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Comment on the sign of the pseudoscalar pole contribution to the muon $g-2$

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Abstract

We correct the error in the sign of the pseudoscalar pole contribution to the muon $g-2$, which dominates the $\mathcal{O}(\alpha^3)$ hadronic light-by-light scattering effect. The error originates from the fact that the algebraic manipulation program FORM treats ϵ-tensor so as to satisfy $\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3}\eta^{\mu_4\nu_4} = 24$, as opposed to the expected value -24 when Minkowski space-time metric $\eta^{\mu\nu}$ is specified (at least in the version available before 1995). Replacing the part $\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}$ by $-\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}\eta_{\mu_4\nu_4} \pm \cdots$ in the FORM-formatted programs, we obtained a positive value for the pseudoscalar pole contribution, which agrees with the recent result obtained by Knecht *et al.*

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In this brief article we report the result of our reexamination of the pseudoscalar pole contribution to the muon $g-2$. In the previous studies [1, 2], the pseudoscalar pole contribution had been noted to be the dominant term of the $\mathcal{O}(\alpha^3)$ hadronic light-by-light scattering contribution to the muon $g-2$, $a_\mu \equiv (g_\mu - 2)/2$. In view of the expected accuracy of the new muon $g-2$ measurement at the Brookhaven National Laboratory (BNL) ($\Delta a_\mu(\exp) = 4 \times 10^{-10}$), the present authors [1] and Bijnens *et al.* [2] have examined this contribution carefully [2, 3], taking account of the experimental data obtained for the $P\gamma^*\gamma$-vertex ($P = \pi^0, \eta, \eta'$) at CLEO [4]. The primary purpose of this paper is to examine the sign of this contribution in light of the two recent papers [5, 6]. We therefore concentrate on the π^0 pole contribution which gives the value

$$a_\mu(\pi^0) = -55.60\ (3) \times 10^{-11}\,, \tag{1}$$

in the naive vector meson dominance (nVMD) model [1]. This model simply modifies the Wess-Zumino term for the $\pi^0\gamma\gamma$-vertex by attaching ρ-meson propagators which carry photon momenta k_1, k_2,

$$-\,i\frac{\alpha}{\pi f_\pi}\epsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta\,\frac{M_\rho^2}{M_\rho^2 - k_1^2}\frac{M_\rho^2}{M_\rho^2 - k_2^2}\,. \tag{2}$$

Both Bijnens *et al.* [2] and Bartos *et al.* [7] obtained the negative value for the pseudoscalar pole contribution independently of our result. Taking this value into account as a part of the standard model prediction, it is found that the current measurement of $g-2$ indicates 2.6 σ discrepancy from the prediction of the standard model [8, 9].

However, the recent papers [5, 6] pointed out that the pseudoscalar pole contribution is positive, which is opposite to the sign of (1). If this is true, it will reduce the deviation of the current BNL measurement from the standard model considerably. In view of these papers [5, 6], and of the significance of the $\mathcal{O}(\alpha^3)$ hadronic light-by-light scattering contribution in interpreting the current measured value of a_μ, we decided to reinvestigate the π^0 pole contribution.

In the following, we summarize the results of our investigation to show various points we have scrutinized closely. A full account is being prepared in a separate paper [10] .

The first phase of examination consists of the following four steps of the original calculation [1]:

(1) We obtained the expression for the contribution to the $\bar{\mu}\mu\gamma$-vertex function $\Gamma^\nu(p_I, p_F)$ ($p_I \equiv p - q/2$ and $p_F \equiv p + q/2$ are the initial and final momenta of the muon) by direct application of Feynman rules.
(2) We evaluated the trace of the γ matrices using the algebraic manipulation program FORM to extract a_μ from $\Gamma^\nu(p_I, p_F)$ by means of the magnetic moment projection [1],

$$
\begin{aligned}
a_\mu &= \lim_{q^2 \to 0} \lim_{\{p\cdot q \to 0;\, p^2 + q^2/4 \to m^2\}} \mathrm{Tr}\left(P_\nu(p,q)\Gamma^\nu(p_I, p_F)\right) , \\
P^\nu(p,q) &\equiv \frac{m}{16\, p^4 q^2} \left(\not{p} - \frac{\not{q}}{2} + m\right) \left(\left(\gamma^\nu \not{q} - \not{q}\gamma^\nu\right) p^2 - 3\, q^2 p^\nu\right) \\
&\qquad \times \left(\not{p} + \frac{\not{q}}{2} + m\right) ,
\end{aligned} \tag{3}
$$

where q is the incoming photon momentum.
(3) The result of Eq. (3) is plugged into a FORTRAN program written in the formalism developed for the numerical evaluation of Feynman diagrams [11].
(4) The numerical evaluation of a_μ is carried out with the help of the Monte Carlo integration routine VEGAS [12].

In addition we confirmed by hand calculation and by MATHEMATICA that the projection operator in Eq. (3) works correctly, by extracting the anomalous magnetic moment $a_\mu = F_M(0)$ from

$$
\Gamma^\nu(p_I, p_F) = \left(F_E(q^2)\gamma^\nu + F_M(q^2)\frac{1}{2m} i\sigma^{\nu\lambda} q_\lambda\right) , \tag{4}
$$

with

$$
\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] . \tag{5}
$$

After going through these steps

(I) we rederived the value given in Eq. (1) for the π^0 pole contribution in the nVMD model.

The next phase was to derive the result obtained in Ref. [5].

We first assumed that the expression given for a_μ in Eqs. (3.4) and (3.5) of Ref. [5] is correct and see if the numerical evaluation of them gives the value of our Eq. (1) but with opposite sign, as claimed in Ref. [5]. We translated Eqs. (3.4)

[1] We correct the typo-error for the expression for $P_\nu(p,q)$ of eq. (3.24) in Ref. [1], which does not affect the result obtained there.

and (3.5) into an expression suitable for the numerical evaluation according to the formalism of Ref. [11]. By carrying out the six-dimensional integration over the Feynman parameters with use of VEGAS, we obtain the value in Eq. (1) but with an opposite sign. This test not only confirms the result of Ref. [5] but also gives an evidence that

(II) both Ref. [5] and our work performed the loop integration part correctly. The disagreement in sign must therefore come from an earlier stage.

We therefore switched our attention to the task of projecting a_μ out from $\Gamma^\nu(p_I, p_F)$ by means of algebraic programs. Using the algebraic manipulation program FORM we pursued the steps described in Ref. [5] in detail to see whether we can reproduce their Eqs. (3.4) and (3.5). We found that

(III) the trace operation by FORM yielded the sign *opposite* to that of $T_{1,2}(q_1, q_2; p)$ in Eq. (3.5) of Ref. [5].

We thus recognized the following two possibilities as most likely:
(a) Ref. [5] made a mistake in picking up the sign of the trace of the γ matrices.
(b) We made a mistake at the stage of picking up the sign of the trace of the γ matrices *systematically*. By systematically we mean that we failed to identify the sign irrespective of what projection operator was used to extract a_μ; we derived the value of Eq. (1) by both our own projection operator (3) and those given in Eqs. (2.9) - (2.11) of Ref. [5]. All projectors gave the same result.

In examining the possibility (b), we noticed one crucial difference between Ref. [5], which leads to the positive value, and ours, which leads to the negative value; while Ref. [5] used the algebraic manipulation program REDUCE to perform the trace calculation of the γ matrices, we used FORM instead. Recall that we have used FORM even for examining the results of Ref. [5].

Thus we decided to check whether FORM works correctly, or whether we handle it properly. The program FORM had been used successfully to calculate the QED corrections to the $g-2$ of the muon and the electron as well as other observables by one of the present authors. However, this does not guarantee that it deals correctly with the ϵ-tensor, the central object of our study of the pseudoscalar contribution [2].

[2] The calculation of Ref. [1] was carried out by a version of FORM available before 1995.

The simplest test of this question is to see if FORM successfully verifies the identity

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3}\eta^{\mu_4\nu_4} = -24\,, \tag{6}$$

which should hold in Minkowski space-time. Unfortunately, the result turned out to be +24, which indicates that the metric fixing statement in FORM (FixIndex $1:-1, 2:-1, 3:-1$;) did not work properly for $\epsilon_{\mu_1\mu_2\mu_3\mu_4}$. In order to see how this affects our calculation, we repeated the trace calculation using the right-hand side of the identity:

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4} = -\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}\eta_{\mu_4\nu_4} \pm (\text{the other 23 terms})\right]\,, \tag{7}$$

where the other 23 terms are obtained by shuffling the order of $\{\nu_1, \nu_2, \nu_3, \nu_4\}$ in all possible ways. Each term contributes in the bracket with the sign $+$ $(-)$ if the even (odd) permutation is performed to reach this order from $\{\nu_1, \nu_2, \nu_3, \nu_4\}$. We found that the method using (7) led to a result opposite in sign to the previous result (1). This means that FORM (at least the version we used) has an internal inconsistency. On the other hand, REDUCE passed the same test without difficulty. Hence, we conclude that it was the case (b) that actually happened [3]. In other words, the π^0 pole contribution in (1) must be changed

$$a_\mu(\pi^0) = 55.60\ (3) \times 10^{-11}\,, \tag{8}$$

in the nVMD model.

The sign of the pseudoscalar pole contribution (8) has returned to that of Ref. [13] and agrees with that of Ref. [5]. But, in the former case, the program containing the product of two ϵ-tensors has also been used to extract a_μ. At the same time, there was an error in the sign of the logarithmic term dominating the pseudoscalar pole contribution, which was corrected in Ref. [1] [4]. The double switching of the sign has led accidentally to the positive value for the pseudoscalar contribution in Ref. [13].

We conclude that the sign of all the results for the pseudoscalar pole contribution as well as the axial-vector meson pole contribution described in Ref. [1, 3] must be reversed. The signs of the charged pseudoscalar loop contribution and the constituent quark loop contribution are not affected by the problem noted here.

[3] The reader should also pay attention when he uses γ_5 in FORM.

[4] The delicate point will be described in Ref. [10].

Collecting those results from Eq. (1.8) and changing the sign of Eqs. (1.9) and (5.1) of Ref. [3], we obtain the new value

$$a_\mu(\mathrm{LL}) = 89.6\ (15.4) \times 10^{-11}\,. \tag{9}$$

as the current value of the $\mathcal{O}(\alpha^3)$ hadronic light-by-light scattering contribution to the muon $g-2$. This reduces the discrepancy between the measurement and the prediction of the standard model to 1.6 σ deviations.

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