THE QCD PHASE DIAGRAM AND THE CRITICAL ENDPOINT

Fabian Rennecke
Brookhaven National Laboratory

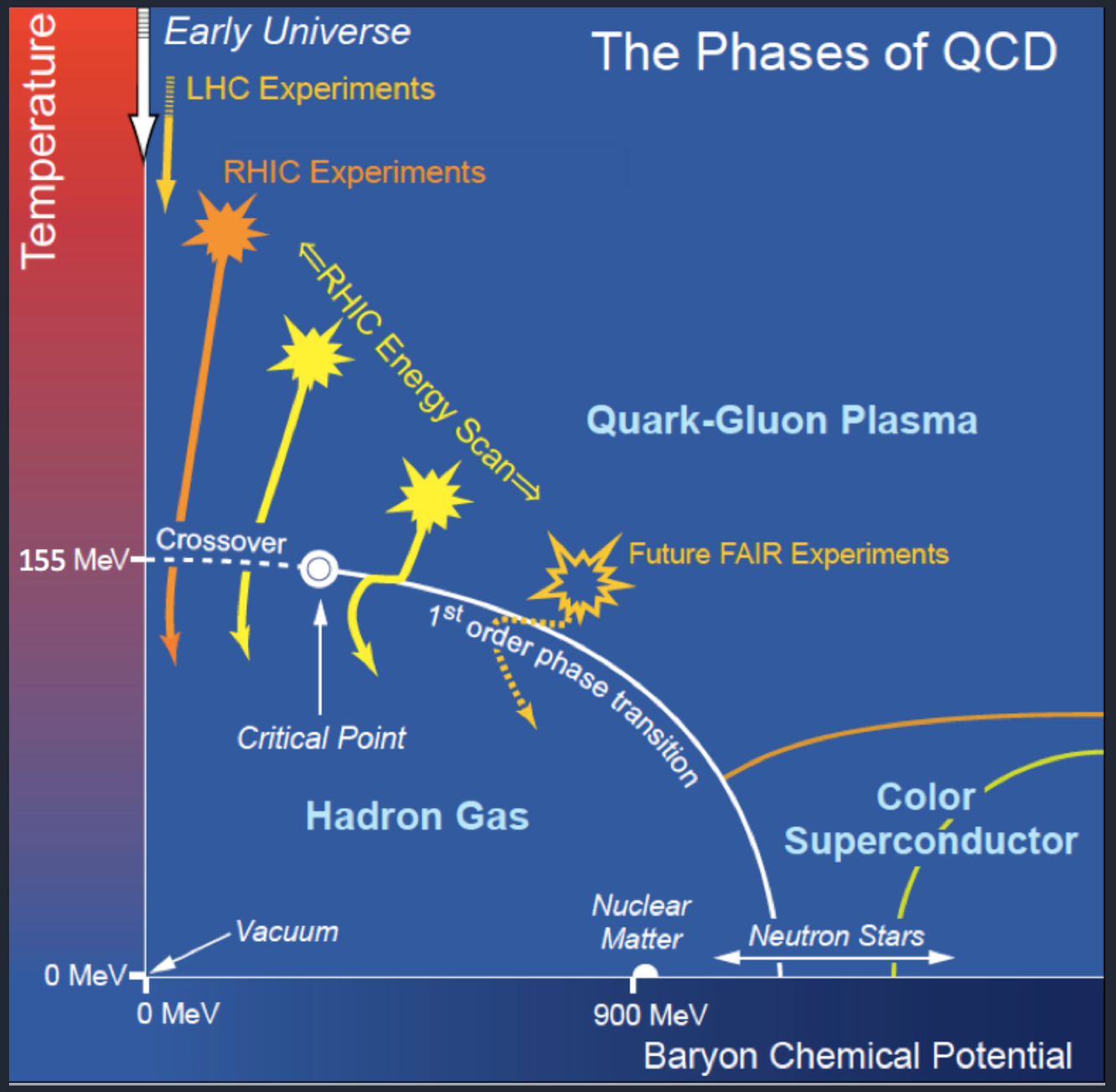
BROOKHAVEN NATIONAL LABORATORY

DFG Deutsche Forschungsgemeinschaft

— RHIC/AGS USERS' MEETING 2019 —

BNL, 04/06/2019

THE QCD PHASE DIAGRAM

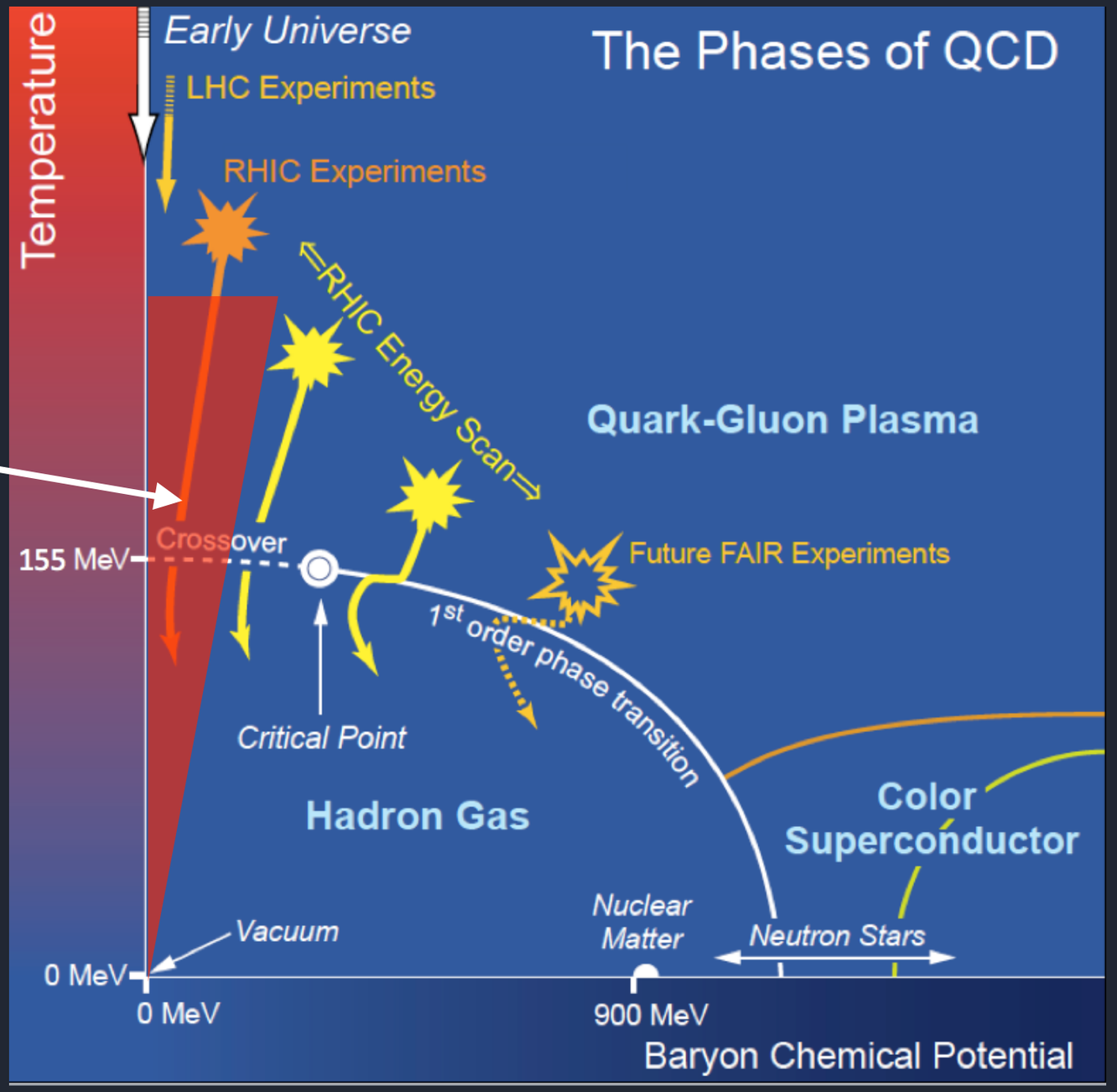


THE QCD PHASE DIAGRAM

CEP unlikely

for $\mu_B/T \leq 2$

[HotQCD, hep-lat/1701.04325]

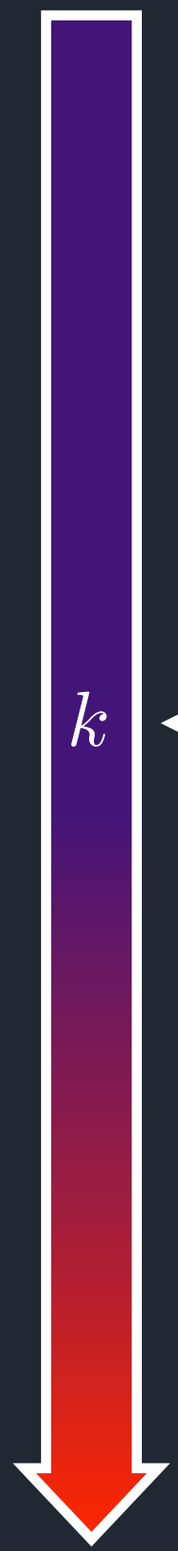


OUTLINE

- The scale evolution of QCD
- Functional RG with emergent bound states
- First (preliminary) results on the phase diagram

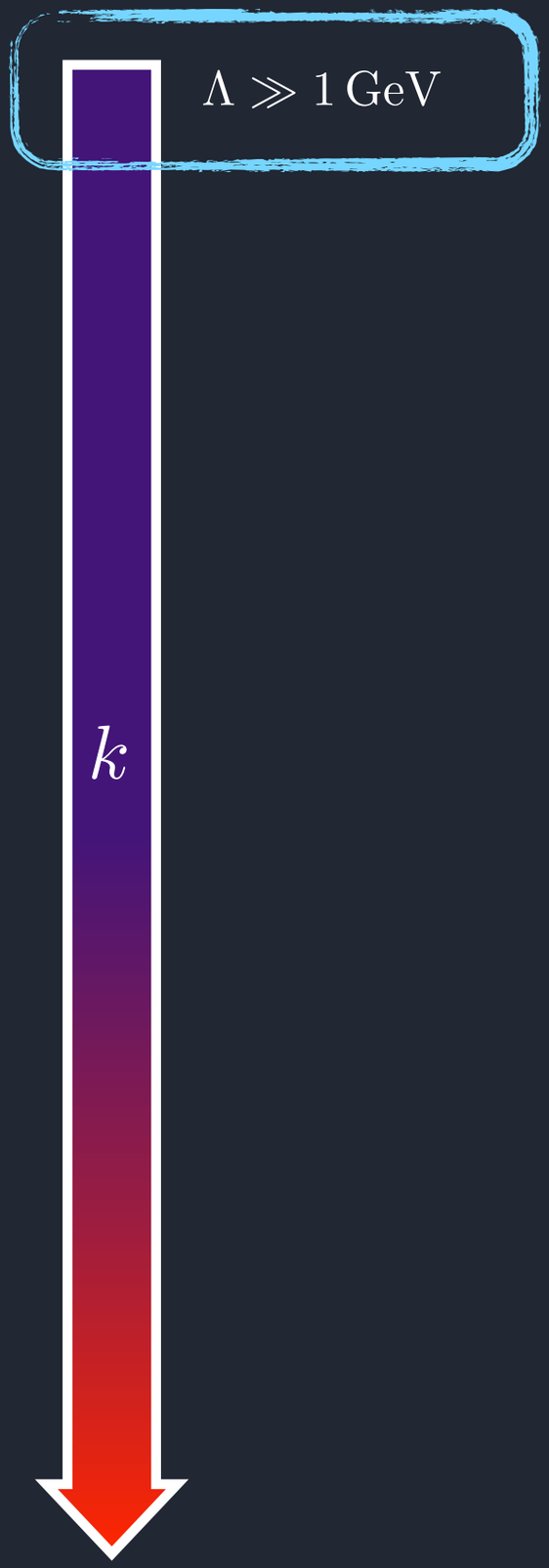
SCALE EVOLUTION OF QCD

integrating out fluctuations from UV to IR



energy / renormalization group scale

SCALE EVOLUTION OF QCD



- initial action: gauge fixed Euclidean QCD

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}(\gamma_\mu D_\mu + m_q)q + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

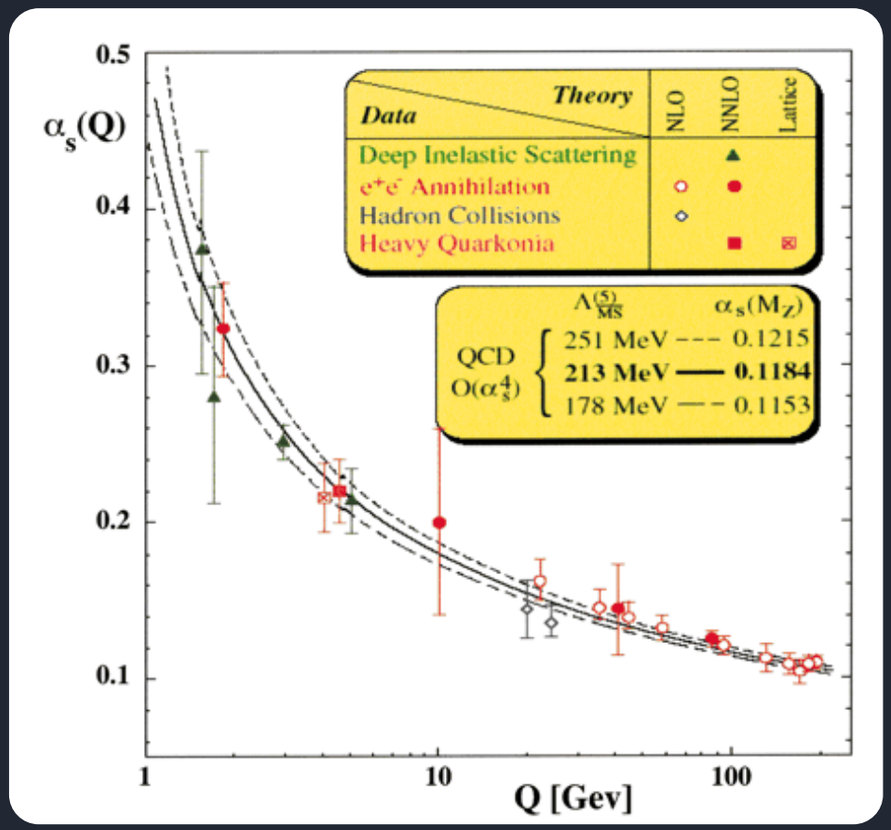
covariant derivative

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

field strength

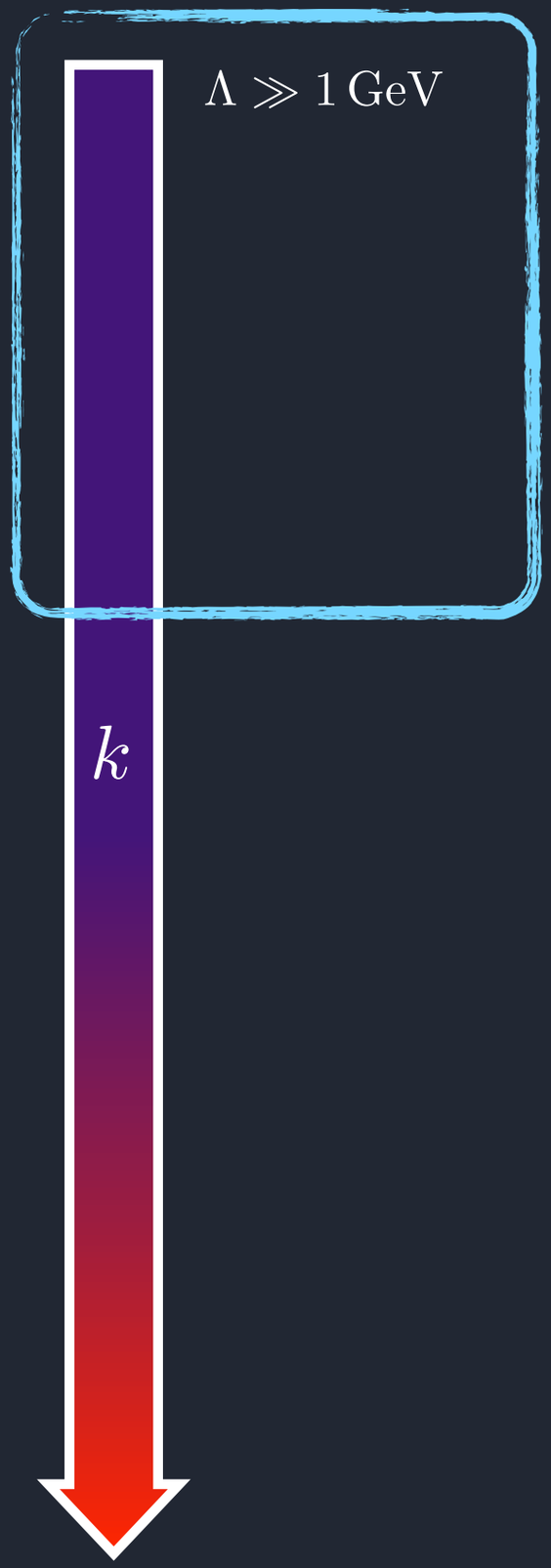
$$F^a_{\mu\nu}t^a = \frac{i}{g}[D_\mu, D_\nu]$$

- asymptotic freedom: fluctuations increase strong coupling

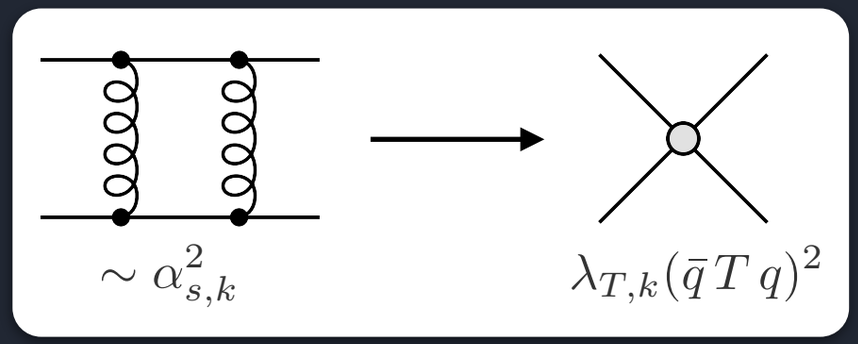


[S. Bethke, hep-ex/0606035]

SCALE EVOLUTION OF QCD

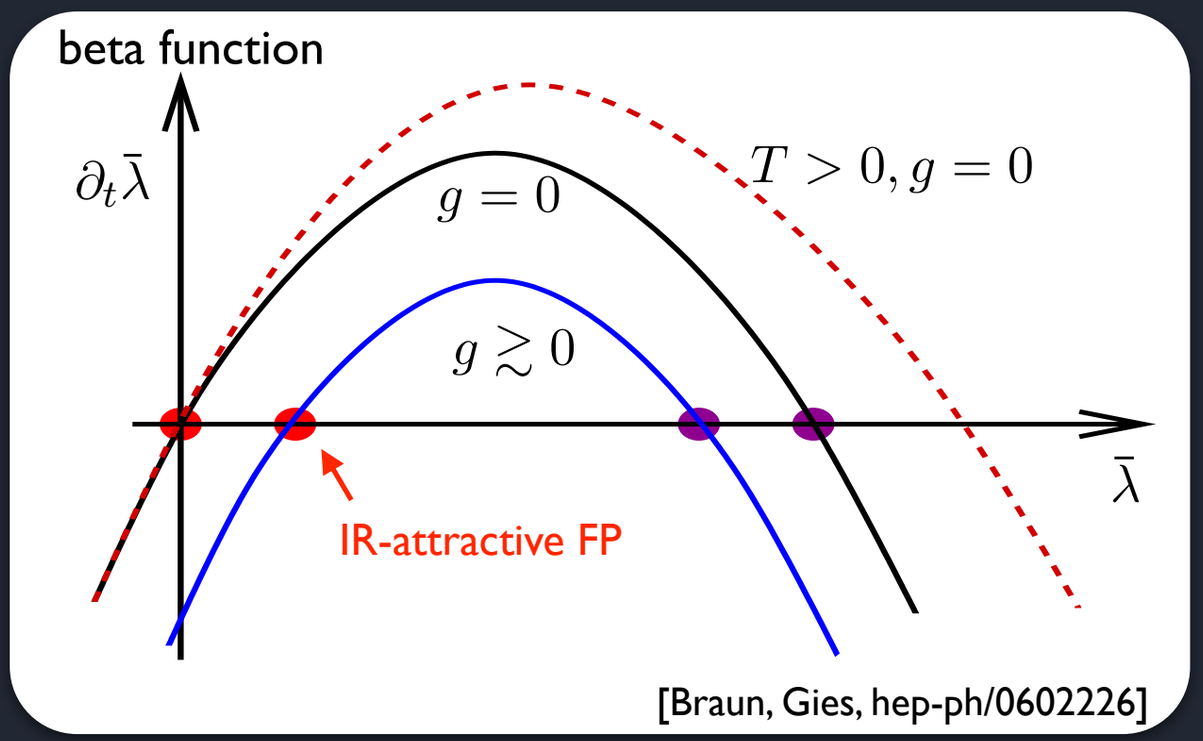


- effective four-quark interactions are generated



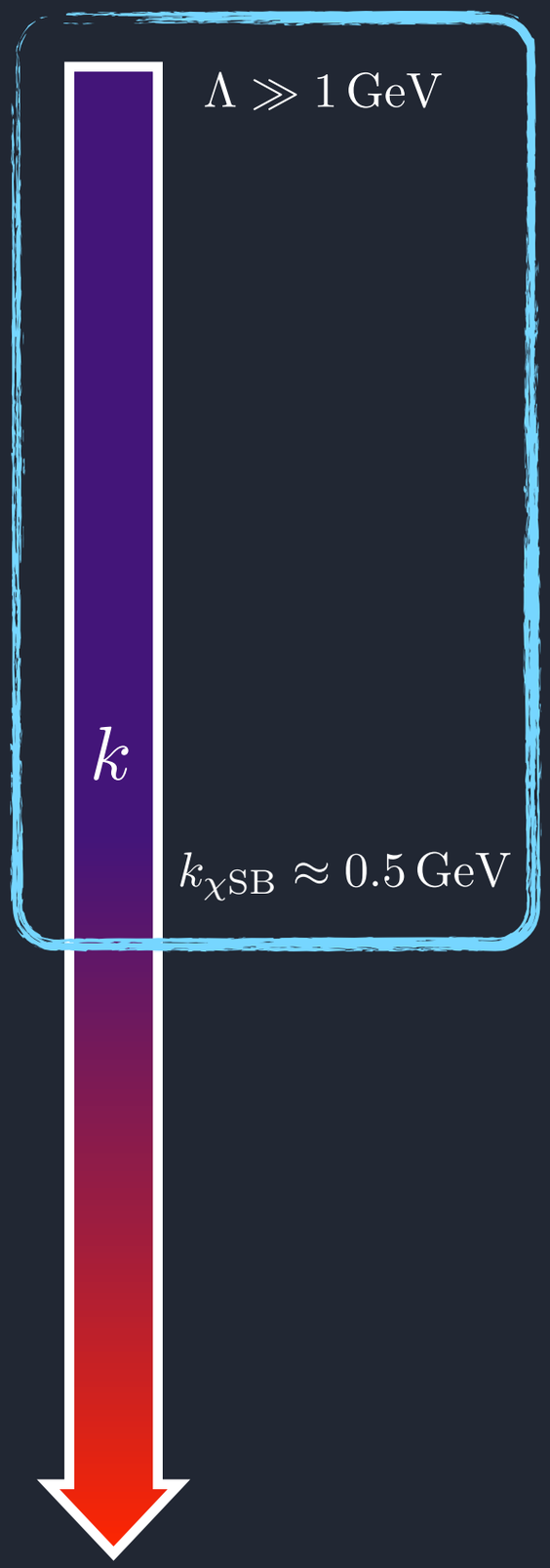
- $\lambda_{T,k}$ grows with decreasing energy scale

- running controlled by IR-attractive fixed point

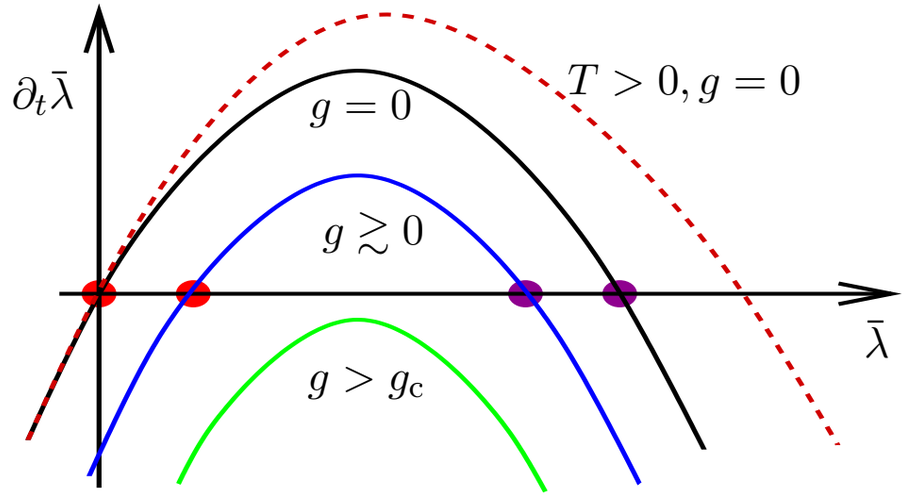


[Braun, Gies, hep-ph/0602226]

SCALE EVOLUTION OF QCD



- strong coupling exceeds critical value: FP vanishes; $\lambda_{T,k}$ diverges

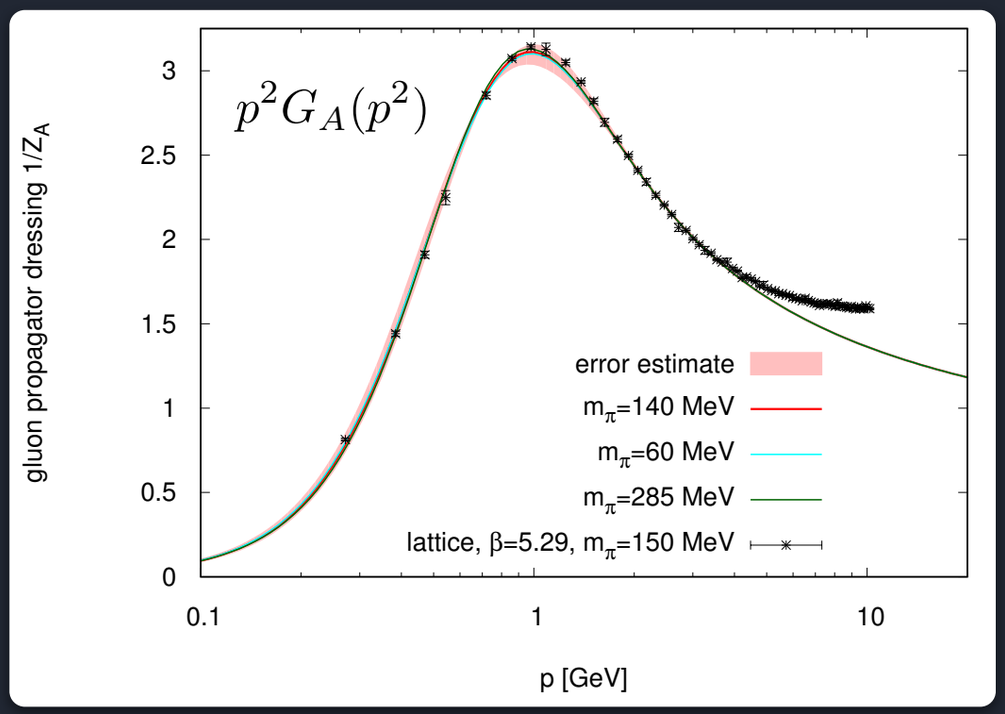


[Braun, Gies, hep-ph/0602226]

- resonance in condensate channel: **chiral symmetry breaking**

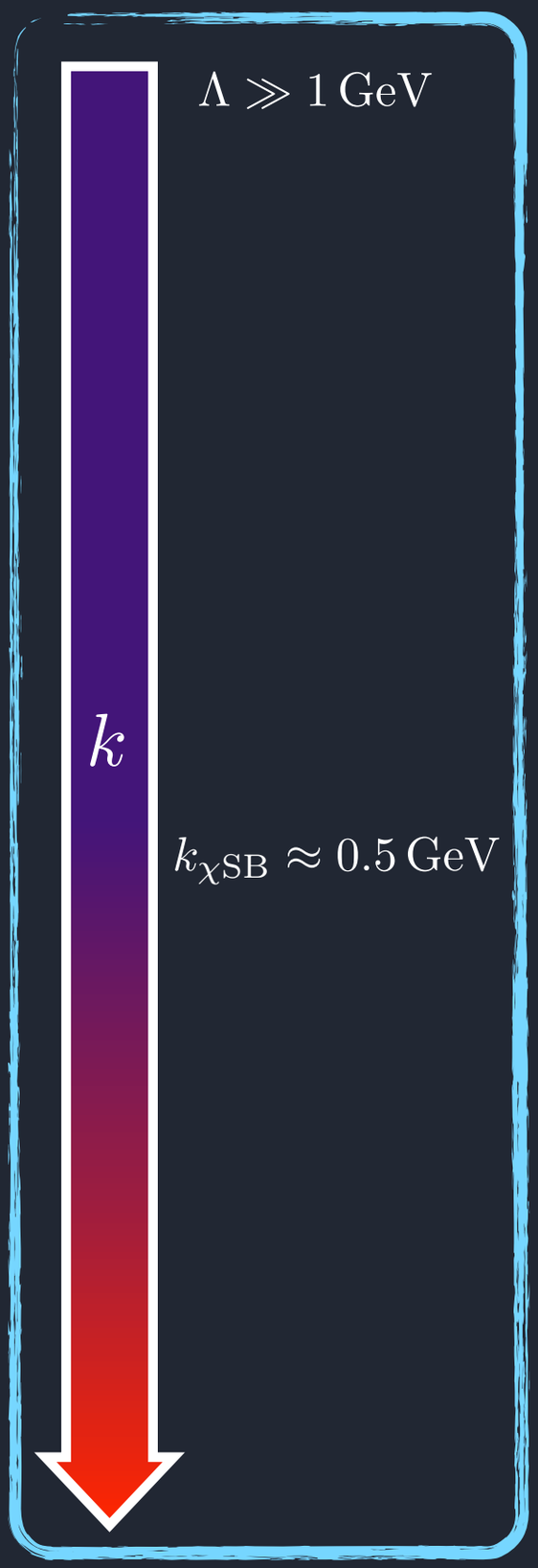
$$\lambda_{S,k}(\bar{q}q)^2 \to \infty \longleftrightarrow \langle \bar{q}q \rangle \neq 0$$

- around the same scale: gluon mass-gap develops, **confinement**



[Cyrol, Mitter, Pawlowski, Strodthoff, hep-ph/1706.06326]

SCALE EVOLUTION OF QCD

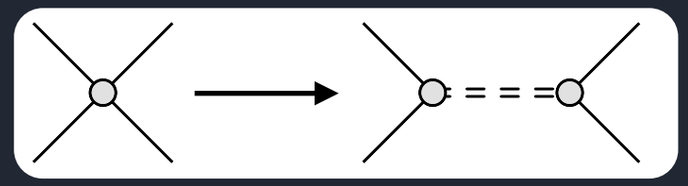


- rewrite resonant 4-quark channels in terms of mesons
  (similar: baryonization from 6-quark interactions)

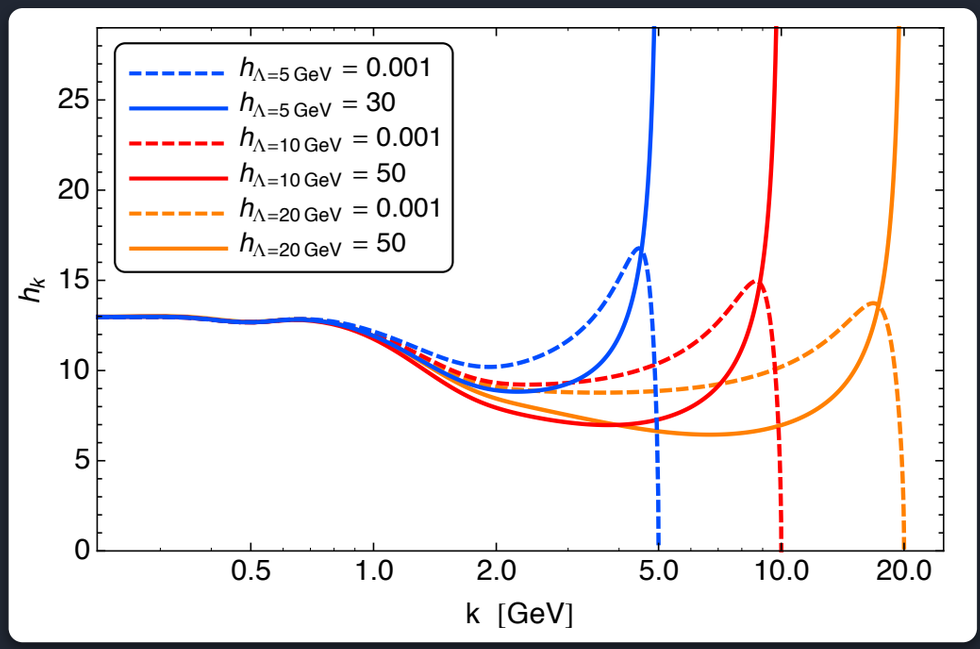
Yukawa coupling

$$\lambda_{T,k} = \frac{h^2_{T,k}}{m^2_{\phi,k}}$$

meson mass parameter



[Braun, Fister, Pawlowski, FR, hep-ph/1412.1045]



- low-energy models **emerge** when gauge sector decouples
- fixed point at small g: low-energy parameters uniquely determined
- different tensor structures of $\lambda_{T,k}$: different bound states

$$(\bar{q}T^a q)^2 + (\bar{q}i\gamma_5 T^a q)^2 \longrightarrow \pi,\ K, \eta,\ \eta',\ a_0,\ \kappa,\ \sigma,\ f_0$$

U(3) generators

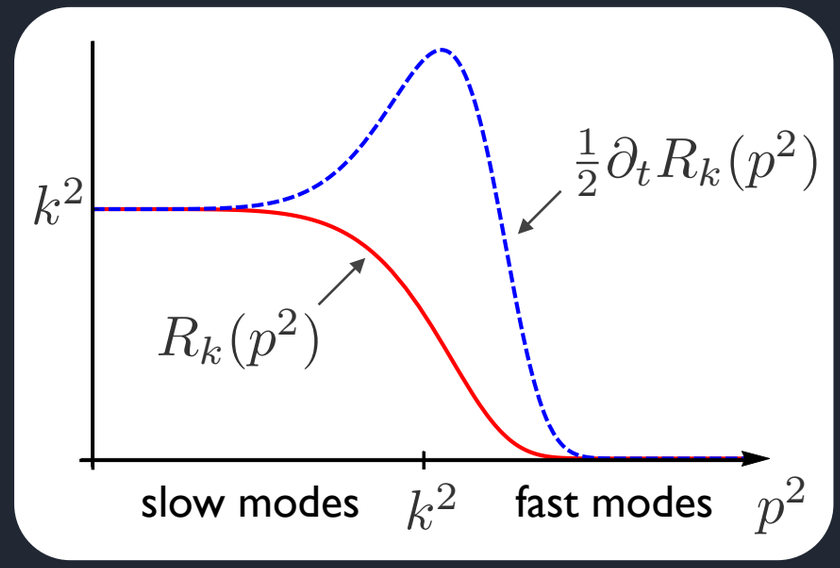
FUNCTIONAL RG

- low-energy QCD inherently strongly interacting: non-perturbative method
- capture fundamental and emergent d.o.f.: scale dependent effective action
- finite chemical potential: no sign problem

- introduce regulator to partition function to suppress momentum modes below energy scale k (Euclidean space):

$$Z_k[J] = \int \mathcal{D}\varphi\, e^{-S[\varphi]-\Delta S_k[\varphi]+\int_x J\varphi}$$

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\,\varphi(-q)R_k(q)\varphi(q)$$



- scale dependent effective action:

$$\Gamma_k[\phi] = \sup_J \left\{ \int_x J(x)\phi(x) - \ln Z_k[J] \right\} - \Delta S_k[J] \qquad \phi = \langle\varphi\rangle_J$$

successively integrate out fluctuations from UV to IR (Wilson RG)

Γ_Λ $\qquad$ Γ_k $\qquad$ $\Gamma_0 = \Gamma$

full quantum effective action (generates 1PI correlators)

→ Γ_k is eff. action that incorporates all fluctuations down to scale k

→ lowering RG-scale k: zooming out / coarse graining

DYNAMICAL HADRONIZATION

[Gies, Wetterich (2002), Pawlowski (2007), Floerchinger, Wetterich (2009)]
[Braun, Fister, Pawlowski, FR, hep-ph/1412.1045]

- k-derivative of Γ_k :

Flow equation
[original: Wetterich 1993]

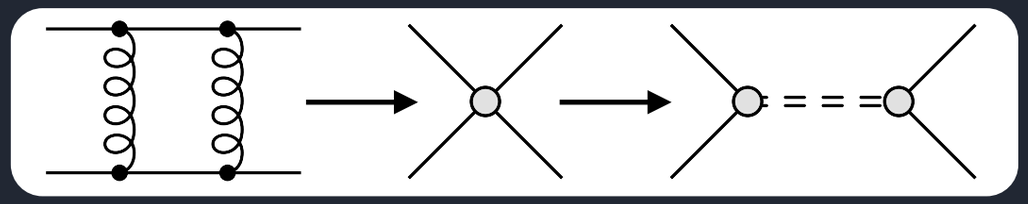
$$\partial_t = k\frac{d}{dk}$$

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\,\text{gluons} - \text{ghosts} - \text{quarks} + \frac{1}{2}\,\text{mesons}$$

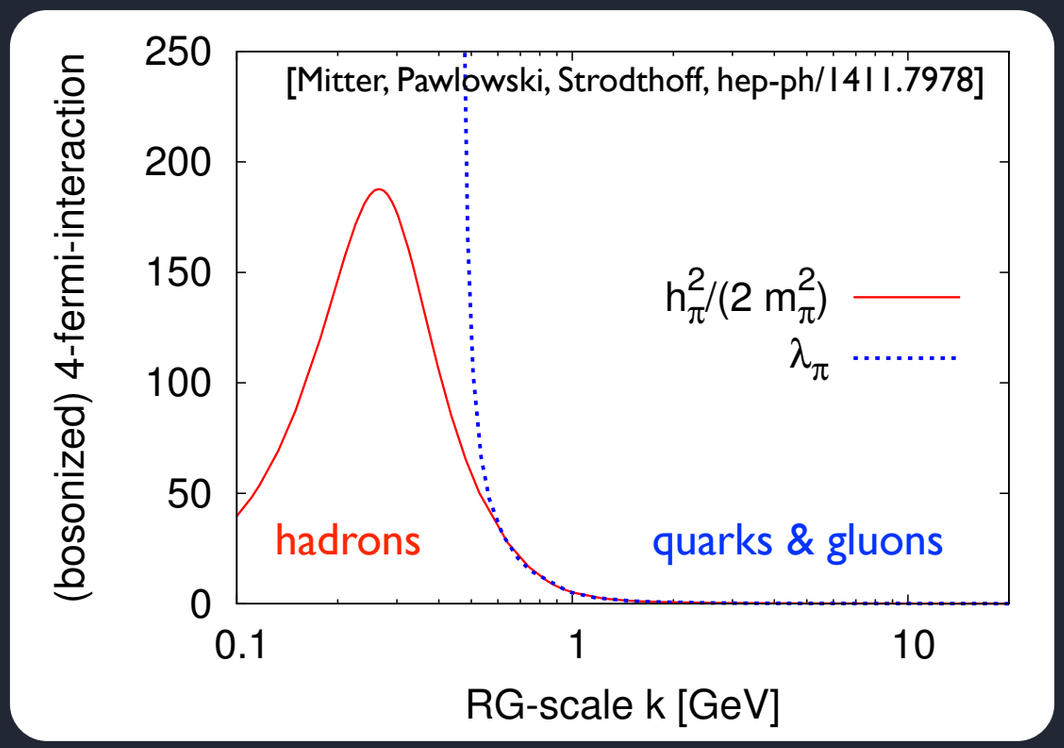
$$\partial_t R_k^\Phi$$

$$\left(\Gamma_k^{(2)}[\Phi] + R_k^\Phi\right)^{-1}$$

- need continuous transition from quarks and gluons to hadrons



→ **bosonize 4-quark interactions in each RG-step**



→ 4-quark interaction encoded in Yukawa coupling

→ unified description of QGP and hadronic phase

hadronic sector emerges dynamically; no model parameters

APPLICATION TO THE PHASE DIAGRAM

Task: construct appropriate Γ_k and solve the resulting flow equation

Difficulty: effective action can contain everything allowed by symmetries

→ truncation necessary

Strategy:

- identify physically most relevant terms and include them: qualitative
  certainly depends on the problem at hand
- systematically add terms and look for convergence: quantitative

Goal:

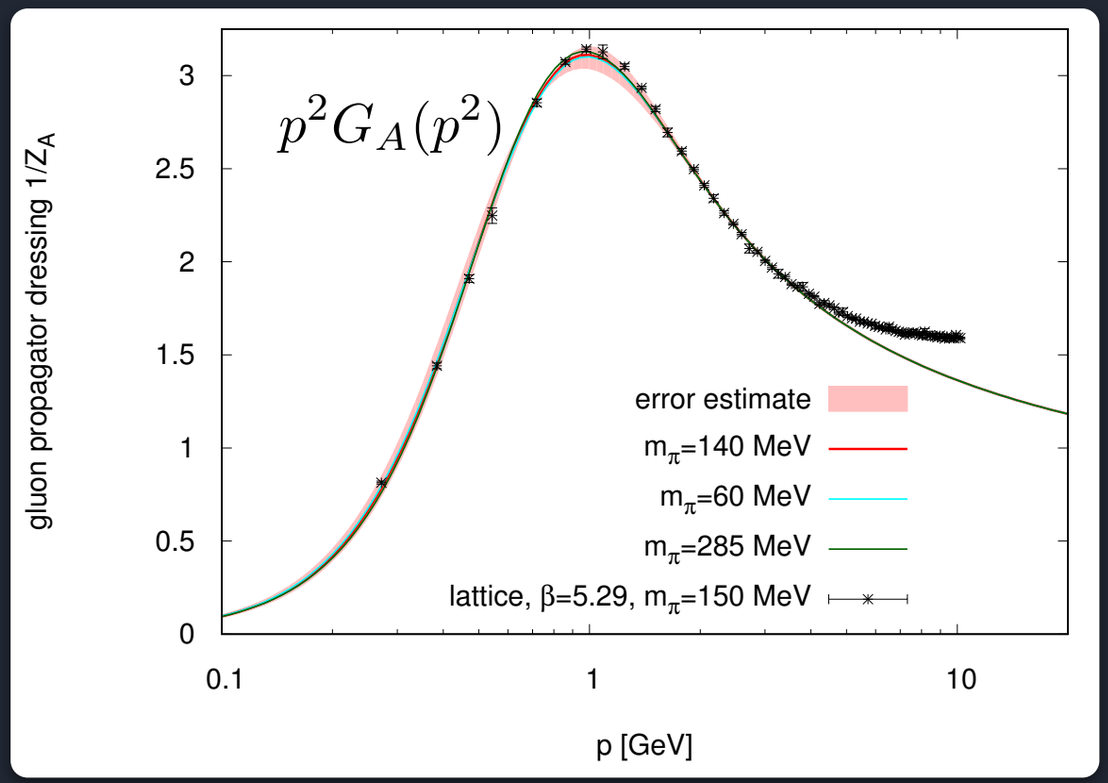
- initialize RG-flow in perturbative regime: microscopic QCD with strong coupling and current quark masses as parameters
- low-energy sector emerges dynamically, fixed from underlying QCD dynamics

VACUUM QCD

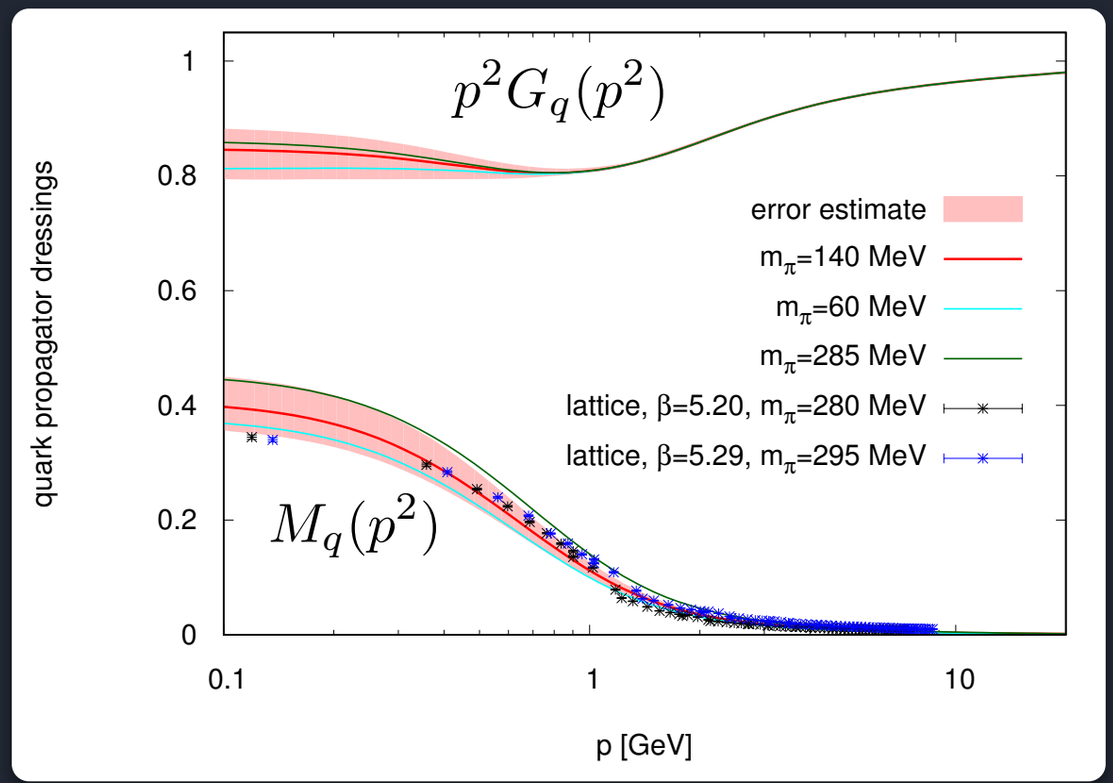
- FRG results in very good agreement with the lattice (YM & $N_f = 2$, Landau gauge)

[Cyrol, Fister, Mitter, Pawlowski, Strodthoff (2011-2018)]

gluon dressing function



quark propagator



[Cyrol, Mitter, Pawlowski, Strodthoff, hep-ph/1706.06326]

VACUUM QCD

- FRG results in very good agreement with the lattice (YM & $N_f = 2$, Landau gauge)

[Cyrol, Fister, Mitter, Pawlowski, Strodthoff (2011-2018)]

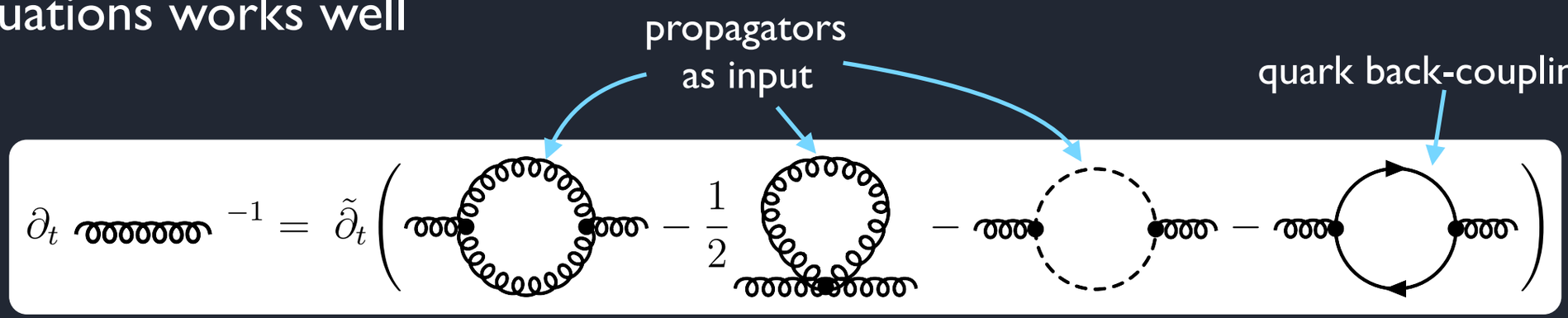
- Also very expensive… (flow equations alone can take GBs in .txt files)

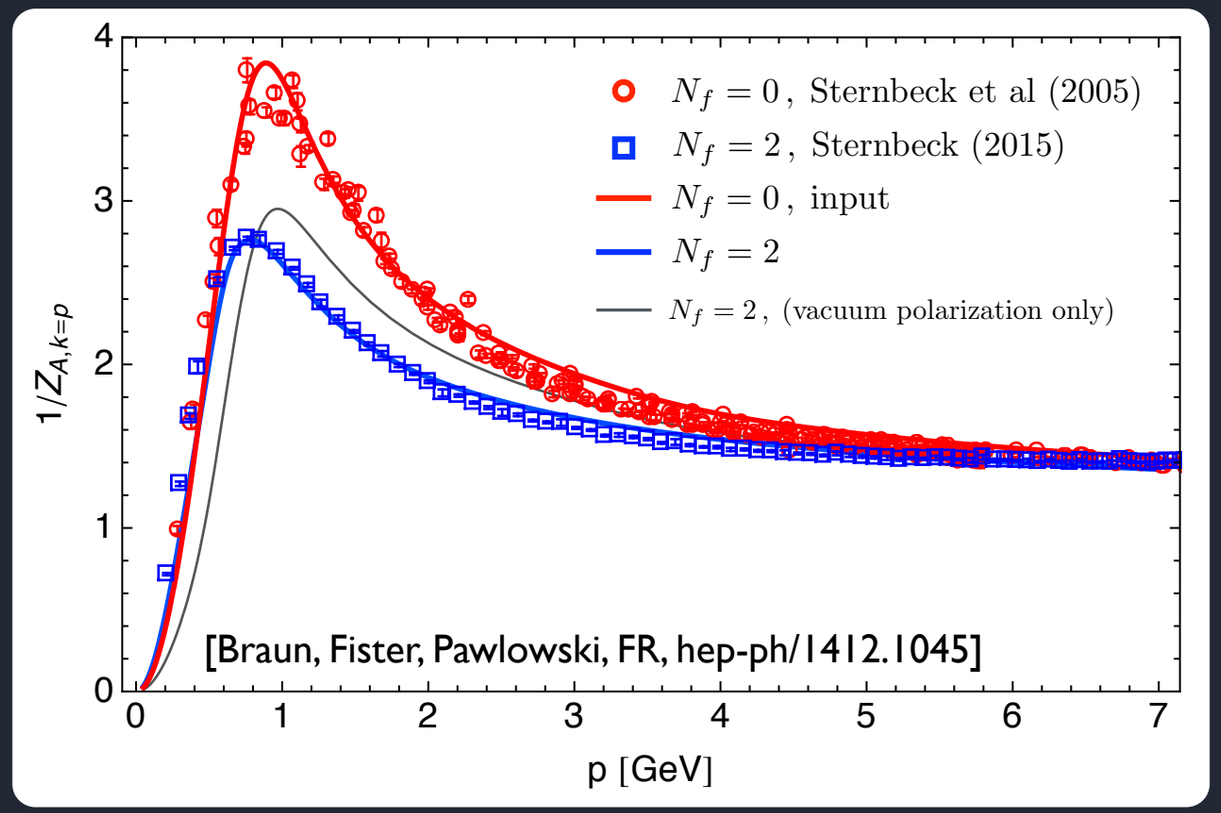
VACUUM QCD

- FRG results in very good agreement with the lattice (YM & $N_f = 2$, Landau gauge)

[Cyrol, Fister, Mitter, Pawlowski, Strodthoff (2011-2018)]

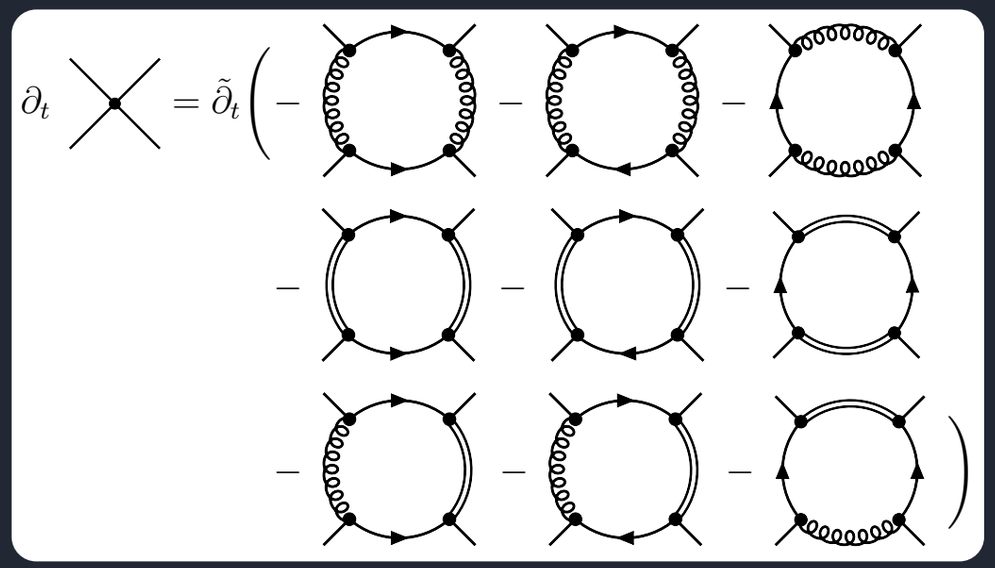
- Also very expensive… (flow equations alone can take GBs in .txt files)
- As it turns out: using gluon and ghost propagators as input and adding matter fluctuations works well





TRUNCATION [Fu, Pawlowski, FR (in preparation)]

- dominant 4-quark channel at low and moderate μ: $\lambda_{\text{S-P,k}}\left[(\bar{q}q)^2+(\bar{q}i\gamma_5\vec{\tau}q)^2\right]$
  [Braun, Leonhardt, Pospiech, hep-ph/1801.08338]

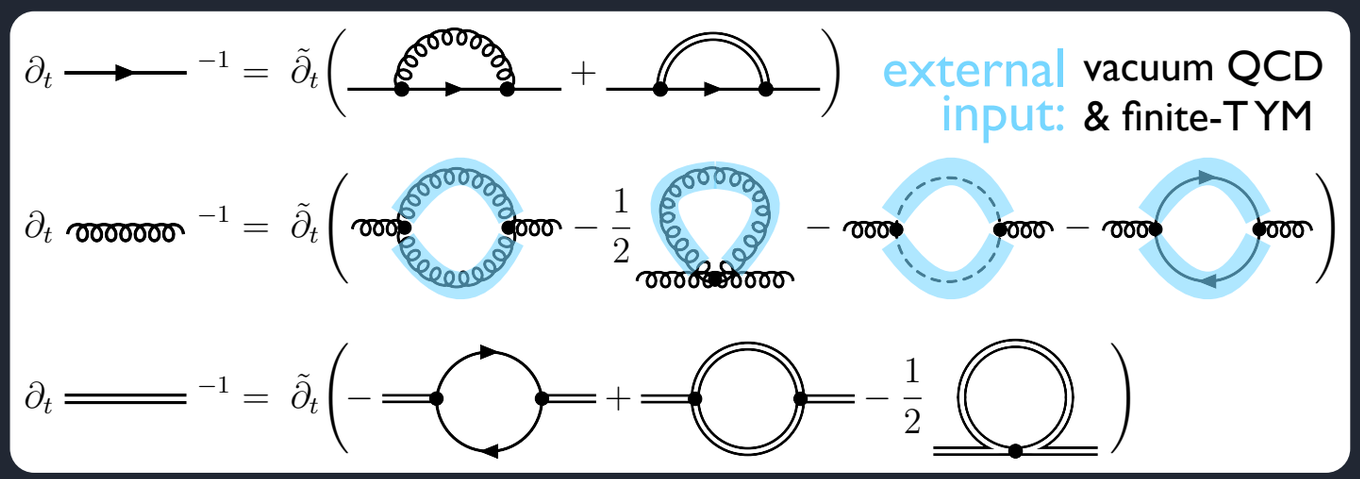


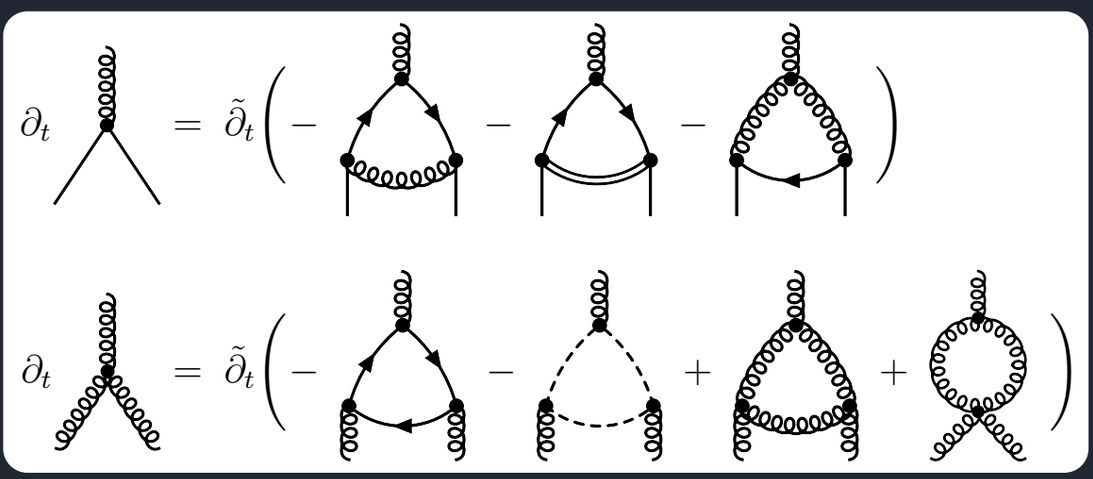
→ if resonant: chiral condensate, $\sigma/f_0(500)$ and pions emerge

→ spontaneous chiral symmetry breaking!

not one-loop!
flow equations:
all propagators and vertices are coupled
solving them:
resummation of loops of infinite order

- dynamical hadronization of this channel: dynamical π,σ + Yukawa interactions
- effective meson potential: arbitrary orders of meson self-interactions
- quark, gluon, meson (& ghost) propagators
- gauge couplings

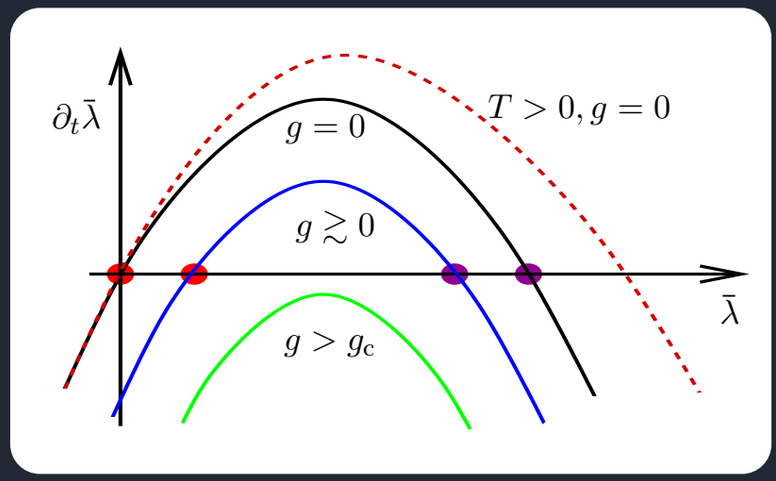


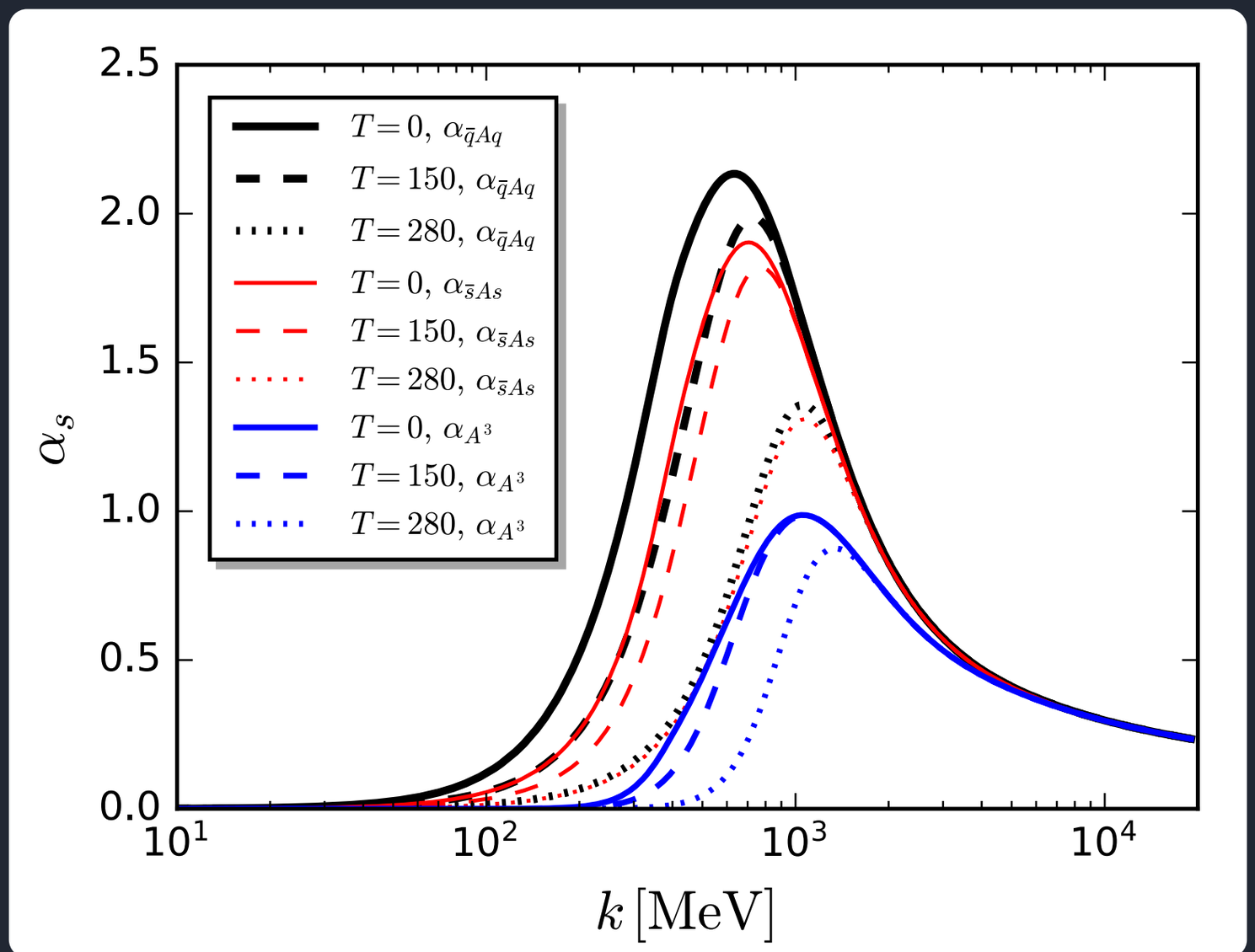


PRELIMINARY RESULTS

[Fu, Pawlowski, FR (in preparation)]

Strong couplings

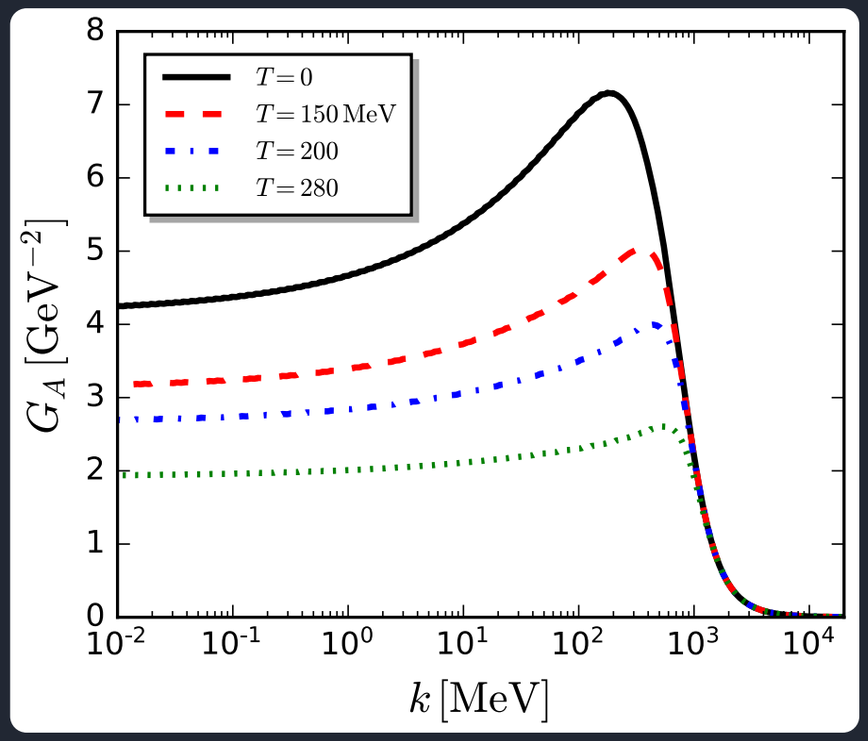


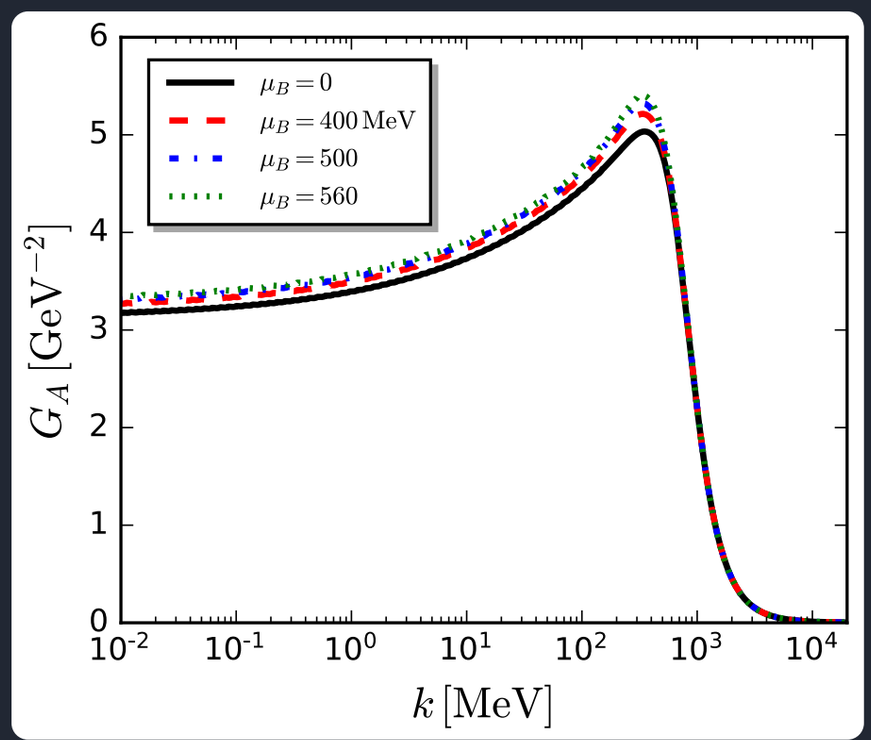


PRELIMINARY RESULTS

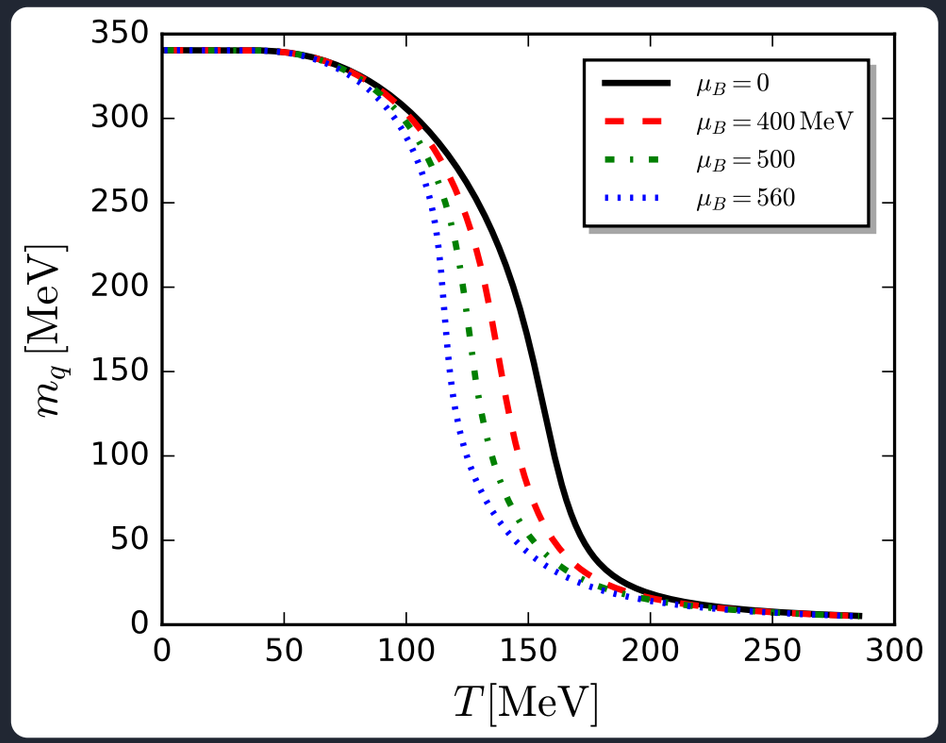
[Fu, Pawlowski, FR (in preparation)]

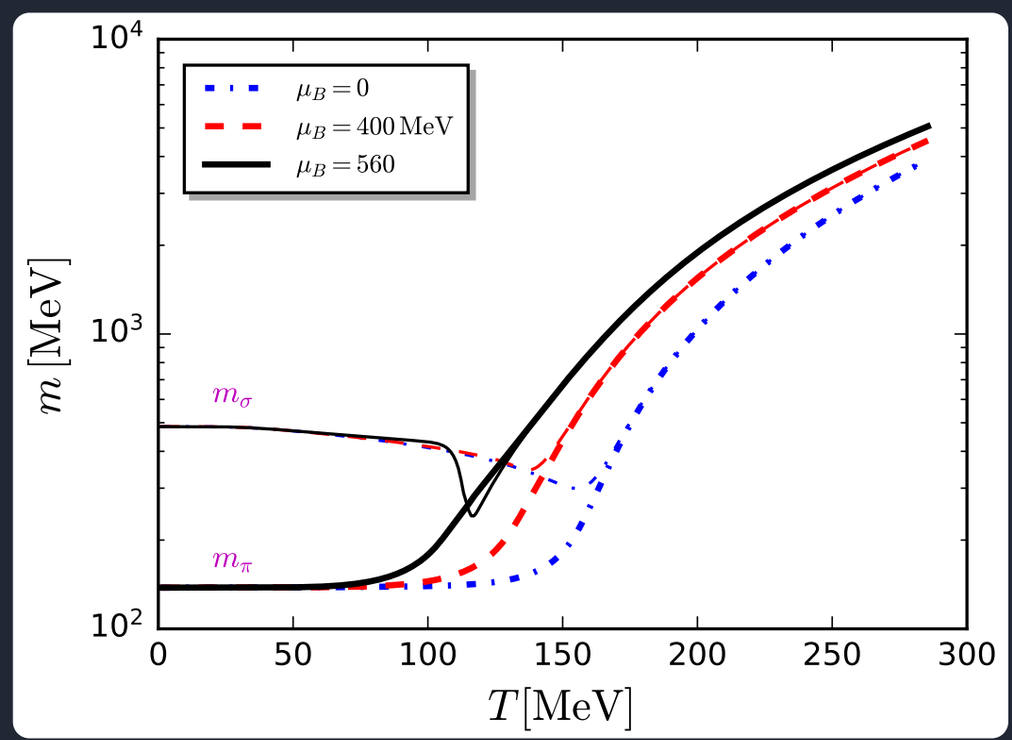
gluon propagator





quark & meson masses

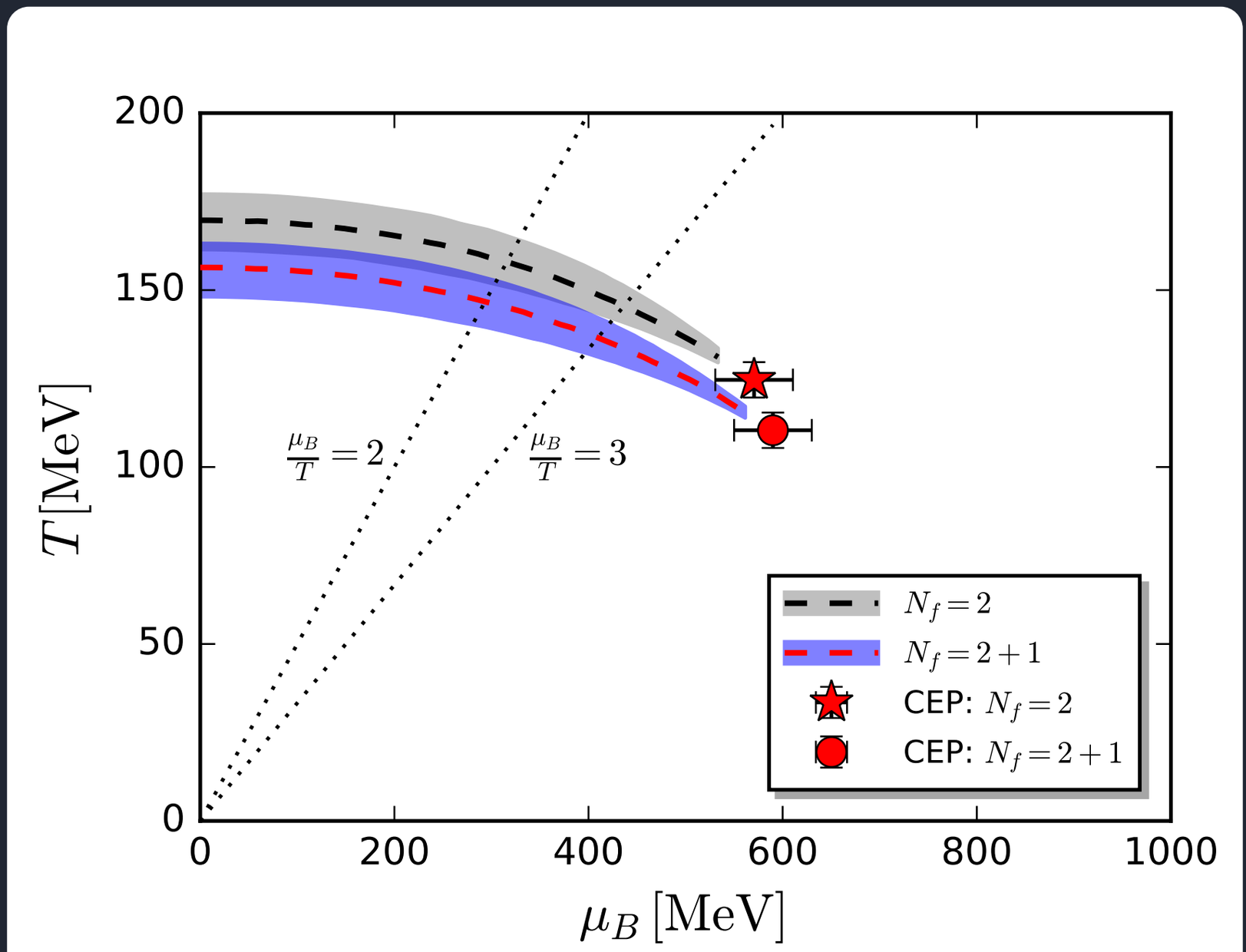




PRELIMINARY RESULTS

[Fu, Pawlowski, FR (in preparation)]

The 2+1 flavor phase diagram



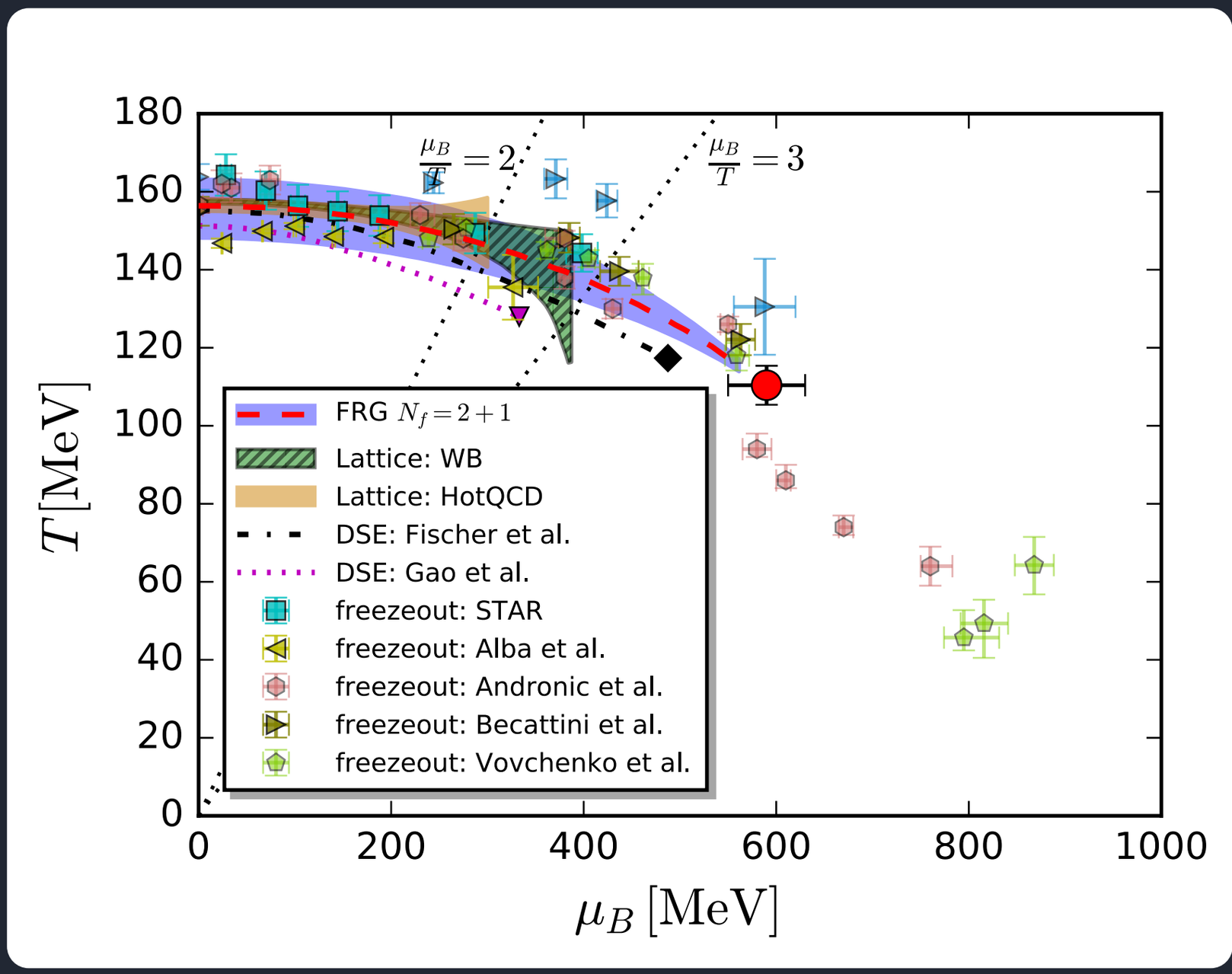
$(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (110^{+5}_{-5}, 590^{+30}_{-20})\,{\rm MeV}$ → $\sqrt{s_{\rm CEP}} \approx 3.8 - 4.5\,{\rm GeV}$

BES Fixed Target!

PRELIMINARY RESULTS

[Fu, Pawlowski, FR (in preparation)]

The 2+1 flavor phase diagram



$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (110^{+5}_{-5}, 590^{+30}_{-20})\,\mathrm{MeV} \longrightarrow \sqrt{s_{\mathrm{CEP}}} \approx 3.8 - 4.5\,\mathrm{GeV}$$

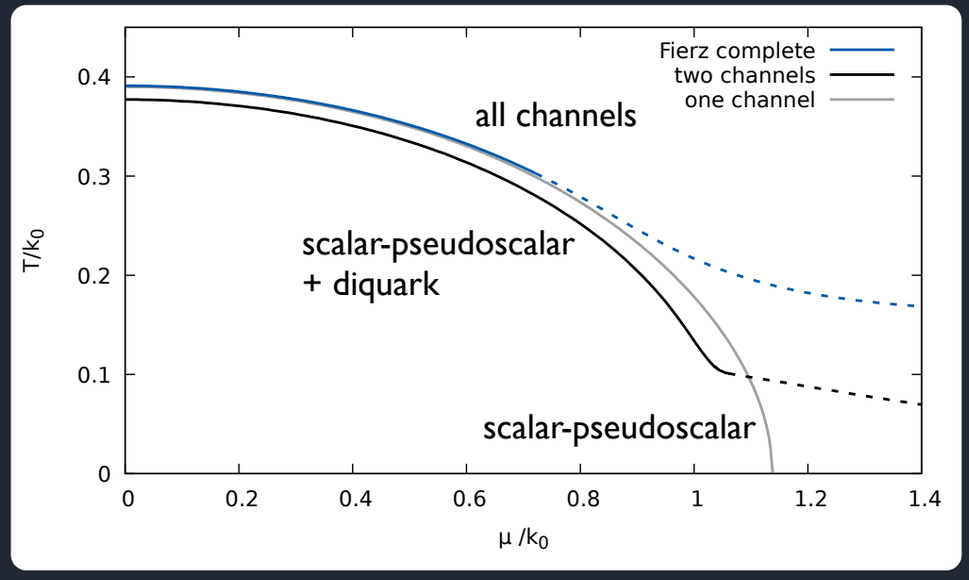
BES Fixed Target!

OUTLOOK

Short- to mid-range goals

- Compute gluon background potential directly [Braun, Gies, Pawlowski, hep-th/0708.2413] [Fischer, Fister, Luecker, Pawlowski, hep-ph/1306.6022]
  - so far: external input from the lattice (introduces a pheno parameter)
  - Polyakov loop potential at finite density

- Include additional 4-quark channels (Fierz-completeness) [Mitter, Pawlowski, Strodthoff, hep-ph/1411.7978] [Braun, Leonhardt, Pospiech, hep-ph/1801.08338]
  - so far: most dominant channel including χSB
  - ω_0 vector channel / diquark channel become relevant at large μ (and `small´ T)
  - access to color-superconducting phases, liquid-gas transition, etc.



- Include additional quark-gluon channels (non-classical) [Cyrol, Mitter, Pawlowski, Strodthoff, hep-ph/1706.06326]
  - crucial for χSB!
  - so far: only classical channel (need additional pheno parameter to compensate for missing channels)

SUMMARY

- First steps towards a systematic computation of the QCD phase diagram from first principles
- indications that CEP is within BES Fixed Target region