Physics of Bunched Beam Stochastic Cooling

Mike Blaskiewicz   C-AD

outline

- History
- Basic ideas
- RHIC system and results for longitudinal cooling
- Simulation algorithms and comparison with data
- Transverse Cooling

History

Herr and Mohl reported cooling bunched beams in ICE (1978)

Chattopadhyay develops bunched beam cooling theory (1983)

$$\theta - \omega_0 t = \varphi(t) \approx a \sin[\omega_s(a)t + \psi_0]$$

Stochastic cooling considered for SPS, RHIC and Tevatron (80s).

Unexpected RF activity pollutes Schottky signal (85s).

Transverse signal suppression seen in Tevatron (1995).

Cooling of long bunches in FNAL recycler (2005).

Proton cooling experiment in RHIC (2006).

Operational cooling of gold in RHIC (2007).

Pasquinelli, PAC95 (3 more?)

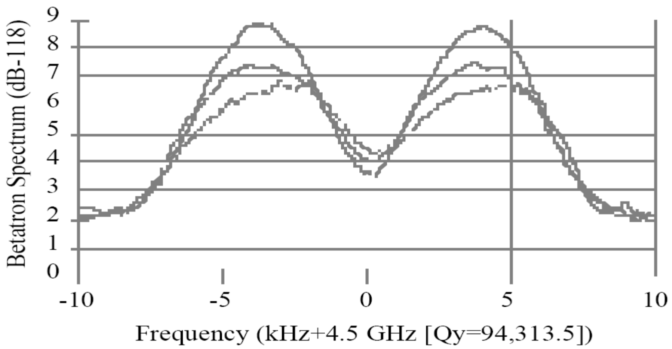
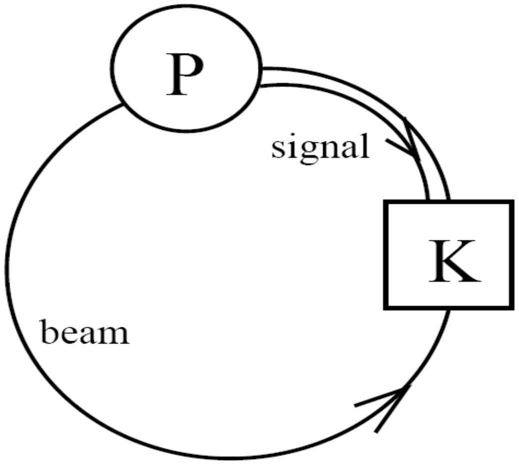
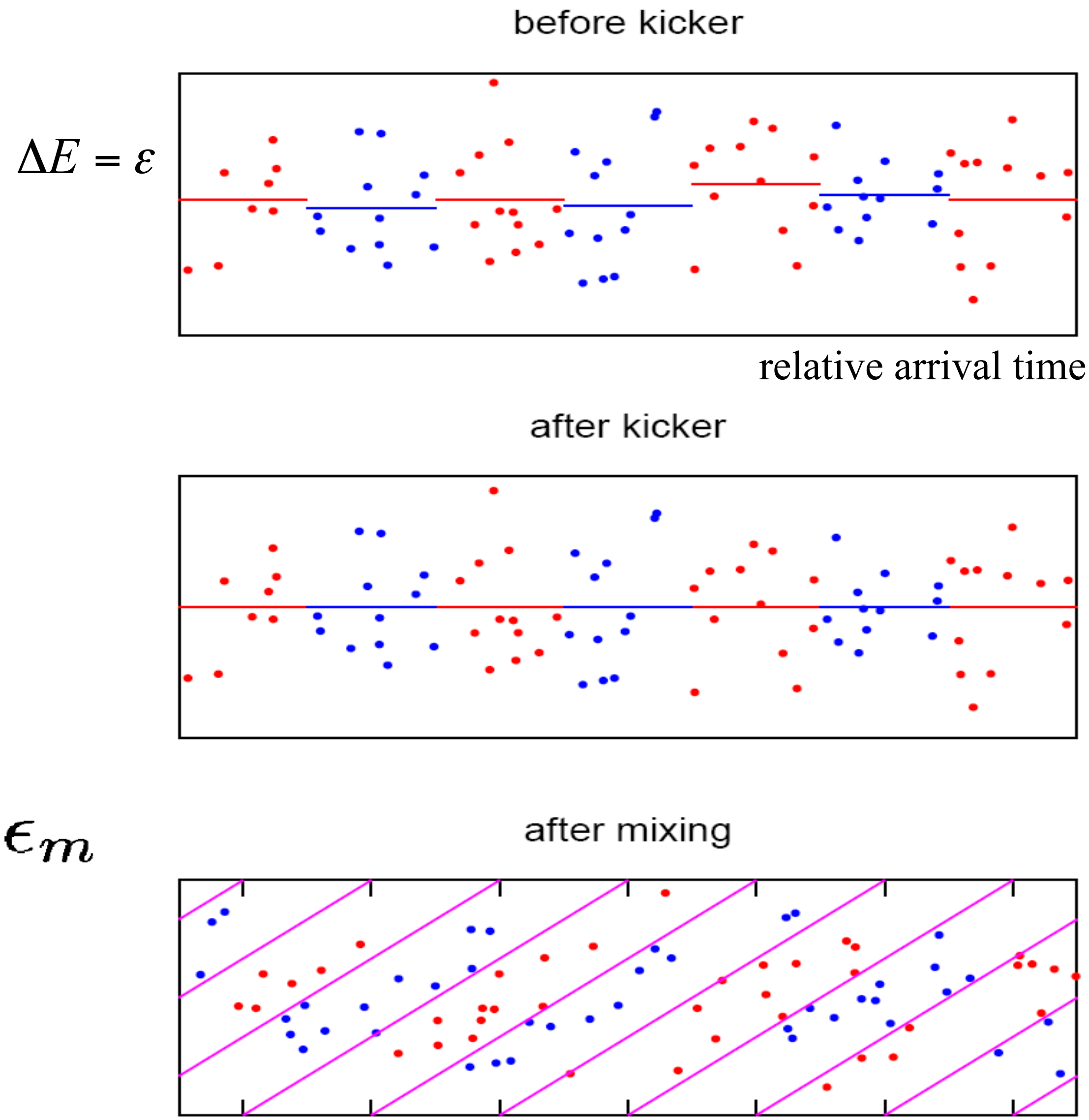


Figure 11: Measurement of signal suppression. Top, signal heating; Middle, open loop; Bottom, Signal Suppression.

Basic idea



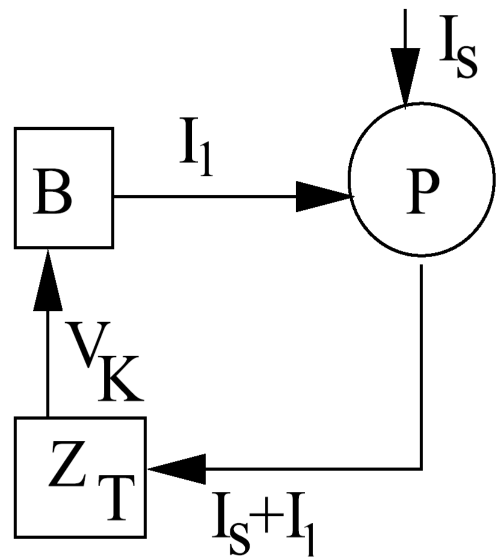


$$\bar{\epsilon}_k = \epsilon_k - \frac{g}{N_s} \sum_{m=1}^{N_s} \epsilon_m$$

$$< \varepsilon_k \varepsilon_m >=< \varepsilon^2 > \delta_{k,m}$$

$$< \bar{\epsilon}^2 > - < \epsilon^2 >= (-2g + g^2) < \epsilon^2 > / N_s$$

Signal Suppression



current at pickup due to voltage at kicker

$$I_1(\tilde{\omega}) = B(\tilde{\omega})V_K(\tilde{\omega}) \propto qN$$

total current at pickup

$$I_P = I_1 + I_S, \qquad I_S \propto q\sqrt{N}$$

voltage at kicker due to current at pickup

$$\begin{aligned} V_K &= -I_P Z_T \\ &= -(I_1 + I_S)Z_T \\ &= -BV_K Z_T - I_S Z_T \end{aligned}$$

net voltage at kicker due to Schottky current

$$V_K = \frac{-I_S Z_T}{1 + BZ_T} \equiv -Z_D I_S$$

Ip is suppressed by the same factor.

Optimal cooling gain for

$$BZ_T \approx 1$$

Mixing…

RHIC RF

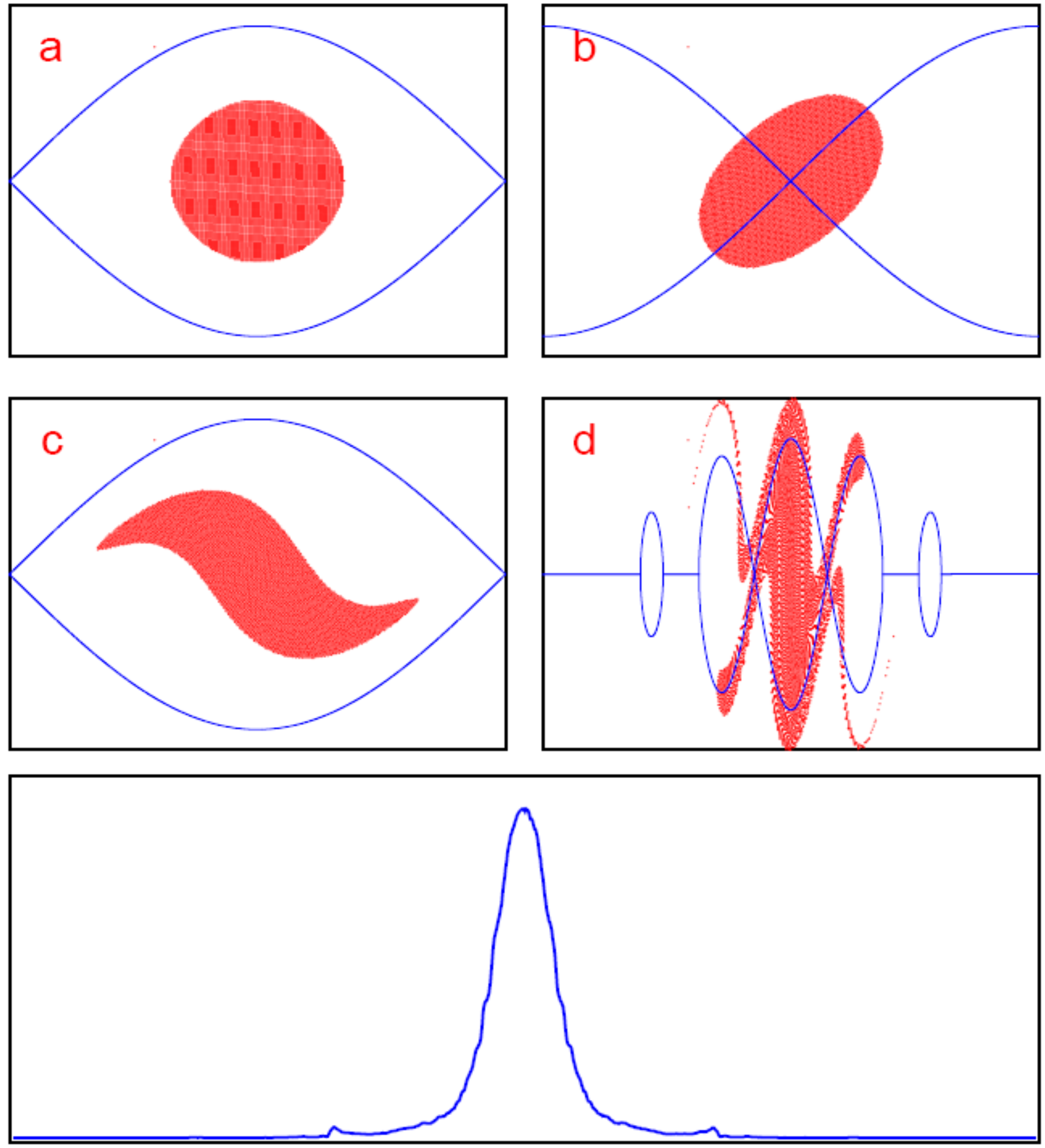
Rebucketing can lead to sharp edges in phase space density

$$\int_{-1}^{1}\sqrt{1-x^2}\cos(kx)dx$$

$$=\pi J_1(k)/k$$

$$\rightarrow k^{-3/2}$$

Lots of high frequency signal

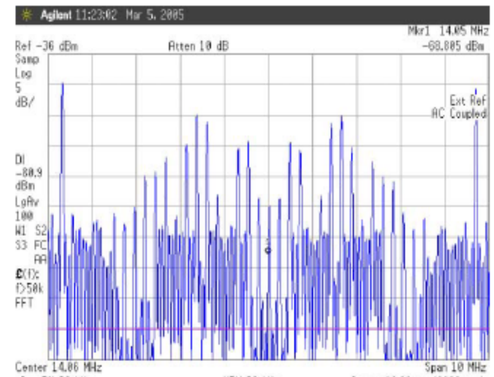


Coherent components are THE technical challenge to bunched beam S.C.

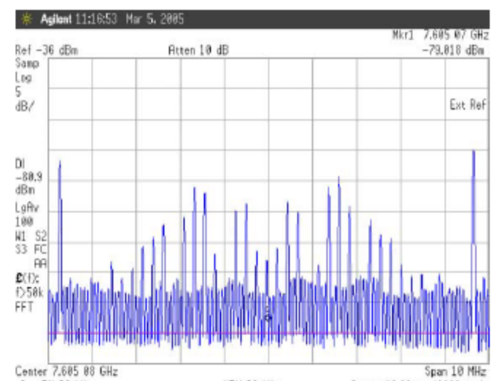
•We know that in RHIC for ions the **bunch shape** has Fourier strength at 8 GHz

- •The low frequency spectrum reflects the bunch filling pattern, abort gap, missing bunches, etc.
- •The high frequency spectrum looks the same
  - •All bunches contribute coherently
  - •Are locked to rf

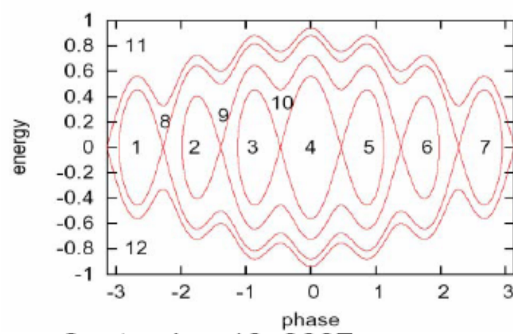
•The bunch shape arises from the dual harmonic rf buckets and the "rebucketing" gymnastic, h=360/2520

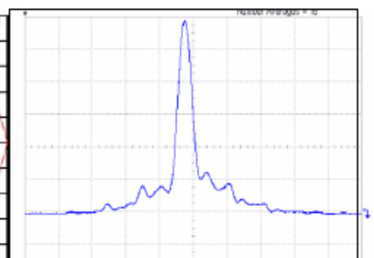


Bunch signal spectrum at 10 MHz



Bunch signal spectrum at 8 GHz

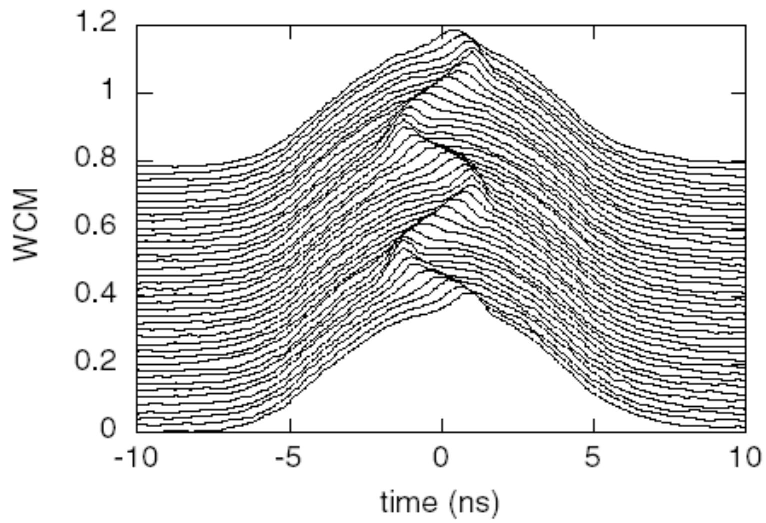




Bunch profile 5 ns/div

September 10, 2007 Mike Brennan 27

Proton coherence is different. PRSTAB 7, 044402 (2004)



$$\frac{d^2\phi}{ds^2} + \sin\phi = \frac{LQ\omega_{\rm rf}^2}{V_{\rm rf}}\frac{\partial\rho(\phi, s)}{\partial\phi}$$

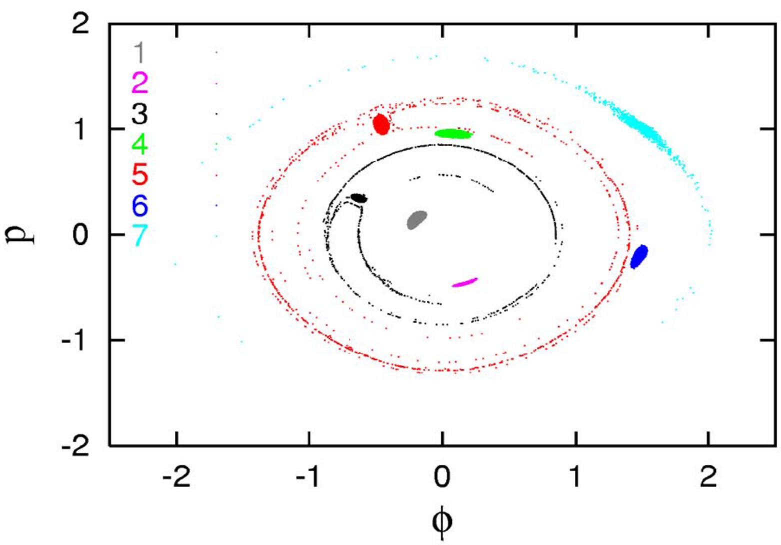
$$H(\phi, p, s) = p^2/2 + 1 - \cos\phi + \ell\rho(\phi, s)$$

$$\phi = \sqrt{2J}\sin(\Psi + s), p = \sqrt{2J}\cos(\Psi + s)$$

$$H_1 = \alpha(J) + \frac{\ell}{\pi}\int d\Psi_1 dJ_1 \frac{f(\Psi_1, J_1, s)}{\sqrt{2J + 2J_1 - 4\sqrt{JJ_1}\cos(\Psi - \Psi_1)}}$$

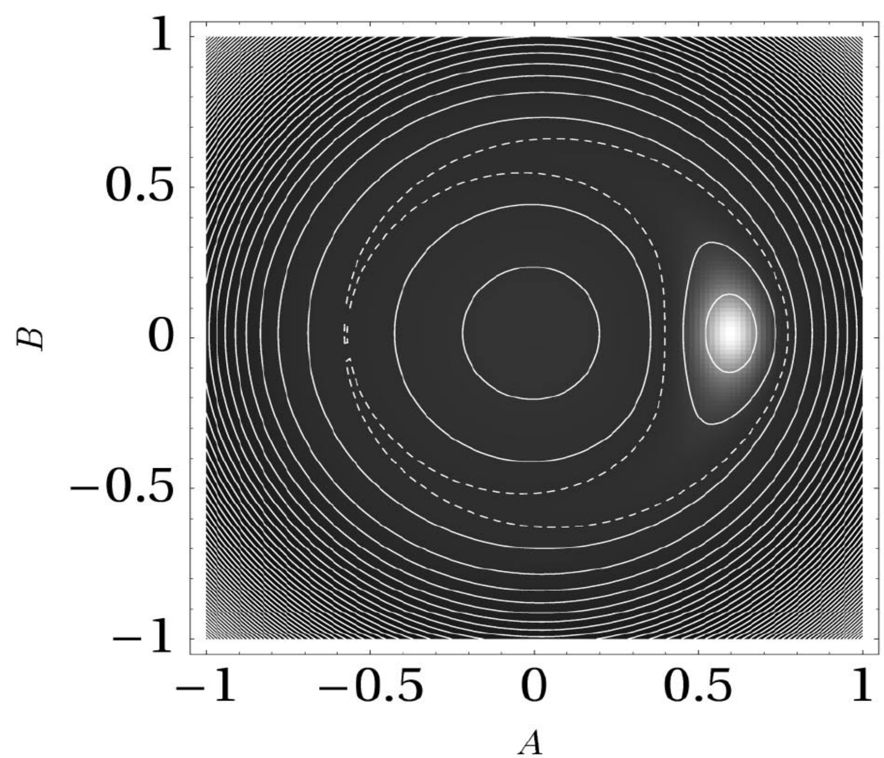
$$f(\Psi, J, s) = g(\Psi + \tilde{r}s, J)$$

$$\frac{2a^3\lambda}{\ell} \approx 0.77r\ln(r)/(1+r)$$



$$\lambda \approx \frac{-\hat{\phi}_0^2}{16}$$

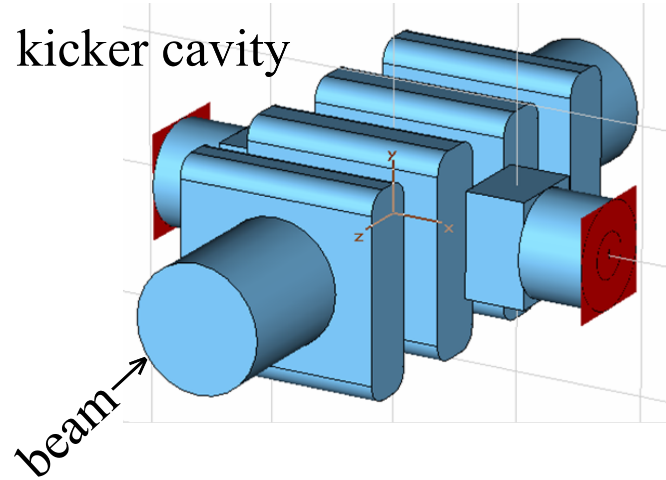
$$r = a/b$$

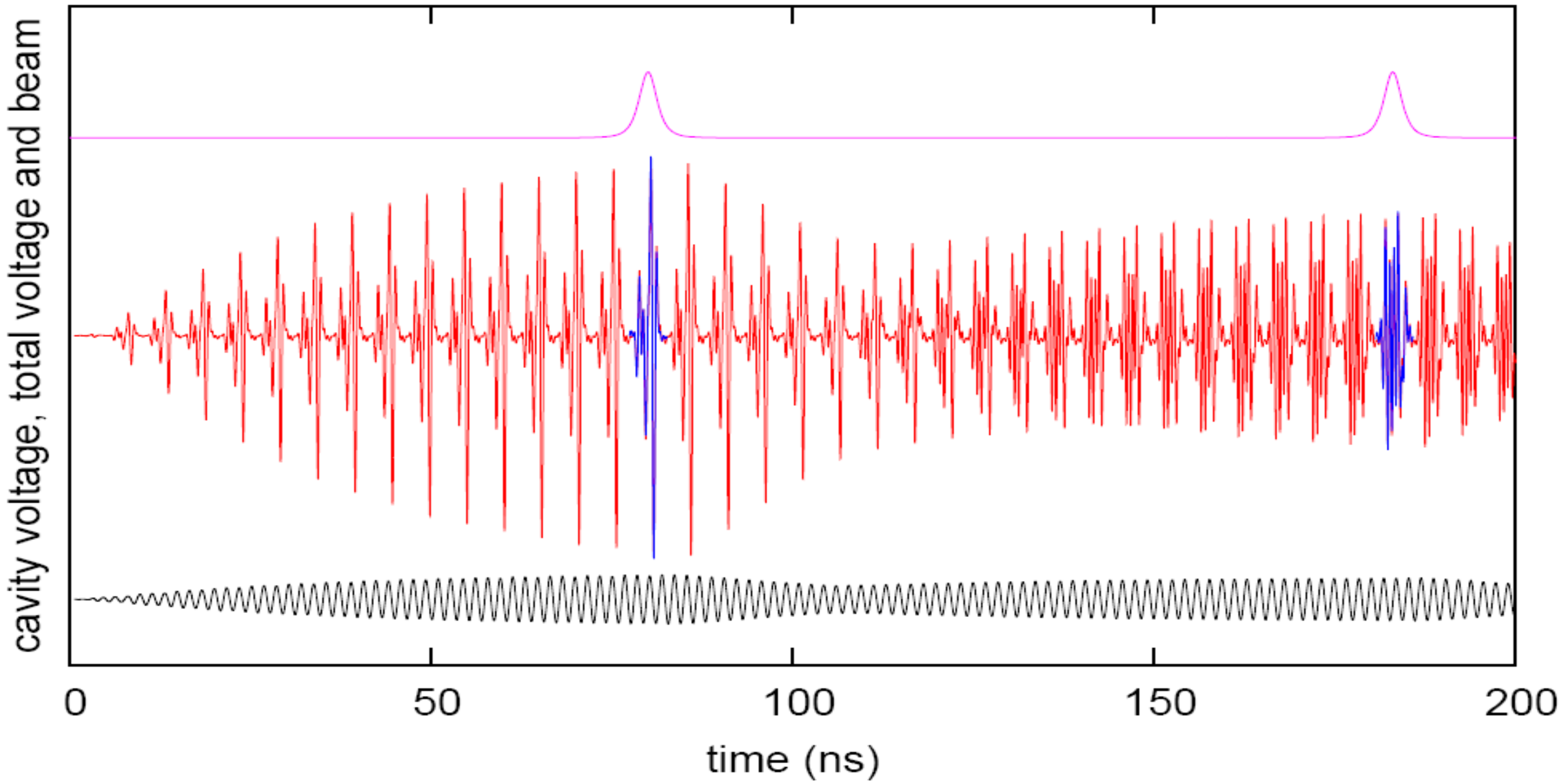


RHIC required a new kicker concept

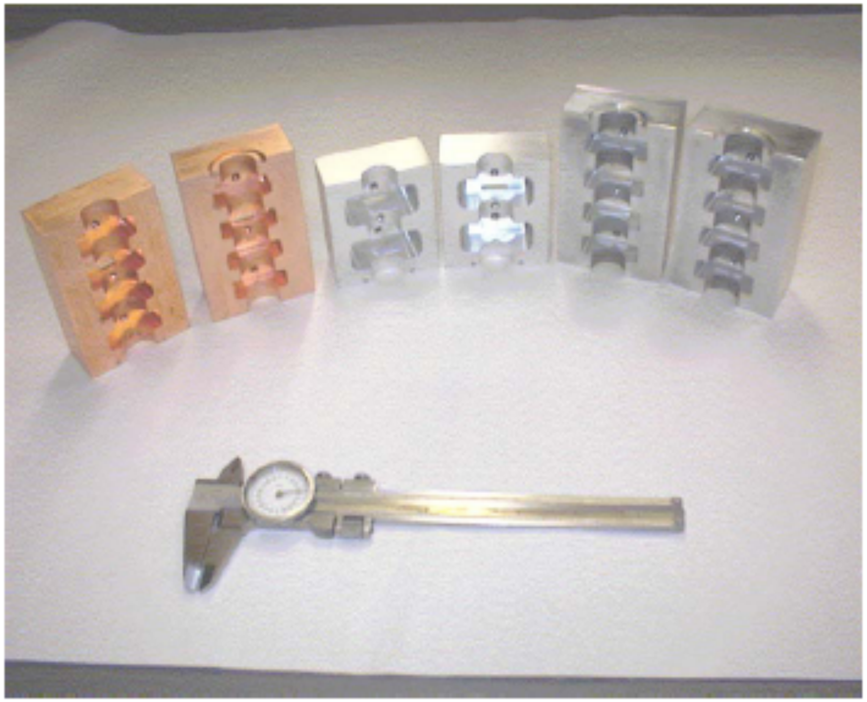
$$V_{rms} \approx \frac{\sigma(E)}{q\sqrt{N_s}} \approx 10kV$$

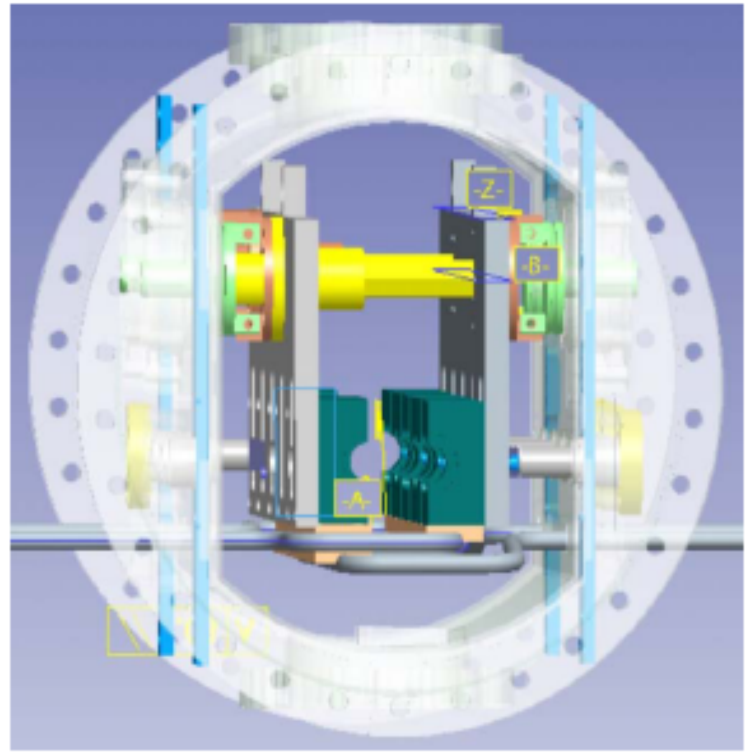
Mixing reduces optimal voltage by a factor of 5 or so





Longitudinal kicker needs to open during the ramp

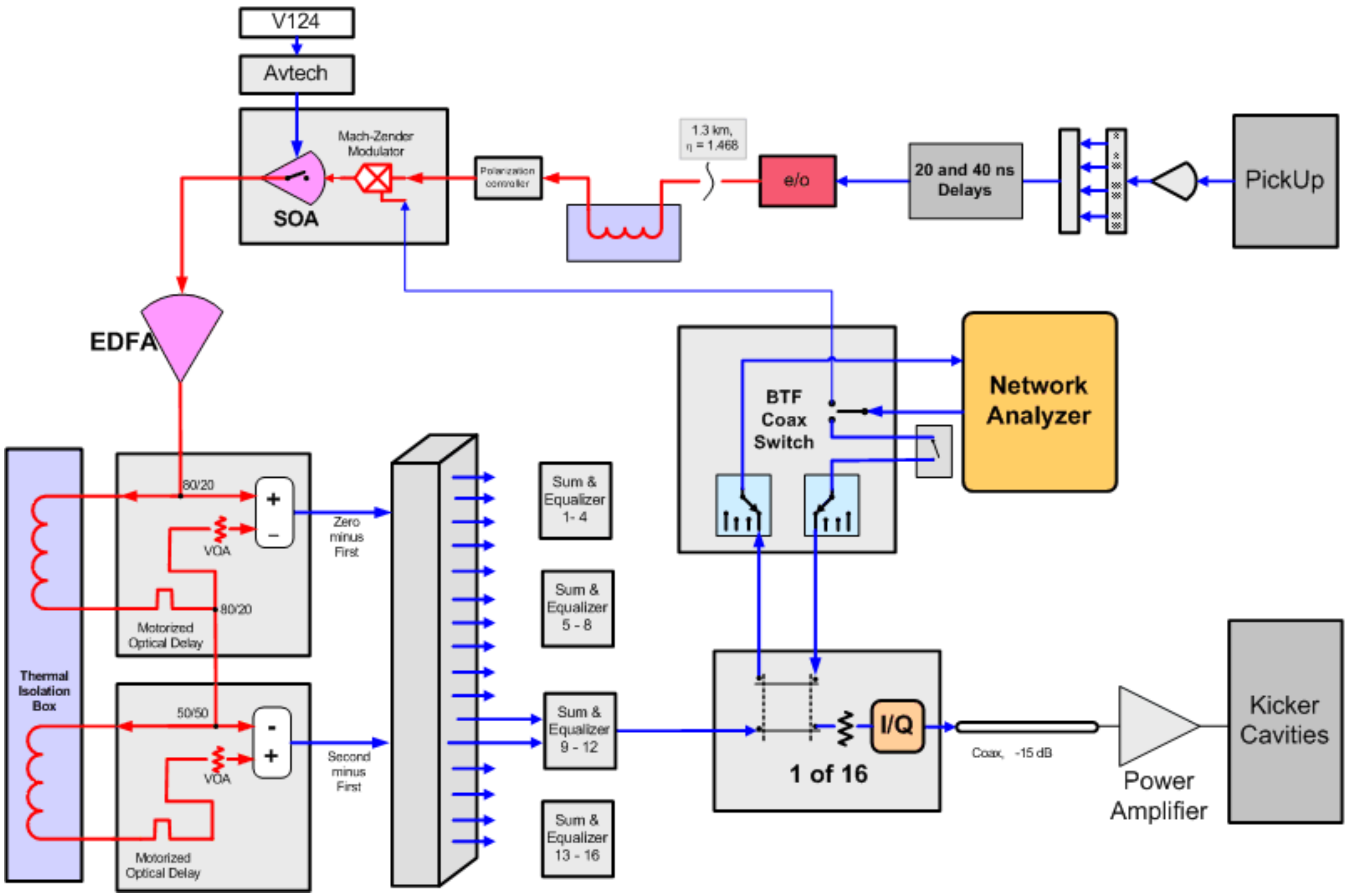




Tolerance for closing is 0.002”
Individual cavities driven by 40 Watt amplifiers
(250 W each for 6, 1k-Ohm kickers with 1 GHz bandwidth)
Amplitude and delay are corrected every 5 minutes.

The actuality is complicated.

Stochastic Cooling Low Level Block Diagram

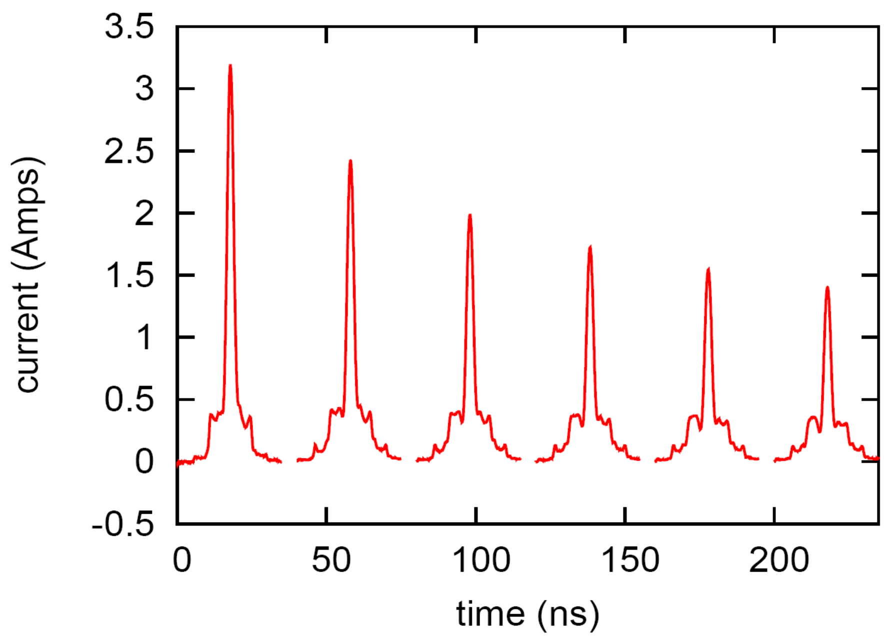


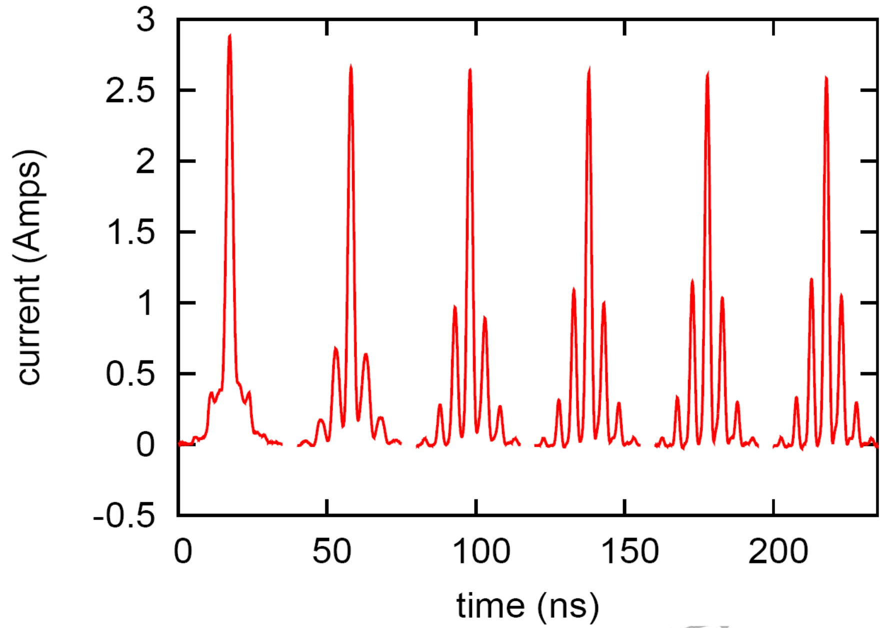
RHIC Results

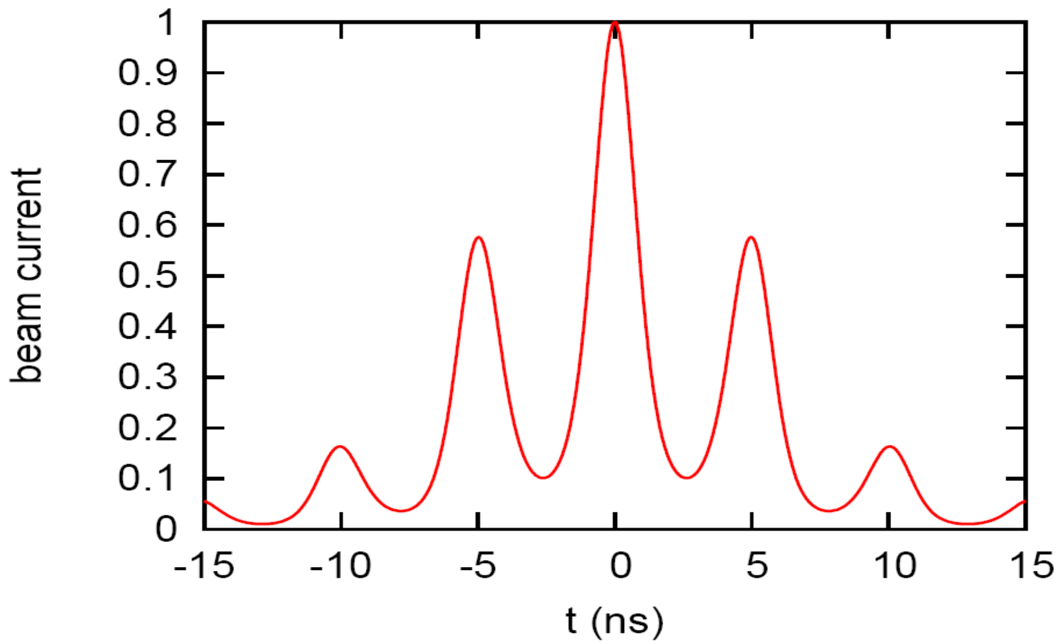
Evolution of a 5 hour RHIC store with 1.E9 Au/bunch.

Top, wall current monitor profiles taken one hour apart without cooling. IBS causes significant loss from the RF bucket.

Bottom, cooling on. Beam loss is consistent with burn off in the interaction regions.







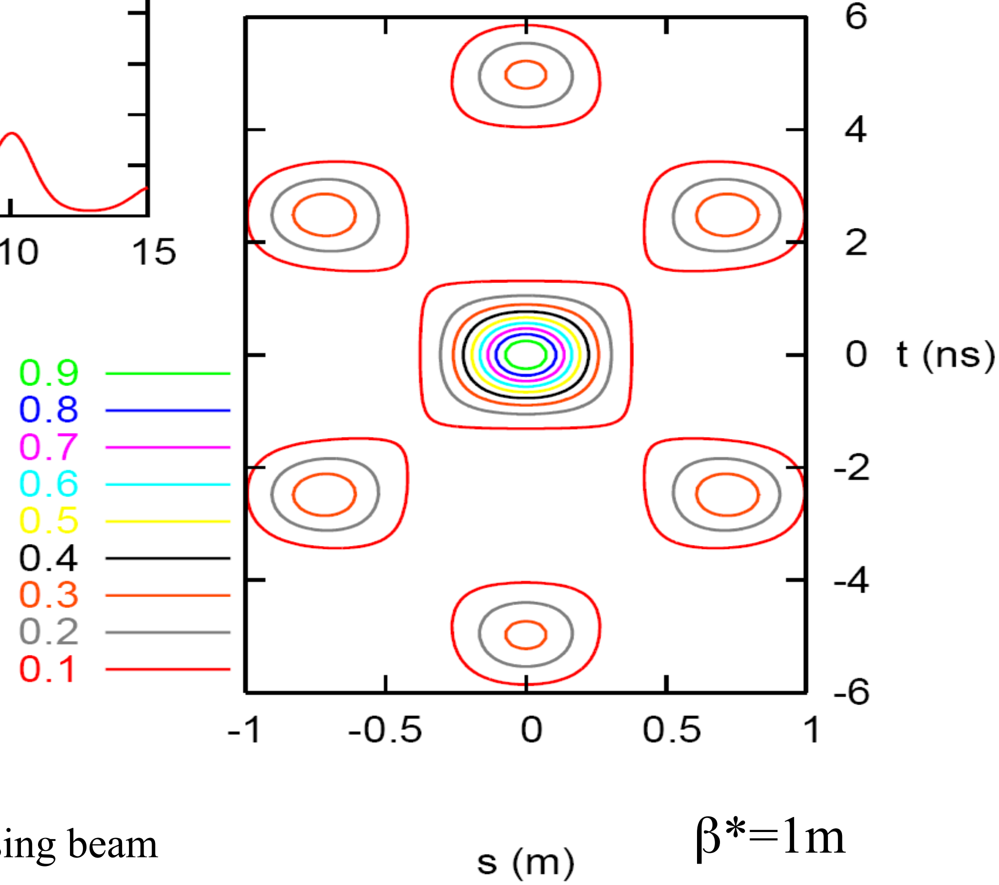
$$\frac{dN_{coll}}{dsdt} = \frac{\sigma}{cq^2} \frac{I(s-ct)I(s+ct)}{2\pi\varepsilon(\beta^* + s^2/\beta^*)}$$

beam in a drift space

$$x(s) = x_0 + s\theta_0$$

$$< x^2(s) > = < x_0^2 > + s^2 < \theta_0^2 > + 2s < x_0\theta_0 >$$

$$= \varepsilon_{rms}\left(\beta^* + \frac{s^2}{\beta^*}\right)$$



β*=1m

Smaller ε allows for smaller β* without increasing beam size in triplets

Simulation Preliminaries I

Tracking a billion macro-particles for a billion turns is not possible.

A robust scaling law accurate over many orders of magnitude is required.

Model intra-beam scattering (IBS) with random kicks drawn from the appropriate distribution. The central limit theorem applies, so the scaling is straightforward.

For stochastic cooling (SC) it is well know that the cooling time scales in proportion to the number of particles.

Define a macro-particle so that the simulated and real beams have the same parameters in the continuum limit. Both could be injected into the same ring and have the same bunch length and rms beam size.

Therefore:

1) Define a real beam and a simulation time for it.
2) Choose the number of macro-particles.
3) Define the number of simulation turns based on SC.
4) Adjust the size of the IBS kicks to coincide with this number of simulation turns.

Simulation Preliminaries II

For example, Consider 1.E9 Au ions over 1.E9 turns.

Take 1.E5 macro-particles. Each particle has 1.E4 Au ion charges.

For SC this implies a simulation time of 1.E5 turns.

The rms IBS kick per turn, in angle and dp/p, is calculated for the actual beam and multiplied by $100 = \sqrt{10^9 / 10^5}$

This degree of scaling requires as much confidence as possible.

Numerical self consistency is checked.

Analytic work based on Fokker-Planck and other statistical methods are known.

Another way of looking at the problem is next.

Economical view of the scaling with N

$$\ddot{x}_j + \Omega_j^2 x_j = -\frac{2g\Omega_0}{N}\sum_{k=1}^{N}\dot{x}_k$$

$$\Omega_j = \Omega_0 + \omega_j, \qquad |\omega_j| << \Omega_0$$

$$x_j = a_j \exp(-\lambda t - i\Omega_0 t)$$

$$(\lambda - i\omega_j)a_j = \frac{g\Omega_0}{N}\sum_{k=1}^{N} a_k$$

$$1 = \frac{g\Omega_0}{N}\sum_{k=1}^{N}\frac{1}{\lambda - i\omega_k}$$

$$\int_{-\infty}^{\omega_k} f(\omega)d\omega = \frac{k-1/2}{N}$$

$$\lambda \approx i\omega_K, \quad \Delta\omega = \frac{1}{Nf(\omega_K)}$$

$$\begin{aligned}
&\sum_{m=1}^{N}\frac{1}{\lambda - i\omega_m} \\
= \quad &\sum_{|m-K|<M}\frac{1}{\lambda - i\omega_m} + \sum_{|m-K|\geq M}\frac{1}{\lambda - i\omega_m} \\
\approx \quad &\sum_{|m|<M}\frac{1}{\lambda - i\omega_K - im\Delta\omega} + \sum_{|m-K|>M}\frac{i}{\omega_m - \omega_K} \\
\approx \quad &\sum_{k=-\infty}^{\infty}\frac{1}{\lambda - i\omega_K - ik\Delta\omega} \\
+ \quad & iN\int_{-\infty}^{\infty}\frac{\omega - \omega_K}{0^+ + (\omega - \omega_K)^2}f(\omega)d\omega. \qquad (13)
\end{aligned}$$

$$\lim_{M\to\infty}\sum_{k=-M}^{M}\frac{1}{z - ik} = \pi\frac{\exp(2\pi z)+1}{\exp(2\pi z)-1},$$

Mixing and signal shielding are fully accounted for.

$$R(\omega_K) = \pi \Omega_0 f(\omega_K) \qquad X(\omega_K) = \Omega_0 \int_{-\infty}^{\infty} \frac{\omega - \omega_K}{0^+ + (\omega - \omega_K)^2} f(\omega) d\omega$$

$$\exp[2\pi N f(\omega_K)(\lambda - i\omega_K)] = \frac{1 + gR - igX}{1 - gR - igX}$$

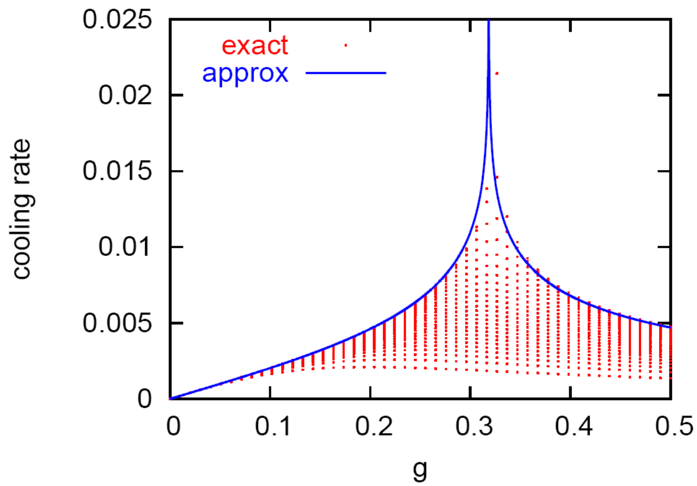


Figure 4: Comparison of actual values of $Re(\lambda)$ versus gain with those obtained from equation (14) with $X = 0$ for a rectangular frequency distribution with $N = 51$. The numerical solution had one eigenmode with a monotonically growing eigenvalue, which is not fully shown.

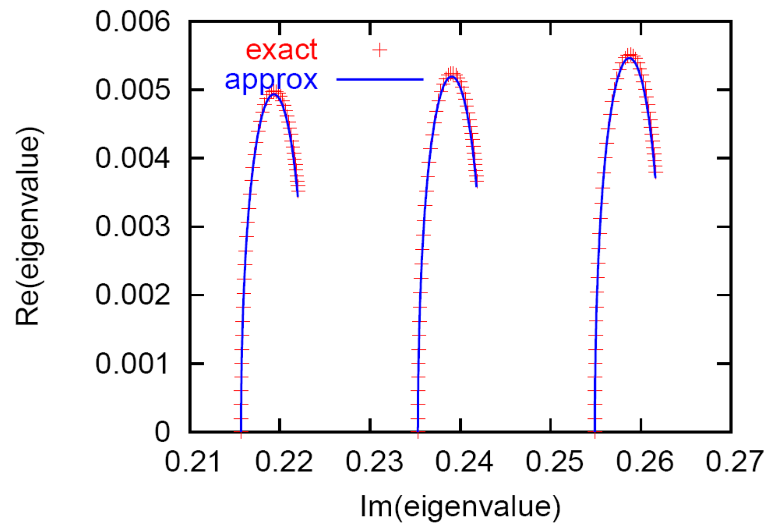
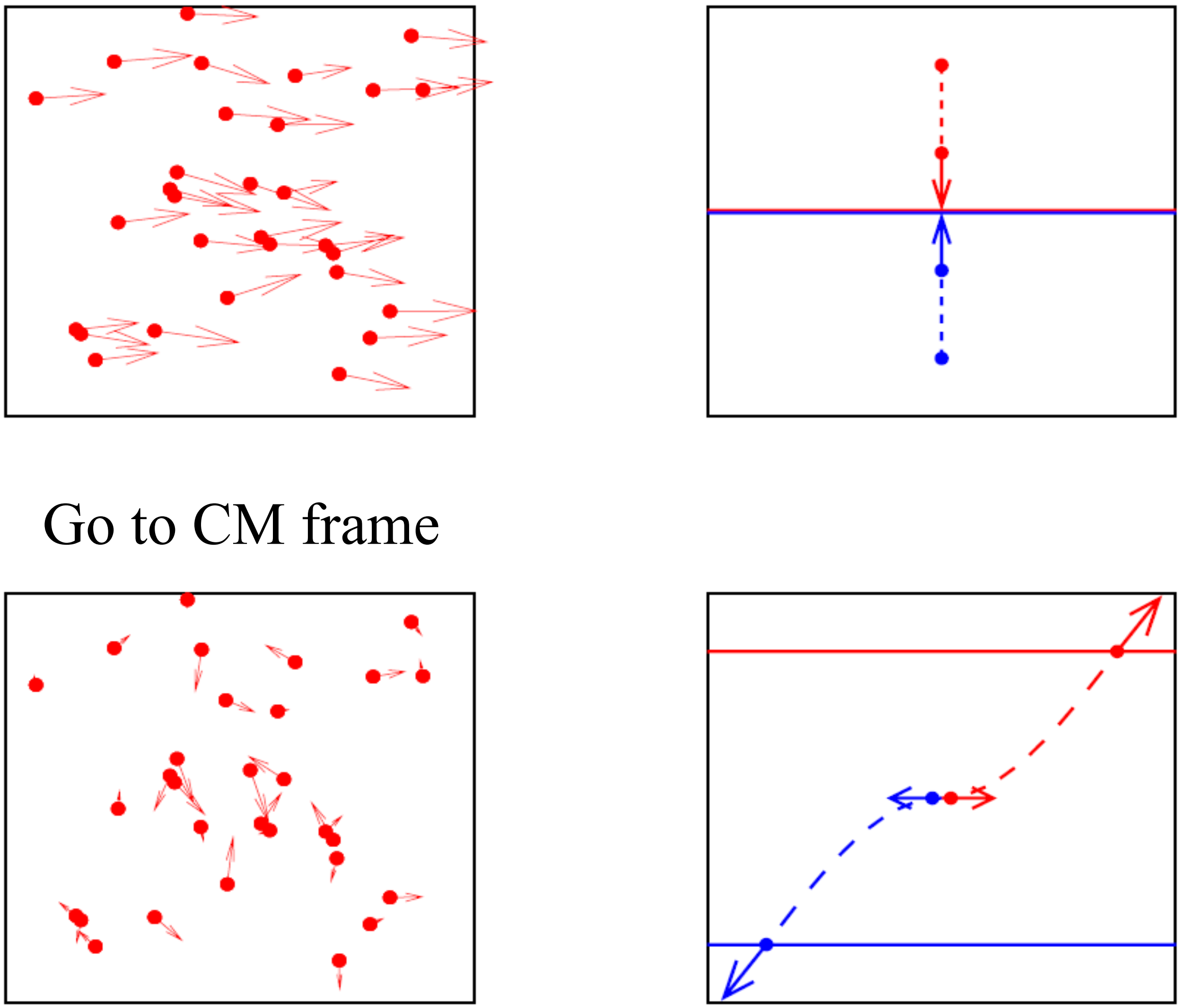


Figure 5: Evolution of λ as a function of gain for the exact, numerical solution and equation (14). The oscillator frequencies were uniformly spaced with $\omega_j = j/N$ and $N = 51$.

Derbenev's picture of intra-beam scattering



Look at 2 particles in the same beam bouncing off each other

Go to CM frame

Dispersion causes average orbit to change with energy.
Can get an increase in transverse speed.

Bunched Beam Simulations I

Dealing with intra-beam scattering

1) Start with Piwinski's formulas using average lattice parms.

$$\frac{1}{\sigma_p^2}\frac{d\sigma_p^2}{dt} = \alpha_{p0}$$

3) Correct for coupling (more later)

$$\alpha_{\perp 0} = (\alpha_{x0} + \alpha_{y0})/2$$

4) Correct for number of macro-particles

$$\alpha_{p1} = R\alpha_{p0}$$

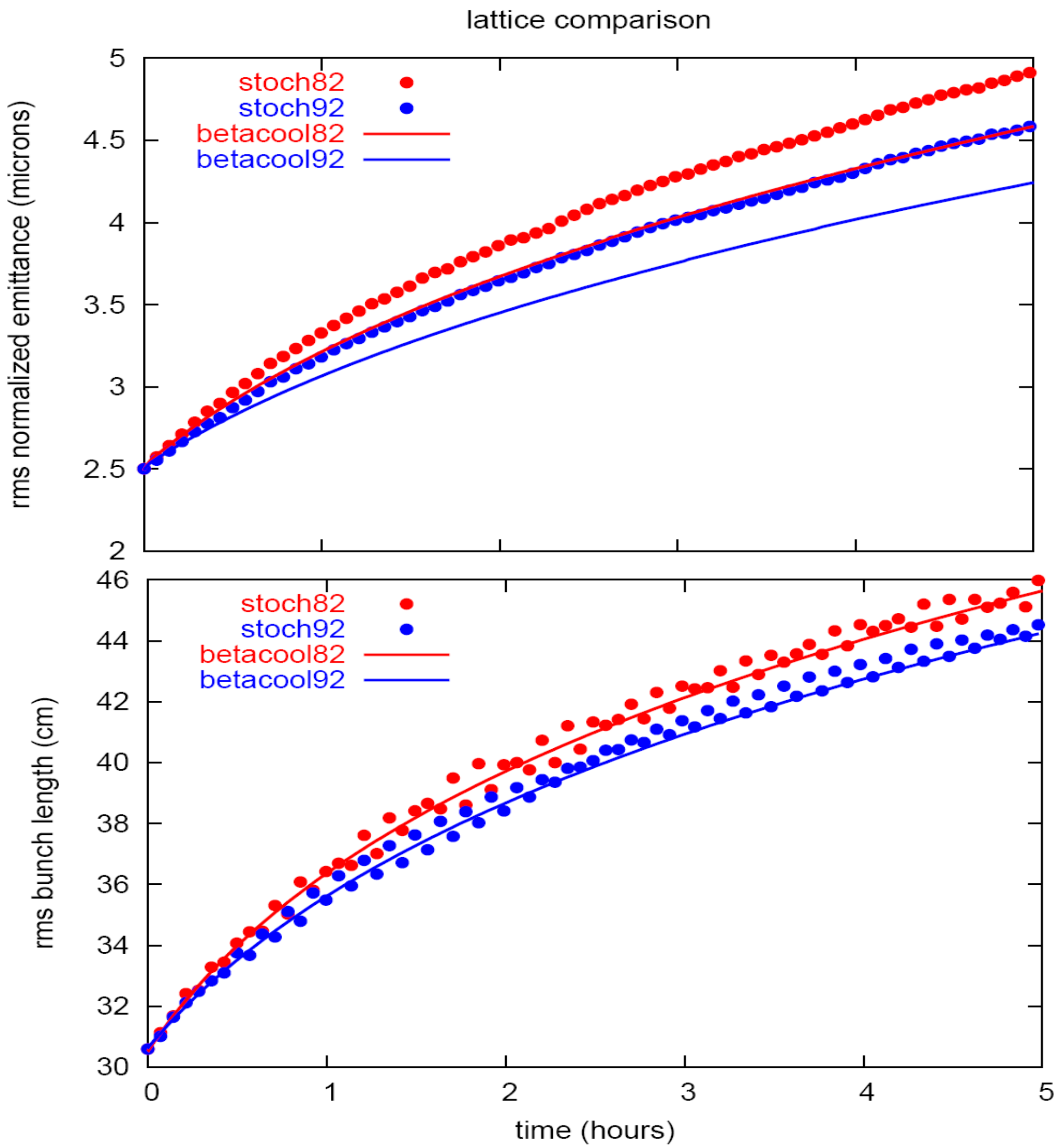
5) Correct for non-gaussian profile

$$F(t) = I(t)\sigma_t 2\sqrt{\pi}/Q$$

6) Langevin kick

$$\Delta p = \sigma_p \sqrt{\alpha_{p1} T_0 F(t)}\chi$$

Comparison with Betacool, courtesy A. Fedotov



Simulations without cooling look good

Data

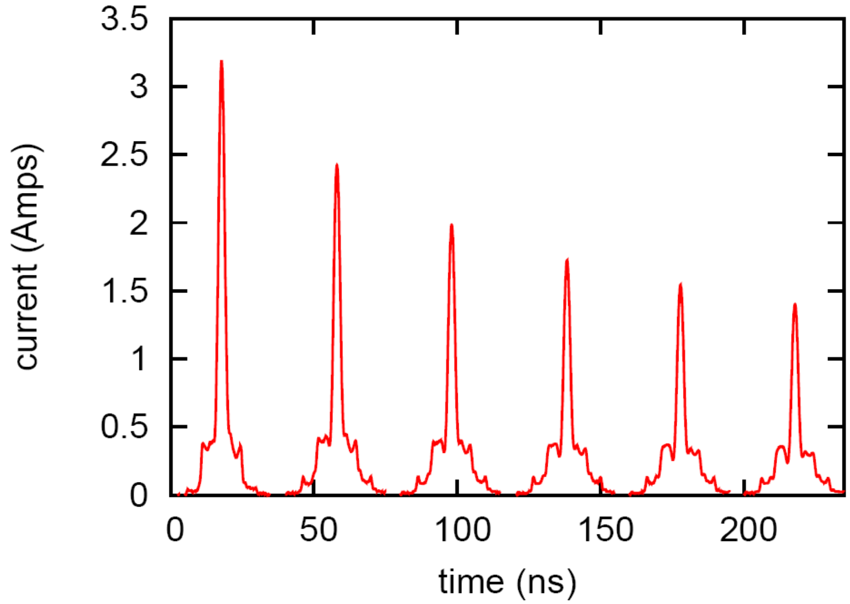


Figure 1: Evolution of the average bunch profile over a five hour RHIC store with gold beam and no cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

Simulation

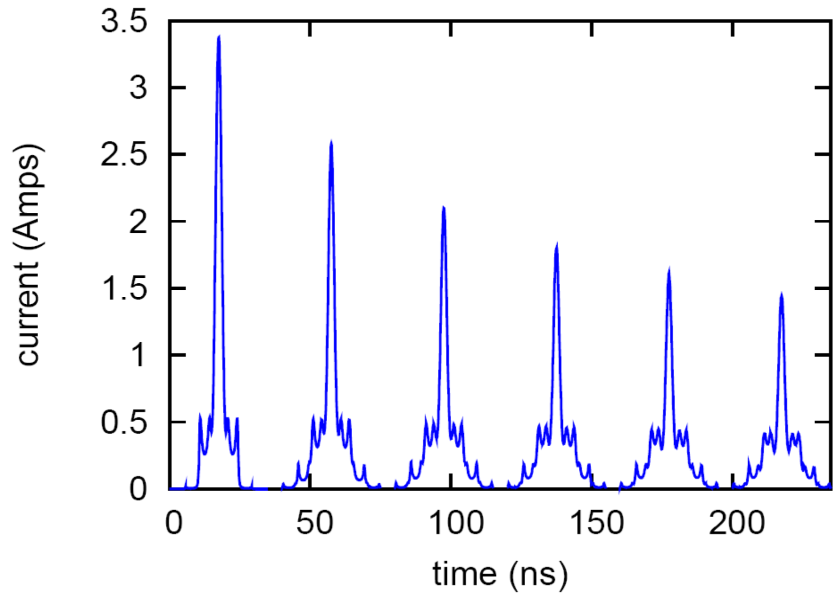


Figure 3: Simulation of the average bunch profile over a five hour RHIC store with gold beam and no cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

Cooling in the time domain

Time domain model of filter cooling.

Very similar to coherent stability problem.

Grid macroparticles on a fine lattice

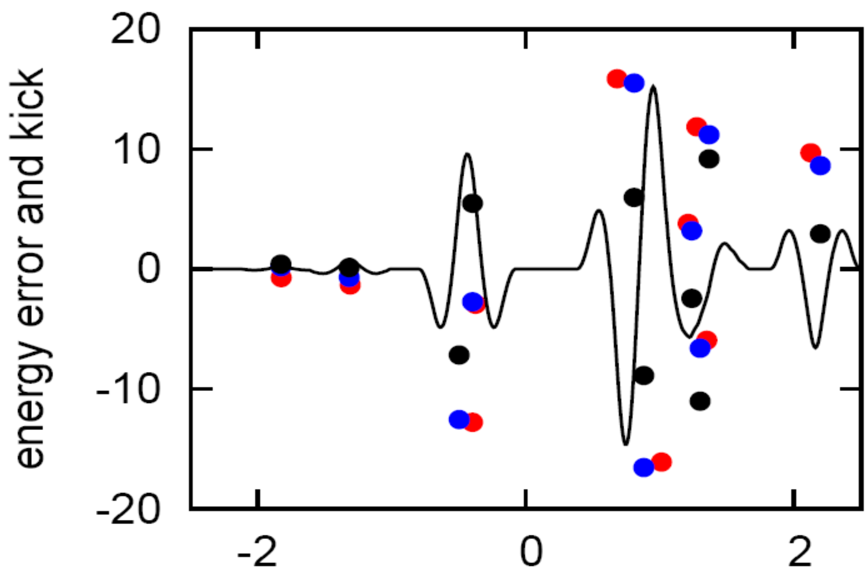
$$I_0(t_k,n) = \frac{q_m}{\Delta t}\sum_{m=1}^{N_m}\hat{\delta}(\tau_m^p(n) - t_k)$$

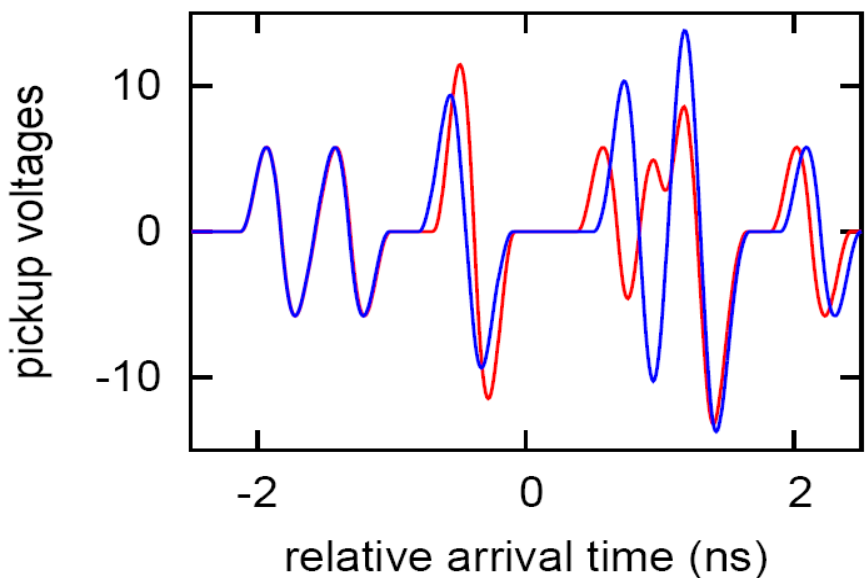
Use cascaded 1 turn delays

$$I_1(t_k,n) = I_0(t_k,n) - 2I_0(t_k,n-1) + I_0(t_k,n-2)$$

8.3 GHz vs. 5.9 for 1 turn delay

Convolve with wake using FFTs





Data .vs. Simulation

Gain calibration in the simulation

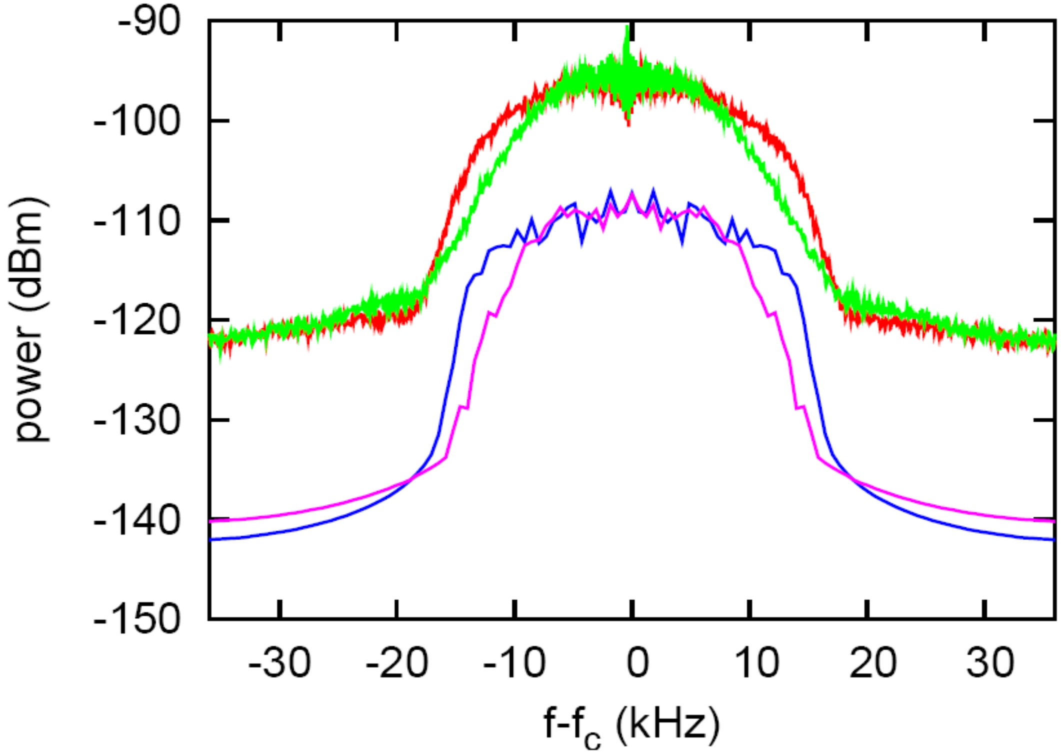


Figure 3: Measured and simulated signal suppression at 6 GHz. The data are the top two traces and the simulation the bottom 2.

Simulation is good, burn-off is a few percent.

Data

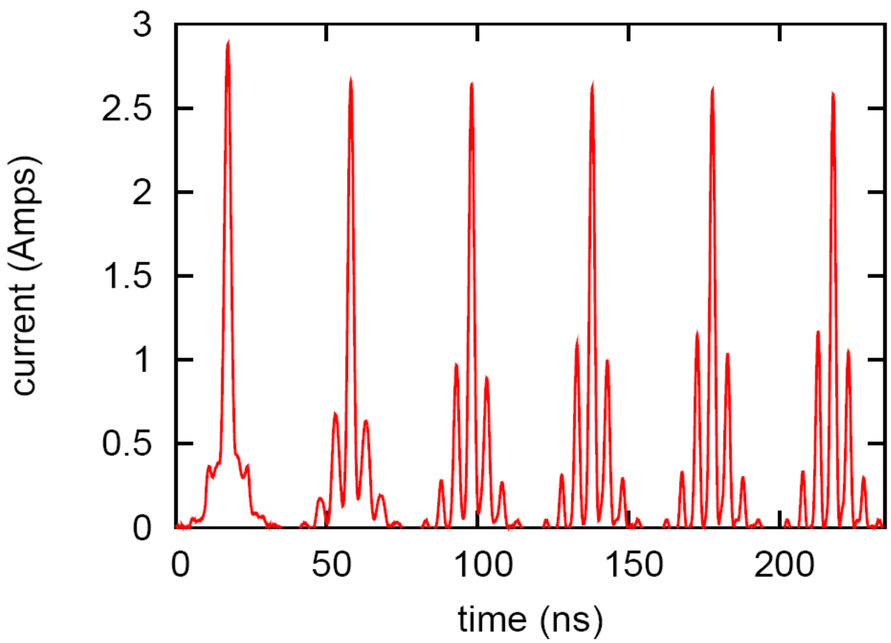


Figure 2: Evolution of the average bunch profile over a five hour RHIC store with gold beam and good longitudinal cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

Simulation

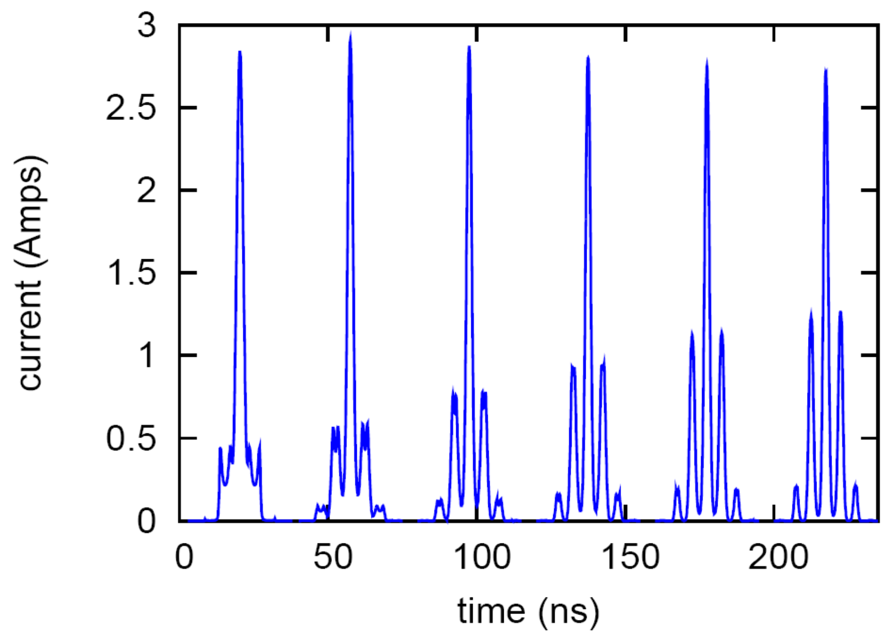


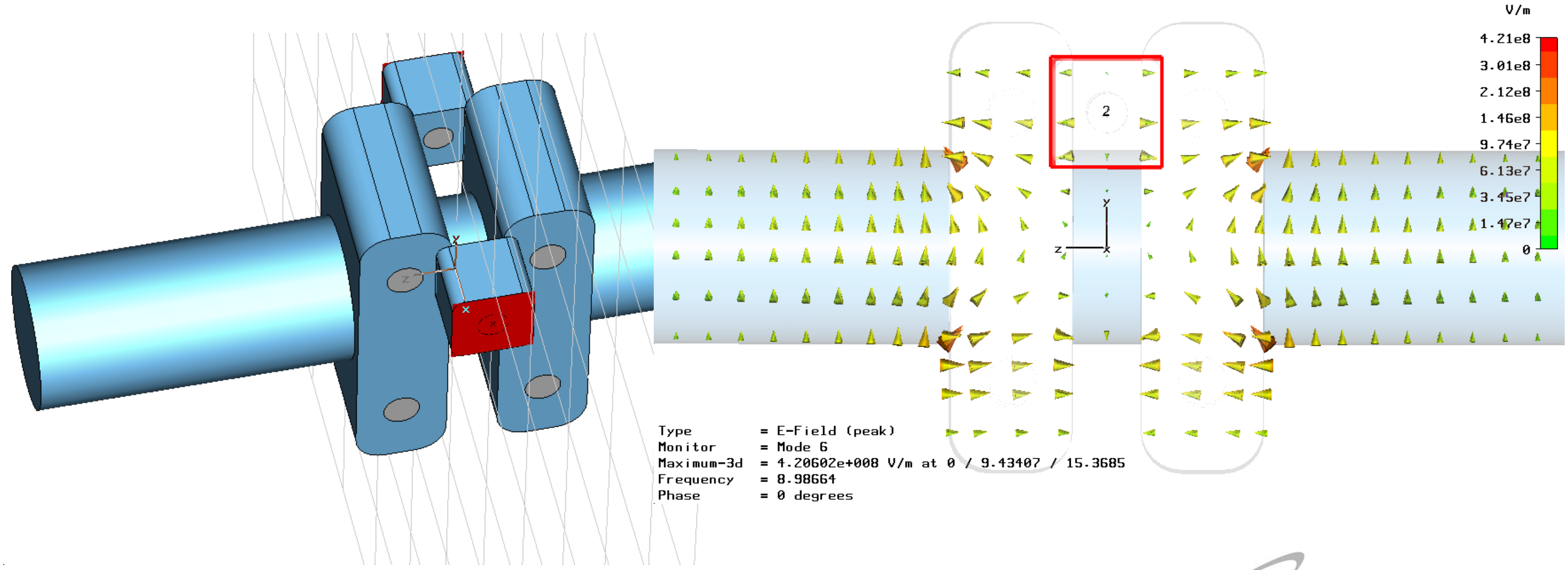
Figure 6: Simulation of the average bunch profile over a five hour RHIC store with gold beam and good cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

Transverse Cooling system

Similar cavities. Low level requires a notch filter (R&D)

40 Watt amplifiers are sufficient.

5-8 GHz keeps aperture reasonable.



Transverse Cooling Simulations

$$H_s(\epsilon,\tau) = \frac{T_0\eta}{2\beta^2 E_0}\epsilon^2 - \int_0^\tau dtqV_{rf}(t)$$

Check of scaling, single harmonic rf, no IBS or longitudinal cooling

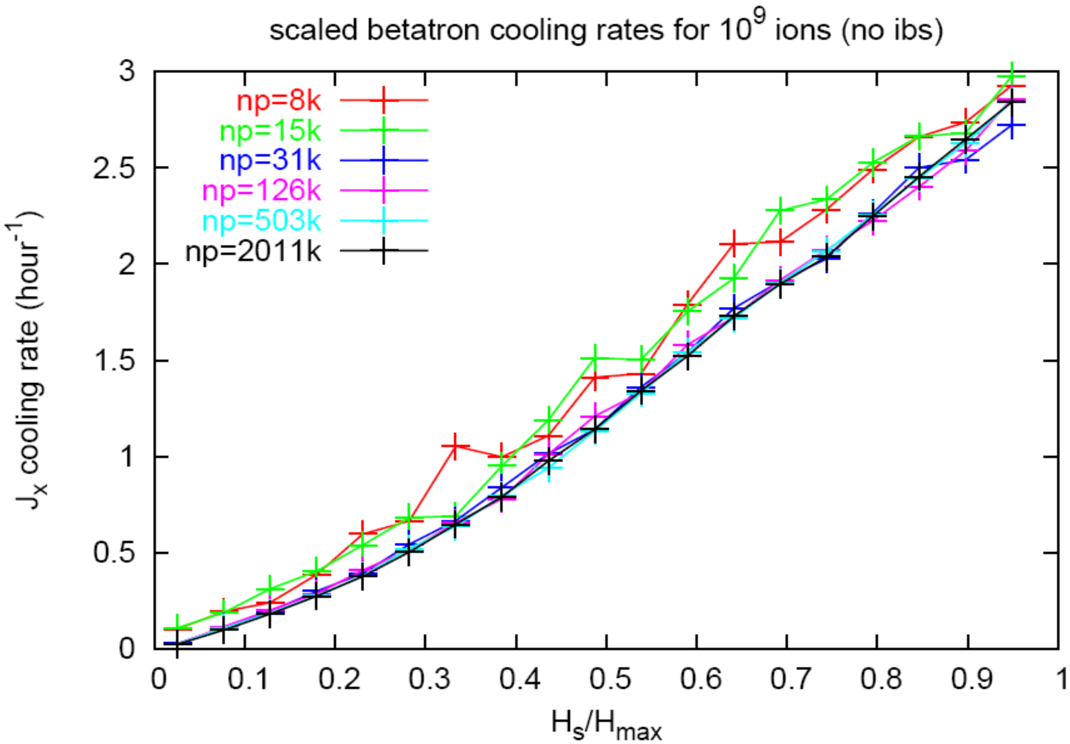


Figure 5: Transverse cooling rate versus the value of the longitudinal hamiltonian. Similar results are shown in [6, 7]

Intra-beam scattering helps cooling

IBS causes diffusion in longitudinal action. Physically important for FNAL Recycler, it's a major source of mixing.

For RHIC, longitudinal cooling keeps the distribution in the bucket, but a given particle will wander in synchrotron amplitude.

The net effect is that all particles have good transverse cooling.

This gives a new simulation time scale to worry about.

One must make sure that the fast mixing from IBS is small compared to the fast mixing from synchrotron motion.

RHIC parameters OK with 50k macroparticles (add transverse picture).

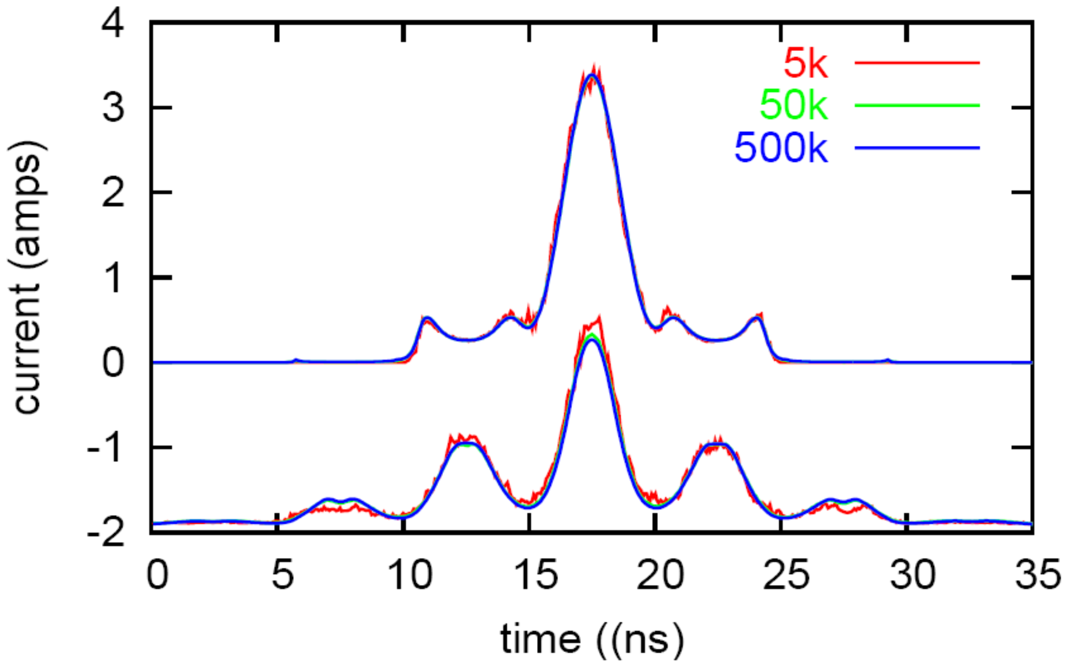


Figure 8: Test of convergence with both cooling and IBS. The initial profiles for 5000, 50,000, and 500,000 macroparticles are shown in the upper traces. The lower traces show the profiles at 2000, 20,000, and 200,000 turns, respectively. This corresponds to 10^9 gold ions evolving over 85 minutes.

Coupling allows for one system to cool both transverse planes

$$\omega_x = \varpi + \delta$$

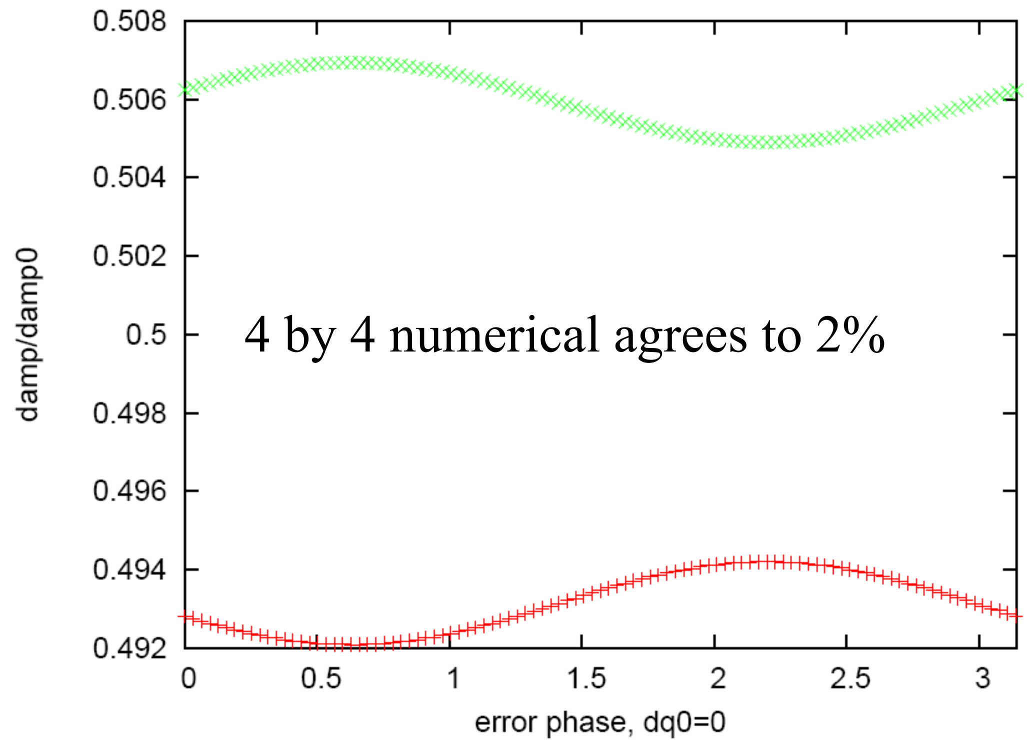
$$\omega_y = \varpi - \delta$$

$$\ddot{x} + \omega_x^2 x = \varpi q y$$

$$\ddot{y} + \omega_y^2 y = \varpi q x - \beta \dot{y}$$

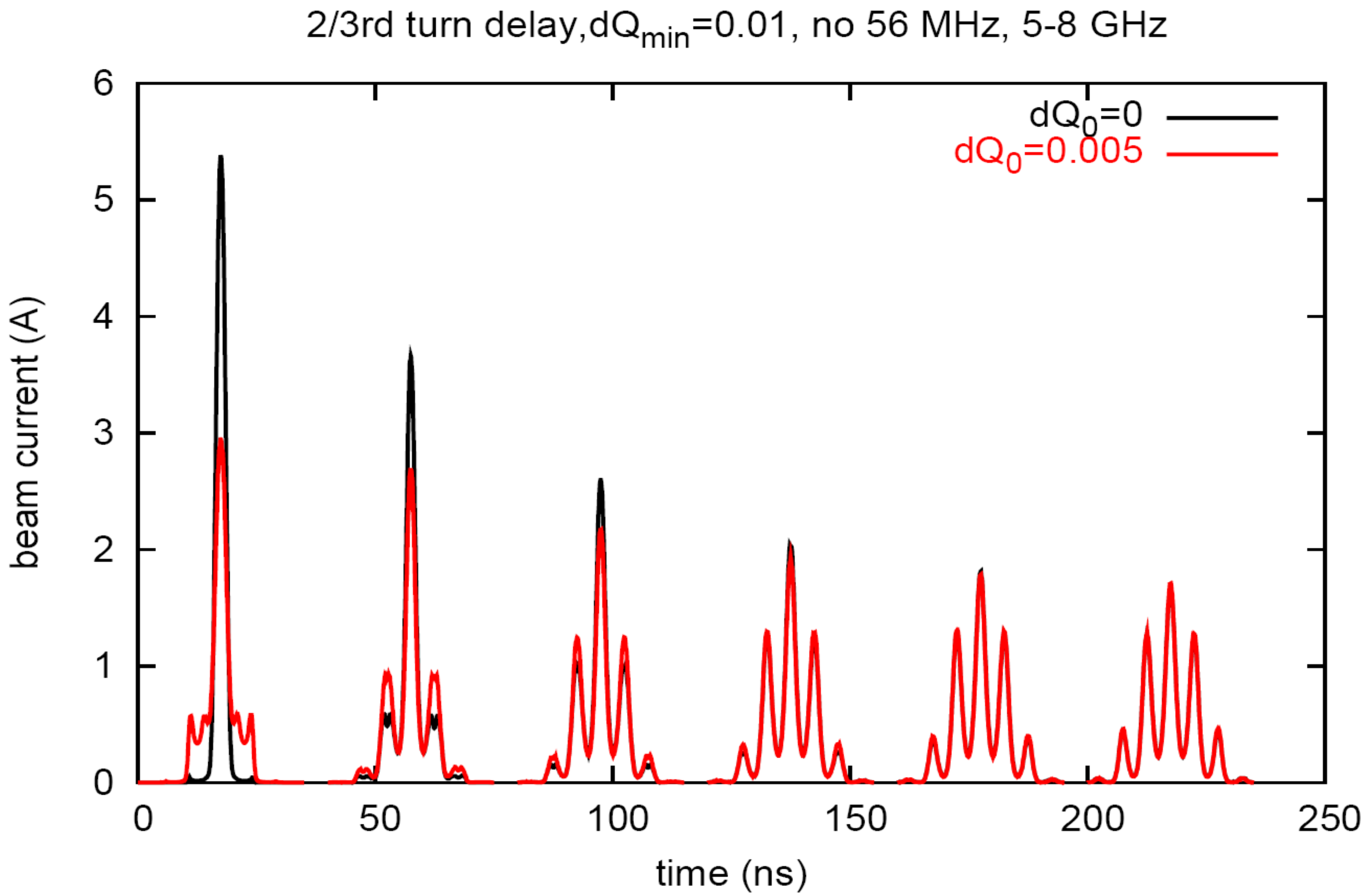
$$x = \hat{x} \exp(-i\varpi t + \lambda t)$$

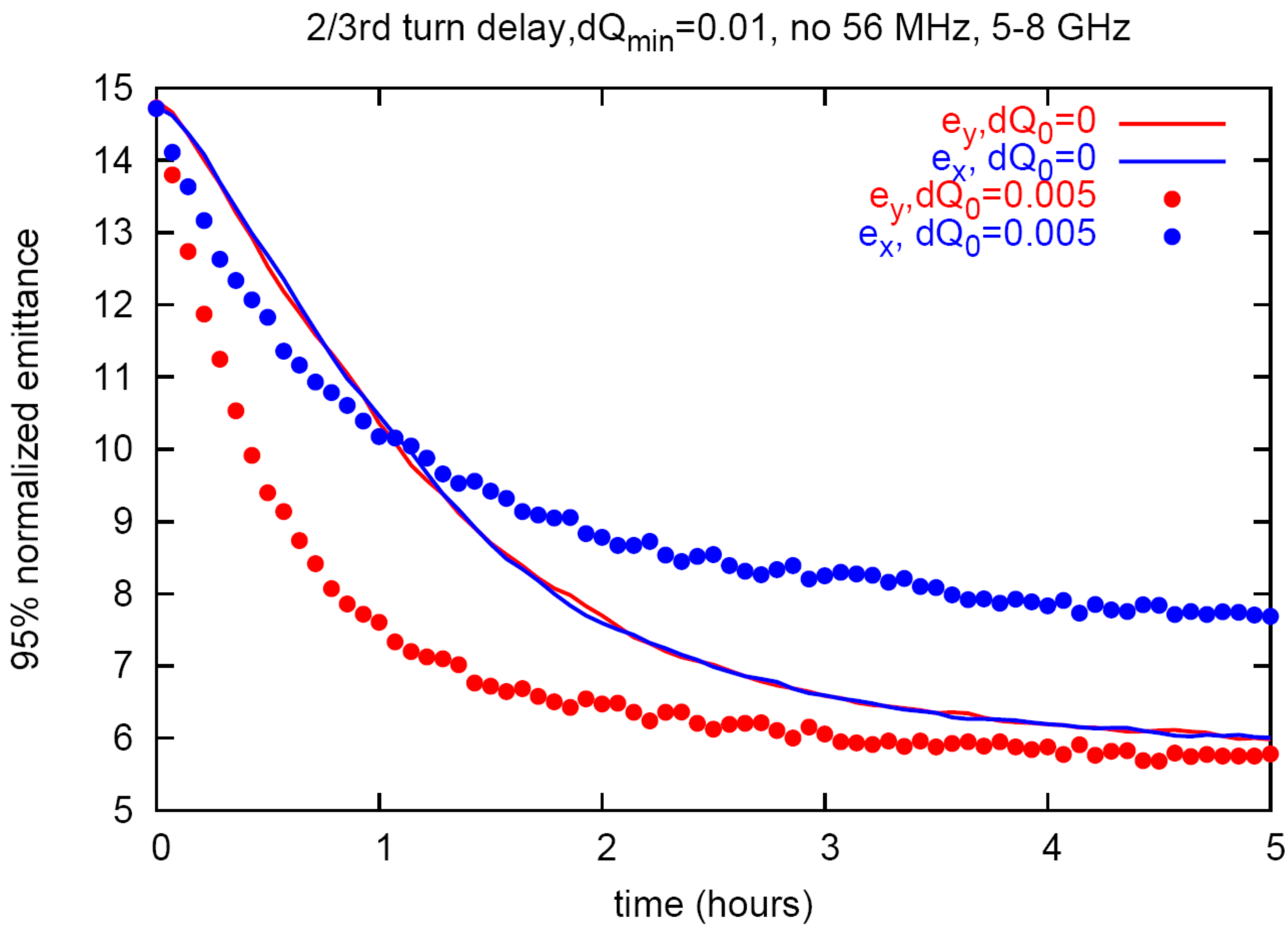
$$|\lambda| << \varpi$$



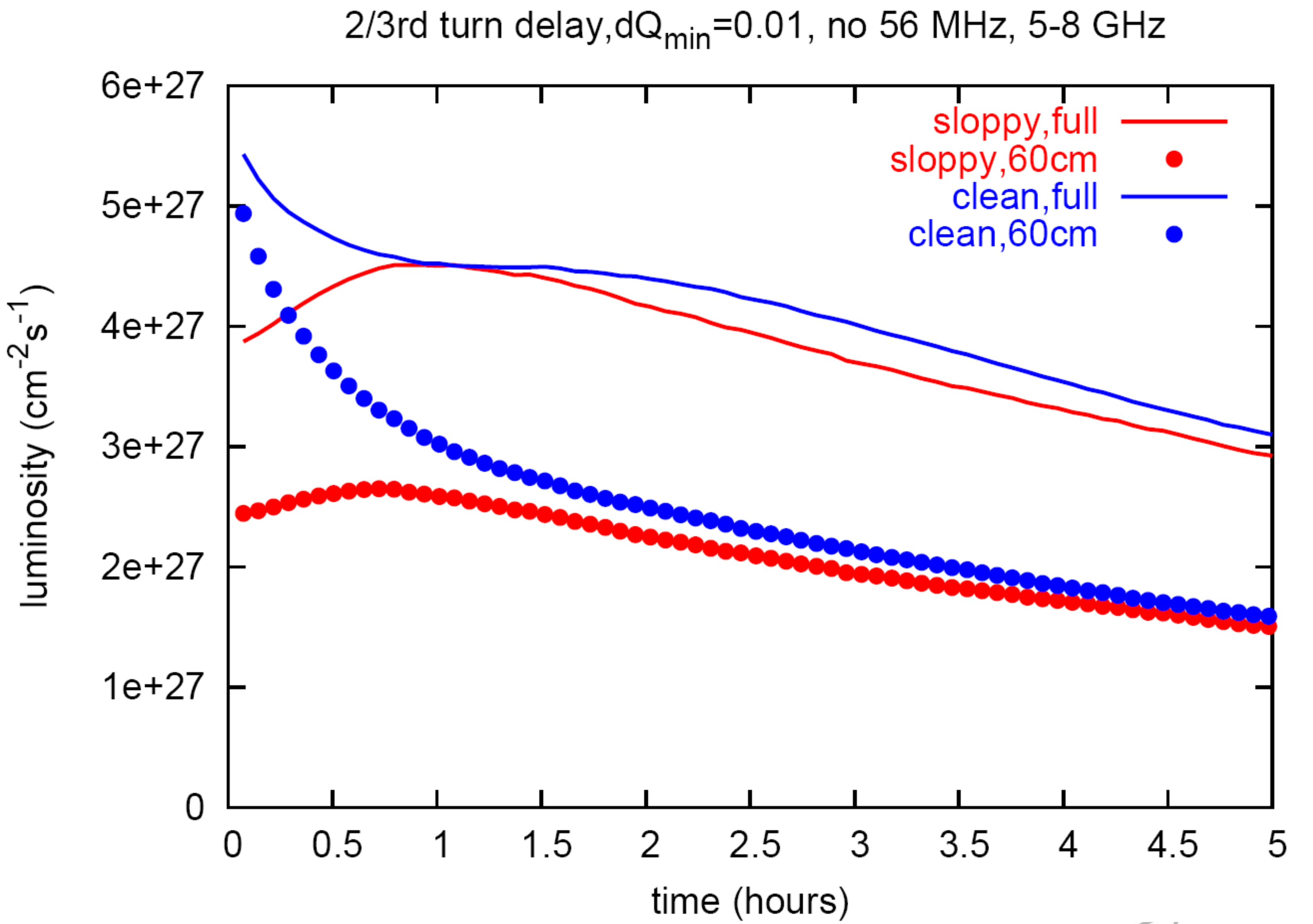
$$\lambda = \frac{-\beta}{4} \pm \left[\frac{\beta^2}{16} - \frac{q^2 + 4\delta^2}{4} - i\frac{\beta\delta}{2} \right]^{1/2}$$

Vertical cooling only (next fall in yellow)

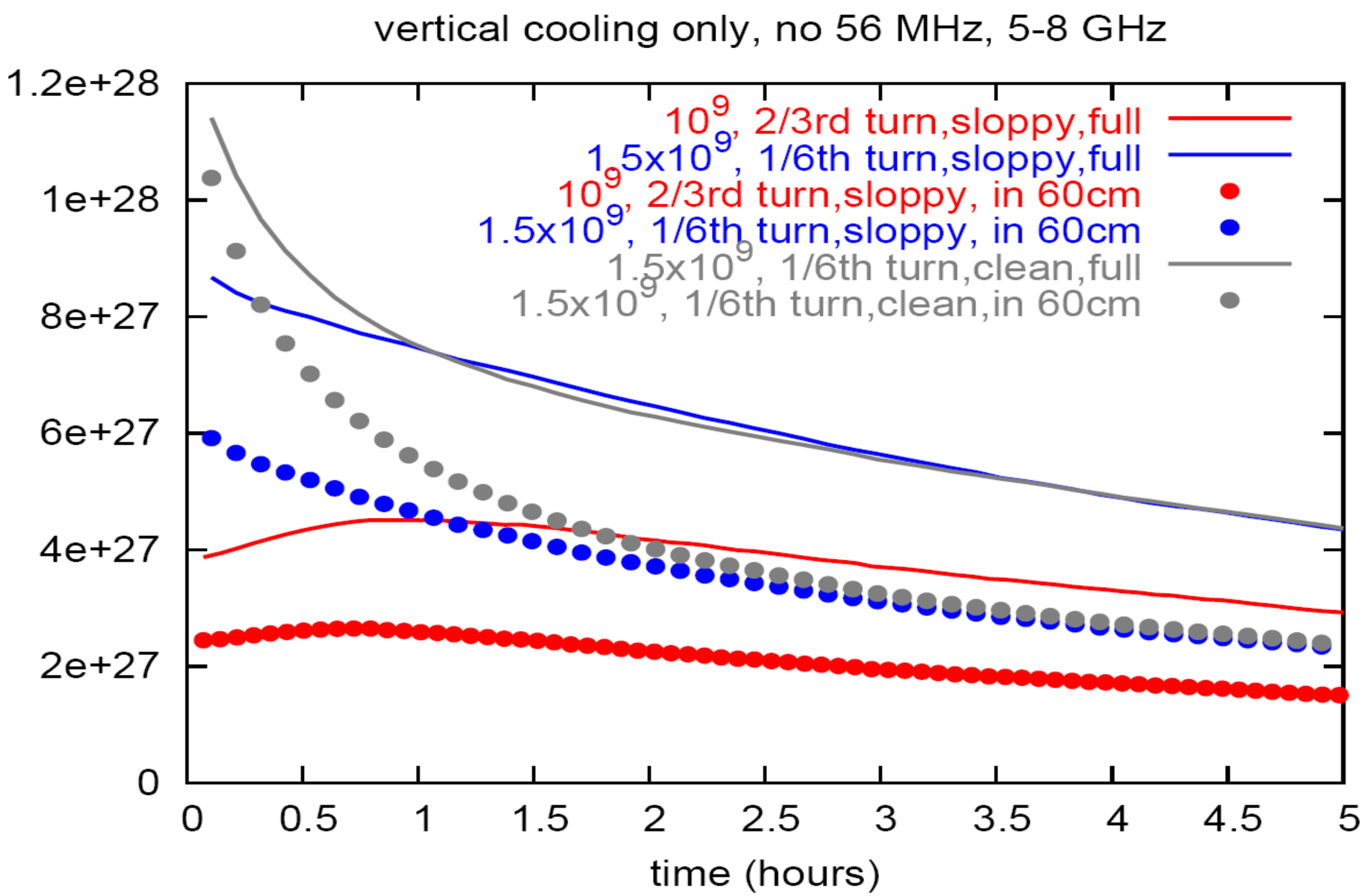



2/3rd turn delay,dQmin=0.01, no 56 MHz, 5-8 GHz
95% normalized emittance
time (hours)
ey,dQ0=0
ex, dQ0=0
ey,dQ0=0.005
ex, dQ0=0.005
15
14
13
12
11
10
9
8
7
6
5
0
1
2
3
4
5

Identical systems in both rings



Higher beam intensity and clean rebucketing



Conclusions

For ion beams in RHIC

1) Longitudinal stochastic cooling worked.
2) Lifetime was improved.
3) Simulations show reasonable agreement with data.
4) Transverse cooling looks straightforward.
5) Expect a big payoff from transverse cooling.