MUSIC with diffusion

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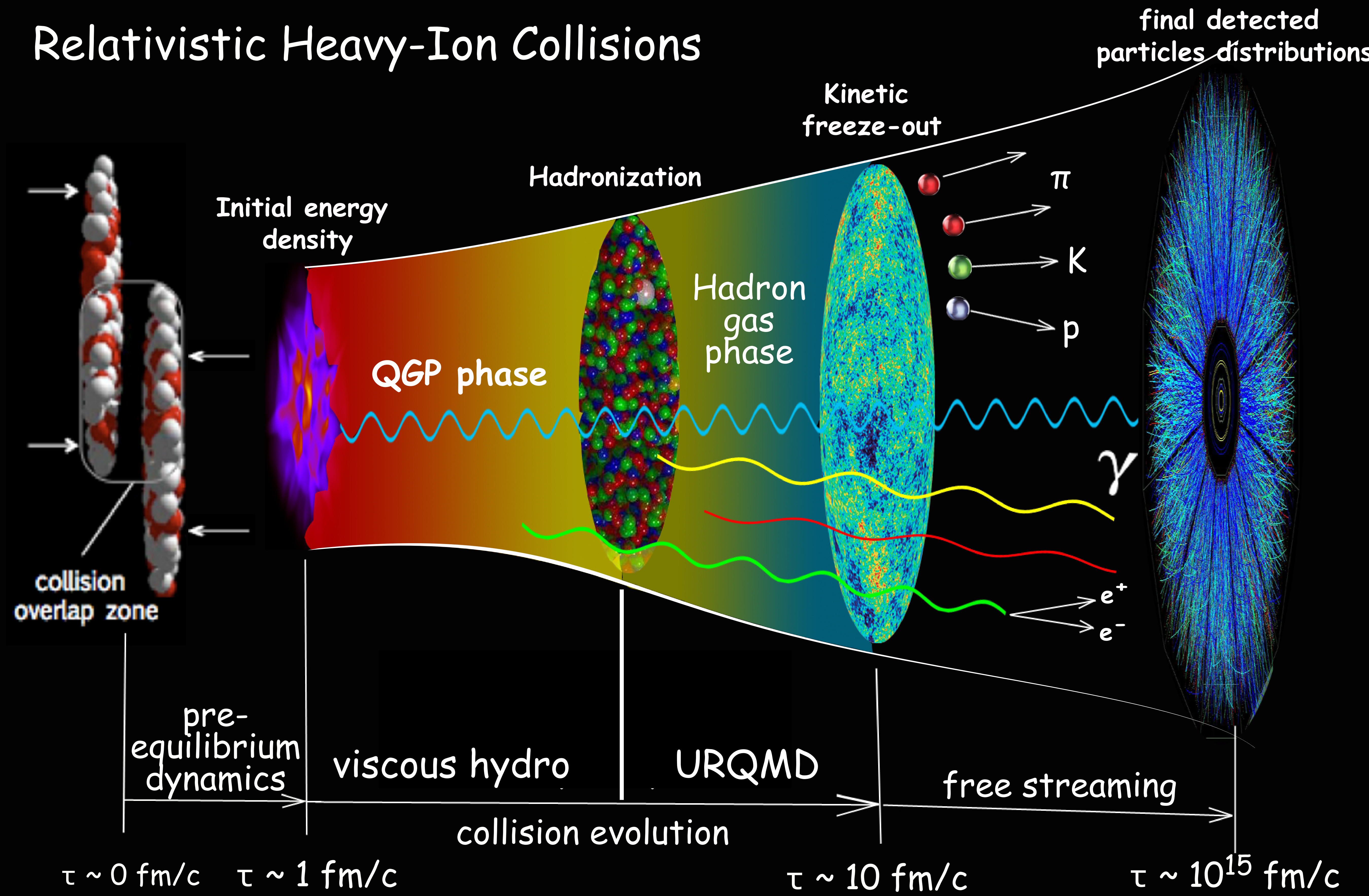
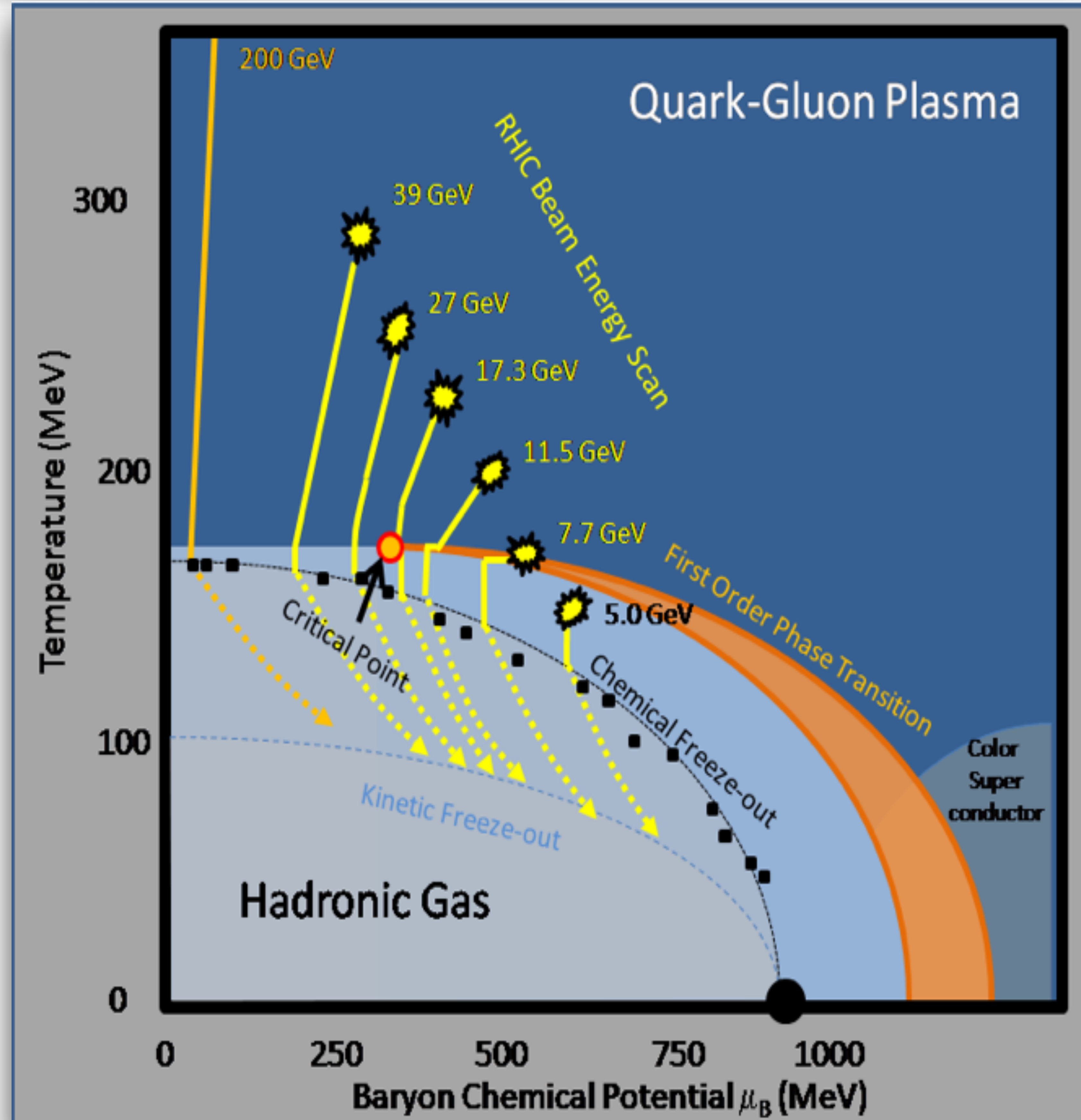
McGill University

In collaboration with Gabriel Denicol, Akihiko Monnai, Bjoern Schenke, Sangyong Jeon, and Charles Gale

Little Bang


Relativistic Heavy-Ion Collisions
final detected particles distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e−
collision overlap zone
pre-equilibrium dynamics
viscous hydro
URQMD
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Exploring the phase of QCD



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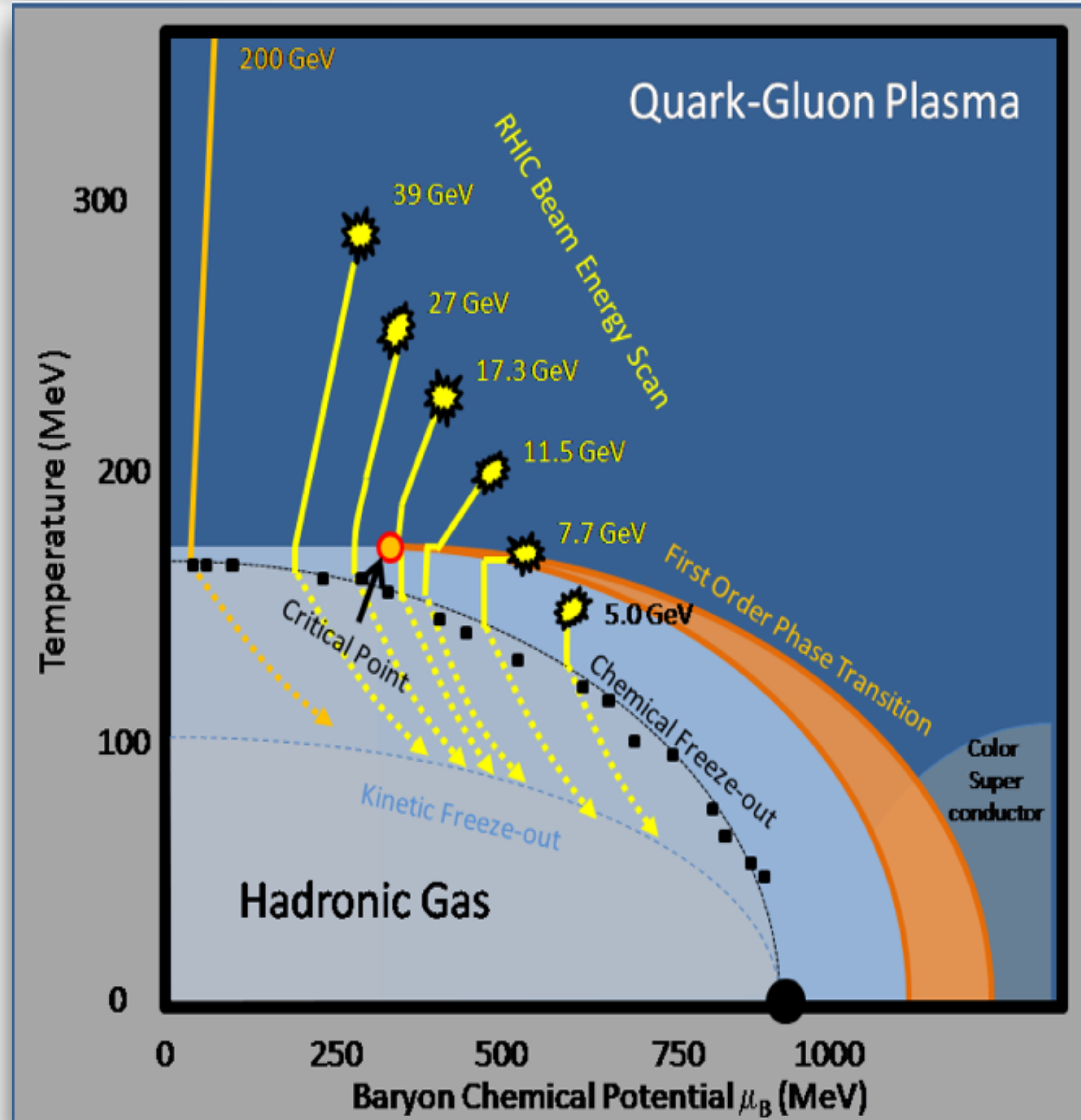

- Event-by-event fluctuating initial conditions
- (3+1)-d dissipative hydrodynamic modelling of the QGP
- Microscopic description for hadronic phase

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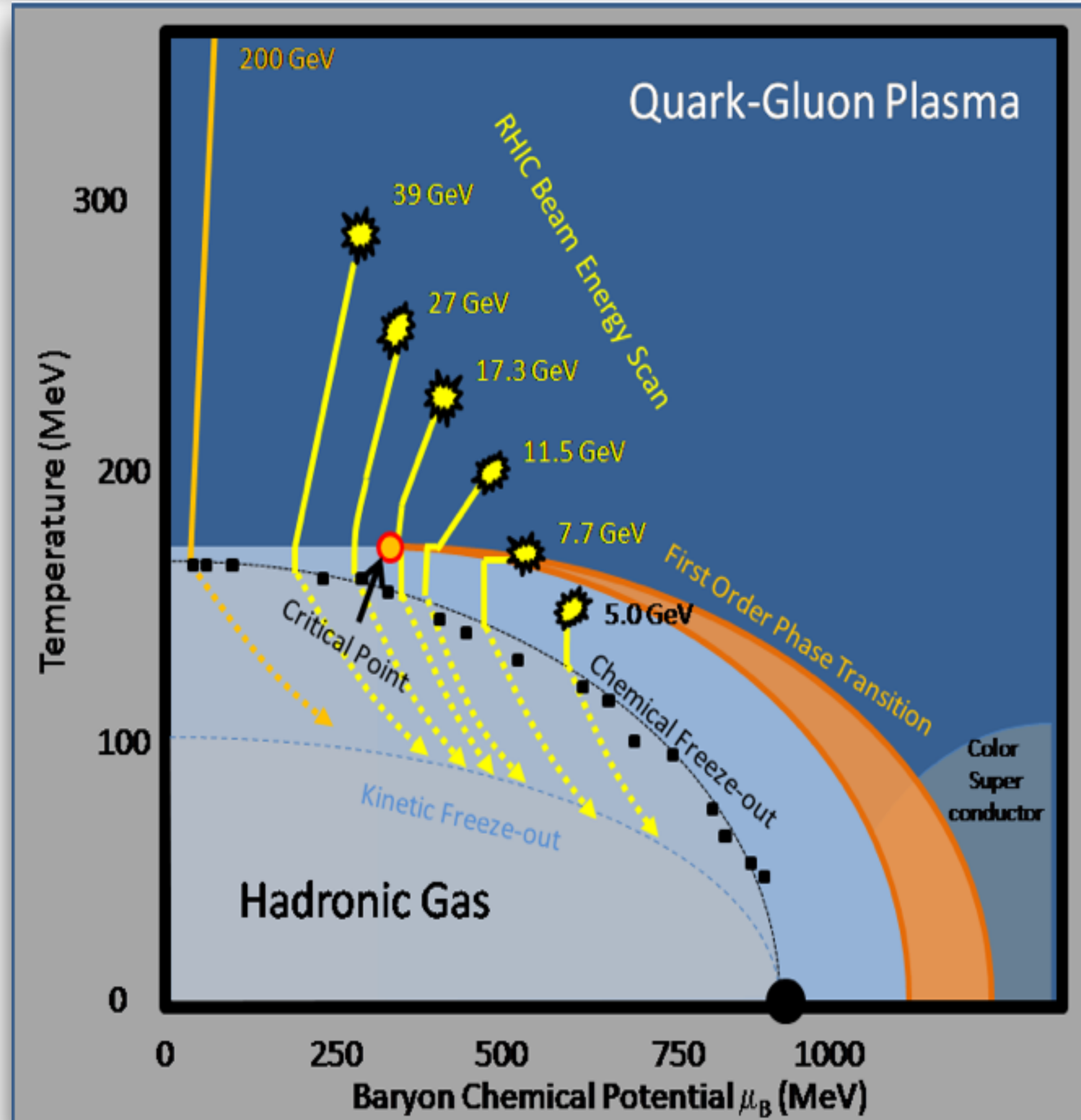

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  (AMPT, UrQMD, MCGlb*, ...)
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  MUSIC
- Microscopic description for hadronic phase
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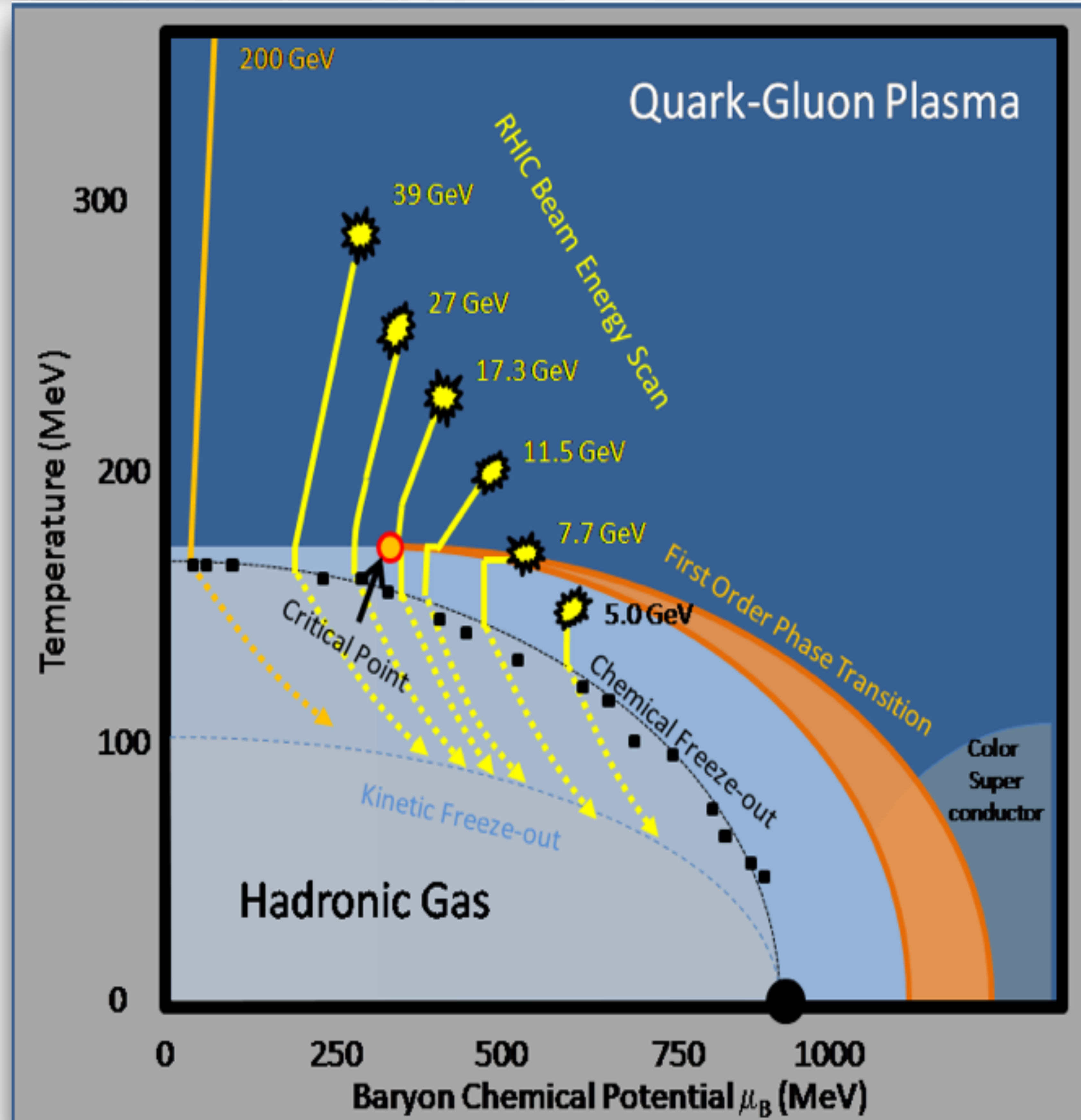

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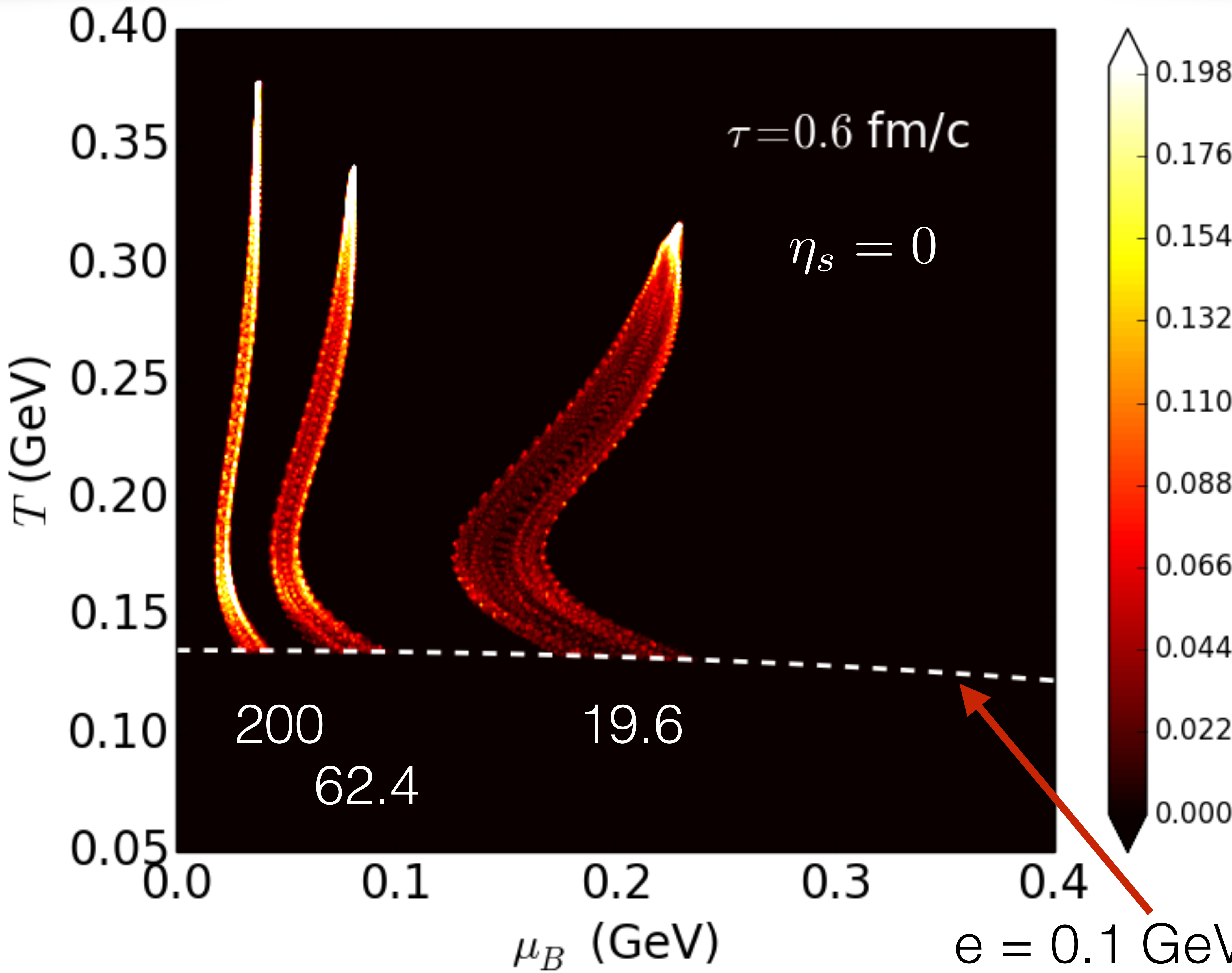

e = 0.1 GeV/fm^3

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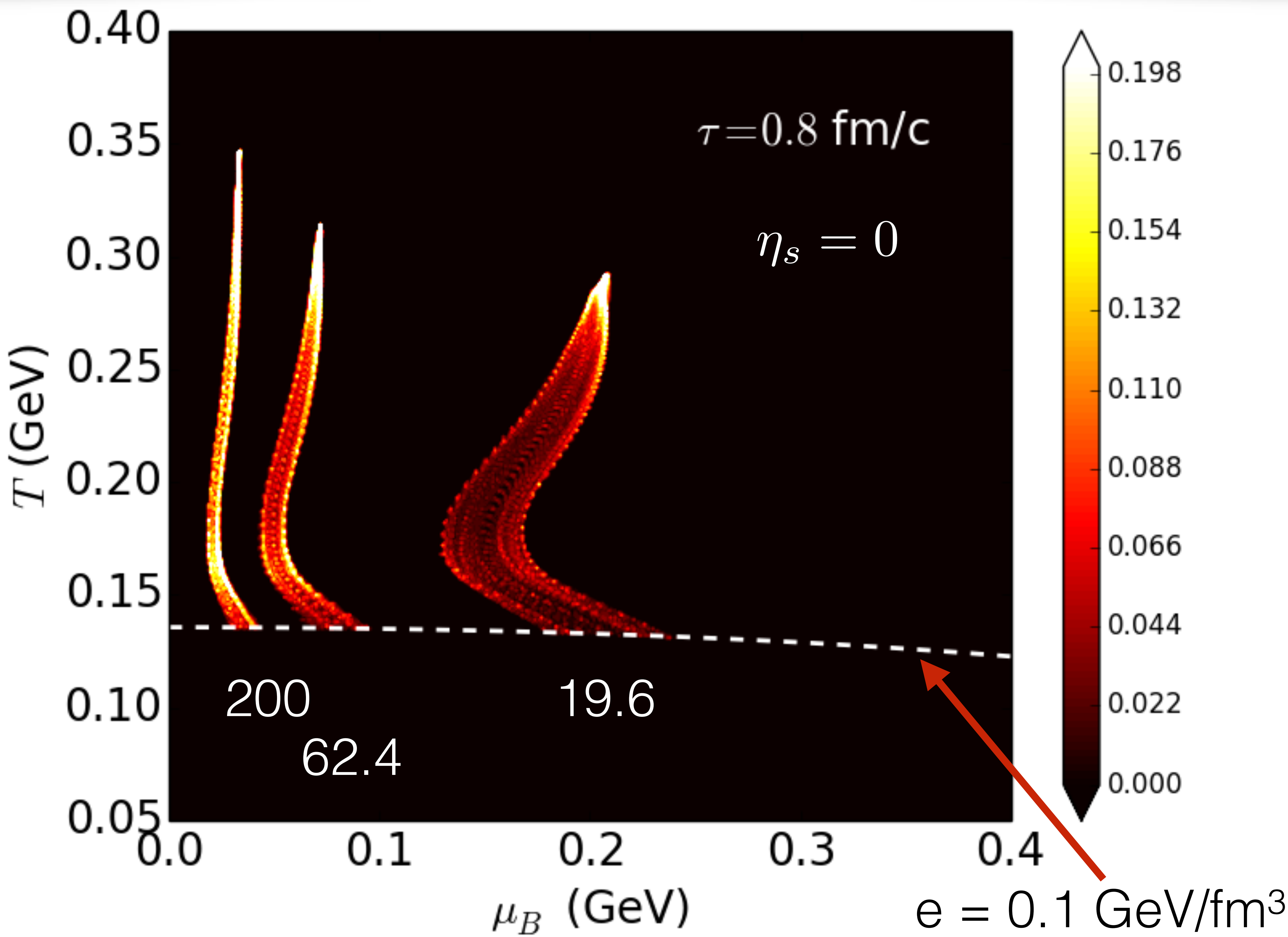

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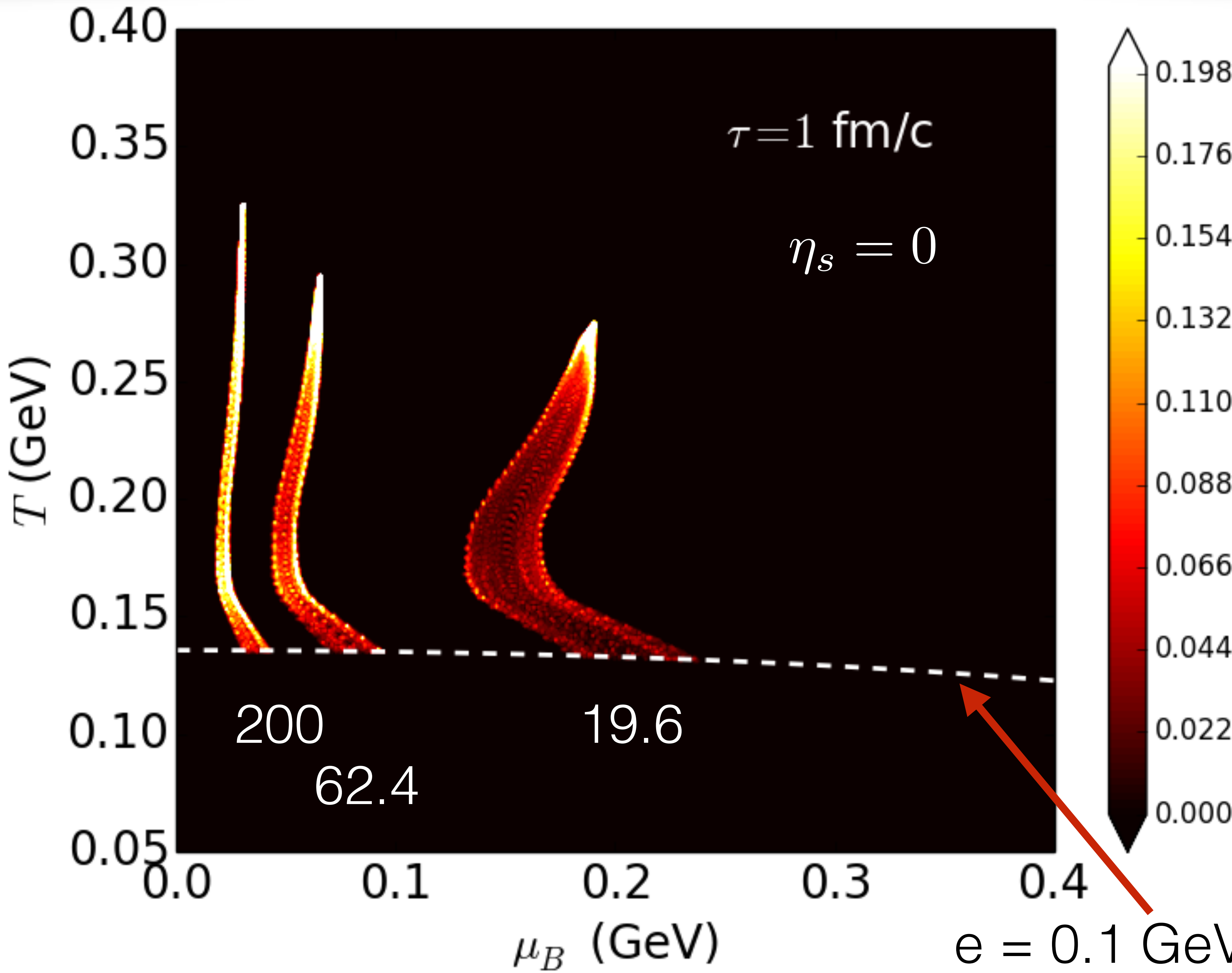

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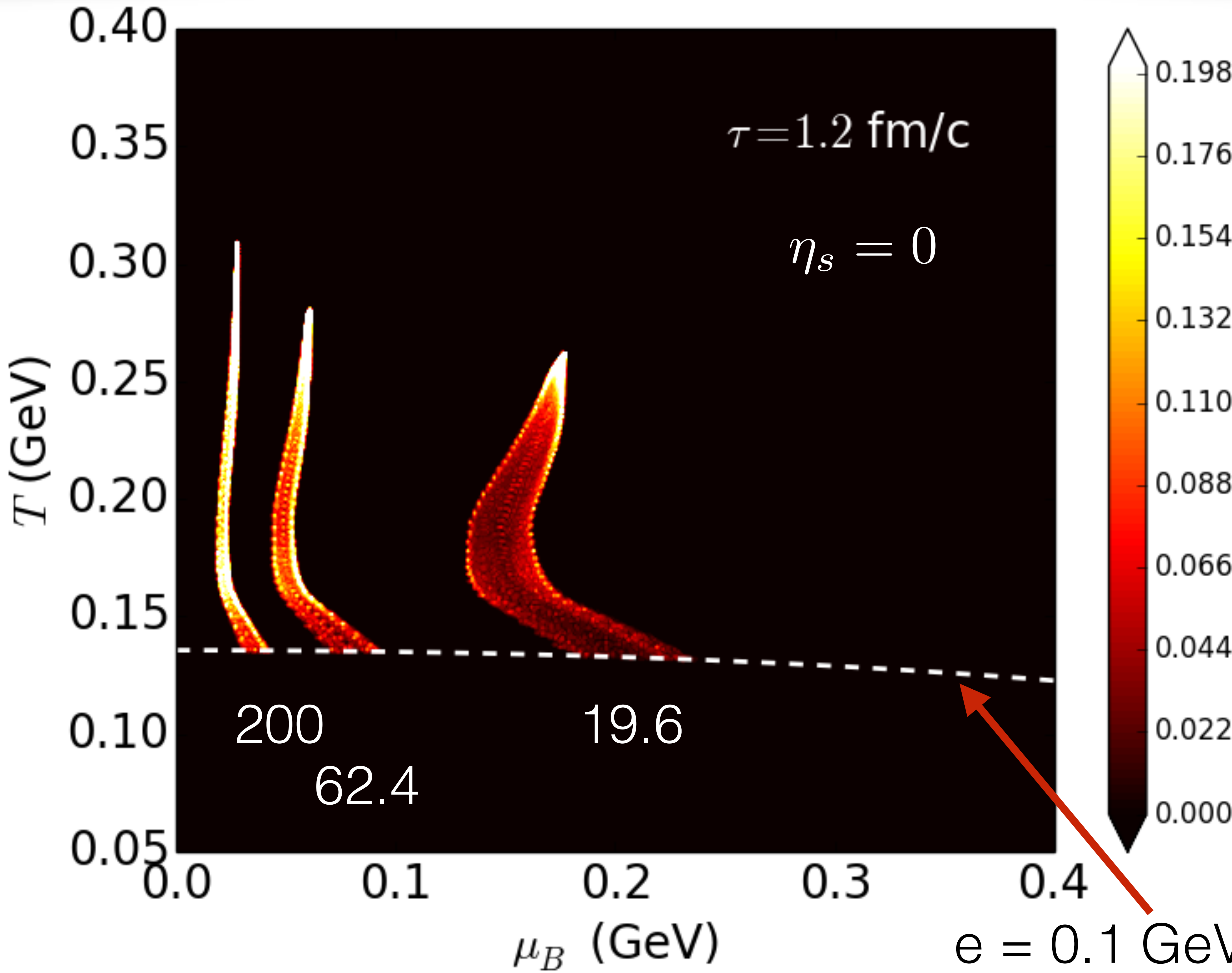

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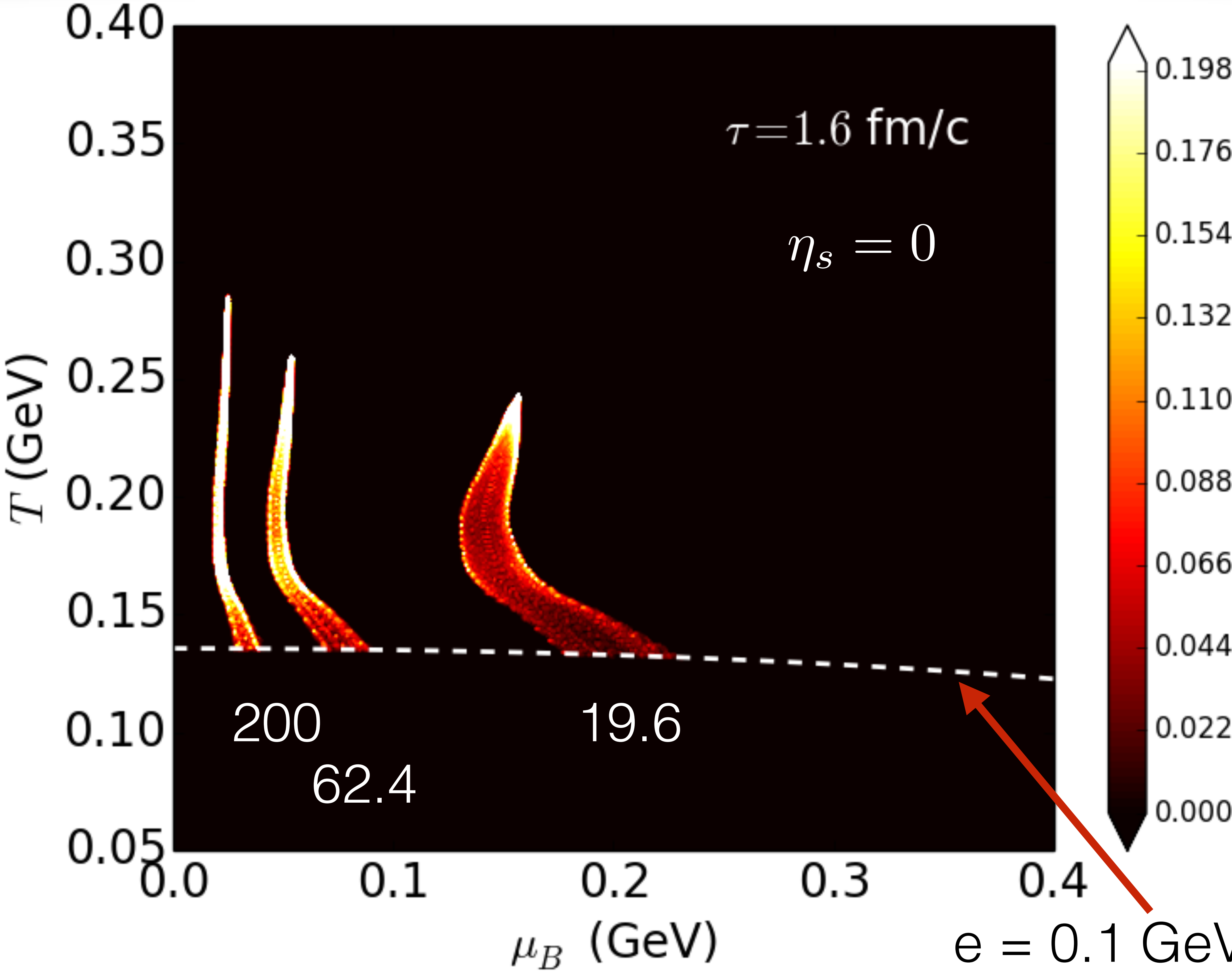

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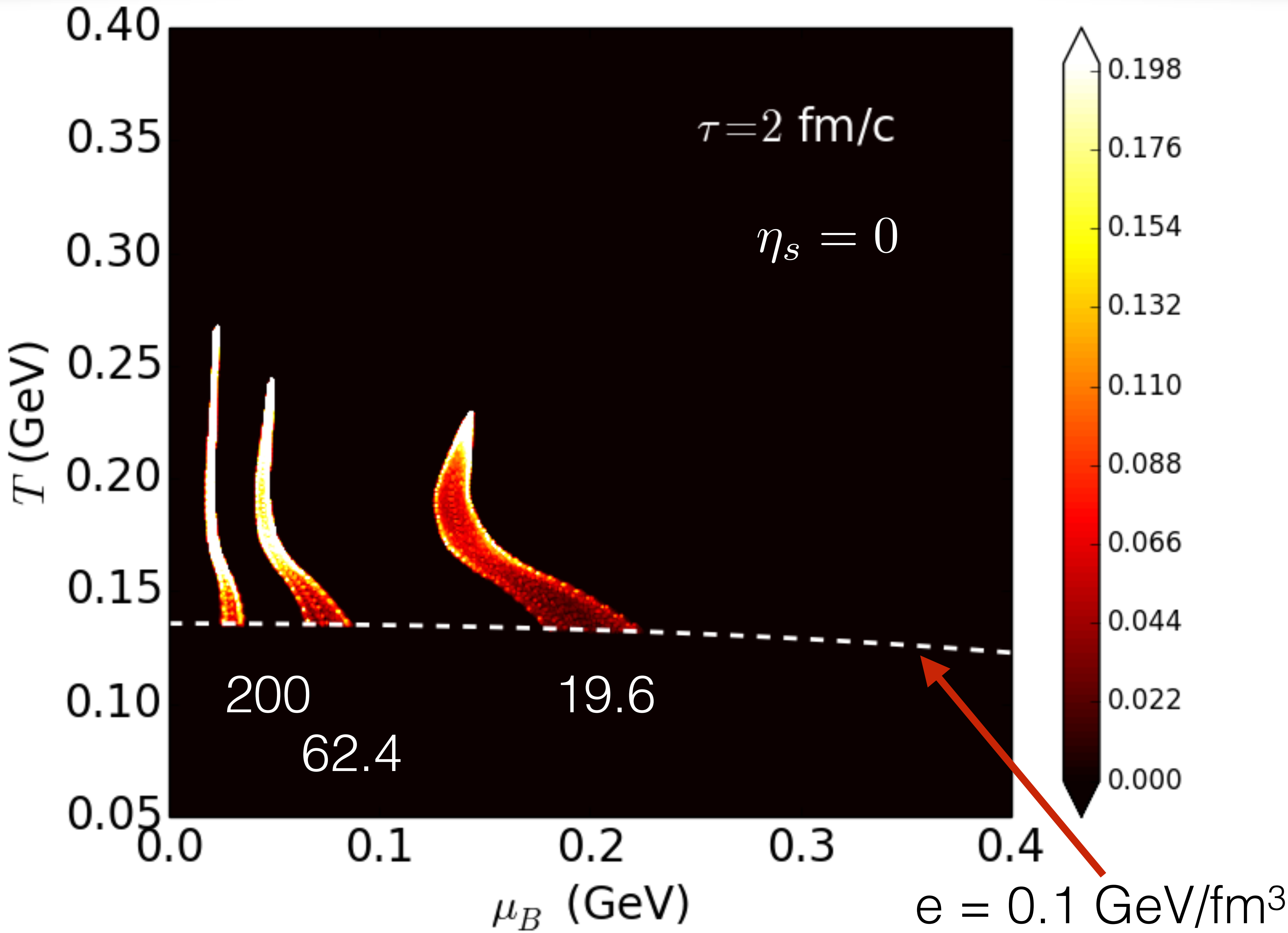

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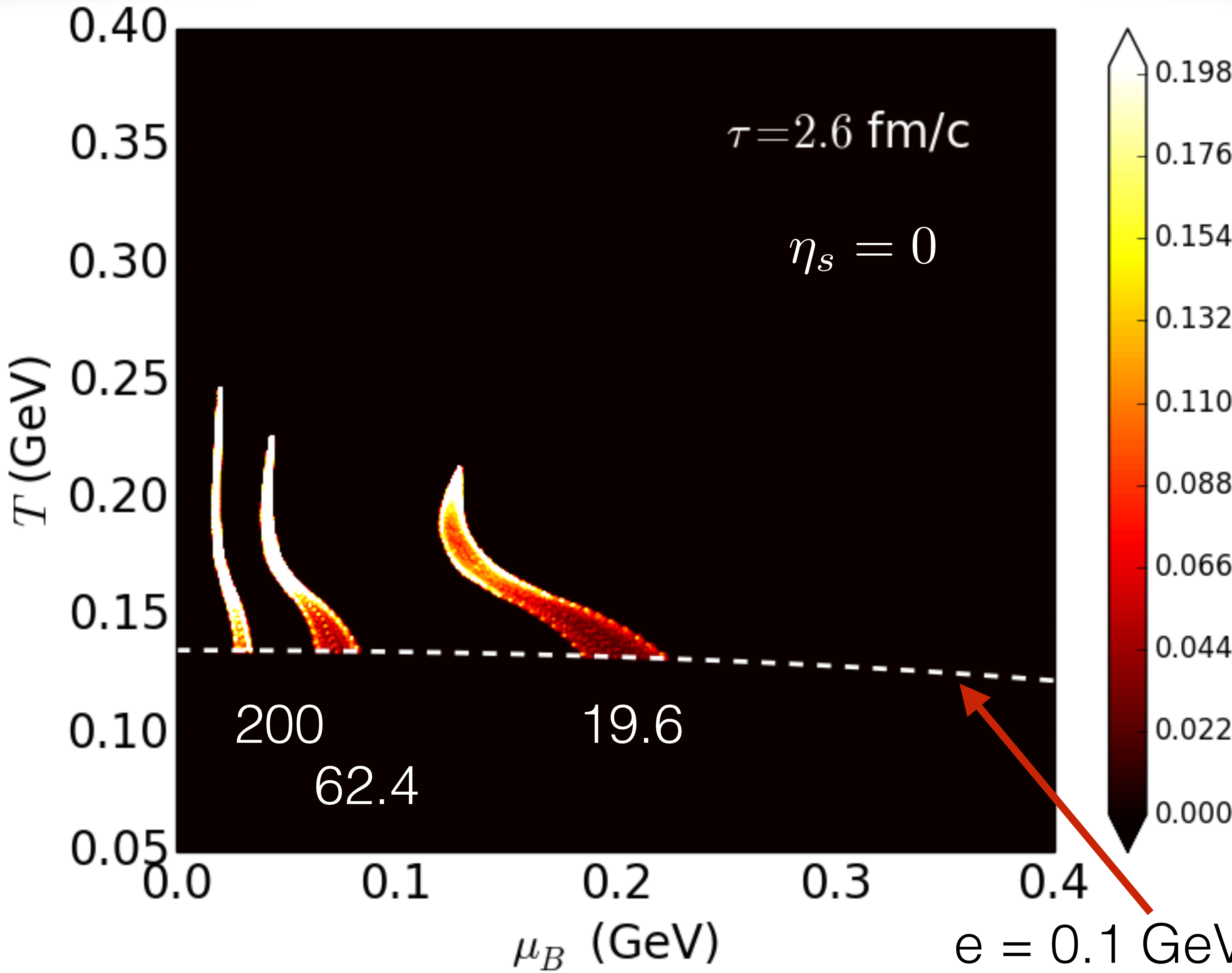

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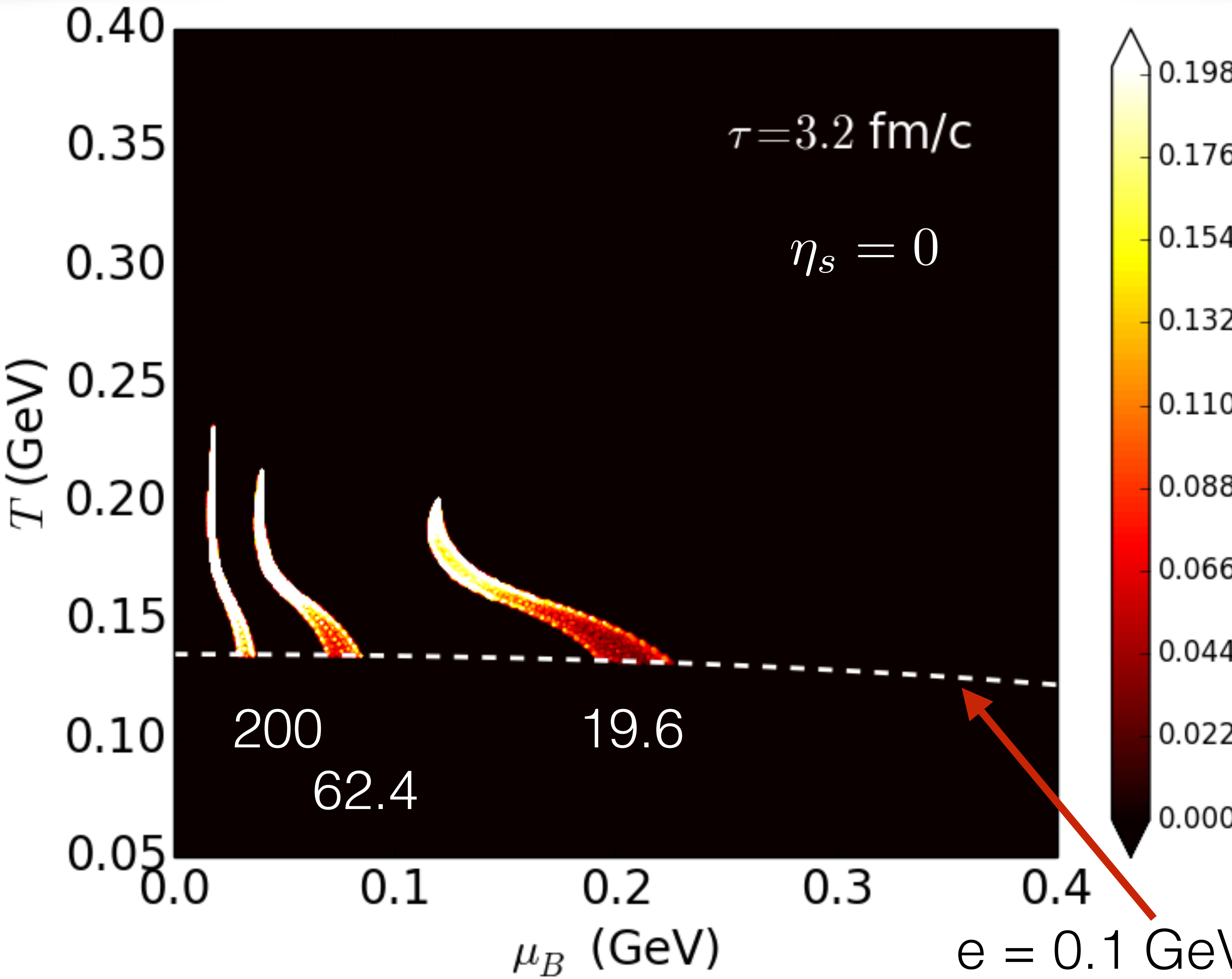

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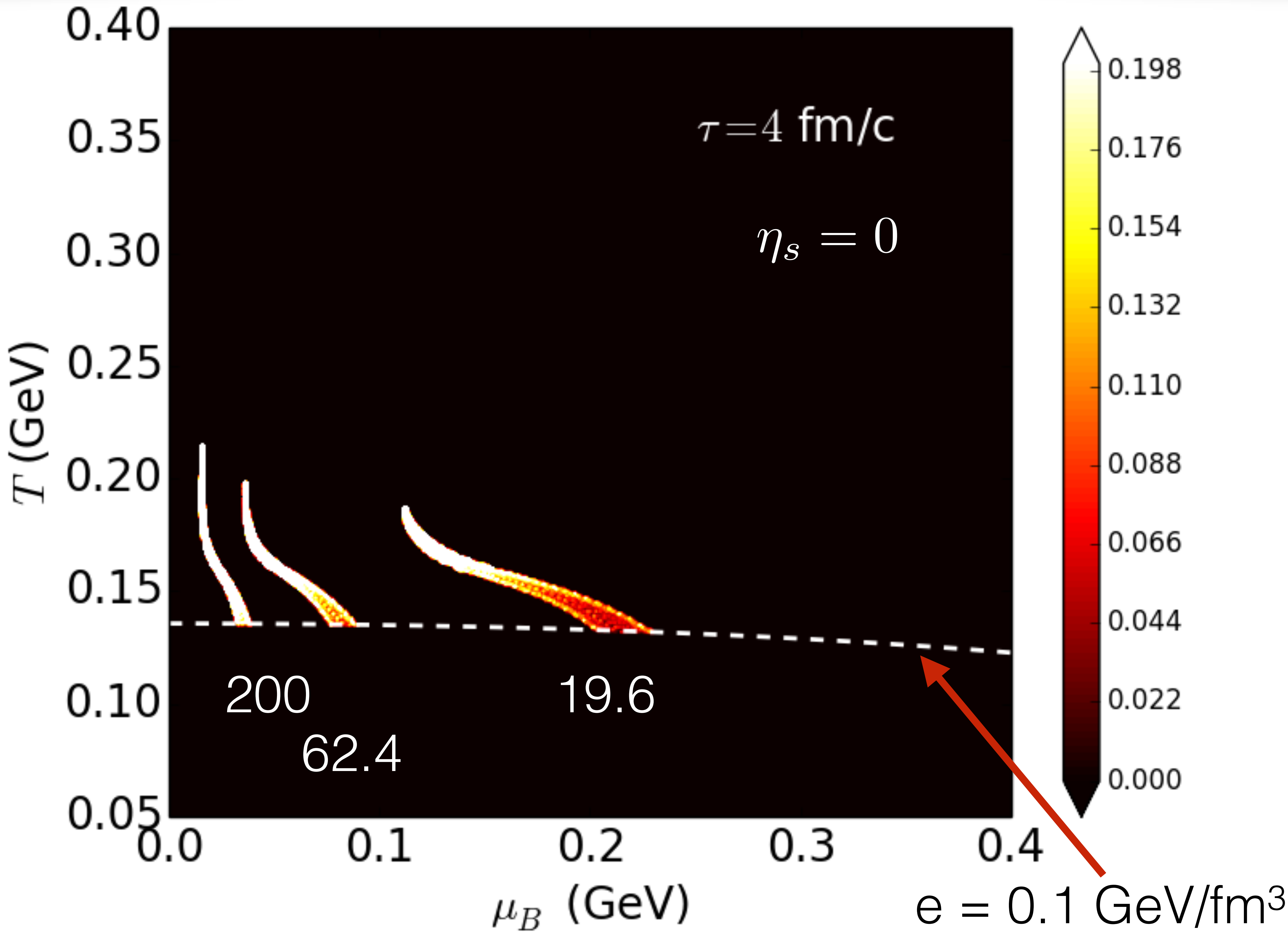

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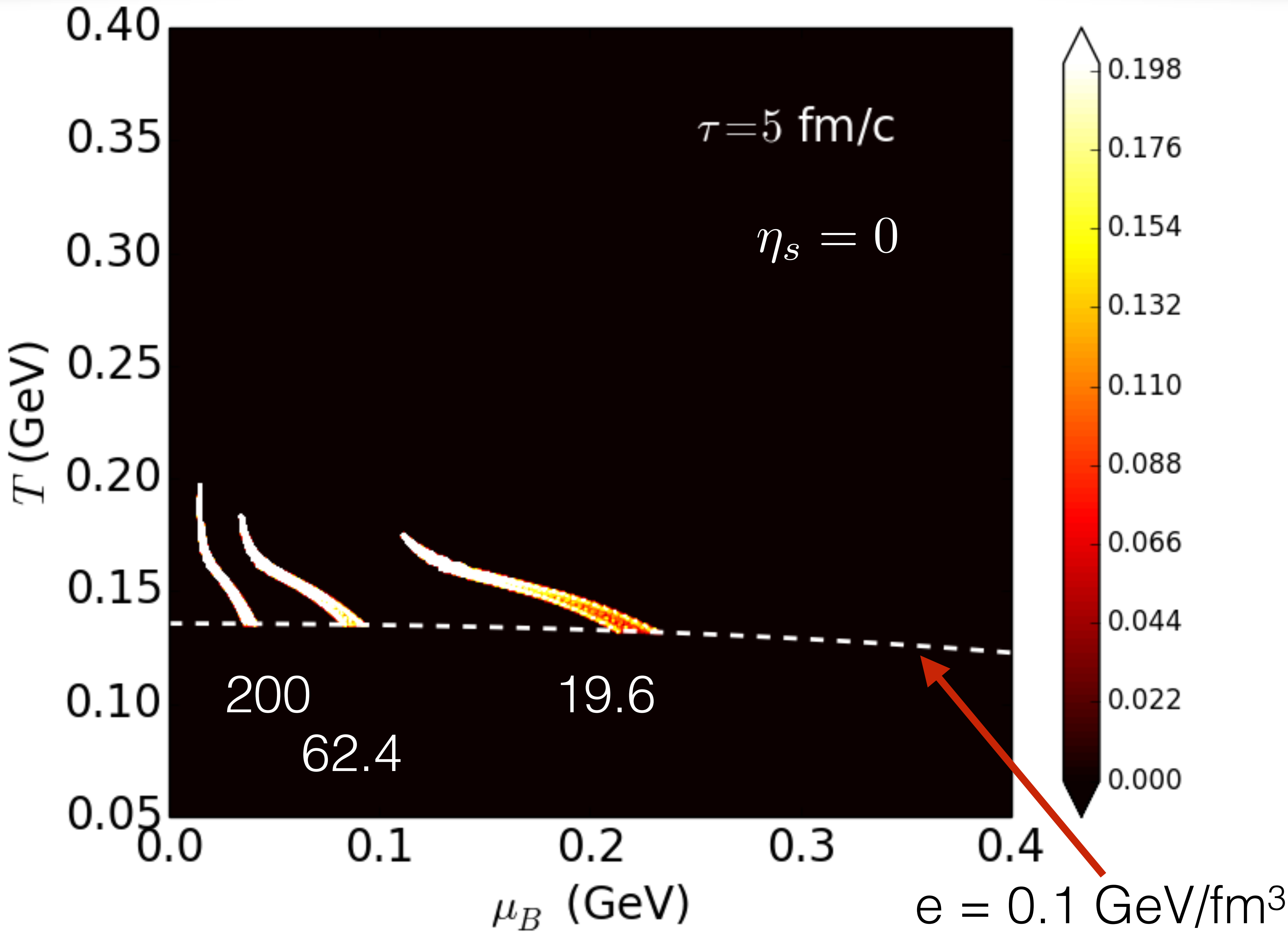

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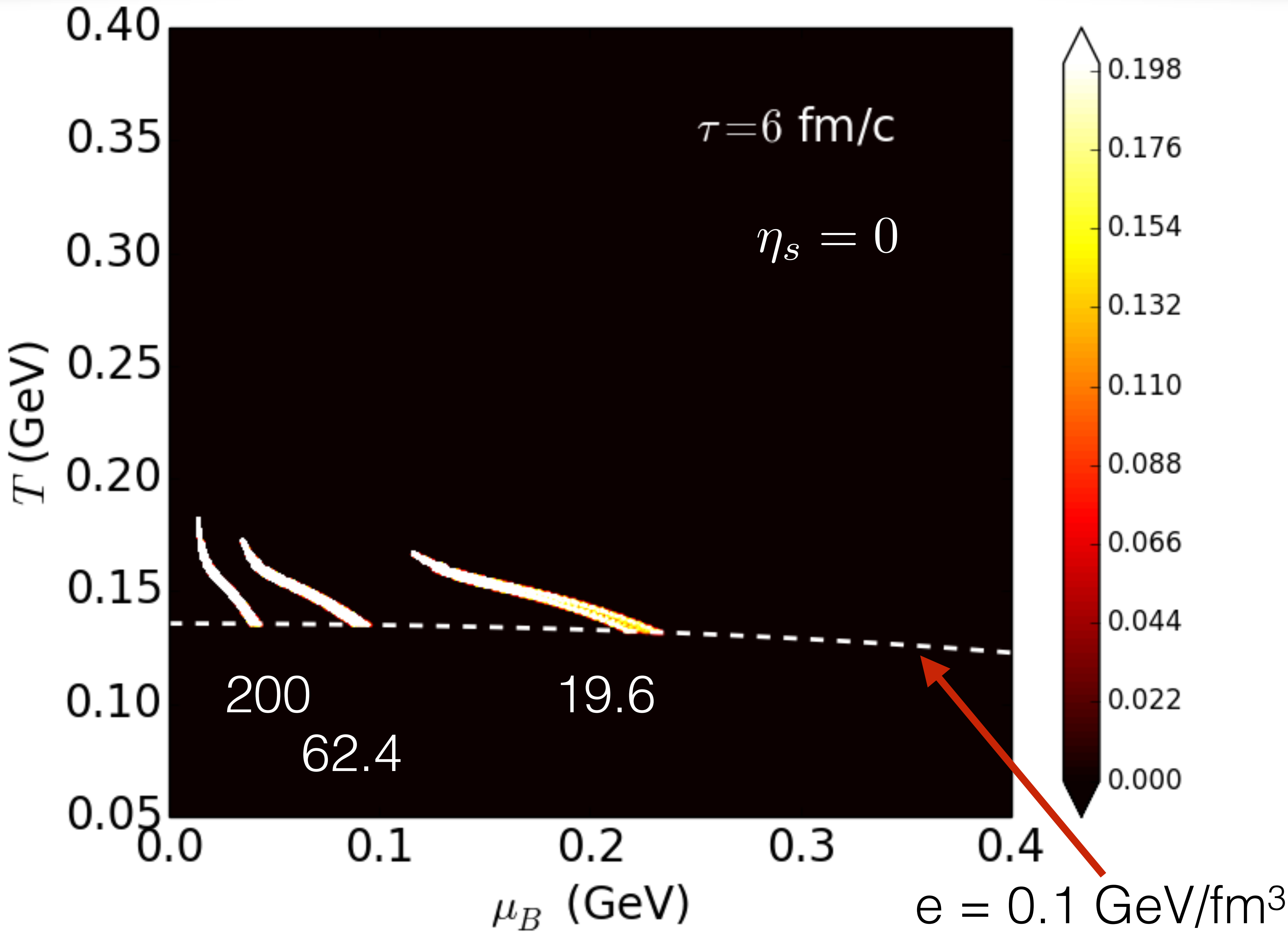

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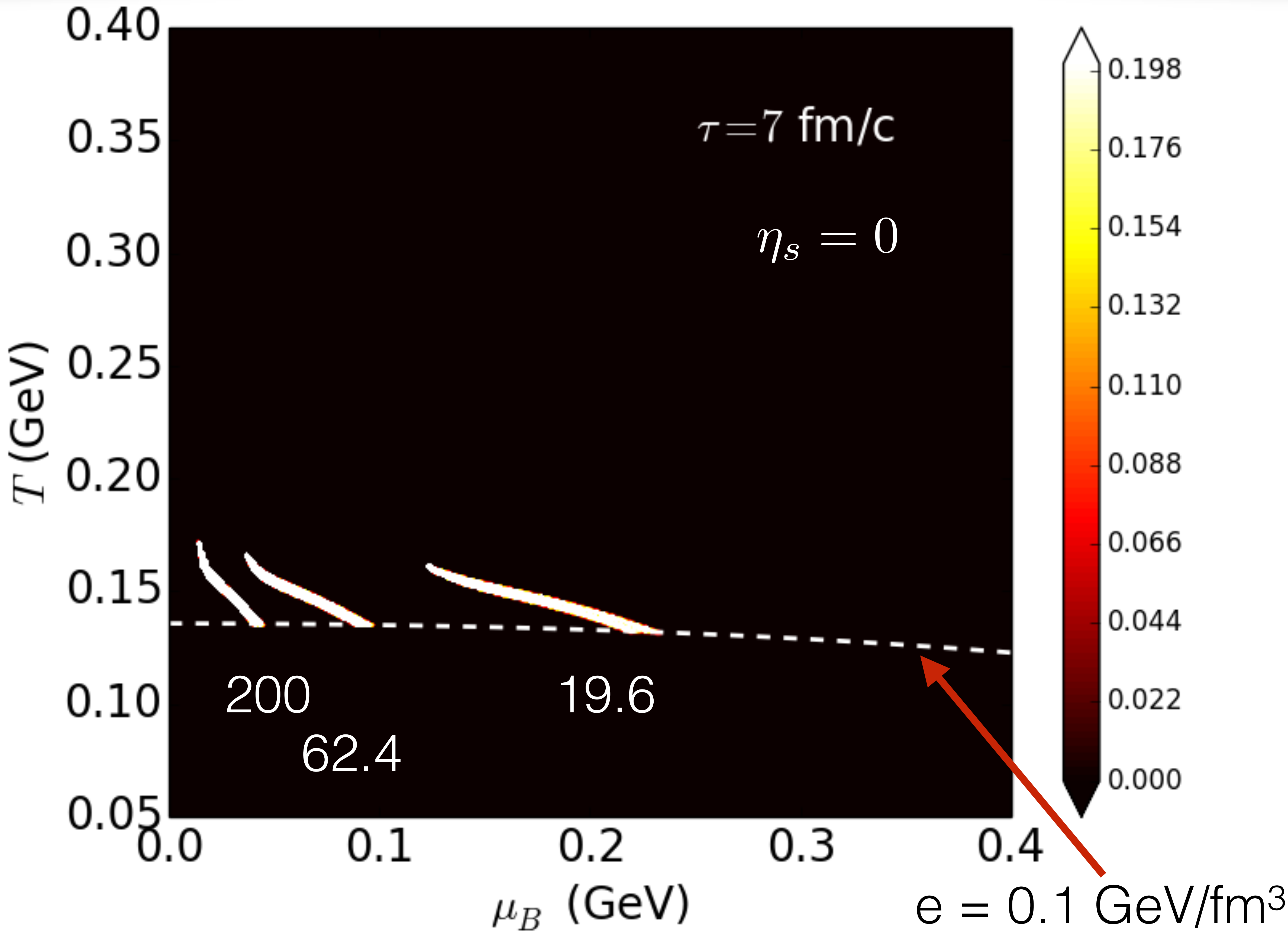

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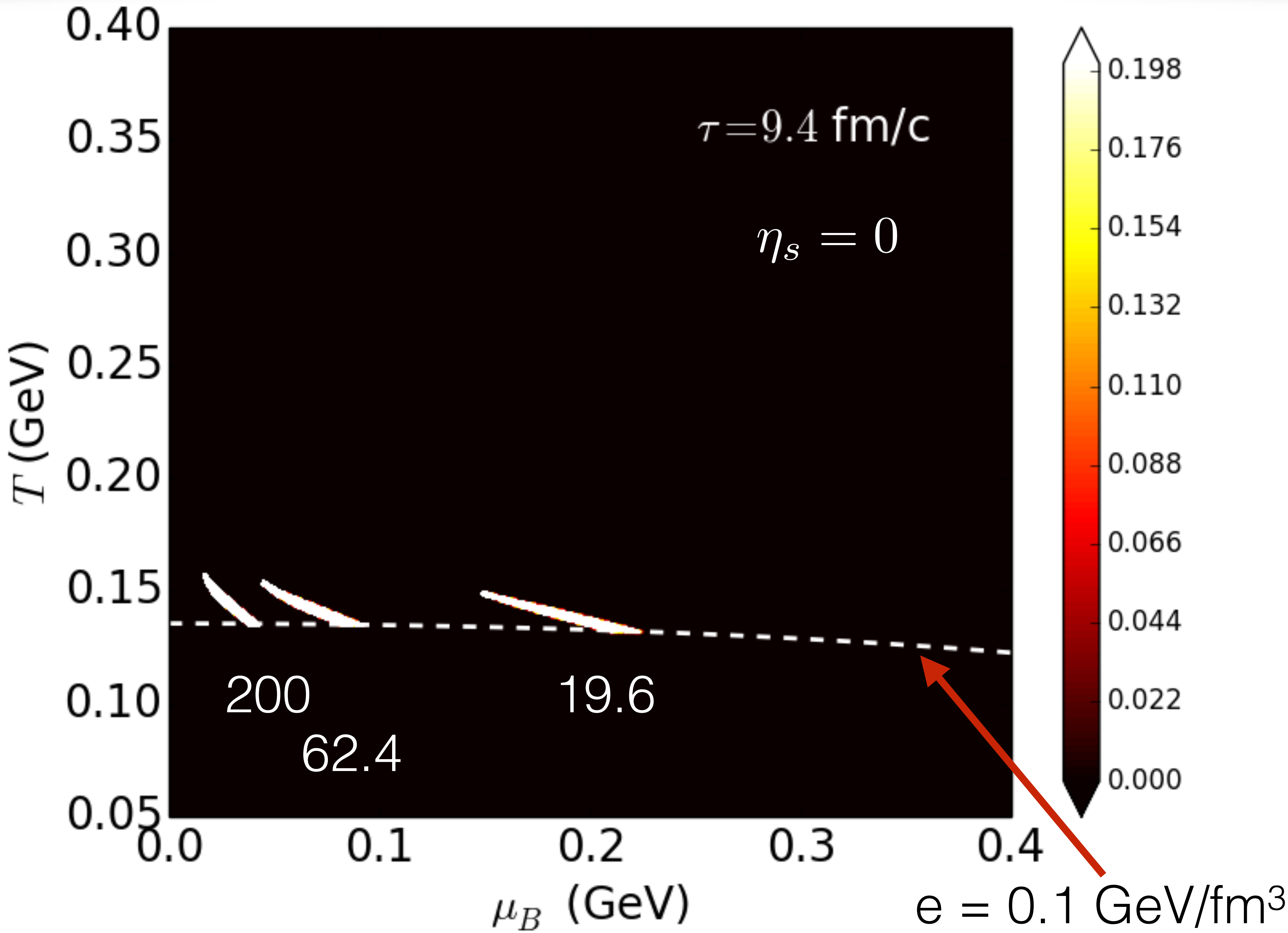

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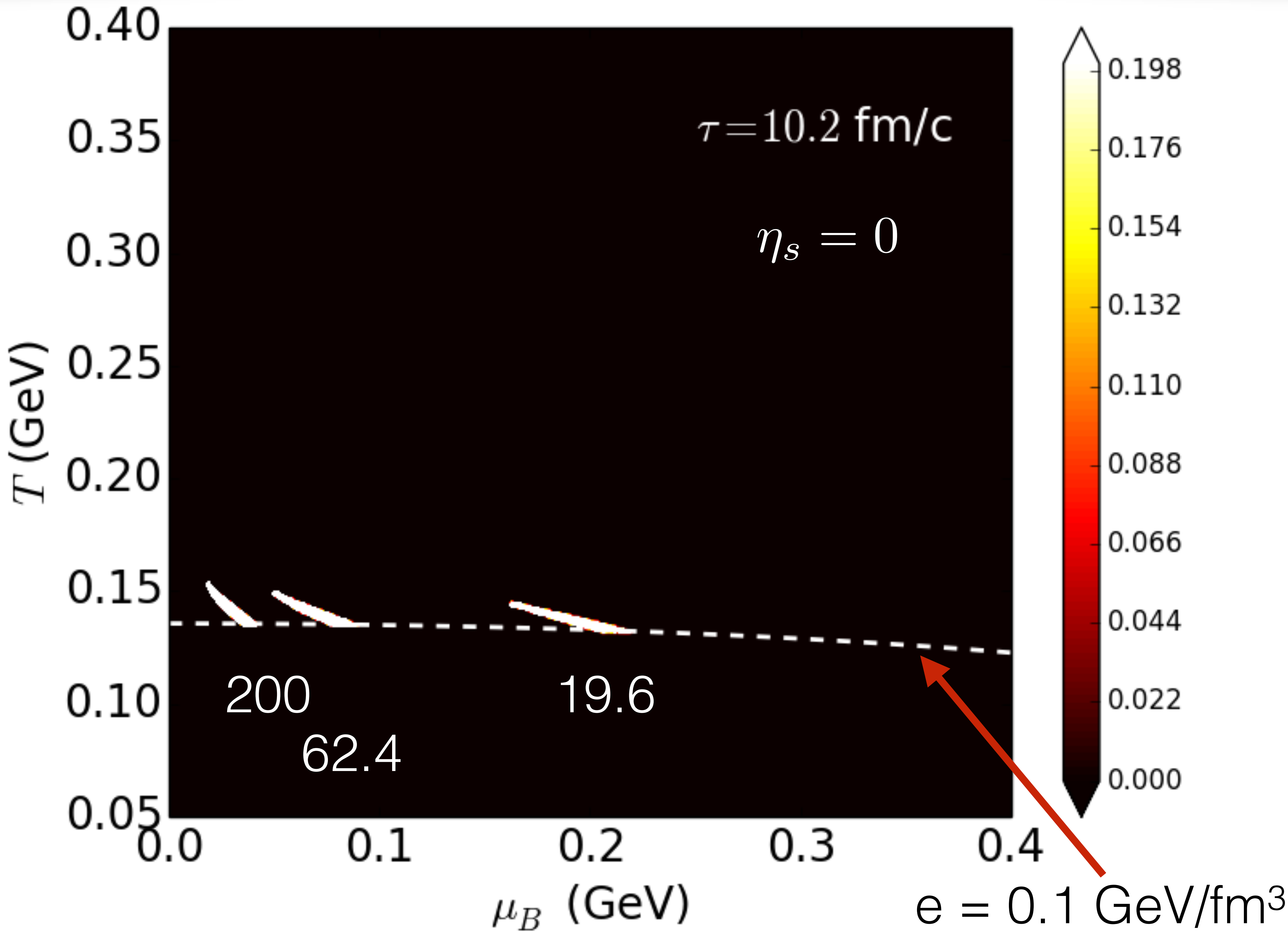

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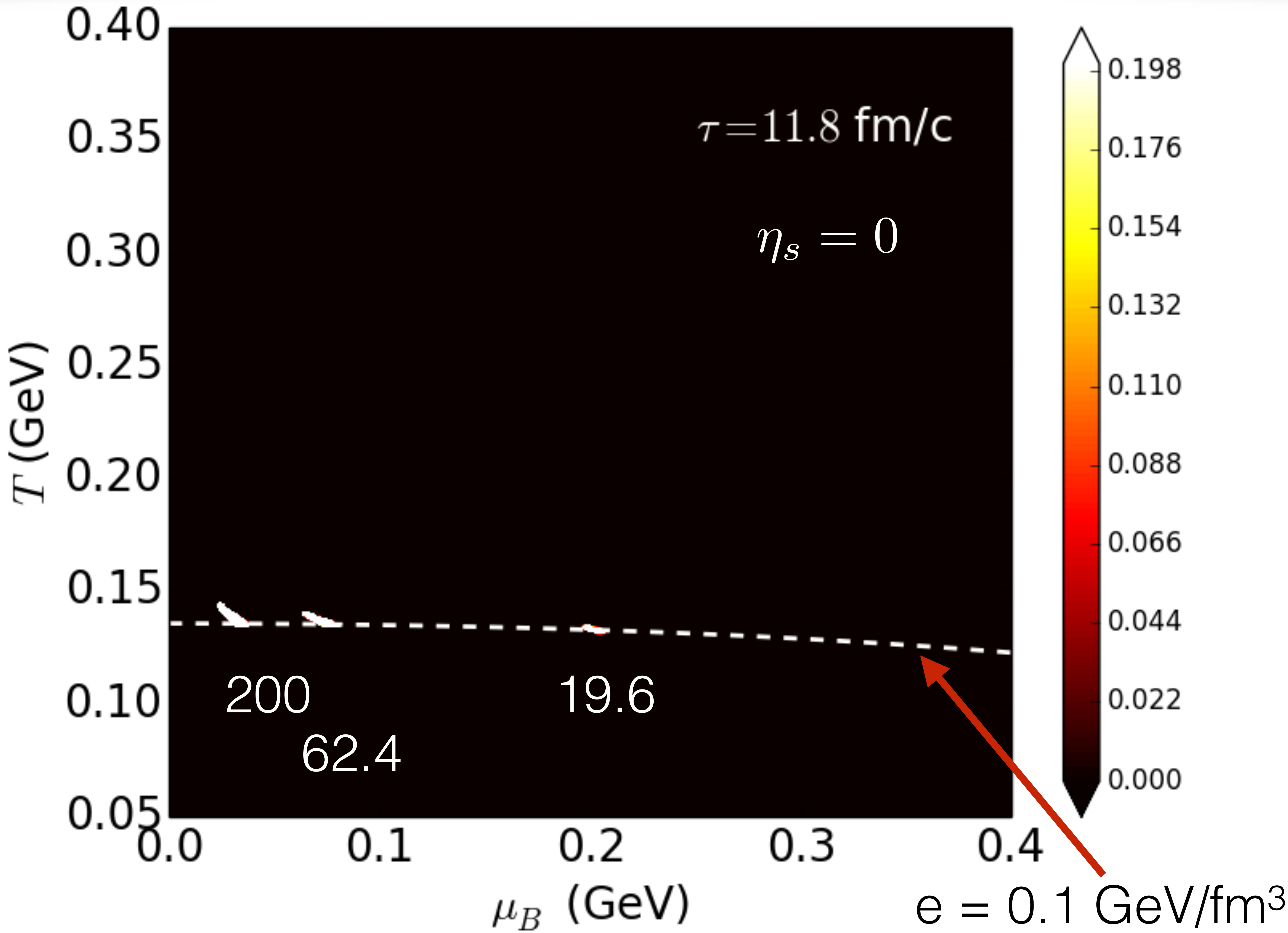

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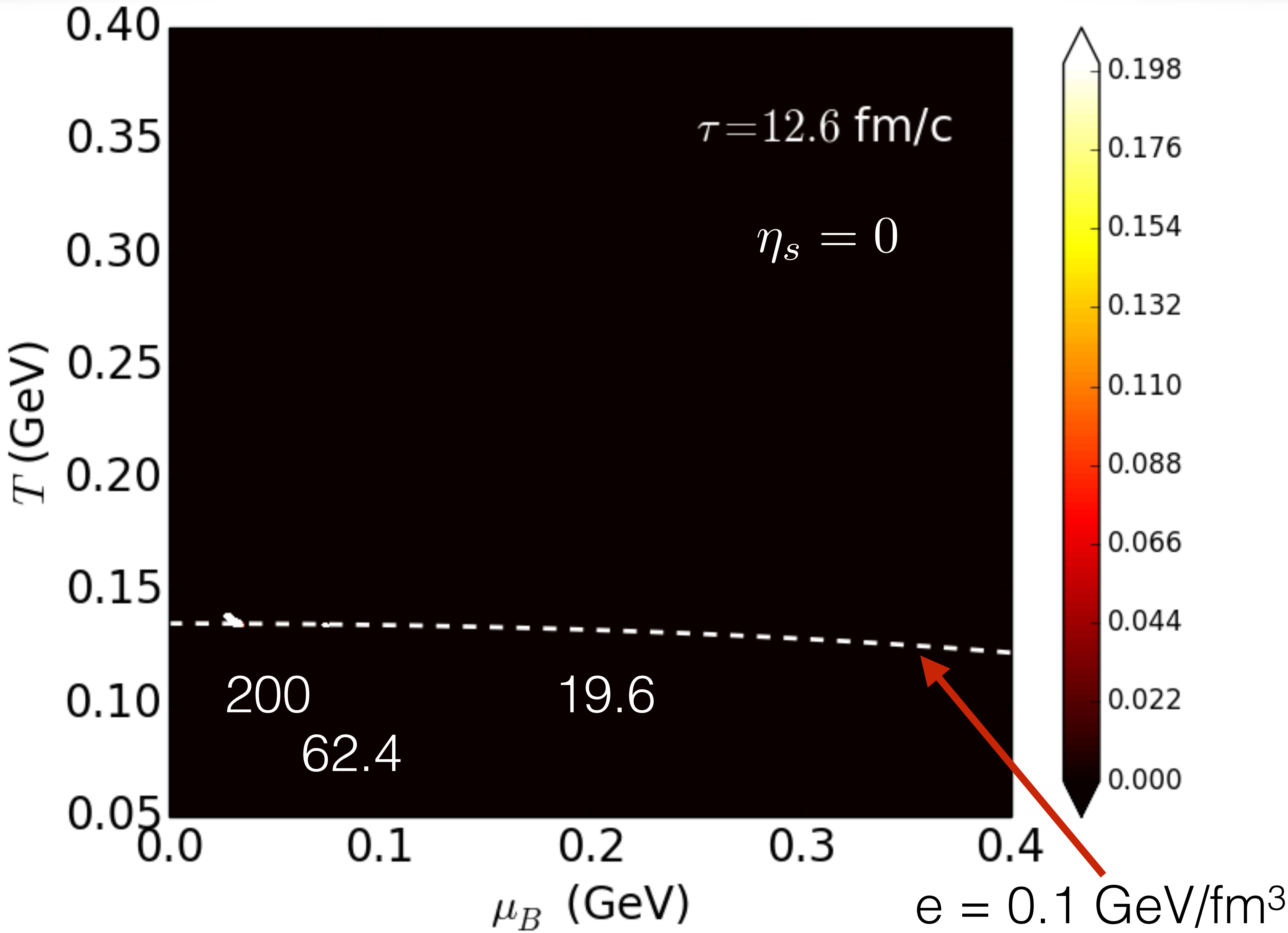

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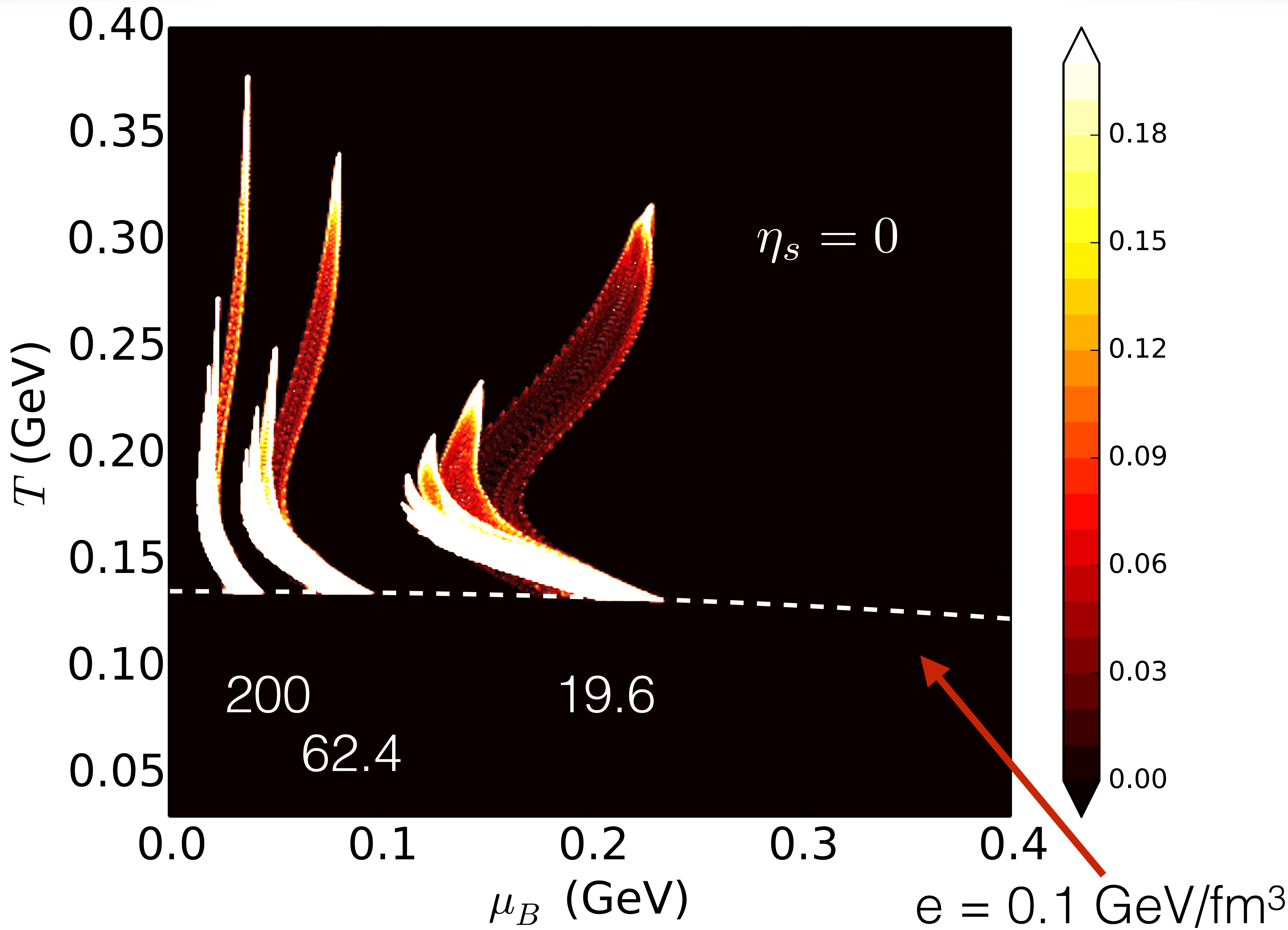


Initialize MUSIC with net baryon density

Since baryon number is conserved,

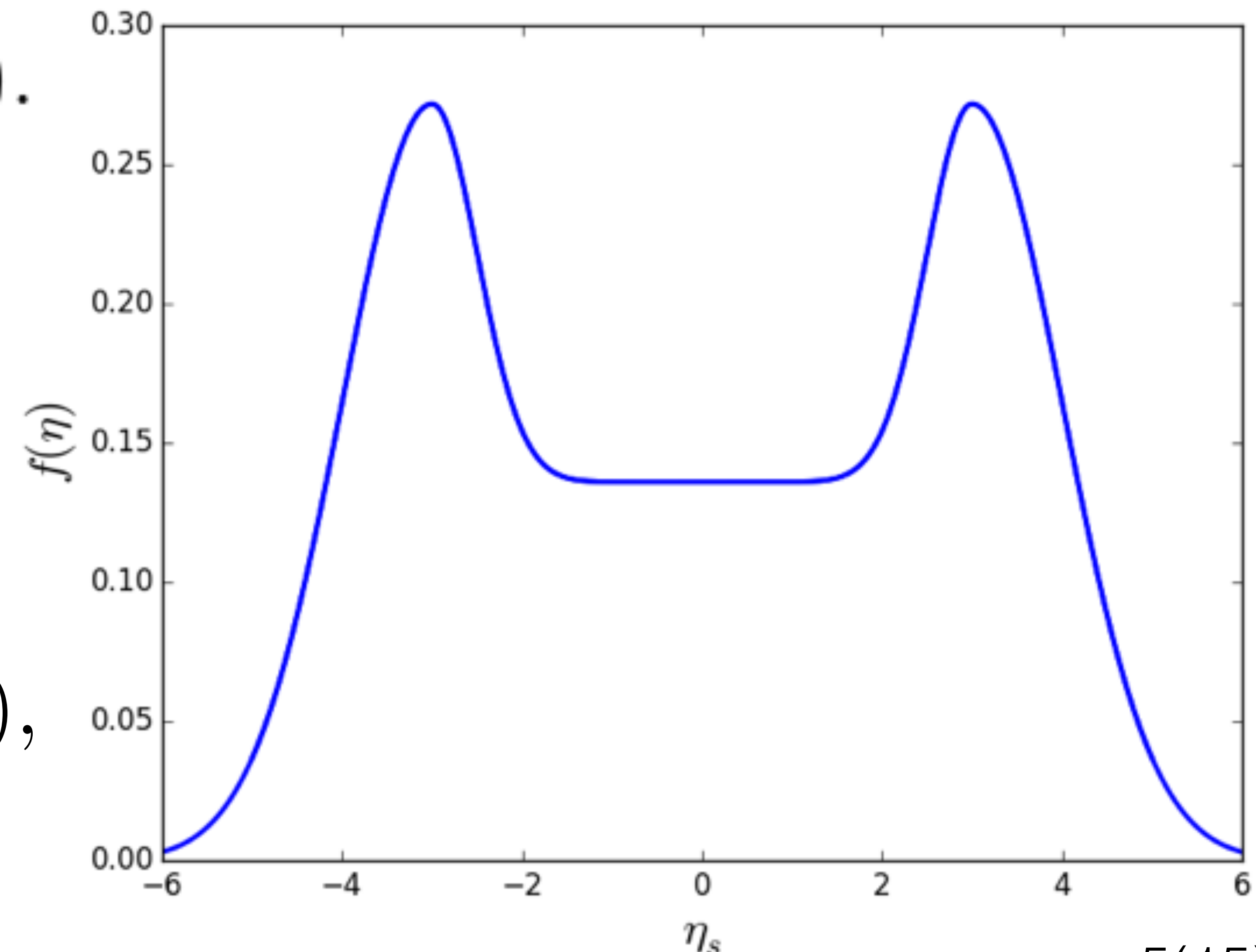
$$\int \tau_0 d\eta_s \int d^2\mathbf{x}_\perp \rho_B(\mathbf{x}_\perp, \eta_s) = N_{\rm part}.$$

For Glauber initial conditions, we assume

$$\rho_B(\mathbf{x}_\perp, \eta_s) = f(\eta_s)\tilde{\rho}_B(\mathbf{x}_\perp).$$

$$\int \tau_0 d\eta_s f(\eta_s) = 1.$$

$$\begin{aligned}\tilde{\rho}_B(\mathbf{x}_\perp) &= n_{\rm part}(\mathbf{x}_\perp)\\ &\equiv T_A(\mathbf{x}_\perp) + T_B(\mathbf{x}_\perp),\end{aligned}$$



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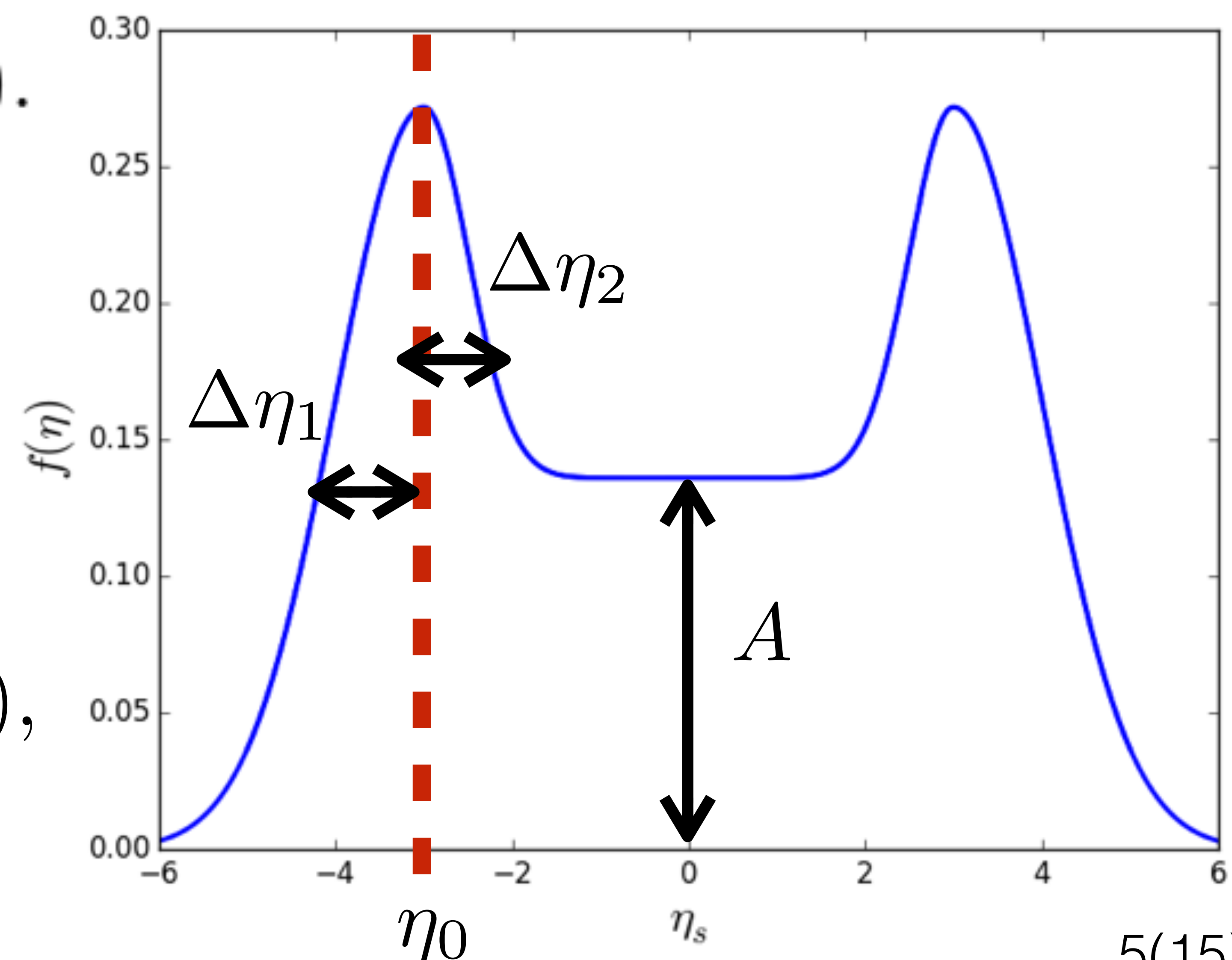


Dissipative hydrodynamics

Energy momentum tensor

$$T^{\mu\nu} = e u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$d_\mu T^{\mu\nu} = T^{\mu\nu}{}_{;\mu} = 0$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Conserved currents

$$J^\mu = n u^\mu + q^\mu$$

$$d_\mu J^\mu = 0$$

$$D = u^\mu d_\mu$$
$$\nabla^\mu = \Delta^{\mu\nu} d_\nu$$
$$\theta = d_\mu u^\mu$$

Dissipative part:

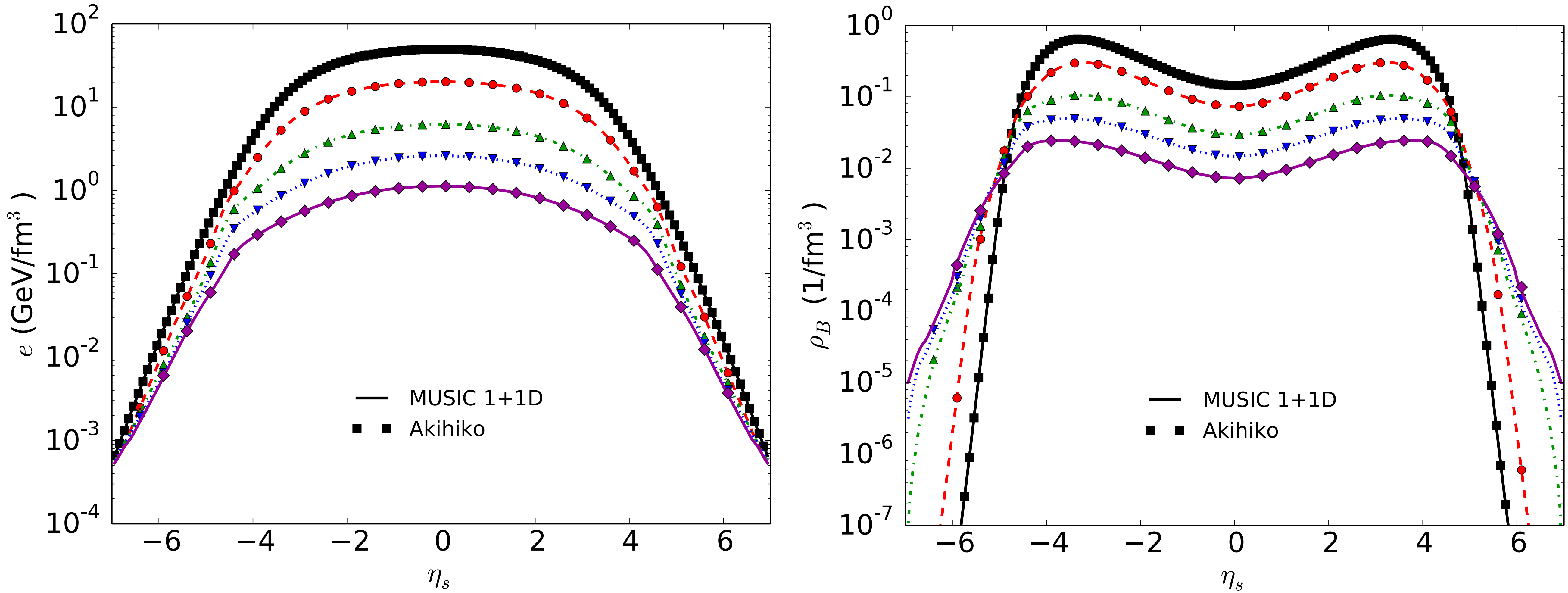
$$\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta$$

$$\Delta^{\mu\nu} D q_\nu = -\frac{1}{\tau_q}\left(q^\mu - \kappa\nabla^\mu\frac{\mu_B}{T}\right) - q^\mu\theta - \frac{3}{5}\sigma^{\mu\nu}q_\nu$$

$$\frac{\eta T}{e + \mathcal{P}} = 0.08 \qquad \kappa = 0.2\frac{n_B}{\rho_B} \qquad \tau_q = \frac{0.2}{T}$$

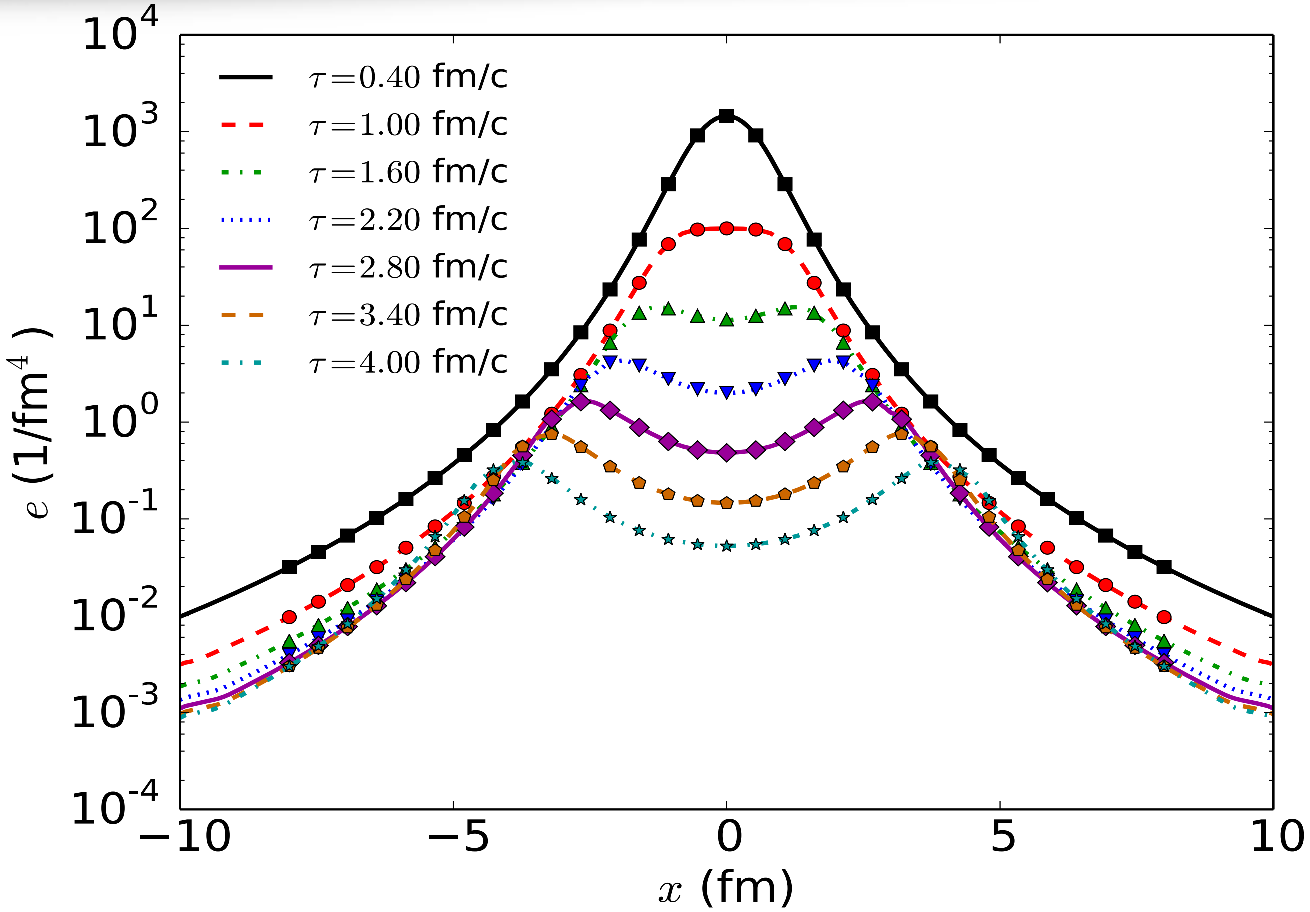
Code Check

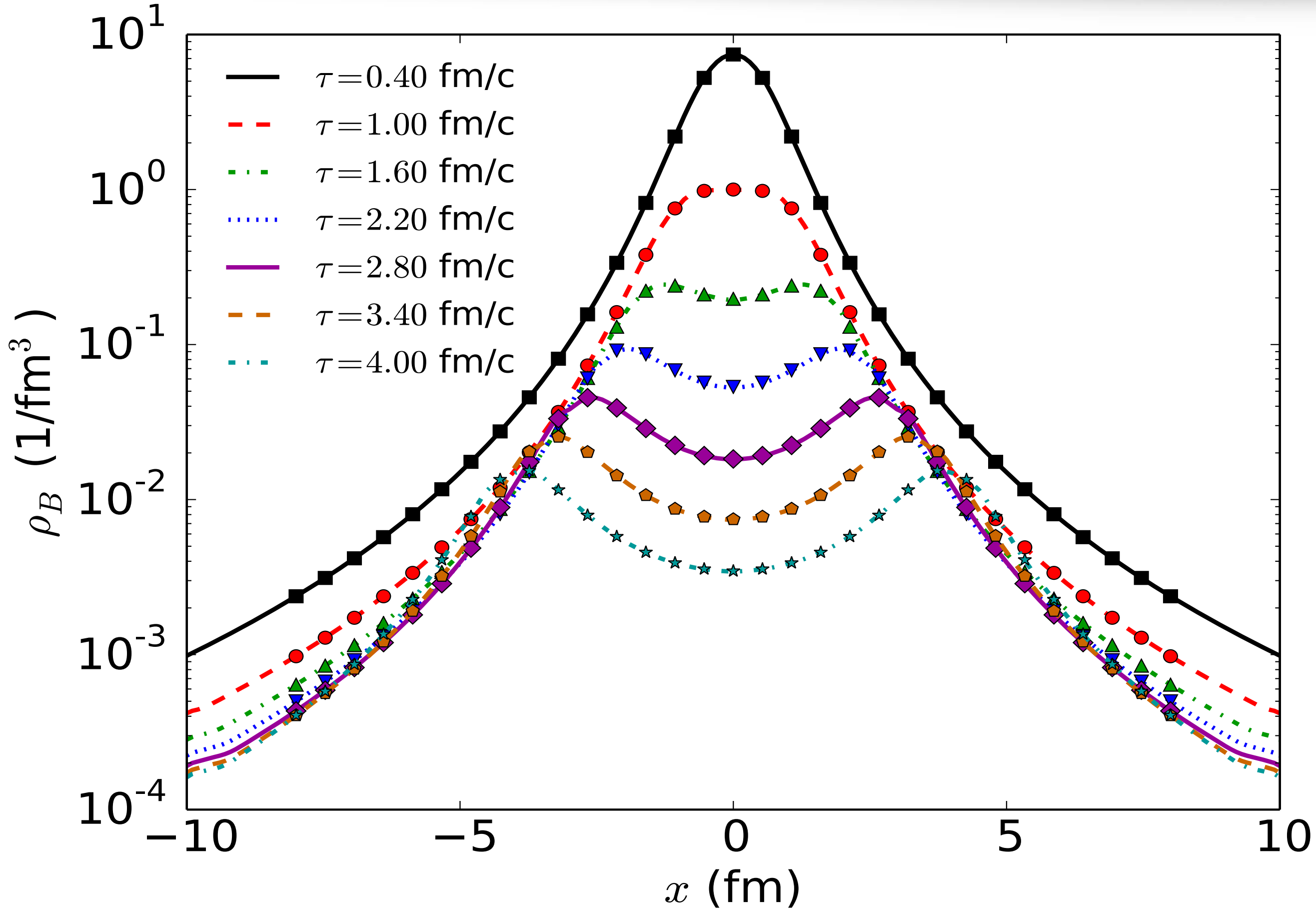
1+1D cross check:

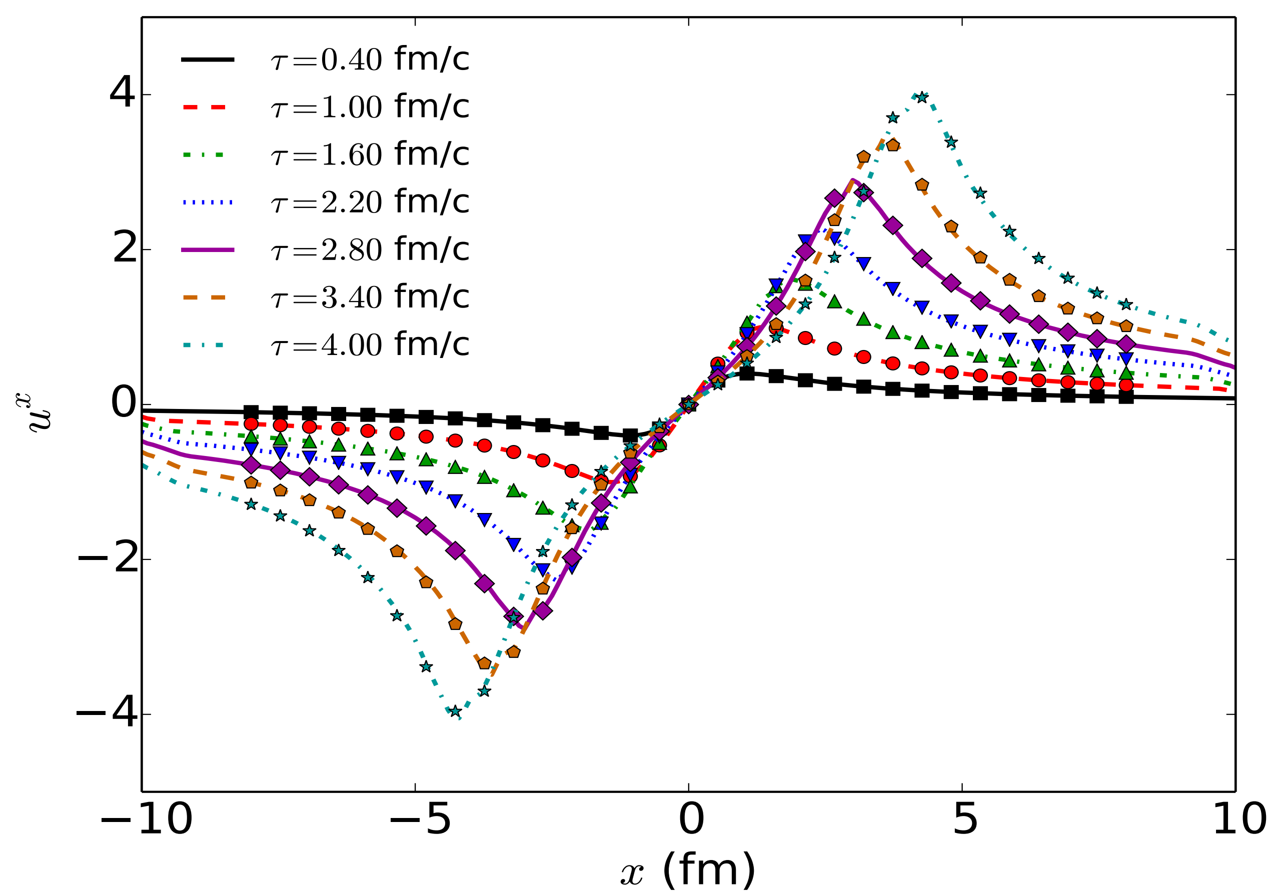


MUSIC results agree very well with Akihiko's results

Code Check

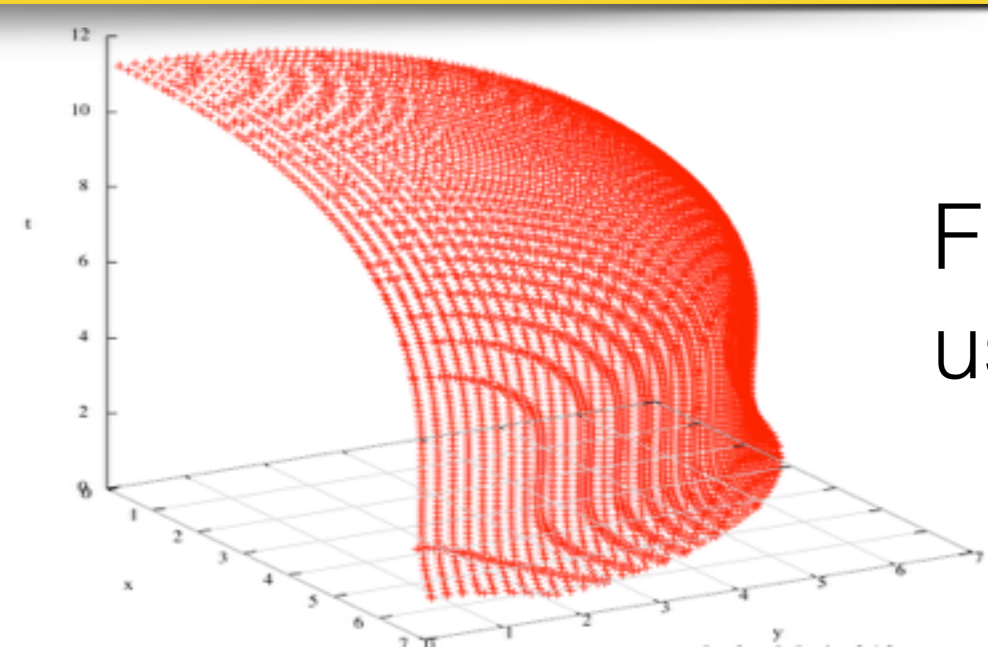






- MUSIC with baryon propagation passed ideal Gubser flow test for the transverse dynamics

Cooper-Frye freeze-out



Freeze-out hyper surface is determined using **Cornelius** freeze-out algorithm

P. Huovinen and H. Petersen, Eur. Phys. J. A **48**, 171 (2012)

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int p^\mu d^3\sigma_\mu(x)\left(f_0(x,p) + \delta f(x,p)\right)$$

$$f_0^i(x,p) = \frac{1}{e^{(E-b_i\mu_B(x))/T(x)} \pm 1}$$

Using relaxation time approximation,

$$\delta f_0^i(x,p) = f_0^i(x,p)(1 \pm f_0^i(x,p))\left(\frac{n_B}{e+\mathcal{P}} - \frac{b_i}{E}\right)\frac{p\cdot q}{\hat{\kappa}}$$

$$\hat{\kappa} = \kappa/\tau_q$$

$\hat{\kappa}(T,\mu_B)$ is calculated using hadron resonance gas model

Cooper-Frye freeze-out

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$$N^B - N^{\bar{B}} = \int d^3\sigma_\mu \sum \frac{g_i}{(2\pi)^3}\int_p p^\mu \left[\left(f_0^B(x,p) - f_0^{\bar{B}}(x,p)\right) + \left(\delta f^B(x,p) - \delta f^{\bar{B}}(x,p)\right)\right]$$

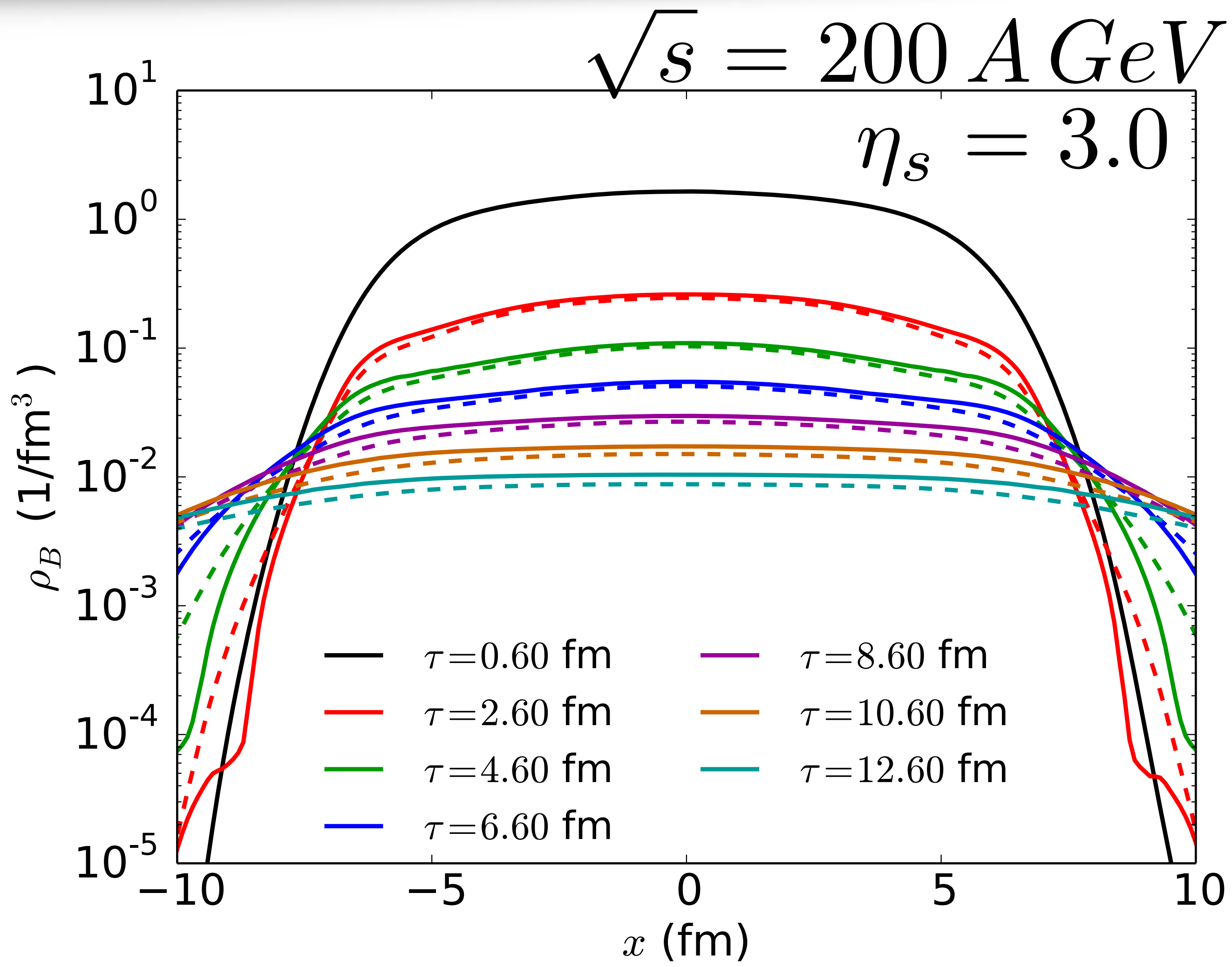
$$= \int d^3\sigma_\mu(n_B u^\mu + q^\mu)$$

$$\partial_\mu(n_B u^\mu + q^\mu) = 0$$

$\Rightarrow$ $N^B - N^{\bar{B}}$ is conserved

- With diffusion, δf is essential to ensure net baryon number conservation

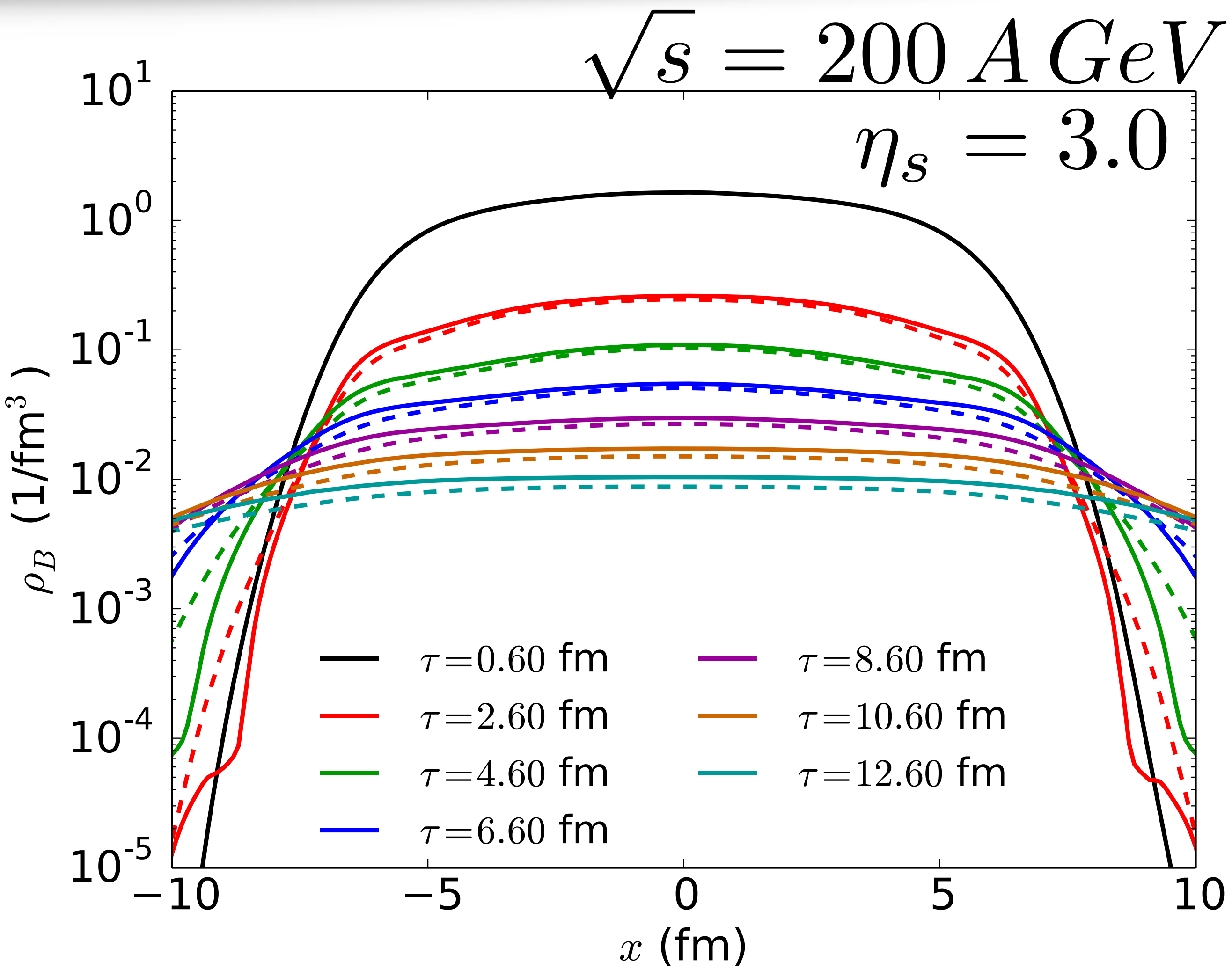
Results

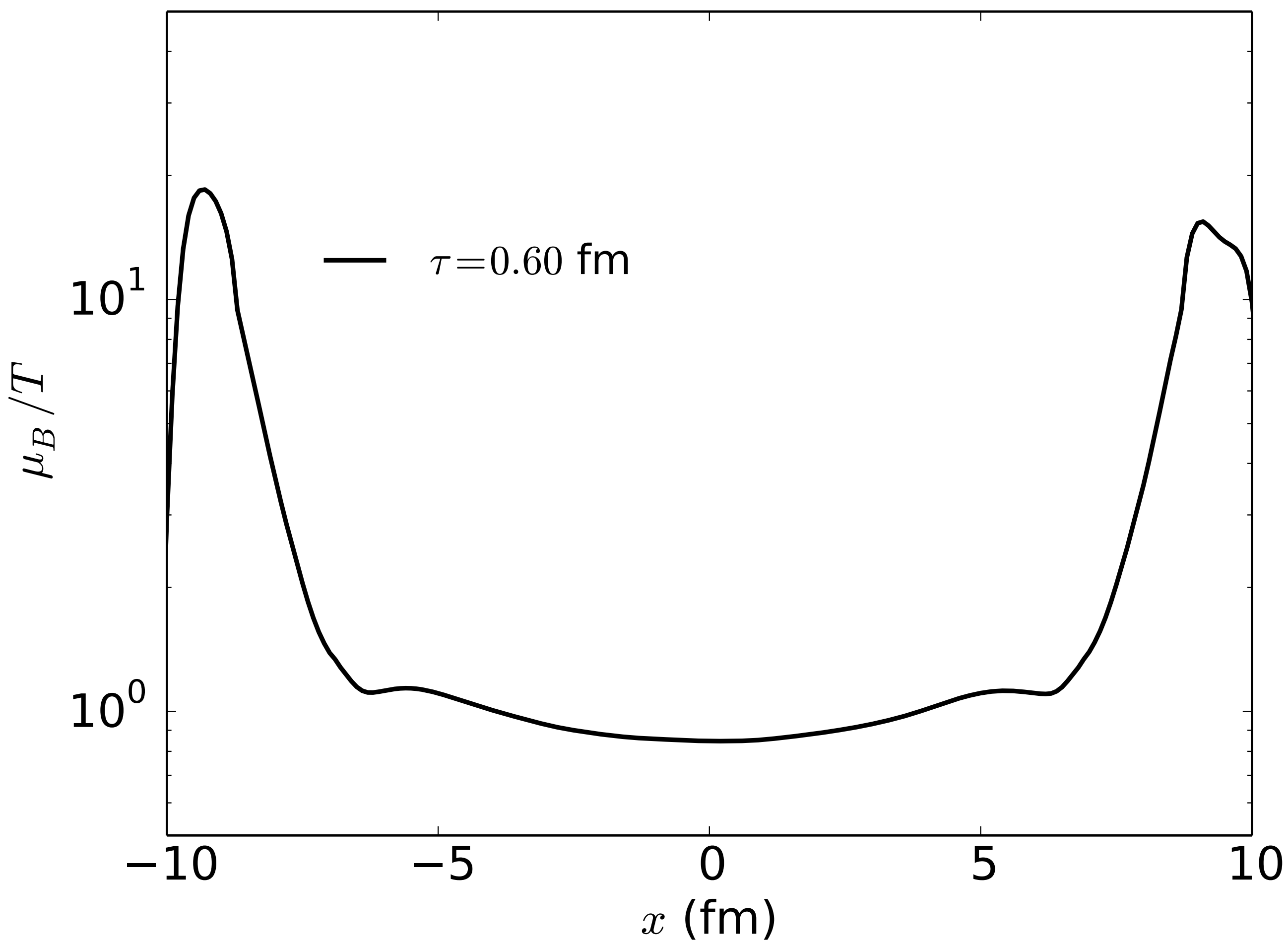


solid: with diffusion dashed: no diffusion

- With diffusion, ρ_B is larger in the center of the transverse plane

Results





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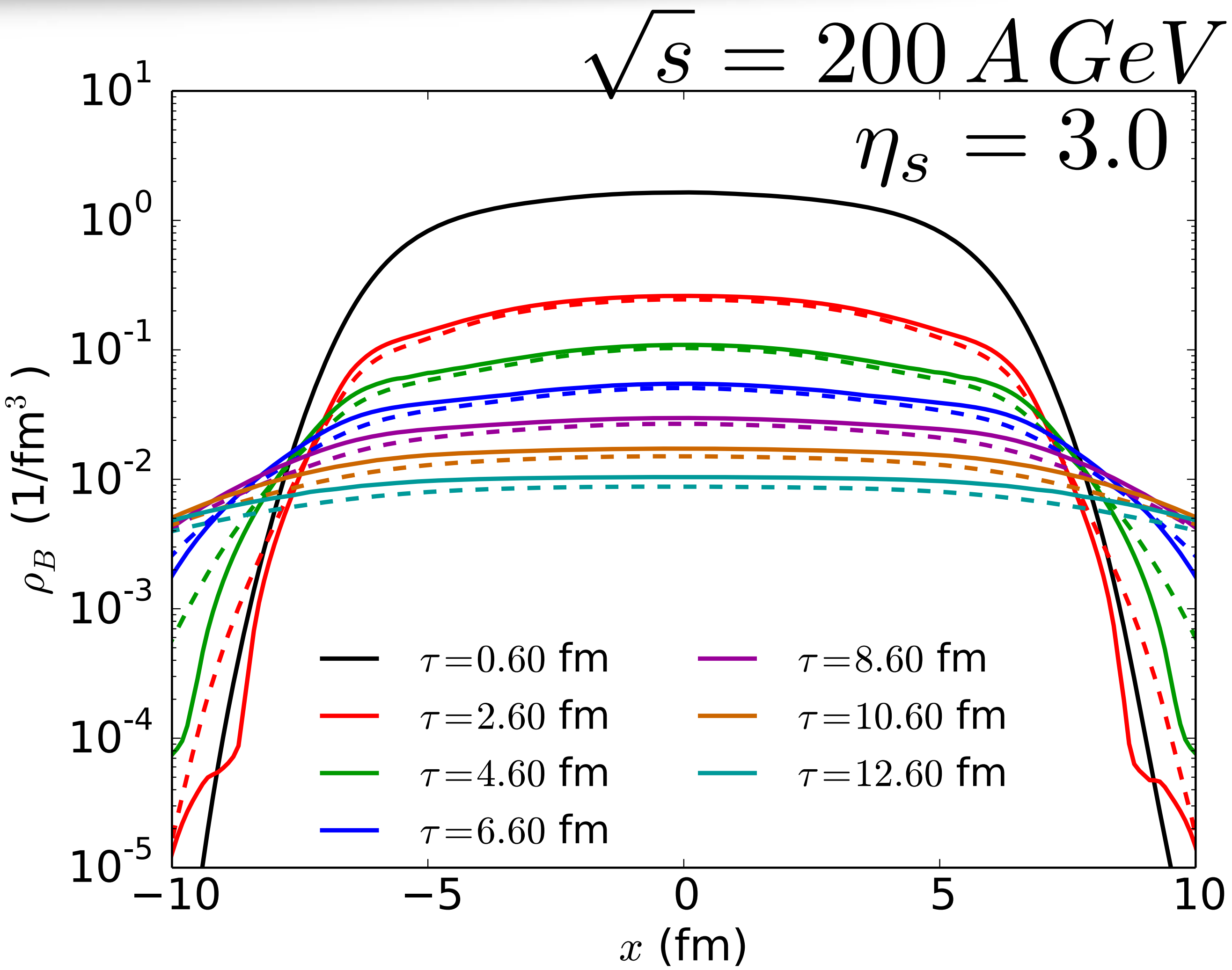
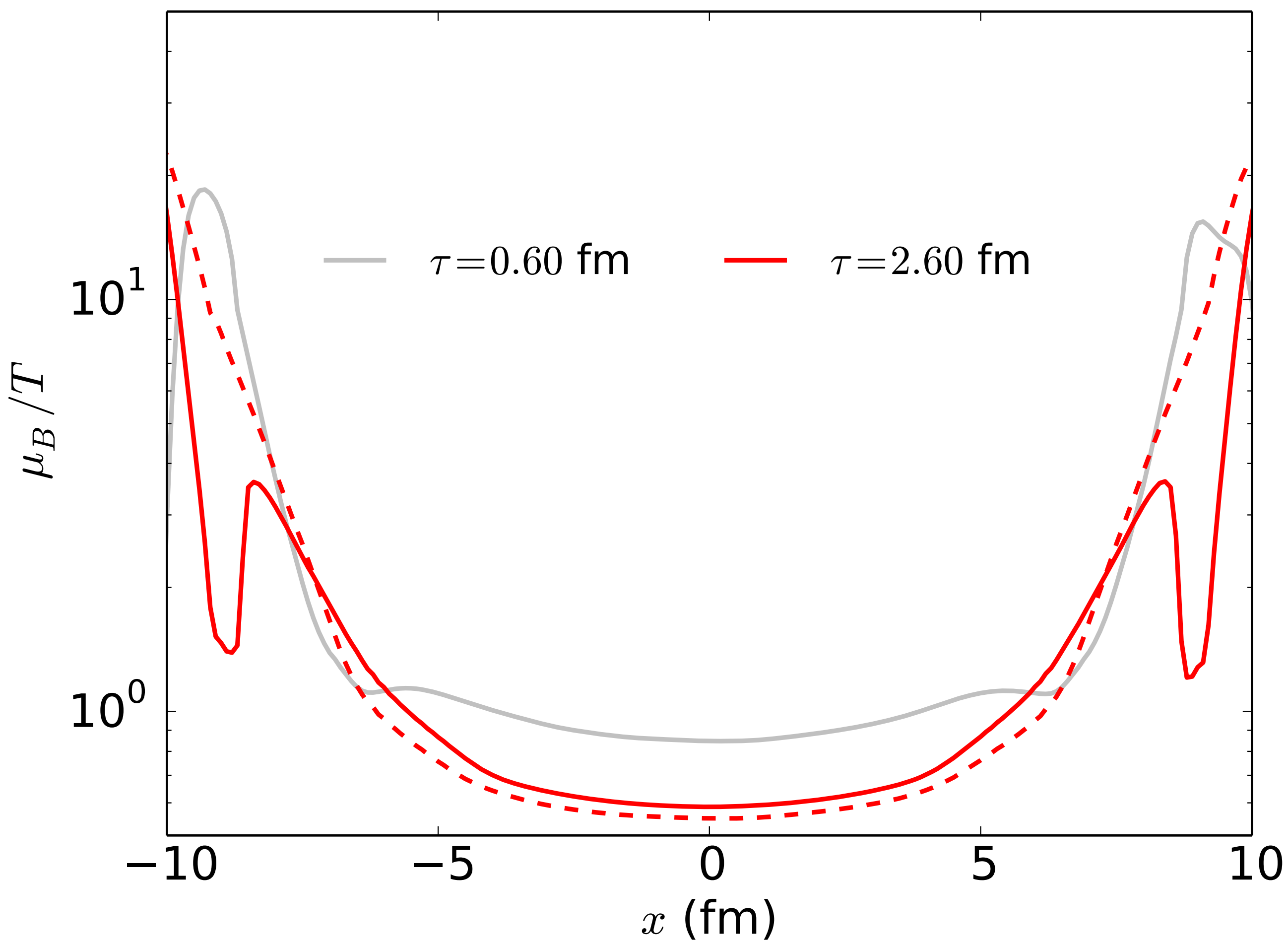
- With diffusion, ρ_B is larger in the center of the transverse plane
- The dynamics of ρ_B is driven by the evolution of u^μ and $\nabla^\mu \frac{\mu_B}{T}$

Results





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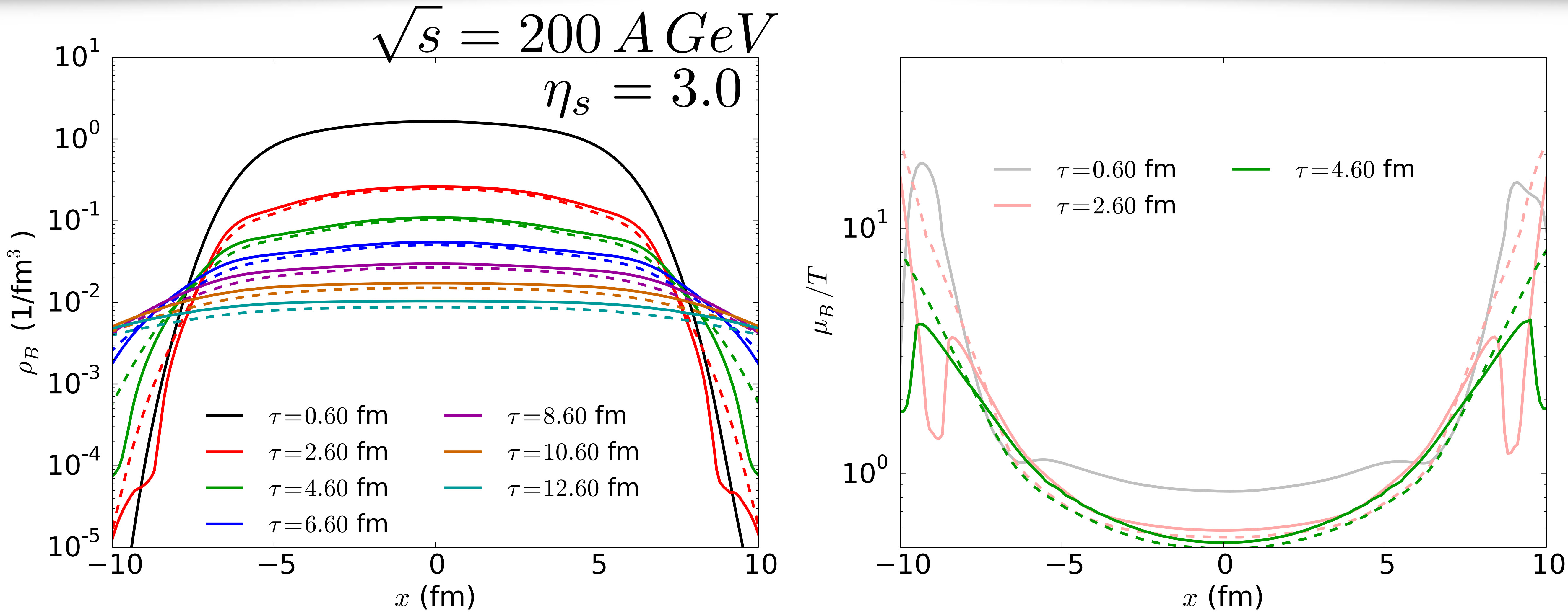

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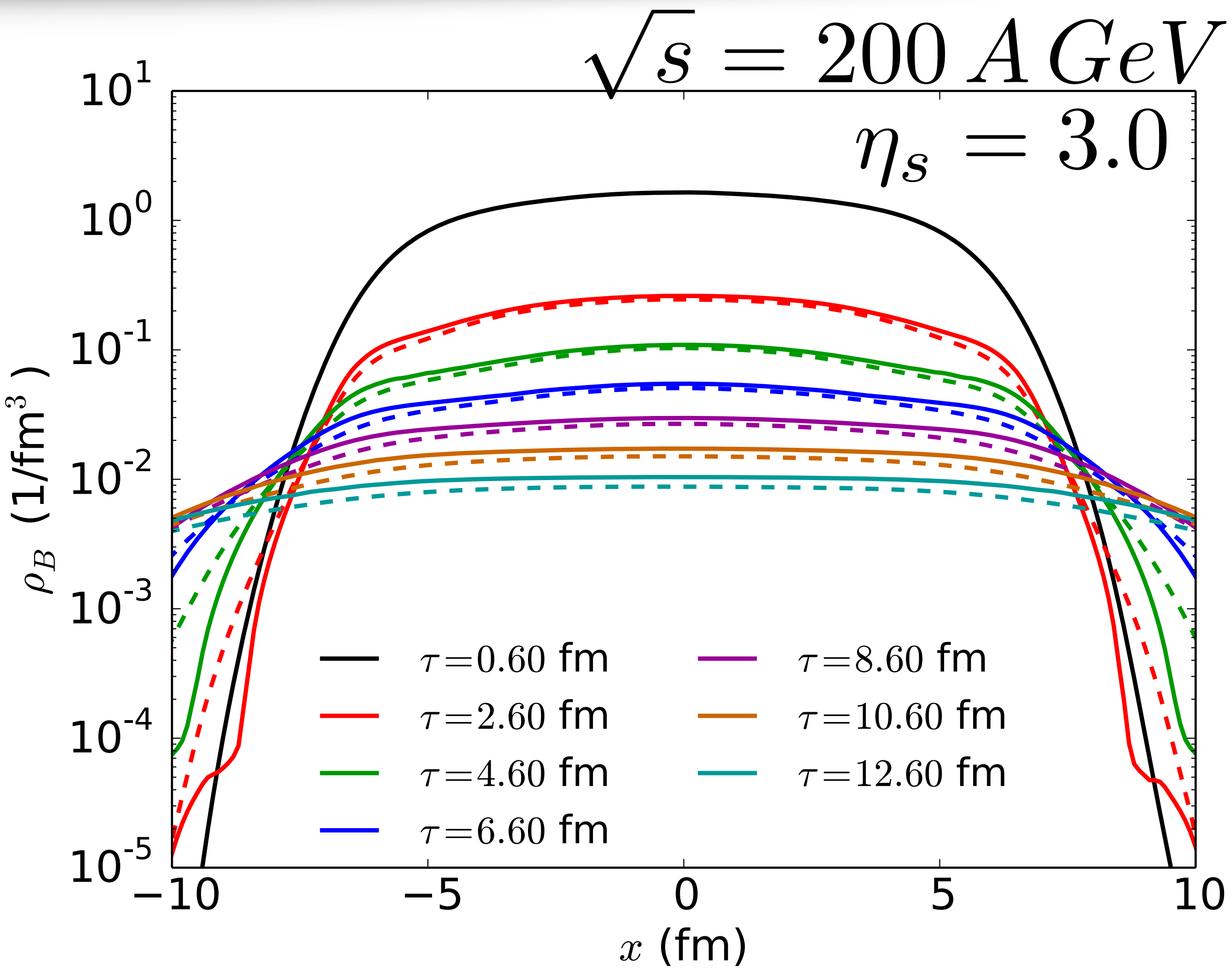

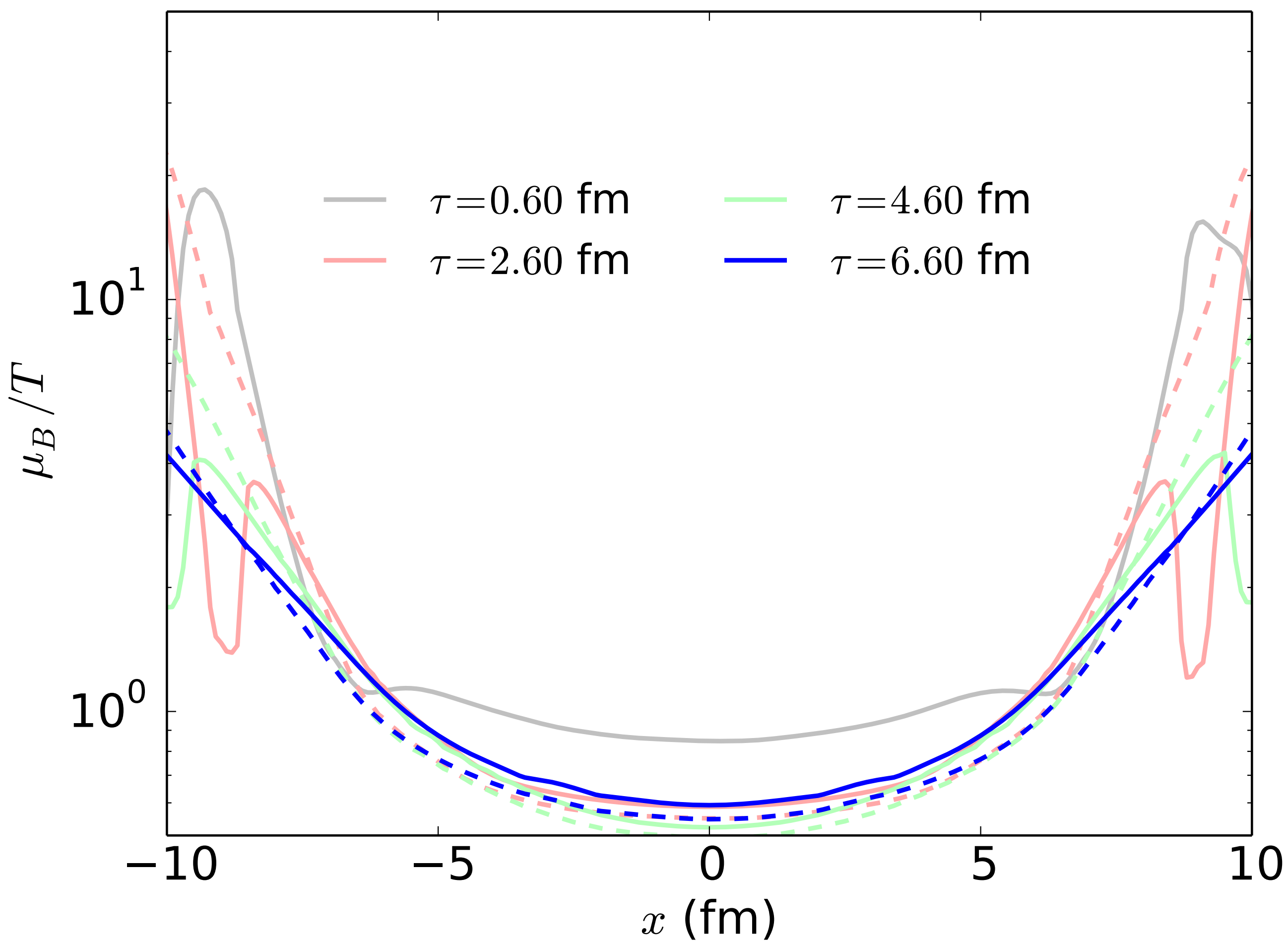

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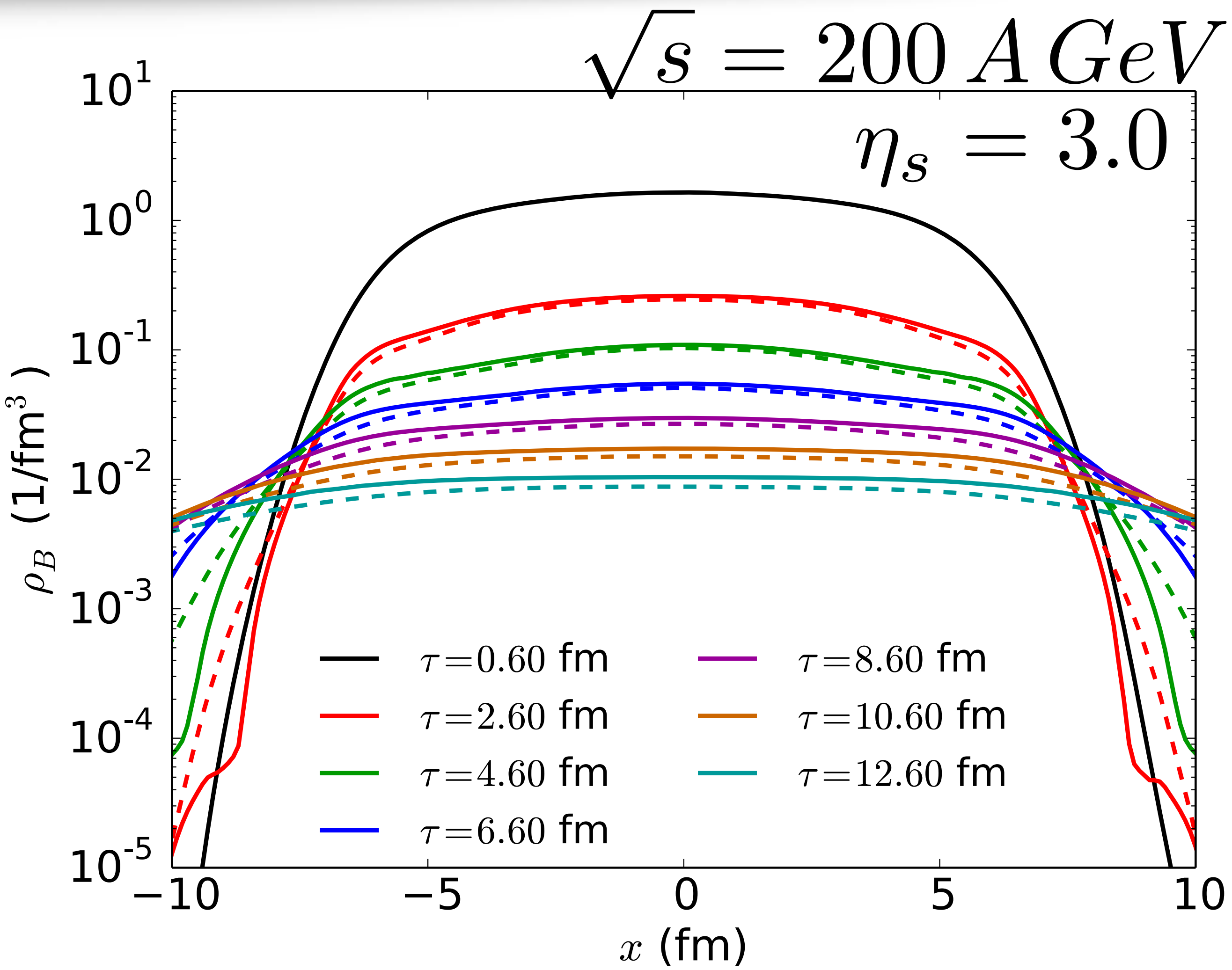

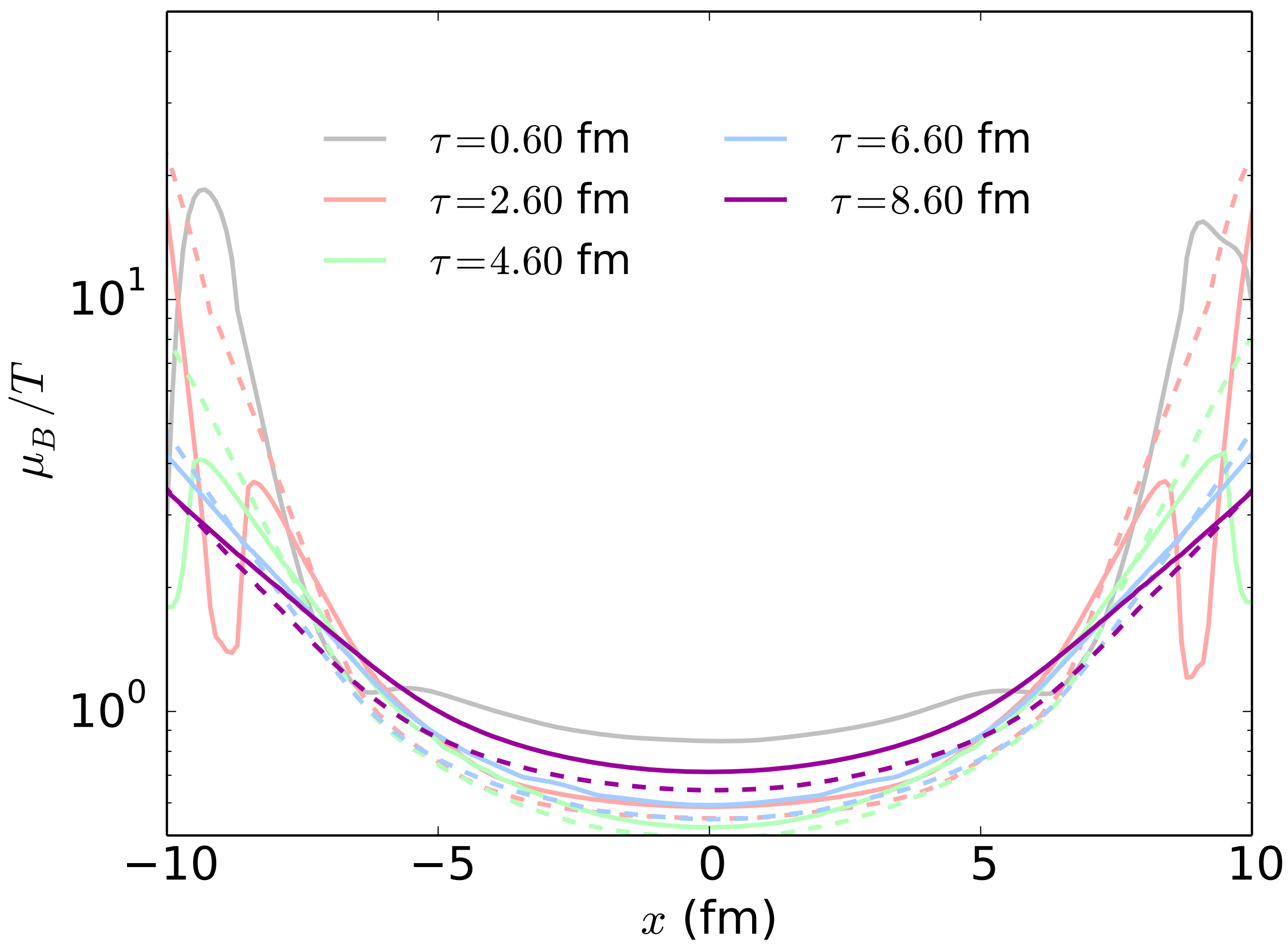

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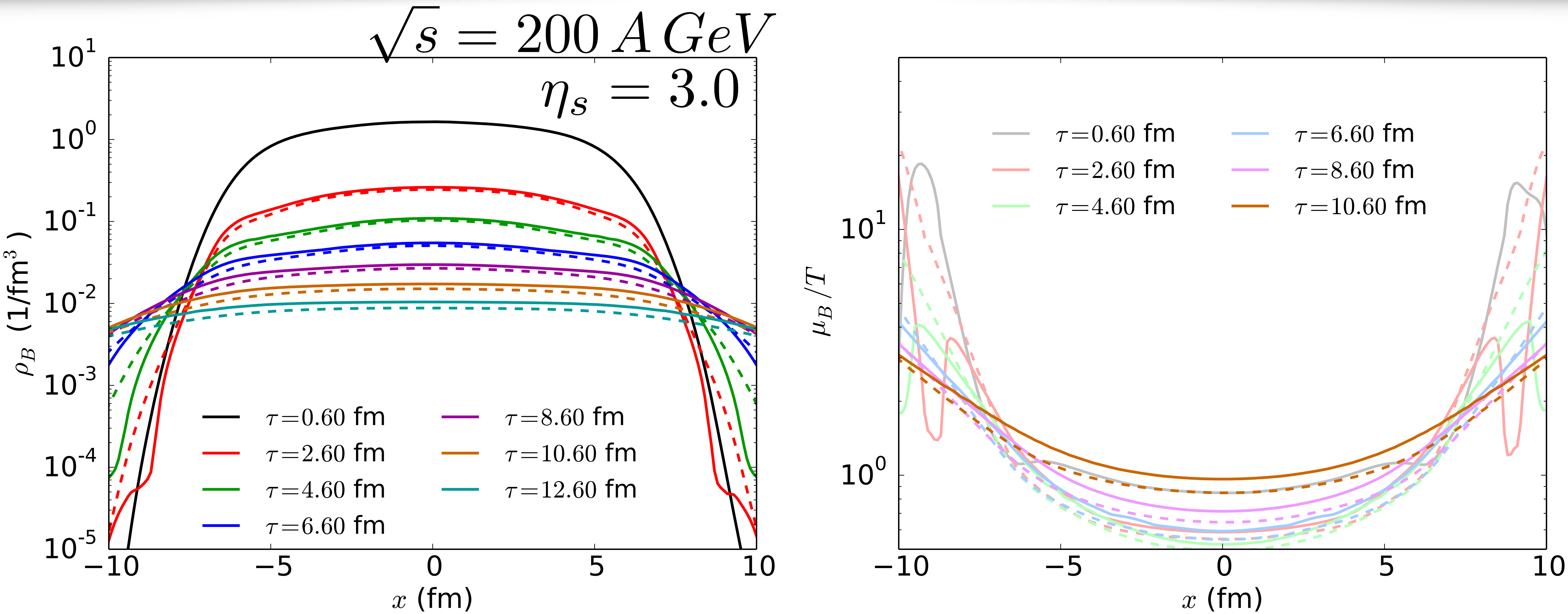

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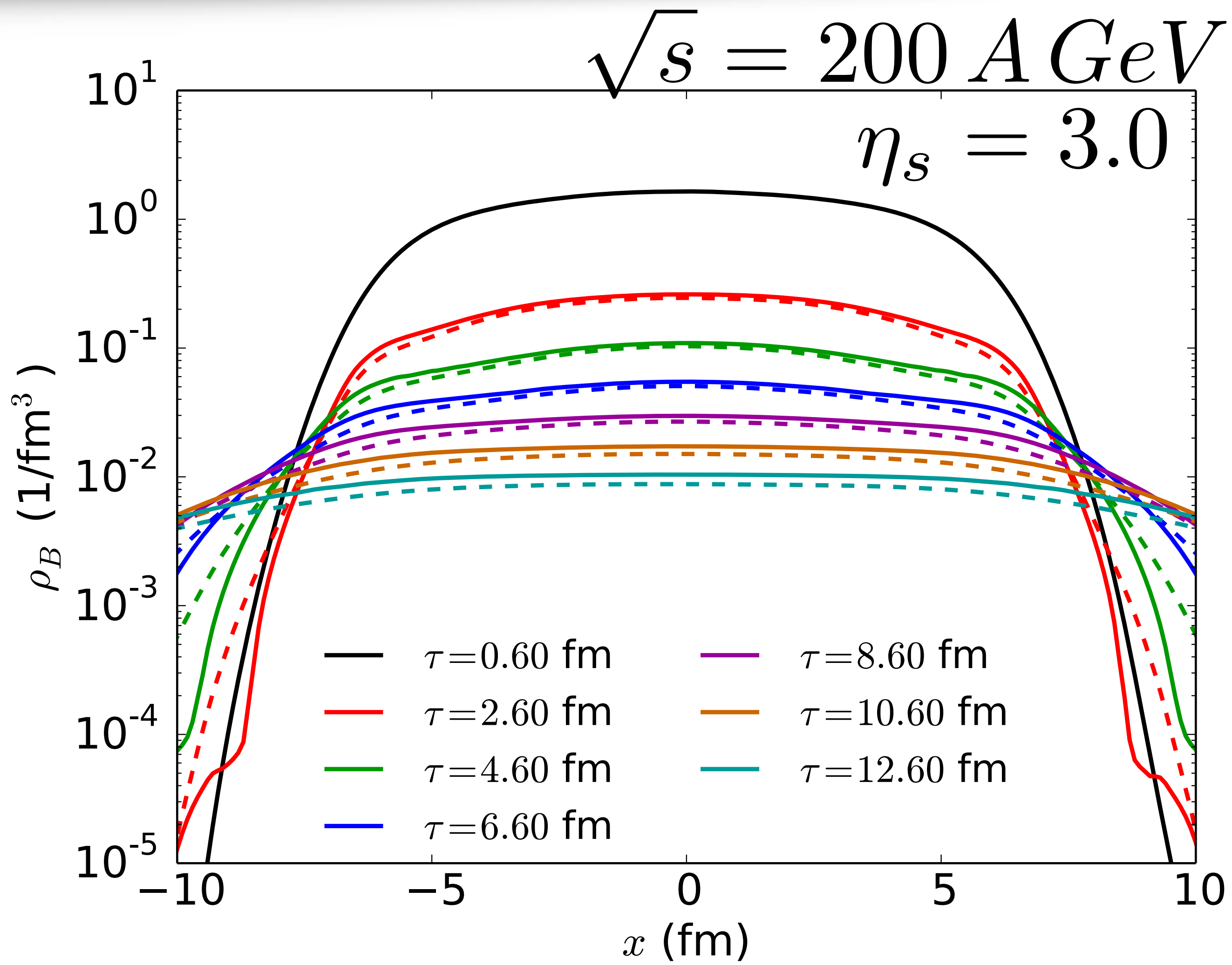

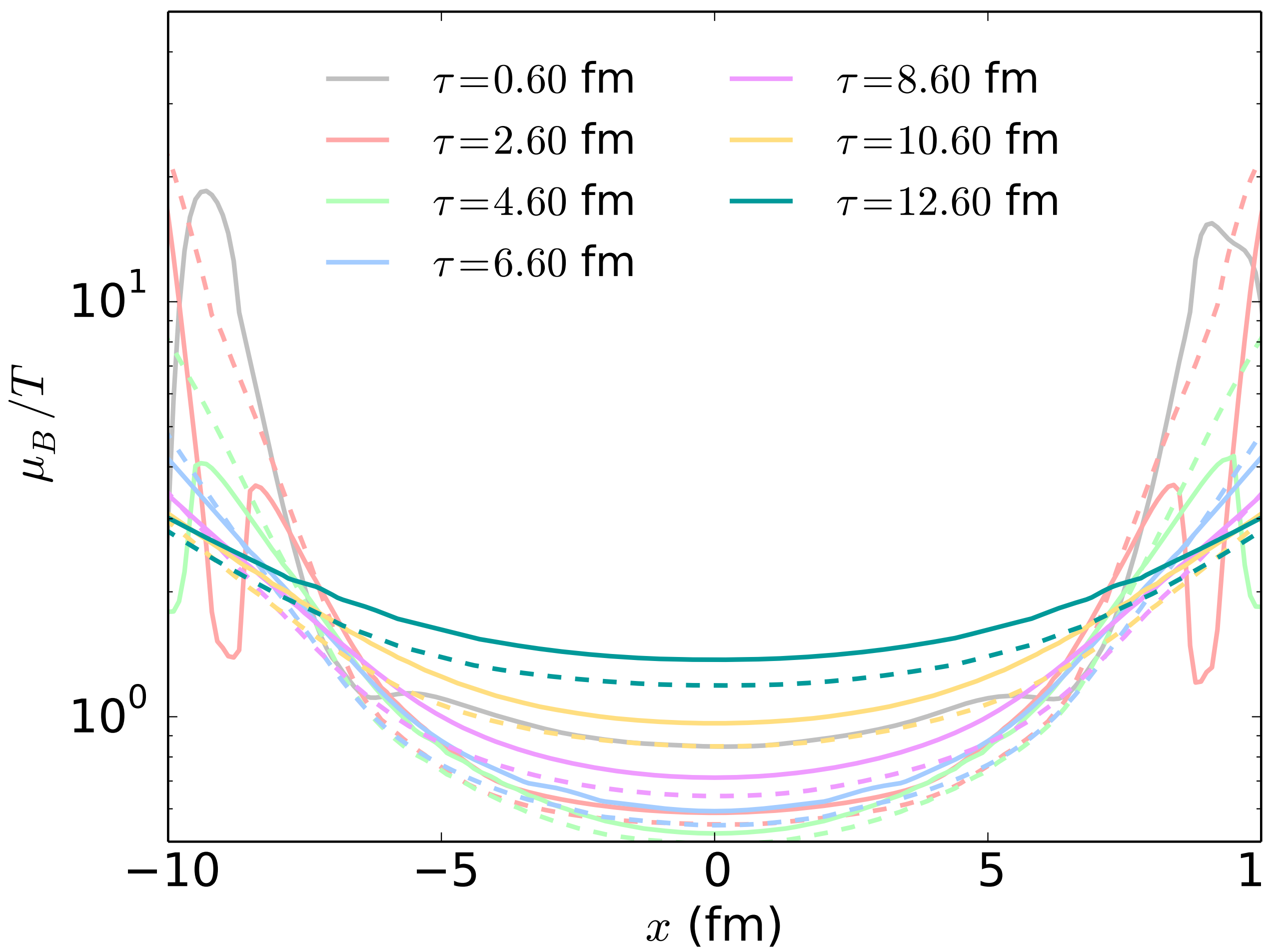

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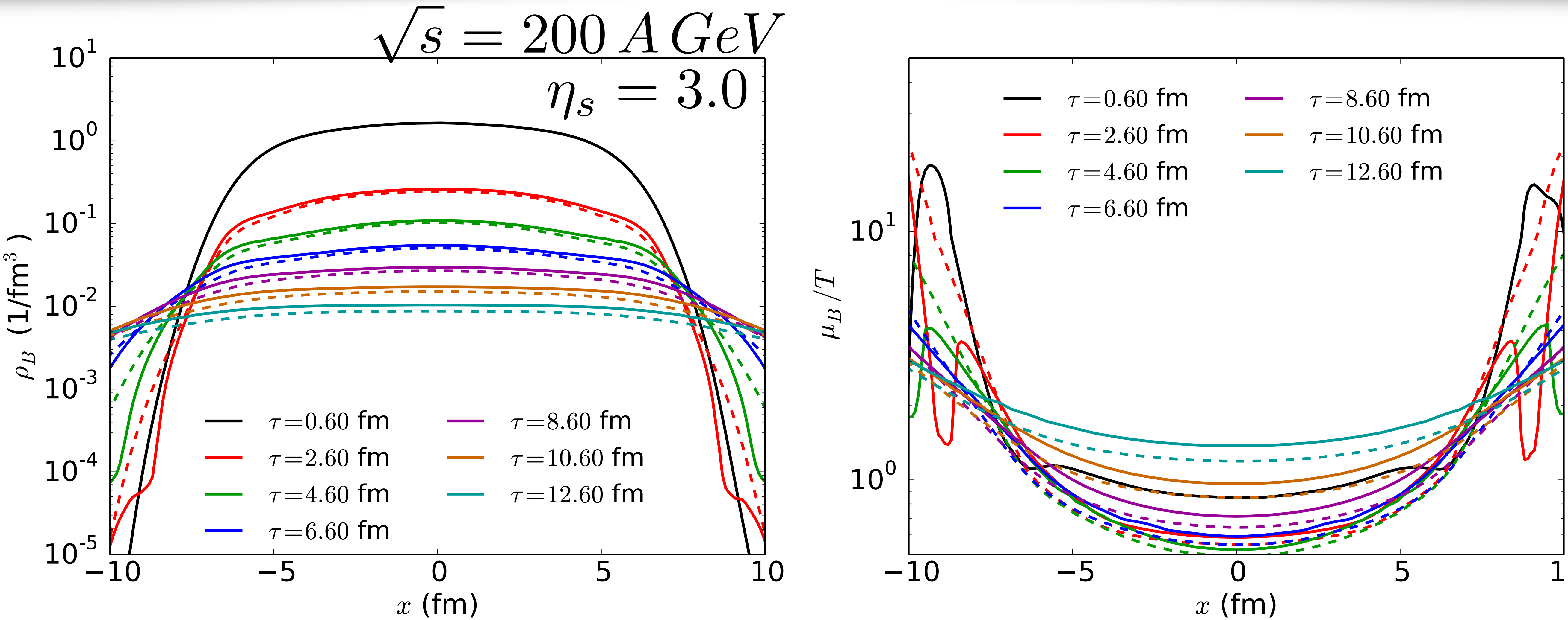

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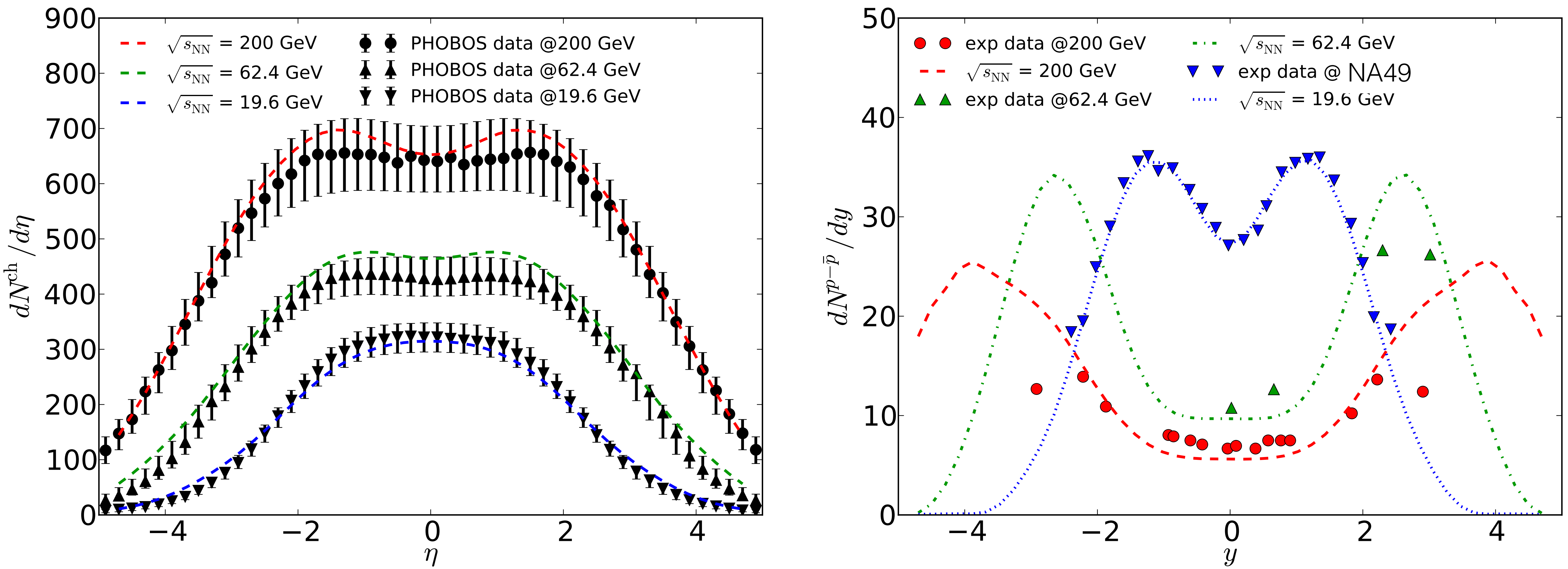

- The initial envelope functions in η_s are tuned to reproduce experimental $dN^{\mathrm{ch}}/d\eta$ and $dN^{p-\bar{p}}/dy$

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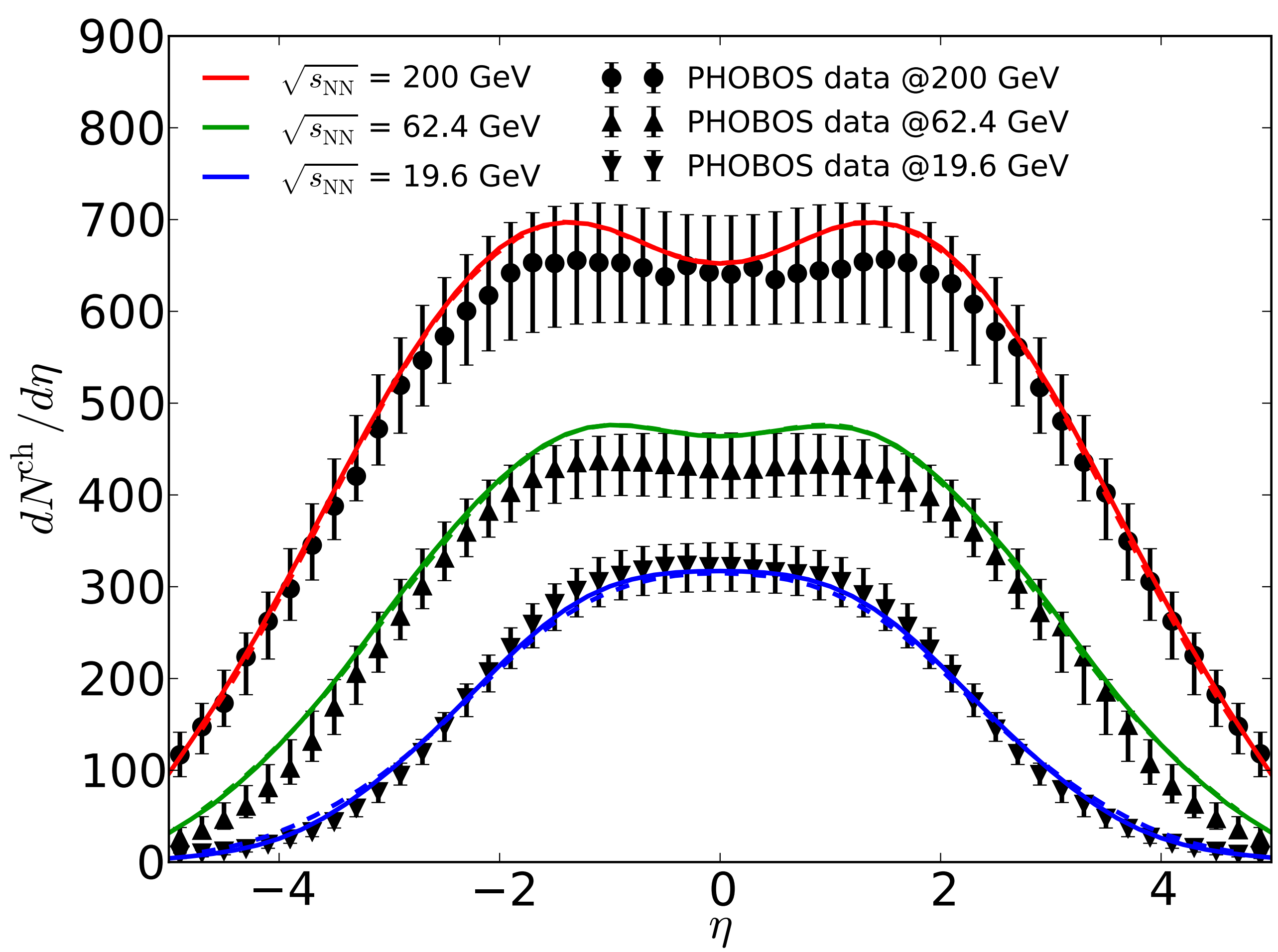
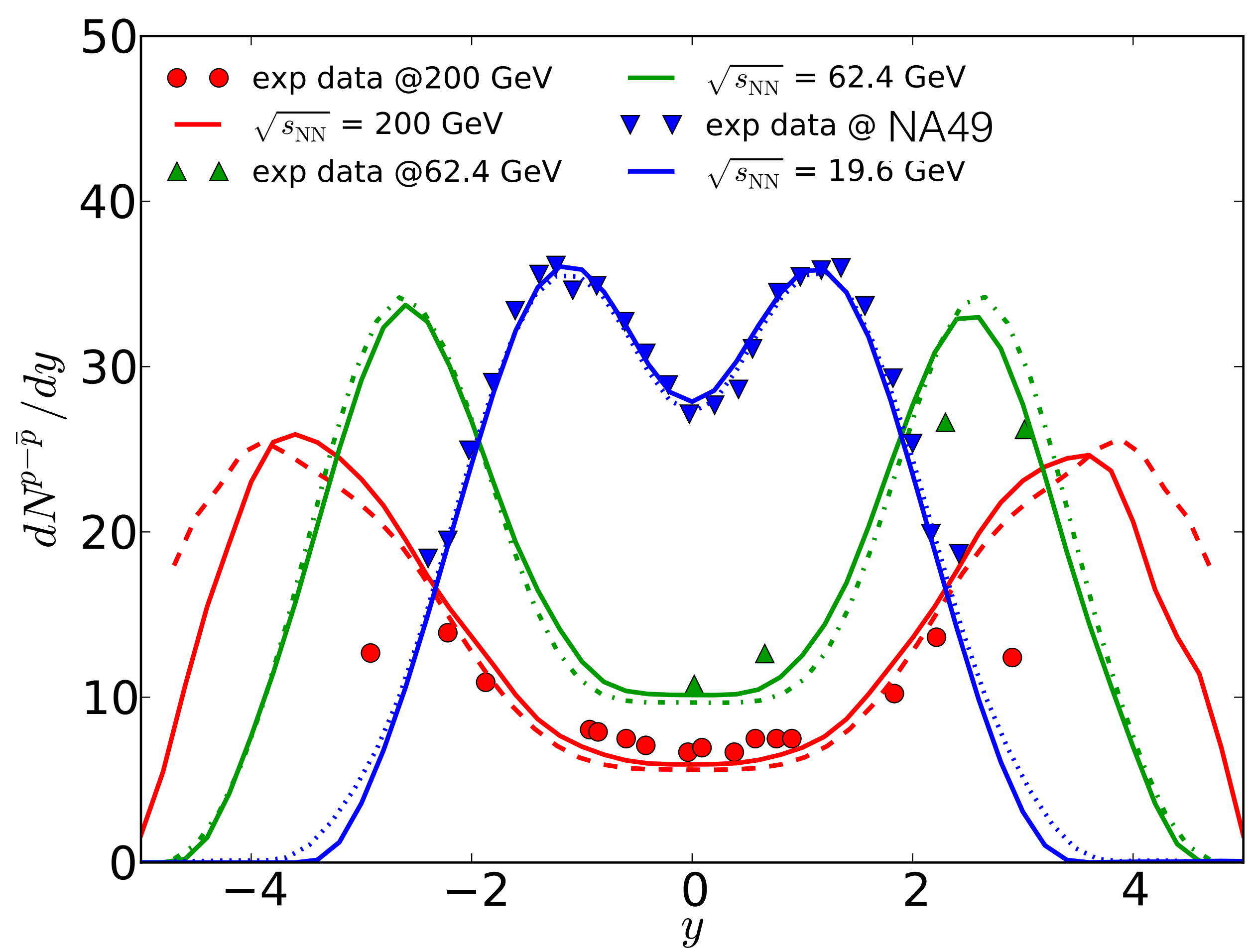
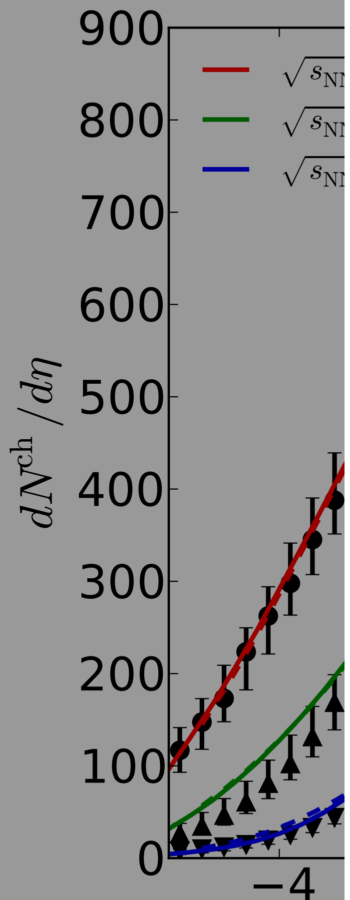
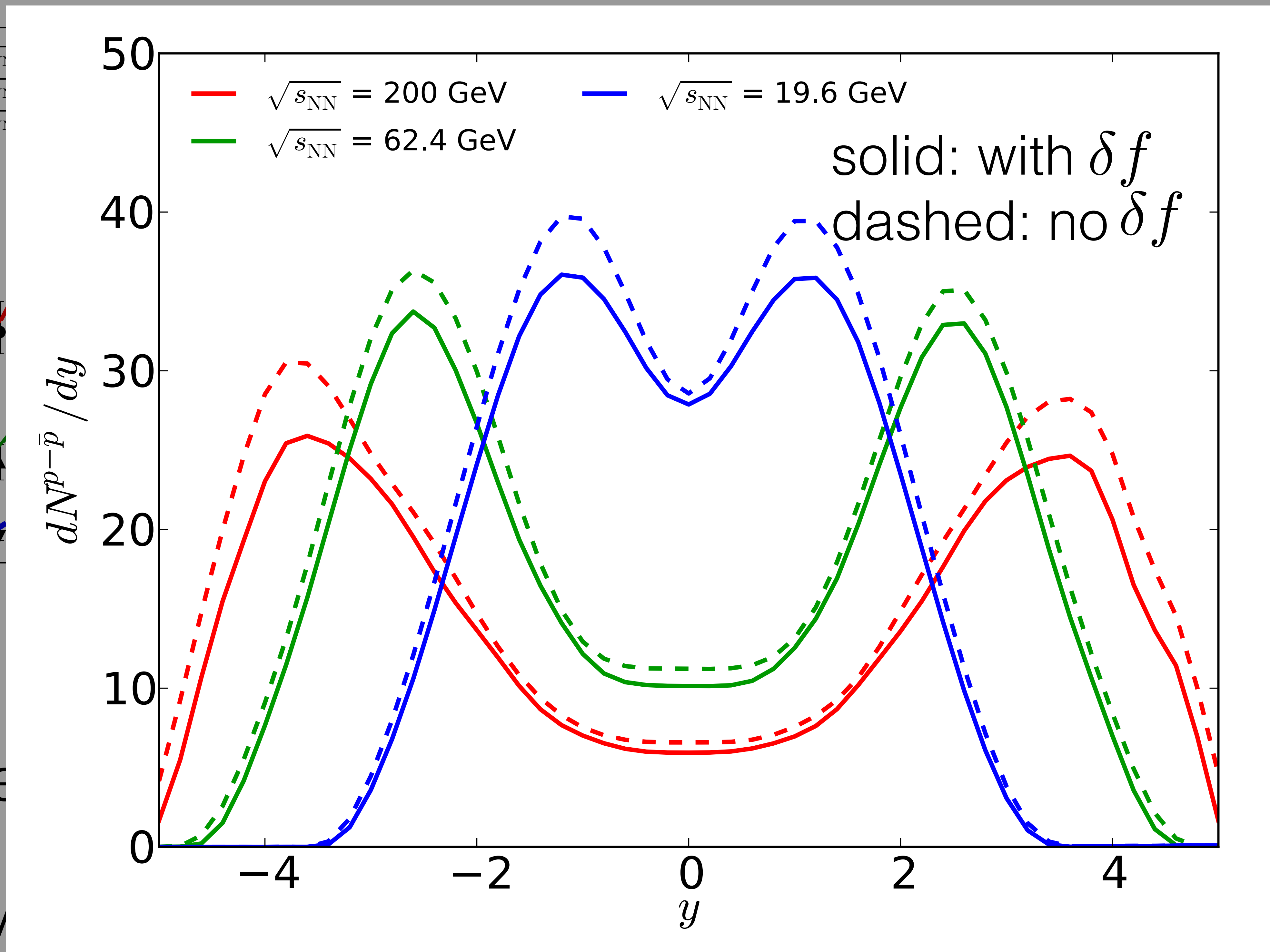


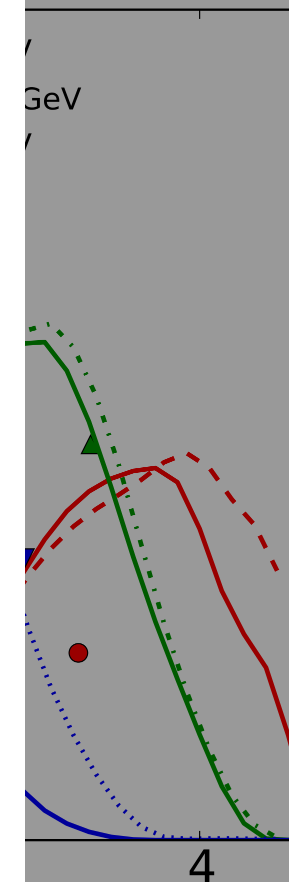


- The initial envelope functions in η_s are tuned to reproduce experimental $dN^{\rm ch}/d\eta$ and $dN^{p-\bar{p}}/dy$
- Baryon diffusion slightly increases $dN^{p-\bar{p}}/dy$ at mid-rapidity and narrows the tail in its distribution.

Results







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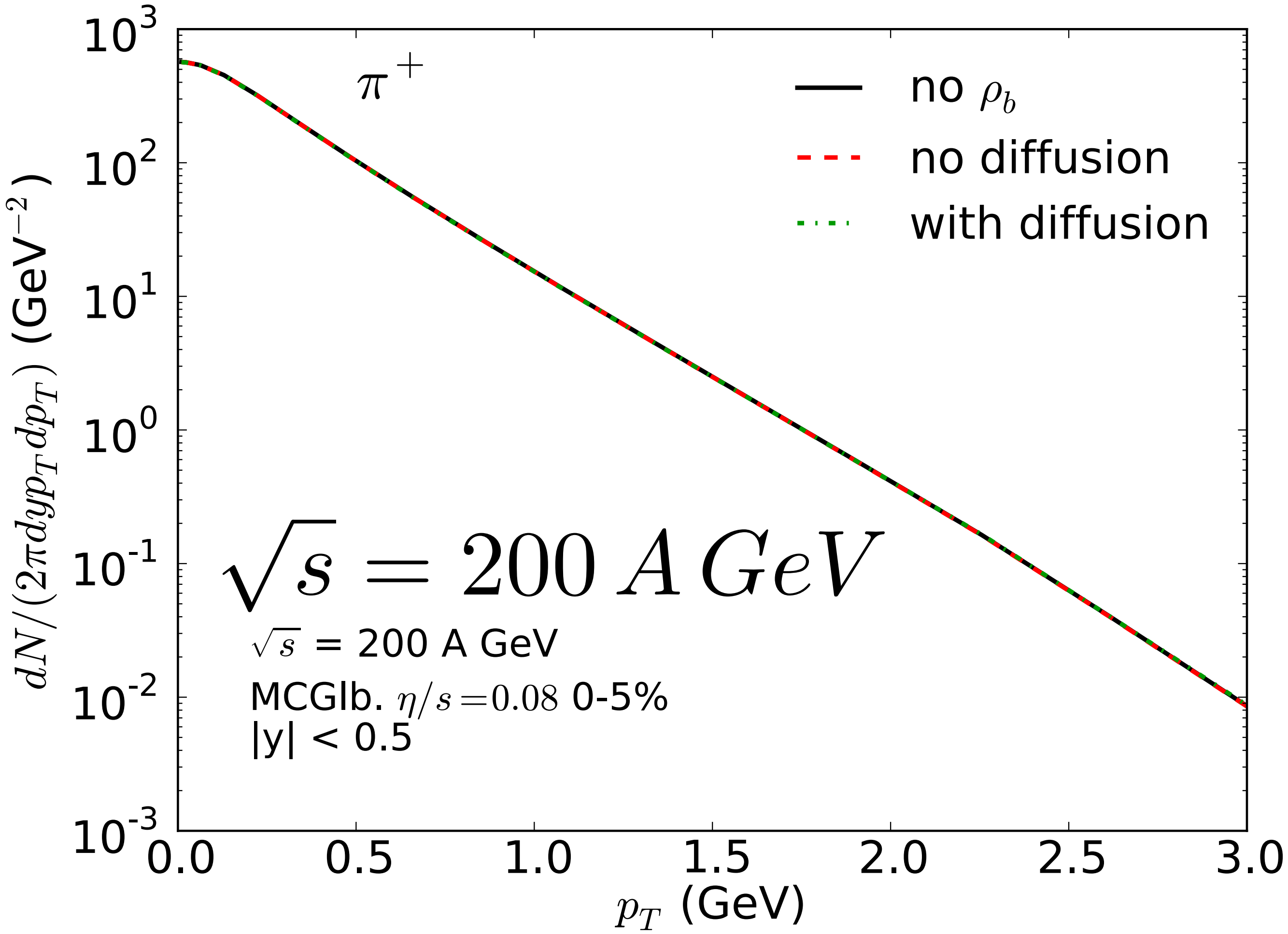
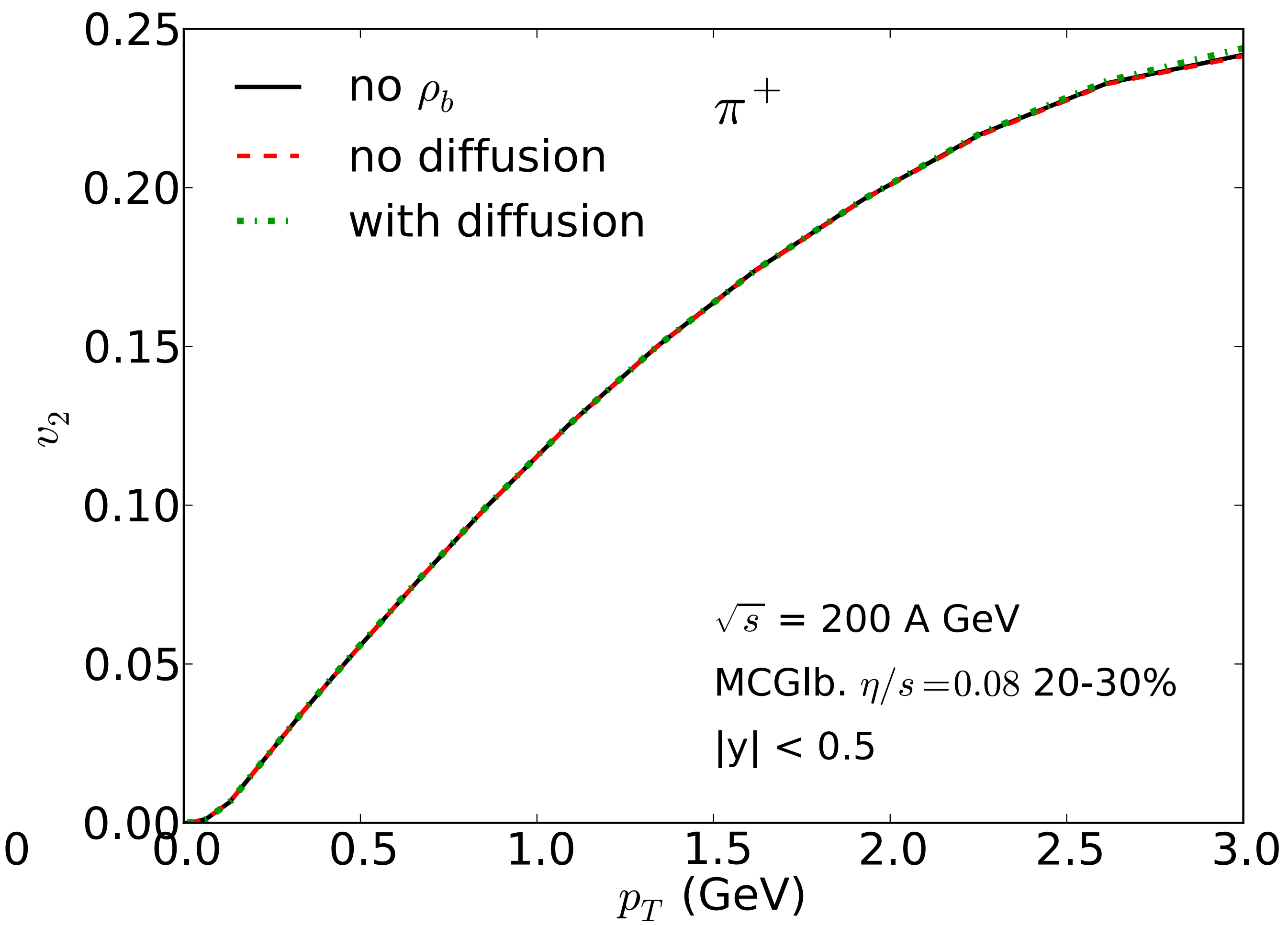
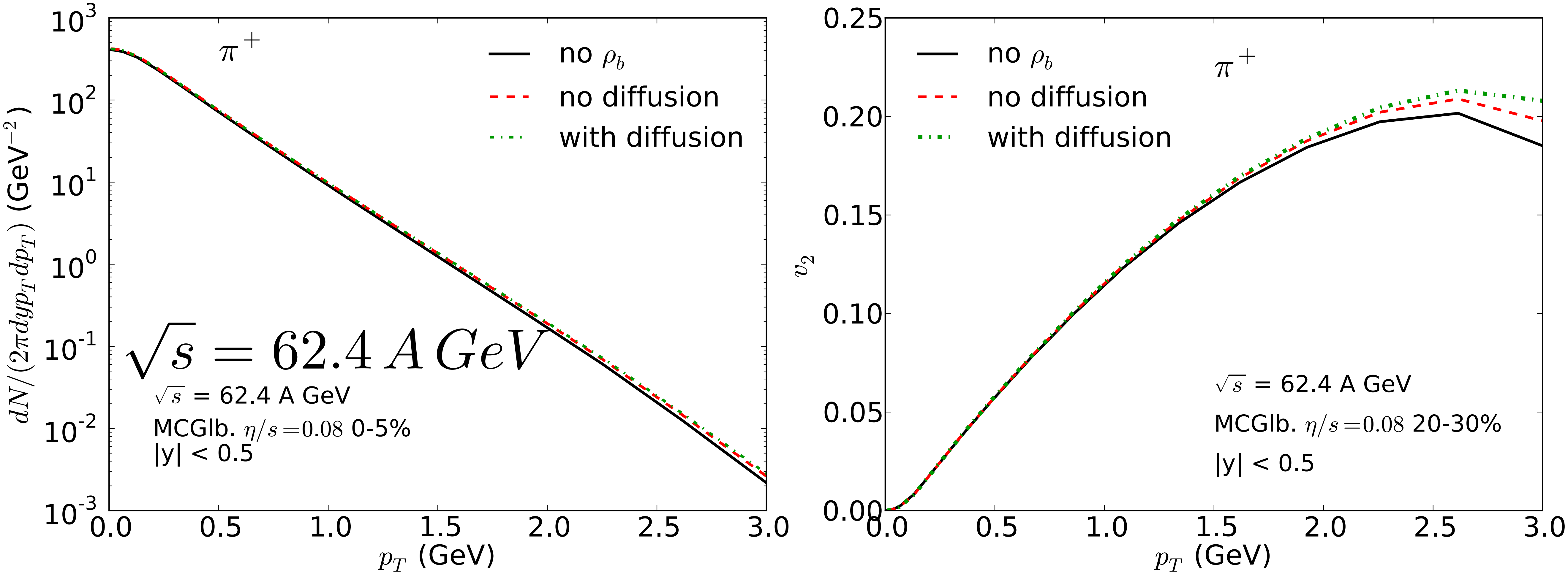
Light meson spectra and v_2





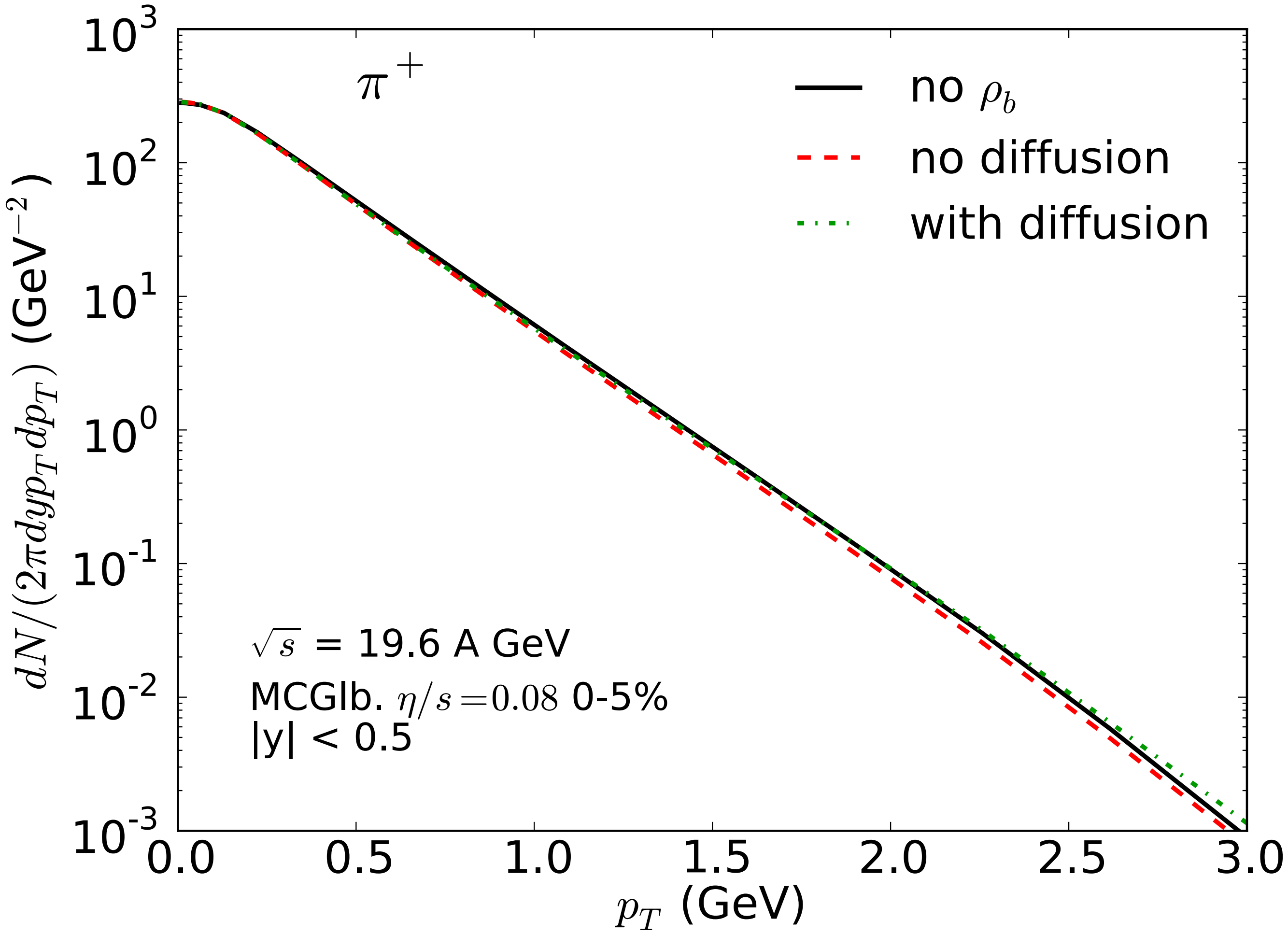
- At top RHIC energy, finite ρ_b and diffusion have little effects on pion spectra and v_2 at mid-rapidity

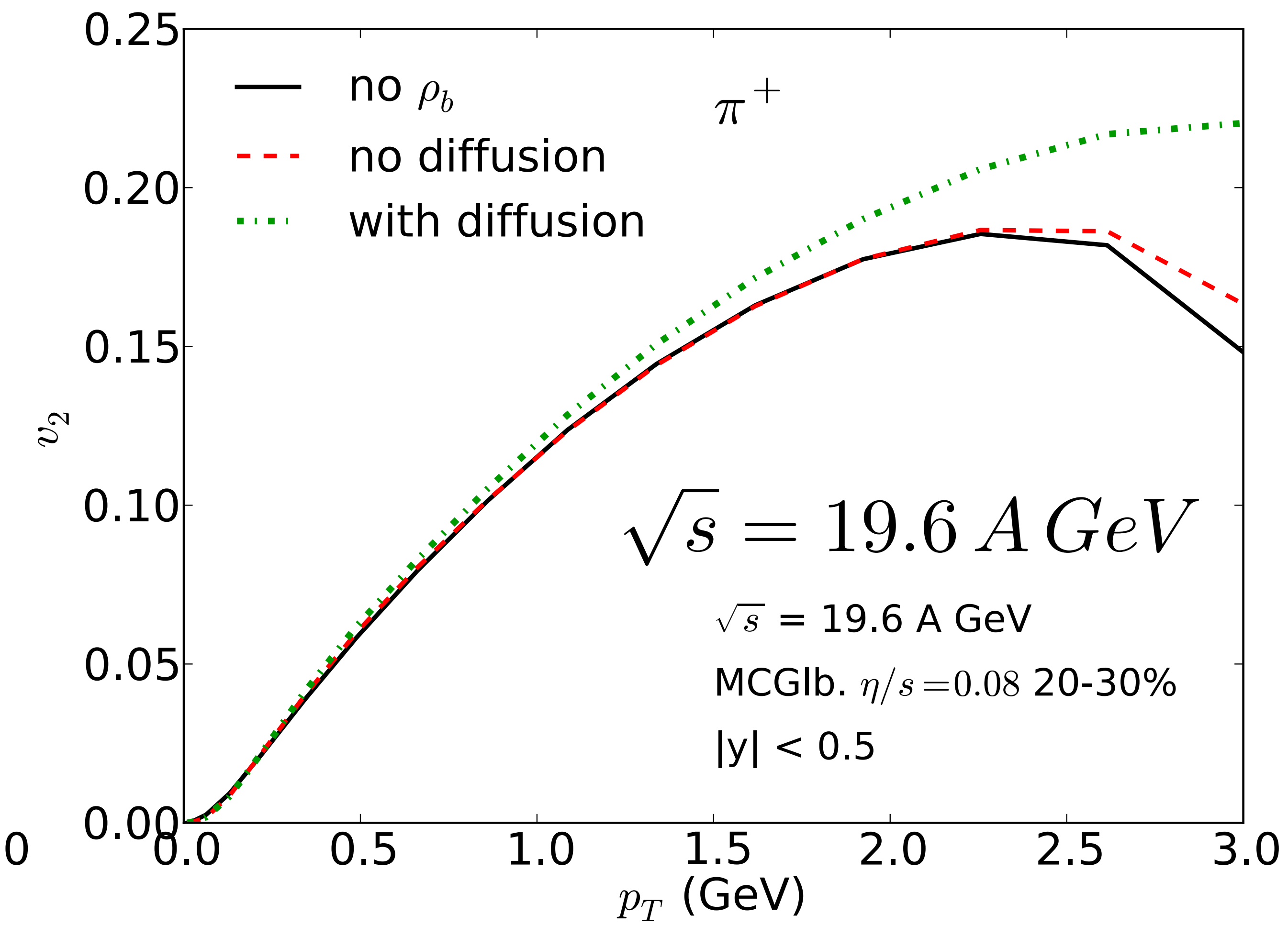
Light meson spectra and v$_2$



- At top RHIC energy, finite ρ_b and diffusion have little effects on pion spectra and v$_2$ at mid-rapidity

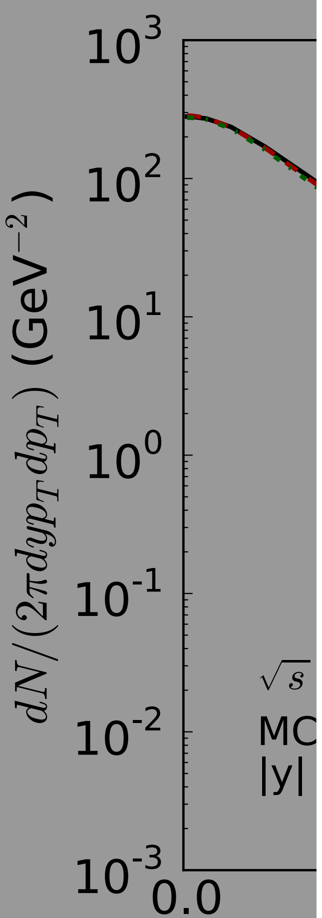
Light meson spectra and v_2

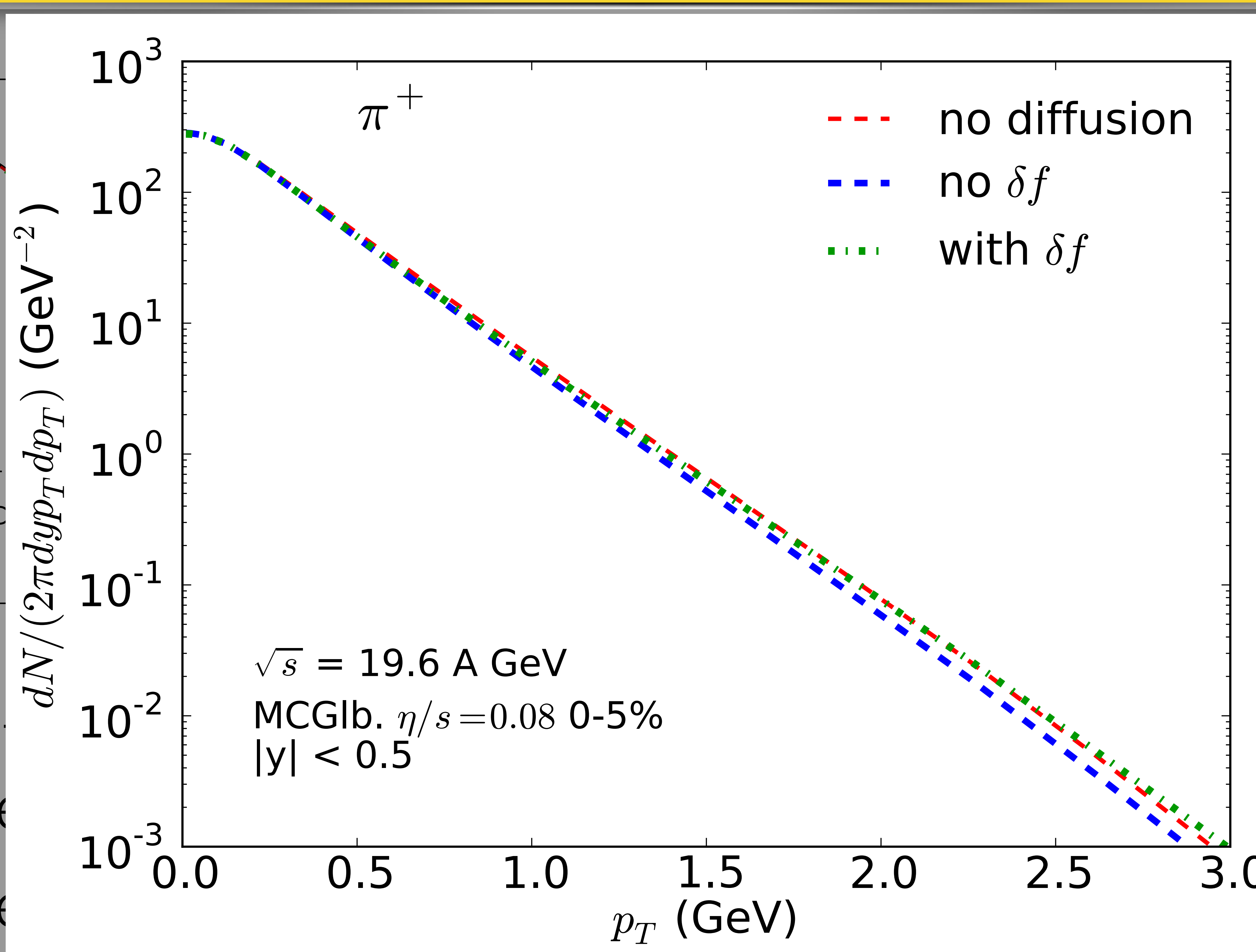


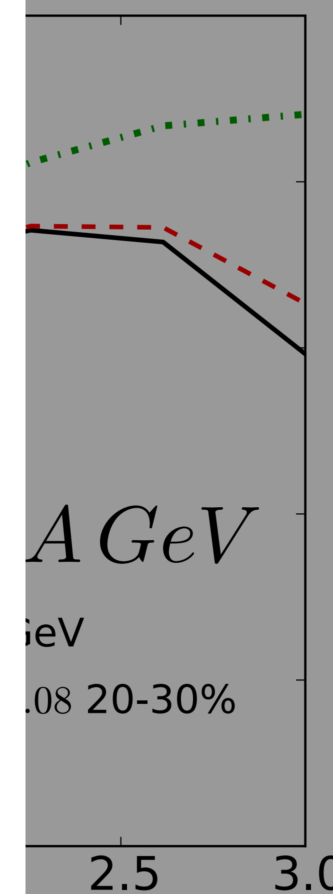


- At top RHIC energy, finite ρ_b and diffusion have little effects on pion spectra and v_2 at mid-rapidity
- The effects increase as collision energy goes down.

Light meson spectra and v_2

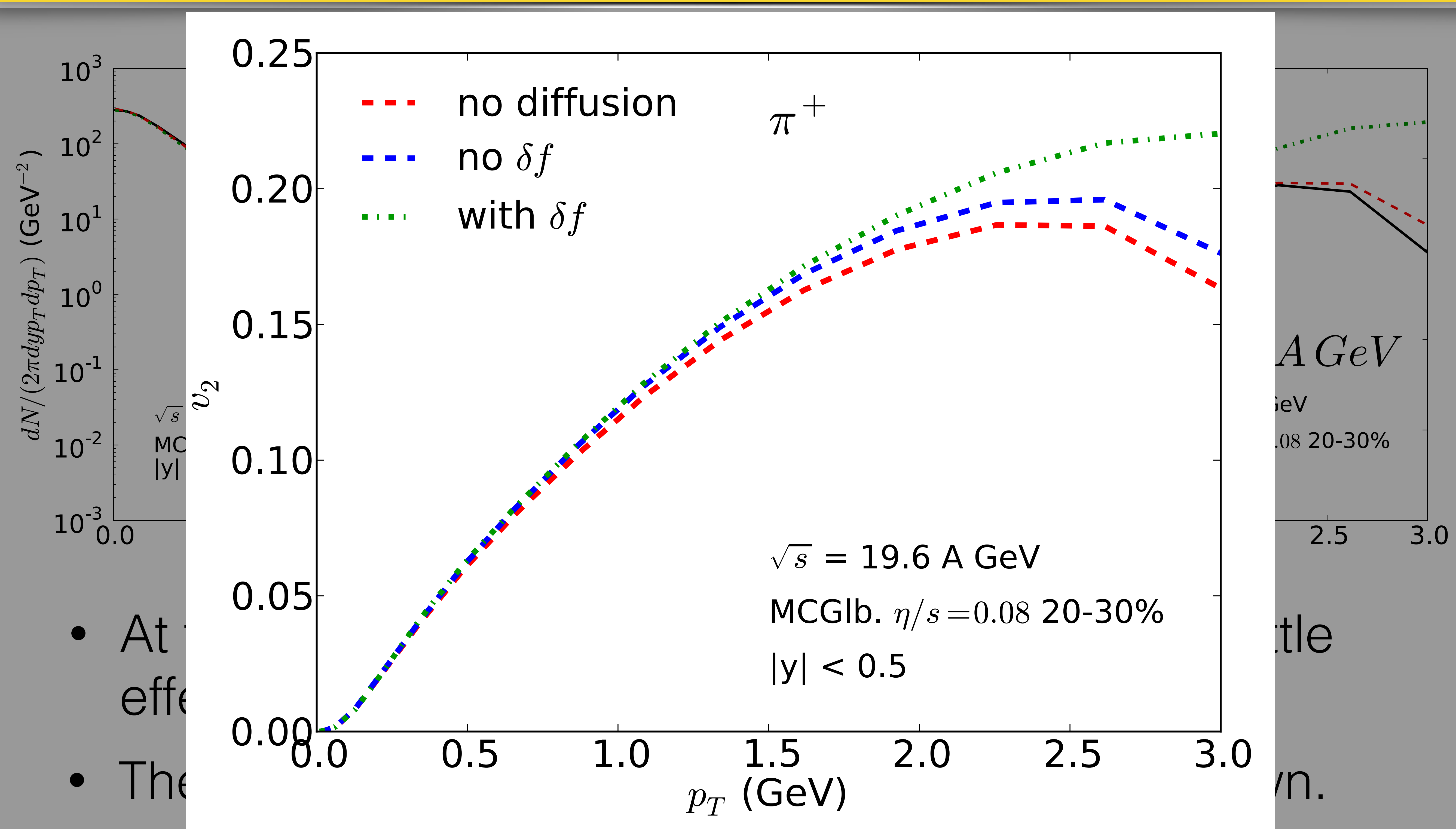






- At … tle effe…
- The … wn.
- Baryon diffusion reduces radial flow; δf makes the pion spectra flatter

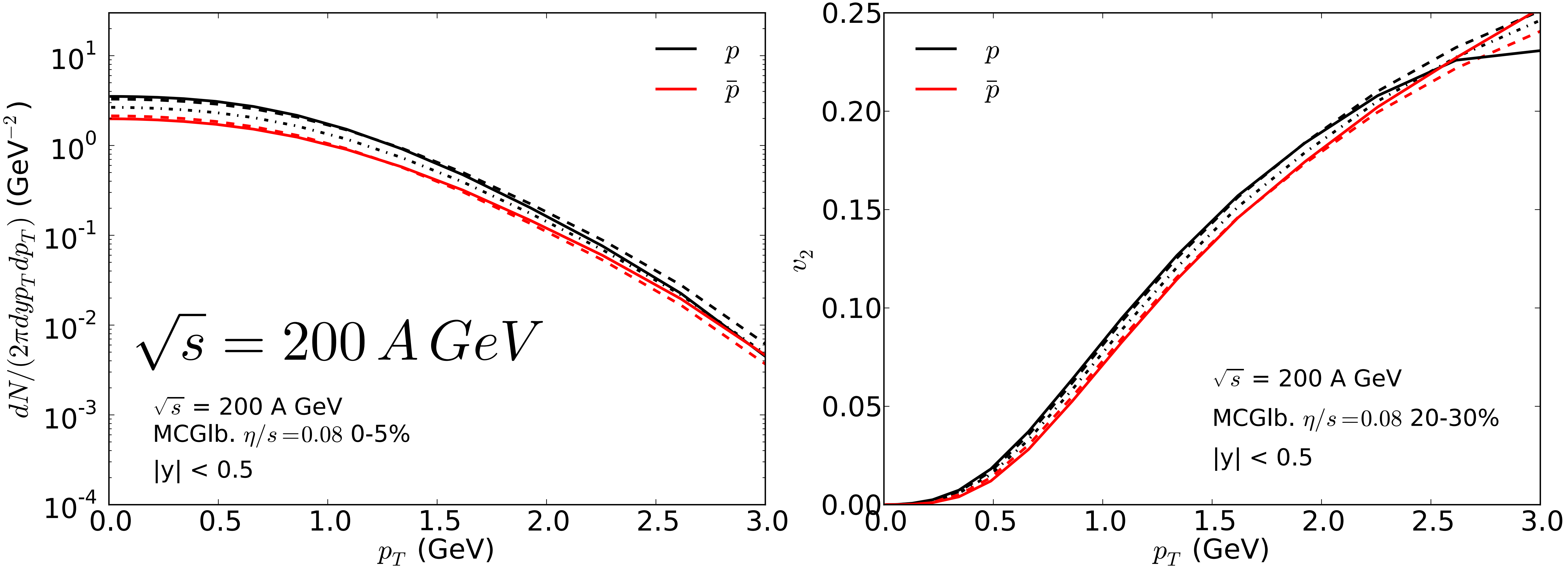
Light meson spectra and v_2



- Baryon diffusion increases pion $v_2(p_T)$; δf increases pion v_2 at high p_T

proton vs anti-proton spectra and v_2

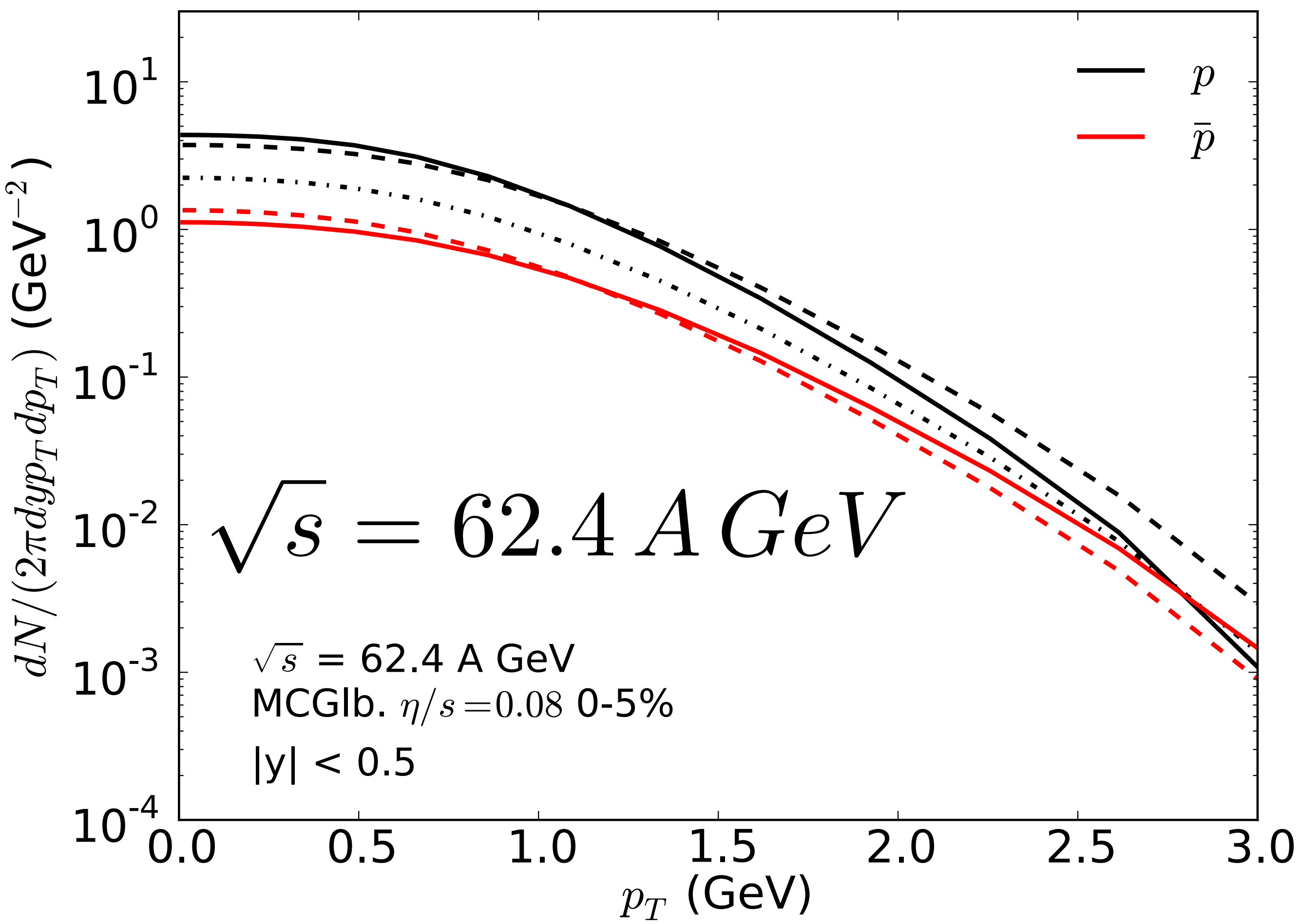
Solid: with diffusion; Dashed: no diffusion; Dash-dotted: no ρ_B

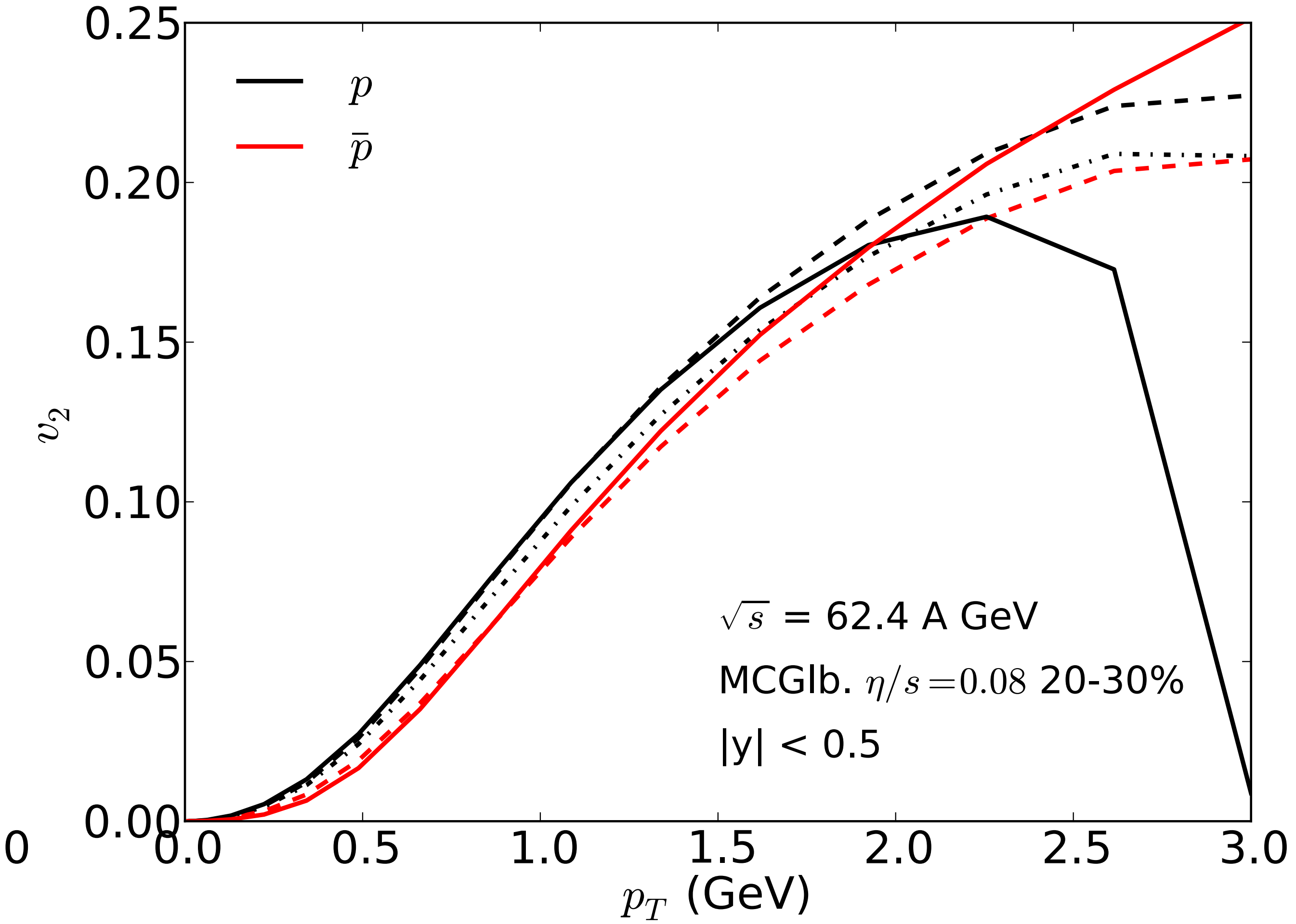


- Baryon diffusion has small effects on proton, antiproton spectra and v_2 at top RHIC energy

proton vs anti-proton spectra and v_2

Solid: with diffusion; Dashed: no diffusion; Dash-dotted: no ρ_B

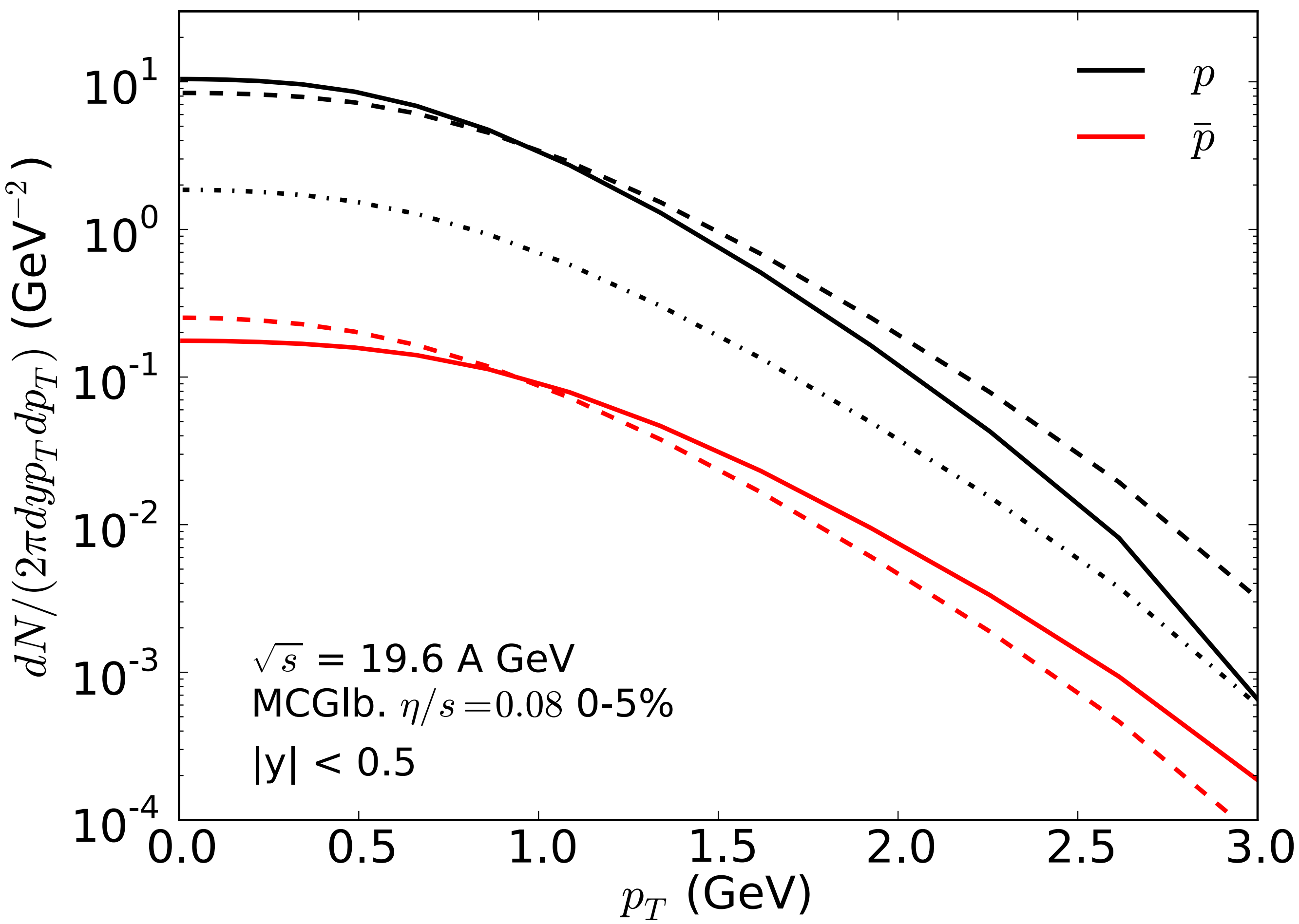


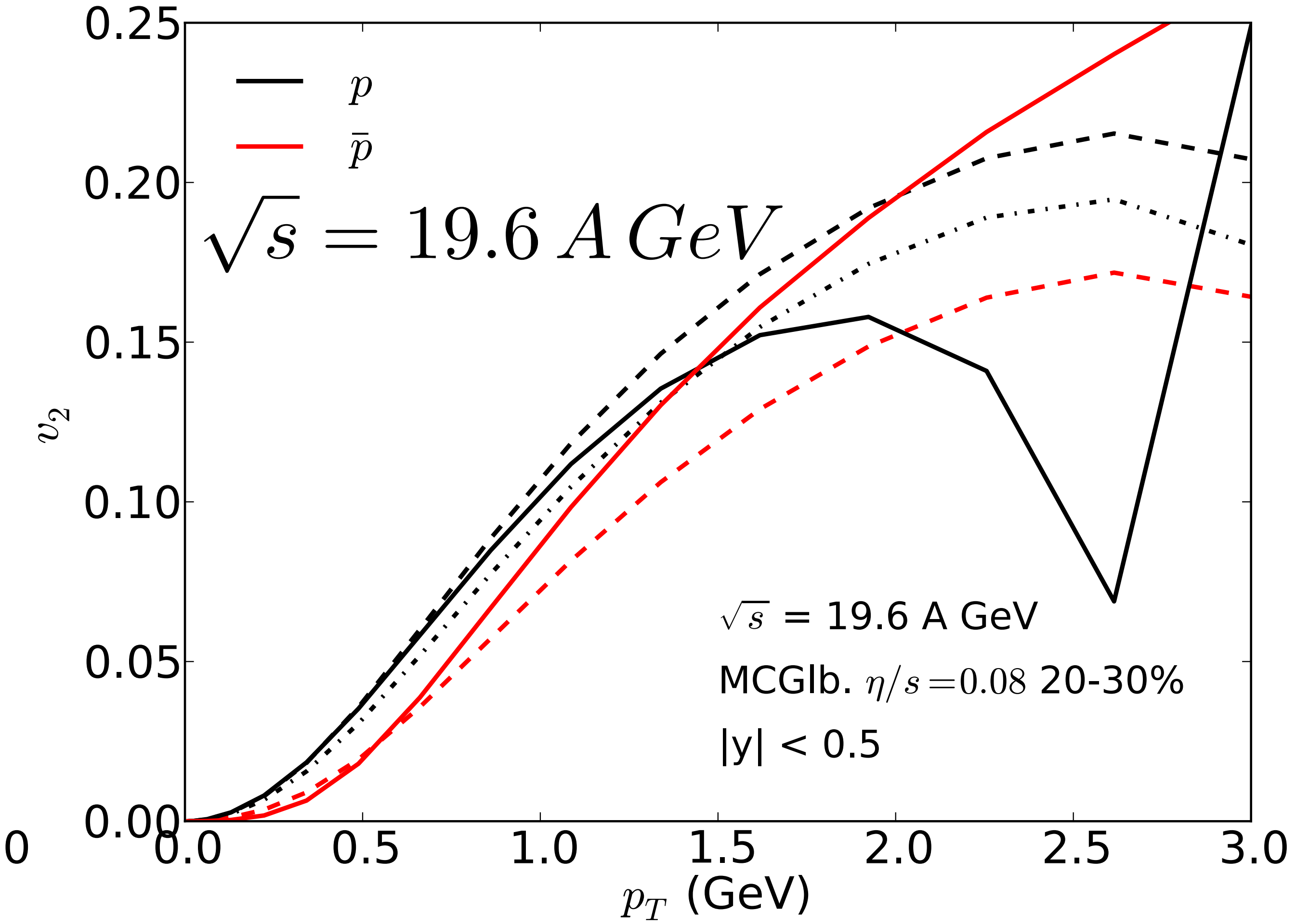


- Baryon diffusion has small effects on proton, antiproton spectra and v_2 at top RHIC energy

proton vs anti-proton spectra and v2

Solid: with diffusion; Dashed: no diffusion; Dash-dotted: no ρ_B

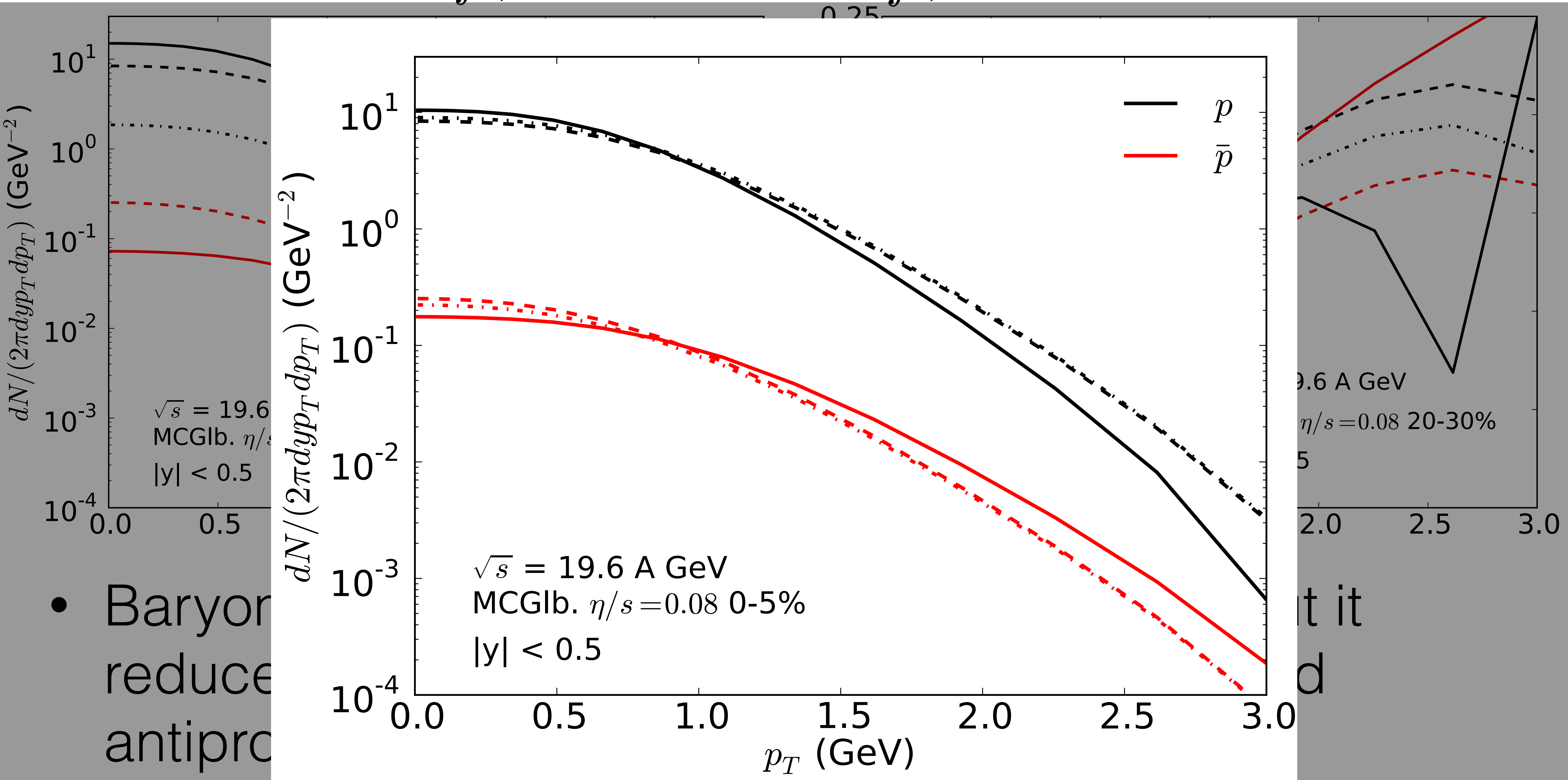




- Baryon diffusion slightly increases $N^p - N^{\bar{p}}$; but it reduces the difference in v2 between proton and antiproton

proton vs anti-proton spectra and v_2

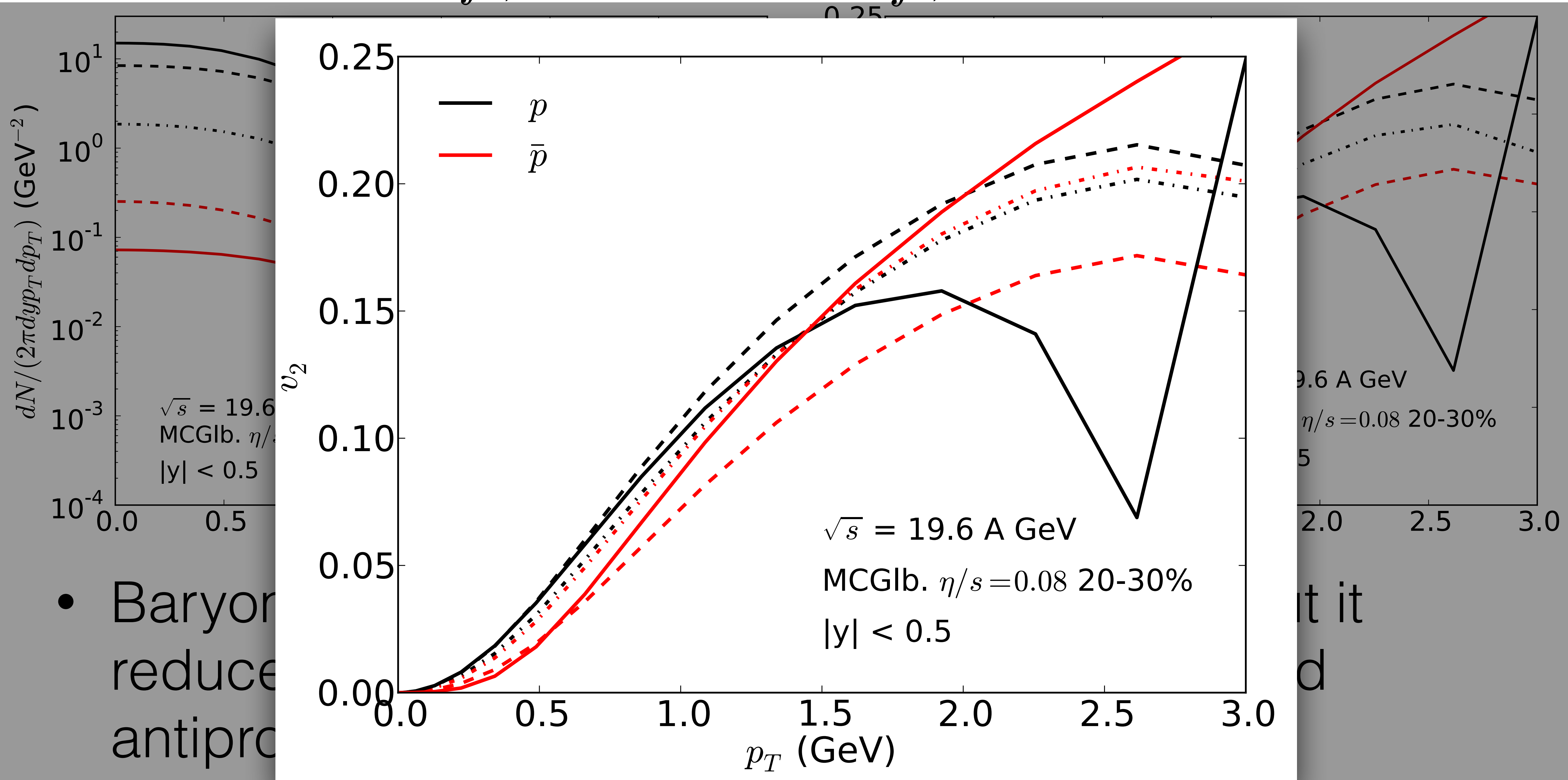
Solid: with δf ; Dash-dotted: no δf ; Dashed no diffusion



- Baryor ... t it reduce ... d antipro...
- Opposite δf corrections to protons and anti-protons

proton vs anti-proton spectra and v_2

Solid: with δf ; Dash-dotted: no δf ; Dashed no diffusion



- Baryon diffusion reduces v_2 asymmetry between protons and anti-protons; δf corrections increase the difference

Conclusion

- We present preliminary **(3+1)-d** viscous hydrodynamic simulations including **net baryon diffusion** for the RHIC BES program
- Out-of-equilibrium δf corrections from baryon diffusion is essential to ensure net baryon number conservation
- Baryons and anti-baryons receive large opposite corrections from baryon diffusion δf
- Baryons diffusion reduce the proton antiproton v_2 asymmetry at the low collision energies
- Evolving more conserved currents, including initial state fluctuations, and coupling to UrQMD will come soon

back up

Stabilizing MUSIC with diffusion

We implement quest_revert for q^μ to stabilize the hydro evolution with diffusion,

$$u^\mu q_\mu = 0 \longrightarrow q^0 = \frac{u^i q^i}{u^0}$$

The size of q^μ

$$\xi_q \equiv \frac{\sqrt{-q^\mu q_\mu}}{|\rho_B|} \frac{1}{\mathrm{prefactor} \times \tanh(e/e_{\mathrm{dec}})}$$

$$\mathrm{prefactor} = 300$$

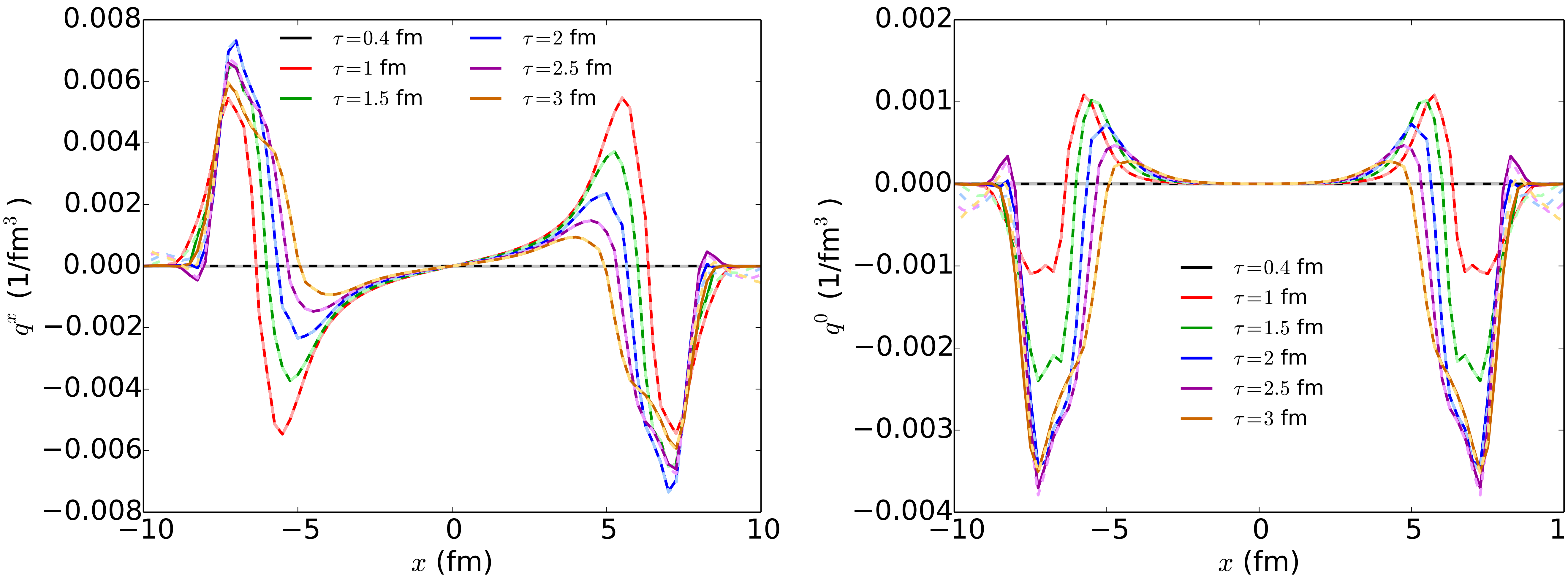
$$\xi_q^{\mathrm{max}} = 0.1$$

If $\xi_q > \xi_q^{\mathrm{max}}$

$$\tilde{q}^\mu = \frac{\xi_q^{\mathrm{max}}}{\xi_q} q^\mu$$

Stabilizing MUSIC with diffusion

We implement quest_revert for q^μ to stabilize the hydro evolution with diffusion,



most of the modifications are at the edges of the fireball