Beam-Beam Effect in eRHIC

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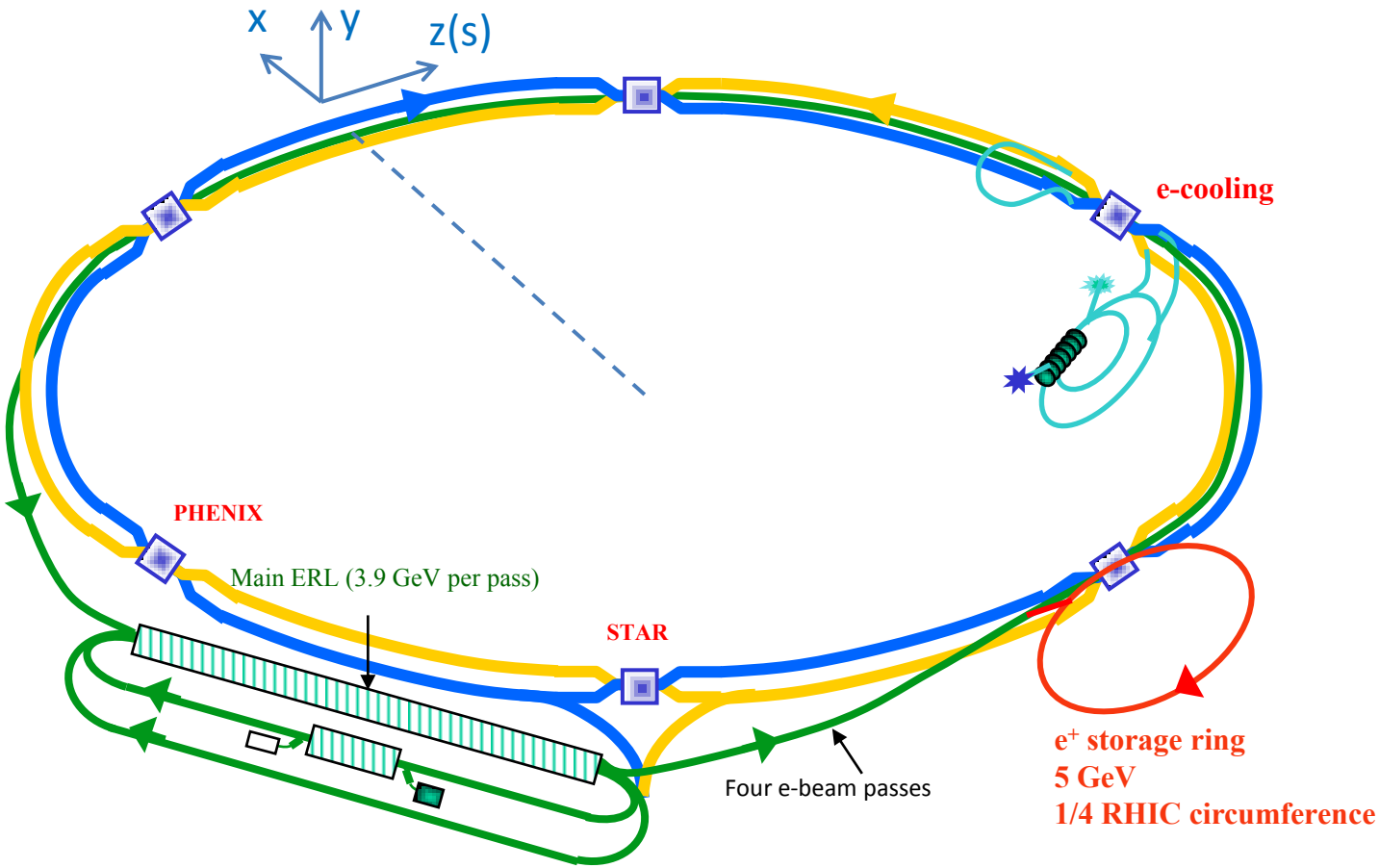
Linac - Ring Scheme Design



Courtesy of V. Ptitsyn

Linac – Ring Parameter Table

	High energy setup		Low energy setup	
	p	e	p	e
Energy (GeV)	250	10	50	3
Number of bunches	166		166	
Bunch spacing (ns)	71	71	71	71
Bunch intensity (10^{11})	2.0	1.2	2.0	1.2
Beam current (mA)	420	260	420	260
95% normalized emittance (π·mm·mrad)	6	115->460	6	115->565
RMS emittance (nm)	3.8	1->4	19	3.3->16.5
β*(cm), x/y	26	100->25	26	150->30
Beam-beam parameters, x/y	0.015	2.6->0.58	0.015	2.6->0.47
RMS bunch length (cm)	20	1	20	1
Polarization(%)	70	80	70	80
Peak Luminosity (1.e33 $cm^{-2}s^{-1}$)	2.6		0.53	

Advantages of Linac-Ring Scheme

- Luminosity enhancement

  0.47×10^{33} cm-2-s-1→ 2.6×10^{33} cm-2-s-1

- Full spin transparency for all energy
- Longer drift space in IR
- Upgradable to higher electron energy (20GeV)

Beam-Beam field

For a transverse Gaussian distribution,

$$\rho(x,y) = \frac{ne}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right)$$

(+/-4 sigma cut-off)

Bassetti-Erskine formular

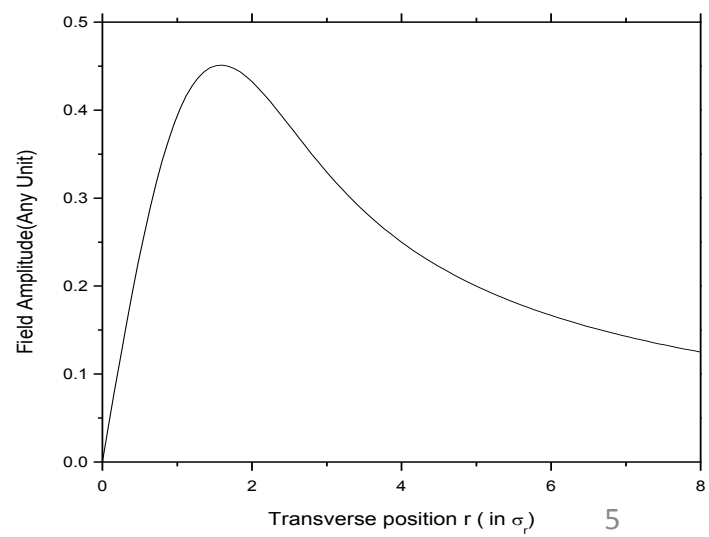
$$E_x - iE_y = \frac{-ine}{2\epsilon_0\sqrt{\pi}\sigma_{x-y}}\left[W\left(\frac{x+iy}{\sigma_{x-y}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right)W\left(\frac{\frac{x\sigma_y}{\sigma_x}+\frac{iy\sigma_x}{\sigma_y}}{\sigma_{x-y}}\right)\right]$$

For round beam case, the field have simple form

$$\vec{E}_r = \frac{ne}{2\pi\epsilon_0 r^2}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right]\vec{r}$$

Near axis, the field is linear.

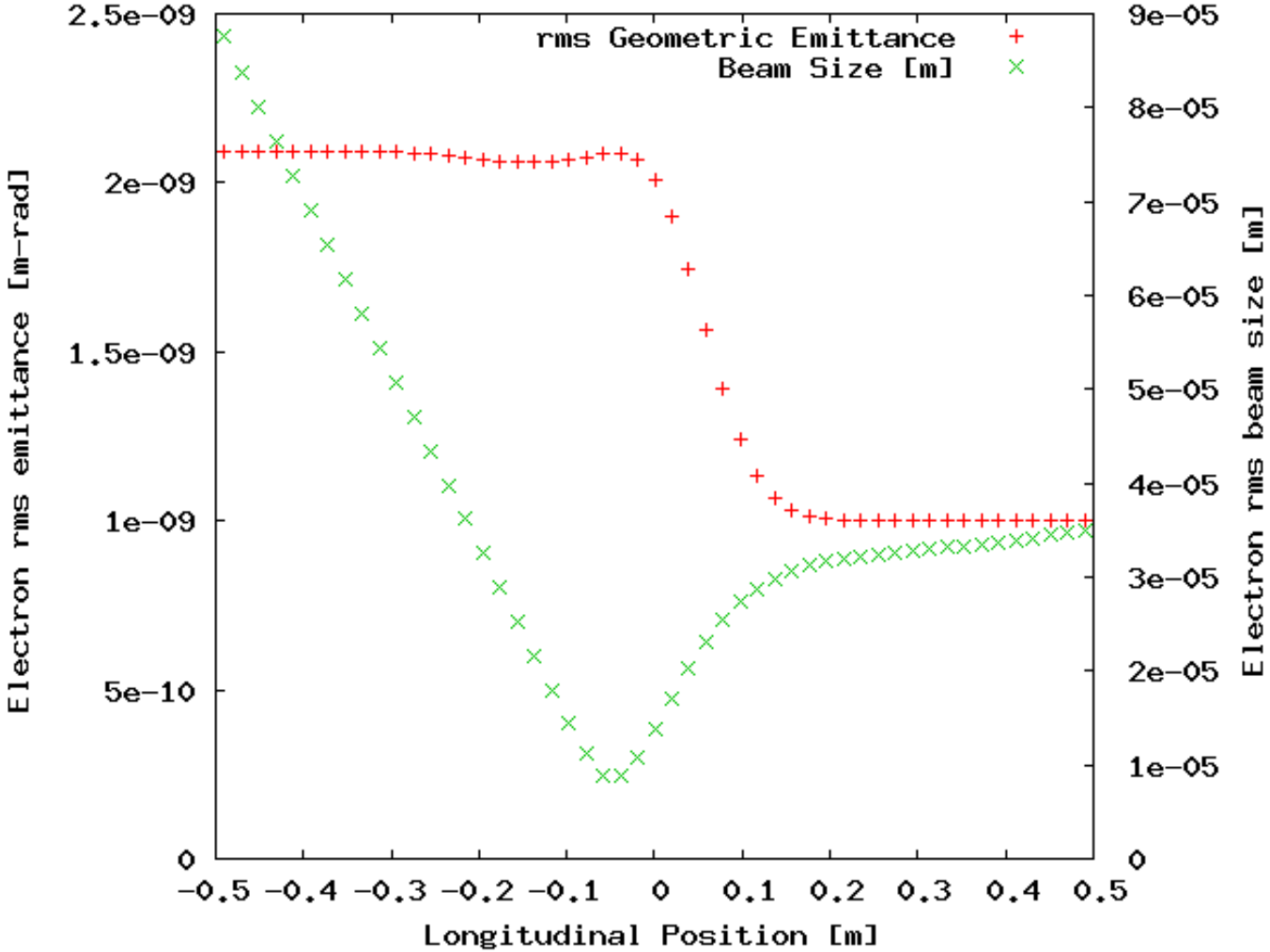
$$\begin{pmatrix}E_x\\E_y\end{pmatrix} = \frac{ne(1+\beta^2)}{2\pi\epsilon_0(\sigma_x+\sigma_y)}\begin{pmatrix}x/\sigma_x\\y/\sigma_y\end{pmatrix}$$



Outline of Beam-Beam Effect Study

- Effect of electron beam
  - Disruption and mismatch
  - Pinch effect
  - Beam loss concerns
- Effect of proton beam
  - Instability of head-tail type (Kink Instability)
  - Proton beam emittance growth from pinch effect
- Other issues, e.g. Noise

Disruption Effects



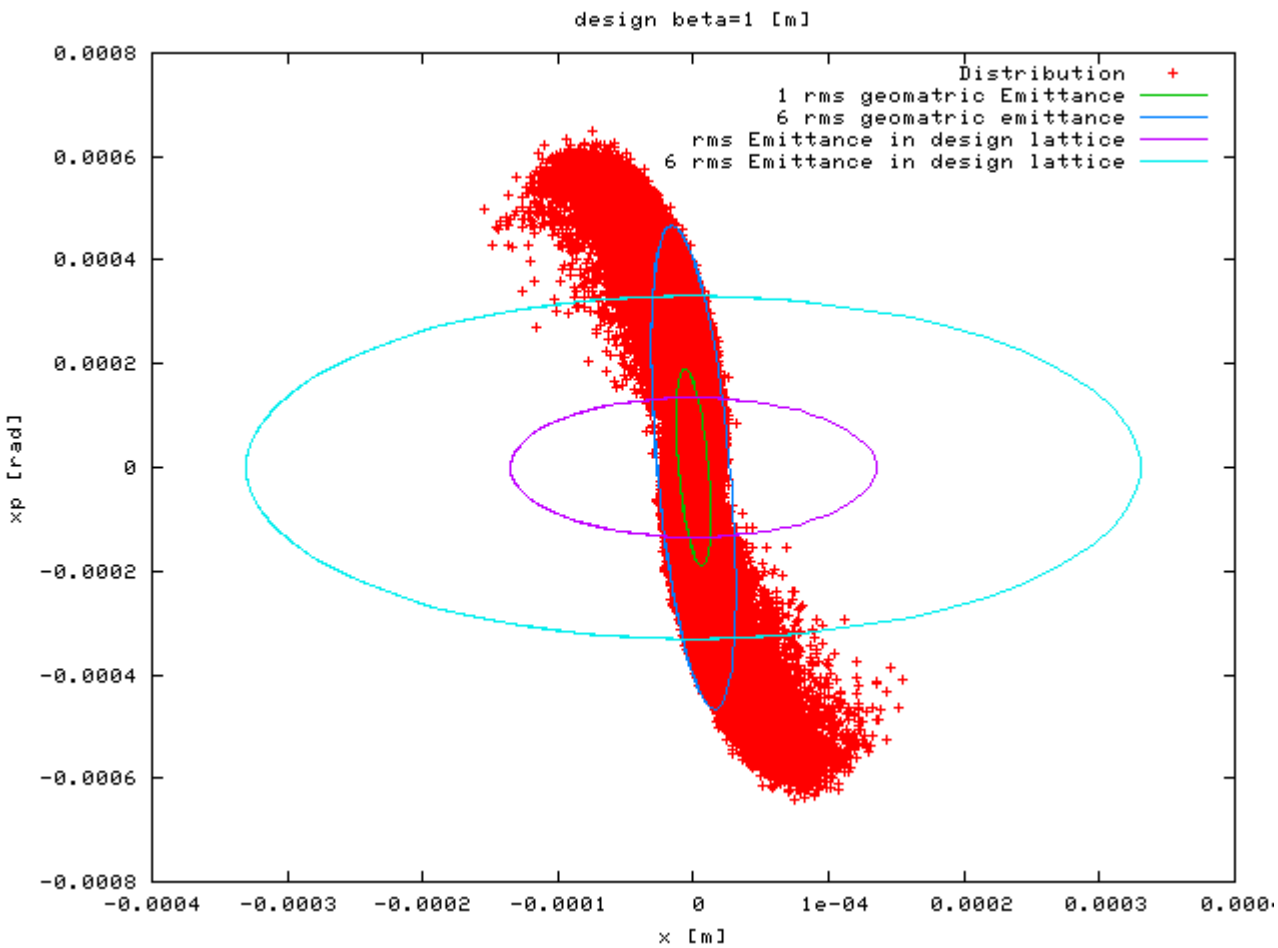
Design β^* = 1m, waist at IP, Initial emittance 1nm

Electron beam travels from positive longitudinal position to negative.

The nonlinear beam-beam force will cause the electron beam geometric emittance growth.

The focusing force will attract the electron to center and form the effect so called 'pinch effect'

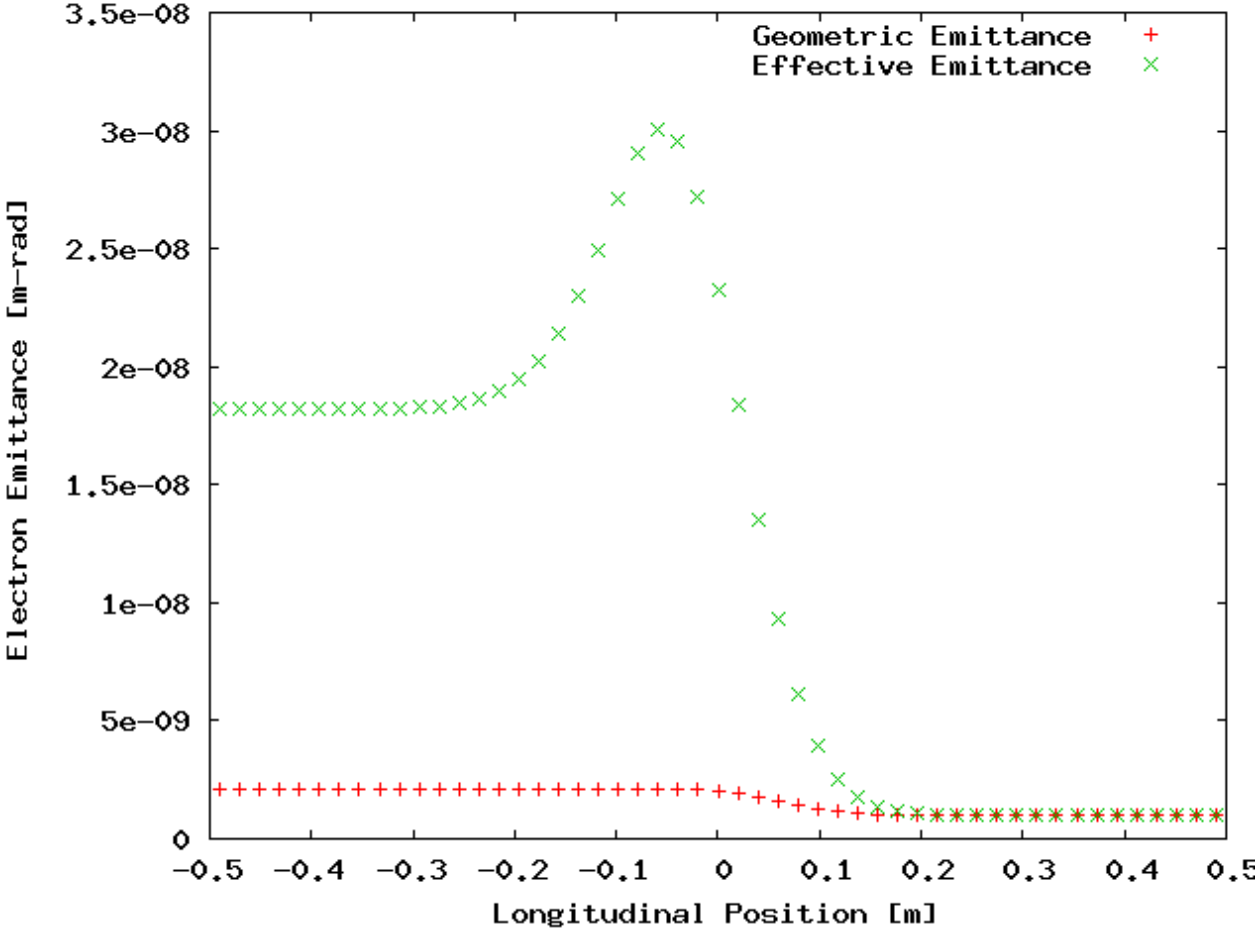
Mismatch



The mismatch due to beam-beam effect also plays a important role. It can enlarge the effective rms emittance in additional to the geometric rms emittance growth.

Design β^* = 1m, waist at IP, Initial emittance 1nm

Effective Emittance vs. Geometric Emittance



Geometric Emittance

$$\varepsilon_x = \sqrt{\left\langle (x-\bar{x})^2 \right\rangle \left\langle (x'-\bar{x}')^2 \right\rangle - \left\langle (x-\bar{x})(x'-\bar{x}') \right\rangle^2}$$

Effective Emittance

$$C(\tilde{x},\tilde{x}') = \gamma \tilde{x}^2 + 2\alpha \tilde{x}\tilde{x}' + \beta \tilde{x}'^2$$

$$\varepsilon_x = \bar{C}/2$$

Minimize the mismatch (Linear approach)

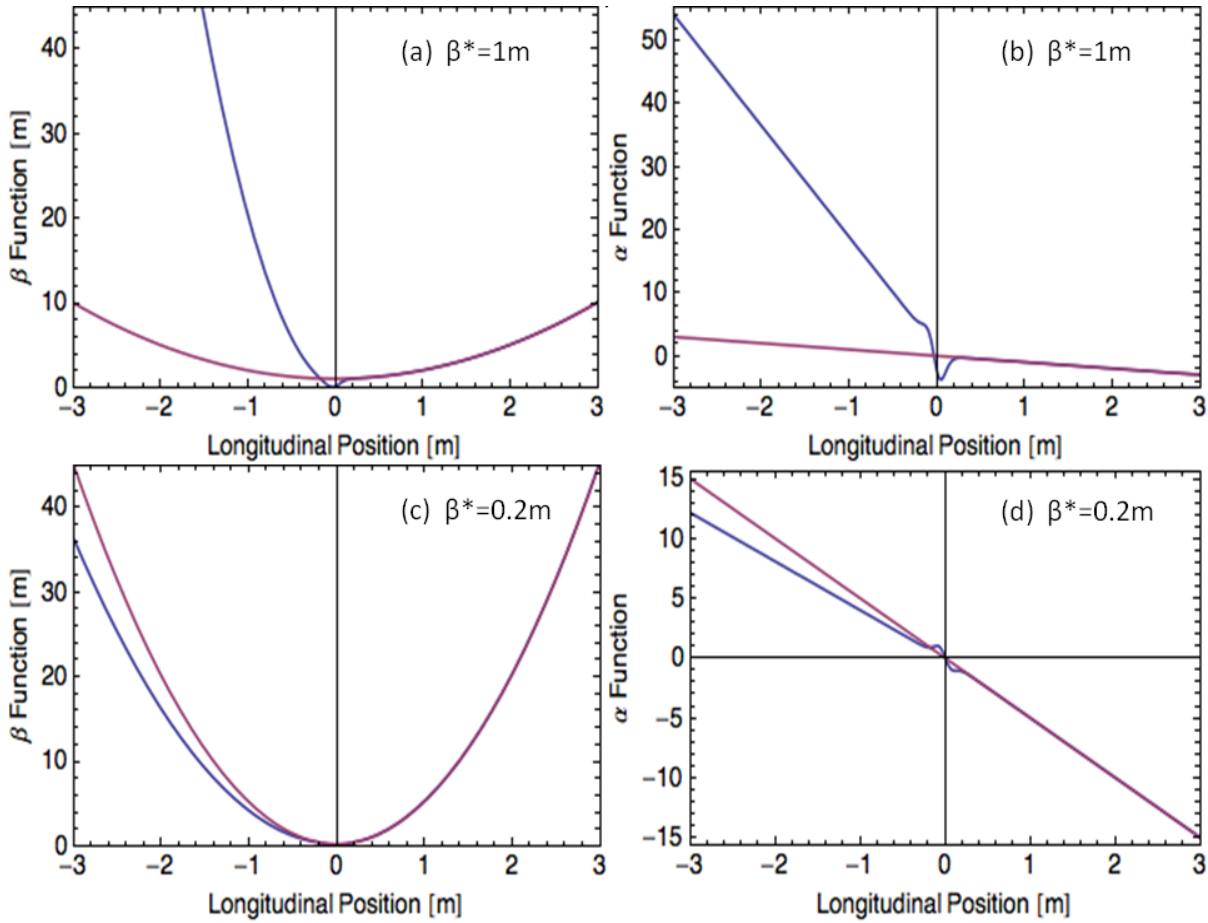
$$w'' + k^2(s)w - 1/w^3 = 0$$

$$\beta(s) = w^2(s)$$

$$\alpha(s) = -w(s)w'(s)$$

The boundary condition of above differential equation is at the negative infinity away from IP before collision theoretically.

Due to quick attenuation of proton longitudinal distribution beyond 3 rms beam length, we can set the boundary condition at the entrance of interaction region where s = 3m.

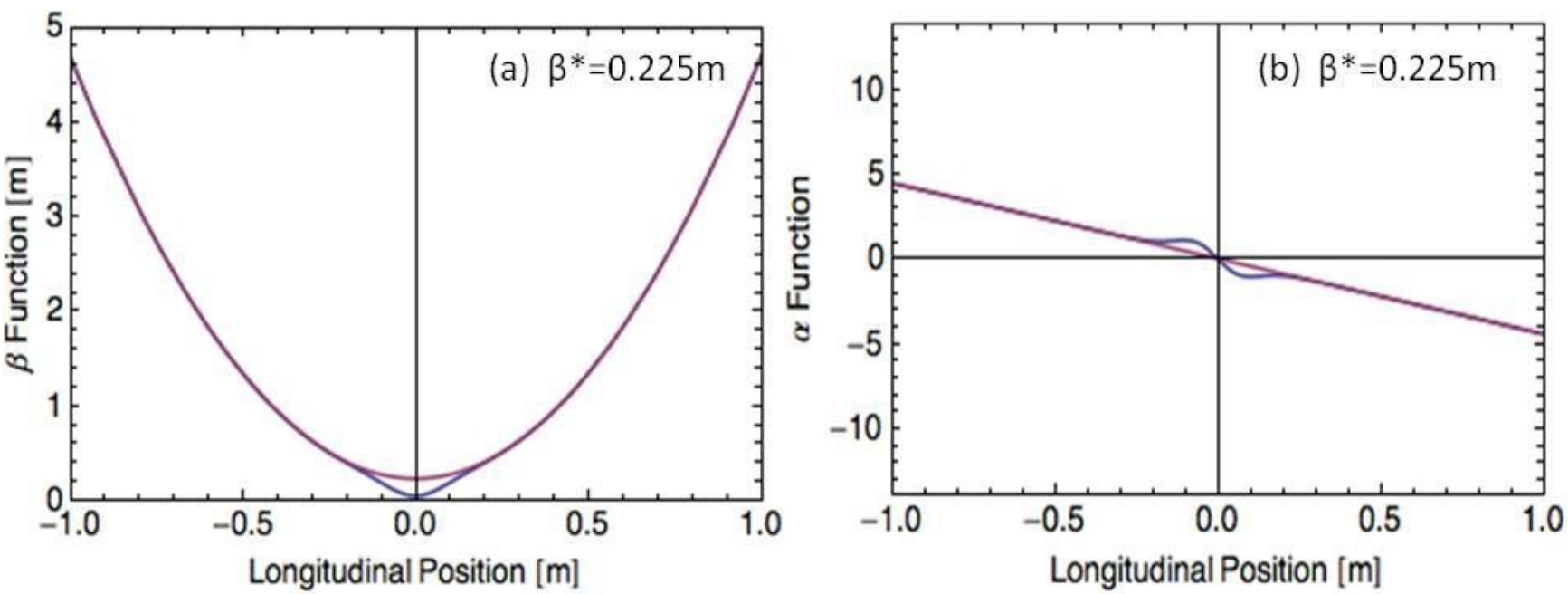


Minimize the mismatch (Linear approach)

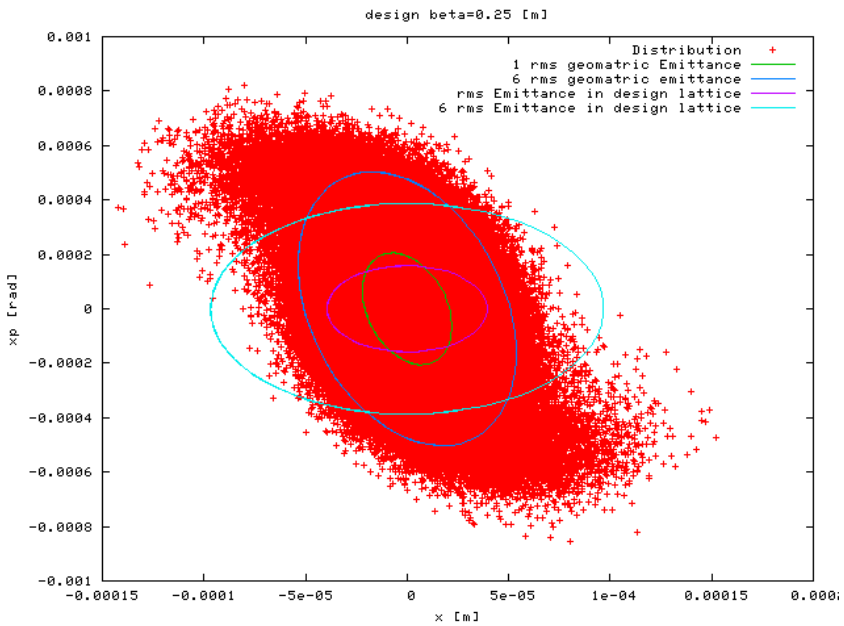
There should a design β^*, which lead the beam after collision to match the optics.
The β^* can be solved from

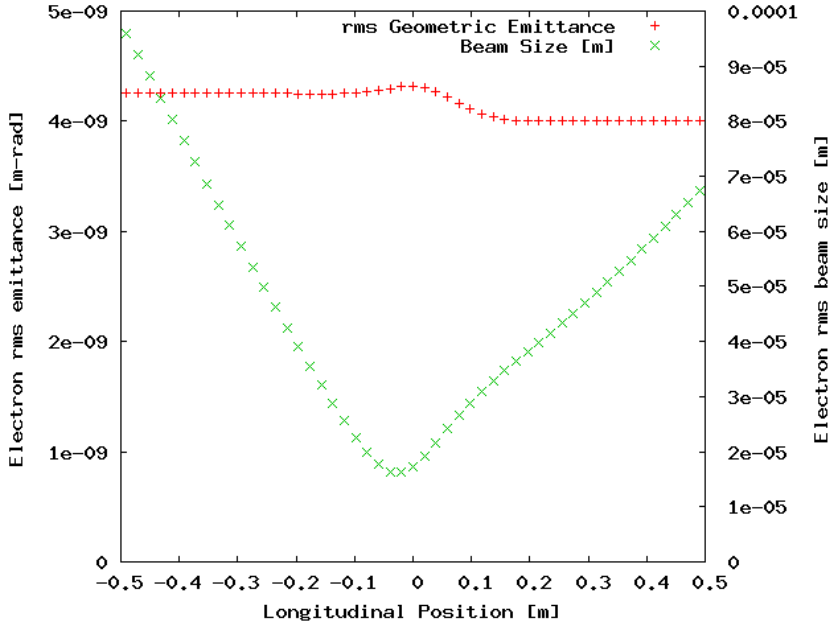
$$\beta\left(\beta^*, s=-3m\right)=\beta^*+\frac{s^2}{\beta^*}$$

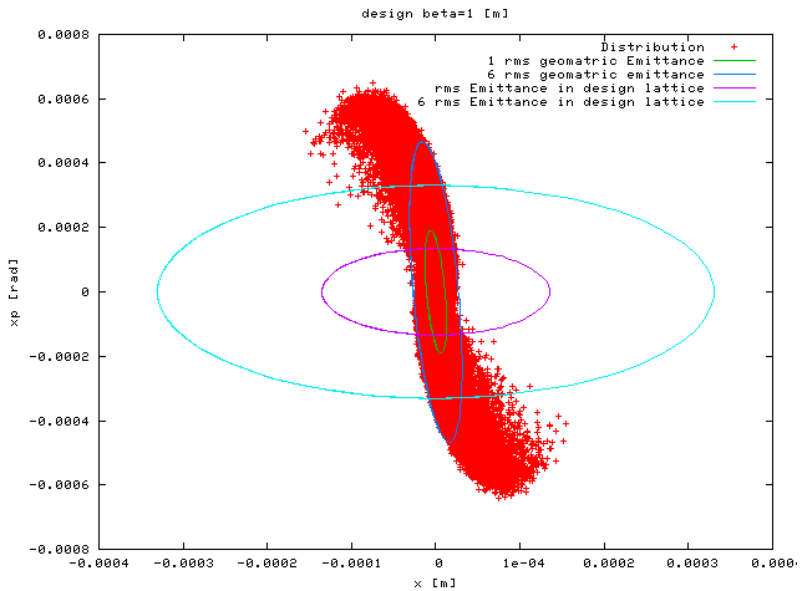
$$\beta^*=0.225\mathrm{m}$$

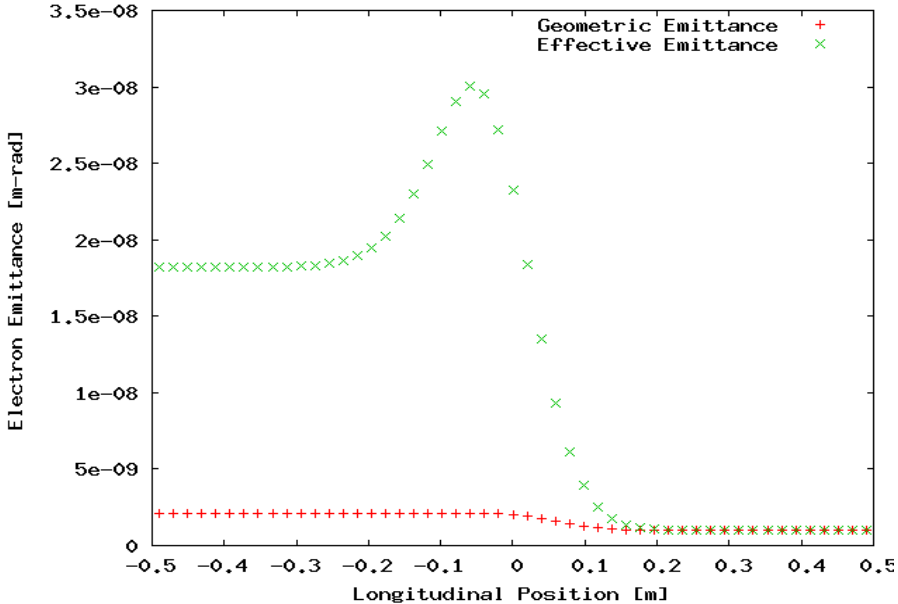


Reduce β* to 0.25m for less mismatch

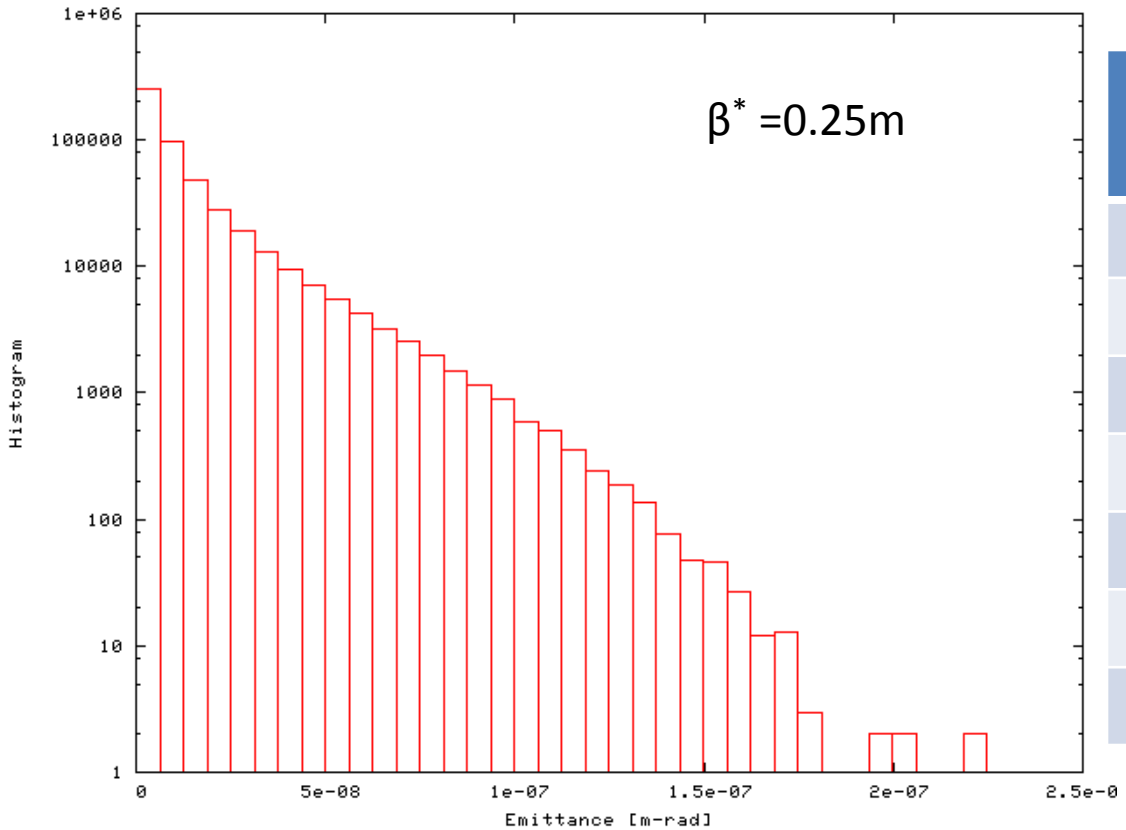








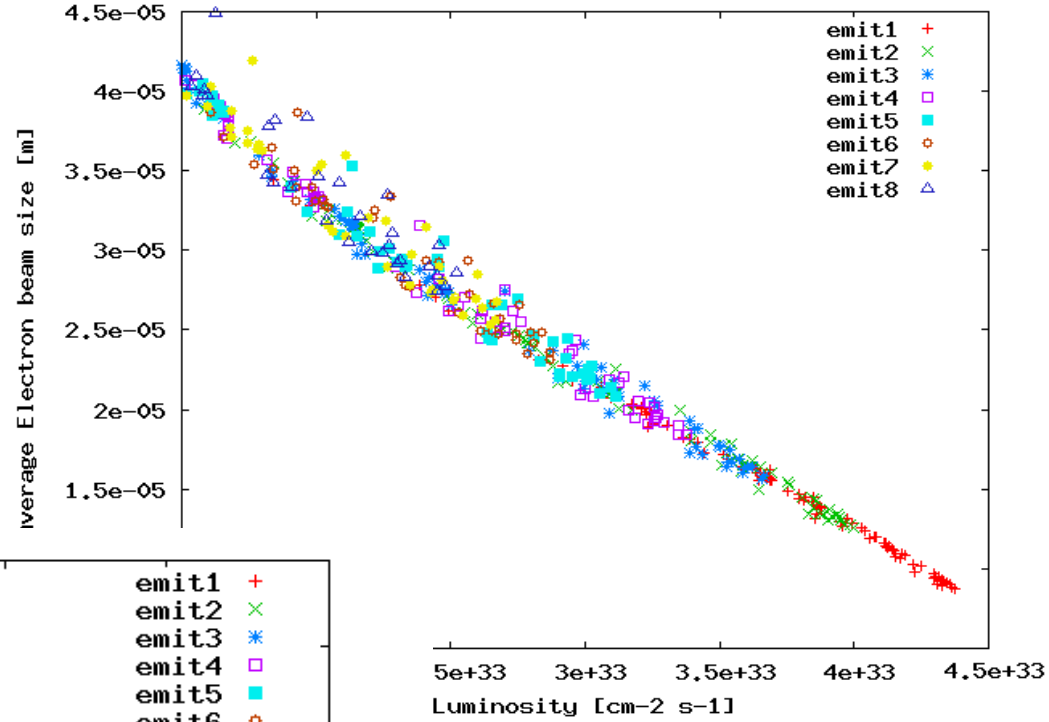
Long-Tail Distribution after Collision

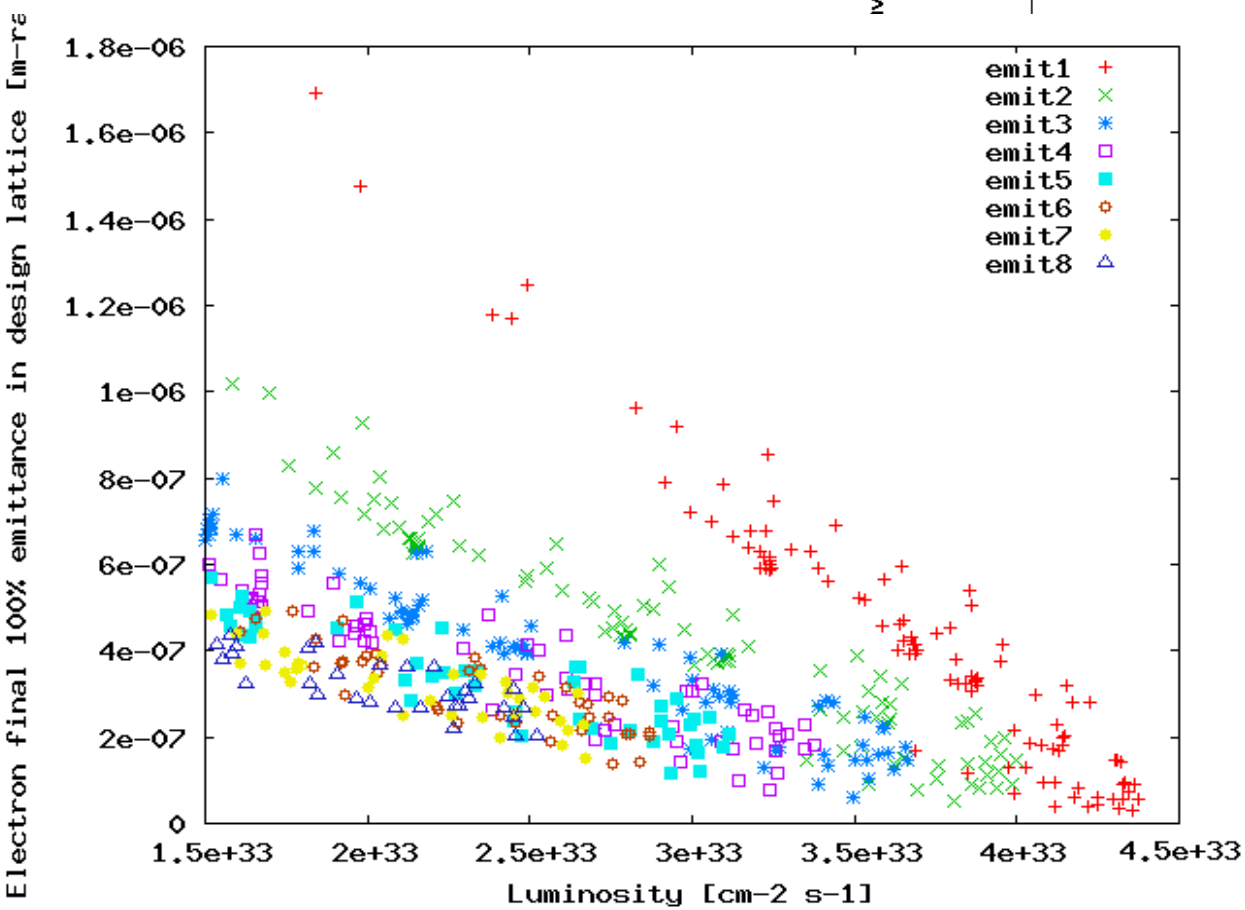


# of RMS Emittance	Gaussian beam	Final dist. At β^* =0.25m
2	63%	70%
4	86%	86%
6	95%	92%
10	99.3%	97.2%
16	100%	99.5%
20	100%	99.9%
30	100%	<1-1e-5

The optics of energy recovery path and linac must have large acceptance from large beam loss.

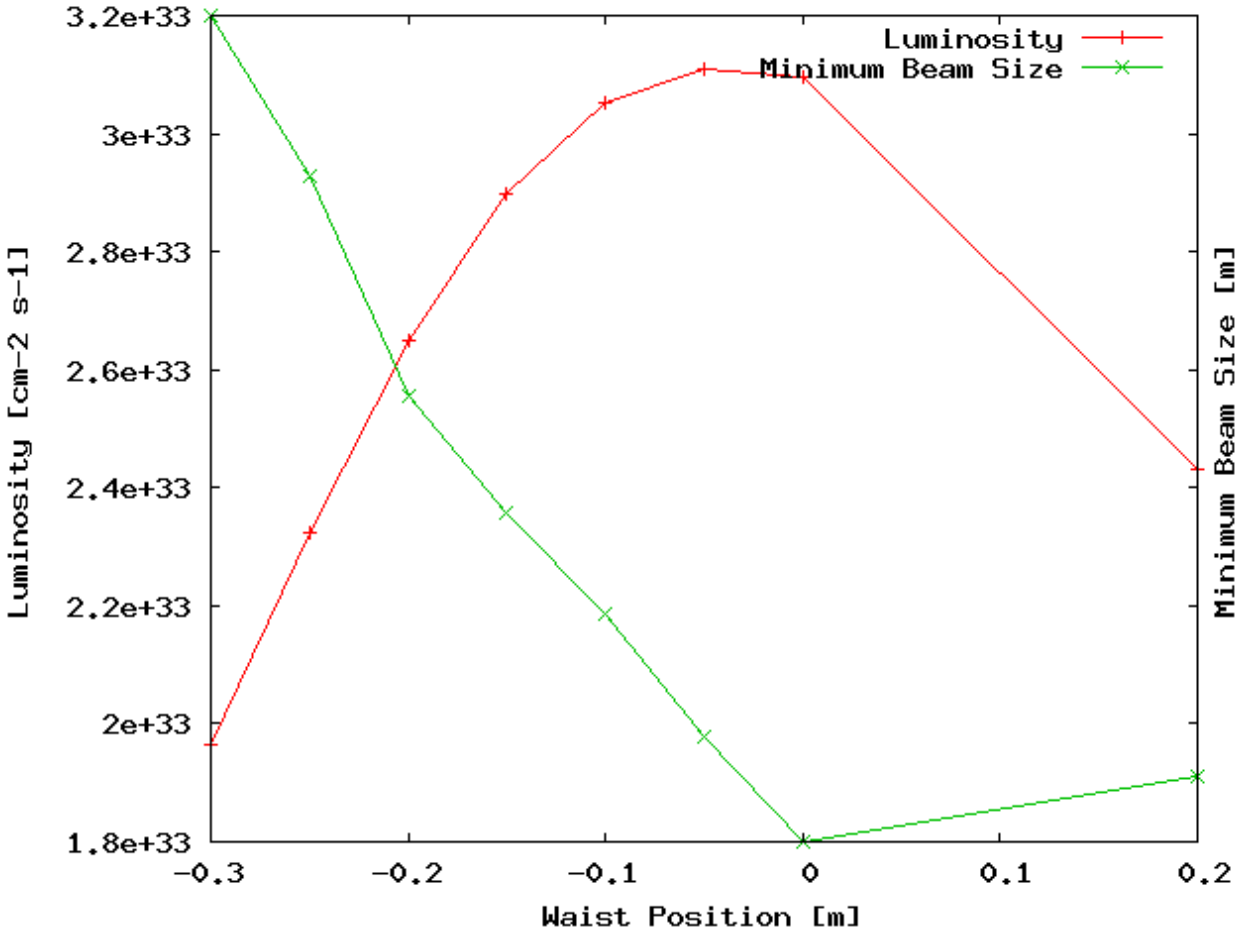
Vary the electron emittance, the optics (beta alpha function) at IP point before collision





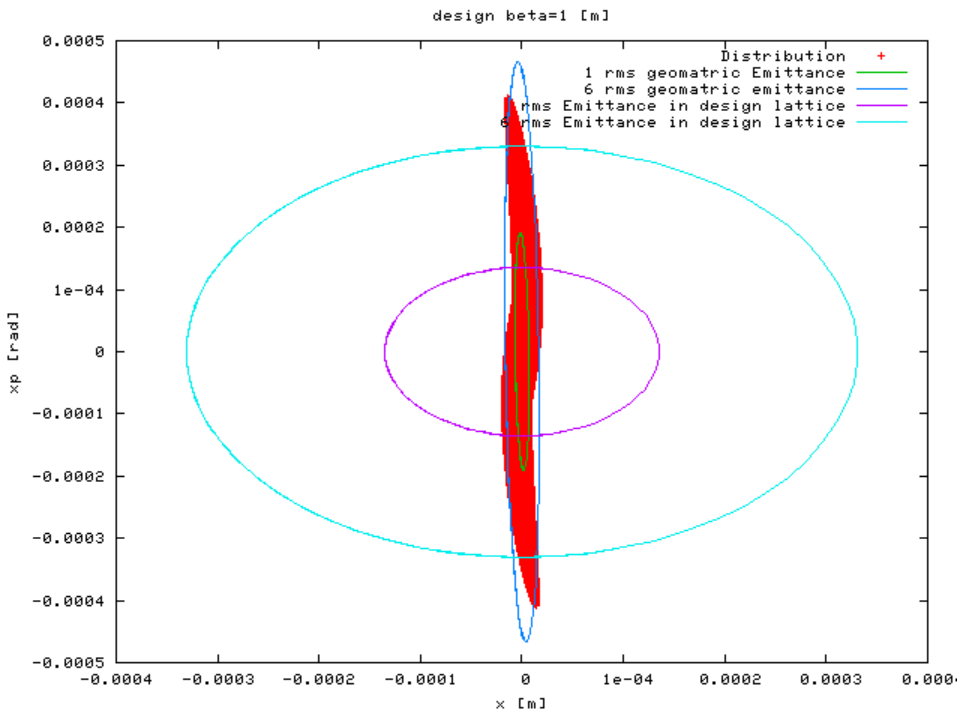
Compromise to get higher luminosity, smaller emittance after collision, and larger average electron beam size.

Luminosity and minimum beam size as function of design waist position

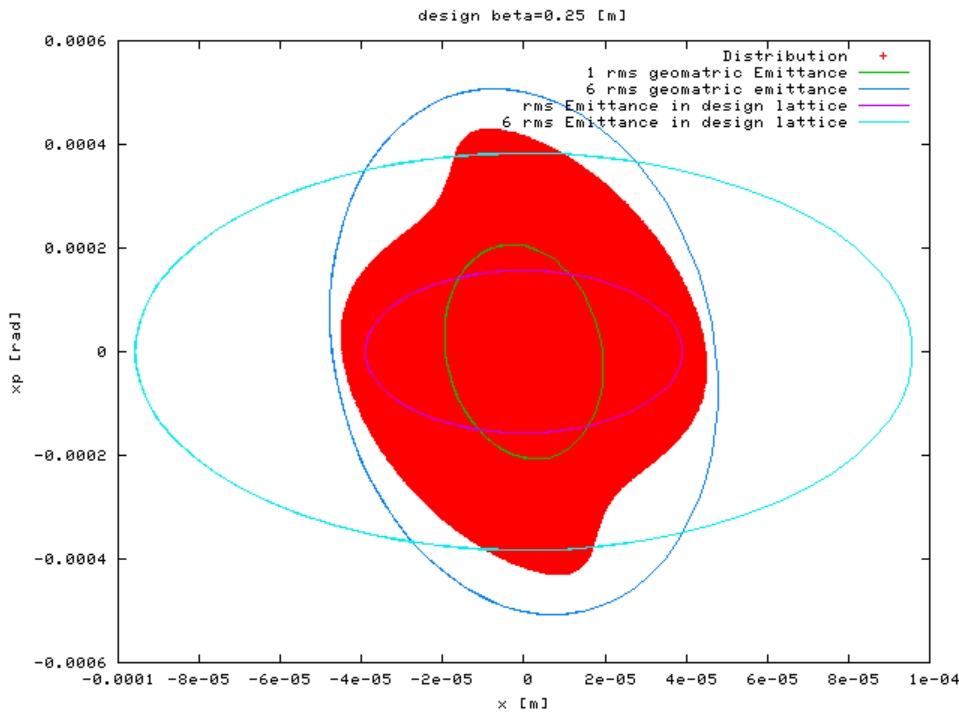


Design β^* = 0.2m, Initial emittance 5nm

Initial Beer-Can Distribution

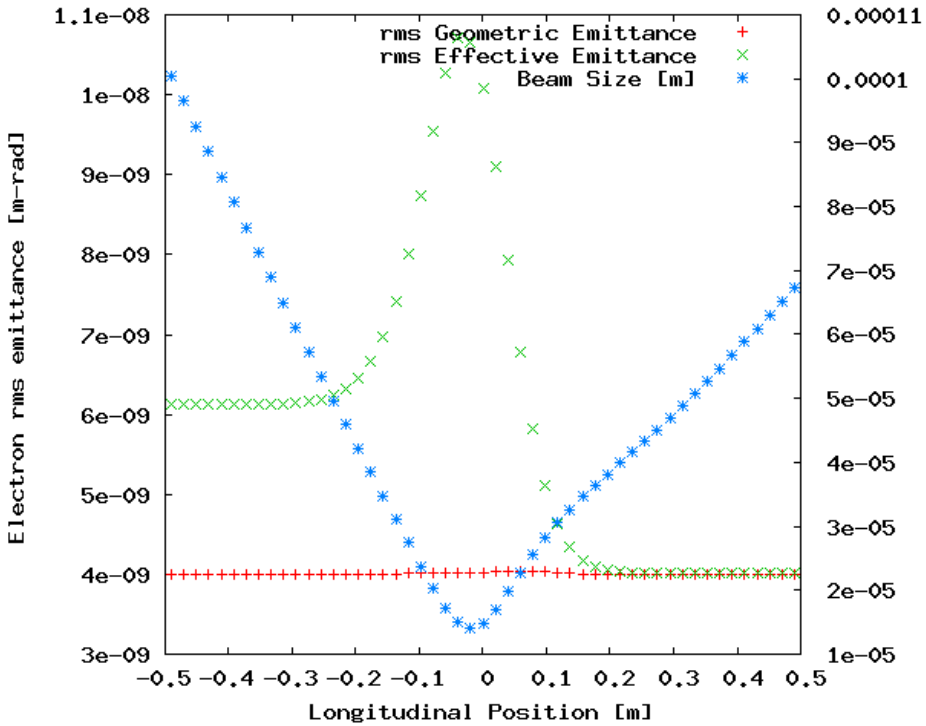


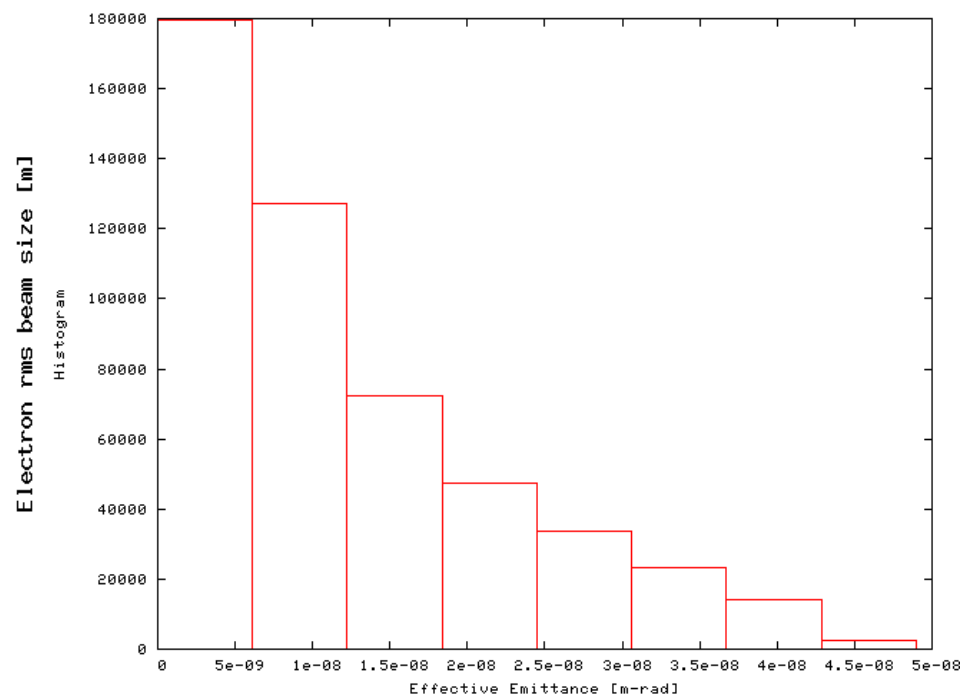
Design β^* = 1m, Initial emittance 1nm



Design β^* = 0.25m, Initial emittance 4nm

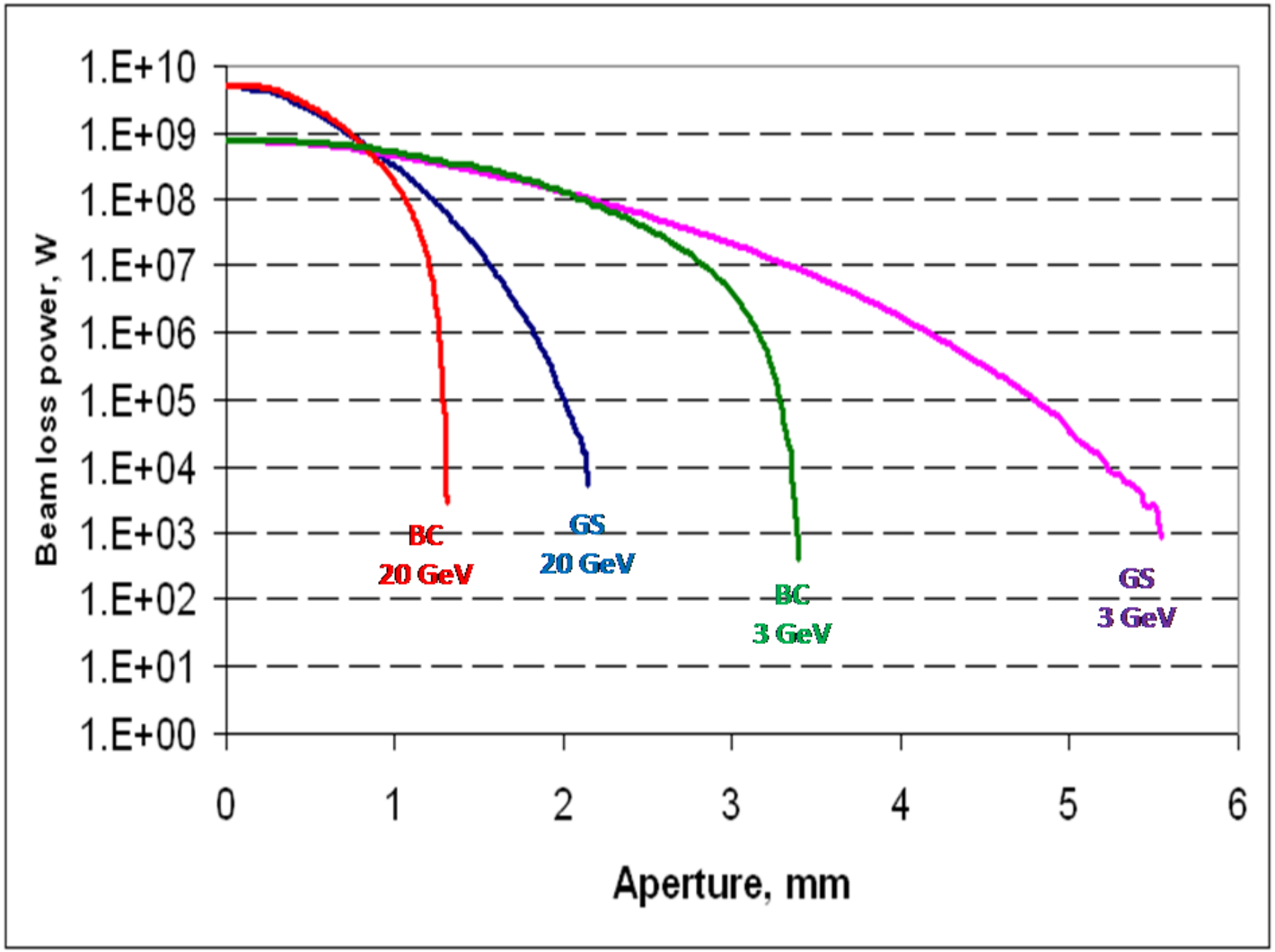
Beer Can case for $\beta^* = 0.25$m





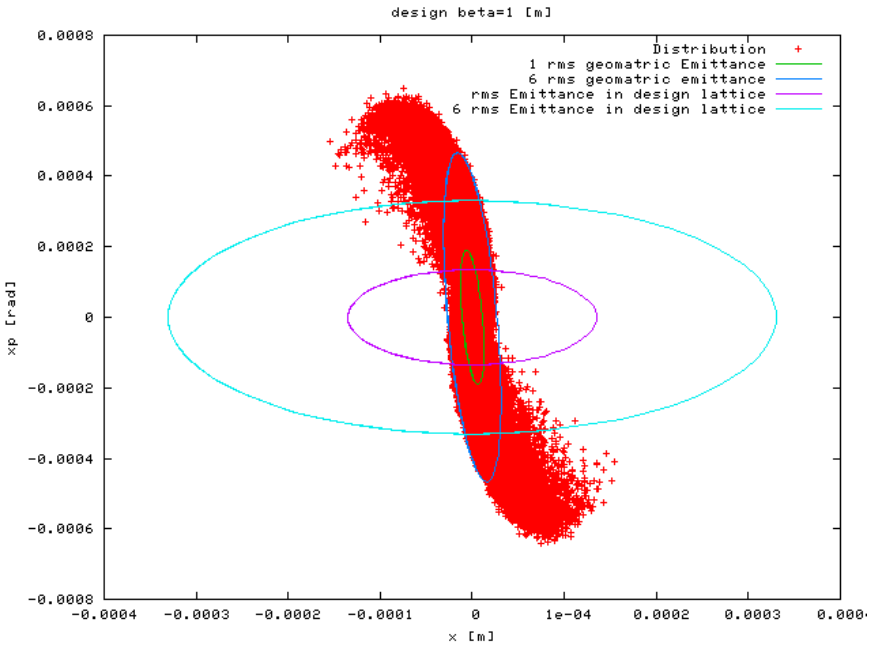
The 100% effective emittance improves 4 times (from 2e-7m-rad to 5e-8 m-rad) if the initial distribution is Beer-Can instead of Gaussian form.

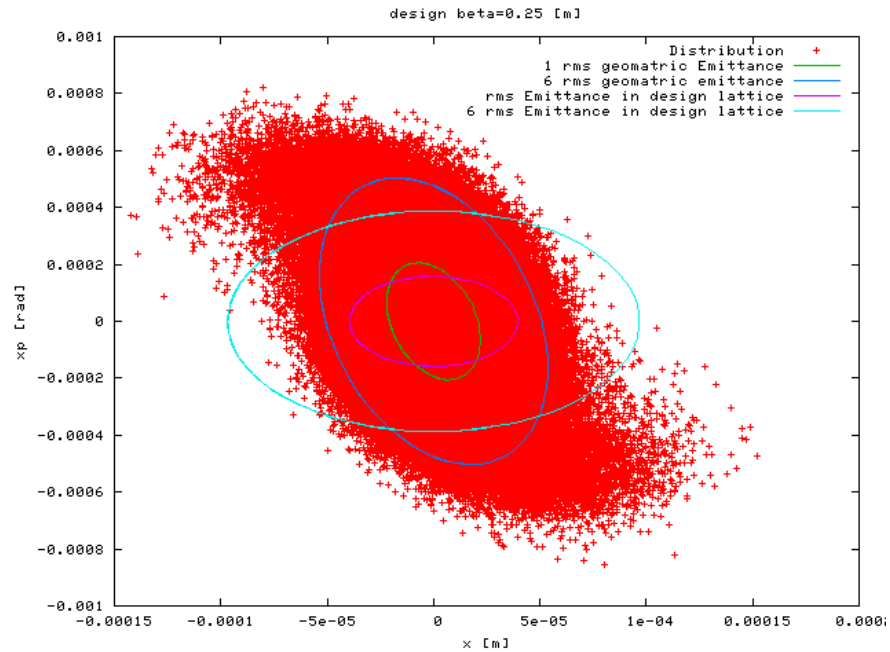
Power Loss calculation



Courtesy of V. Ptitsyn

Effective β* and waist





Design β* = 1m at IP
Initial emittance 1nm

$$\tilde{\beta}(s=0)=0.078\text{m}$$

$$\tilde{\alpha}(s=0)=0.59$$

$$\tilde{\beta}^{*}=0.058\text{m at waist } s=-0.034$$

$$\beta(s=-3\text{m})=112.8\text{m}$$

Design β* = 0.25m at IP
Initial emittance 4nm

$$\tilde{\beta}(s=0)=0.112\text{m}$$

$$\tilde{\alpha}(s=0)=0.34$$

$$\tilde{\beta}^{*}=0.1\text{m at waist } s=-0.034$$

$$\beta(s=-3\text{m})=88\text{m}$$

Effective β^* and waist

Design emittance and β^*	Design waist (m)	Effective β^*(m)	Effective waist(m)	Design waist (m)	Effective β^*(m)	Effective waist(m)
ε=1nm β^*=1m	0	0.0577	-0.0341	1	0.0745	-0.0849
ε=4nm β^*=0.25m	0	0.101	-0.0344	0.2	0.0102	-0.0965
ε=5nm β^*=0.2m	0	0.113	-0.0280	0.2	0.127	-0.115
ε=10nm β^*=0.1m	0	0.110	0.0222	β^*=0.2m s=0.2m#	0.216	-0.114

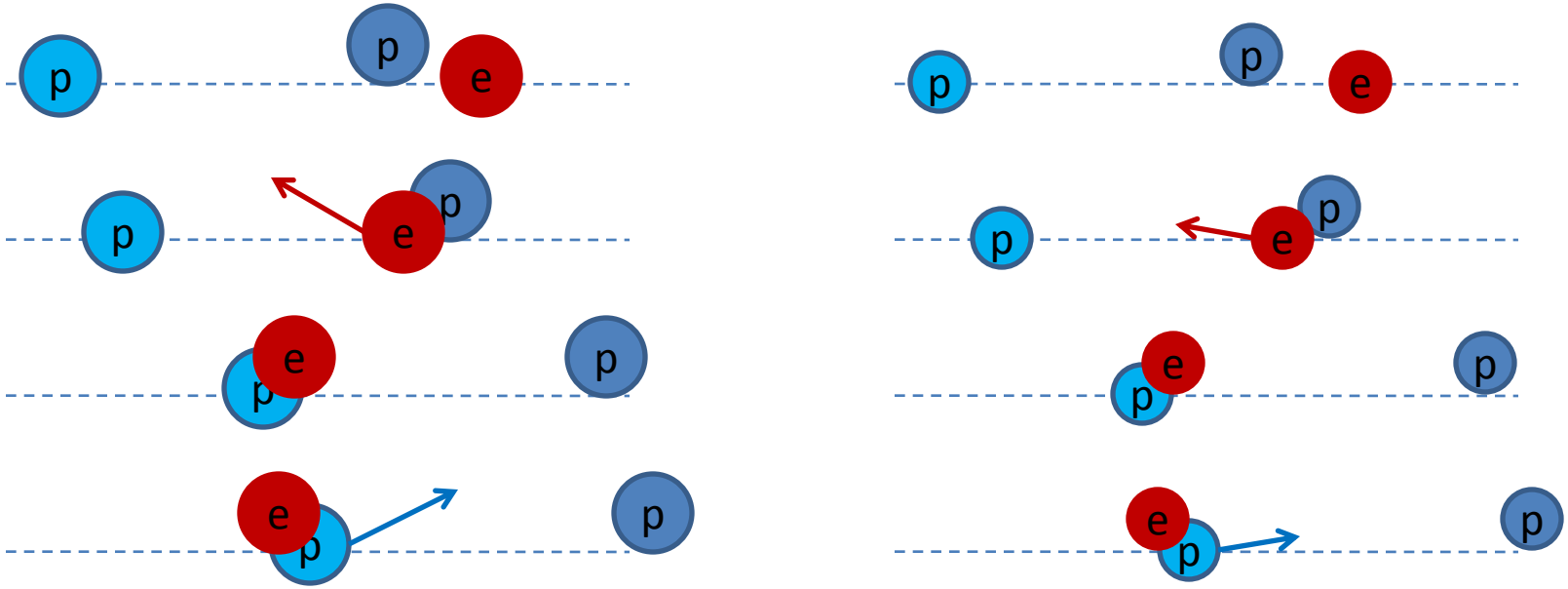
Luminosity here is 1.5×10^{33}

Beam-Beam effect on Proton beam

- Tune shift and tune spread
  - Need proper working point.
  - (0.672, 0.678) is used in simulation.
- May introduce single bunch transverse instability (Kink instability).
  - Beam-beam force acts as wake field.
  - Threshold
  - Possible way to suppress the emittance growth

Kink instability

Use 2-Particle model to illustrate kink instability, The two particles have same synchrotron amplitude but opposite phase. Let T be the synchrotron period.



After T/2, the head and tail exchange there positions

Threshold (Two-particle model)

One turn map for two particles:

$$M = \begin{pmatrix} \cos\phi + \alpha\sin\phi & \beta\sin\phi & 0 & 0 \\ -\gamma\sin\phi & \cos\phi - \alpha\sin\phi & 0 & 0 \\ 0 & 0 & \cos\phi + \alpha\sin(\phi) & \beta\sin\phi \\ 0 & 0 & -\gamma\sin\phi & \cos\phi - \alpha\sin\phi \end{pmatrix}$$

Kick from the leading particle to trailing one.

$$K_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ a & 0 & 0 & 1 \end{pmatrix}$$

with $a = \dfrac{\sigma_{pz}}{2 f_p f_z} = \dfrac{N_p N_e r_p r_e \sigma_{pz}}{2\sigma_{px}^2 \sigma_{ex}^2 \gamma_p \gamma_e}$

After half synchrotron period

$$K_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Here we assume the wake field is constant. More precisely, the electron beam performs a oscillation inside the proton beam

Threshold (Two-particle model)

The total matrix for one synchrotron oscillation gives:

$$T = \left(K_2 M\right)^{N_s/2} \cdot \left(K_1 M\right)^{N_s/2} \qquad N_s = 1/\nu_s$$

If synchrotron oscillation is slow enough and transverse tune is not too close to integer and half integer.

$$N_s \gg 1 \quad \text{and} \quad N_s \gg \left|\cot\phi\right|$$

$$T = \begin{pmatrix} 1 - \left(\frac{aN_s\beta}{4}\right)^2 M\left(N_s\phi\right) & \frac{aN_s\beta}{4} X\left(N_s\phi\right) \\ \frac{aN_s\beta}{4} X\left(N_s\phi\right) & M\left(N_s\phi\right) \end{pmatrix}$$

$$M\left(\phi\right) = \begin{pmatrix} \cos\phi + \alpha\sin\phi & \beta\sin\phi \\ -\gamma\sin\phi & \cos\phi - \alpha\sin\phi \end{pmatrix}$$

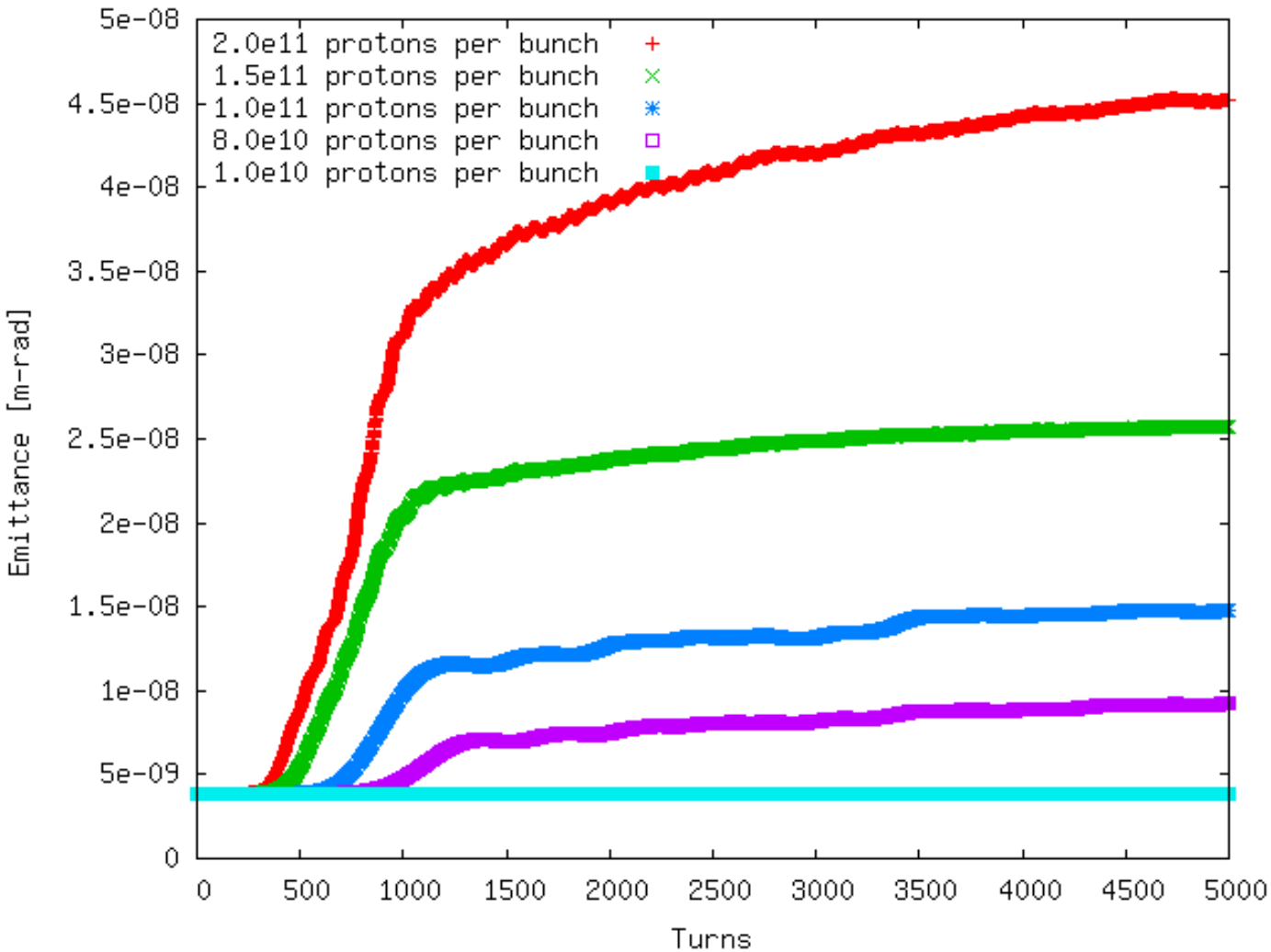
$$X\left(\phi\right) = \begin{pmatrix} -\alpha\cos\phi + \sin\phi & -\beta\cos\phi \\ \gamma\cos\phi & \alpha\cos\phi + \sin\phi \end{pmatrix} = M\left(\phi - \frac{\pi}{2}\right)$$

Threshold:

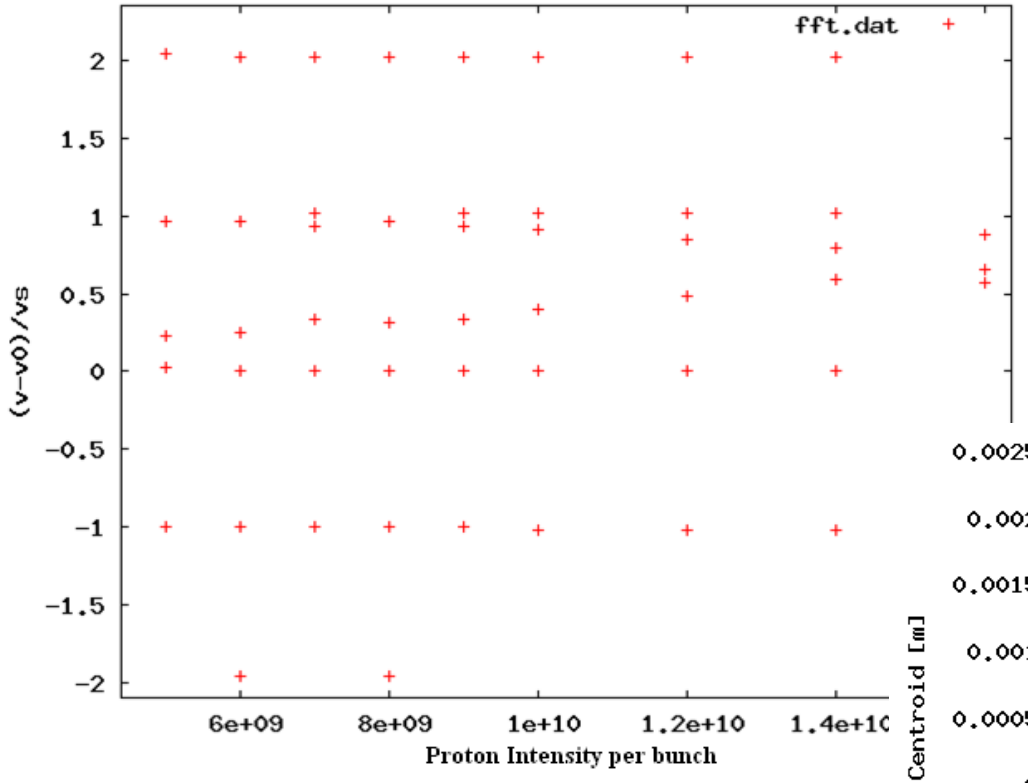
$$\frac{aN_s\beta}{4} < 2 \quad \Rightarrow \quad a < \frac{8\nu_s}{\beta}$$

$$N_p < 1.2 \times 10^{10} \text{/bunch}$$

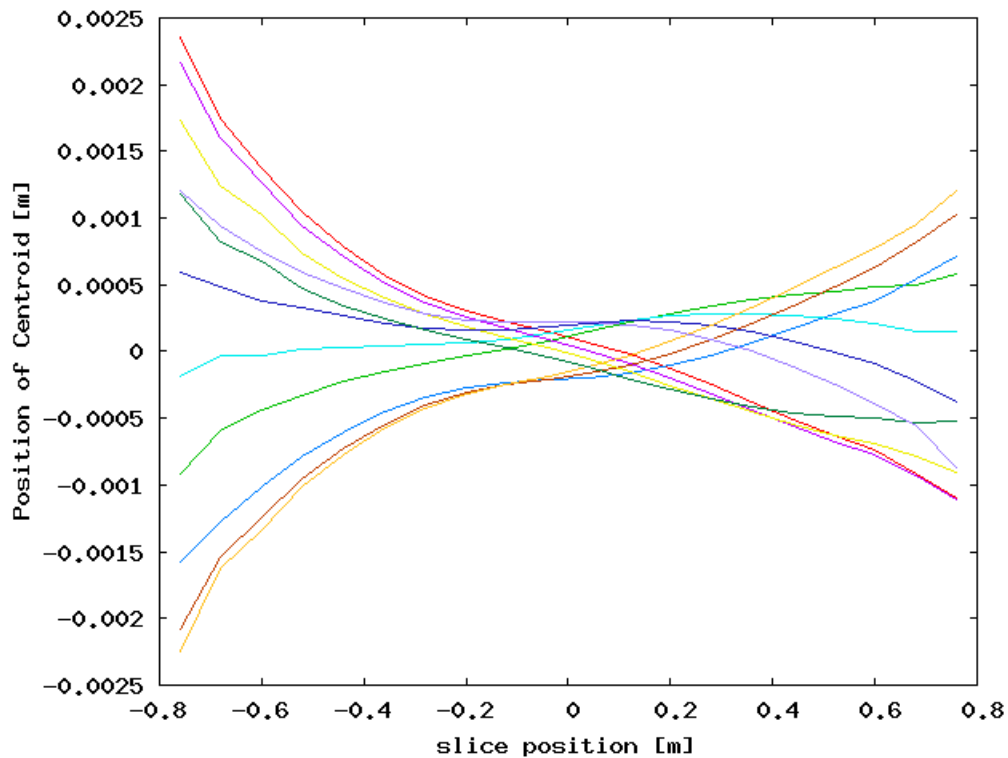
Proton emittance growth due to kink instability



Threshold and modes

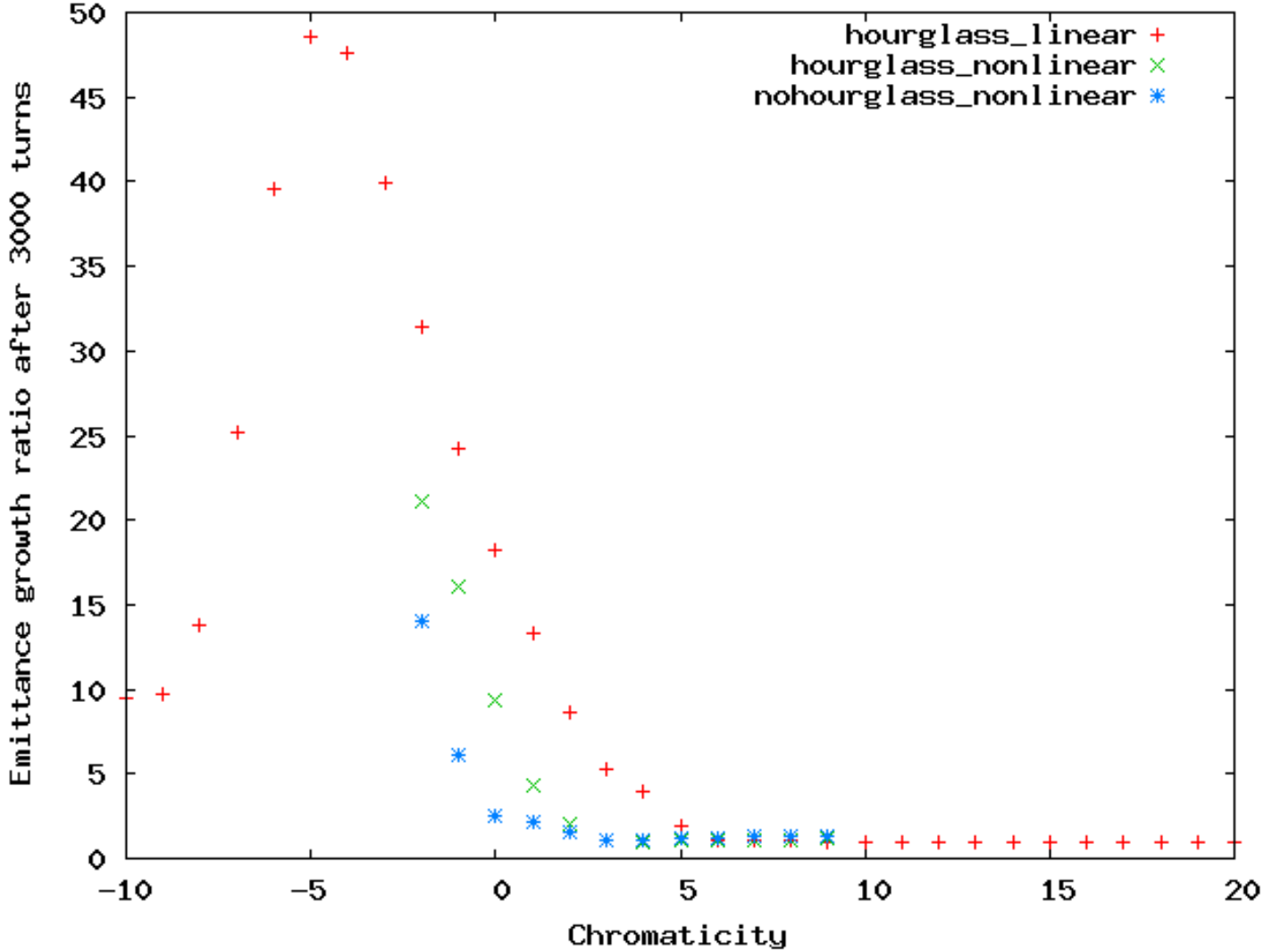


For fixed electron intensity (1.2e11 per bunch), the threshold of proton intensity for kink instability is about 1.6e10 proton per bunch.



The longitudinal snapshot shows mode 1 pattern.

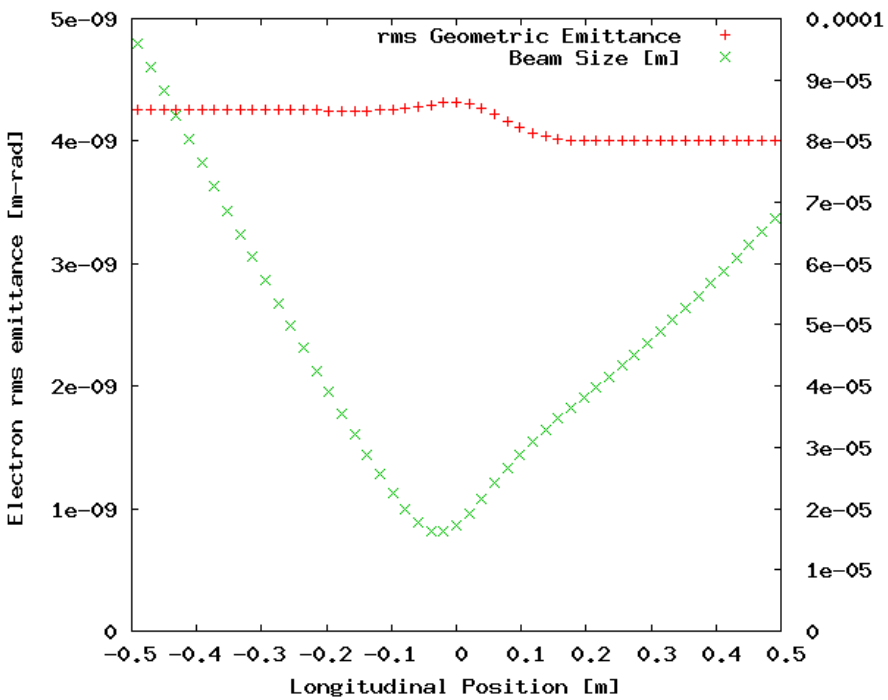
Increase tune spread to suppress emittance growth

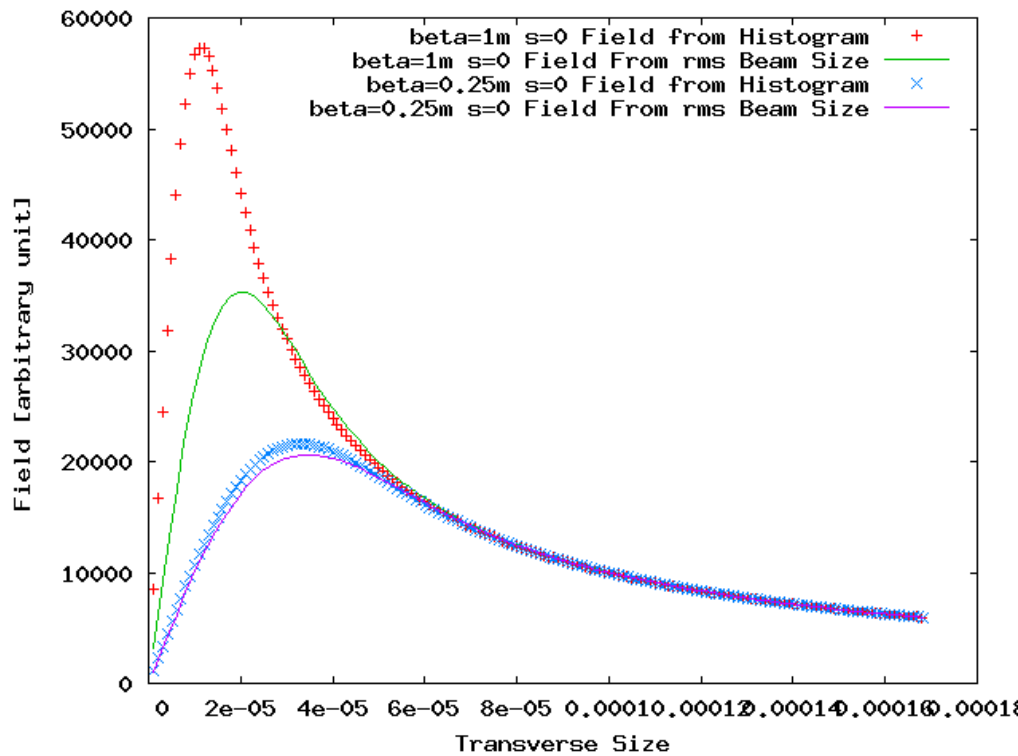


With Energy spread 1e-3

Pinch Effect on Proton Beam

- Source
  - Electron rms beam size shrinks by proton beam.
  - Electron beam distribution has a dense core.



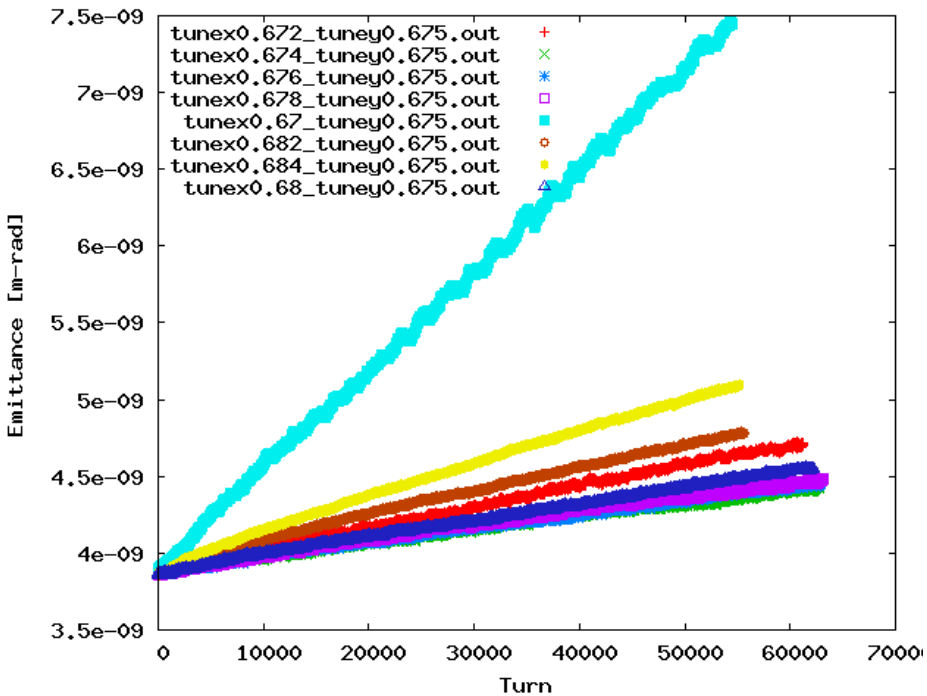


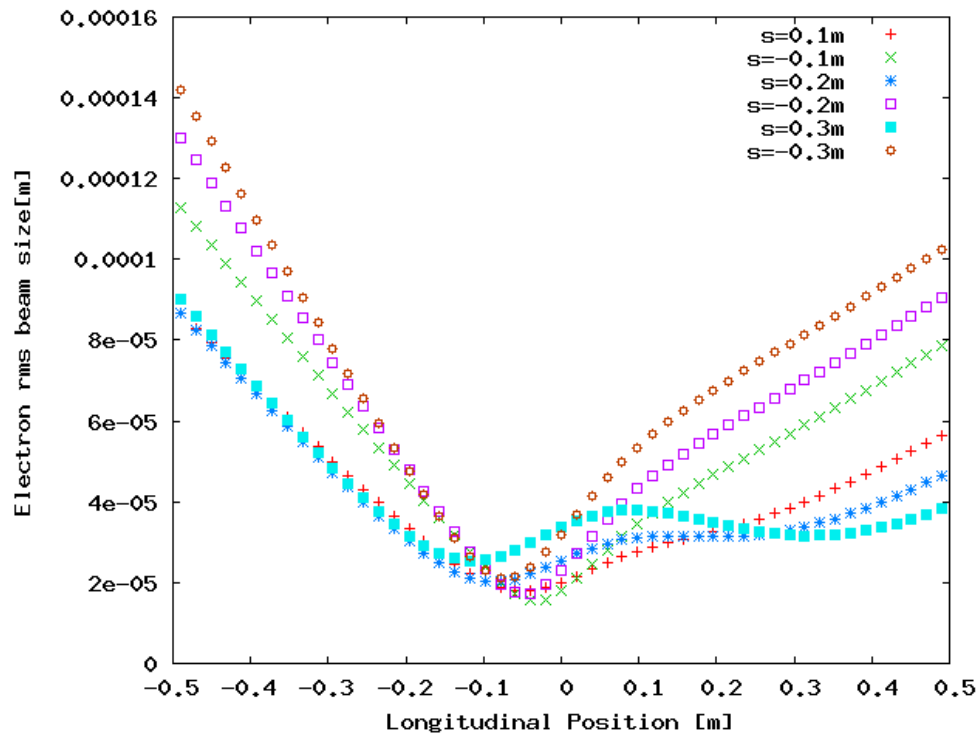
Pinch Effect on Proton Beam

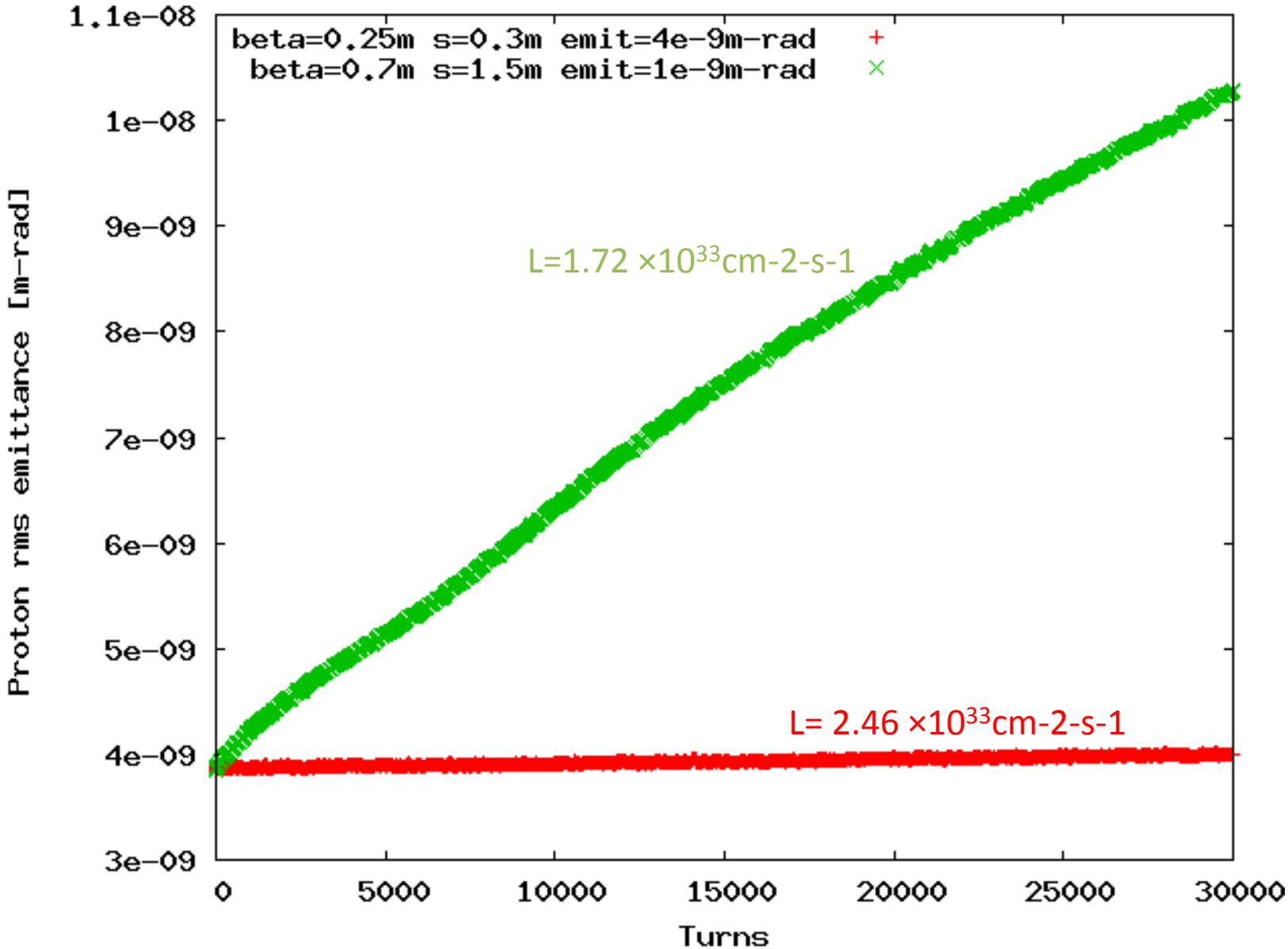
- Main Factors
  - Working Points (avoid nonlinear resonance)

    (0.674,0.675)
  - Electron optics and initial emittance

    (reduce synchro-betatron oscillation)
  - Difference of Gaussian and Beer-Can distribution of electron beam

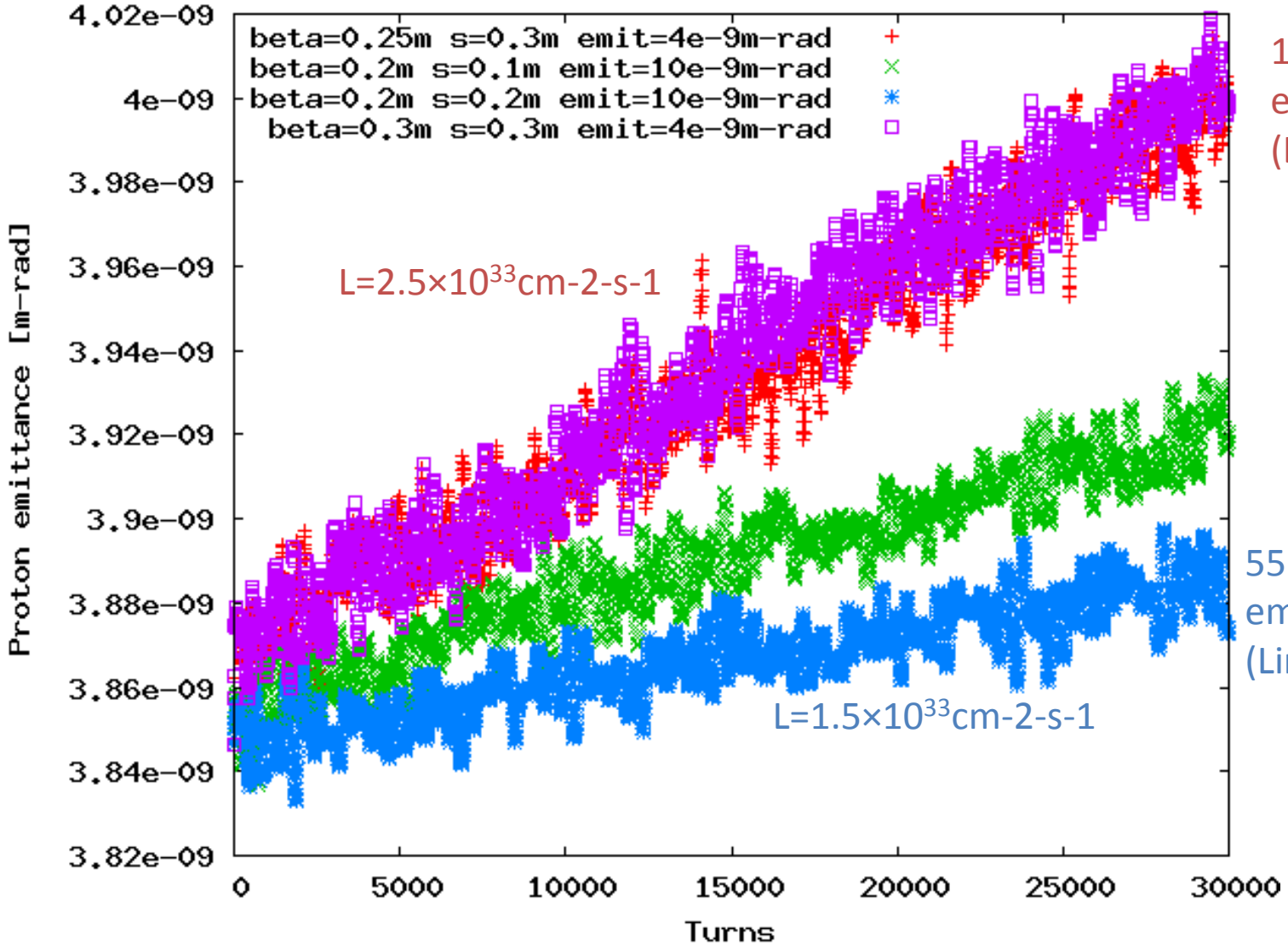






This shows the dense core of electron beam plays a very important role in proton beam emittance growth. Large emittance and small design beta* is preferred for electron beam.

Possible electron candidates



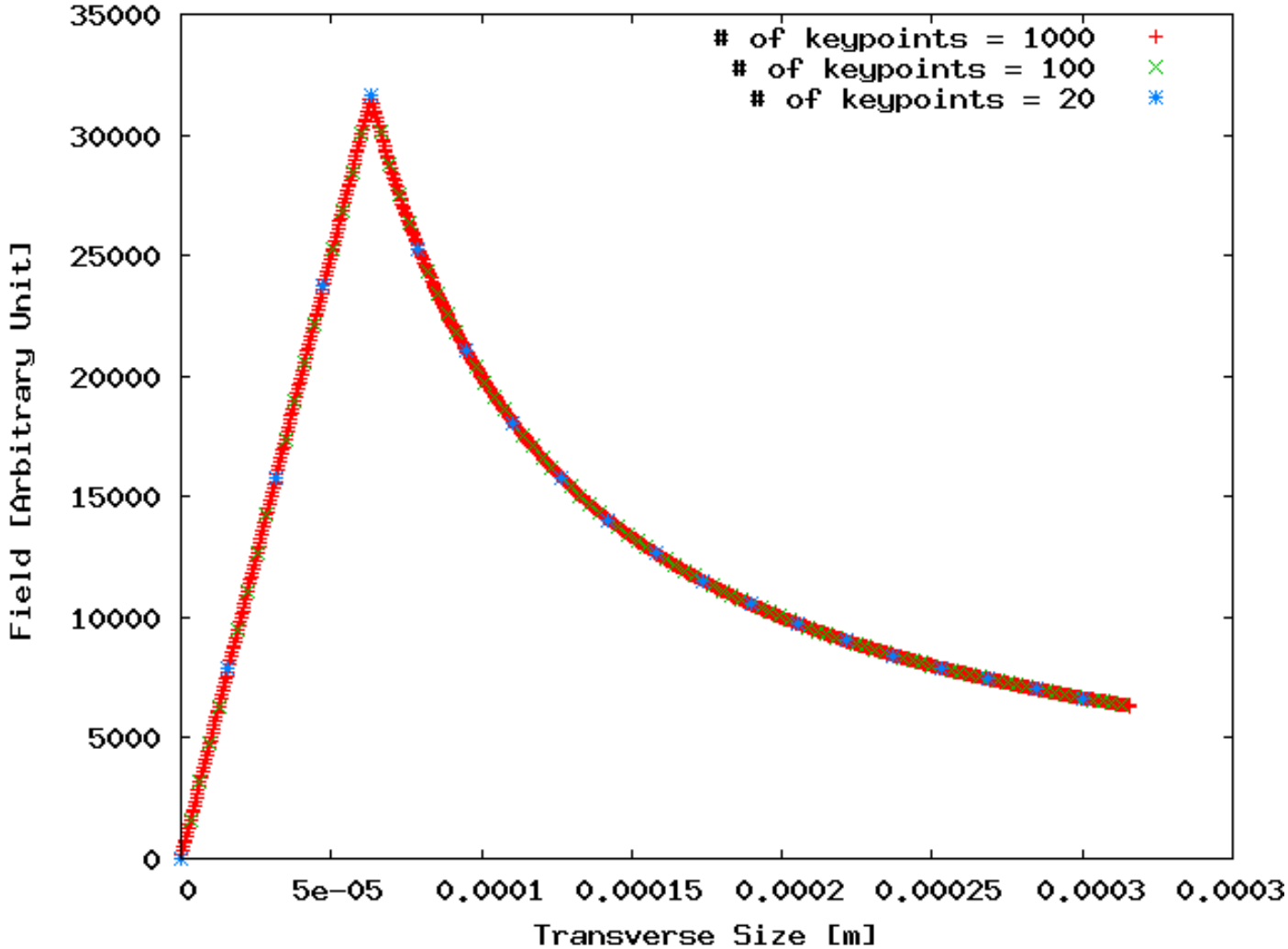
16 seconds for emittance double (Linear fit)

55 seconds for emittance double (Linear fit)

Conclusion

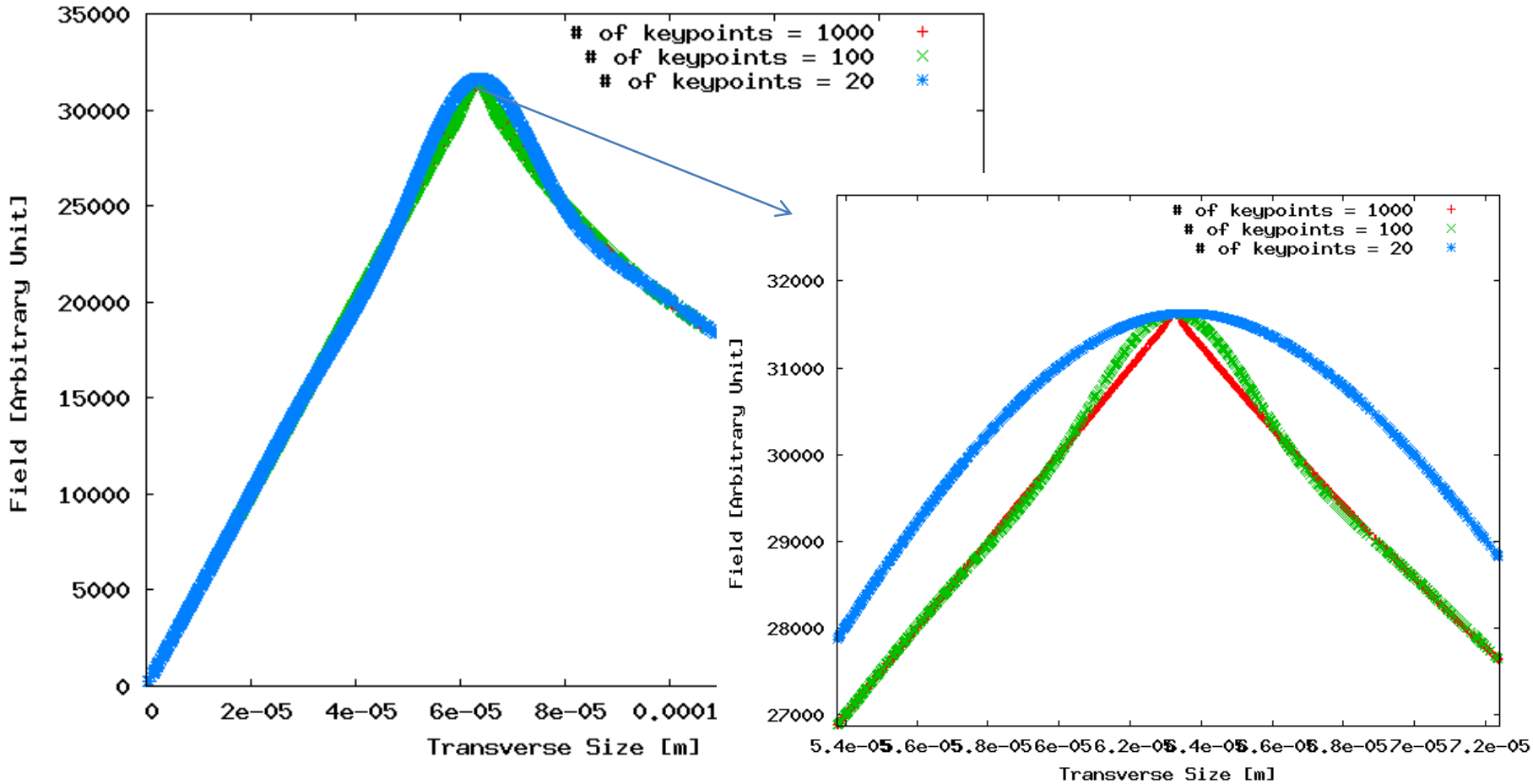
- We can optimize the design parameter to prevent beam loss after collision.
- R&D of beam-beam effect provide information after collision for both IR design and linac design.
- Kink instability can be suppressed by proper energy spread
- Small beta and large emittance for electron beam is preferred for proton beam quality
- More results are coming for pinch effect.

Smoothed BeerCan

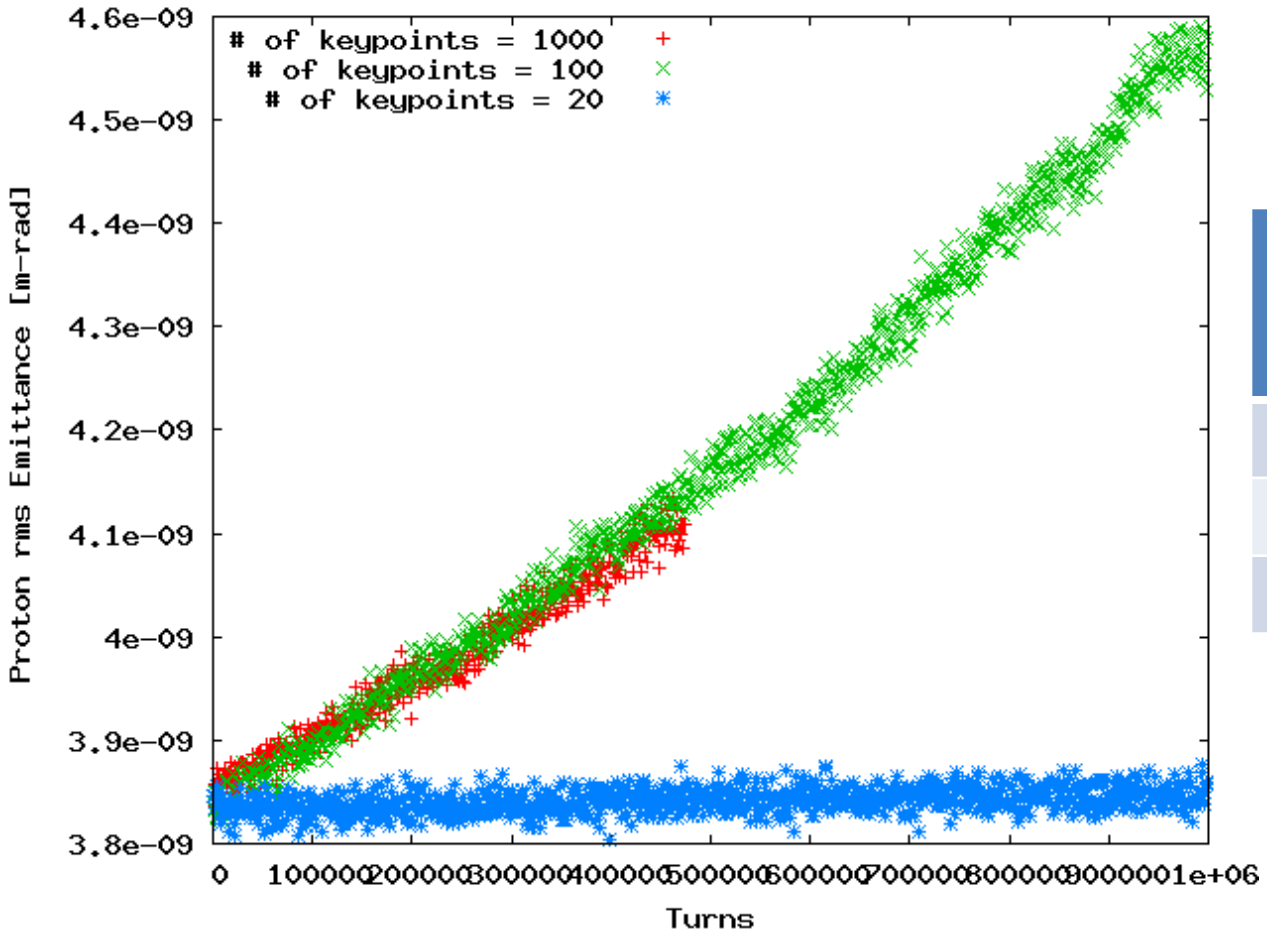


Use Key points and spline interpolation instead of explicit function to represent field

The field after interpolation



Emittance Growth



Key points	Time of emittance double (s)
20	3000
100	66
1000	N/A