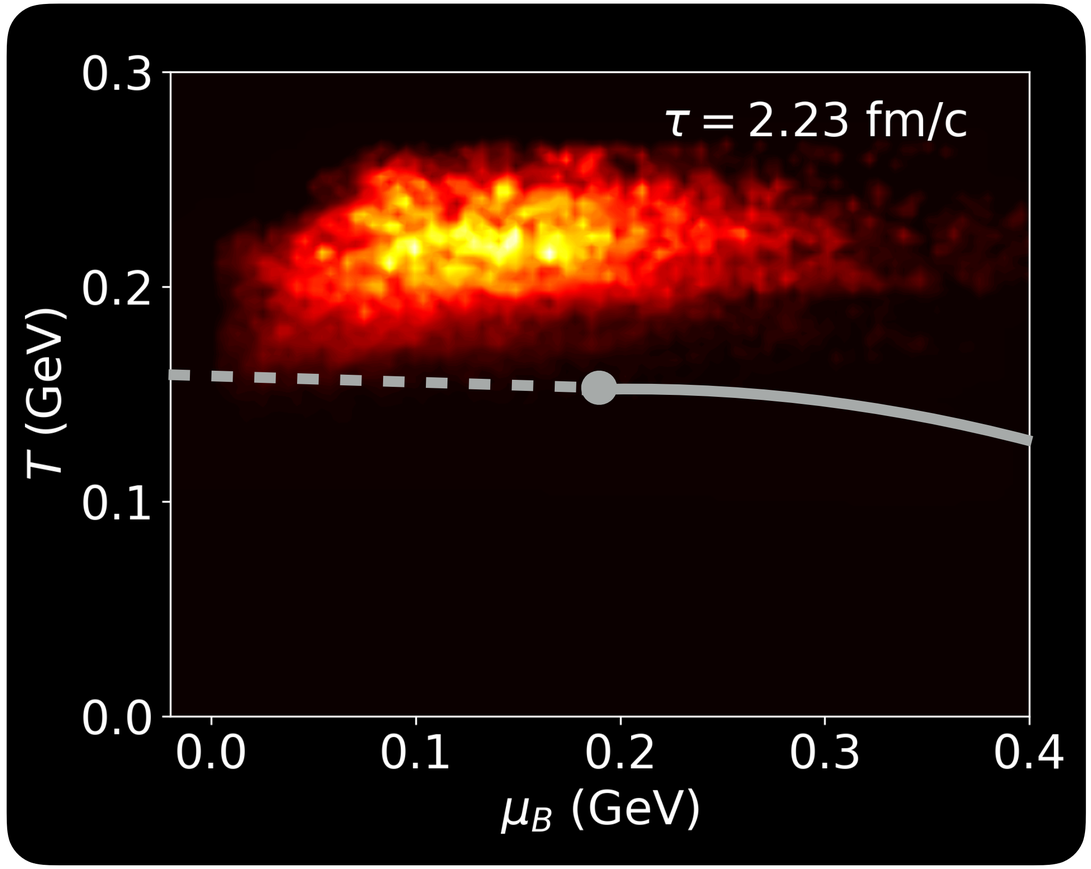

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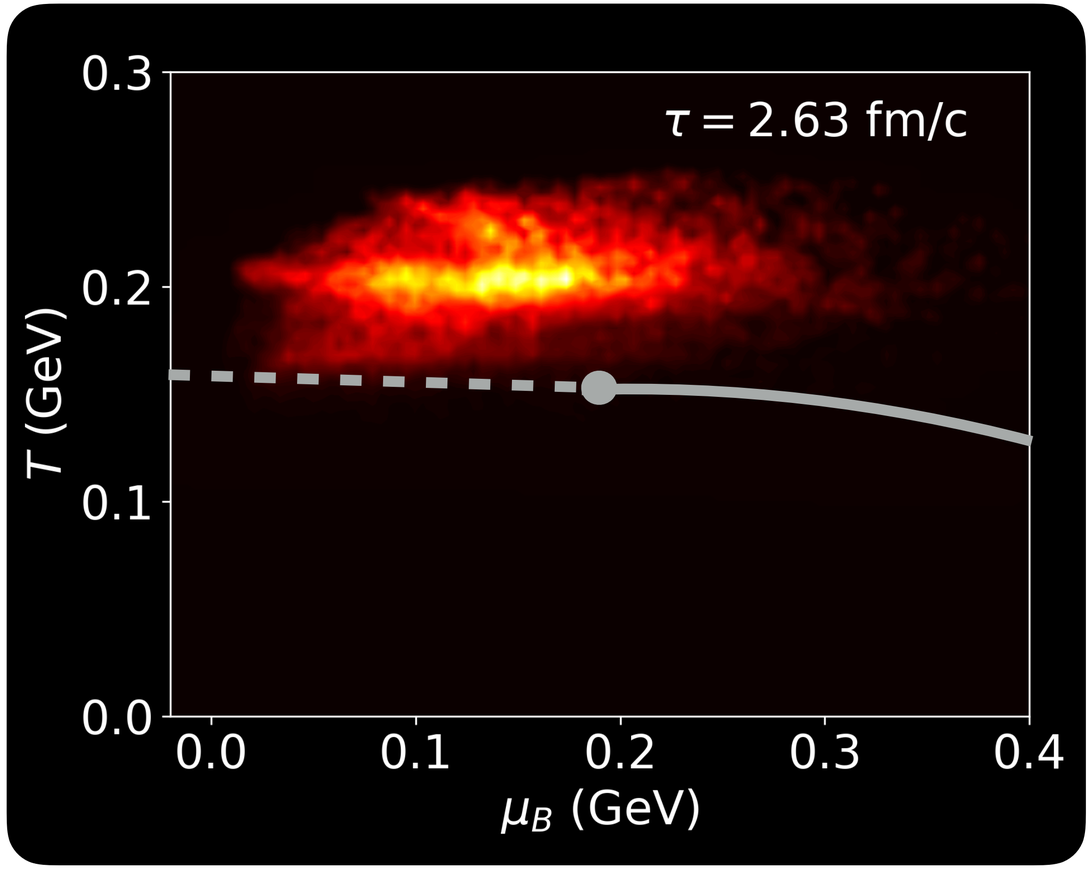

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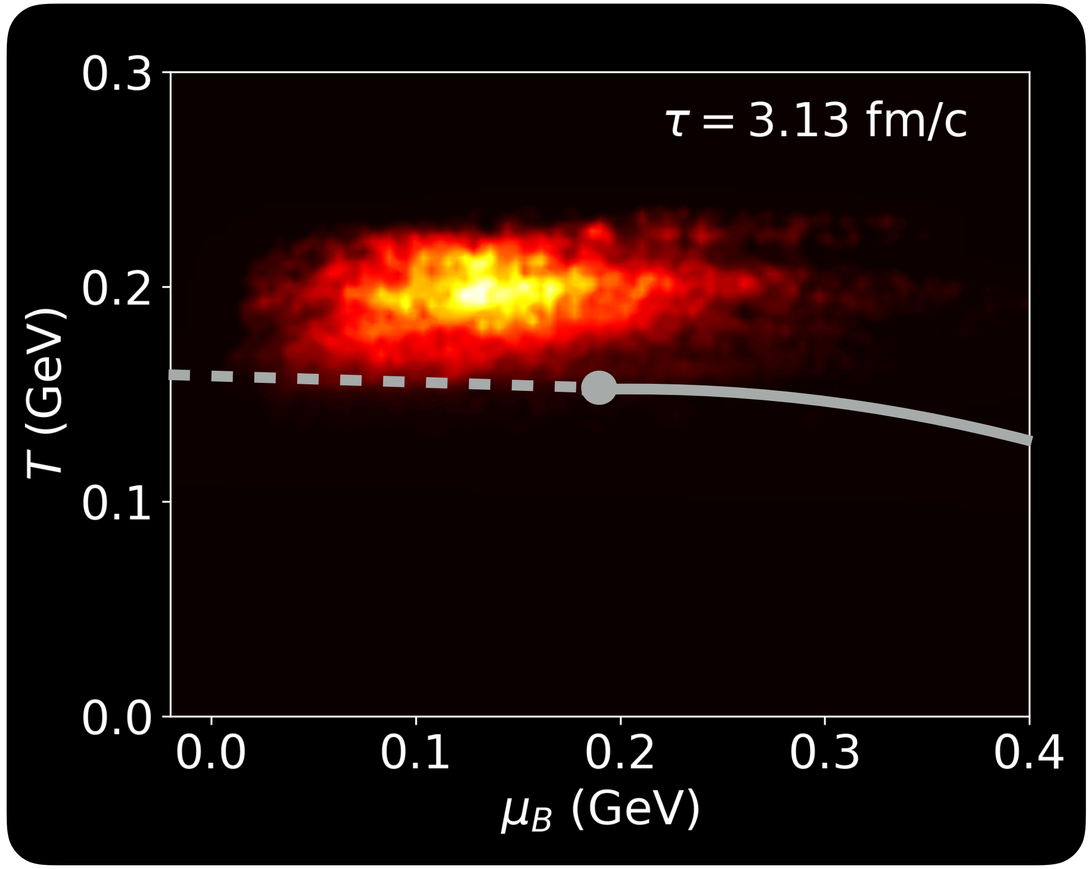

C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

0-5% AuAu@19.6 GeV

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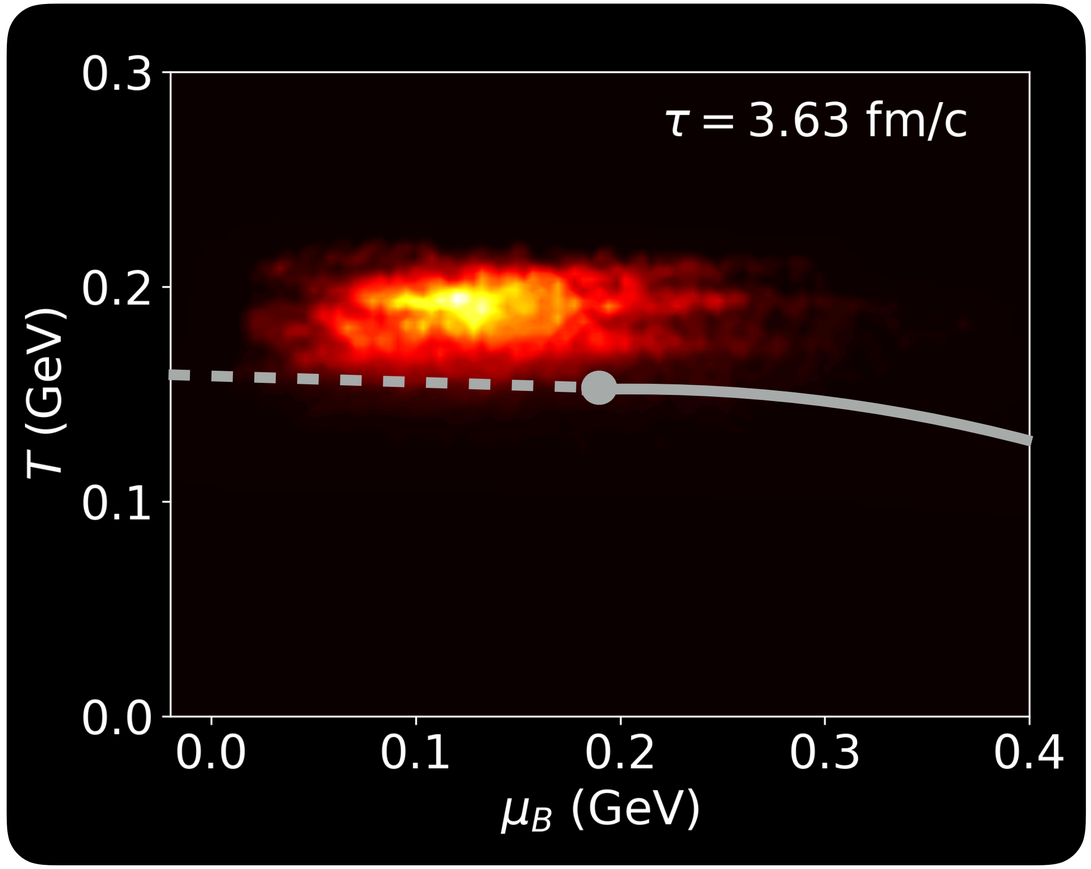

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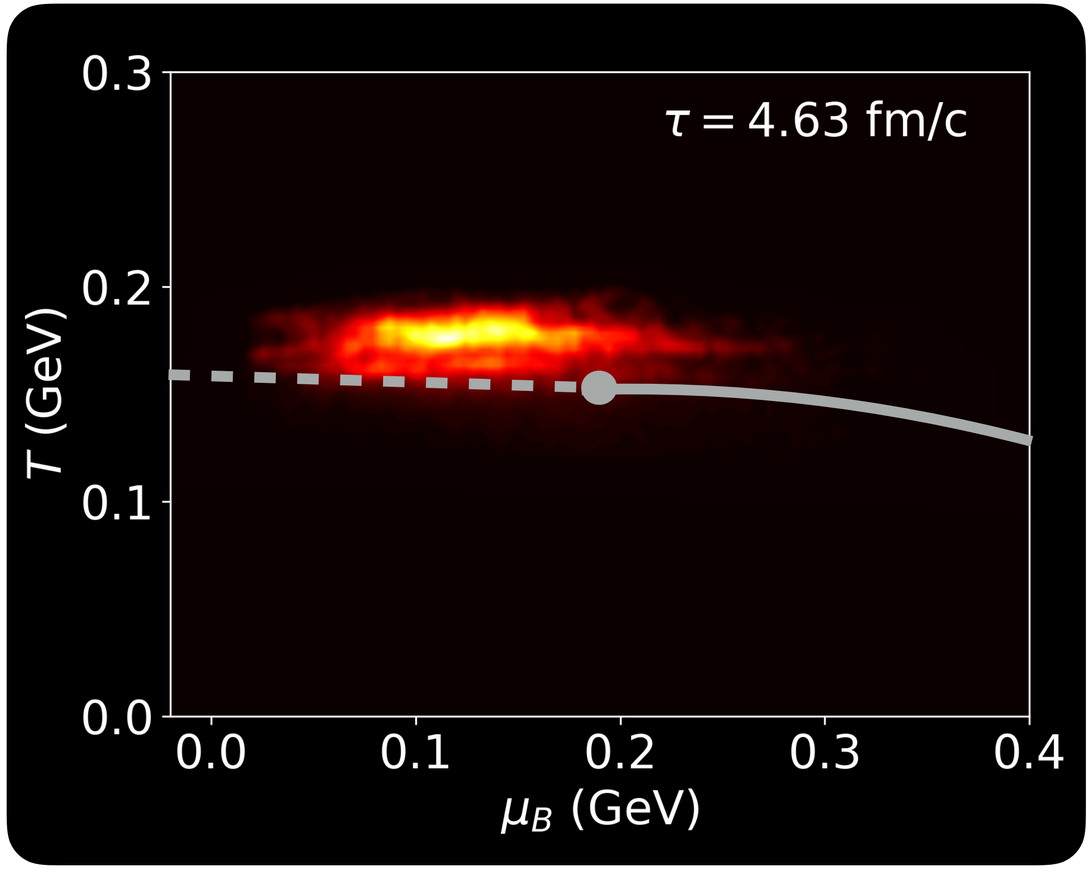

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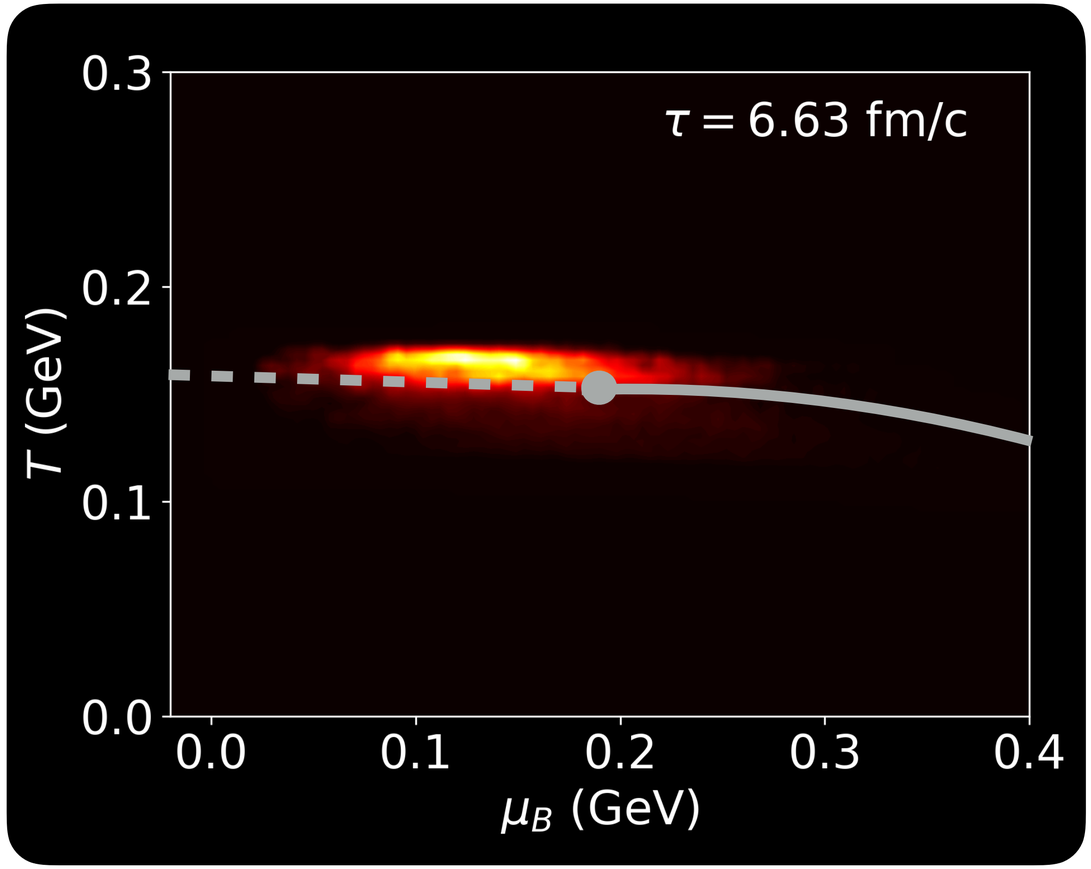

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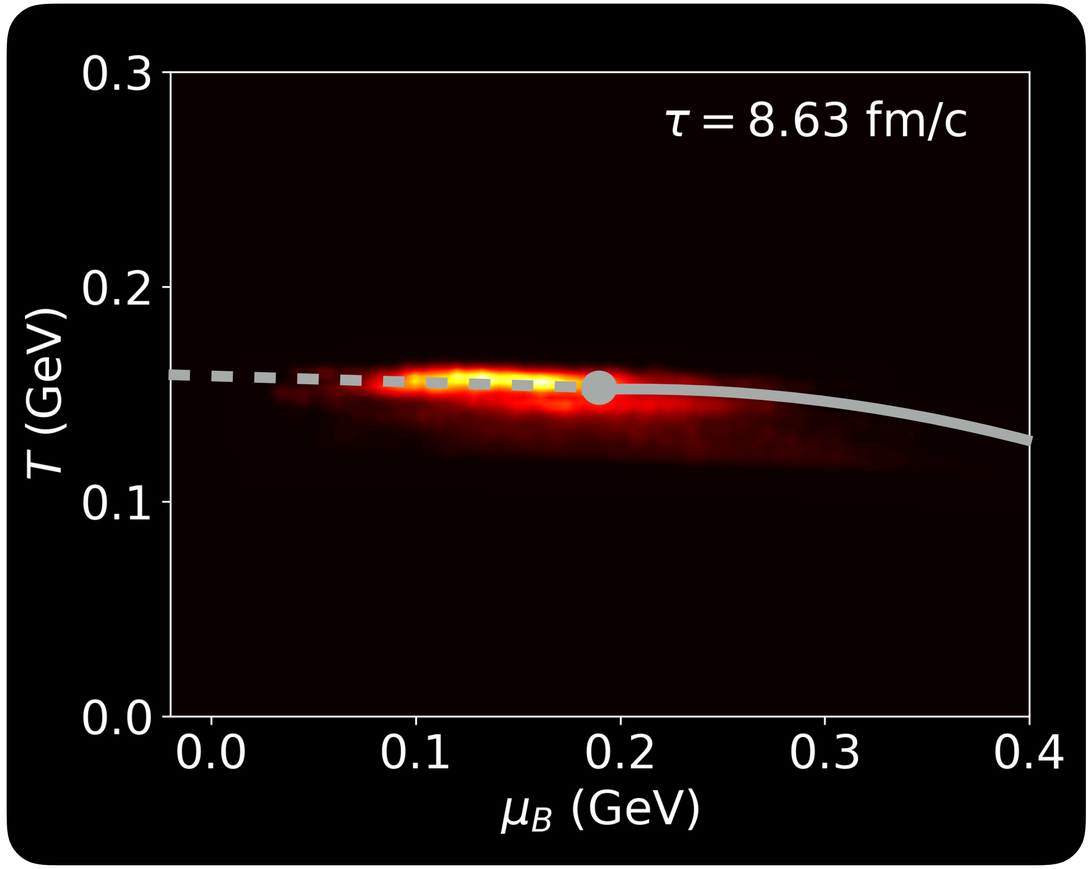

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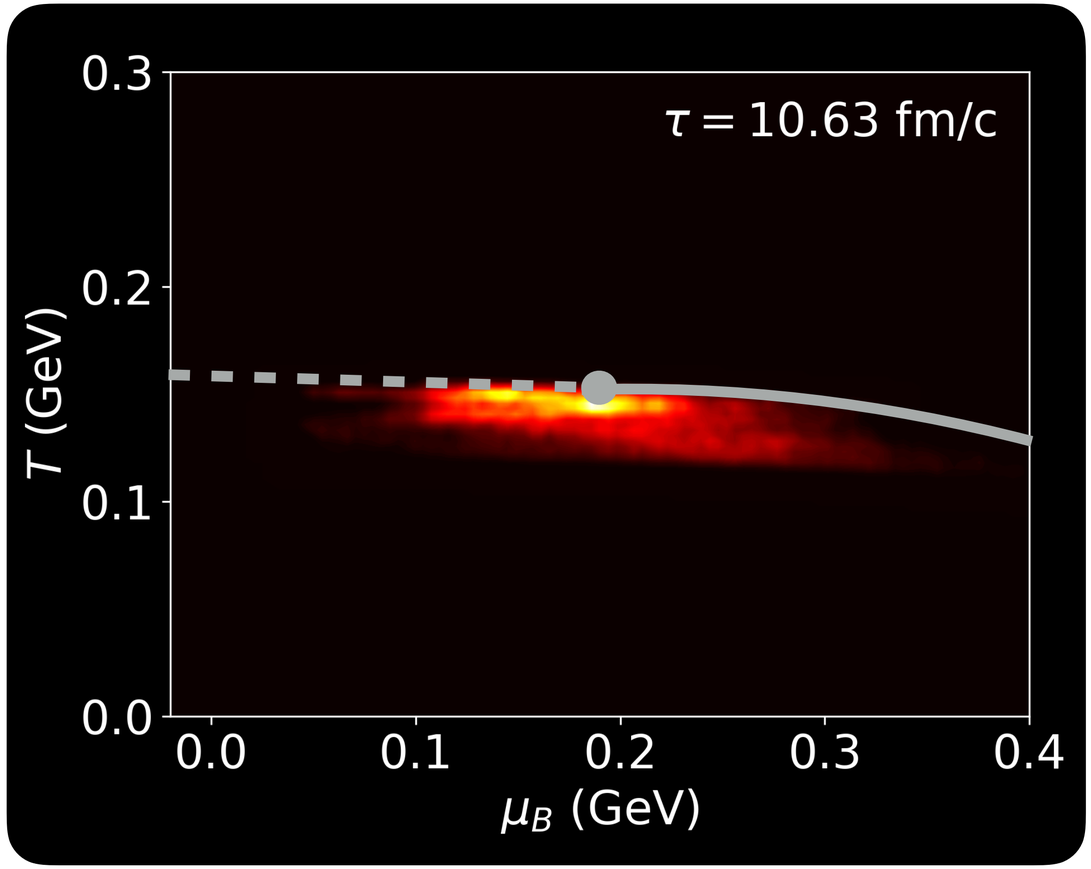

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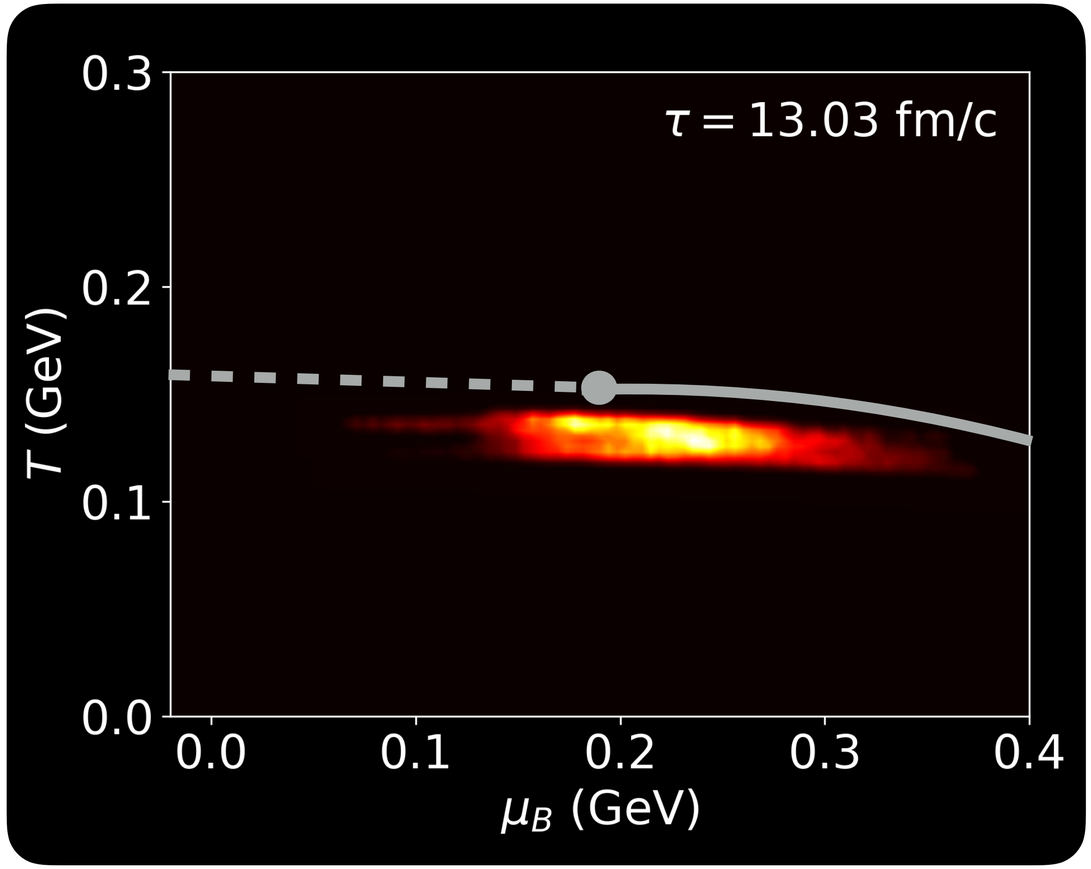
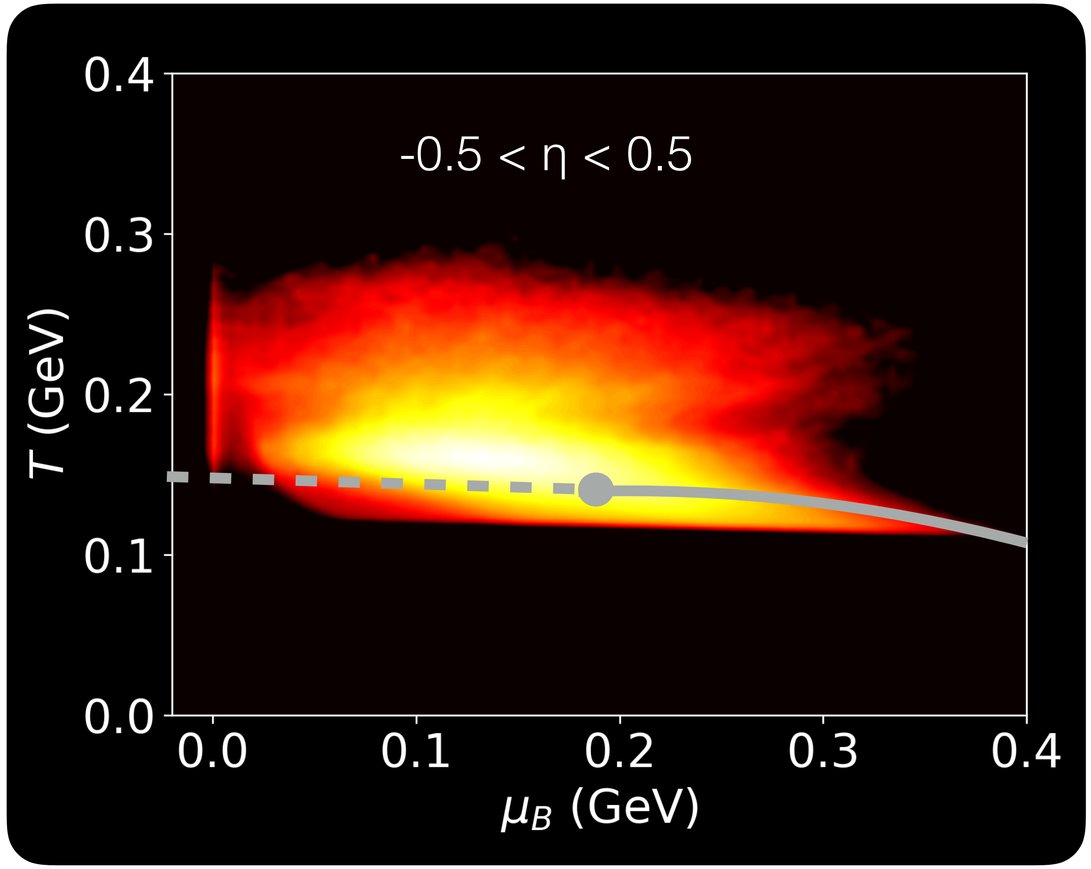


C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

0-5% AuAu@19.6 GeV

EVOLUTION IN THE T-μ_B PLANE

- Fireball trajectory in the T-μ_B is a wide distribution



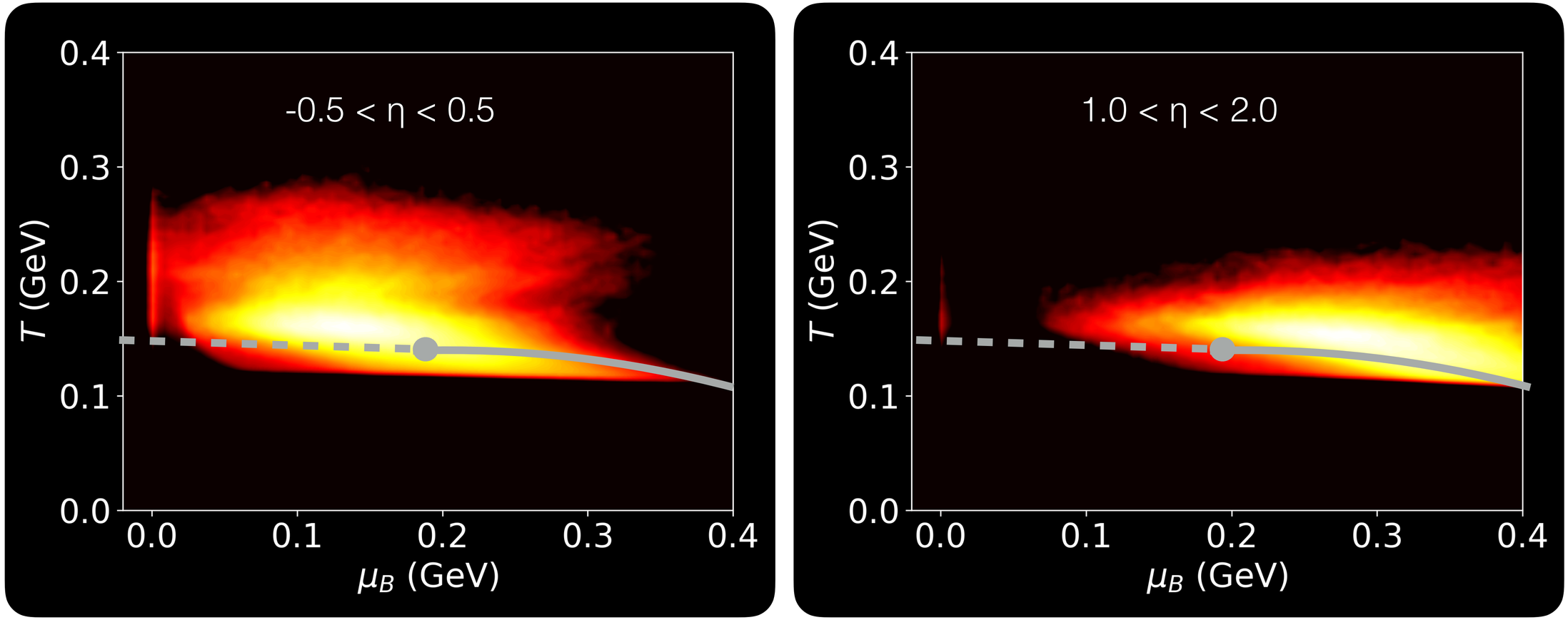
C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

time and space integrated

0-5% AuAu@19.6 GeV

EVOLUTION IN THE T-μ_B PLANE

- Trajectory changes significantly with rapidity

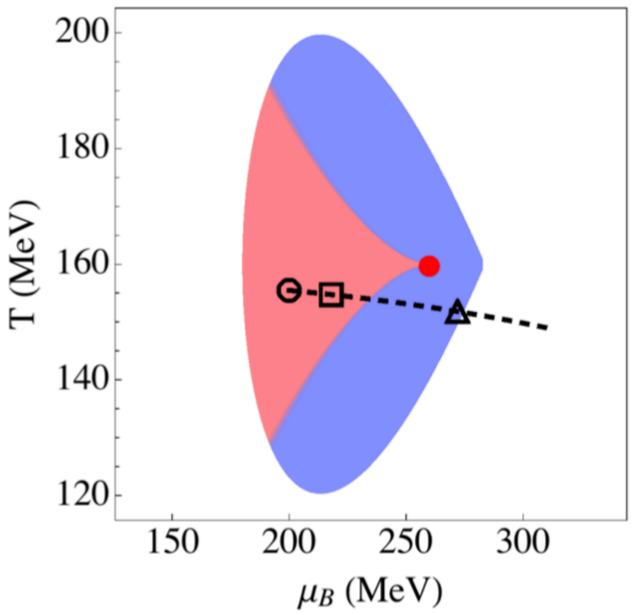


C. Shen, B. Schenke, Phys.Rev. C97 (2018) 024907

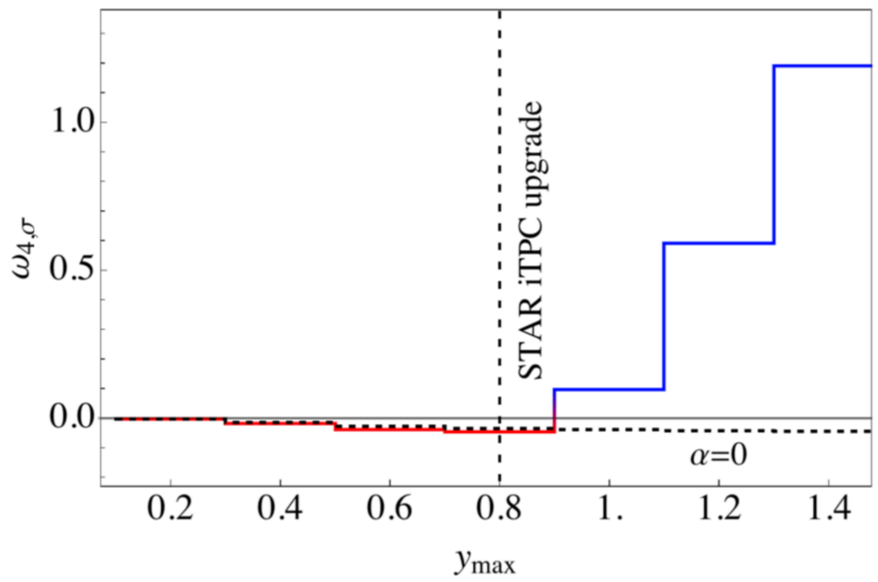
REFINING THE SEARCH: BINNING IN RAPIDITY

J. Brewer, S. Mukherjee, K. Rajagopal, Y. Yin, arXiv:1804.10215

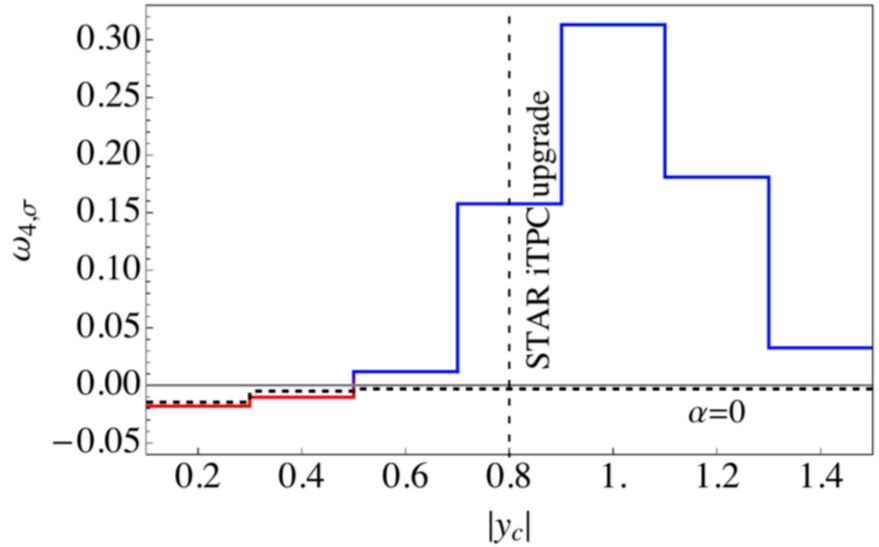
color: sign of kurtosis



Varying y_{max}

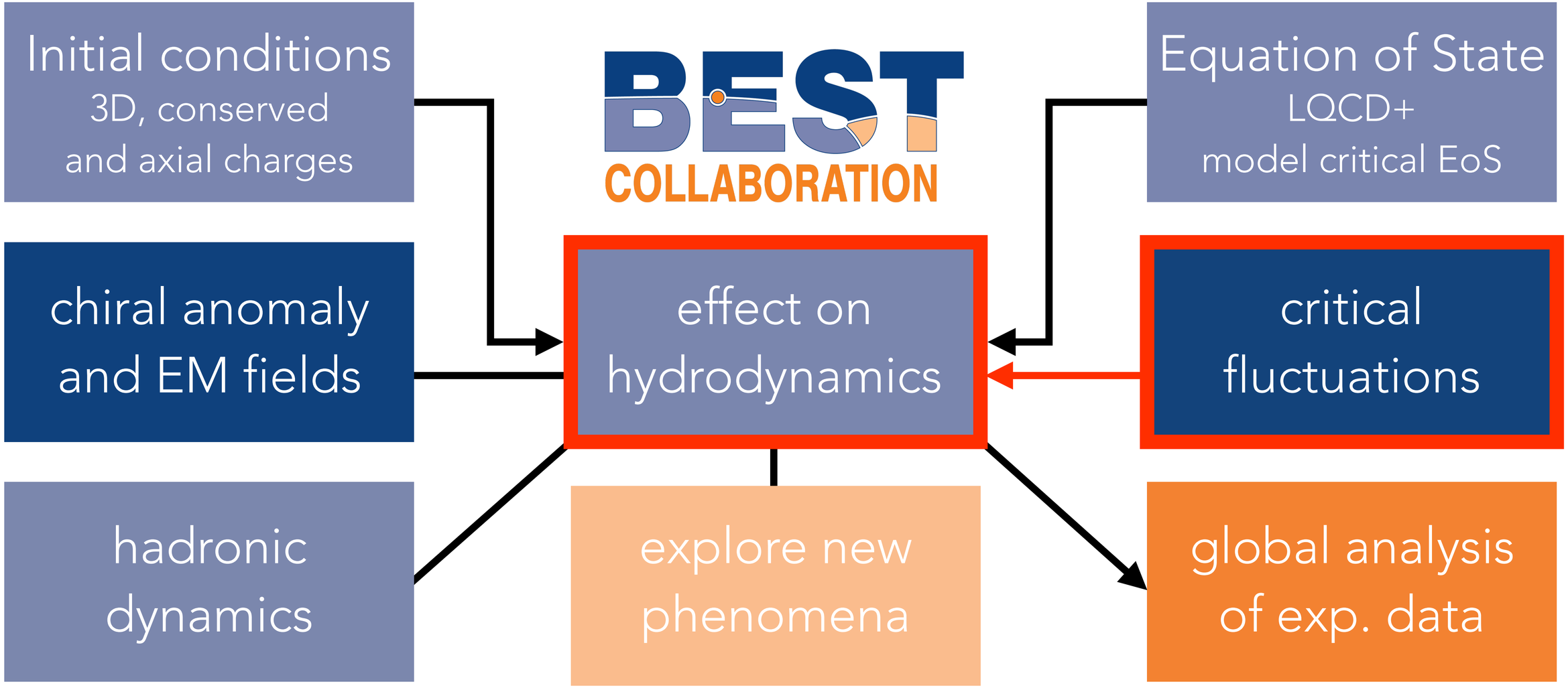


Binning in y



Critical signatures easier to detect at lower rapidity

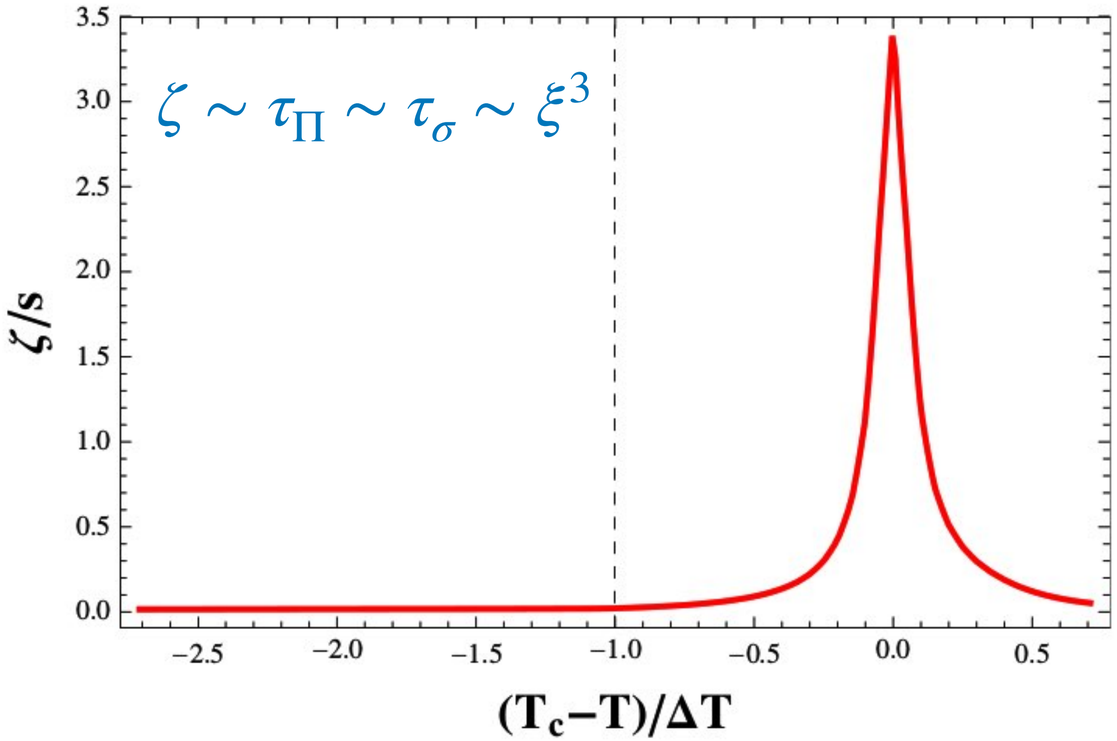
THE BEAM ENERGY SCAN THEORY COLLABORATION

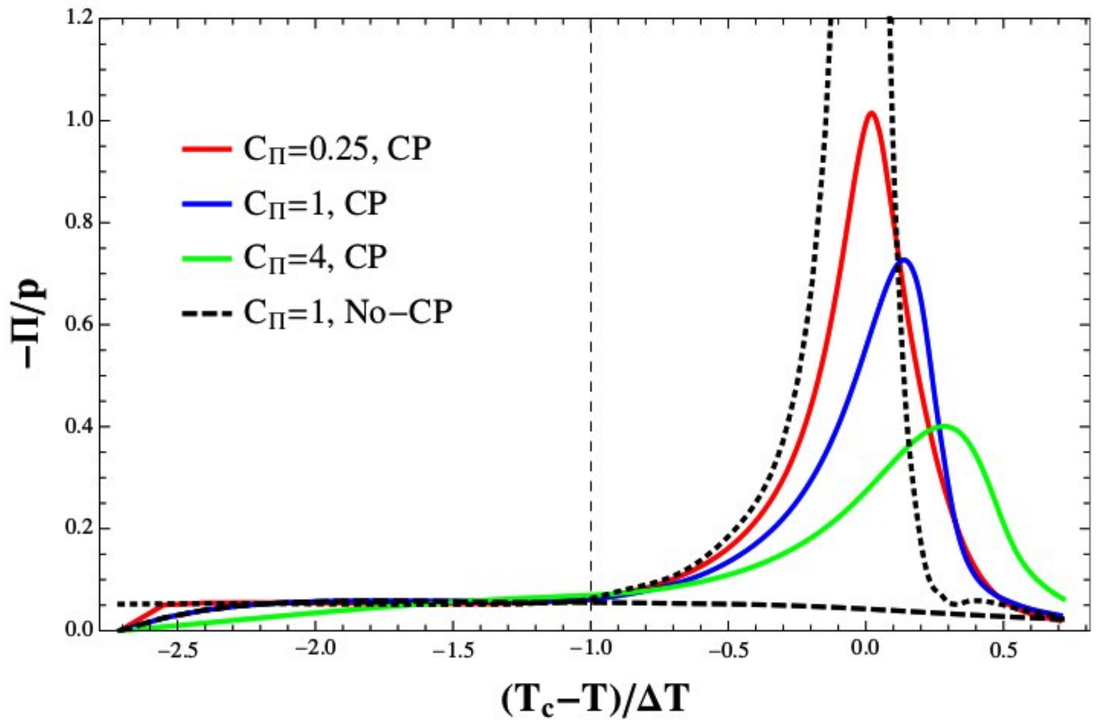


DIVERGING BULK VISCOSITY AT CRITICAL POINT

Monnai, Mukherjee, Yin: Phys. Rev. C95, 034902 (2017)

Critical fluctuations leads to break down of ordinary hydrodynamics for: $k \sim \xi^{-3}$





bulk viscous pressure

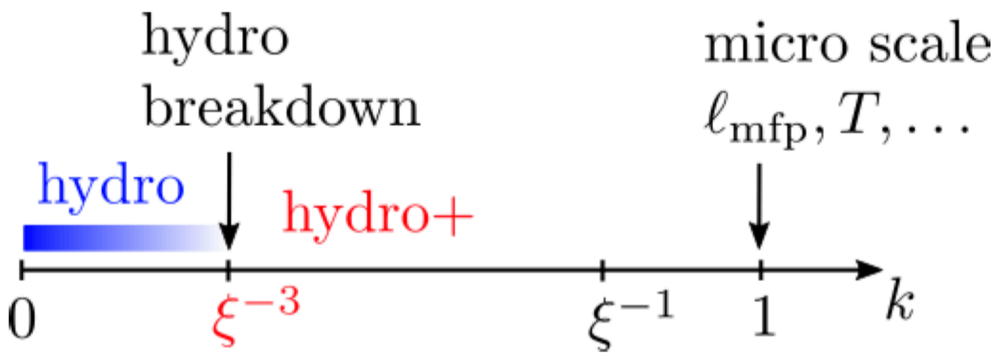
(1+1)D, Israel-Stewart hydro

HYDRO+ COUPLING TO CRITICAL MODE

M. Stephanov, Y. Yin, arXiv:1704.07396, arXiv:1712.10305

$\zeta \sim \tau_\Pi \sim \tau_\sigma \sim \xi^3$ Hydro breaks down because of large relaxation time

Treating the slow mode Φ with $\tau_\Phi \sim \xi^3$ separately (relaxation equation) will increase range of validity:

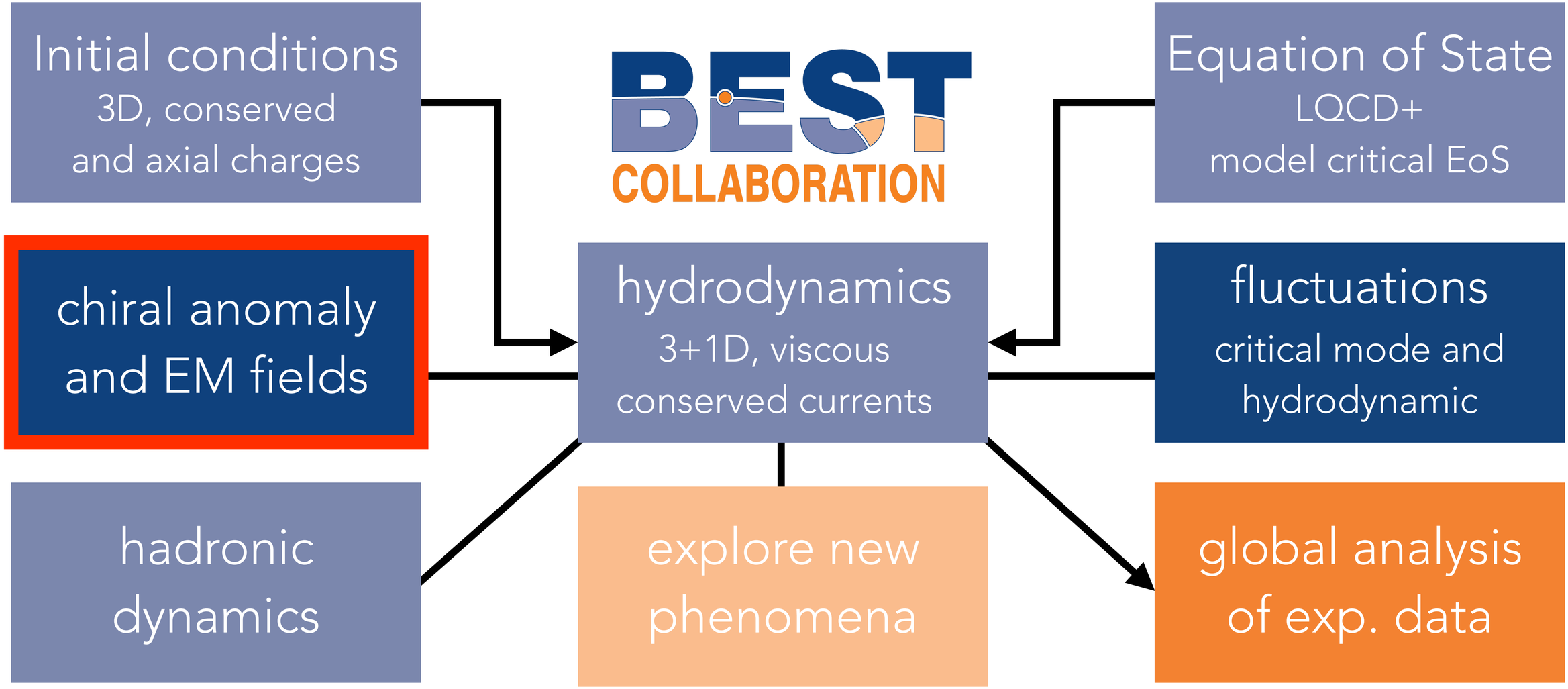


Major effects of critical fluctuations on the evolution:

- Strong frequency dependence of the anomalously large bulk viscosity
- Stiffening of the EoS with increasing frequency

Explicit implementation and simulation to be done

THE BEAM ENERGY SCAN THEORY COLLABORATION



ANOMALOUS VISCOUS FLUID DYNAMICS

S. Shi, Y. Jiang, E. Lilleskov, J. Liao, Annals Phys. 394 (2018) 50-72

Framework for linearized evolution of fermion currents in the QGP, on top of neutral background described by hydrodynamic simulations

$$\hat{D}_\mu J^\mu_{f,R} = +\frac{N_c Q_f^2}{4\pi^2} E_\mu B^\mu$$

$$\hat{D}_\mu J^\mu_{f,L} = -\frac{N_c Q_f^2}{4\pi^2} E_\mu B^\mu$$

$$J_R^\mu = n_R\, u^\mu + \nu_R^\mu + \frac{N_c q}{4\pi^2}\, \mu_R\, B^\mu$$

$$J_L^\mu = n_L\, u^\mu + \nu_L^\mu - \frac{N_c q}{4\pi^2}\, \mu_L\, B^\mu$$

chiral magnetic effect

viscous effect

$$\Delta^\mu{}_\nu\, d\, \nu_{R,L}{}^\nu = -\frac{1}{\tau_{rlx}}(\nu_{R,L}{}^\mu - \nu_{NS}{}^\mu)$$

$$\nu_{NS}{}^\mu = \frac{\sigma}{2} T \Delta^{\mu\nu} \partial_\nu \frac{\mu}{T} + \frac{\sigma}{2} q\, E^\mu$$

also see:

Chiral transport equations from Wigner function formalism

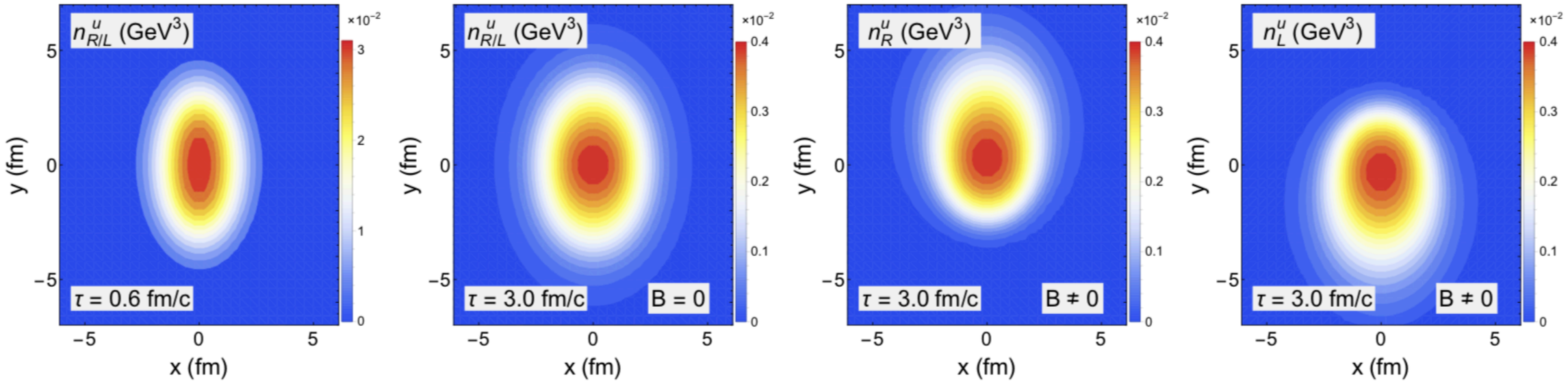
A. Huang, S. Shi, Y. Jiang, J. Liao, P. Zhuang, arXiv:1801.03640

ANOMALOUS VISCOUS FLUID DYNAMICS

S. Shi, Y. Jiang, E. Lilleskov, J. Liao, Annals Phys. 394 (2018) 50-72

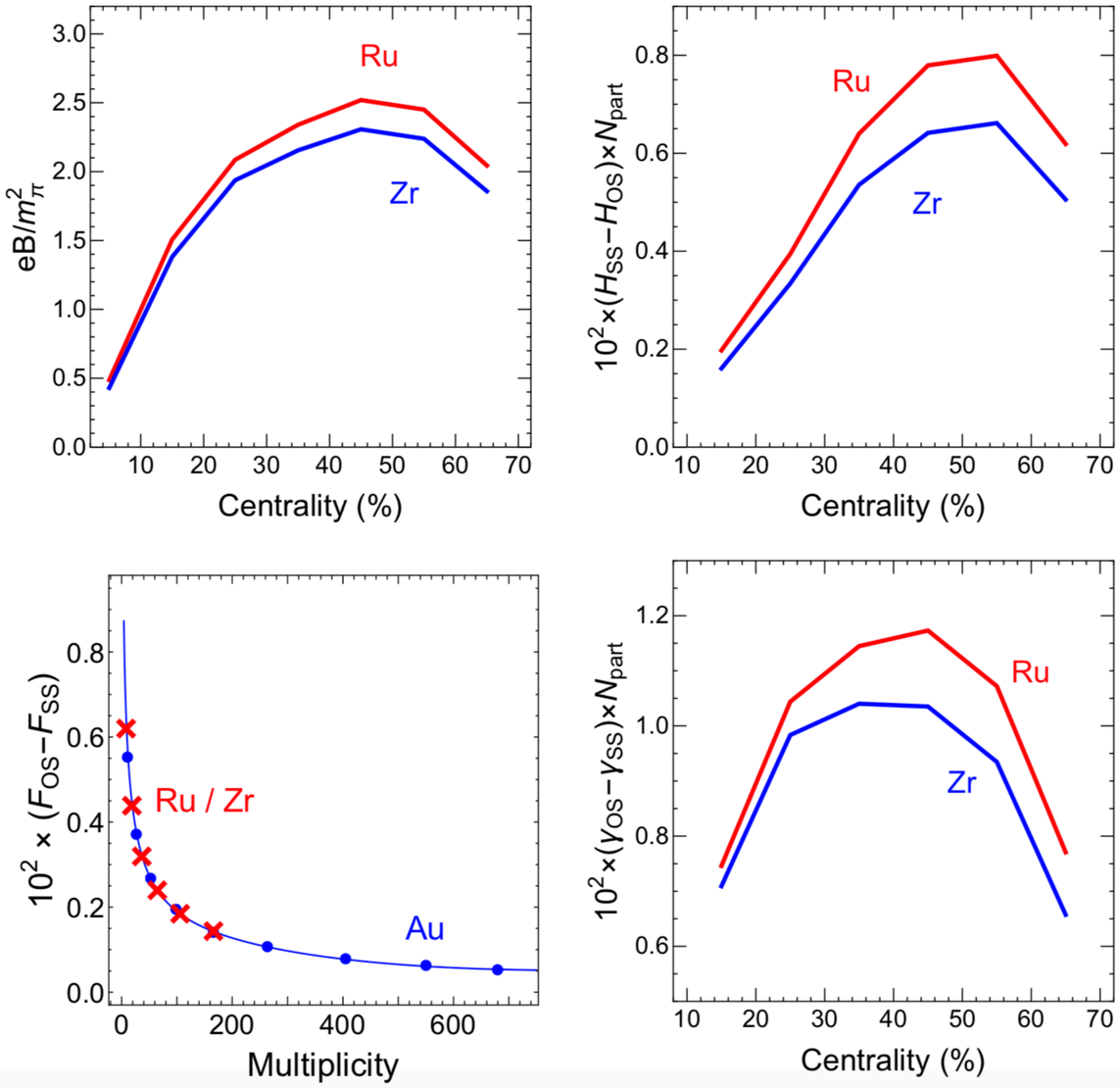
u-flavor densities:

initial | $B_y=0$ | $B_y>0$ right-handed | $B_y>0$ left-handed



PREDICTIONS FOR THE ISOBAR RUN

S. Shi, Y. Jiang, E. Lilleskov, J. Liao, Annals Phys. 394 (2018) 50-72



$$\gamma_{\alpha\beta} \equiv \langle \cos(\phi_i + \phi_j - 2\Psi_{RP}) \rangle_{\alpha\beta}$$

α, β = +,- SS=++,-- OS=+-

H is pure CME contribution to γ correlator

F is pure background, here estimated from Au+Au data

~15% difference between the two systems

SUMMARY AND OUTLOOK

- Significant progress on many aspects:
  Equation of state, initial condition, 3+1D viscous fluid dynamics, (critical) fluctuations, anomalous fluid dynamics, …
- Comprehensive theoretical framework to support BES II
- To be addressed:
  - Numerical implementation of Hydro+
  - Event-by-event conservation of conserved charges in statistical freeze-out
  - How to match critical correlations to hadronic transport model
  - Early-time dynamics of chiral anomaly

- More information and additional publications at:
  `https://www.bnl.gov/physics/best/`