Linear Betatron Coupling: general theory and applications to RHIC

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1. Quick view of coupling theory
2. RHIC decoupling on the ramp

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1.1 What's linear betatron coupling

- Transverse motion

$$\begin{cases} x'' + k_x x &= f(x,y) \\ y'' + k_y y &= g(x,y) \end{cases}$$

Linear betatron coupling

$$\begin{cases} x'' + k_x x &= k_1 y \\ y'' + k_y y &= k_2 x \end{cases}$$

- Sources of linear betatron coupling:

  Skew quadrupoles or rotated normal quadrupoles

  Solenoids

- Linear betatron motion can be solved with matrix

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{(n+1)} = \mathbf{T} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{n} \qquad \mathbf{T} = \begin{pmatrix} \mathbf{M} & \mathbf{m} \\ \mathbf{n} & \mathbf{N} \end{pmatrix}$$

1.2 Matrix parameterization

- E.D. Courant and H.S. Snyder, Ann. Phys. 3, 1958
- G. Ripken, DESY Internal Report No. R1-70/04, 1970
- D. Edwards and L. Teng, IEEE Trans. Nucl. Sci. 20, p885, 1973
- R. Talman, in Proc. of Joint US-CERN School on Particle Accelerators, Italy, 1988
- D. Sagan and D. Rubin, Phys. Rev. ST Accel. Beams 2, 074001,1999

- V. Lebedev and S. Bogacz, JLAB Report No. JLAB-ACC-99-19 (unpublished)
- Y. Luo, Phys. Rev. ST Accel. Beams 7, 124001,2004
- A. Wolski, Phys. Rev. ST Accel. Beams 9, 024001,2006

1.3 Perturbation theory

Hamiltonian approach:

- A. Schoch, CERN Report No. 57-21, 1957
- G. Guignard, CERN Report No. 76-06, 1976
- G. Guignard, Phys. Rev. E 51, p6104, 1995

Matrix approach:

- S. Peggs, IEEE Trans. Nucl. Sci. 30, 2460, 1983
- M. Billing, Cornell Report CBN 85-2, 1985
- Y. Luo, NIM A 566, p218-226, 2006
- R. Calaga and R. Tomas, Phys. Rev. ST Accel. Beams 8, 034001, 2005

1.4 Edwards-Tengs' approach

$$\mathbf{T} = \begin{pmatrix} \mathbf{M} & \mathbf{m} \\ \mathbf{n} & \mathbf{N} \end{pmatrix} \xrightarrow[\text{frame change}]{\text{diagonalization}} \mathbf{T} = \mathbf{V}\begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix}\mathbf{V}^{-1}$$

$$\mathbf{V} = \begin{pmatrix} r\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^{+} & r\mathbf{I} \end{pmatrix}, \quad \mathbf{C}^{+} = \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix} \qquad \gamma^2 + \|\mathbf{C}\| = 1$$

$$\mathbf{A} \text{ and } \mathbf{B}: \quad \begin{pmatrix} \cos(2\pi\nu_i) + \alpha_i \sin(2\pi\nu_i) & \beta_i \sin(2\pi\nu_i) \\ -\gamma_i \sin(2\pi\nu_i) & \cos(2\pi\nu_i) - \alpha_i \sin(2\pi\nu_i) \end{pmatrix}$$

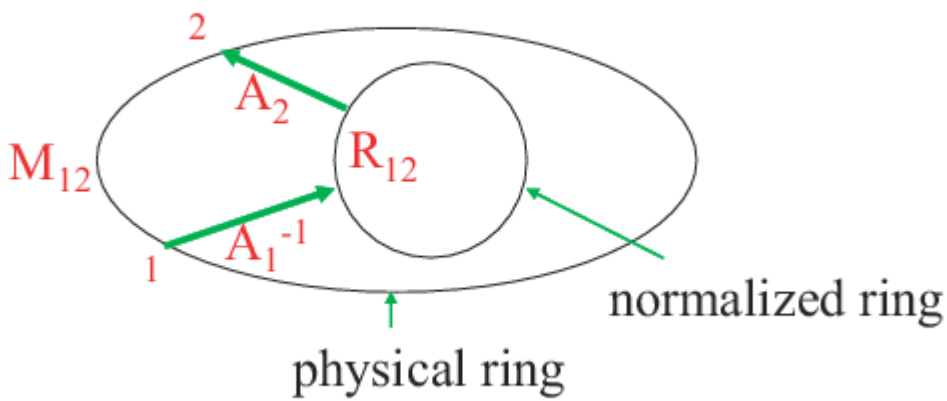
$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \mathbf{V} \cdot \begin{pmatrix} \sqrt{2\beta_I J_I}\cos\Phi_I \\ -\sqrt{\frac{2J_I}{\beta_I}}[\sin\Phi_I + \alpha_I\cos\Phi_I] \\ \sqrt{2\beta_{II} J_{II}}\cos\Phi_{II} \\ -\sqrt{\frac{2J_{II}}{\beta_{II}}}[\sin\Phi_{II} + \alpha_{II}\cos\Phi_{II}] \end{pmatrix}$$

1.5 Norm Form approach

$$\mathbf{T} = \mathbf{P}\mathbf{R}(2\pi\mu_I, 2\pi\mu_{II})\mathbf{P}^{-1}$$

$$\mathbf{R}(\Delta\Phi_I, \Delta\Phi_{II}) = \begin{pmatrix} \mathbf{R}(\Delta\Phi_I) & \mathbf{0} \\ \mathbf{0} & \mathbf{R}(\Delta\Phi_{II}) \end{pmatrix}$$

$$\mathbf{R}(\Delta\Phi_i) = \begin{pmatrix} \cos\Delta\Phi_i & \sin\Delta\Phi_i \\ -\sin\Delta\Phi_i & \cos\Delta\Phi_i \end{pmatrix}$$



$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \mathbf{P}\begin{pmatrix} \sqrt{2J_{\mathrm{I}}}\cos\Phi_{\mathrm{I}} \\ -\sqrt{2J_{\mathrm{I}}}\sin\Phi_{\mathrm{I}} \\ \sqrt{2J_{\mathrm{II}}}\cos\Phi_{\mathrm{II}} \\ -\sqrt{2J_{\mathrm{II}}}\sin\Phi_{\mathrm{II}} \end{pmatrix}$$

P can be directly obtained from eigenvectors of **T**

$$\begin{cases} \mathbf{e}_1 = \frac{i}{\sqrt{2}}(\mathbf{v}_1 - \mathbf{v}_1^*) \\ \mathbf{e}_2 = \frac{1}{\sqrt{2}}(\mathbf{v}_1 + \mathbf{v}_1^*) \\ \mathbf{e}_3 = \frac{i}{\sqrt{2}}(\mathbf{v}_2 - \mathbf{v}_2^*) \\ \mathbf{e}_4 = \frac{1}{\sqrt{2}}(\mathbf{v}_2 + \mathbf{v}_2^*) \end{cases} \qquad \mathbf{P} = (\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\ \mathbf{e}_4).$$

If you still want Edwards-Teng's beta, alpha, gamma, etc., then

$$\mathbf{P} = \begin{pmatrix} r\sqrt{\beta_I} & 0 & c_{11}\sqrt{\beta_{II}} - \frac{c_{12}\alpha_{II}}{\sqrt{\beta_{II}}} & -\frac{c_{12}}{\sqrt{\beta_{II}}} \\ -\frac{\alpha_I r}{\sqrt{\beta_I}} & \frac{r}{\sqrt{\beta_I}} & c_{21}\sqrt{\beta_{II}} - \frac{c_{22}\alpha_{II}}{\sqrt{\beta_{II}}} & \frac{c_{22}}{\sqrt{\beta_{II}}} \\ -\frac{c_{12}\alpha_I}{\sqrt{\beta_I}} - c_{22}\sqrt{\beta_I} & \frac{c_{12}}{\sqrt{\beta_I}} & r\sqrt{\beta_{II}} & 0 \\ \frac{c_{11}\alpha_I}{\sqrt{\beta_I}} + c_{21}\sqrt{\beta_I} & -\frac{c_{11}}{\sqrt{\beta_I}} & -\frac{\alpha_{II} r}{\sqrt{\beta_{II}}} & \frac{r}{\sqrt{\beta_{II}}} \end{pmatrix}$$

1.6 Hamiltonian perturbation theory

Coupling coefficient:

$$C^- = |C^-|e^{i\chi} = \frac{1}{2\pi}\int_0^L \sqrt{\beta_x\beta_y}k_s e^{i[\Phi_x-\Phi_y-2\pi\Delta\cdot s/L]}dl$$

Eigentunes:

$$Q_1 = Q_{x,0} - \frac{\Delta}{2} + \frac{1}{2}\sqrt{\Delta^2+|C^-|^2},$$

$$Q_2 = Q_{y,0} + \frac{\Delta}{2} - \frac{1}{2}\sqrt{\Delta^2+|C^-|^2},$$

The fractional eigentune split:

$$\begin{aligned}|\Delta Q| &= |Q_1 - Q_2 - p| \\ &= \sqrt{\Delta^2+|C^-|^2}.\end{aligned}$$

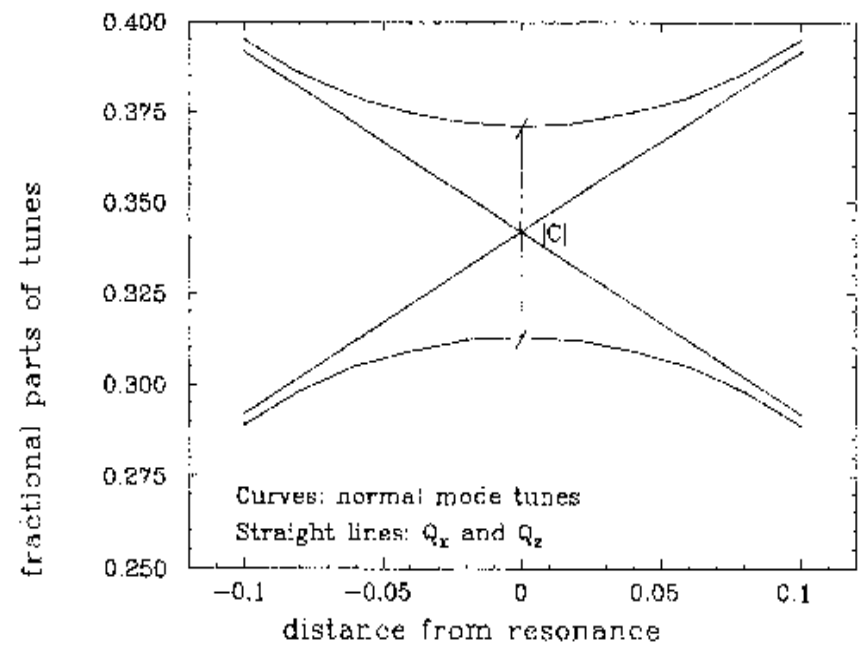


FIG. 3. Coupled tunes as functions of the distance from the difference resonance (dimensionless quantities).

2.1 Decoupling on the RHIC ramp

- Requirement:

  1) Tunes on the ramp:

     design tune split: ~0.01

     flat tunes for spin polarization preservation

  2) Requirement:

     dQmin < 0.005

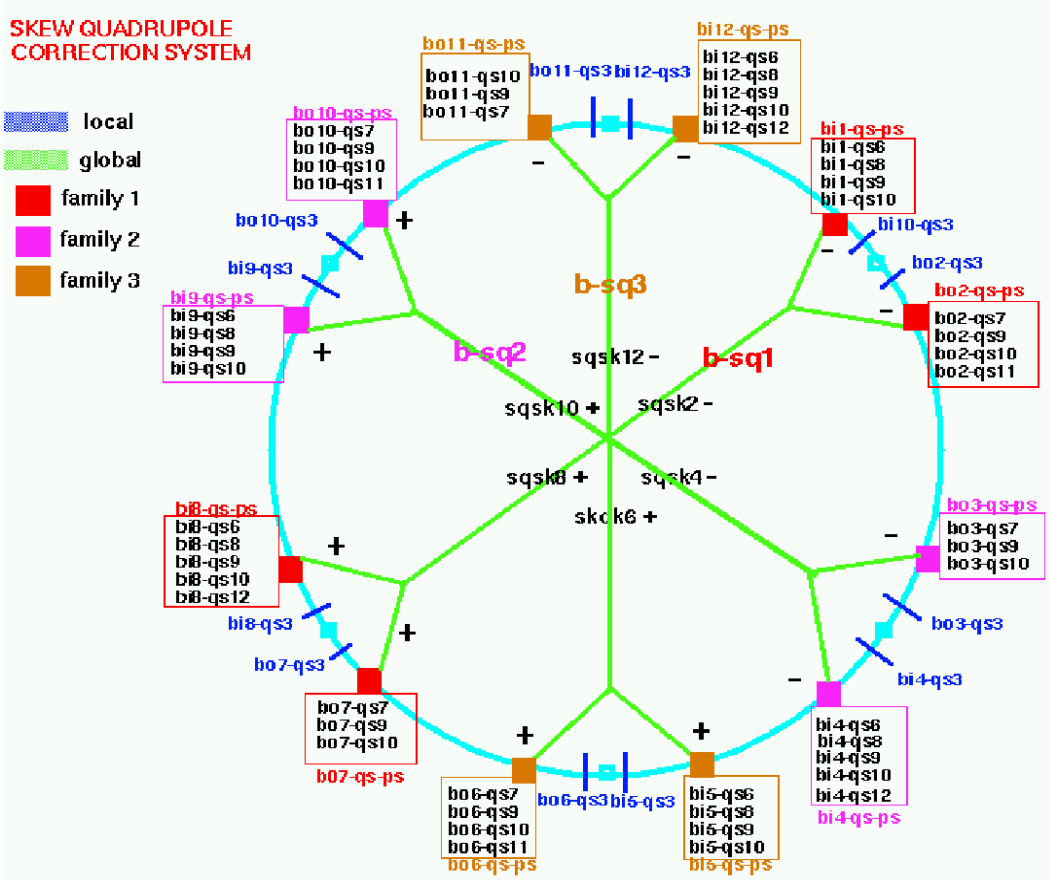
- Methods:

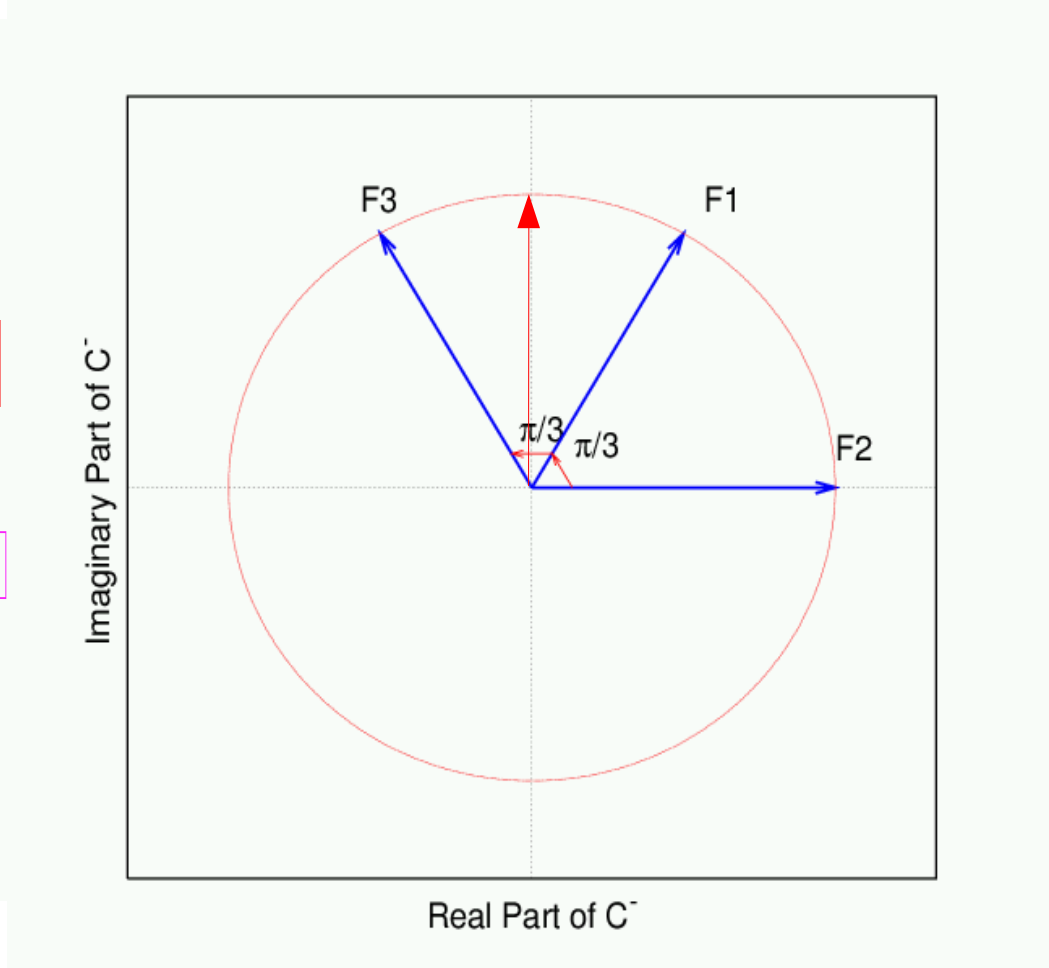
  1) N-turn transfer maps
  2) Three-ramp correction
  3) Skew quadrupole modulations

     Coupling amplitude modulation ==> tested in run'04

     Coupling angle modulation ==> used in pp run'05
  4) Eigenmode projection parameters ==> decoupling feedback

2.2 RHIC skew quadrupole families

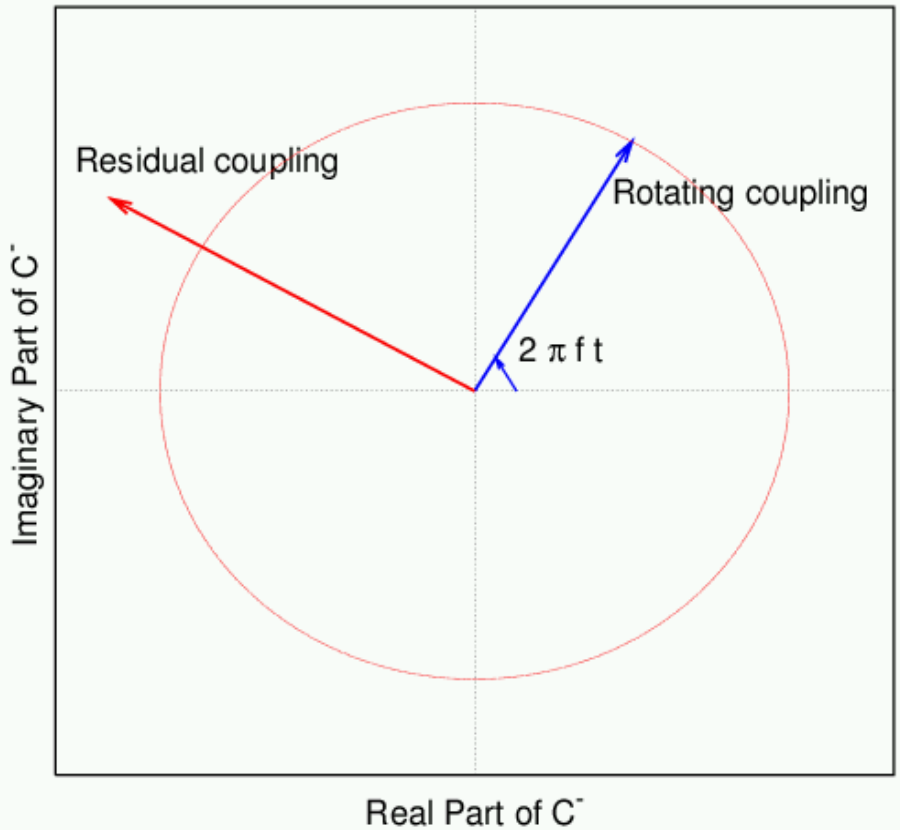




2.3 RHIC global decoupling timeline

- F. Pilat, etc., “ Coupling measurement and correction during RHIC run 2001 and development for 2003”, C-D/AP /77, 2002.
- W. Fischer, “Robust linear coupling correction with N-turn maps”, PRST AB 5, 54001, 2002
- Y. Luo, etc., “Measurement of global betatron coupling with skew quadrupole modulation”, PRST AB 8, 014001, 2005
- Y. Luo, etc., “ Fast and robust global decoupling with coupling angle modulation”, PRST AB 8, 074002, 2005
- Y. Luo, etc., “ Continuous measurement of global difference coupling using a phase-locked loop tune meter in the RHIC”, PRST AB 9, 124001, 2006
- P. Cameron, etc., “Simultaneous tune and coupling feedback in the RHIC, and possible implications for the LHC”, PRST AB 9, 122801, 2006

2.4 Coupling angle modulation



Assuming:

$$C^{-}_{mod} = |C^{-}_{mod,amp}| \cdot e^{i2\pi ft}$$

$$|C^{-}_{res,amp}| = k\ |C^{-}_{mod,amp}|$$

- During the modulation,

$$\Delta Q^2_{min} = \Delta^2 + (k-1)^2 \cdot |C^{-}_{mod,amp}|^2$$

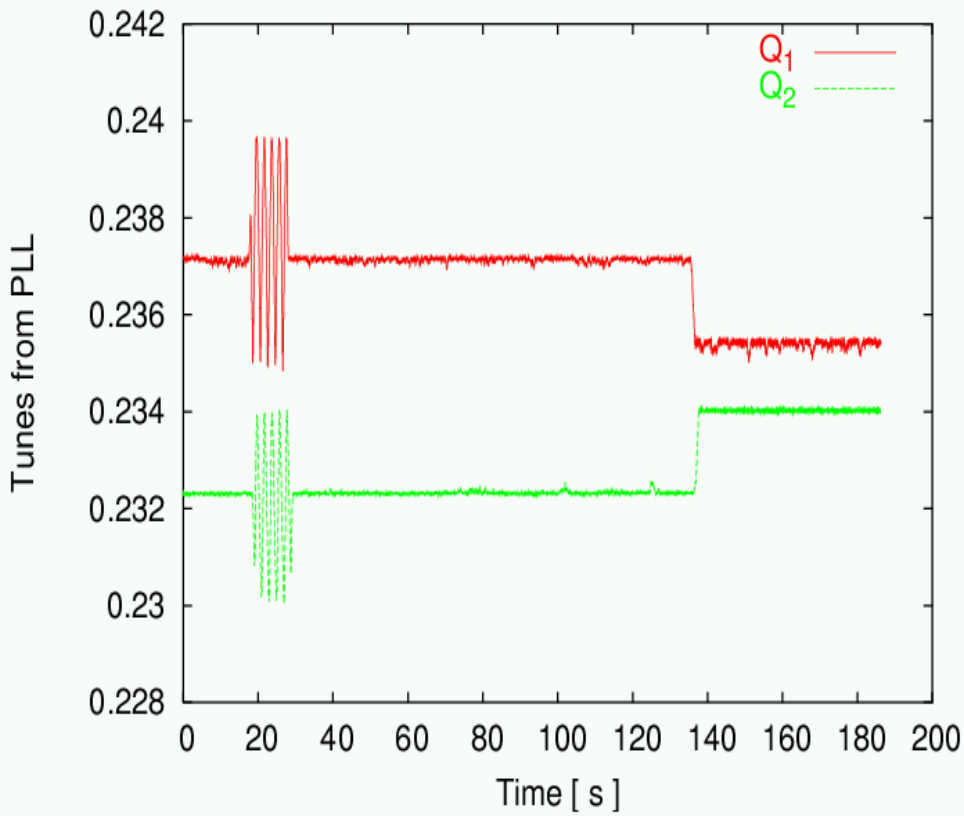
$$\Delta Q^2_{max} = \Delta^2 + (k+1)^2 \cdot |C^{-}_{mod,amp}|^2$$

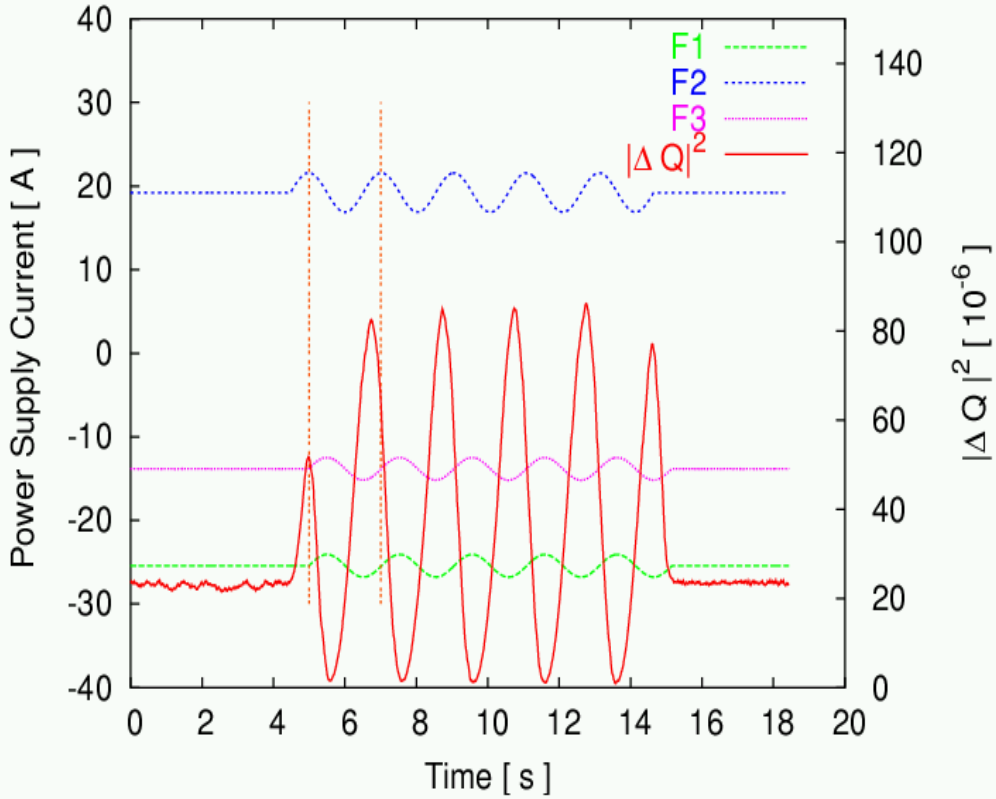
- Without modulation,

$$\Delta Q^2_0 = \Delta^2 + k^2 \cdot |C^{-}_{mod,amp}|^2$$

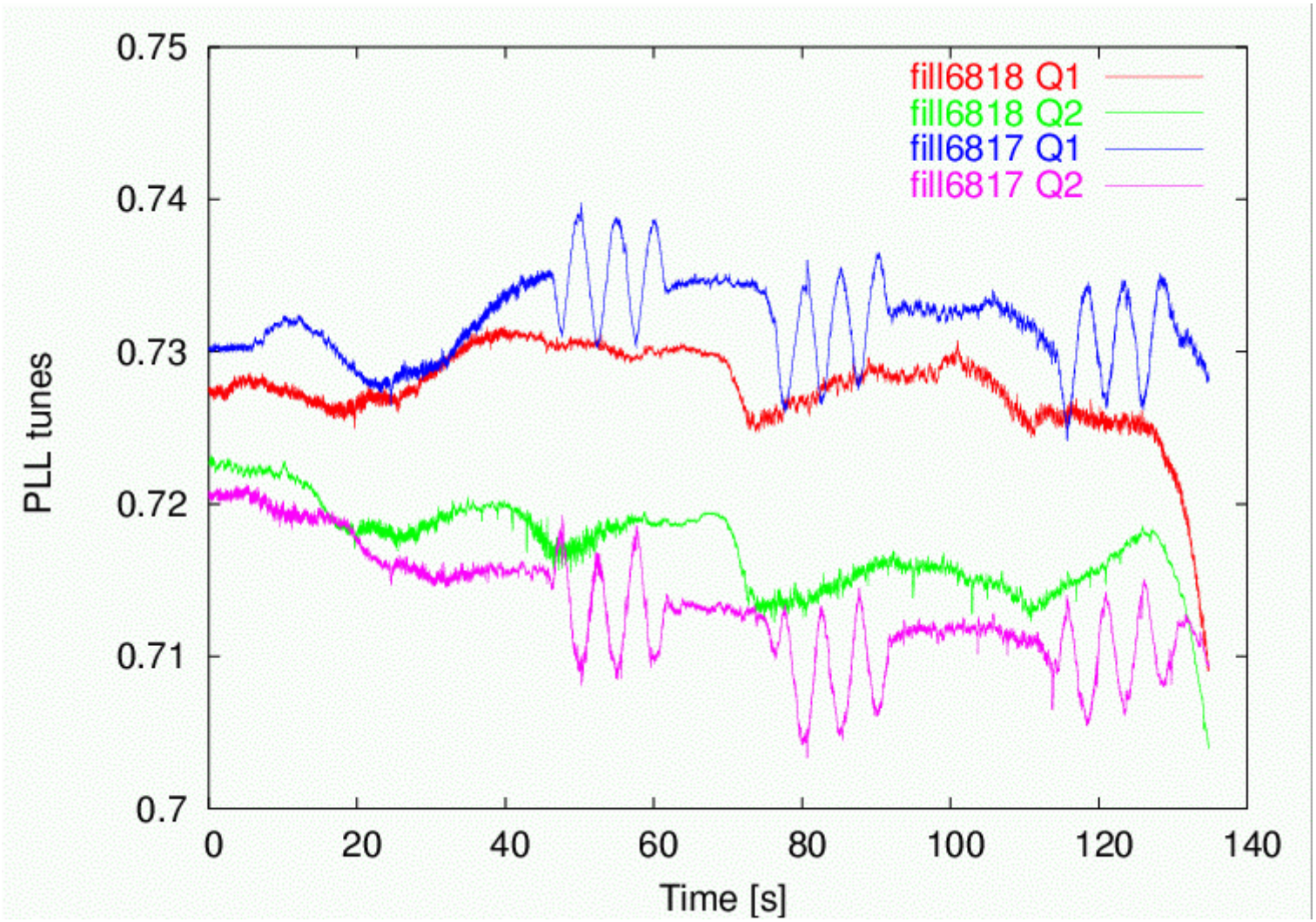
- Then, k is given by

$$k = [4(\frac{\Delta Q^2_{max} - \Delta Q^2_0}{\Delta Q^2_{max} - \Delta Q^2_{min}} - \frac{1}{2})]^{-1}$$





One example at the RHIC Blue ring store



One example on the Yellow ring ramp

2.5 Continuous global coupling measurement

- Eigenmode projection parameters

$$\begin{cases} x_n = A_{I,x}\cos[2\pi Q_1(n-1)+\phi_{I,x}] + A_{II,x}\cos[2\pi Q_2(n-1)+\phi_{II,x}] \\ y_n = A_{I,y}\cos[2\pi Q_1(n-1)+\phi_{I,y}] + A_{II,y}\cos[2\pi Q_2(n-1)+\phi_{II,y}] \end{cases},$$

We define four eigenmode projection parameters:

$$\begin{cases} R_I = \dfrac{A_{I,y}}{A_{I,x}}, \\ R_{II} = \dfrac{A_{II,x}}{A_{II,y}}. \end{cases} \qquad \begin{cases} \Delta\phi_I = \phi_{I,y} - \phi_{I,x}, \\ \Delta\phi_{II} = \phi_{II,x} - \phi_{II,y}. \end{cases}$$

From Hamiltonian perturbation theory:

$$\begin{cases} R_I = \sqrt{\dfrac{\beta_y}{\beta_x}} \cdot \dfrac{|C^-|}{\sqrt{\Delta^2+|C^-|^2}+\Delta} \\ R_{II} = \sqrt{\dfrac{\beta_x}{\beta_y}} \cdot \dfrac{|C^-|}{\sqrt{\Delta^2+|C^-|^2}+\Delta} \end{cases}, \qquad \begin{cases} \Delta\phi_I = \chi \\ \Delta\phi_{II} = \pi - \chi \end{cases}.$$

- Knowing projection parameters, we then have

$$C^- = |C^-|e^{i\chi} = \frac{1}{2\pi}\int_0^L \sqrt{\beta_x\beta_y}k_s e^{i[\Phi_x-\Phi_y-2\pi\Delta\cdot s/L]}dl$$

Coupling coefficient:

$$|C^-| = \frac{2\sqrt{R_I R_{II}}}{1+R_I R_{II}}\,|Q_I - Q_{II} - p|,$$

$$\Delta = \frac{1-R_I R_{II}}{1+R_I R_{II}}\,(Q_I - Q_{II} - p).$$

Set tunes (tunes without coupling) :

$$Q_{x,0} = \frac{Q_I + Q_{II}}{2} + \frac{\Delta}{2},$$

$$Q_{y,0} = \frac{Q_I + Q_{II}}{2} - \frac{\Delta}{2}.$$

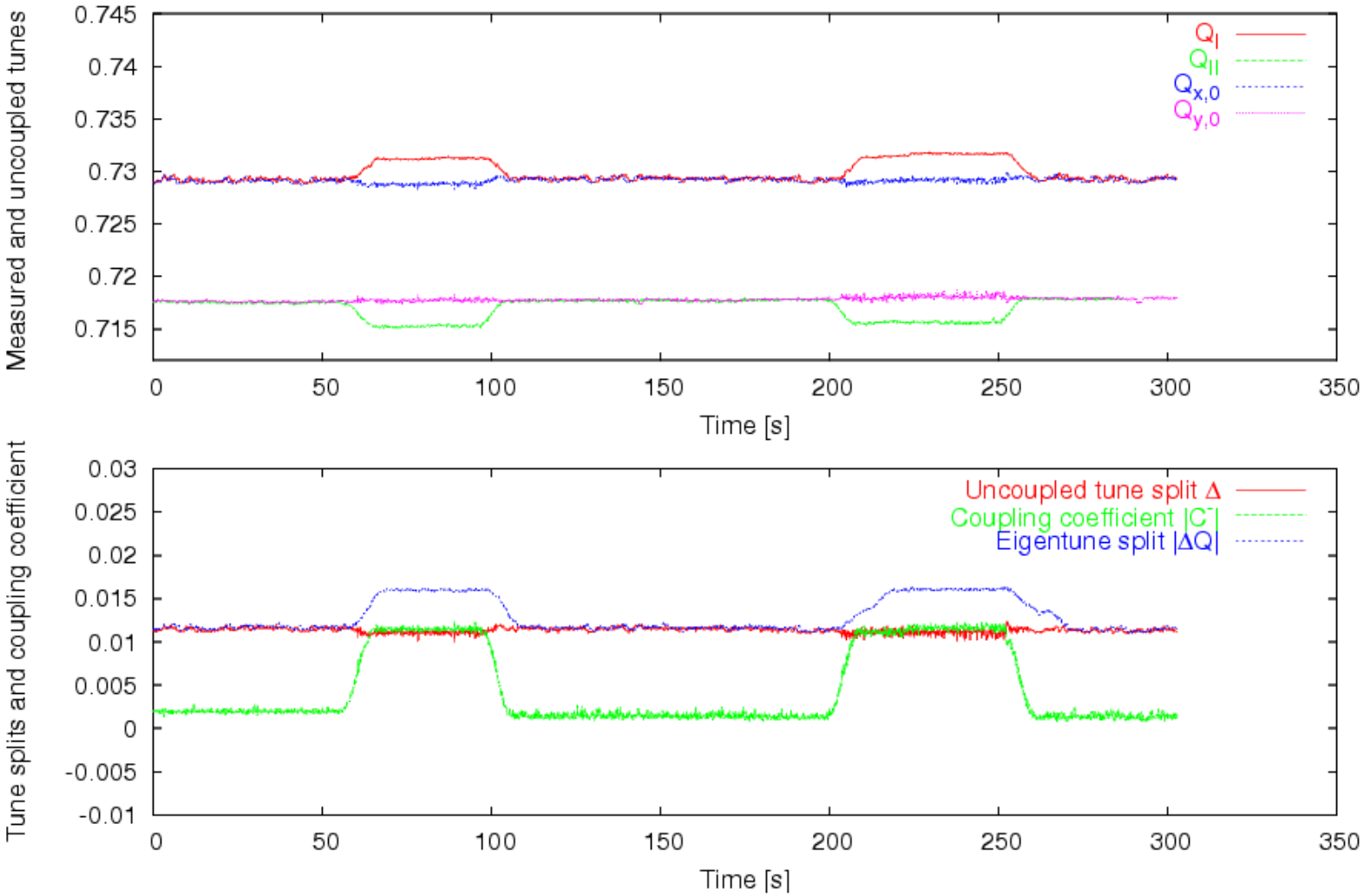
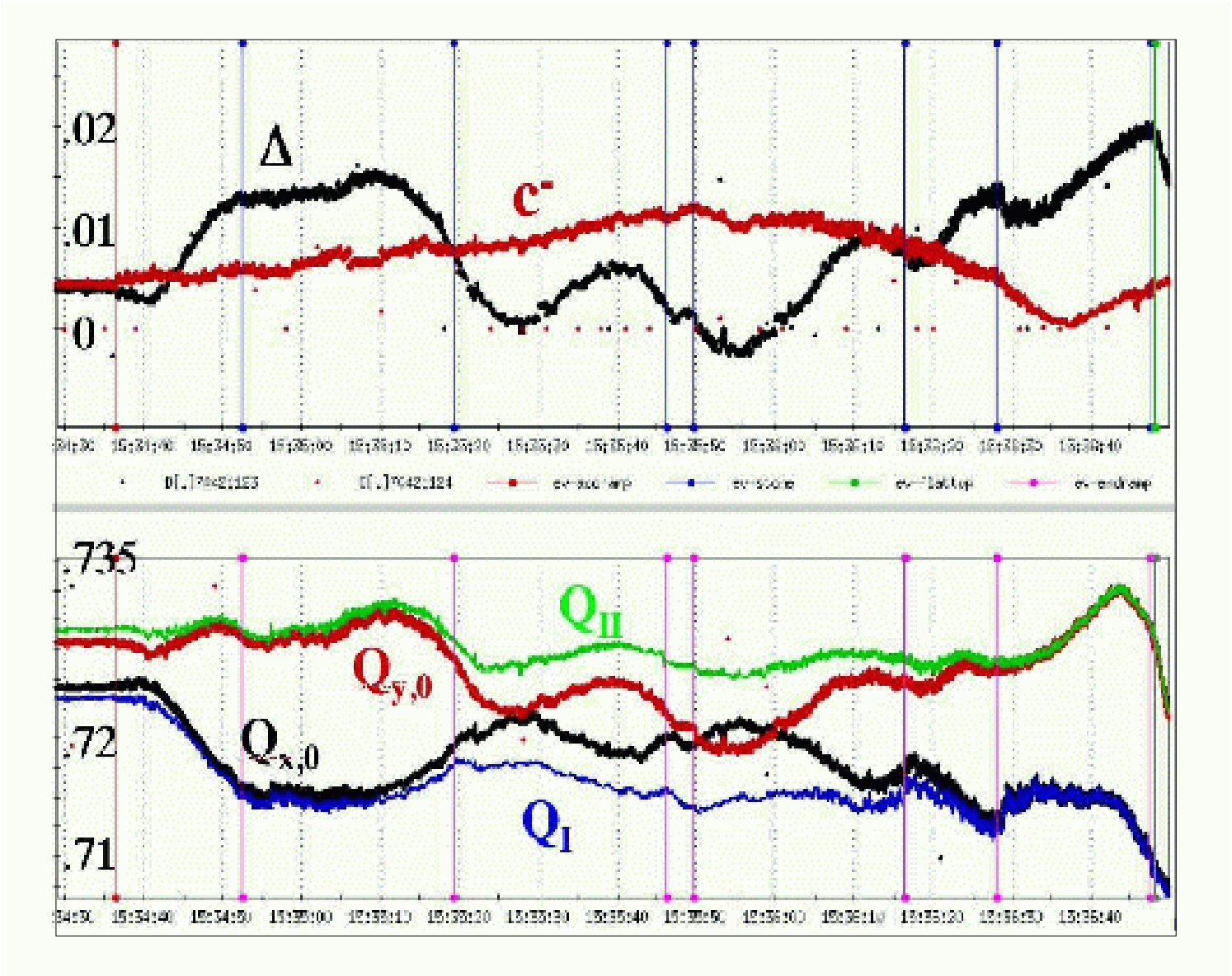


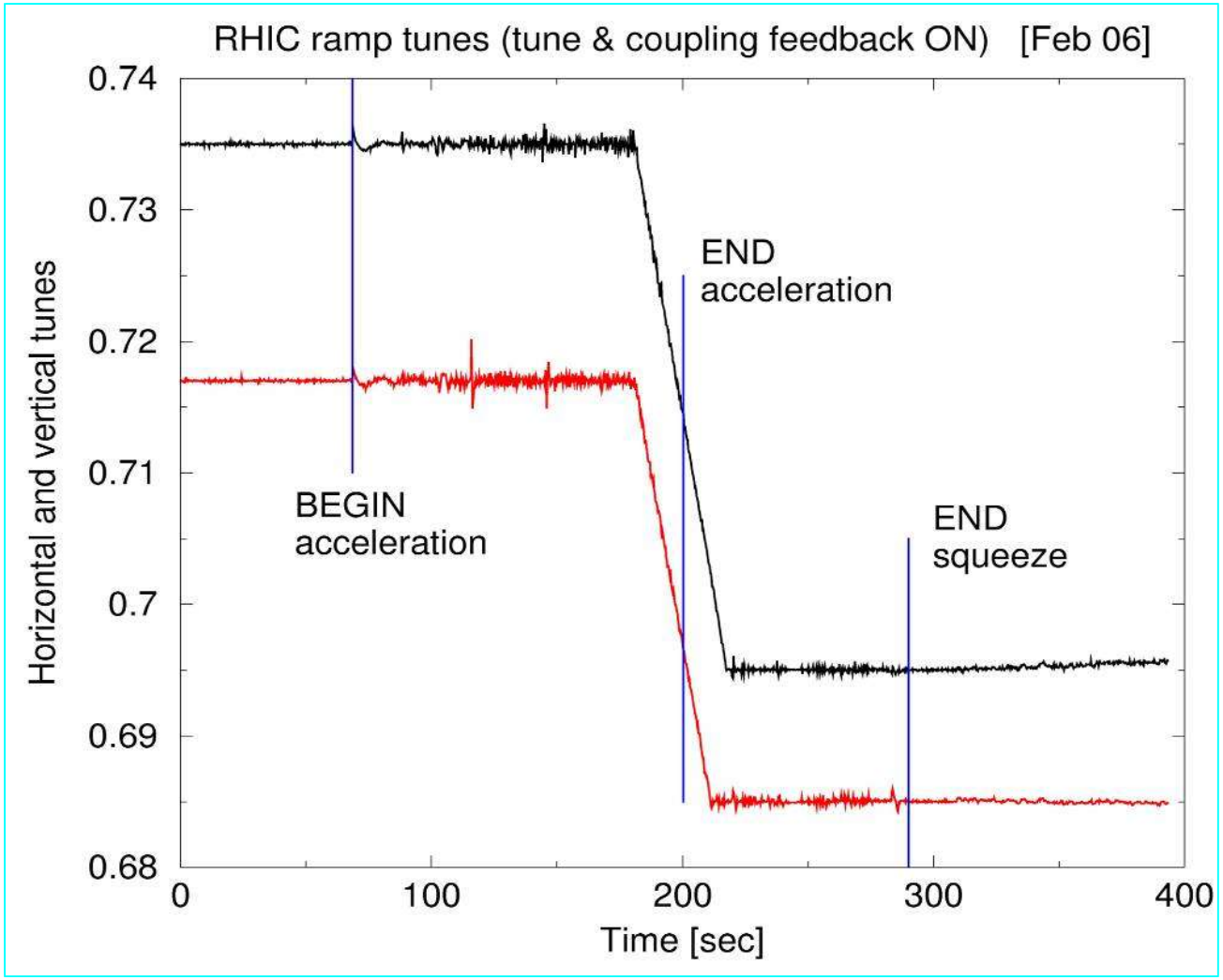
FIG. 5: The fractional tune splits and the coupling coefficient.

One example of coupling measurement at injection

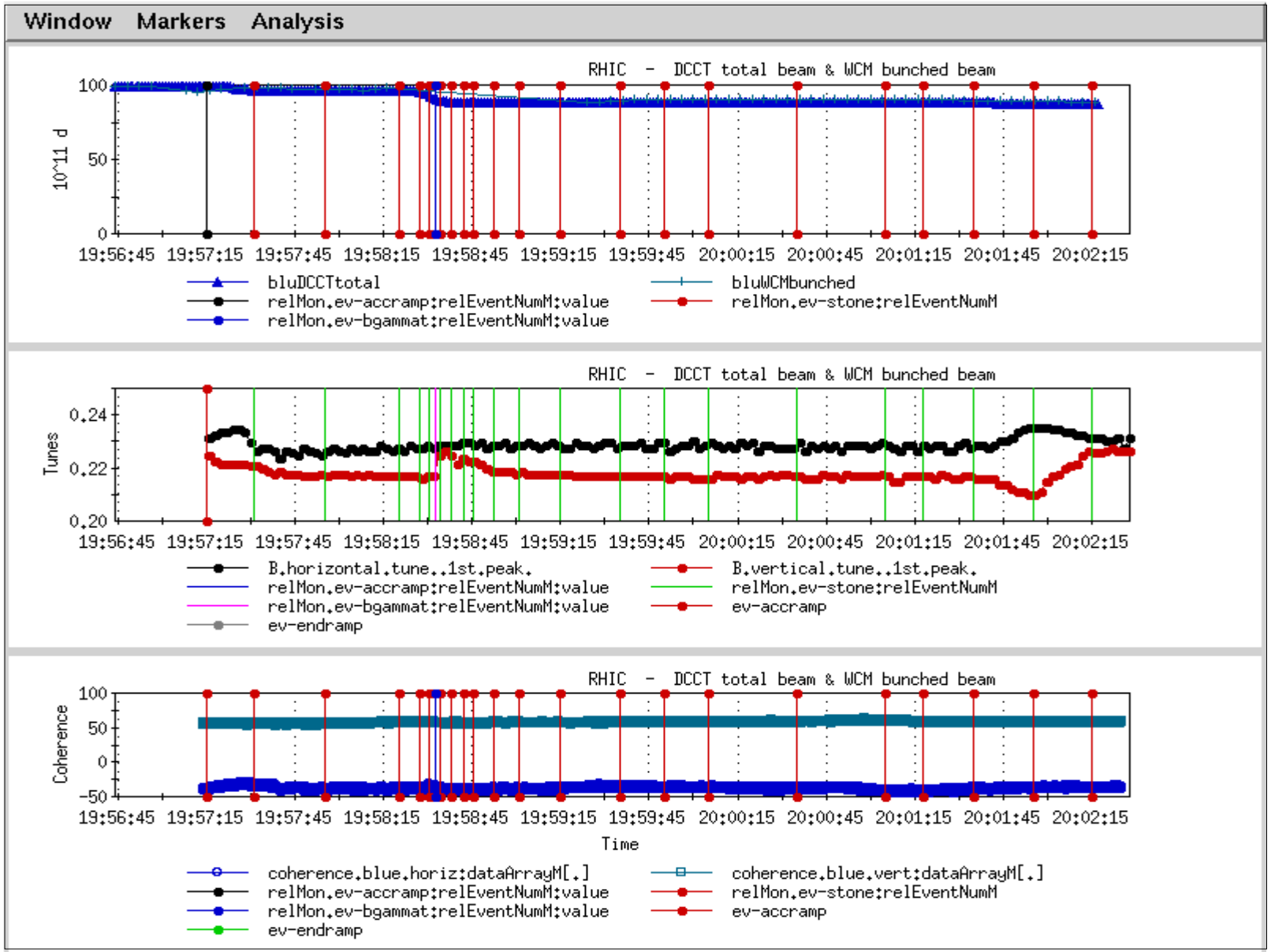


One example of coupling measurement on ramp (fill 7401)

2.6 Tune & decoupling feedback

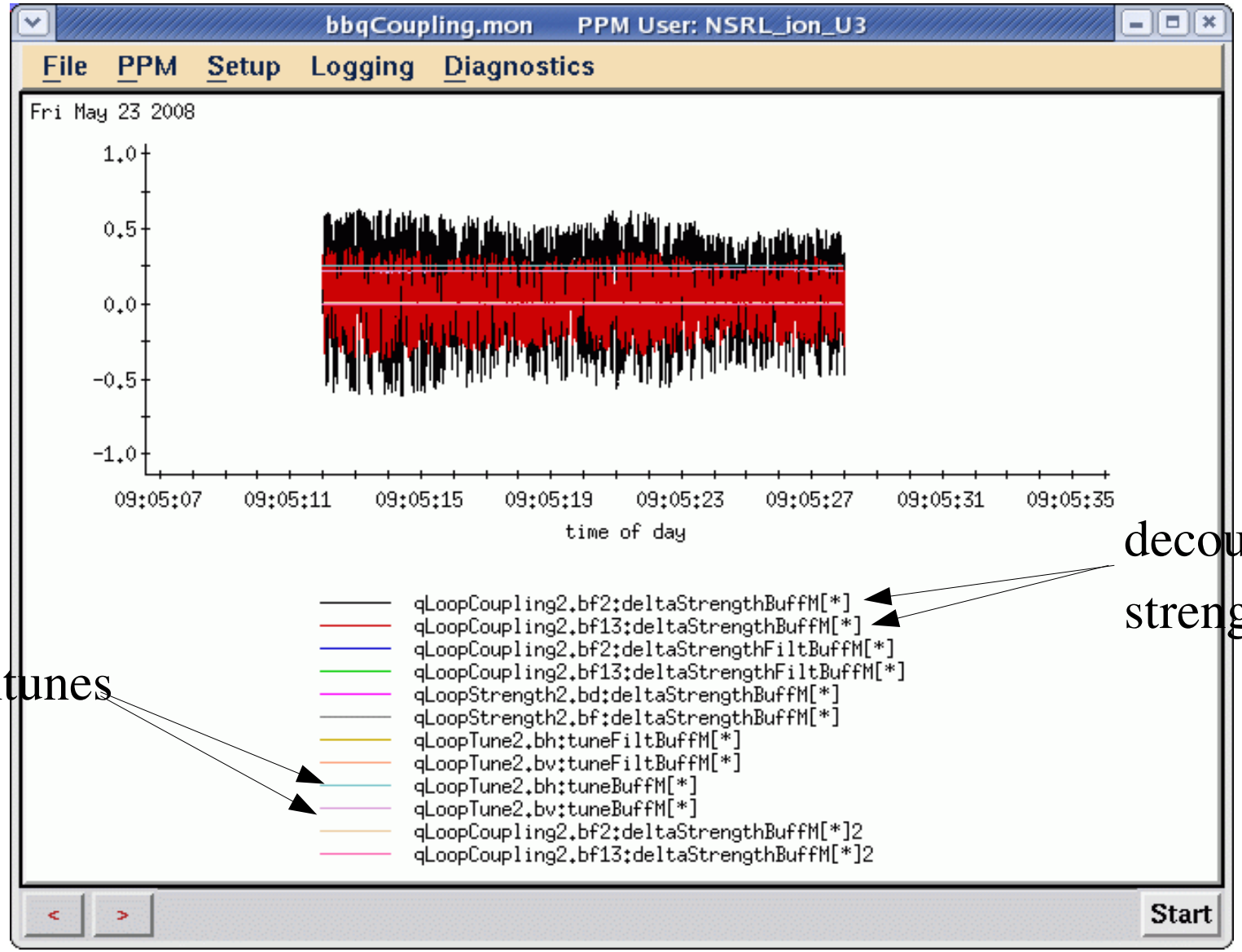


2.7 Ramp replay



One example of ramp replay(fill 9487)

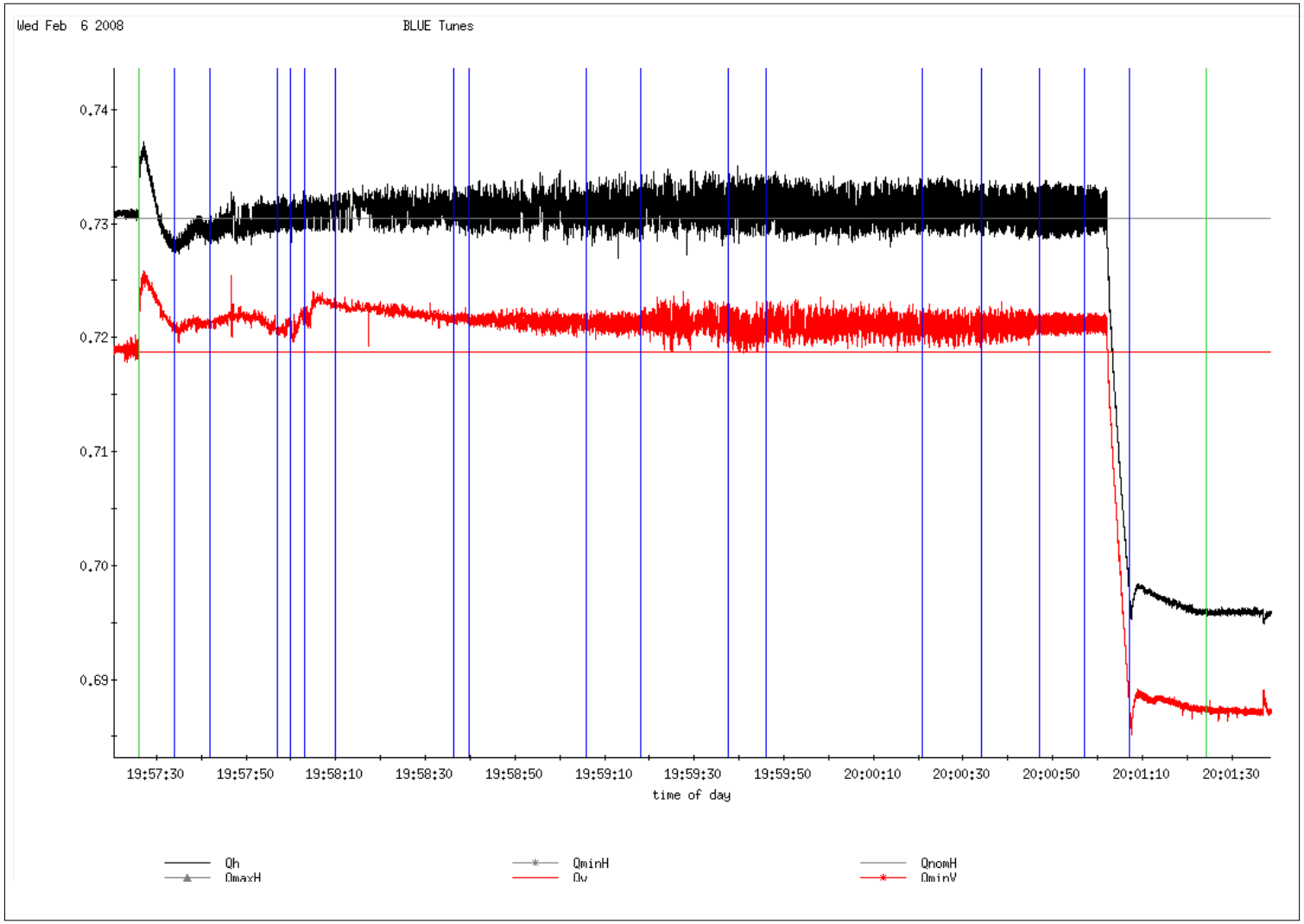
2.8 BBQcoupling.mon



decoupling strengths

eigentunes

2.9 The problem is being addressed



We are waiting for hybrid BBQ (245Mhz + BBQ electronics)

2.10 The remain theoretical problems to be solved

BTF of coupling

Interpret PLL readings in presence of coupling

Interpret AC Dipole readings in presence of coupling

BTF of beam-beam

Many people contributed to the RHIC ramp decoupling, tune/coupling feedback, and ramp replay and so on.

P. Cameron, J. Cupolo, W. Dawson, C. Degen, A. Dellappena, W. Fischer, M. Gasior(CERN), L. Hoff, J. Laster, R. Jones (CERN), Y. Luo, A. Marusic, J. Morris, S. Peggs, F. Pilat, T. Roser, C. Schulththeiss, C.Y. Tan (FNAL), S. Tepikian, D. Trbojevic ,.....