Hadronic transport approach for collisions at RHIC BES energies

Dmytro (Dima) Oliinychenko

June 12, 2018

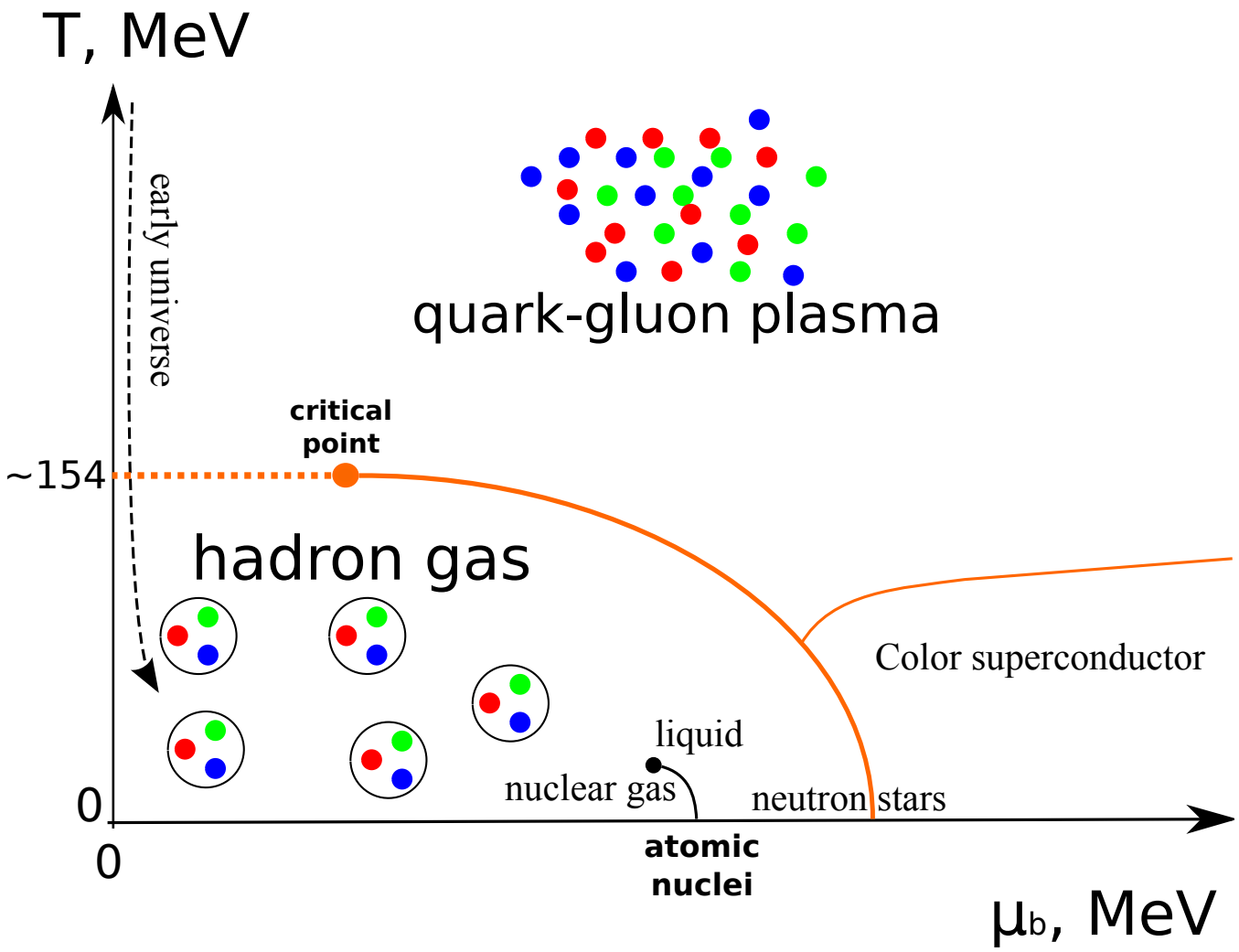
LBNL with Prof. Volker Koch
FIAS with Prof. Hannah Petersen

DO, Huovinen, Petersen, PRC 91 (2015) no.2, 024906
DO, Petersen, PRC 93 (2016) no.3, 034905
DO, Petersen, J.Phys. G44 (2017) no.3, 034001
Weil et al, PRC 94 (2016) no.5, 054905
Schwarz, DO, Pang, Ryu, Petersen, arXiv:1707.07026

BEST COLLABORATION

Nuclear matter in equilibrium: phase diagram

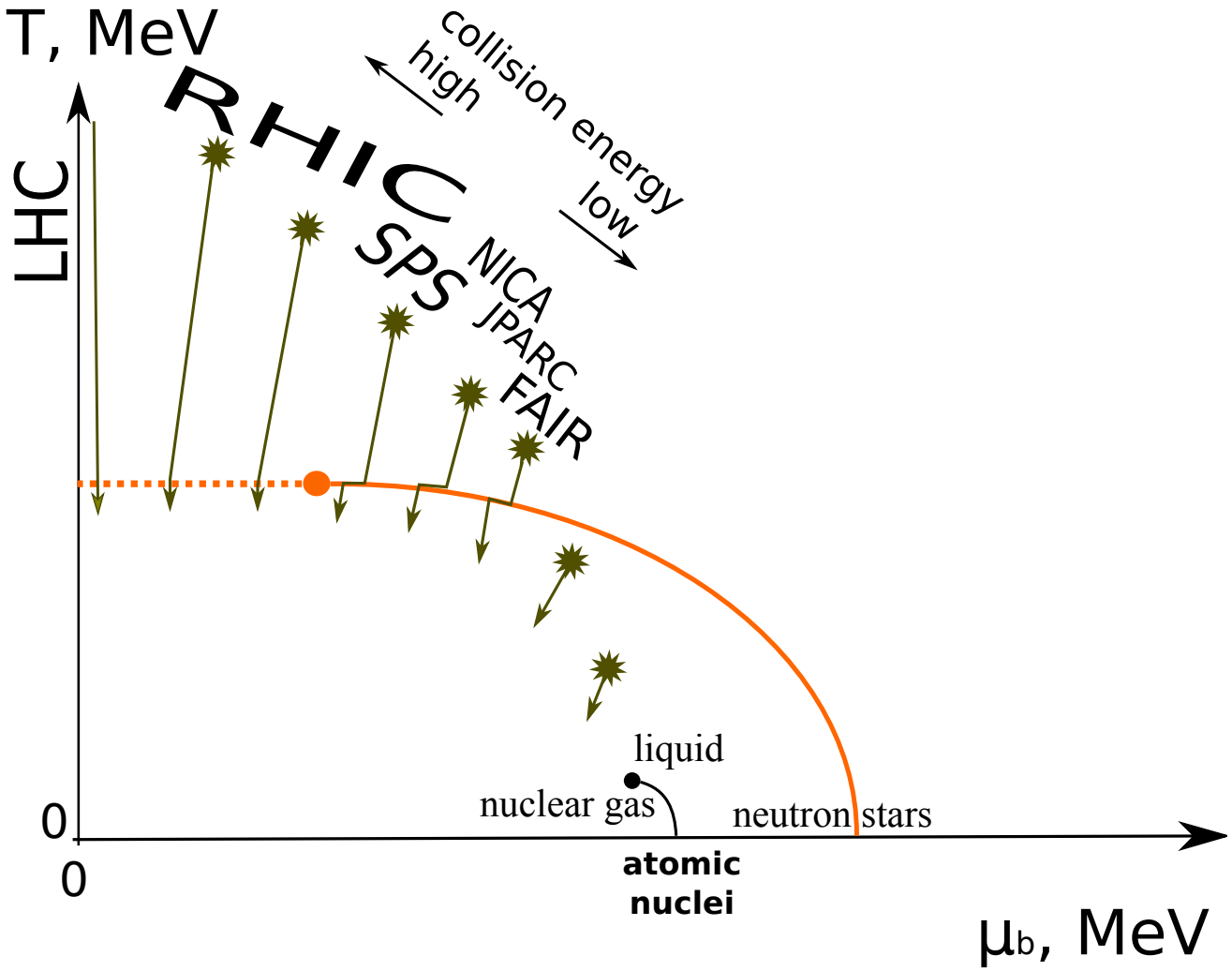
Does phase transition/critical point exist? (T^{cr}, μ_b^{cr})?



Heavy ion collisions: experiments

Crossing critical point expected at intermediate collision energies

$(\sqrt{s_{NN}} \simeq 5 - 20$ GeV$)$



Heavy ion collisions: experiments

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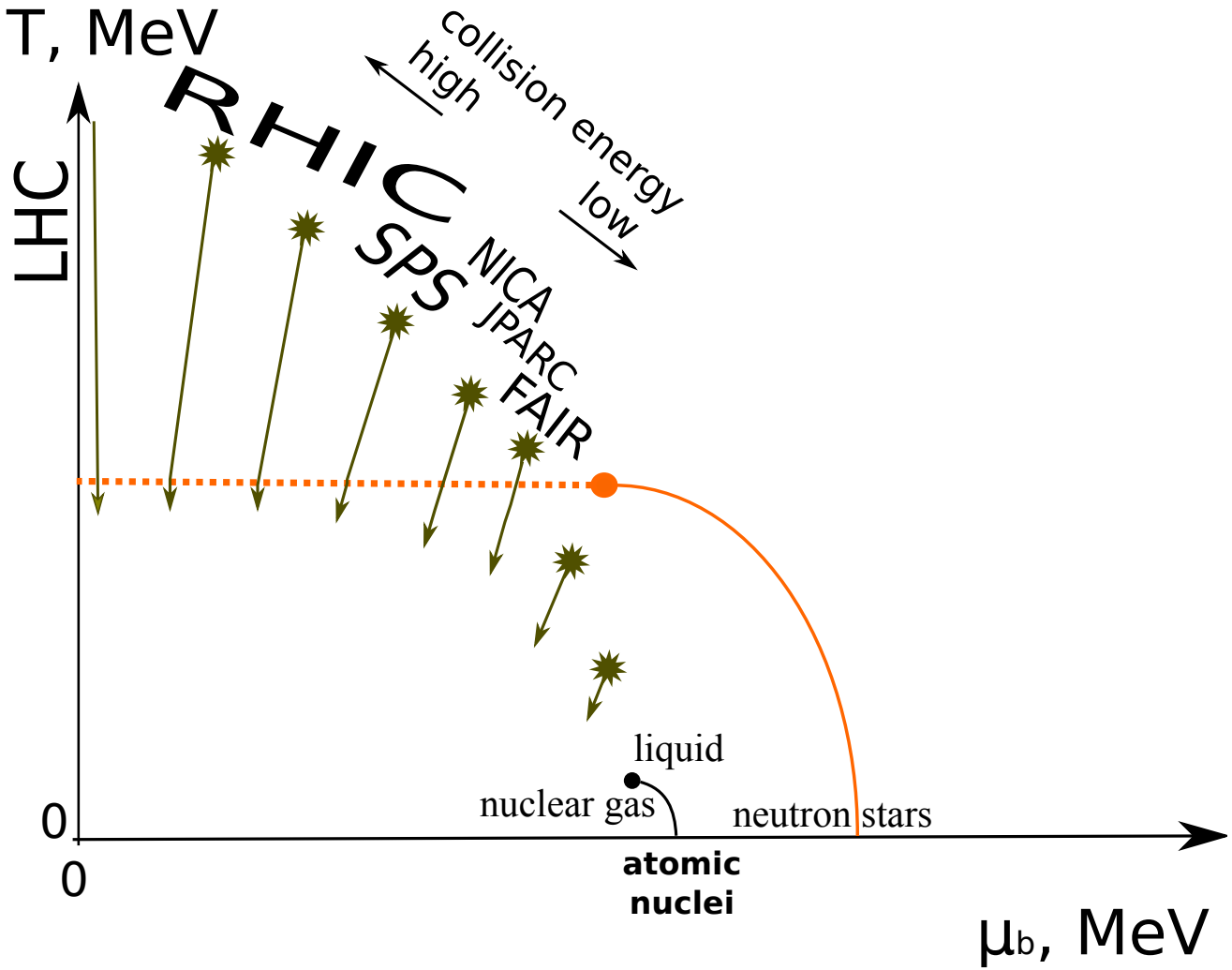
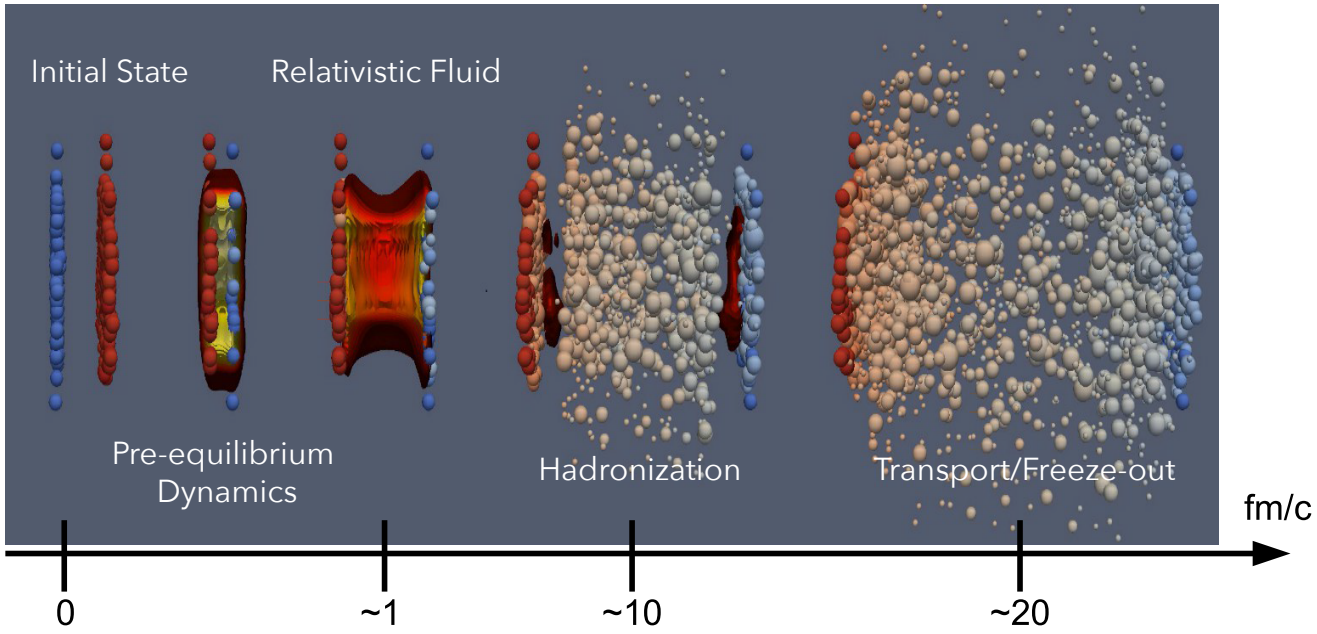


Phase transition search scenario at $\sqrt{s_{NN}} = 5 - 20$ GeV

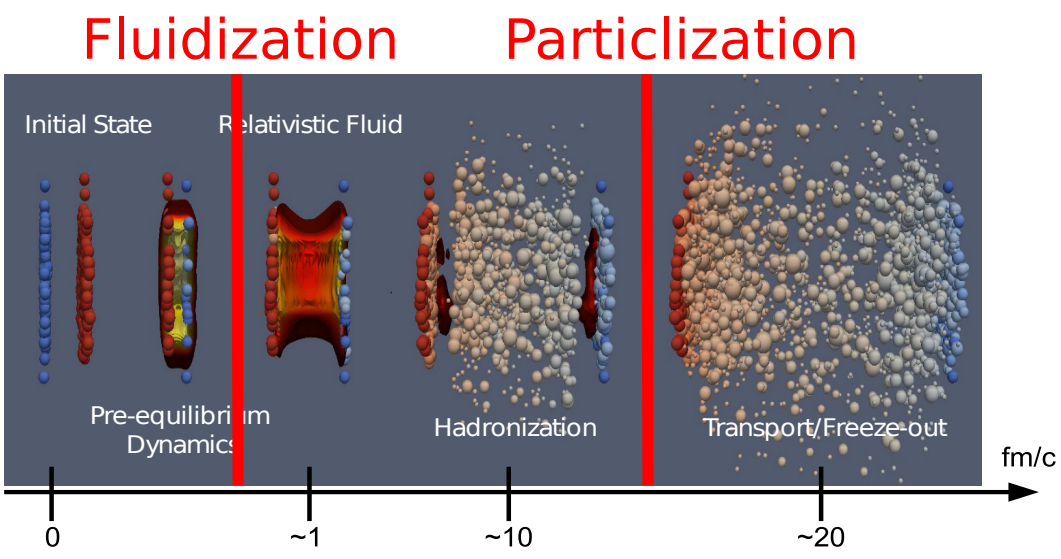
- Model with variable Equation of State
  M(EoS, other parameters)
  - Pure transport with tunable interactions?
    e.g. works of Y. Nara (JAM model)
  - Pure hydro — tunable EoS?
    e.g. Ivanov (3-fluid hydro), L.-G. Pang (CLVisc)
  - Hybrid (hydro + transport) approach?
    EoS tuned in hydro? or in transport too?
- Finding observable(s) O sensitive to EoS
  | O(1st order PT) - O(cross-over) | $\gg$ error
  for all other parameters varied
  - Kink, Step, Horn?
  - $dv_1/dy(\sqrt{s})$?
  - HBT $R_{\mathrm{side}}/R_{\mathrm{out}}$?
  - proton dN/dy?
- Bayesian analysis: extracting EoS parameters $\rightarrow$ critical point

Hybrid (hydrodynamics + transport) approaches



- Hydrodynamics: local thermal equilibrium,
  $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu j^\mu = 0$, EoS, boundary conditions
  Applicability: mean free path $\ll$ system size $\implies$ high density
- Transport: Monte-Carlo solution of Boltzmann equations
  Applicability: mean free path $\gg \lambda_{Compton}$ $\implies$ low density
- Hybrid: hydro at high density + transport at low density

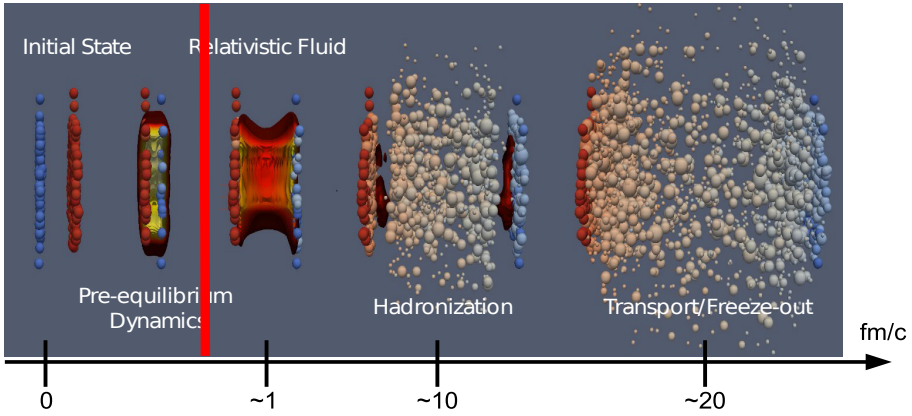
What changes for hybrids at intermediate energies?



- Initial state for hydrodynamics
  - Nuclei not flat, baryon stopping
  - How thermalized is it?
  - Initial state via source terms works of C. Shen; Akamatsu et al [arXiv:1805.09024]
- Baryon charge conservation is important
- Larger negative Cooper-Frye contributions at particlization
- Core-corona separation works of J. Steinheimer; Akamatsu et al [arXiv:1805.09024]

Initial state for hydrodynamics from transport

Fluidization



Petersen et al., PRC 78 (2008) 044901;
Skokov, Toneev, PRC 73 (2006) 021902;
Werner et al., PRC 82 (2010) 044904;
Anrade et al., Eur.Phys.J. A 29 (2006) 23-26;
Gale et al., PRL 110 (2013) no.1, 012302;
Karpenko et al., PRC 91 (2015) no.6, 064901;
Pang et al., PRC 86 (2012) 024911;
Bhalerao et al., PRC 92 (2015) no.1, 014903

- Geometrical criterion: CM-frame $t_{start} = \frac{2R}{\beta\gamma} = 2R\sqrt{\frac{2m_N}{E_{lab}}}$
- Non-equilibrium $T^{\mu\nu}$ from coarse-graining
- Forcing ideal-fluid $T^{\mu\nu}$
- Deviation of $T^{\mu\nu}$ from equilibrium neglected

How far is $T^{\mu\nu}$ from equilibrium?

Coarse-grained microscopic transport approach

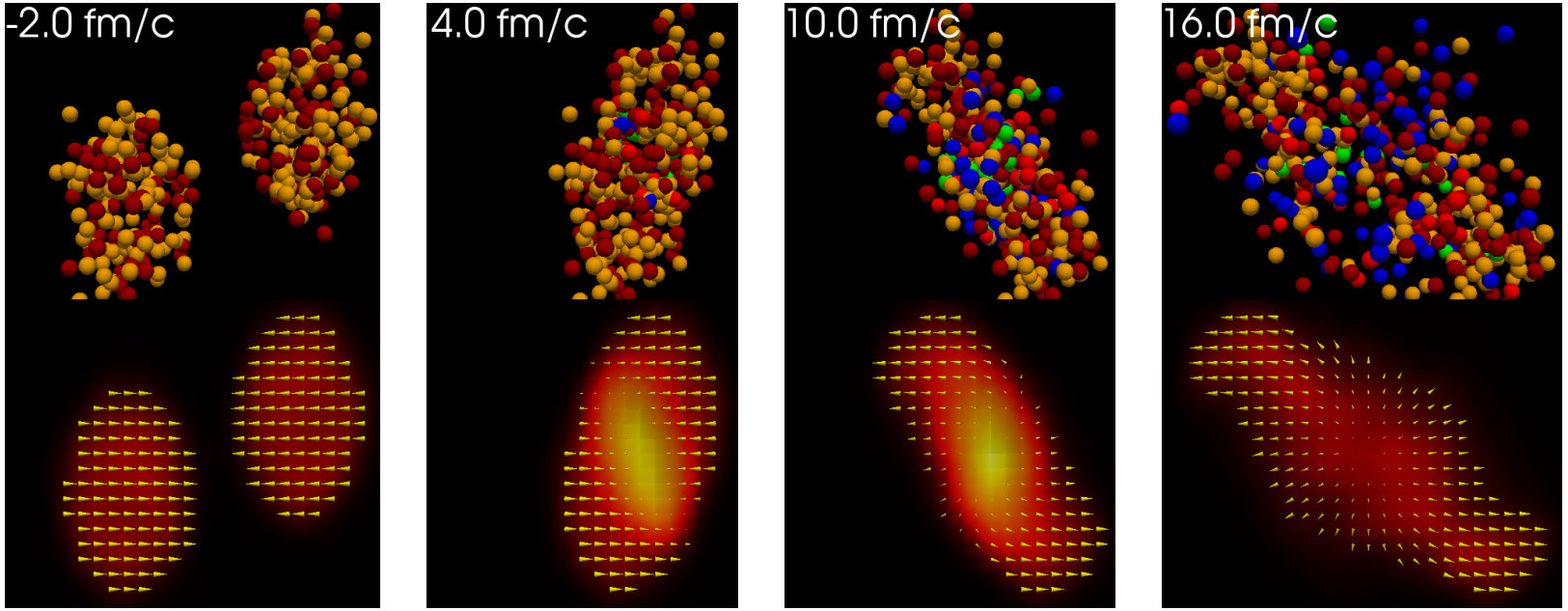
- Generate many events from transport approach
- On a (t,x,y,z) grid compute

$$T^{\mu\nu}(\vec{r}) = \frac{1}{N_{ev}} \sum_{events} \sum_{i} \frac{p_i^\mu p_i^\nu}{p_i^0} K(\vec{r} - \vec{r_i}, u_i); \quad u^\mu = (u^0, \vec{u}) = \frac{p^\mu}{m}$$

$$K(\vec{r} - \vec{r_i}, u, \sigma) = \frac{u_0}{(2\pi\sigma^2)^{3/2}} exp\left(-\frac{(\vec{r} - \vec{r_i})^2 + (\vec{u} \cdot (\vec{r} - \vec{r_i}))^2}{2\sigma^2}\right)$$

- In each cell go to Landau frame: $T_L^{0\nu} = (\epsilon_L, 0, 0, 0)$
- Obtain T, μ_b, μ_s from ideal hadron gas EoS

Au+Au, $\sqrt{s_{NN}} = 3$ GeV, $b = 5$ fm

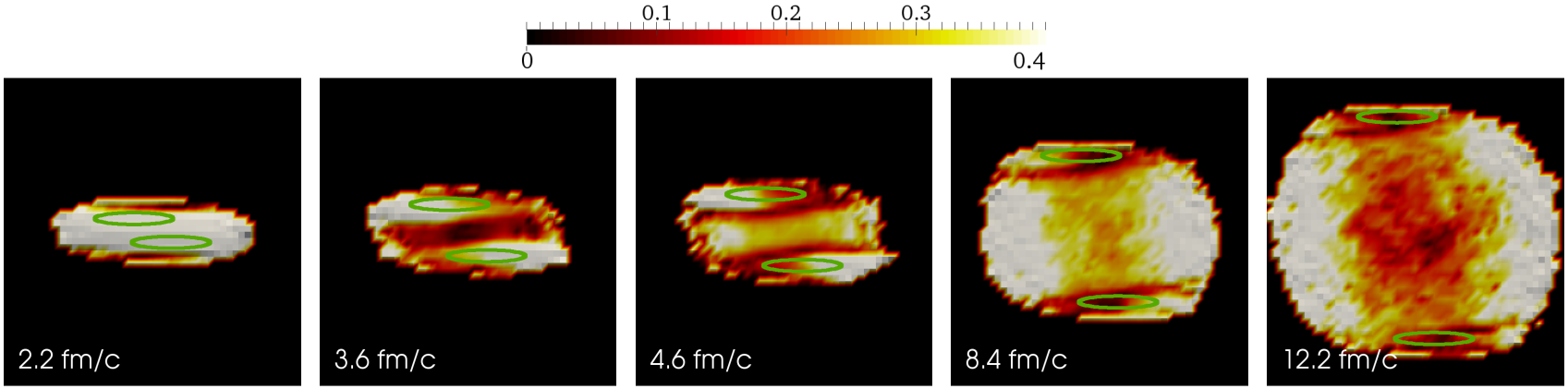


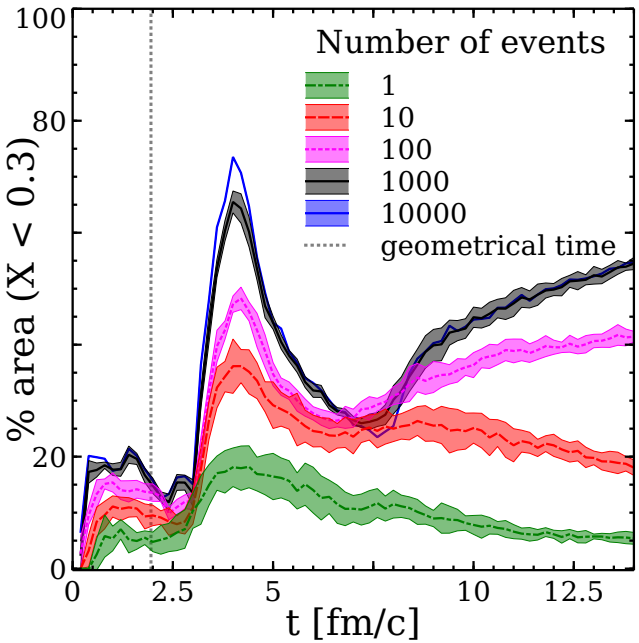
Investigating $T^{\mu\nu}$ in a coarse-grained UrQMD transport

- Transport simulation of Au+Au at $E_{lab} = 5$ - 160 A GeV
- Using UrQMD:
  - Ultra-Relativistic Quantum Molecular Dynamics
  - Hadronic degrees of freedom, strings
  - 2→2 scatterings, $2 \to 1$ resonance formations,
    $1 \to 2$ and $1 \to 3$ resonance decays, string fragmentation
  - Cascade mode: potentials are off
- Pressure anisotropy $x = \frac{|T_L^{xx} - T_L^{yy}| + |T_L^{yy} - T_L^{zz}| + |T_L^{zz} - T_L^{xx}|}{T_L^{xx} + T_L^{yy} + T_L^{zz}} \leq 2$
  $x = 0$ - ideal hydro
  $x <\sim 0.3$ - viscous hydro

Pressure anisotropy X

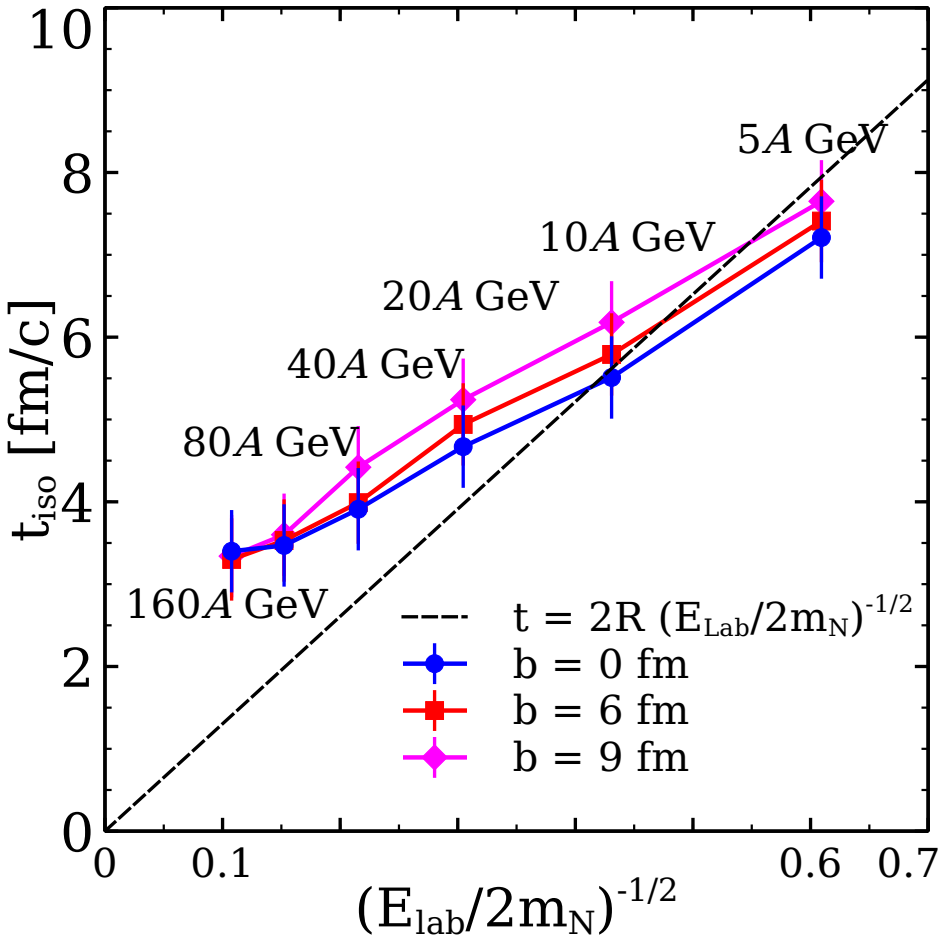
Au+Au collision, $E_{lab} = 80\ A$ GeV, $b = 6$ fm





Event-by-event hydro: less events for initial state - larger anisotropy

Isotropization time

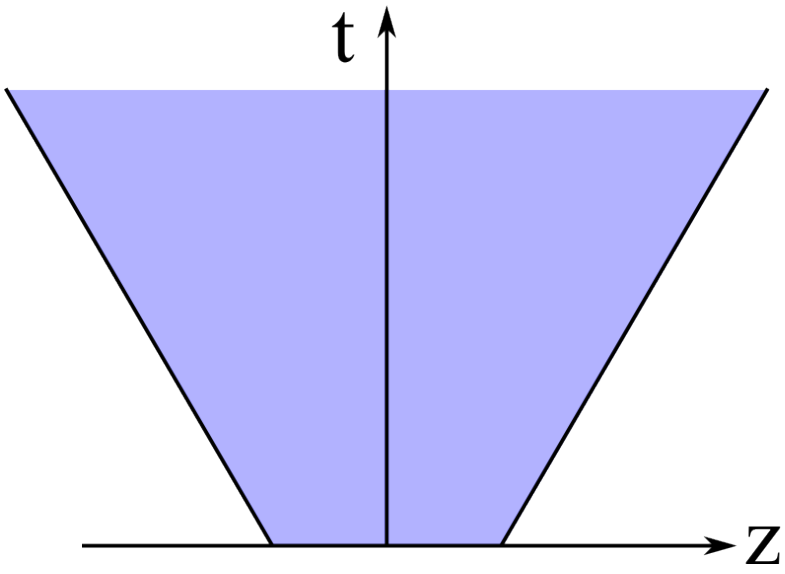


- Geometrical criterion is partly justified
- Core-corona splitting in hydrodynamics is required

Cooper-Frye negative contributions

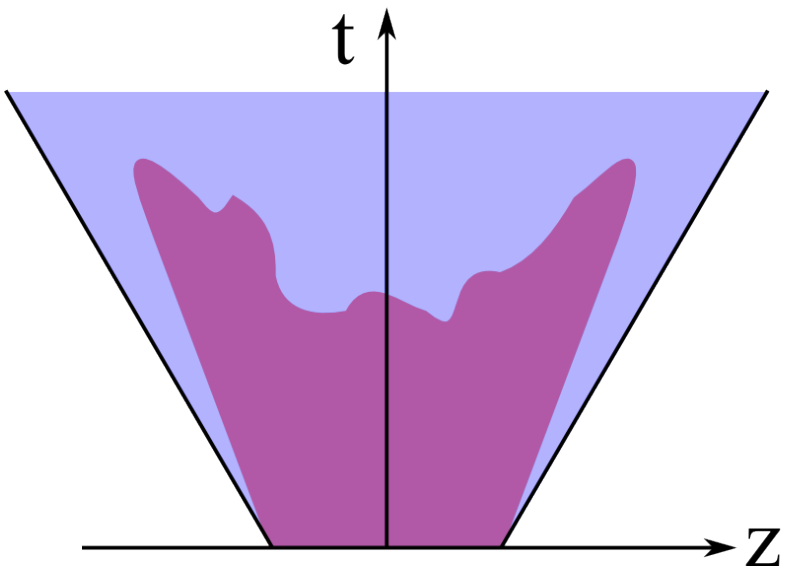
What are they? Why do they arise?
What makes them more important at smaller energies?

Conventional hybrid approaches



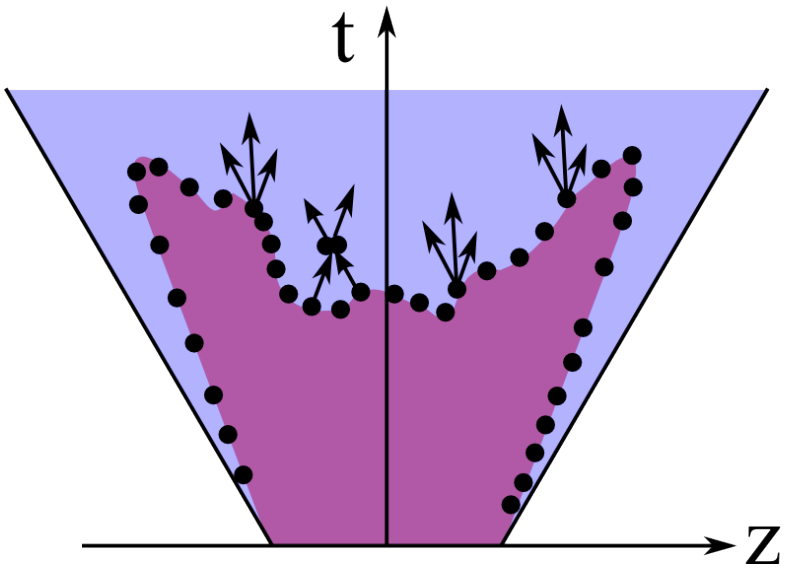
- Solve hydro equations in the light cone

Conventional hybrid approaches



- Solve hydro equations in the light cone
- Find freeze-out hypersurface *aposteriori*

Conventional hybrid approaches



- Solve hydro equations in the light cone
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- Particlization (Cooper-Frye formula)

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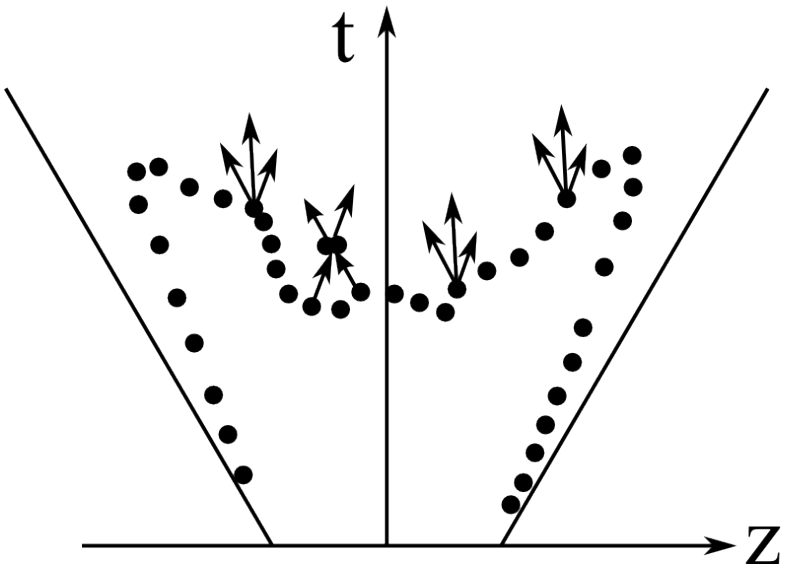

- Solve hydro equations in the light cone
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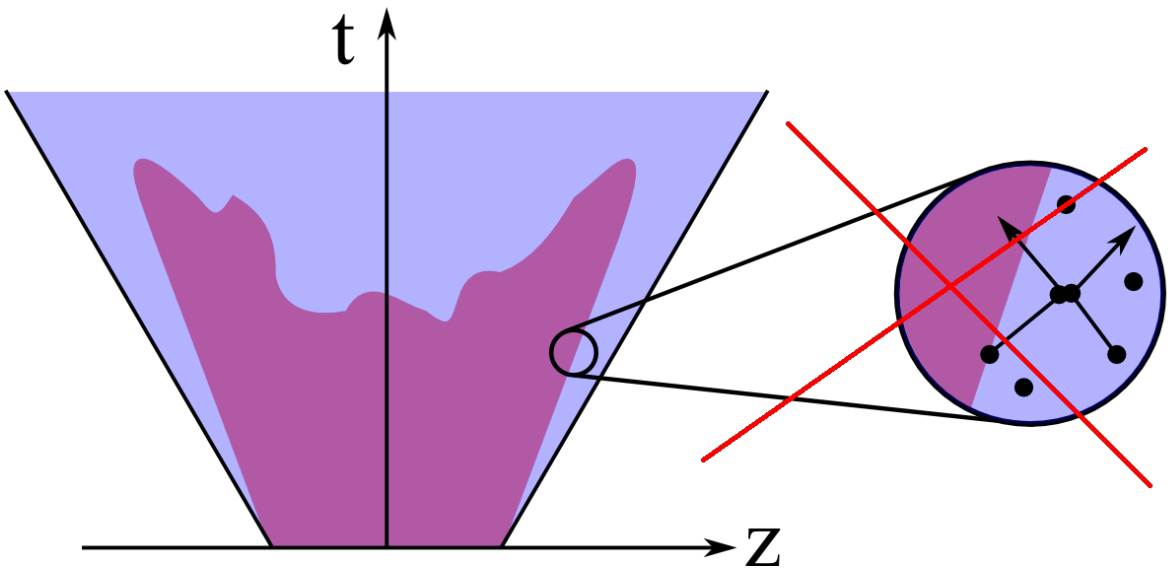
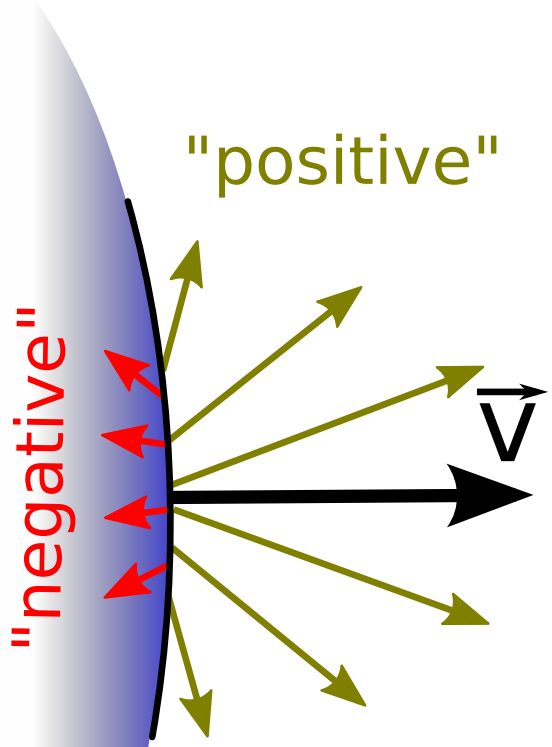
- Particles are decoupled from hydro,
  but can scatter with each other

Conventional hybrid approaches



- Solve hydro equations in the light cone
- Find freeze-out hypersurface *aposteriori*
- Particlization (Cooper-Frye formula)
- Particles are decoupled from hydro,
  but can scatter with each other
- No feedback to hydrodynamics

Particlization and negative contributions



$d\sigma_\mu$ - normal 4-vector

$u_\mu = (\gamma, \gamma\vec{v})$ - 4-velocity

T - temperature

μ - chemical potential

- Particlization
  - know ϵ, p, u_μ on the surface
  - from EoS - T, μ
  - want particles
- "Cooper-Frye formula"

  $$d^3N(p) = f(p)\frac{d^3p}{(2\pi\hbar)^3}\frac{p^\mu}{p^0}d\sigma_\mu$$

  $\frac{p^\mu}{p^0} \cdot d\sigma_\mu$ - analog of $n \cdot V$

  e.g. ideal hydro

  $$f(p) = \left(e^{\frac{p^\mu u_\mu - \mu}{T}} \pm 1\right)^{-1}$$
- $p^\mu d\sigma_\mu > 0$: positive contribution, particles fly out
- $p^\mu d\sigma_\mu < 0$: negative contribution, particles fly in

Negative contributions: options and alternatives

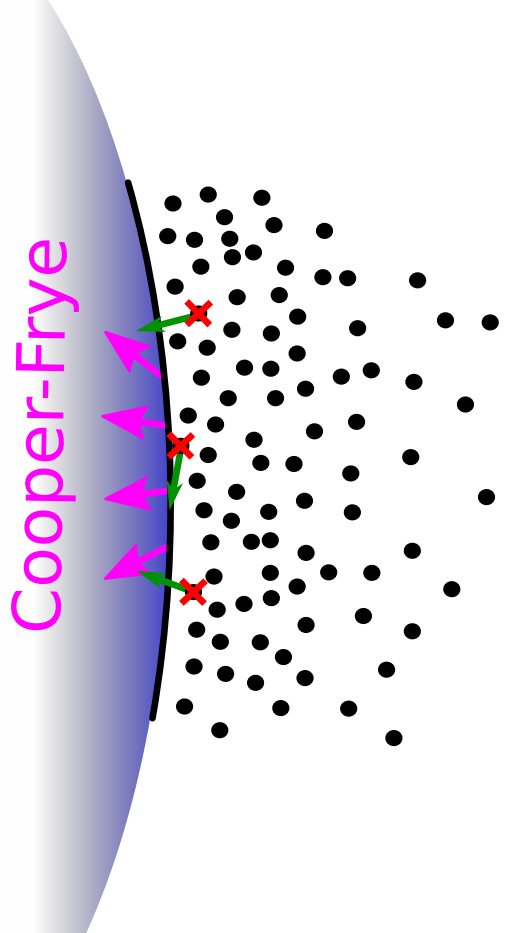
- Account feedback to hydro - great increase in complexity
  Bugaev, PRL, 2003; Czernai, Acta Phys. Hung., 2005
- Transition using rate equation: "hydrokinetic approach"
  Sinyukov, Akkelin, Hama, PRL, 2002; Akkelin, Hama, Karpenko, Sinyukov, PRC, 2008
  Still no particles return to hydrodynamical region
- Cutting $f(p)$ and readjusting to fulfill conservation laws
  Bugaev, Nucl. Phys. A, 1996; Anderlik et al, PRC 1999
  Sharp discontinuity at particlization
- Transition layer of finite thickness
  Anderlik et al, PRC, 1999; Csernai, Eur. Phys. J. A, 2005;
- Account effectively - artificial constructions
  Pratt, PRC, 2014
- Neglect - violate conservation laws

How large are negative contributions?

What changes if we neglect them and how much?

Negative contributions: possible solution

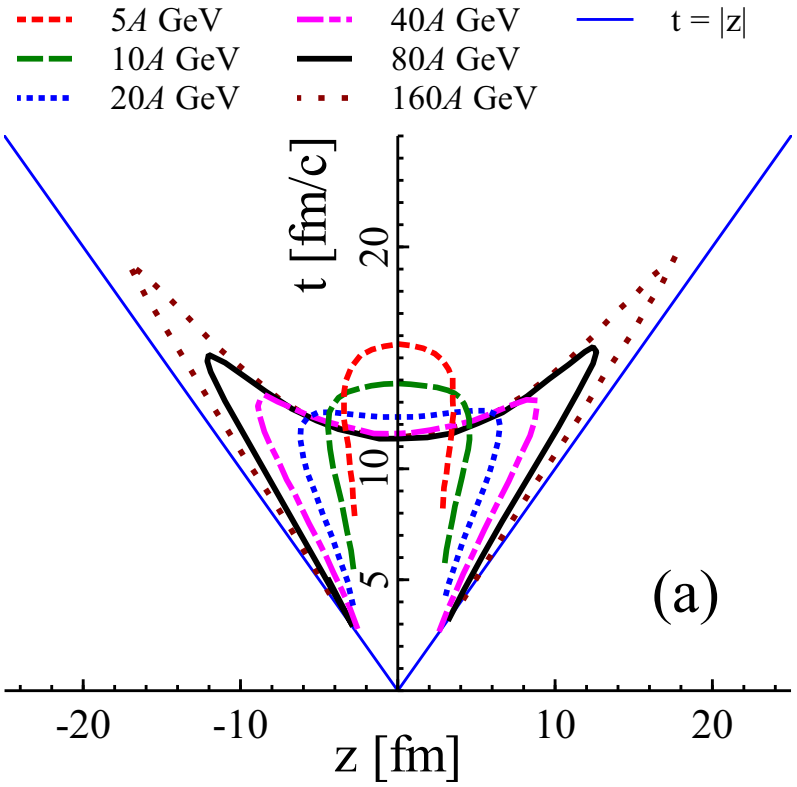
- Hybrid approach
- Assume: transport equivalent to hydrodynamics
- Neglect negative Cooper-Frye contributions + remove particles from cascade if they fly to hydrodynamical region



Is it possible to compensate negative Cooper-Frye contributions? That would solve problem with conservation laws.

Negative contributions with coarse-grained UrQMD

DO, Huovinen, HP, PRC 91 (2015)



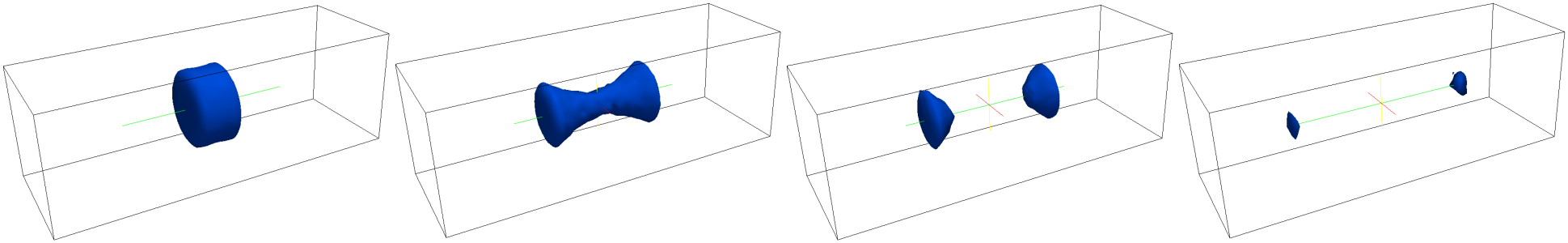
Au+Au central collisions

typical particlization

hypersurface

$\epsilon_{LRF} = 0.3$ GeV/fm^3

$E = 160$ A GeV



t = 4 fm/c t = 11 fm/c t = 13 fm/c t = 18 fm/c

Definitions for negative contributions

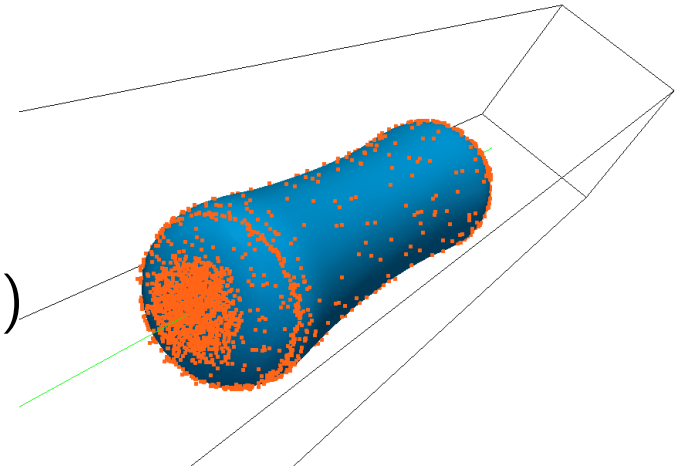
- Hypersurface Σ: $\epsilon_L(t, x, y, z) = const$
  - A) Cooper-Frye formula on Σ
  - B) count UrQMD particles crossing Σ
- $A \equiv B$ if particle distribution from UrQMD is exactly equilibrated

A) Cooper-Frye

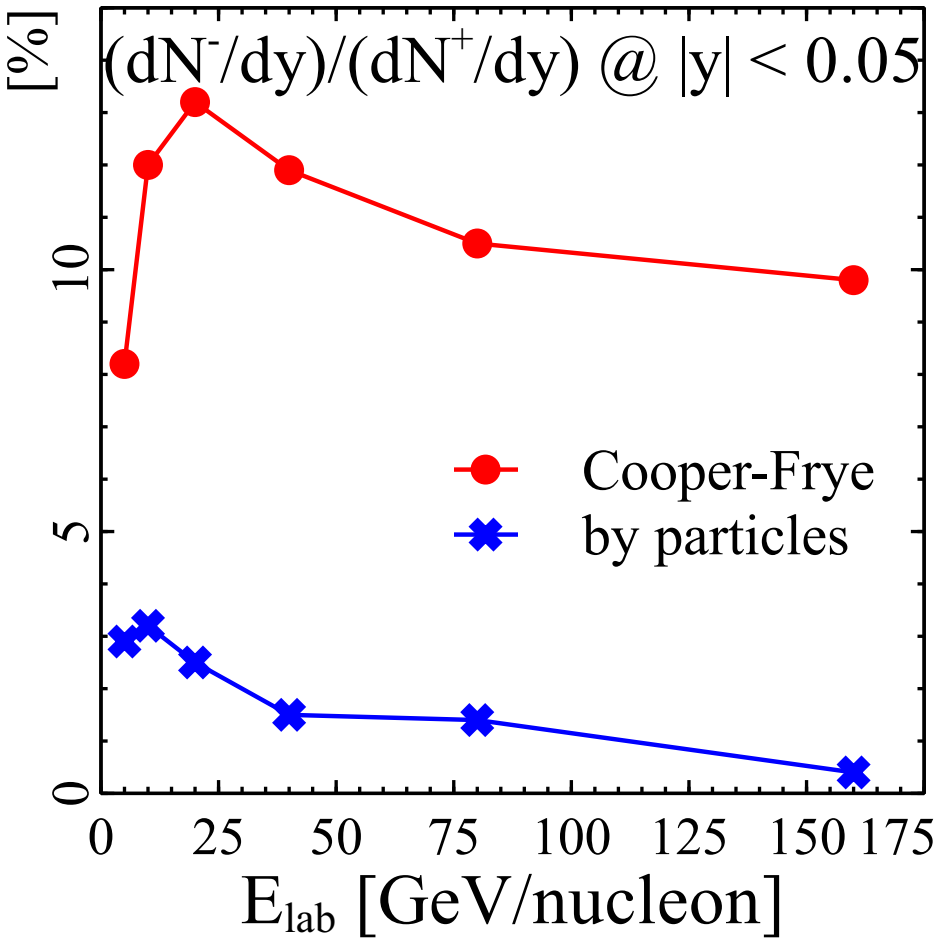
$$p^0 \frac{d^3 N^+}{dp^3} = \frac{p^\mu d\sigma_\mu}{exp(p^\nu u_\nu / T) \pm 1} \theta(p^\nu d\sigma_\nu)$$

$$p^0 \frac{d^3 N^-}{dp^3} = \frac{p^\mu d\sigma_\mu}{exp(p^\nu u_\nu / T) \pm 1} \theta(-p^\nu d\sigma_\nu)$$

B) "by particles"



Negative contributions: energy dependence

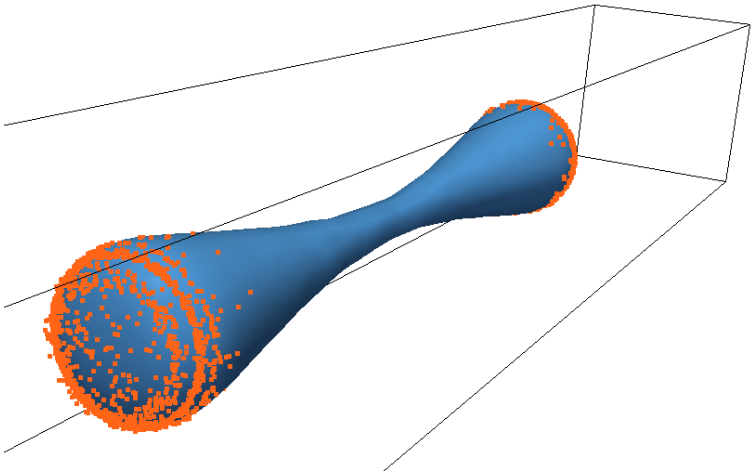


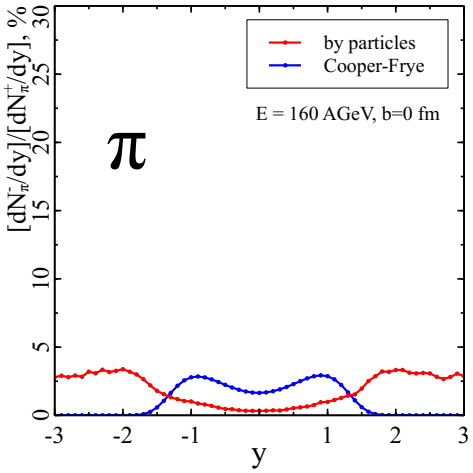
↓ collision energy - slower expansion - larger negative contributions

Non-equilibrium calculation gives much smaller values

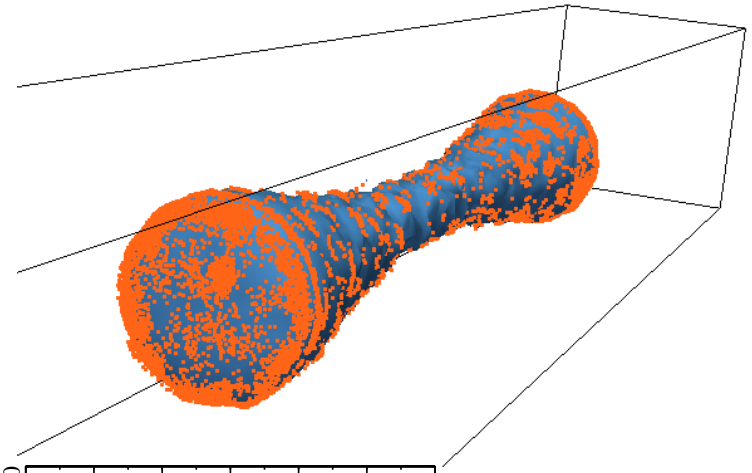
Negative contributions: surface lumpiness

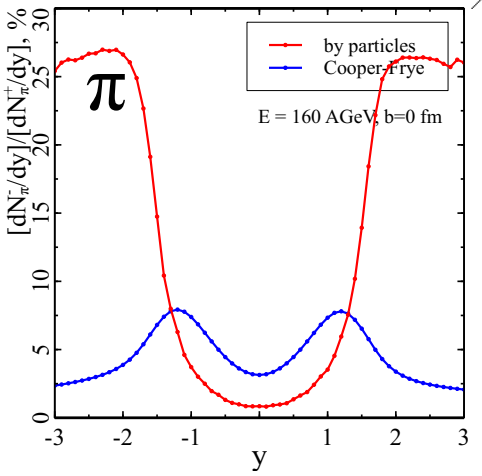
Smooth surface





Lumpy surface





Lumpier surface - larger negative contributions

Event-by-event conservation laws at particlization

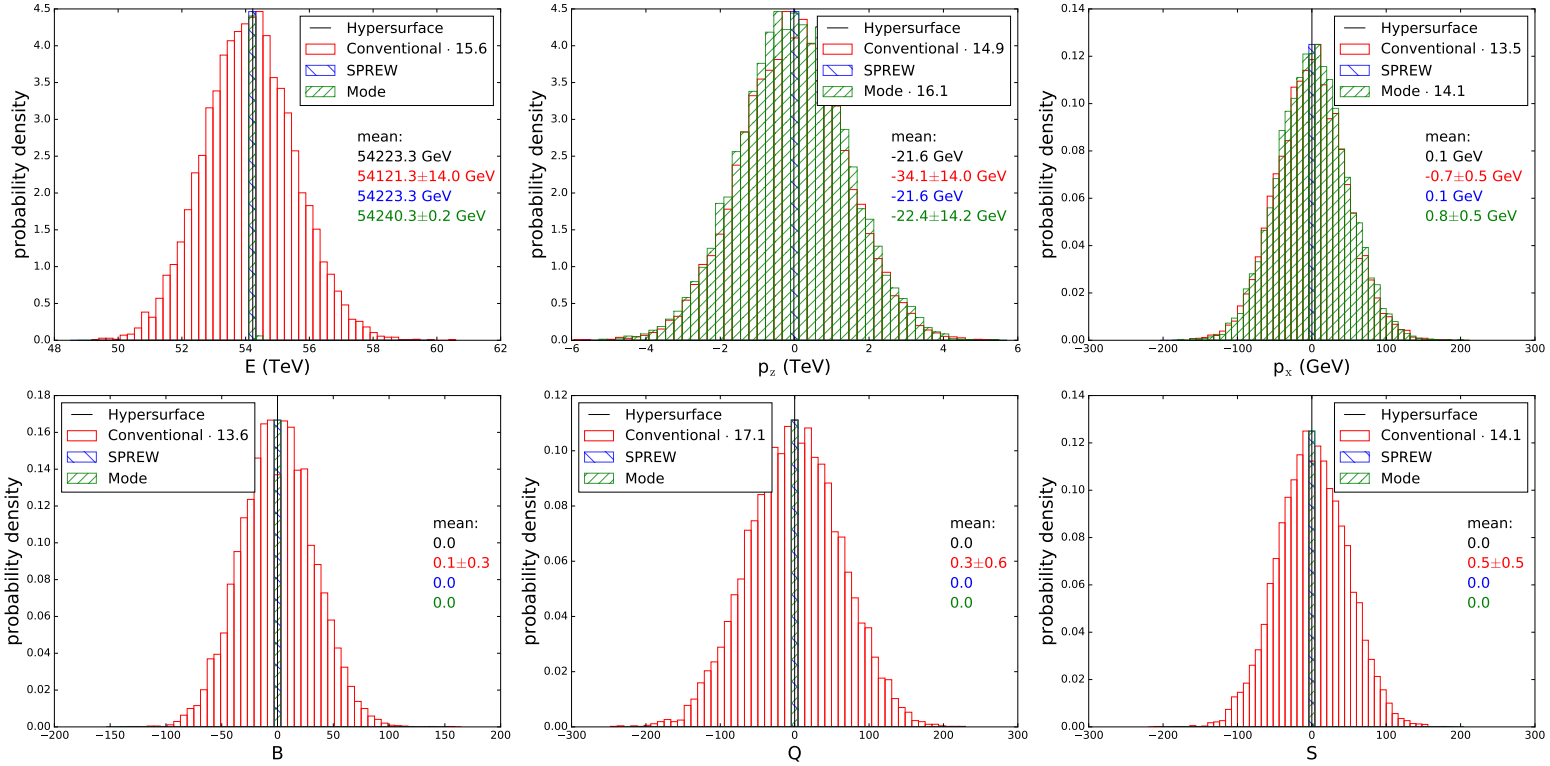
Are they important?

When can they be neglected?

Hadron multiplicity fluctuations

Cooper-Frye formula assumes independent particles

Particles are correlated due to conservation laws



comparing sampling algorithms with conservation laws

C. Schwarz, DO, L.-G. Pang, S. Ryu, H. Petersen, J Phys G 45 (2018), 015001

E-by-e conservation laws are necessary to study fluctuations

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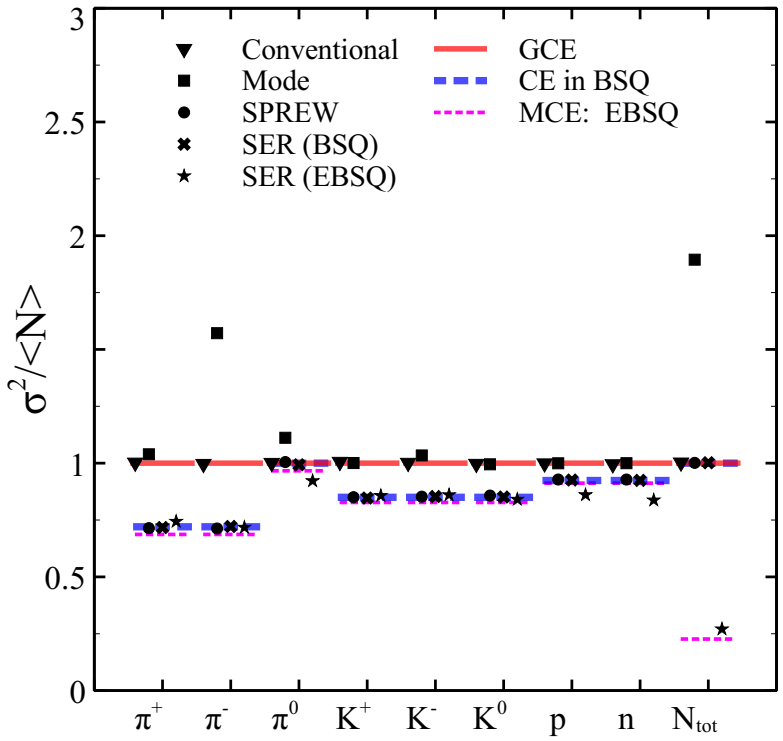


comparing sampling algorithms with conservation laws

C. Schwarz, DO, L.-G. Pang, S. Ryu, H. Petersen, J Phys G 45 (2018), 015001

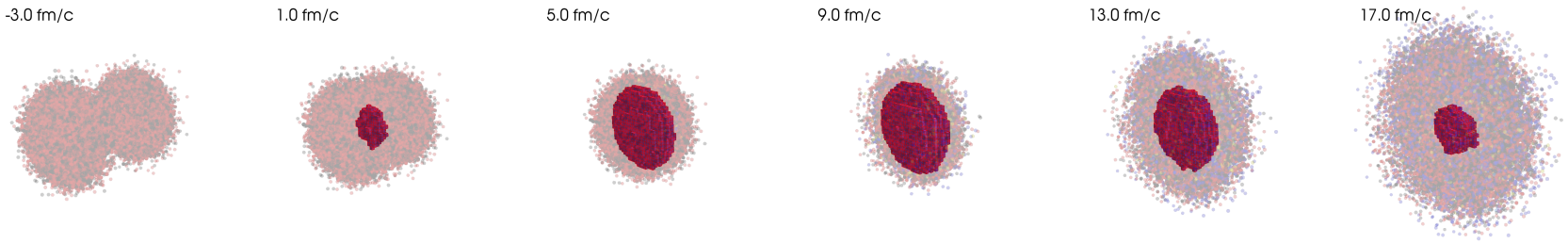
E-by-e conservation laws are necessary to study fluctuations

Alternative approach: thermal bubbles

- Applicability of Boltzmann equation breaks at large density
  $l_{mfp} \gg \lambda_{Compton}$ does not hold anymore

- Switching to hydrodynamics at high density?
  A possible and established way, however leads to Cooper-Frye negative contributions
  DO, Petersen, Phys.Rev. C91 (2015) no.2, 024906

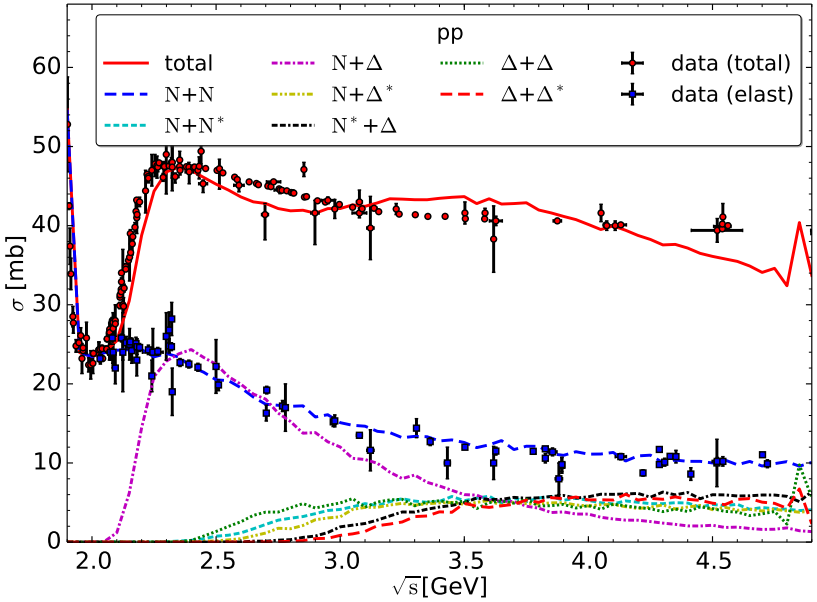
- Thermal bubbles:
  local forced thermalization in the regions of high density
  Effectively simulates intensive interactions and/or formation of quark-gluon plasma
  DO, Petersen, J.Phys. G44 (2017) no.3, 034001

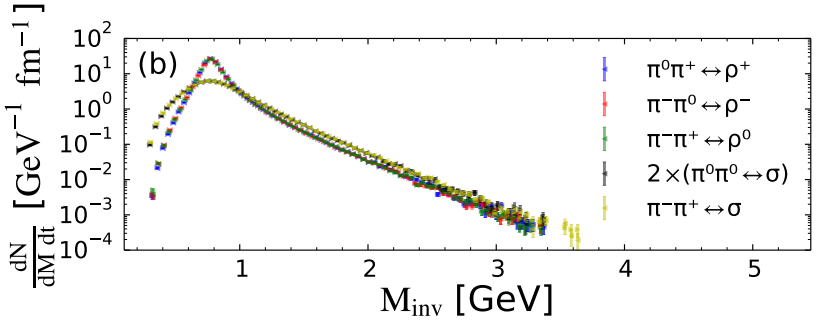


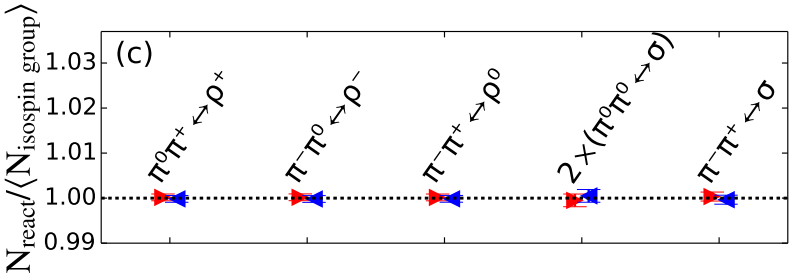
central Au+Au collision, $E_{\mathrm{kin}} = 2$ GeV, $N_{test} = 100$, purple region - thermalization region, energy density of switching 0.3 GeV/fm^3

SMASH transport model

- Used as a basis to implement thermal bubbles
- Most of hadrons from PDG with $M < 2.5$ GeV
- $2 \to 2$, $2 \to 1$ interactions and string fragmentations
- Resonances have relativistic Breit-Wigner spectral function with $\Gamma(m)$
- Detailed balance strictly fulfilled
- Used in the cascade mode in this study

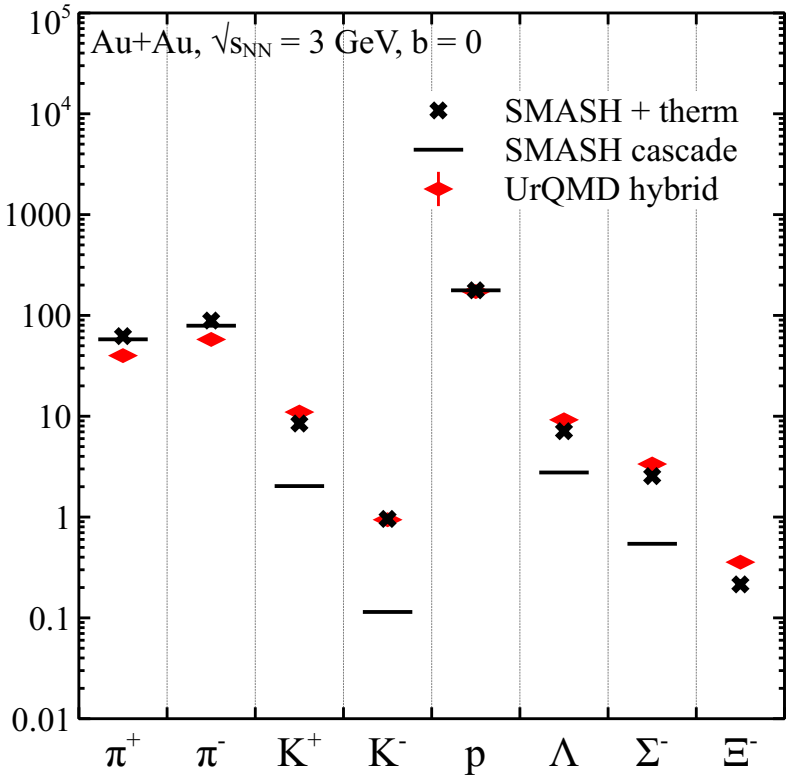






Effects of forced thermalization: multiplicities

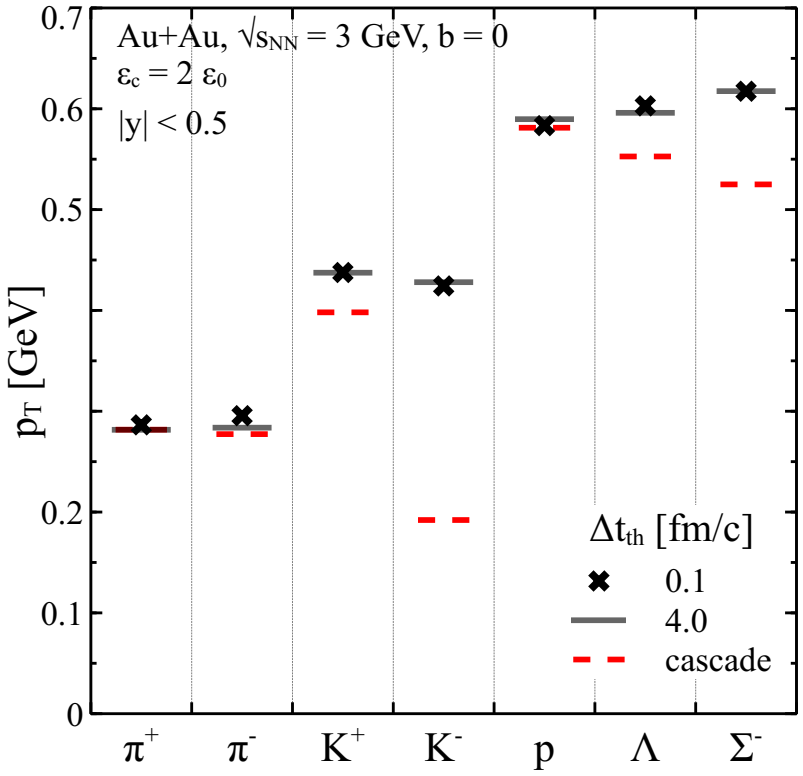
Au+Au central collision, $\sqrt{s} = 3$ GeV



Thermalization leads to strangeness enhancement - similarly to UrQMD hybrid

Effects of forced thermalization: transverse momenta

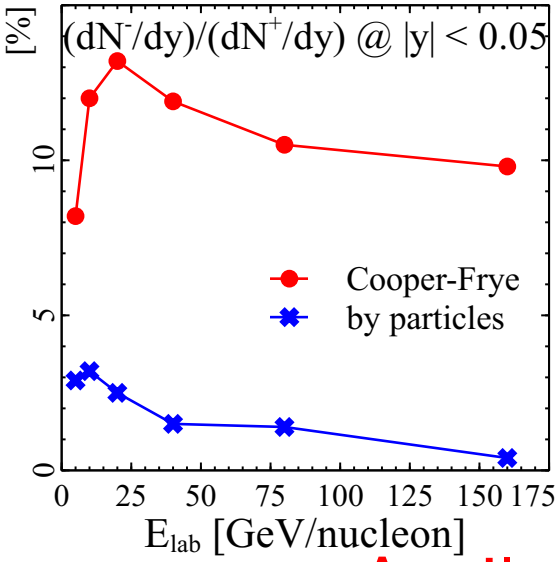
Au+Au central collision, $\sqrt{s} = 3$ GeV

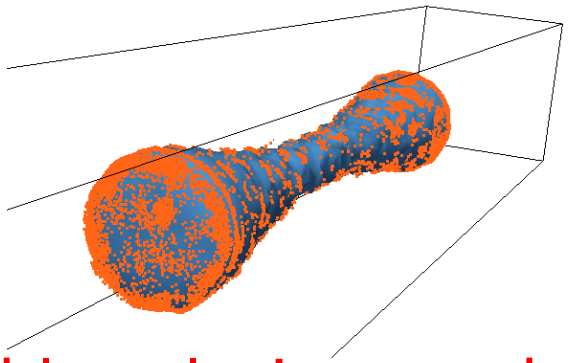


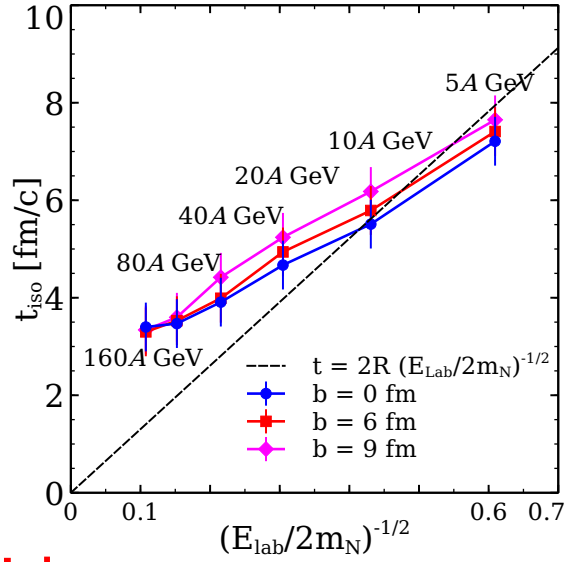
Thermalization leads to pressure isotropization and transversve push

Summary

Conventional hybrid approaches meet challenges at intermediate energies





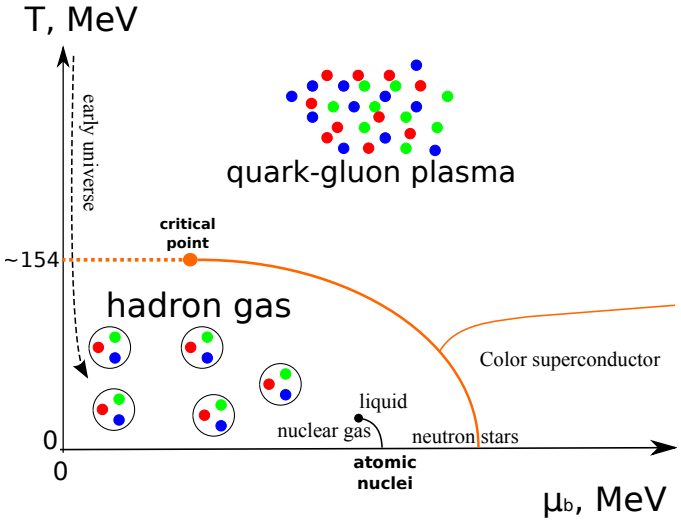


Available solutions and ideas:

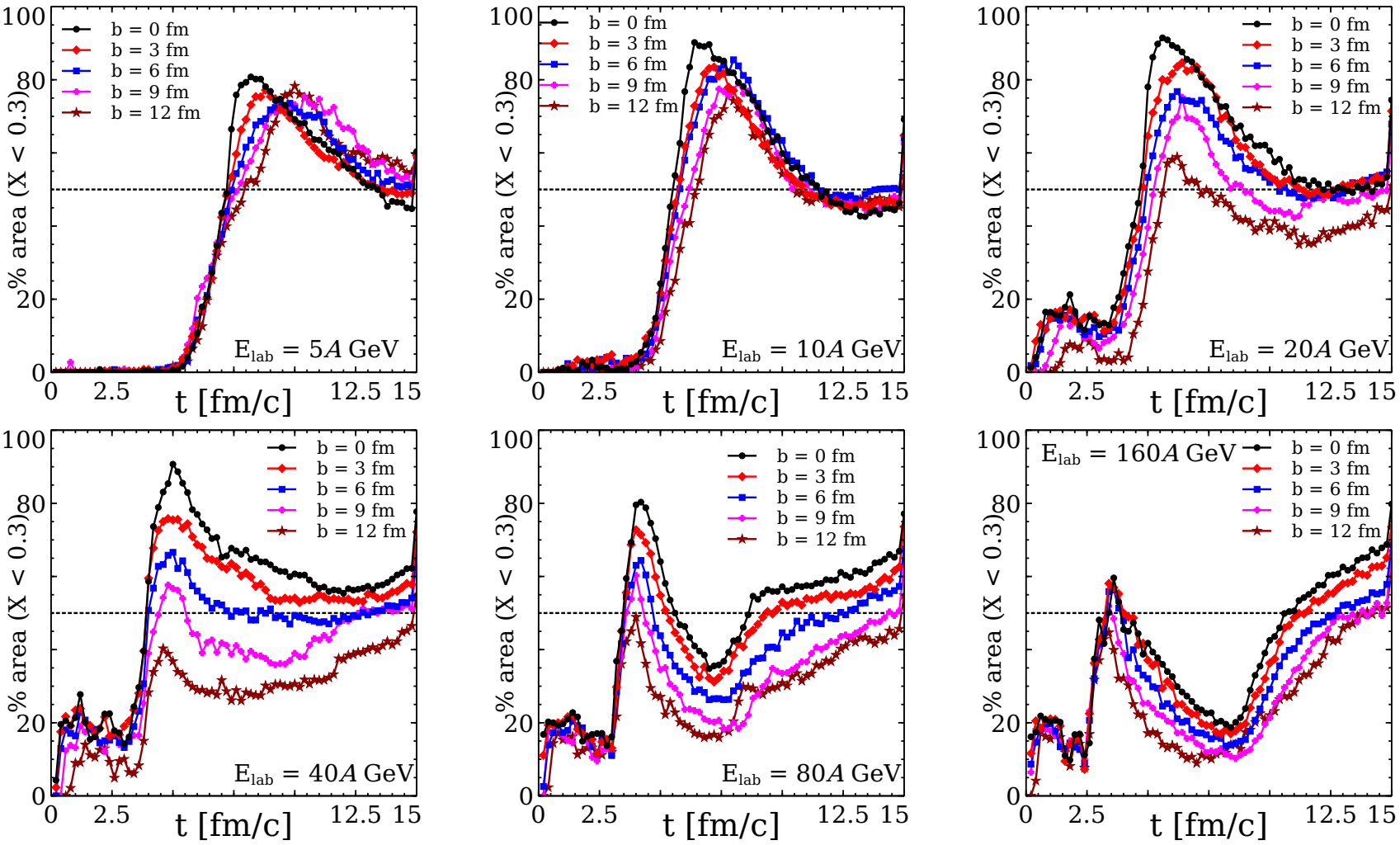
- Core-corona separation
- Initial state via source terms
- Thermal bubbles model
- Pure transport with tunable potentials
- Event-by-event conservation laws

Phase transition and cross-over

- Crossover at $\mu_b/T < 2$, no phase transition
  Borsanyi et al, Phys. Lett. B, 2014; Bazavov et al, PRD, 2014; Bazavov et al, PRD, 2017
- Indications of 1st order phase transition at $\mu_b > 0$
  Stephanov, Prog. Theor. Phys. Suppl., 2004 - overview
  - massless 2-flavour QCD
    Berges, Rajagopal, Nucl.Phys. B, 1999; Halasz et al, PRD, 1998
  - 2-flavour linear sigma model and NJL model
    Scavenius, Moscy, Mishustin, Rischke, PRC, 2001
  - model based on the statistical bootstrap
    Antoniou, Kapoyannis, PLB, 2003



Pressure anisotropy X versus collision energy



Define "isotropization time" t_{iso}: more than 50% of reaction plane area has $X < 0.3$.

SMASH transport model J. Weil et al., Phys.Rev. C94 (2016) no.5, 054905

- Monte-Carlo solver of relativistic Boltzmann equation

  BUU type approach, testparticles ansatz: $N \to N \cdot N_{test}$, $\sigma \to \sigma / N_{test}$

- Degrees of freedom
  - most of established hadrons from PDG up to mass 3 GeV
  - mesons: 44 non-strange mesons, 12 strange mesons
  - baryons: 17 N, 8 Δ, 14 Λ, 10 Σ, 6 Ξ, 2 Ω
  - strings via Pythia 8

- Interactions: $2 \leftrightarrow 2$ and $2 \to 1$ collisions, decays, potentials

- Initial conditions:
  - "collider" - elementary or AA reactions, $E_{beam} \gtrsim 0.5\ A$ GeV
  - "box" - infinite matter simulations

    detailed balance tests, computing transport coefficients, thermodynamics of hadron gas
  - "sphere" - expanding system

    comparison to analytical solution of Boltzmann equation,
    Tindall et al., Phys.Lett. B770 (2017) 532-538
  - "list" - hadronic afterburner after hydrodynamics

Interactions in SMASH

- Resonance formation and decay

  Ex. $\pi\pi \to \rho \to \pi\pi$, quasielastic scattering
  $\pi\pi \to f_2 \to \rho\rho \to \pi\pi\pi\pi$

- (In-)elastic $2 \to 2$ scattering

  Ex. $NN \to NN$, $NN \to N\Delta$, $NN \to \Delta\Delta$, $KN \to \Lambda\pi$
  parametrized cross-sections $\sigma(\sqrt{s}, t)$ or
  isospin-dependent matrix elements $|M|^2(\sqrt{s}, I)$

- String formation/fragmentation

  Via Pythia 8

- Potentials

  only change equations of motion

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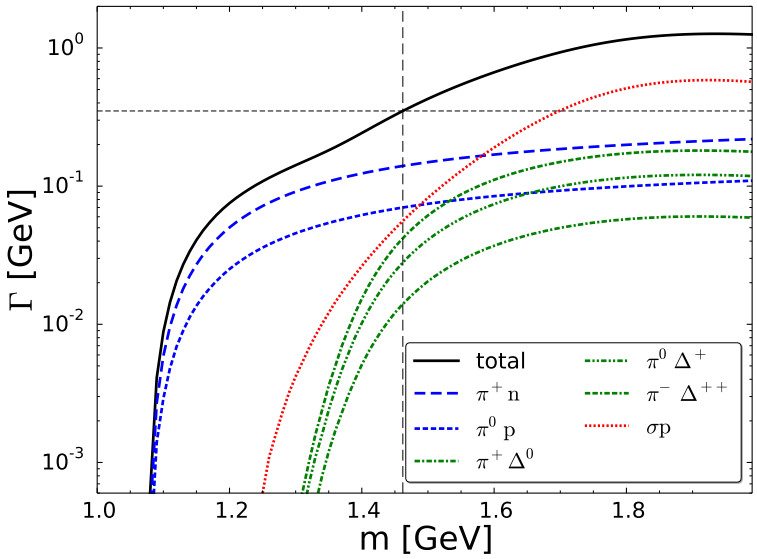
$N(1440)^+$



For every resonance:

- Breit-Wigner spectral function $\mathcal{A}(m) = \frac{2\mathcal{N}}{\pi} \frac{m^2\Gamma(m)}{(m^2 - M_0^2)^2 + m^2\Gamma(m)^2}$
- Mass dependent partial widths $\Gamma_i(m)$

  Manley formalism for off-shell width Manley and Saleski, Phys. Rev. D 45, 4002 (1992)
  Total width $\Gamma(m) = \sum_i \Gamma_i(m)$

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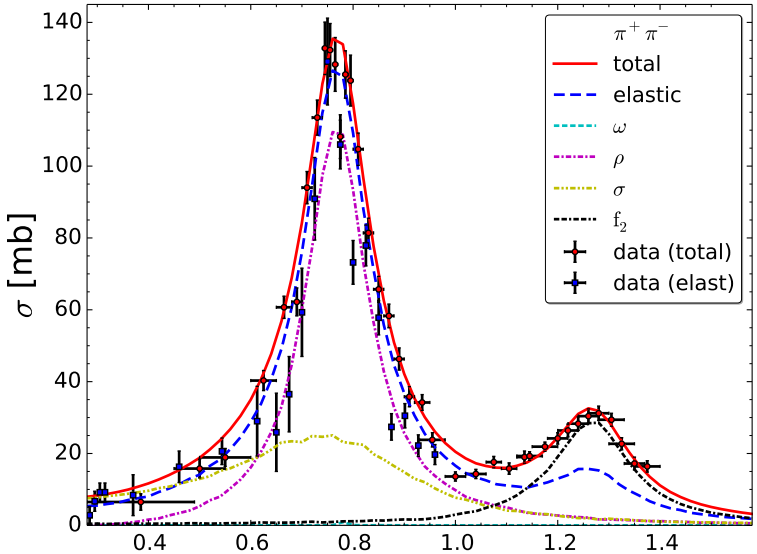

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Interactions in SMASH

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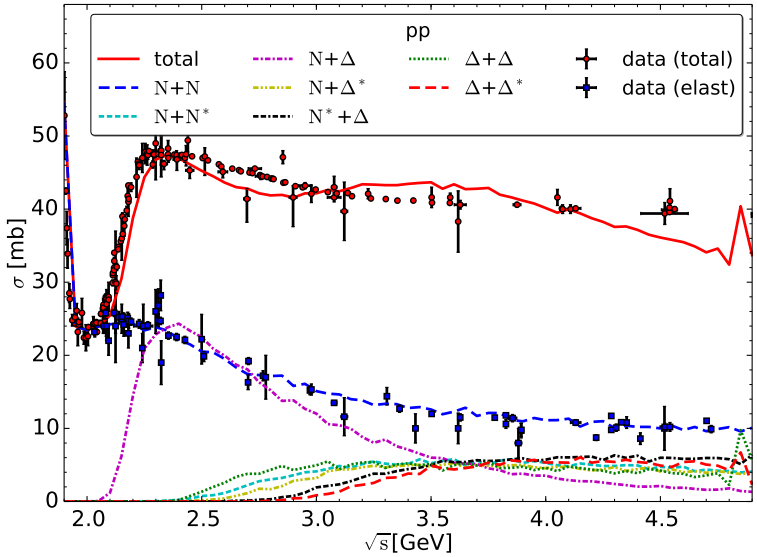

- Potentials

  only change equations of motion

  - $NN \to NN^*$, $NN \to N\Delta^*$, $NN \to \Delta\Delta$, $NN \to \Delta N^*$, $NN \to \Delta\Delta^*$

    angular dependencies of $NN \to XX$ cross-sections implemented

  - Strangeness exchange $KN \to K\Delta$, $KN \to \Lambda\pi$, $KN \to \Sigma\pi$

Interactions in SMASH

- Resonance formation and decay

  Ex. $\pi\pi \to \rho \to \pi\pi$, quasielastic scattering
  $\pi\pi \to f_2 \to \rho\rho \to \pi\pi\pi\pi$

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- String formation/fragmentation

  Via Pythia 8

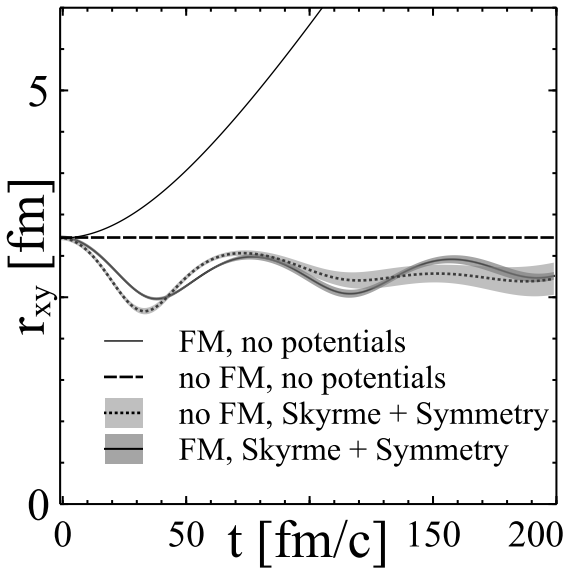
- Potentials

  only change equations of motion

  - Skyrme and symmetry potential
  - $U = a(\rho/\rho_0) + b(\rho/\rho_0)^\tau \pm 2S_{\text{pot}}\frac{\rho_{I3}}{\rho_0}$

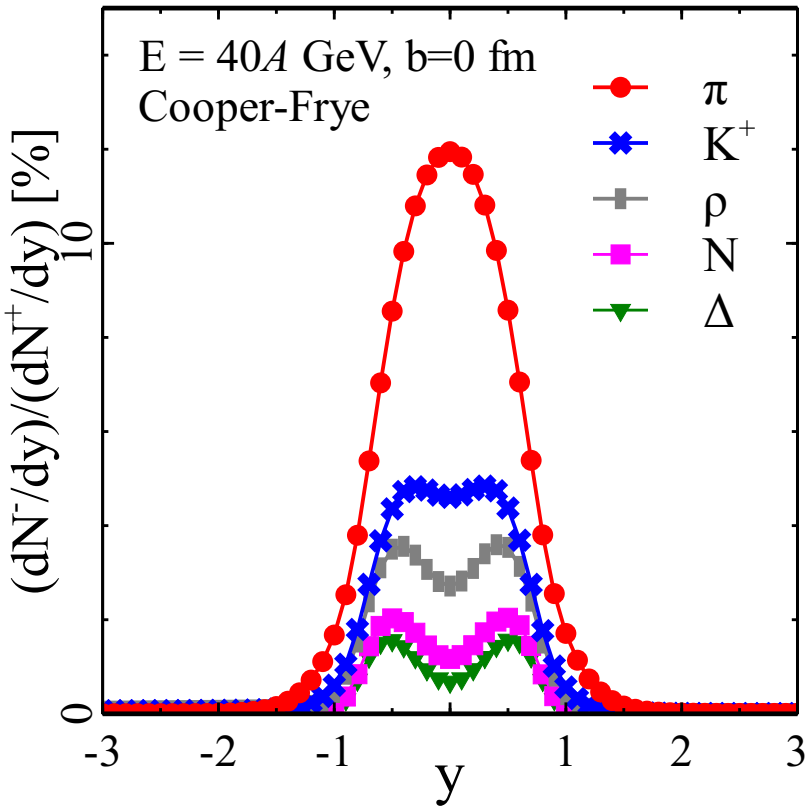
    ρ - Eckart rest frame baryon density
    ρ_{I3} - Eckart rest frame density of I_3/I
    $a = -209.2$ MeV, $b = 156.4$ MeV, $\tau = 1.35$, $S_{\text{pot}} = 18$ MeV
    corresponds to incompressibility $K = 240$ MeV
    assures stability of a nucleus with Fermi motion

Transverse radius of Cu nucleus.



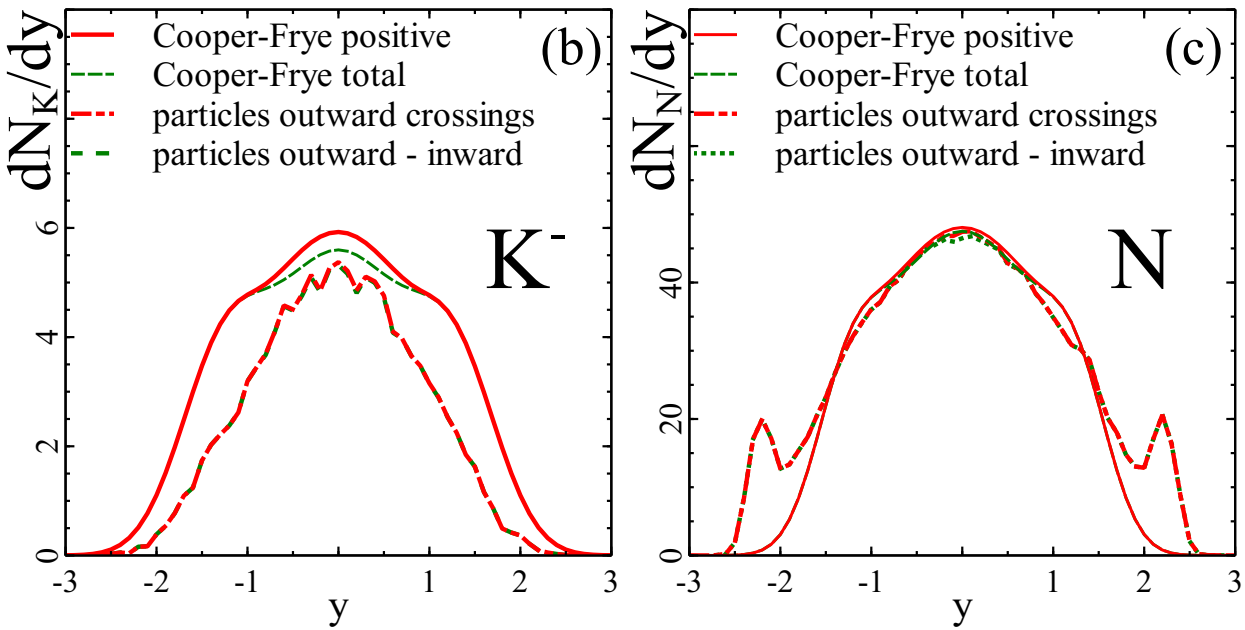
Negative contributions: particle mass dependence

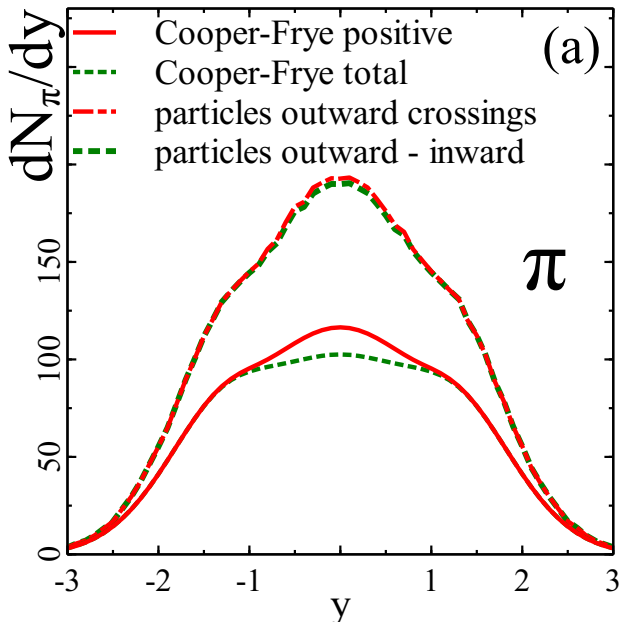
E = 40 AGeV, b = 0, $\epsilon_0 = 0.3$ GeV/fm^3, dN/dy distributions



Smaller mass - larger negative contribution

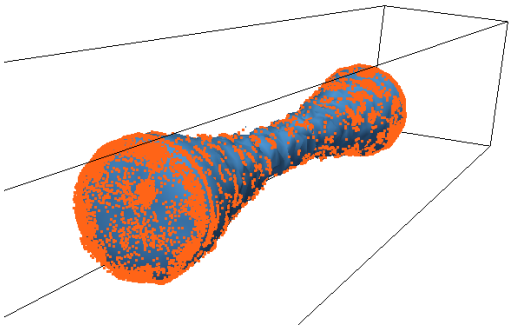
Cooper-Frye positive contributions

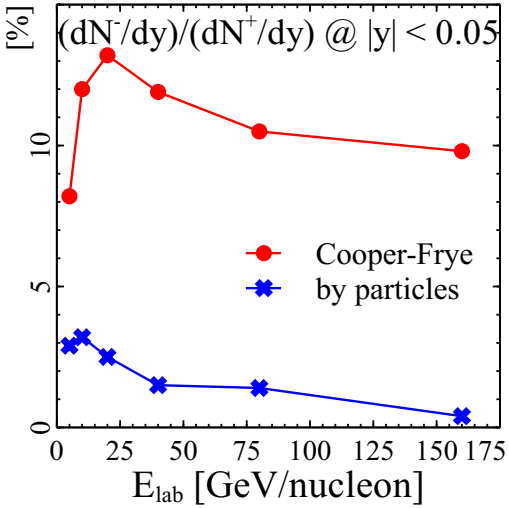


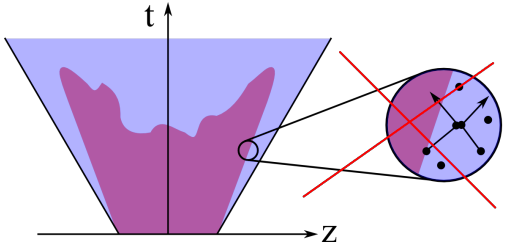


Summary of "Cooper-Frye" part

- Hybrid approaches adopt approximations:
  - aposteriori determination of particlization surface
  - particles decouple from hydrodynamics once and cannot get back into it
- These approximations become inadequate for
  - low collision energies
  - large fluctuations (event-by-event/fluctuating hydro)







Smearing kernel DO, HP, Phys.Rev. C93 (2016) no.3, 034905

The energy-momentum tensor $T^{\mu\nu}$ is constructed as

$$T^{\mu\nu}(\vec{r}) = \frac{1}{N_{ev}} \sum_{events} \sum_{i} \frac{p_i^\mu p_i^\mu}{p_i^0} K(\vec{r} - \vec{r_i}, p_i) \tag{1}$$

Smearing kernel $K(r)$: $K(r)d^3r$ should be Lorentz scalar

$$\Delta x^i = \Lambda^i_j \Delta x'^j \tag{2}$$

$$\Lambda^i_j = \delta^i_j + (u^i u_j)/(1+\gamma) \tag{3}$$

$$(\Delta x^i)^2 = \Lambda^i_j \Delta x'^j \Lambda^i_k \Delta x'^k \tag{4}$$

$$\Lambda^i_j \Lambda^i_k = \delta_{jk} + u_j u_k \tag{5}$$

$$(\Delta \vec{x})^2 = (\Delta \vec{x'})^2 + (\Delta \vec{x'} \cdot \vec{u})^2 \tag{6}$$

$$K(\vec{r}) = \gamma (2\pi\sigma^2)^{-3/2} exp\left(-\frac{\vec{r}^2 + (\vec{r}\cdot\vec{u})^2}{2\sigma^2}\right) \tag{7}$$

Normalization using $\int \left(\prod_{i=1}^n dx_i\right) e^{-x_i A^{ij} x_j} = \pi^{n/2} \left(detA\right)^{-1/2}$

Fluidization: introduction

- Energy-momentum tensor from particles:
  - $T^{\mu\nu}(r) = \int \frac{p^\mu p^\nu}{p^0} f(r,p) \frac{d^3p}{(2\pi\hbar)^3}$
  - Molecular dynamics: $f(\vec{r},\vec{p}) = \sum_i g_i \delta(\vec{r}-\vec{r}_i)\delta(\vec{p}-\vec{p}_i)$
    $T^{\mu\nu}(r) = \frac{1}{V}\sum_{\in V} \frac{p^\mu p^\nu}{p^0}, \quad V \to 0$
  - Gaussian smearing: for particle in its rest-frame
    $\delta(\vec{r}) \to N\, exp\left(-r^2/\sigma^2\right)$
- $T^{\mu\nu}$ from particles does not necessarily have required form for fluid dynamics
- In the Landau rest frame (by definition $T^{0i} = 0$):

$T_L^{\mu\nu}$(ideal hydro)

$$\begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$T_L^{\mu\nu}$(from particles)

$$\begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p+\pi^{xx} & \pi^{xy} & \pi^{xz} \\ 0 & \pi^{xy} & p+\pi^{yy} & \pi^{yz} \\ 0 & \pi^{xz} & \pi^{yz} & p+\pi^{zz} \end{pmatrix}$$

- Applicability of (ideal or viscous) hydrodynamics requires $\pi^{ij} \ll p$

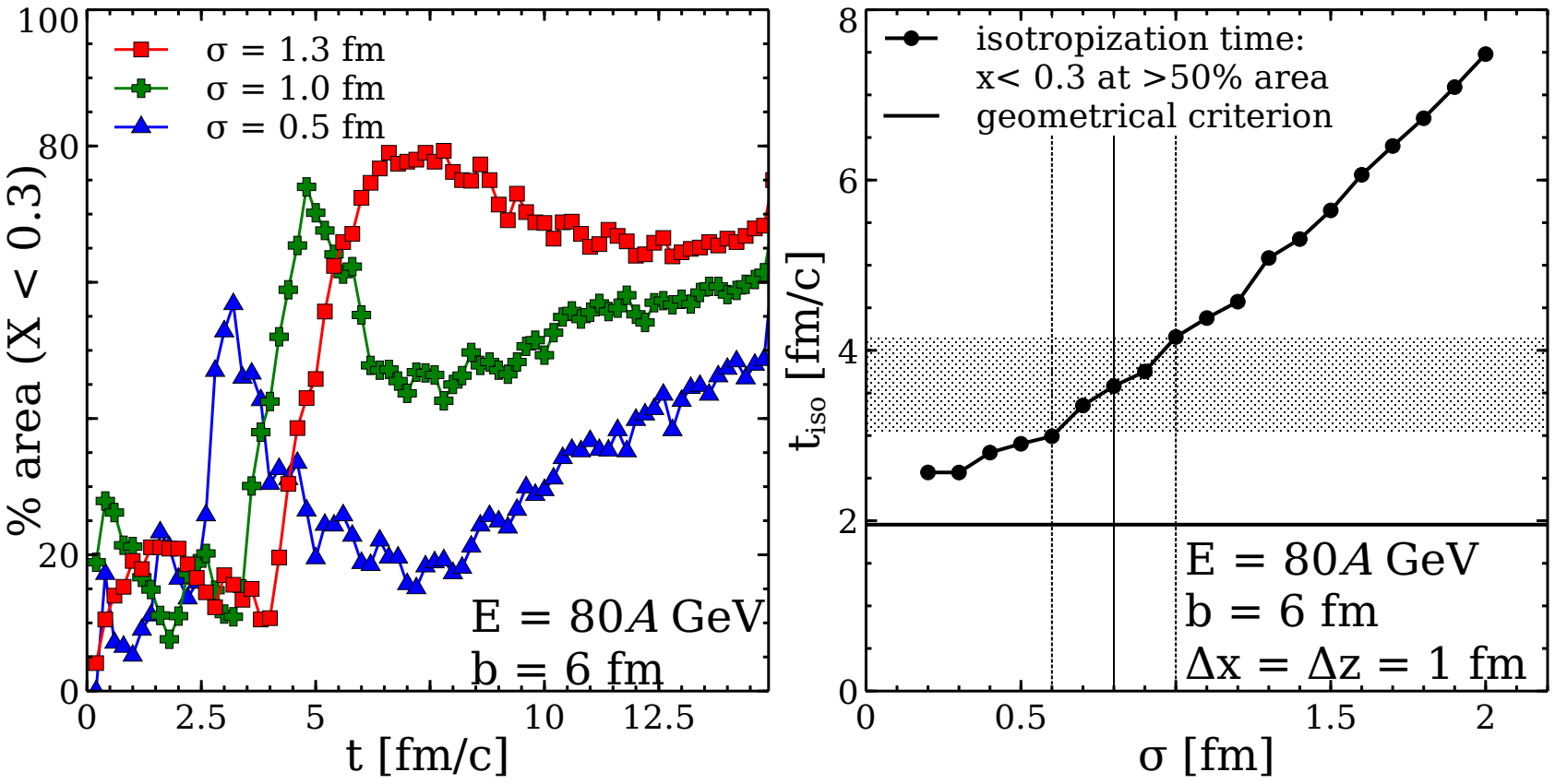
Is it possible to separate p from $T^{\mu\nu}$ unambiguously?

Fluidization in UrQMD hybrid

- Performed at fixed time t_{start} in center of mass frame
- $t_{start} = \frac{2R}{\beta\gamma} = 2R\sqrt{\frac{2m_N}{E_{lab}^{kin}}}$, estimate for time when nuclei pass through each other
- Gaussian smearing: $\sigma = 1$ fm
- Ideal hydrodynamics:
  - Calculate in computational frame: $T^{\mu 0}_{comp}$, j^0_{comp}
  - Assume that
    $T^{\mu\nu} = T^{\mu\nu}_{ideal} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$, $u^\mu = \gamma(1, \vec{v})$
    $$\begin{cases} \epsilon_{lab} & \equiv T^{00} = \gamma^2(\epsilon + p) - p \\ \vec{M} & \equiv T^{0i} = \gamma^2(\epsilon + p)\vec{v} \end{cases} \Rightarrow \begin{cases} p & = M/v - \epsilon_{lab} \\ \epsilon & = \epsilon_{lab} - Mv \end{cases}$$
  - Substitute to equation of state, $p = p(\epsilon, n)$, and obtain v:
    $M/v - \epsilon_{lab} = p(\epsilon_{lab} - Mv, j^0_{comp}/\gamma)$
- Only 4 components of $T^{\mu\nu}$ are used in the calculation

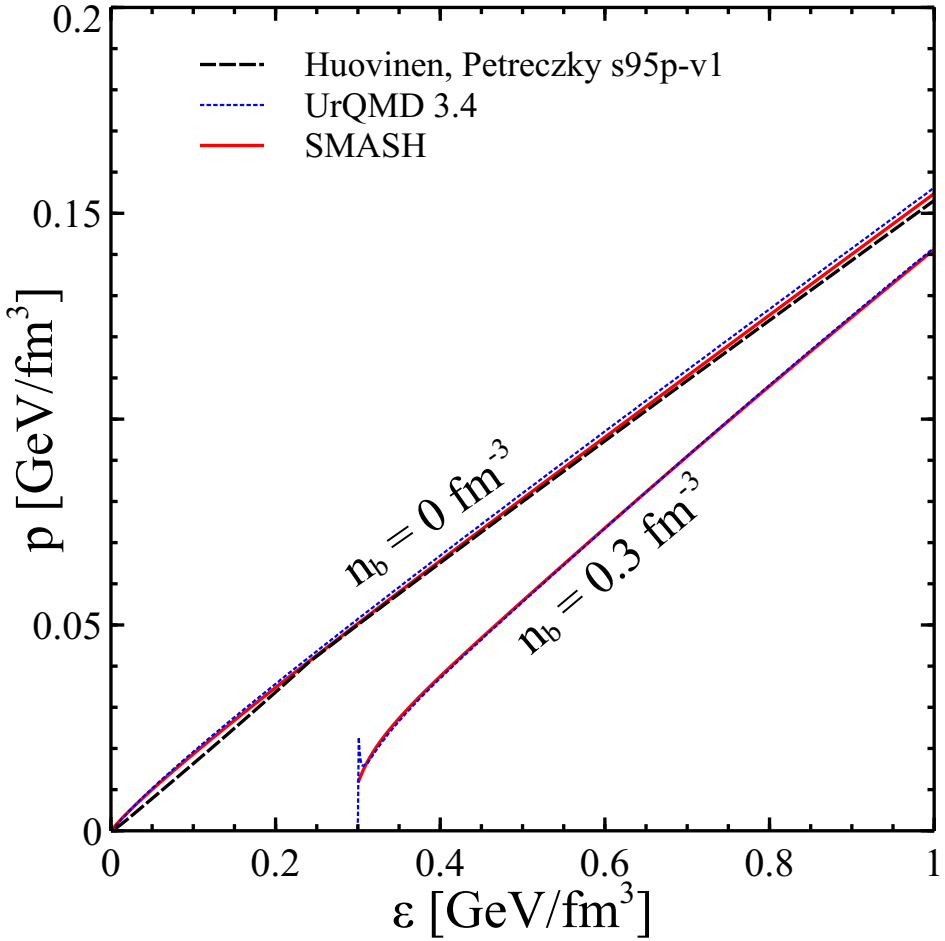
How large are the neglected components of $T^{\mu\nu}$?
Are they small enough for applicability of (ideal/viscous) hydro?

Sensitivity of pressure anisotropy to smearing σ



SMASH ideal hadron gas EoS

More hadron sorts - smaller pressure at given energy density



P. Huovinen, P. Petreczky, Nucl.Phys. A837 (2010) 26-53

UrQMD ≡ Hadron Gas EoS from UrQMD tables by J. Steinheimer

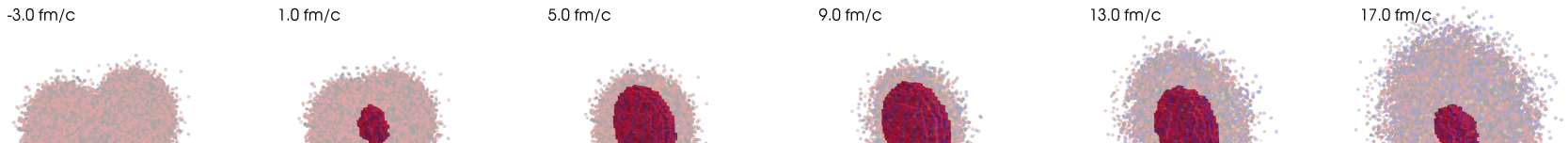
Forced grand-canonical thermalization

Every time interval Δt_{th}:

- Span a lattice and compute $T^{\mu\nu}$, j_B^μ, j_S^μ on it
- Find rest frame ϵ, n_b, n_S, T, μ from EoS in every cell
- Remove particles from $\epsilon > \epsilon_c$ region
- Sample new particles in $\epsilon > \epsilon_c$ region
  - Thermal distribution function
  - Conserving charges, energy and momentum globally
- Run usual transport model until next thermalization

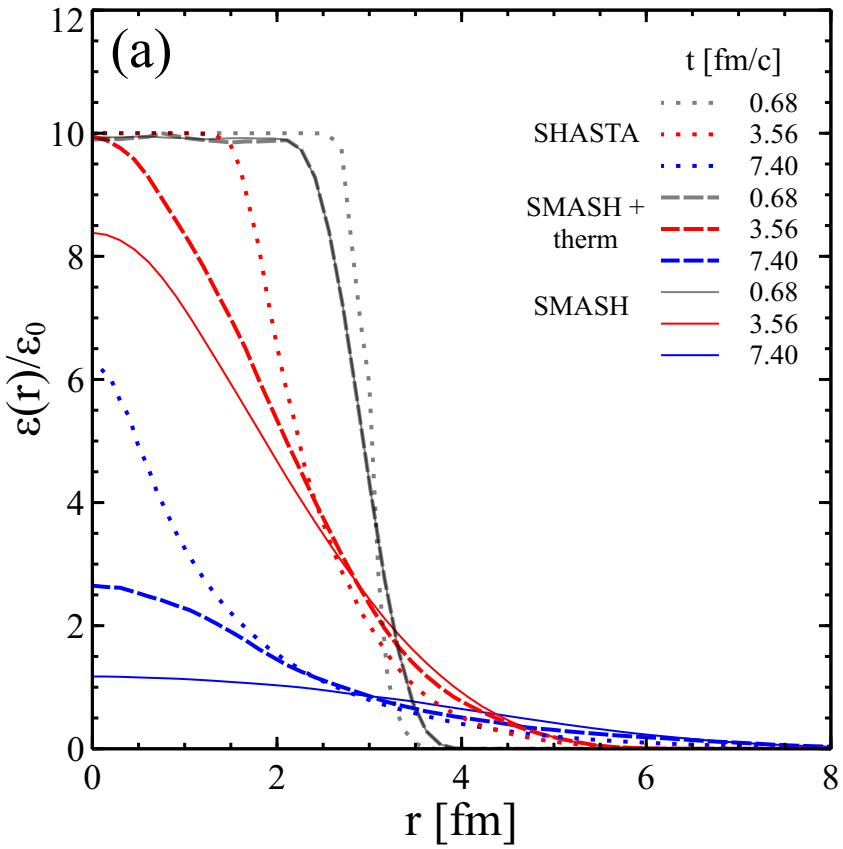
Ways to interpret the procedure:

- changing local distribution function to the thermal one
- effective treatment of multi-particle collisions
- "Zero-time hydro" = fluidization + immediate particlization



Forced thermalization: expanding sphere setup

Evolution of $R = 3$ fm sphere, initial $\epsilon = 10\epsilon_0$, $n_B = 0$.
Thermalization at $\epsilon > 2\epsilon_0$ every $\Delta t_{th} = 1$ fm.



SMASH + thermalization interpolates between transport and hydro

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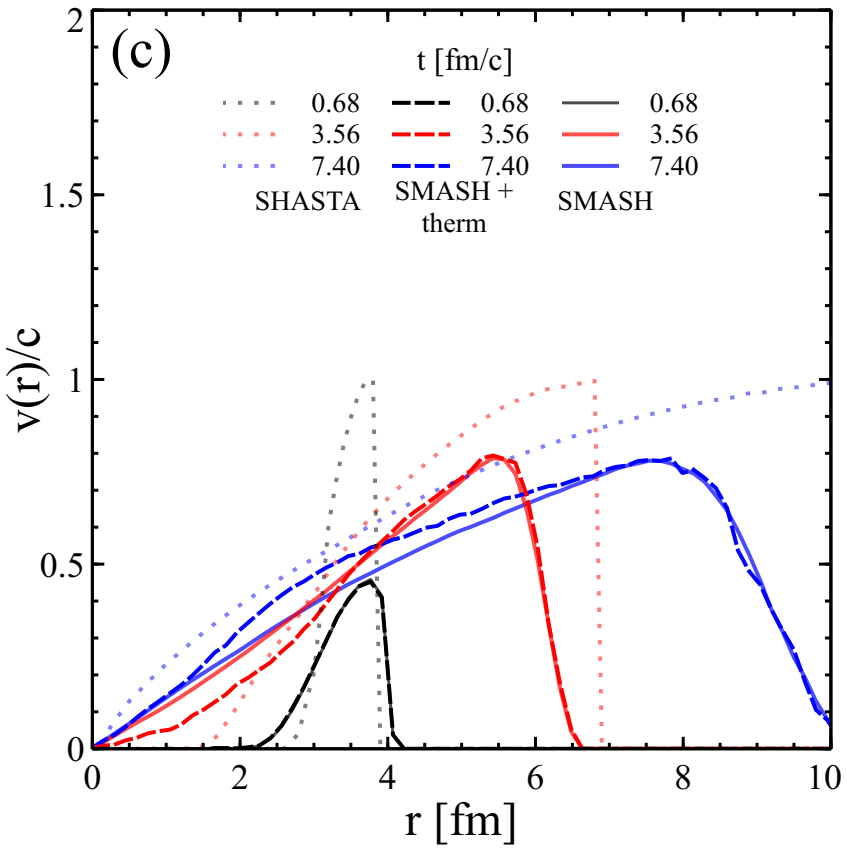
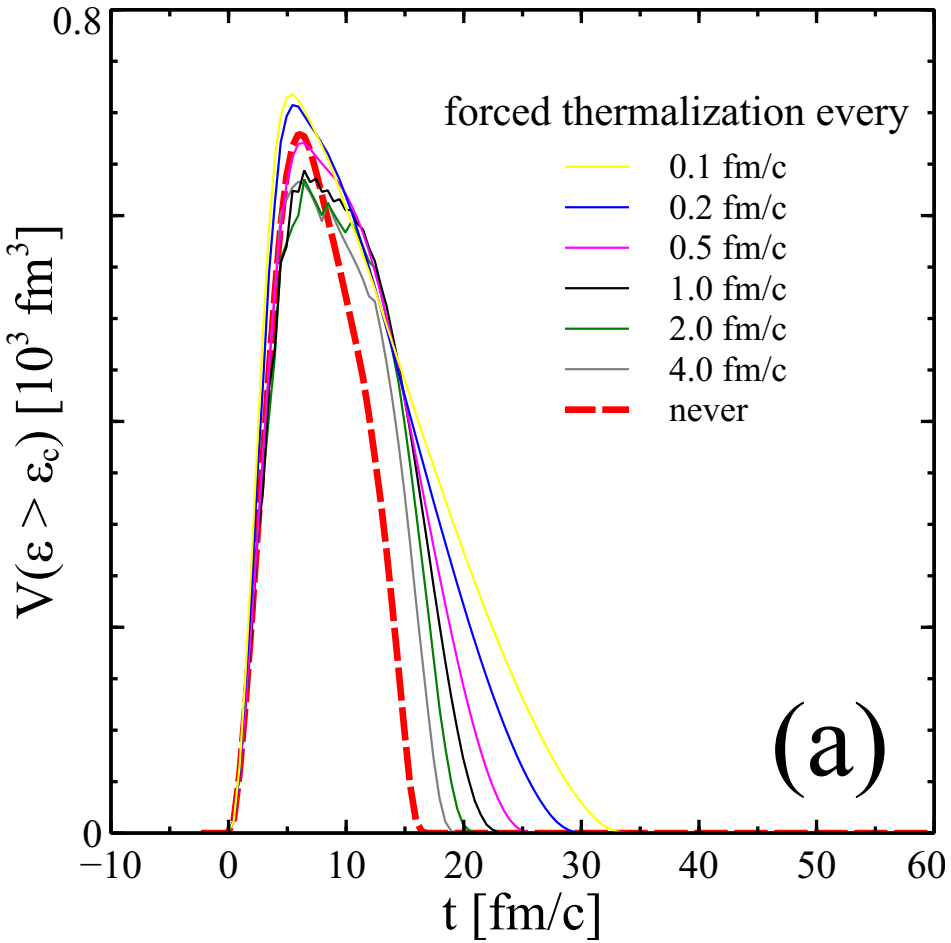


SMASH + thermalization interpolates between transport and hydro

Effects of forced thermalization

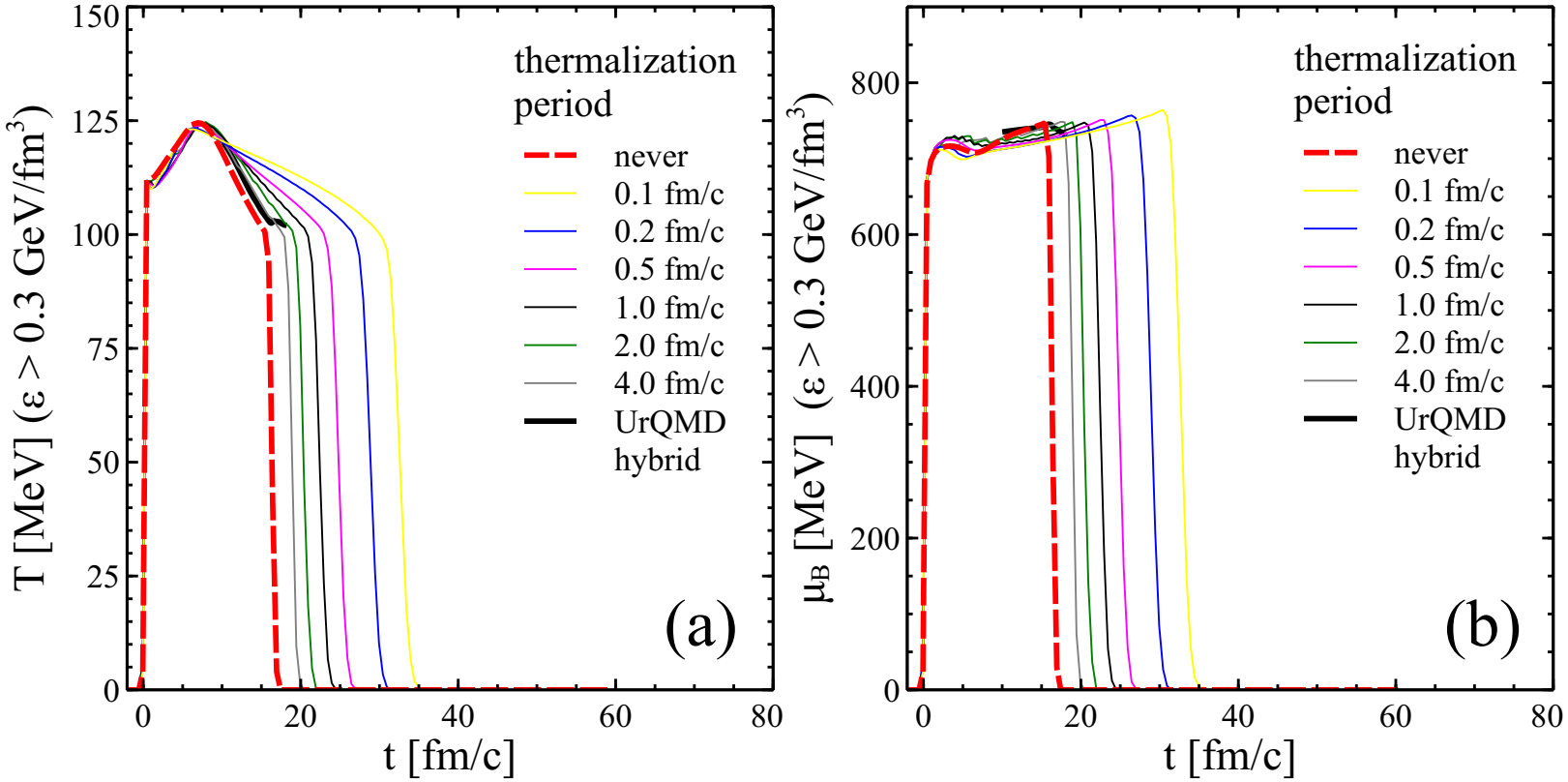
Au+Au central collision, $\sqrt{s} = 3$ GeV, $\epsilon_c = 2\epsilon_0$



High-density region exists longer

$\langle T \rangle$ and $\langle \mu_B \rangle$ in the thermalization region

Au+Au central collision, $\sqrt{s} = 3$ GeV

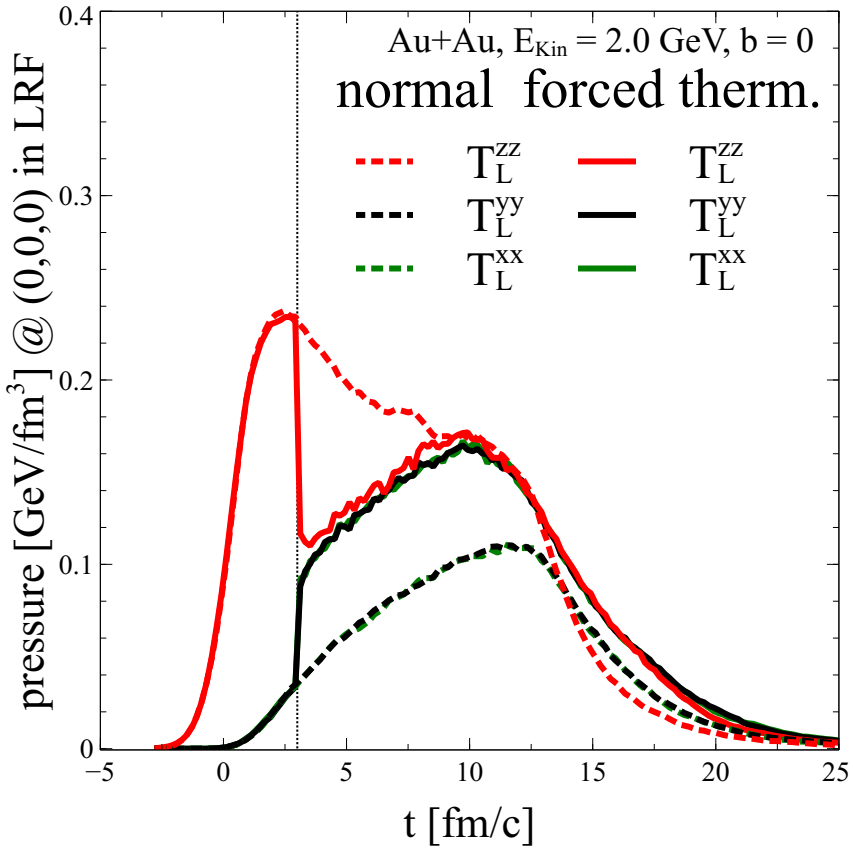


Reasonable behavior of T and μ_b

Effects of forced isotropization: pressure

Forced isotropization at $t > 3$ fm/c, where $\epsilon > 0.3$ GeV/fm^3.

Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$

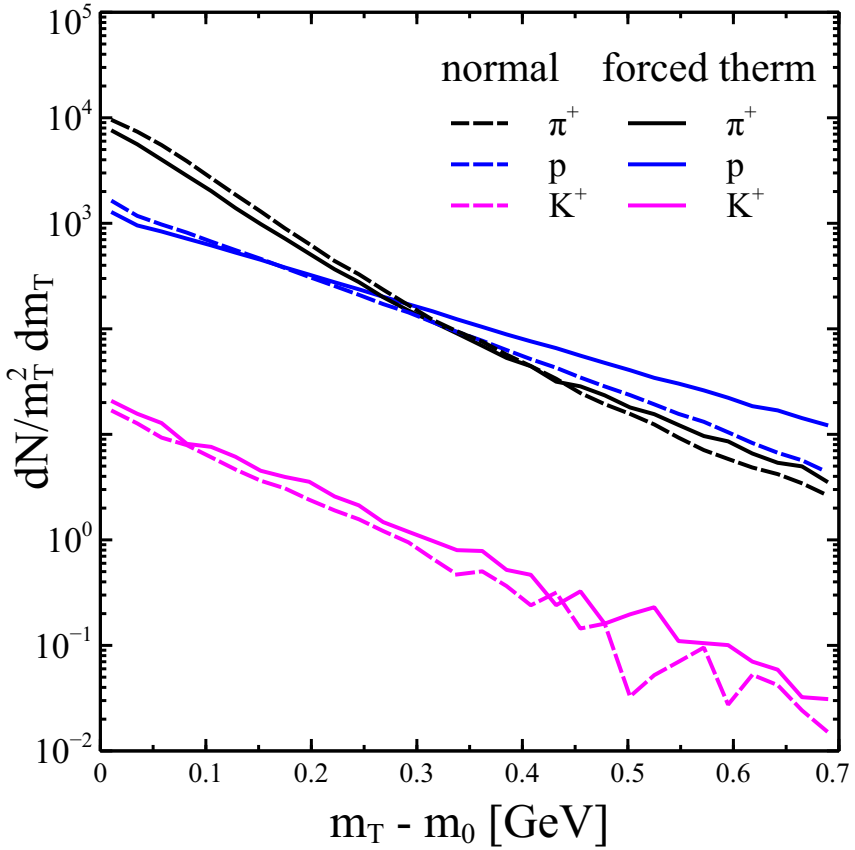


Pressures rapidly equalize

Effects of forced isotropization: m_T spectra

Forced isotropization at $t > 1$ fm/c, where $\epsilon > 0.3$ GeV/fm^3.

Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$



Particles move from low p_T to high p_T: "transverse push"

Effecs of forced isotropization: y spectra

Forced isotropization at $t > 1$ fm/c, where $\epsilon > 0.3$ GeV/fm^3.

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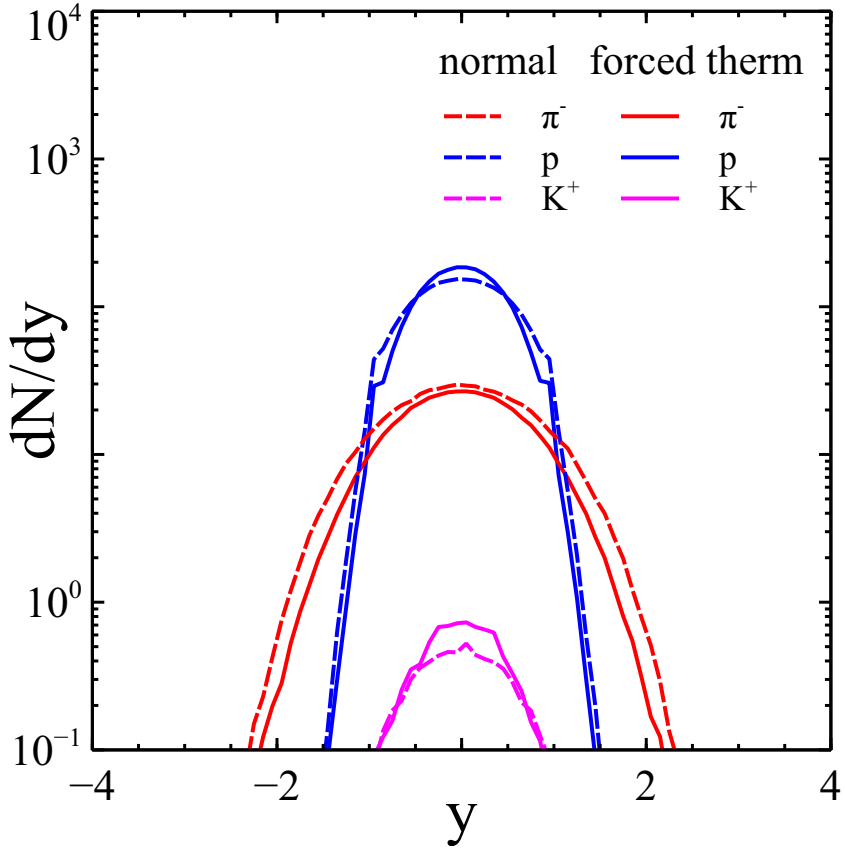
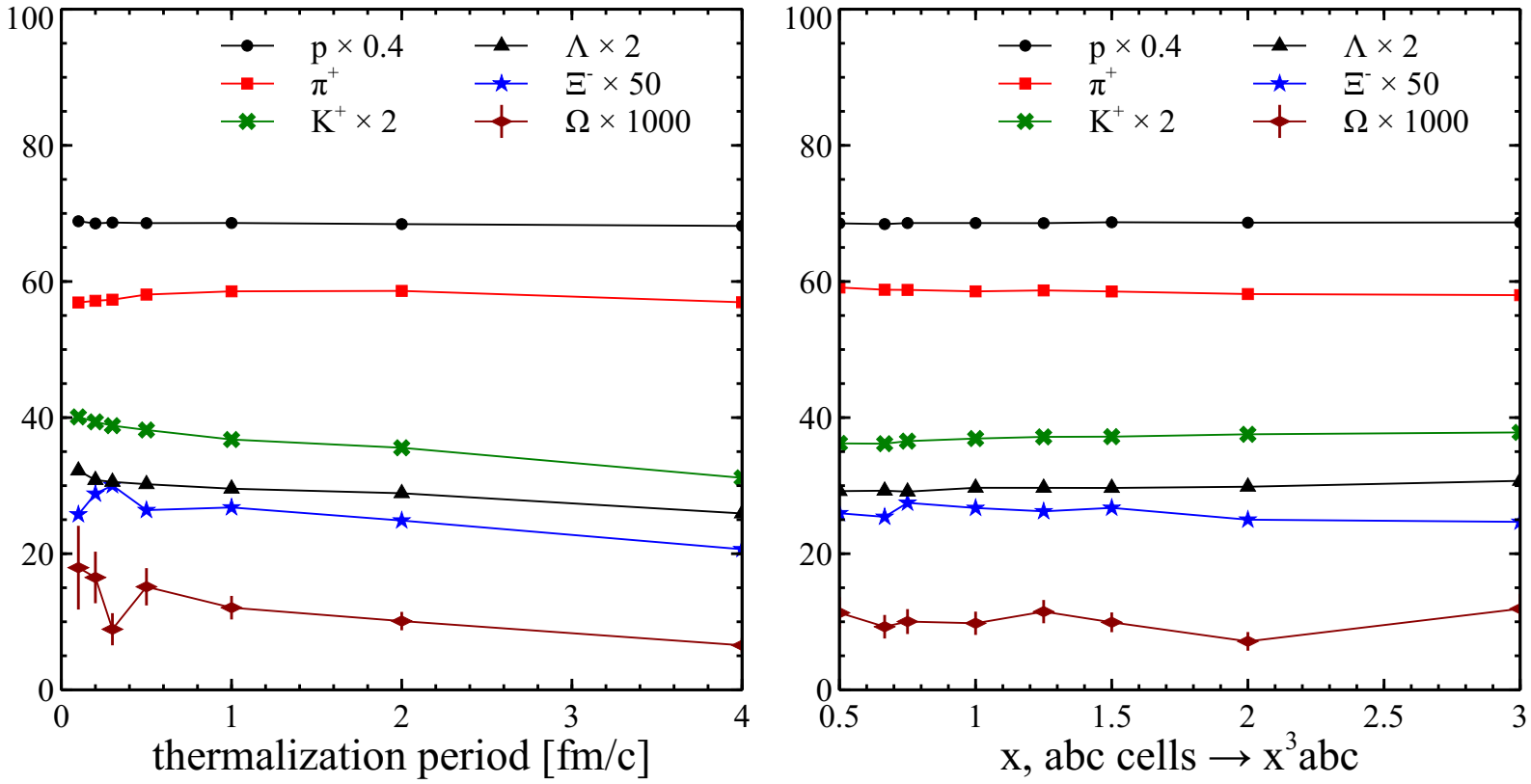


Nucleons move to midrapidity, less pions, more kaons

Effects of forced thermalization: multiplicities

Au+Au central collision, $\sqrt{s} = 3$ GeV, $[1.0 \times 1.0 \times 0.5]$ lattice spacing



Thermalization period and lattice spacing are not important for multiplicities