Talk at User's meeting, BNL, June 4 2019

Baryon clustering at the critical line

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based on two papers with Juan Torres-Rincon:

1. Baryon clustering at the critical line and near the hypothetical critical point in heavy-ion collisions arXiv:1805.04444

2. Baryon pre-clustering at freezeout stage of heavy-ion collisions and light ion production, in progress

At this meeting there is no need to do introduction to why do the Beam Energy Scan

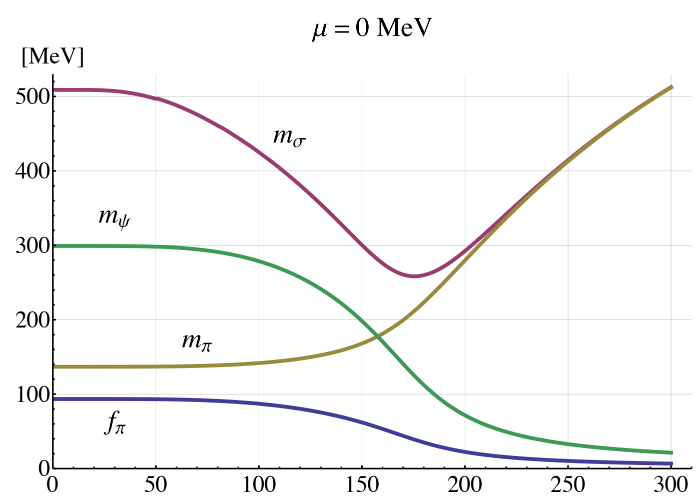
. M. A. Stephanov, K. Ra jagopal and E. V. Shuryak, Phys. Rev. Lett. 81, 4816 (1998) [hep-ph/9806219].

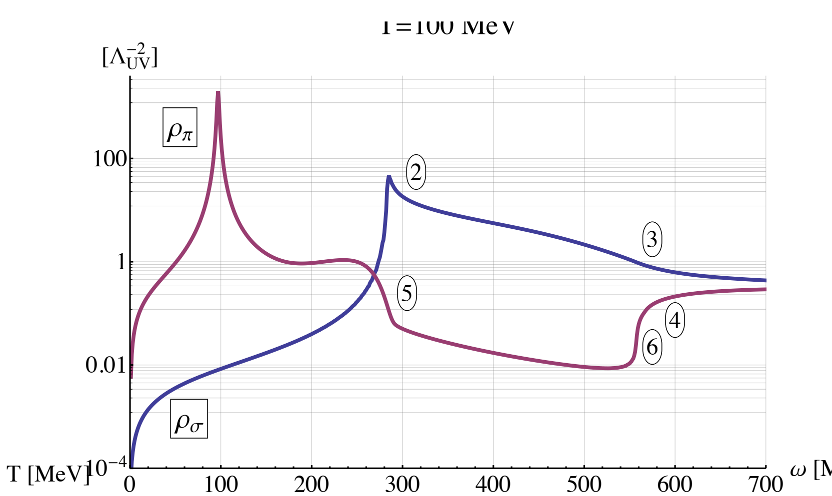
but perhaps some words about why nucleon clustering?

The nuclear forces are extremely well tuned: two subsequent cancellations
(i) sigma vs omega (attraction versus repulsion) and (ii) kinetic vs potential energies
result in small binding (0-16 MeV)
from light to heavy nuclei

multi-neutron systems are all unbound

With or without QCD critical point, we expect sigma meson spectral density strongly modified by the chiral transition





T=150 MeV

<u>Spectral Functions for the Quark-Meson Model Phase Diagram from the Functional Renormalization Group</u>,
Nils Strodthoff, Lorenz von Smekal, Jochen Wambach Phys.Rev. D89 (2014) no.3, 034010 <u>arXiv:1311.0630</u>

The setting: the Walecka model sigma and omega exchanges only (isospin-neutral)

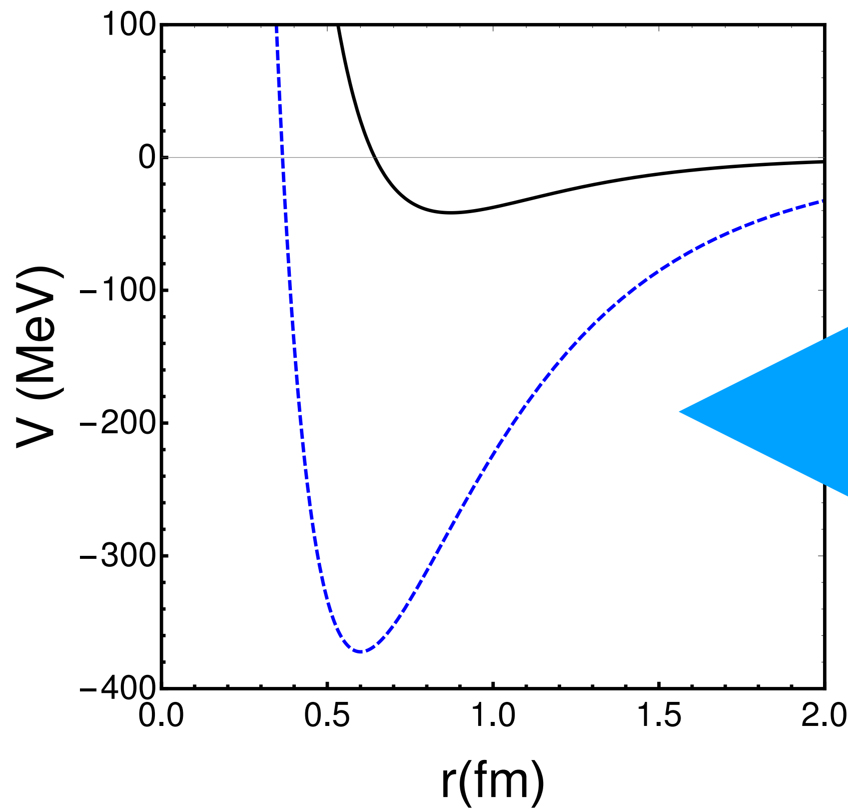
$$\tilde{V}(r) = -\frac{g_\sigma^2}{4\pi r} e^{-m_\sigma r} + \frac{g_\omega^2}{4\pi r} e^{-m_\omega r}$$

The coupling values selected by Serot and Walecka [4] are

$$g_\sigma^2 = 267.1 \left(\frac{m_\sigma^2}{m_N^2}\right), \quad g_\omega^2 = 195.9 \left(\frac{m_\omega^2}{m_N^2}\right)$$

$$\langle P \rangle = (\frac{n}{2})(-\frac{g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2})$$

we modified it a bit to correspond to other binary potentials



effective sigma mass is 285 MeV

FIG. 1: The effective nuclear potentials (MeV), in the vacuum (black solid line) and a modified one at the freezeout conditions (blue dashed line).

The setting: the Walecka model sigma and omega exchanges only (isospin-neutral)

But, at the freezeout of heavy ion collisions, we are
close to critical Tc,
and this delicate balance can
be strongly modified:
a decrease of m(sigma) is expected

$$\tilde{V}(r) = -\frac{g_\sigma^2}{4\pi r}e^{-m_\sigma r} + \frac{g_\omega^2}{4\pi r}e^{-m_\omega r}$$

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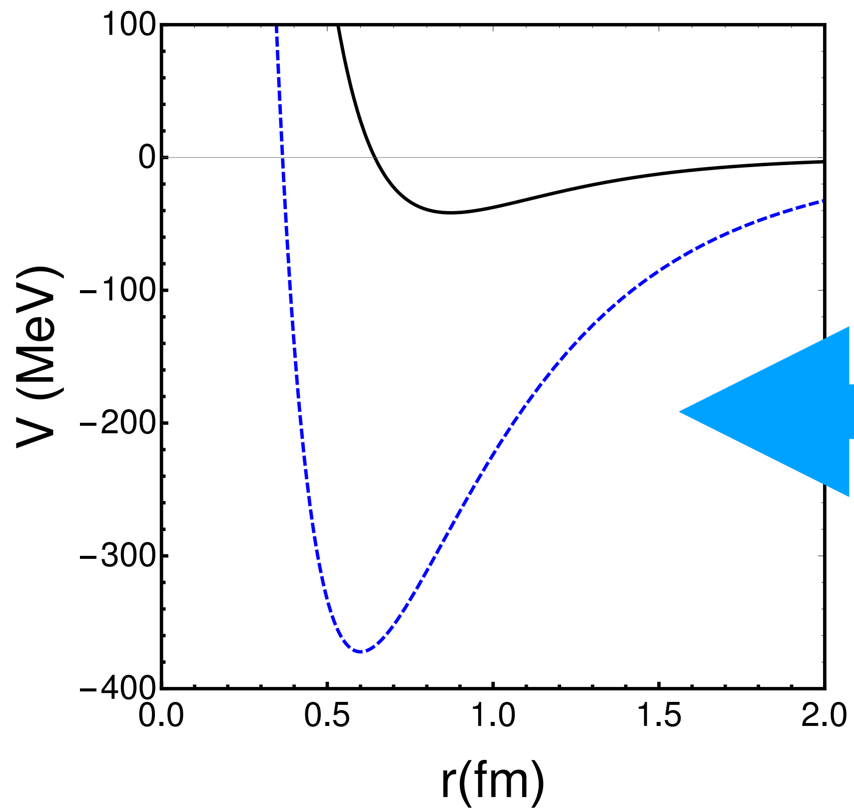

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Our first paper was based on general study of clustering:
globular clusters in Galaxies,
kinetics near liquid-gas transition
snow production etc
using rather simple tool,
classical Molecular dynamics
(excellent way to study out-of equilibrium situations)

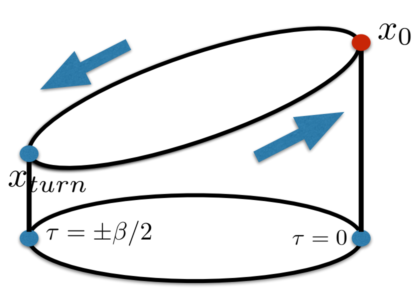
Our second paper was about quantum effects
in equilibrium clustering:
(i) semiclassical "flucton" method at finite T;
(ii) QM with hyper spherical coordinates for He4;
(iii) path integral Monte-Carlo ;

So, whether the particular effects we discuss
do or do not happen in real life,
we worked out interesting methodical tools…

Semiclassical approach to clusters:
fluction paths at nonzero T in equilibrium
(first developed by Turbiner and ES in quantum mechanics
Unlike WKB works in multimentional cases
And corrections are down by Feynman diagrams
Where for several problems calculated up to 3 loops

Density matrix dominated by fluction paths which should have correct period

$$\beta = \frac{\hbar}{T}$$



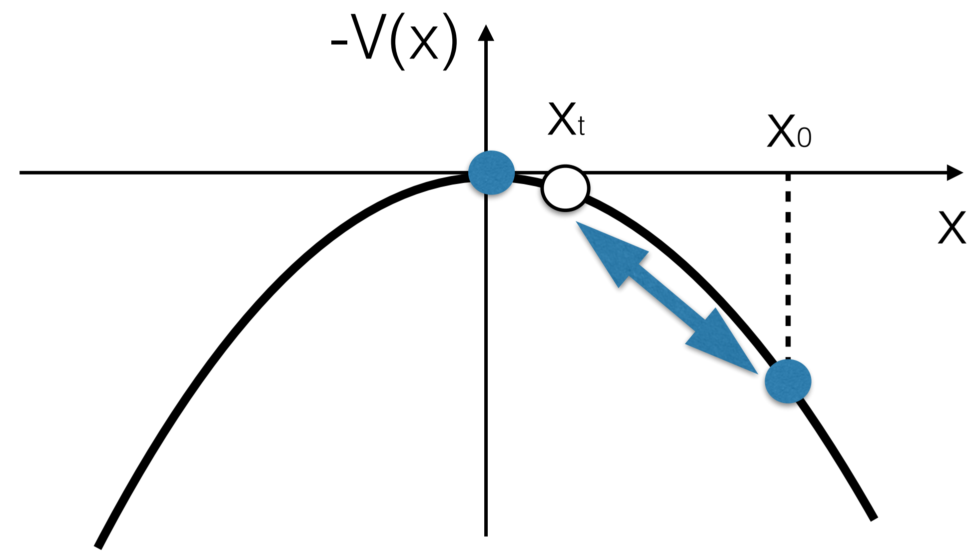


FIG. 1: The sketch of the inverted potential $-V$ versus coordinate x. Flucton is the classical trajectory starting and ending at the same initial point x_0. At non-zero temperature it goes through the turning point x_t, see text. At zero temperature x_t coincides with the location of the maximum, $x_t = 0$.

(the first time ever) testing the flucton method at finite T

Fluctons for anharmonic oscillator at $T \neq 0$

$$S_E = \oint d\tau \left(\frac{\dot{x}^2}{2} + \frac{x^2}{2} + \frac{g}{2} x^4 \right) .$$

the usual density matrix (line, 60 states)

$$P(x_0) = \sum_i |\psi_i(x_0)|^2 \; e^{-E_i/T}$$

$$P(x_0) \sim exp\big(- S_E[x_{flucton}(\tau)] \big)$$

(points on the plot)
so, the method works very well

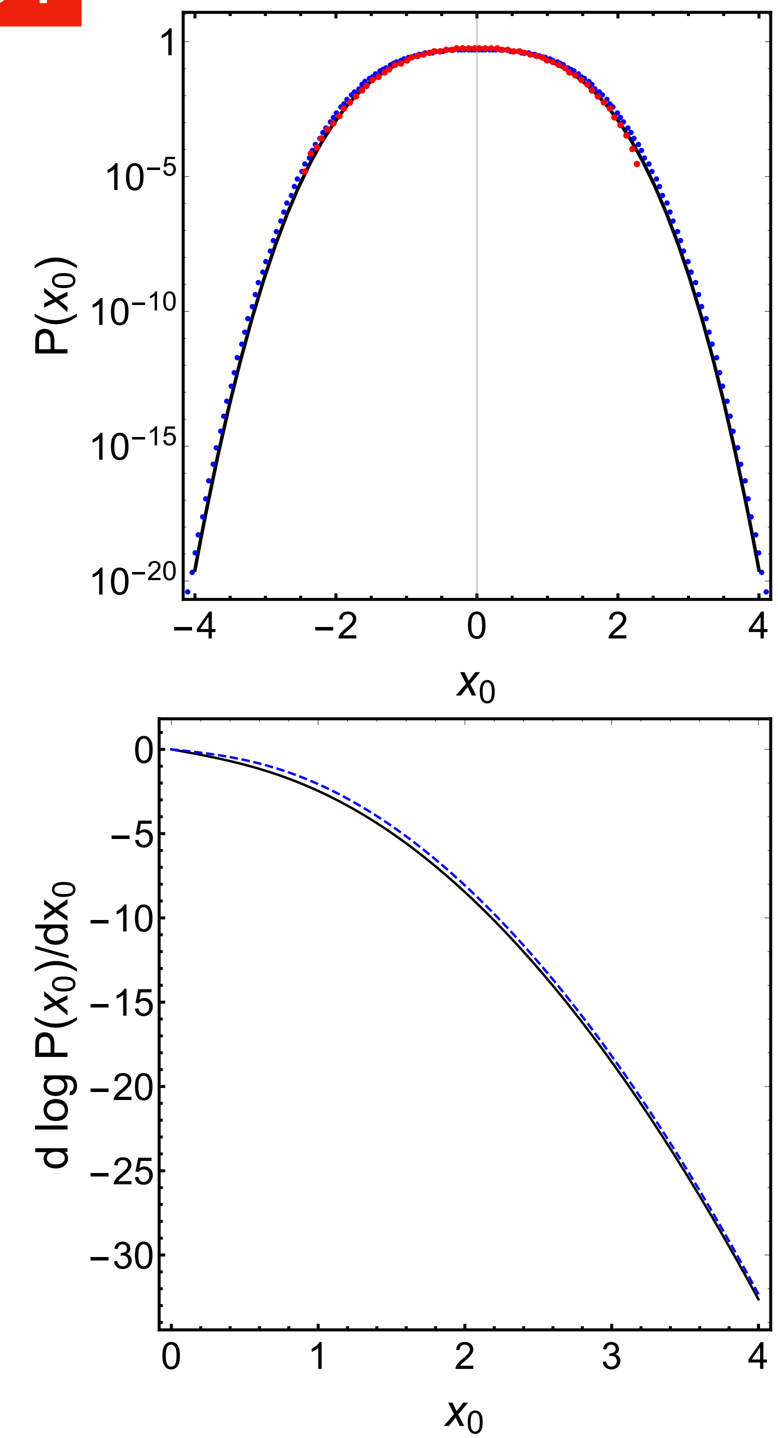


FIG. 3: Top panel: Density matrix $P(x_0)$ vs x_0 for anharmonic oscillator with the coupling $g = 1$, at temperature $T = 1$, calculated via the definition (1) (line) and the flucton method (points). The line is based on 60 lowest state wave functions found numerically. Bottom panel: Comparison of the logarithmic derivative of the density matrix of the upper panel.

K-harmonics applied to He4 (not a new method, and yet we found something new with it...)

Jacobi coordinates for 4 particles

$$\vec{\xi}[1] = \frac{\vec{x}[1]-\vec{x}[2]}{\sqrt{2}}, \quad \vec{\xi}[2] = \frac{\vec{x}[1]+\vec{x}[2]-2\vec{x}[3]}{\sqrt{6}},$$

$$\vec{\xi}[3] = \frac{\vec{x}[1]+\vec{x}[2]+\vec{x}[3]-3\vec{x}[4]}{2\sqrt{3}}$$

hyperdistance in 9 dimensional space

$$\rho^2 = \sum_{m=1}^{3} \vec{\xi}[m]^2 = \frac{1}{4}\Big(\sum_{i\neq j}(\vec{x}[i]-\vec{x}[j])^2\Big)$$

redefining the wave function and the radial Schreodinger eqn Note, the first derivative is gone but some new repulsive potential remains (not orbital!)

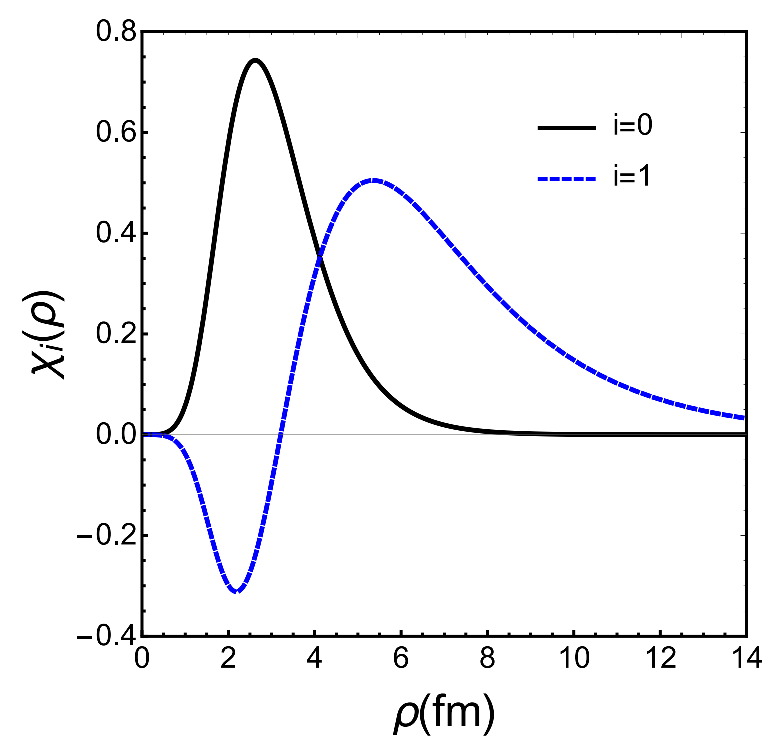
$$\psi(\rho) = \chi(\rho)/\rho^4$$

$$\frac{d^2\chi}{d\rho^2} - \frac{12}{\rho^2}\chi - \frac{2M}{\hbar^2}(W(\rho)+V_C(\rho)-E)\chi = 0$$

Solving the eigenvalue problem in App. A we have obtained 40 lowest eigenstates for Eq. (A3) using the simplest potential V_1 from Ref. [17] and the Coulomb term between the two protons. The ground state energy we find is $E_0 = -27.8$ MeV, very close to the experimental value of -28.3 MeV.

Rather unexpectedly, we also find a second bound state (missed in [17]) with energy $E_1 = -2.8$ MeV. To determine whether this state is physical, we show in Table ?? the excited states of ^{4}He. Among them there is just one 0^+ state, with a binding energy of

$$B = -28.3 \text{ MeV} + 20.2 \text{ MeV} = -8.1 \text{ MeV}$$



here are experimentally observed excited states of He4

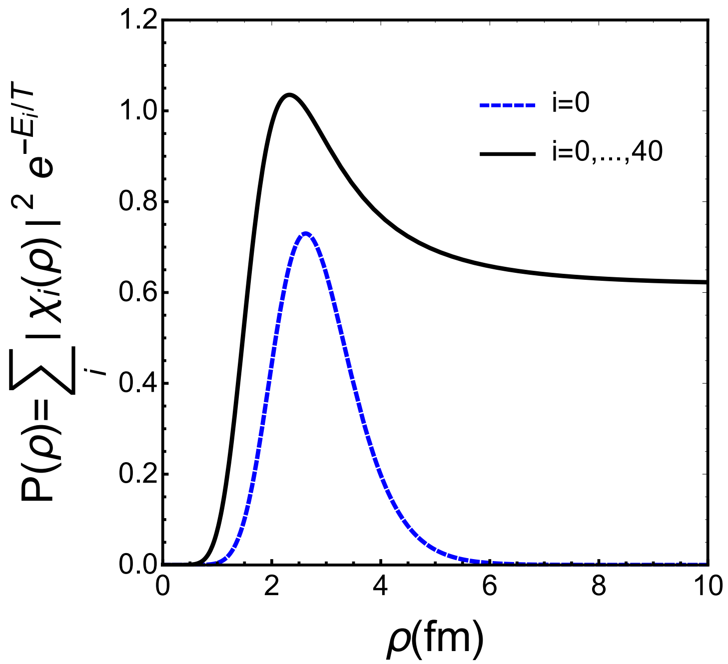
the first one fits well to our second bound state

Now, getting convinced that we understand quantum mechanics of 4 nucleons in He4, we proceed to calculate the density matrix at finite T and check how it changes when the nuclear potential changes

TABLE I: Low-lying resonances of ^{4}He system, from BNL properties of nuclides listed in *nndc.bnl.gov* web page. J^P is total angular momentum and parity, Γ is the width. The last column is the decay channel branching ratios, in percents. p, n, d correspond to emission of proton, neutron or deuterons.

E (MeV)	J^P	Γ (MeV)	decay modes, in %
20.21	0^+	0.50	p =100
21.01	0^-	0.84	n =24, p =76
21.84	2^-	2.01	n = 37, p = 63
23.33	2^-	5.01	n = 47, p = 53
23.64	1^-	6.20	n = 45, p = 55
24.25	1^-	6.10	n = 47, p = 50 , d=3
25.28	0^-	7.97	n = 48 , p = 52
25.95	1^-	12.66	n = 48 ,p = 52
27.42	2^+	8.69	n = 3 , p = 3 ,d = 94
28.31	1^+	9.89	n = 47 , p = 48 , d = 5
28.37	1^-	3.92	n = 2, p = 2, d = 96
28.39	2^-	8.75	n = 0.2, p = 0.2 , d = 99.6
28.64	0^-	4.89	d=100
28.67	2^+	3.78	d=100
29.89	2^+	9.72	n = 0.4 , p = 0.4, d = 99.2

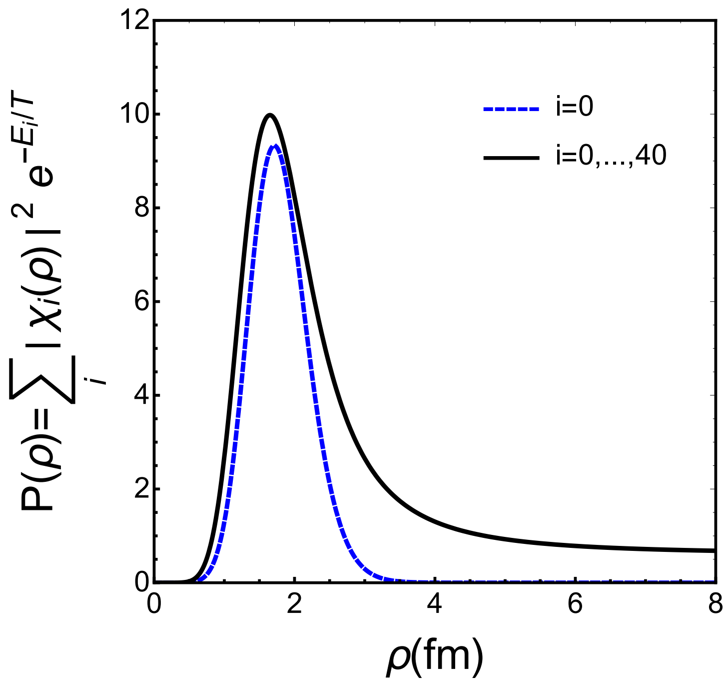
How the clustering changes as a function of the effective nuclear potential?



unmodified nuclear potential

total and

contribution of the ground state



modified nuclear potential

This modified potential now has several radial bound states: their energies are

$$E(\text{ MeV}) = -226.1, -120.1, -52.6, -17.3, -3.4, -0.1\ .$$

So, clustering changes from effect of 0.5 to about 10, factor 20

FIG. 9: Solid lines: Boltzmann-weighted density matrix, at $T = 100$ MeV, using 40 lowest states of the K-harmonics radial equation, for the unmodified nuclear potential V_1 used in Ref. [17] (upper plot) and a modified one (lower plot). In both cases the blue dashed lines show the contribution of the lowest bound state.

the flucton method, and Boltzmann show similar results, once effective repulsion is included

"pre-clusters" versus "fragments"

At T=O(10 MeV) a "multifragmentation",
production of various isotopes in wide range of A

We discuss freezeout of higher energy collisions
in which T=O(100 MeV)
fragments heavier than He4 are not produced

Yet even under such conditions one may have "pre-clusters"
Of several nucleons held together by inter nucleon potential

They have size of the order of 1.5 fm, the nuclear force radius

$$\Delta E \sim T \gg |E_{binding}|$$

Therefore after freezeout they decay mostly into free nucleons,
But, with certain (projected) probability, also into d,t,He3,(pnLambda)

The large size of the bound states is important,
but so is their compact component
(without which they would not be bound)

what effect of pre-clustering can we see experimentally?
this is what is already observed

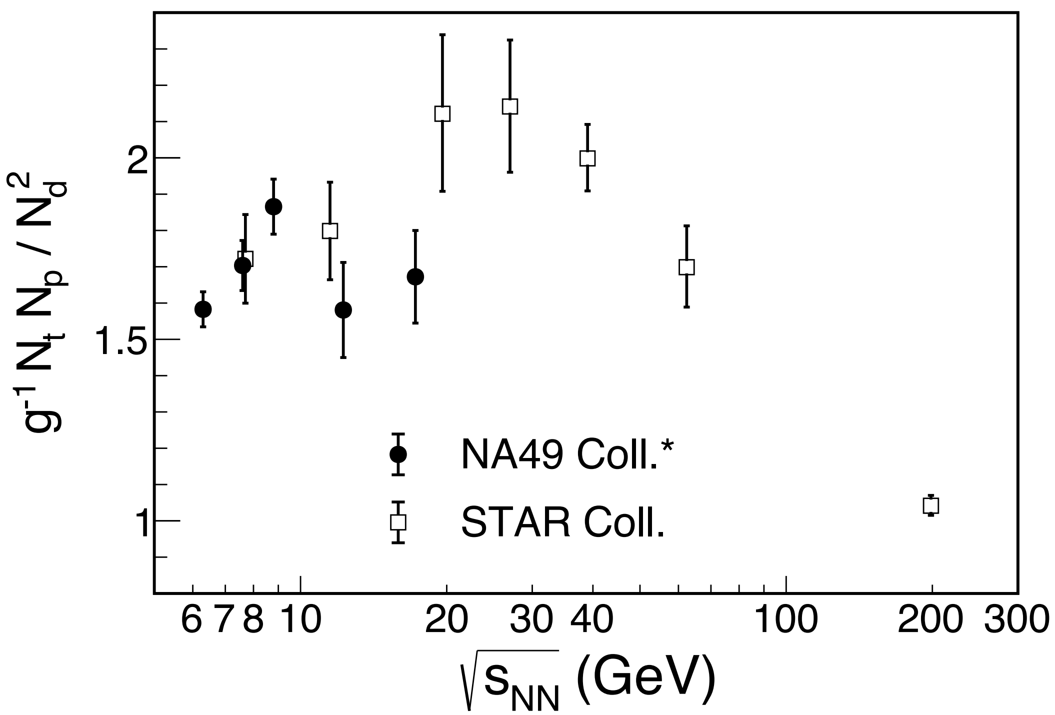


FIG. 10: The ratio (25) as a function of collision energy. The ratio is normalized by the corresponding statistical weights: note that the high energy RHIC point at the r.h.s. of the plot give the ratio value consistent with 1. Deviation from 1 is related to nonzero interaction as shown in (27).

$$\mathcal{O}_{tpd} = \frac{N_t N_p}{N_d^2}$$

note that exp[(\mu-mN)/T] cancels out, as well as thermal kinetic energy
only interaction remains
3 pairs in t, against 1 in d

Note that the point at 200 GeV (at the r.h.s.) gives 1, same as at LHC (ALICE)
it is not a random number but
the prediction of the statistical model !

Note further, the effect of binding is negligible B<<T:
but modified potential gives binding comparable to T

some predictions for ratios, from he4-like pre-clusters

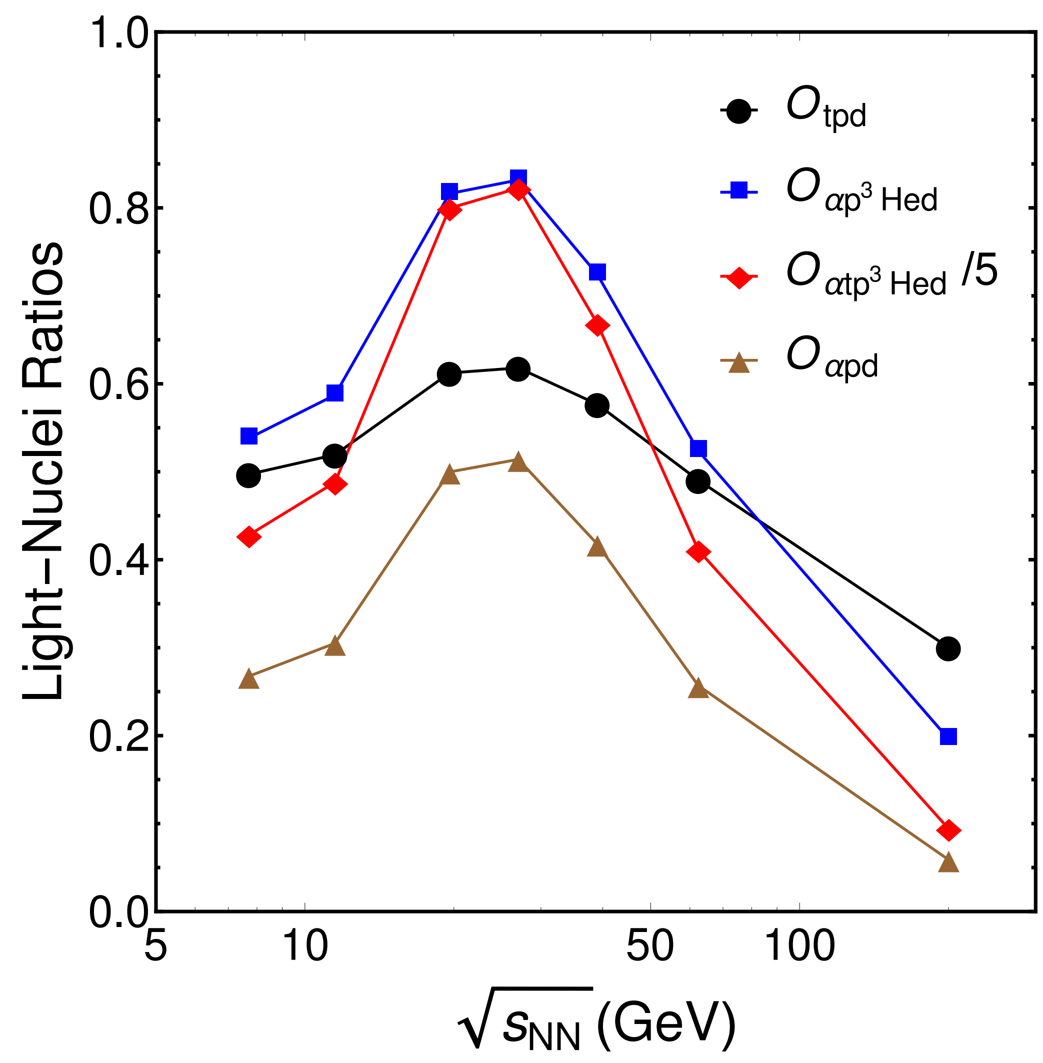
$$\mathcal{O}_{tpd} = \frac{N_t N_p}{N_d^2}$$

$$\mathcal{O}_{\alpha p^3\mathrm{He}d} = \frac{N_\alpha N_p}{N_{^3\mathrm{He}} N_d}\,, \qquad \mathcal{O}_{\alpha t p^3\mathrm{He}d} = \frac{N_\alpha N_t N_p^2}{N_{^3\mathrm{He}} N_d^3}$$

$$\mathcal{O}_{tpd} = \frac{4}{9}\left(\frac{3}{4}\right)^{3/2} \frac{\langle e^{-3V/T}\rangle_t}{\langle e^{-V/T}\rangle_d^2} \approx 0.29\langle e^{-V/T}\rangle$$

$$\mathcal{O}_{\alpha p^3\mathrm{He}d} = \frac{1}{3}\left(\frac{2}{3}\right)^{3/2} \frac{\langle e^{-6V/T}\rangle_\alpha}{\langle e^{-3V/T}\rangle_{^3\mathrm{He}}\langle e^{-V/T}\rangle_d}$$
$$\approx 0.18\langle e^{-2V/T}\rangle$$

$$\mathcal{O}_{\alpha t p^3\mathrm{He}d} = \frac{8}{54} 2^{3/2} \frac{\langle e^{-6V/T}\rangle_\alpha \langle e^{-3V/T}\rangle_t}{\langle e^{-3V/T}\rangle_{^3\mathrm{He}}\langle e^{-V/T}\rangle_d}$$
$$\approx 0.42\langle e^{-3V/T}\rangle$$



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Let us consider as an example a *ppnn* pre-cluster. Apart of forming a single bound state, the alpha particle or ^4He, it can decay into (i) 4 individual nucleons; (ii) 1+3 channels $p + t, n + He^3$; (iii) 2+2 channel d+d. The question then is whether one can experimentally observe pre-clusters looking at these two-body channels.

let us apply statistical model of the pre-cluster decays using these resonances

note long lifetimes, well beyond the lifetime of the fireball

one further finds that decays of a single ppnn pre-cluster should produce, in average, 0.24(p+ tritium), 0.27(n+He^3) and 0.97 deuterons (0.49 dd pairs). Detector resolution permitting, one should search for evidences of these p + t, d + d resonances in heavy ion datasets. If such “feed down” be found, ould obviously be the direct evidence for 4-nucleon pre-clustering we advocate in this work.

summary

1. **We studied pre-clustering of baryons at freeze out conditions,**

$$T \sim 100\, MeV, \mu_b = 0...500\, MeV$$

including quantum and thermal fluctuations

2. **K-harmonics and semiclassical methods work consistently, they describe e.g. He4 at T=0 and nonzero T**

3. **pre-clustering is very sensitive to the underlying nuclear potential, changes in sigma spectral density due to chiral transition or QCD critical point (if we are lucky)**

4. **If such modification does happen, as we expect, it will strongly enhance ratios related with production of light nuclei (via feed-down from reclusters e.g. of 4 nucleons)**

Comment on phenomena in the BES energy range

Most emphasized are enhanced fluctuations
Kurtosis of baryon distribution
Perhaps a signal of QCD critical point
Seen at lowest energies

$$\sqrt{s_{NN}} \sim 7\,GeV$$

Another well documented is the lowest lived fireball
Related to ``the softest point” of the EOS

$$\sqrt{s_{NN}} \sim 47\,GeV$$

The ratio below, sensitive to clustering, peaks at $\sqrt{s_{NN}} \sim 20\,GeV$

$$R = \frac{n(t)n(p)}{n(d)^2}$$