# Challenges and advantages of Fixed Field Alternating-Gradient (FFAG) accelerators

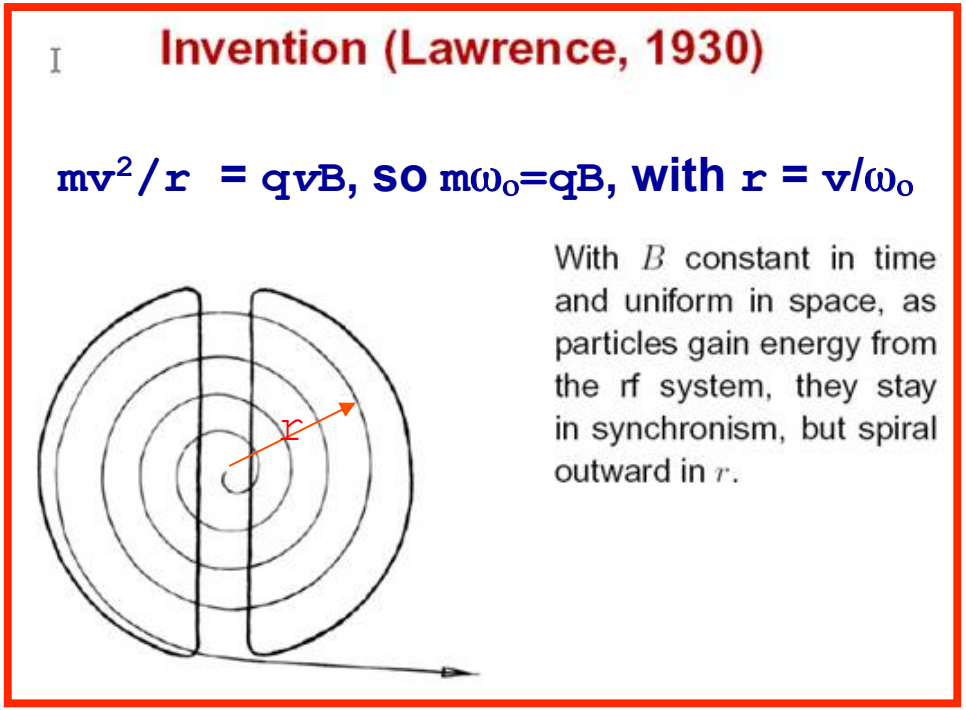
Dejan Trbojevic

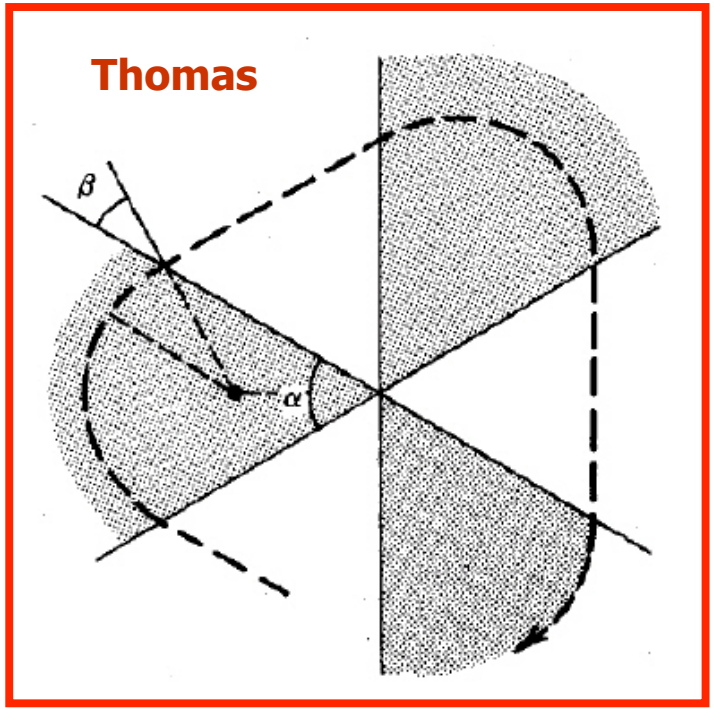
- **Introduction**
  - History of accelerators: Cyclotrons, Scaling FFAG, why synchrotrons won?
  - Why today one can build FFAG easier – TOSCA, magnet design
  - Where does the basic idea of non-scaling FFAG come from?
- **Non-scaling FFAG basic properties:**
  - Very strong focusing, small dispersion function, tune varies with energy, time of flight is parabolic function of energy, fixed gradient $B(x)=Bo+G\cdot x$, maximum energy range $\delta p/p=\pm 50\text{-}60$ %.
- **Tools? Correct Hamiltonian: PTC, TEAPOT**
  - Two old masters: Ernest Courant and All Garren were just smilling: only their SYNCH passed the test.
- **Fresh from the press:** few recent examples from different applications:, electron acceleration, muon acceleration, proton therapy, Carbon/proton gantry, proton drivers, heavy ions (RIA) ect.
- **Advantages and problems of the non-scaling FFAG's:**
  - Tunes vary – in many turns applications 0.3 to be avoided.
  - Magnetic field at the end of the magnets.

# A little bit of accelerator history:

**[Slides from the Rick Baartman presentation: 'Cyclotrons: Classic to FFAG' – 2004]**

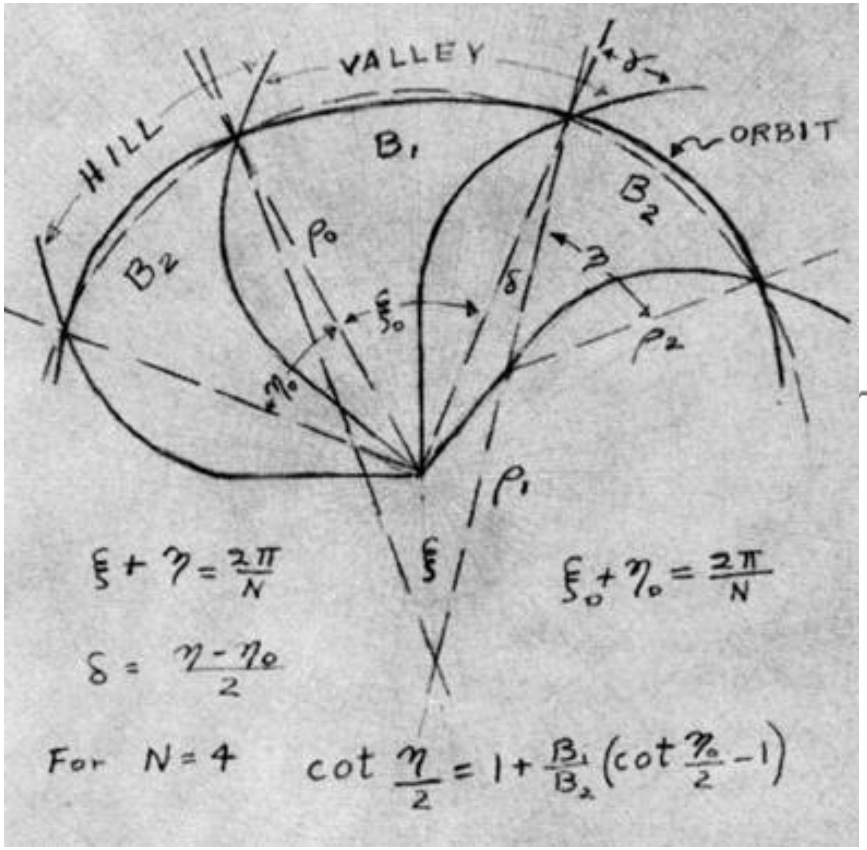
***FF but not FFAG***

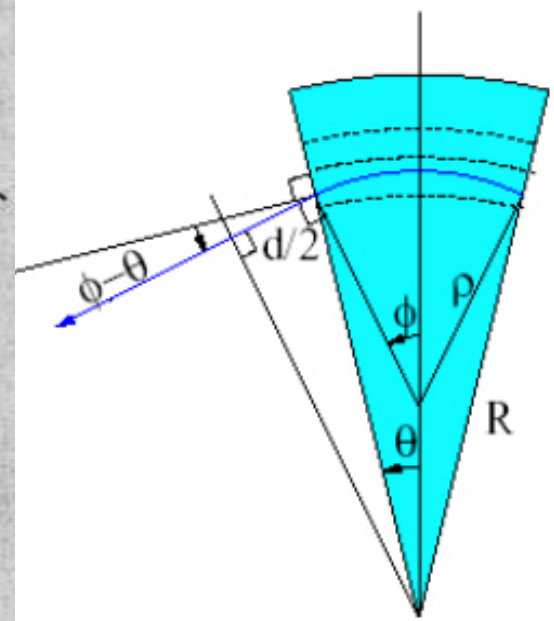



**They are also FF (Fixed Field) with weak focusing – no opposite bend but not AG (Alternating Gradient) with strong focusing –with opposite bend.**

# From CYCLOTRONS to FFAG's:

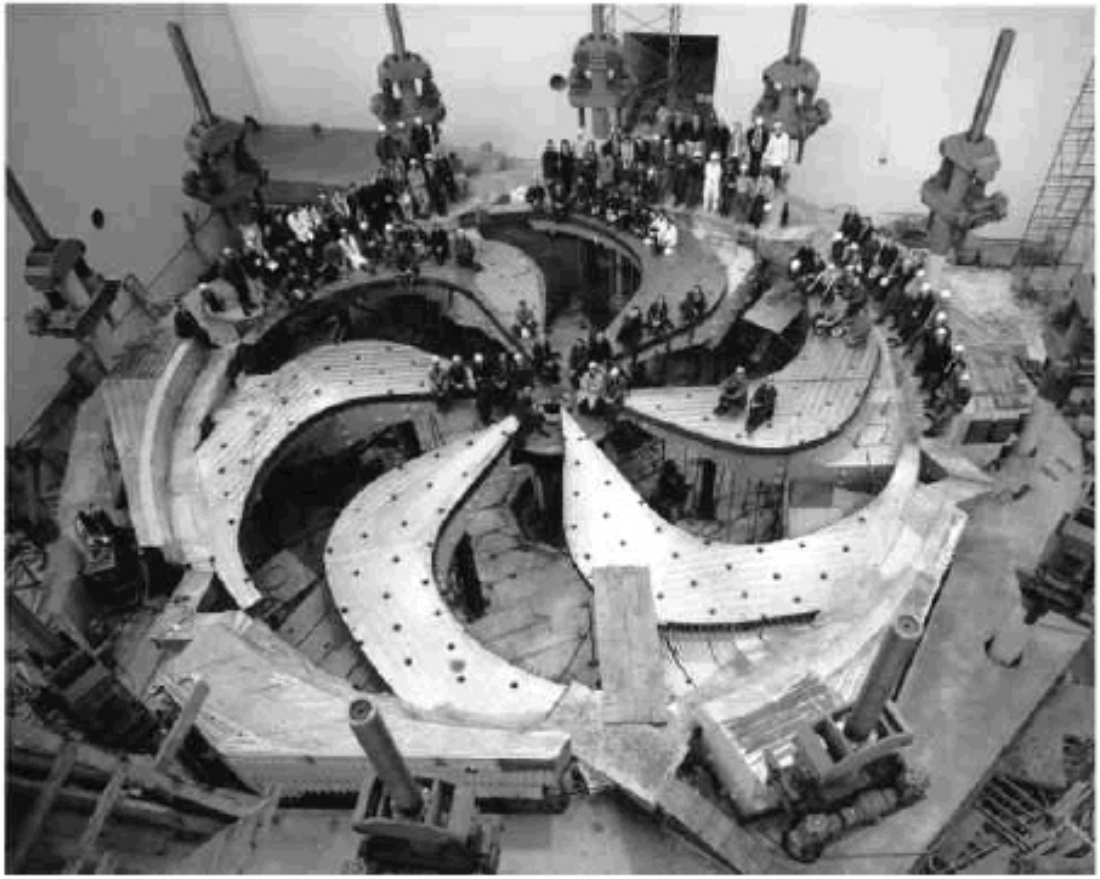



$$\frac{\sin(\theta)}{\rho} = \frac{\sin(\phi)}{R}$$

$$d/2 = R\sin(\phi-\theta)$$

# Example: TRIUMF cyclotron



| Energy | $R$ | $\beta\gamma$ | $\xi$ | $1+2\tan^2\xi$ | $F^2$ | $\nu_z$ |
|---|---|---|---|---|---|---|
| 100 MeV | 175 in. | 0.47 | 0° | 0.0 | 0.30 | 0.28 |
| 250 MeV | 251 in. | 0.78 | 47° | 3.3 | 0.20 | 0.24 |
| 505 MeV | 311 in. | 1.17 | 72° | 20.0 | 0.07 | 0.24 |

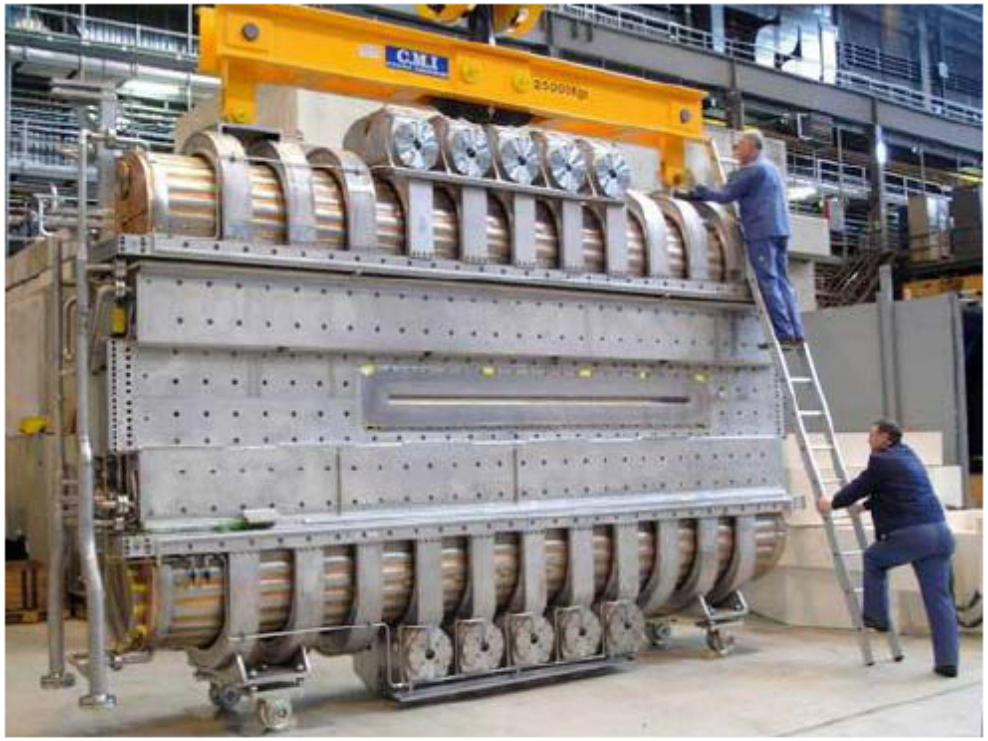
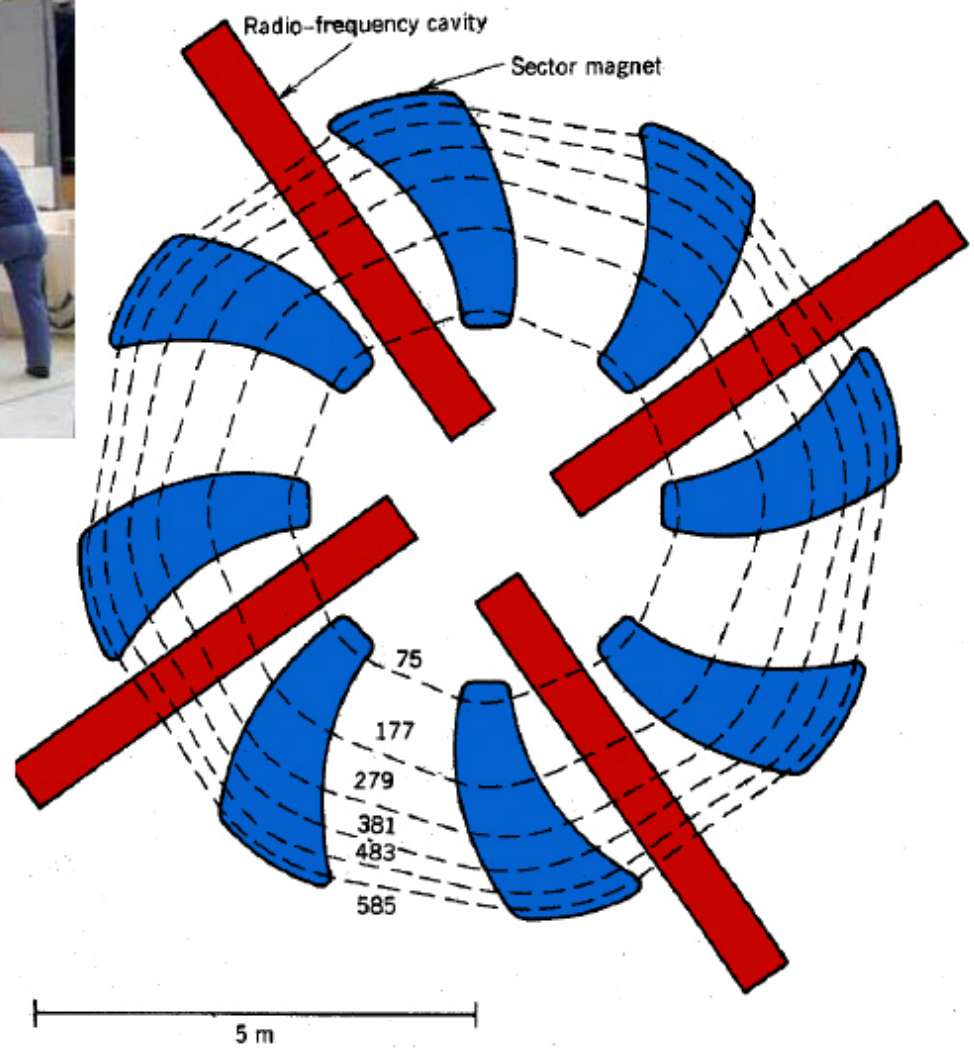


Fig. 7: New copper resonator for the Ring Cyclotron. 7.6m.

PSI – (Paul Scherrer Institute)
CYCLOTRON 4.5 meter outer orbit
Compared to TRIUMF 7.6 m




**Presently runs at 590 MeV**
**with energy on target of 1.2 MW**
**2 mA in CW mode**

Upgrade to **1.8 MW** at the beam energy of 590 MeV

– Typeset by FoilTEX –

# The first SCALING FFAG

**MURA-KRS-6** Phys.Rev. **103,** 1837 (1956)
**November 12, 1954**
**K. R. Symon: The FFAG SYNCHROTRON – MARK I**

**This is why FFAG had lost:** $\Delta r \sim 60\text{-}100$ cm

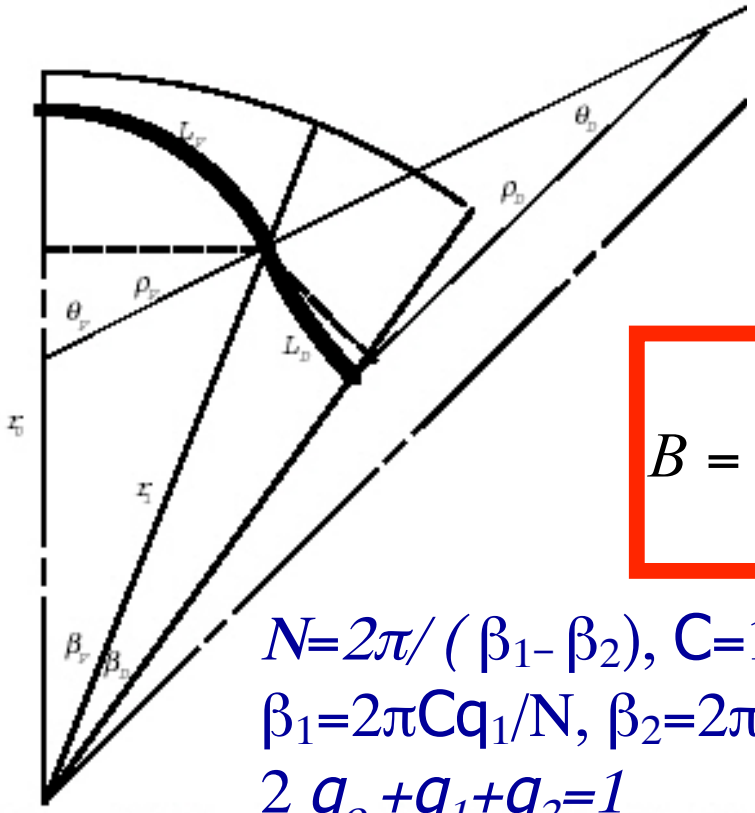


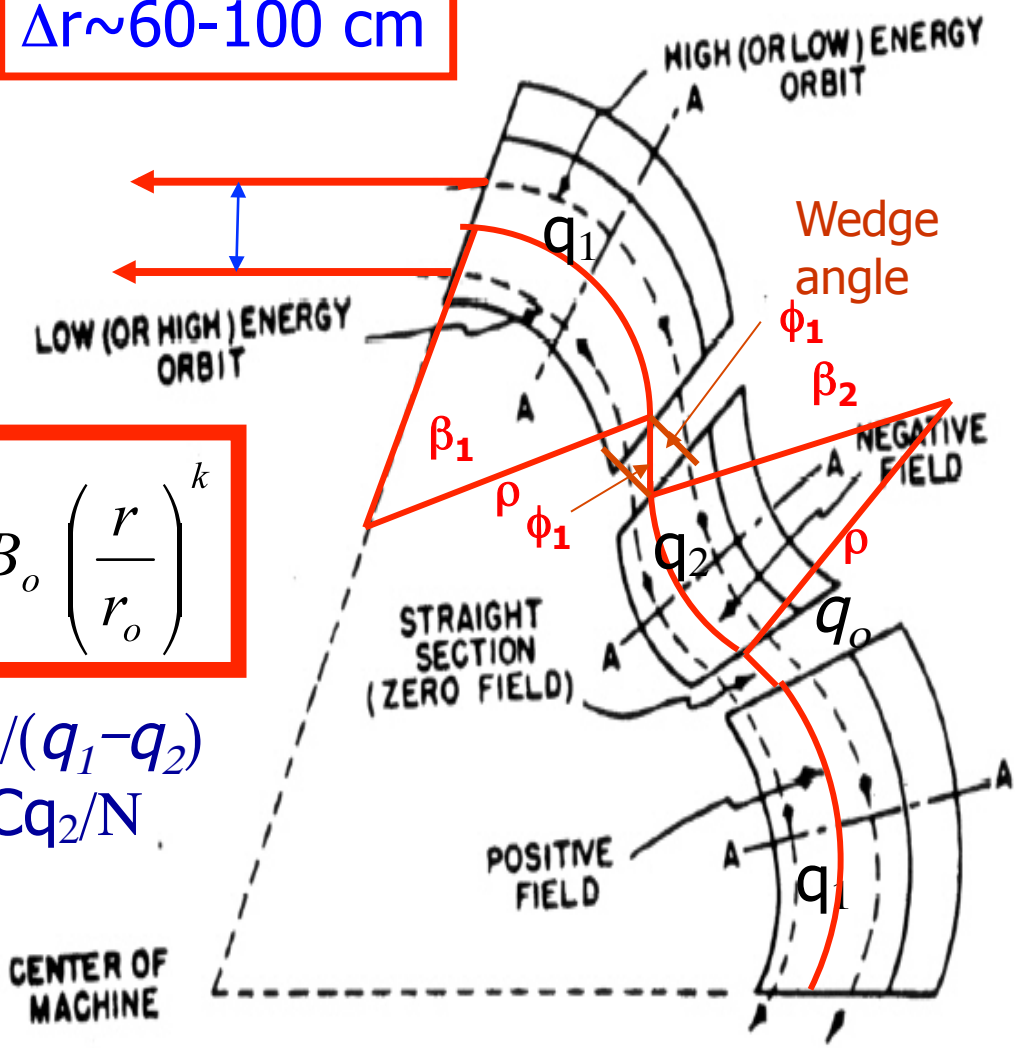
Figure 1: Closed orbit of a triplet focusing FF half cell; a half of F magnet, D magnet of one half straight section, is depicted.

$$B = B_o \left( \frac{r}{r_o} \right)^k$$

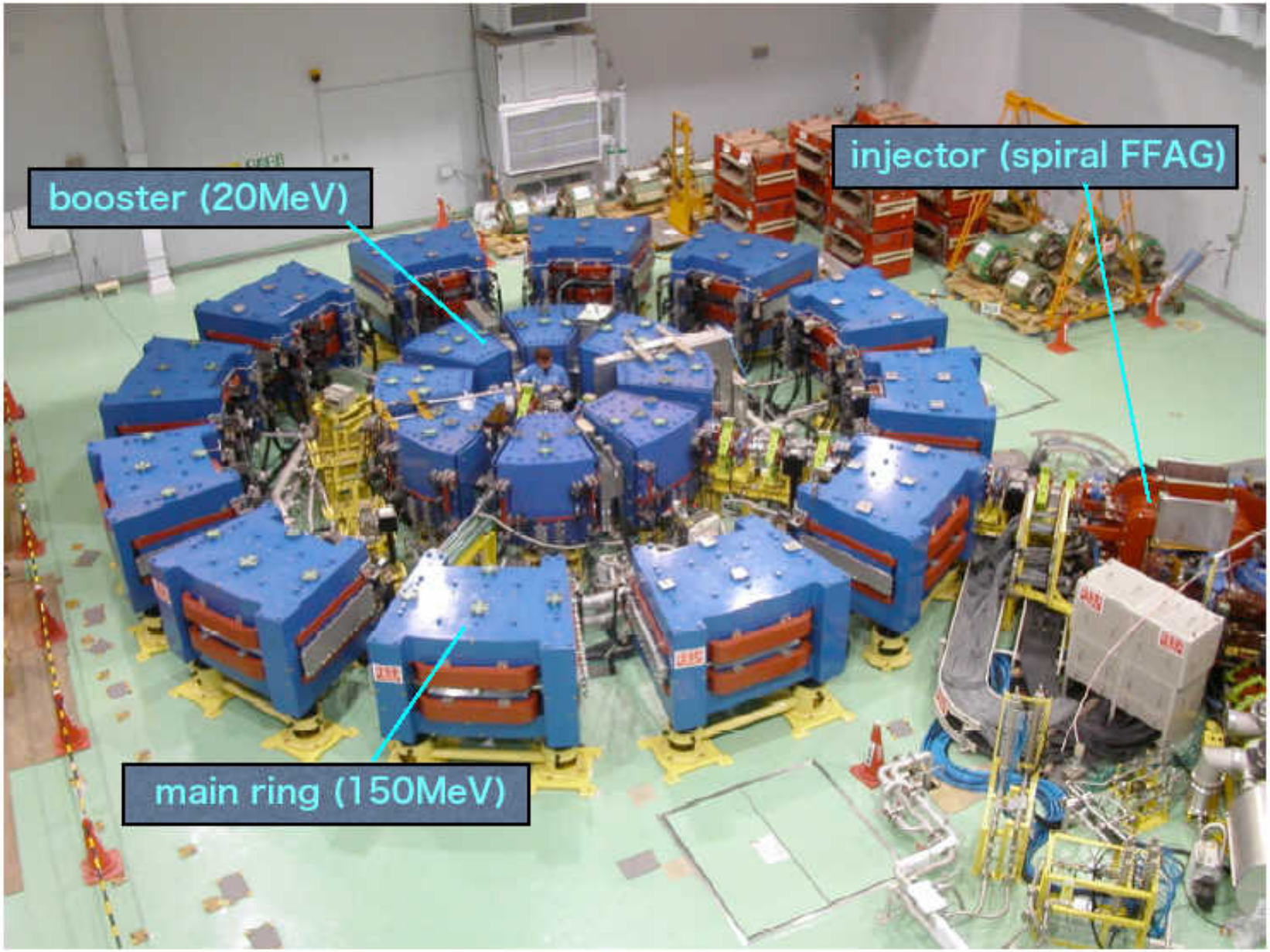
$N = 2\pi / (\beta_1 - \beta_2)$, $C = 1/(q_1 - q_2)$

$\beta_1 = 2\pi C q_1 / N$, $\beta_2 = 2\pi C q_2 / N$

$2 q_o + q_1 + q_2 = 1$



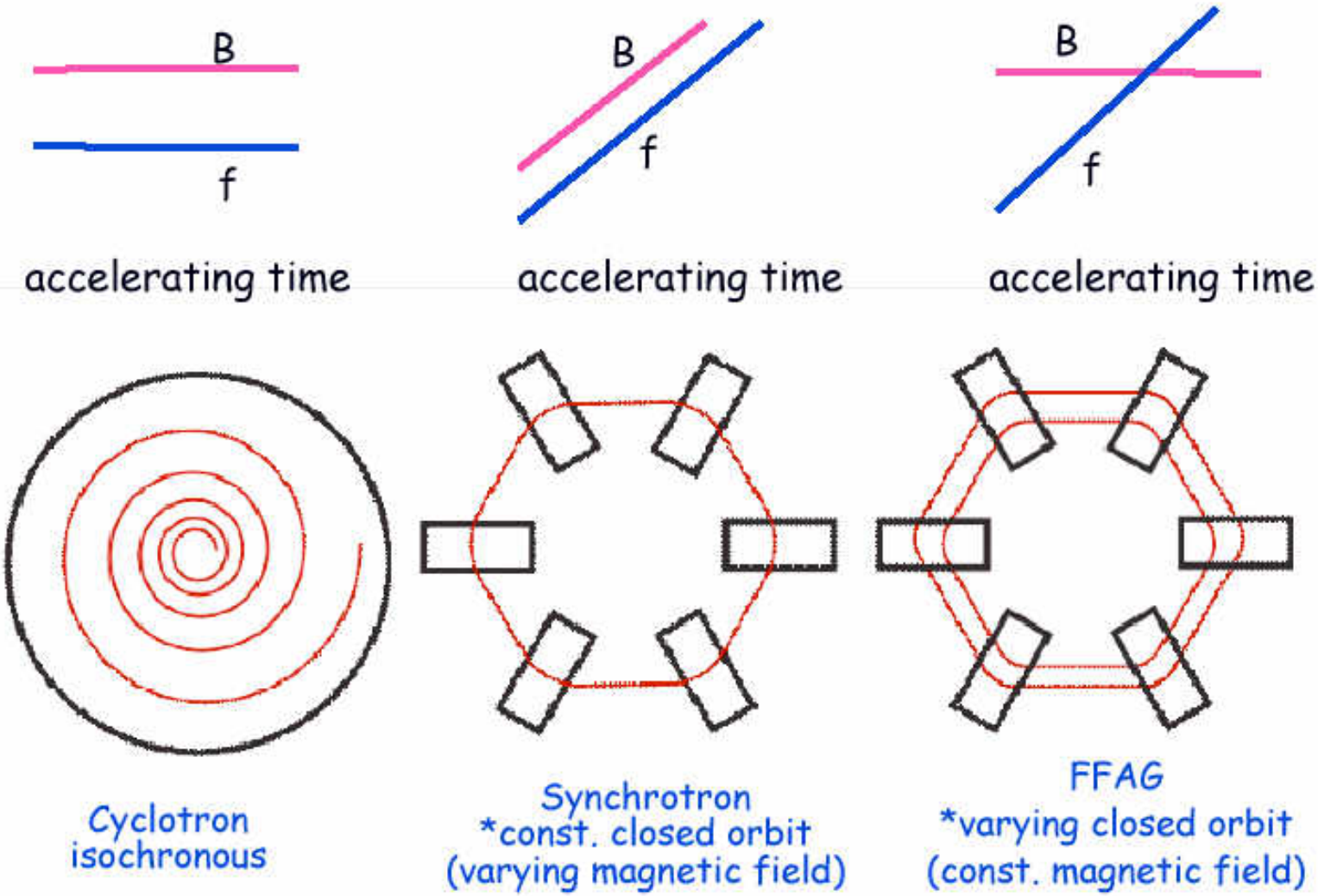
# FFAG complex at KURRI



# 150 MeV FFAG at KEK

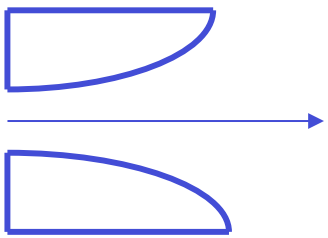
Yoshimaru Mori – EPAC 06 presentation

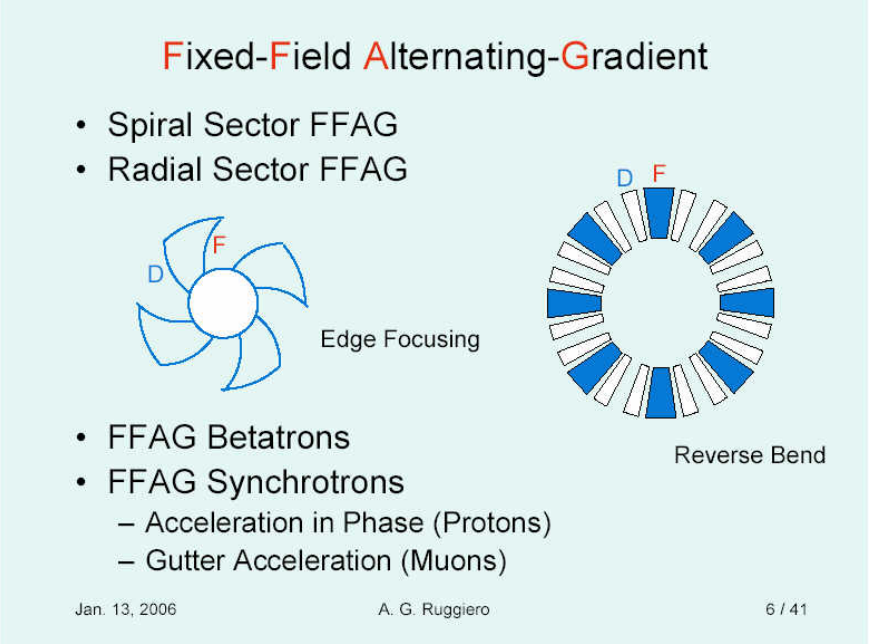
## Comparison among Cyclotron, Synchrotron and FFAG



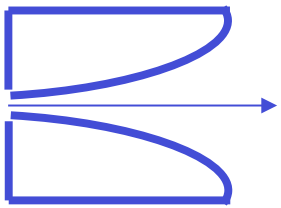
# Scaling FFAG – Non scaling FFAG

$B = B_o(r/r_o)^k$






$B = Bo + r\,G_o$



**Scaling FFAG properties:**

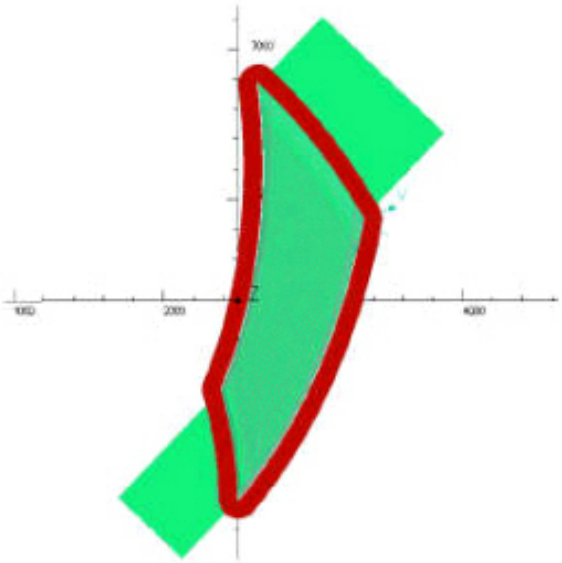
- **Zero chromaticity.**
- **Orbits parallel for different $\delta p/p$**
- **Relatively large circumference.**
- **Relatively large physical aperture (80 cm – 120 cm).**
- **RF - large aperture**
- **Tunes are fixed for all energies.**
- **Negative momentum compaction.**
- **$B = B_o(r/r_o)^k$ non-linear field**

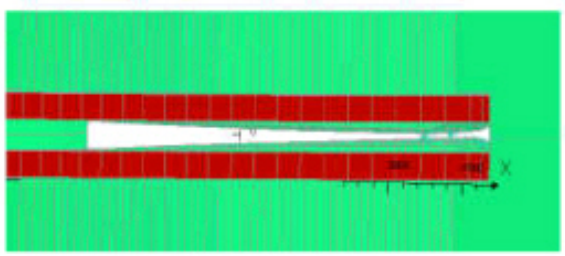
**Non-Scaling FFAG properties:**

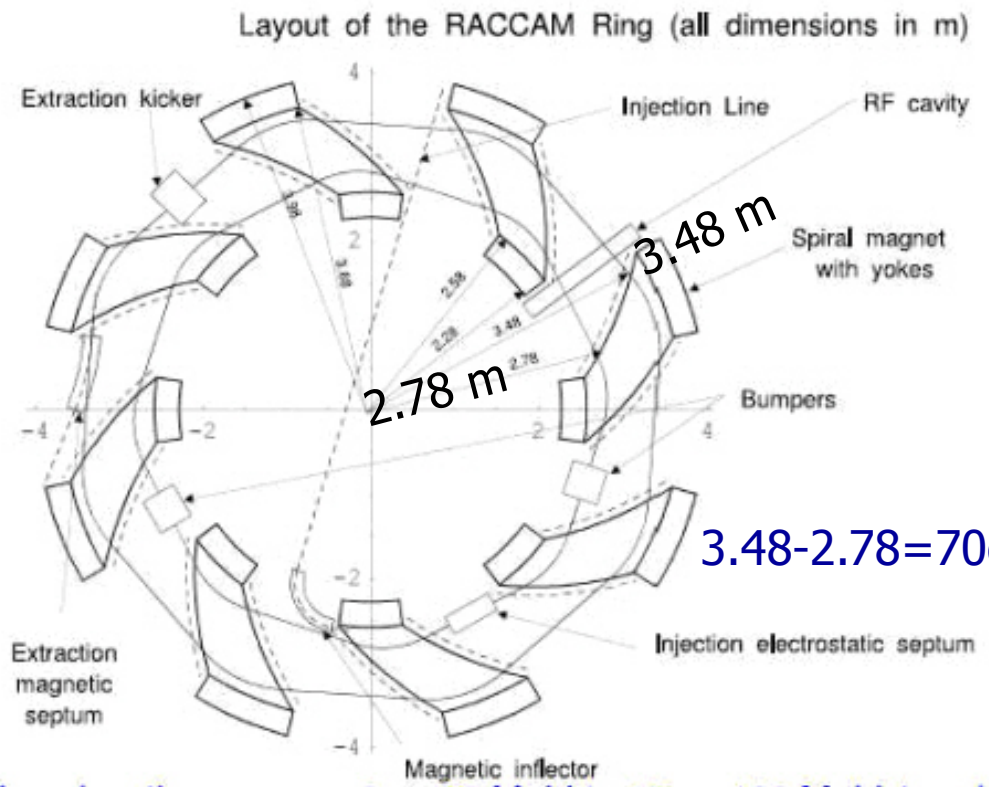
- **Chromaticity is changing.**
- **Orbits are not parallel.**
- **Relatively small circumference.**
- **Relatively small physical aperture (0.50 cm – 10 cm).**
- **RF - smaller aperture.**
- **Tunes move 0.4-0.1 in basic cell.**
- **Momentum compaction changes.**
- **$B = Bo + x\,G_o$ linear field**

# Spiral scaling FFAG magnets

$$B(r,\theta) = B_0 \times (r/r_0)^k \times flutter(r,\theta)$$








3.48-2.78=70cm

Acceleration range : $6 \rightarrow 70$ MeV to $17 \rightarrow 180$ MeV, tunable

Prospects for bunch to bunch energy variation

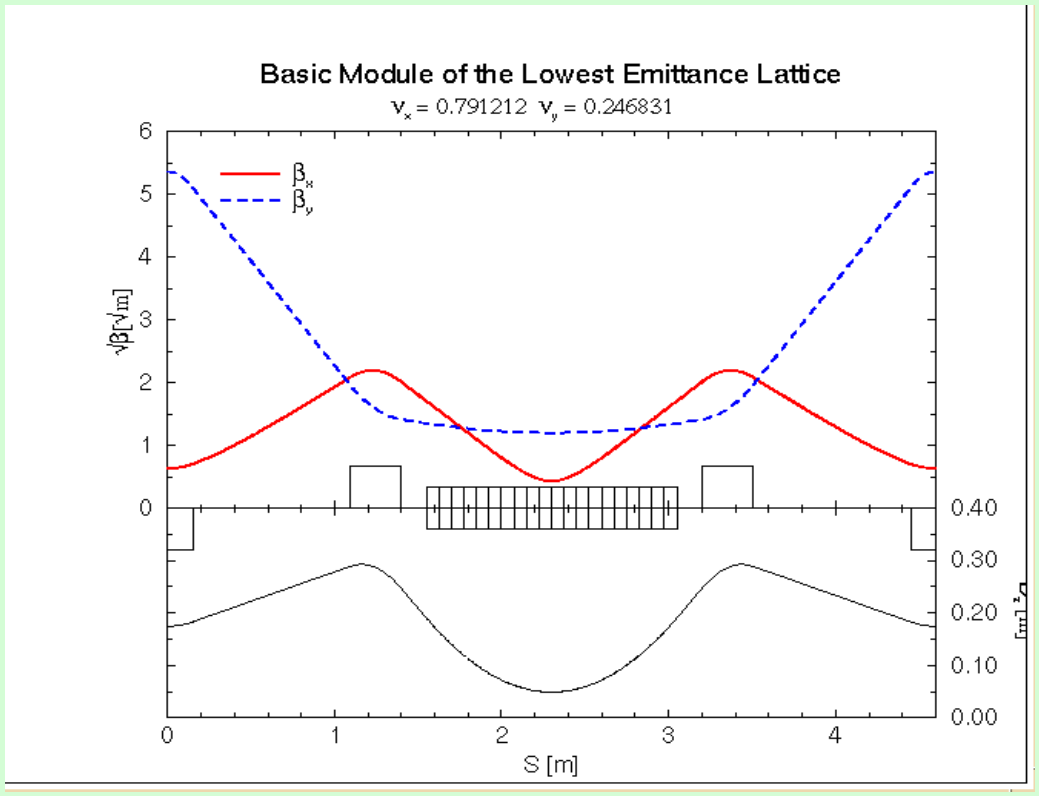
Dejan Trbojevic
September 30, 1999
Montauk, Long Island
Muon Collider Workshop

**FFAG Lattice Presented as a Trigonometrical Problem**

- **Introduction**
- **Why the K.R. Symon FFAG proposal finally got the attention after 44 years (1954)??**
- **Basic Parameters - trigonometry**
- **An Example**
- **Possible Improvements**

## The minimum emittance lattice:

- The minimum emittance lattice requires reduction of the function H:
  - The normalized dispersion amplitude corresponds to the $\langle H \rangle^{1/2}$
  - Conditions are for the minimum of the betatron function $\beta_x$ and dispersion function $D_x$ to have small values at the middle of the dipole (combined function dipole makes it even smaller).

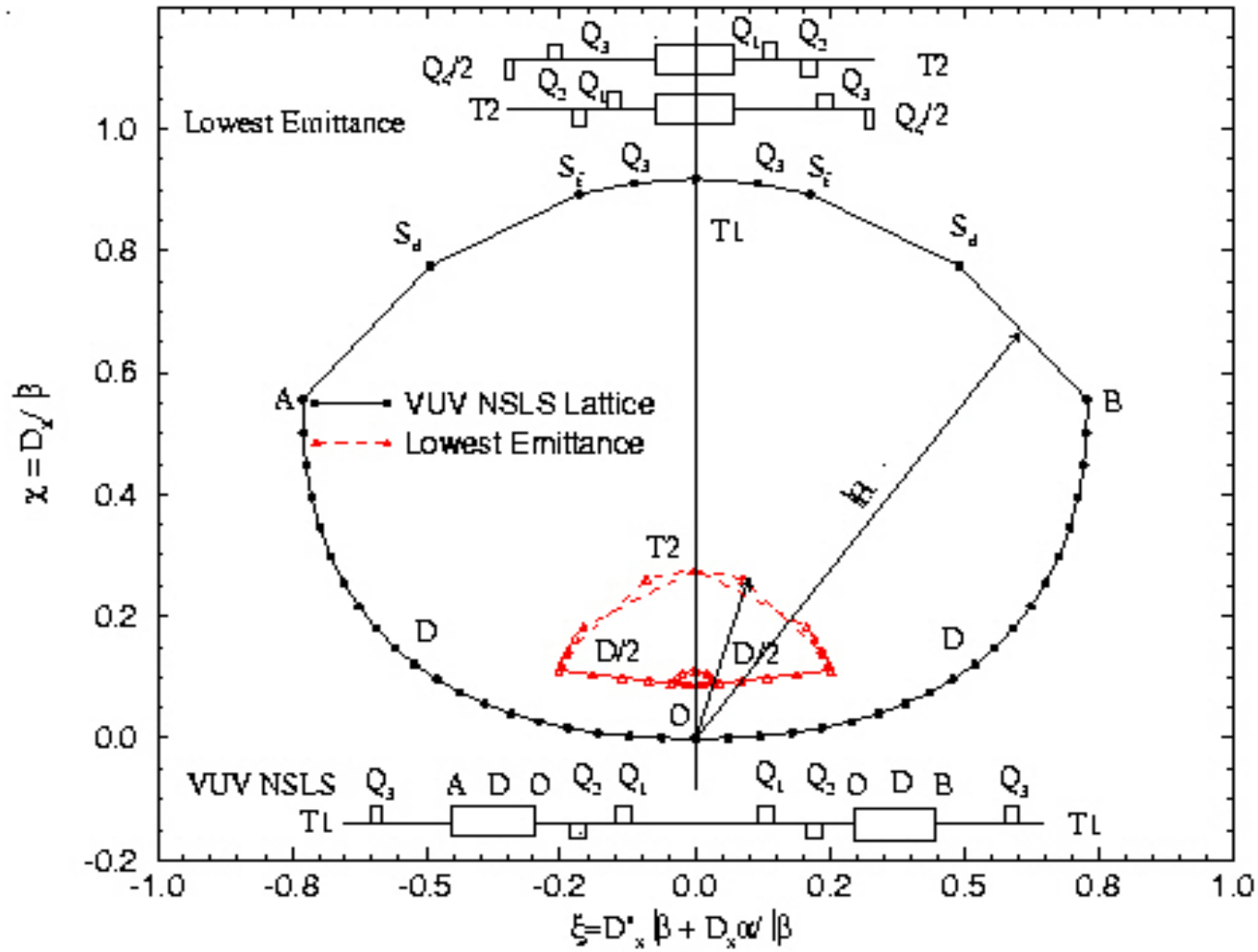



$$\beta_{min} = Ld/2\sqrt{15}$$

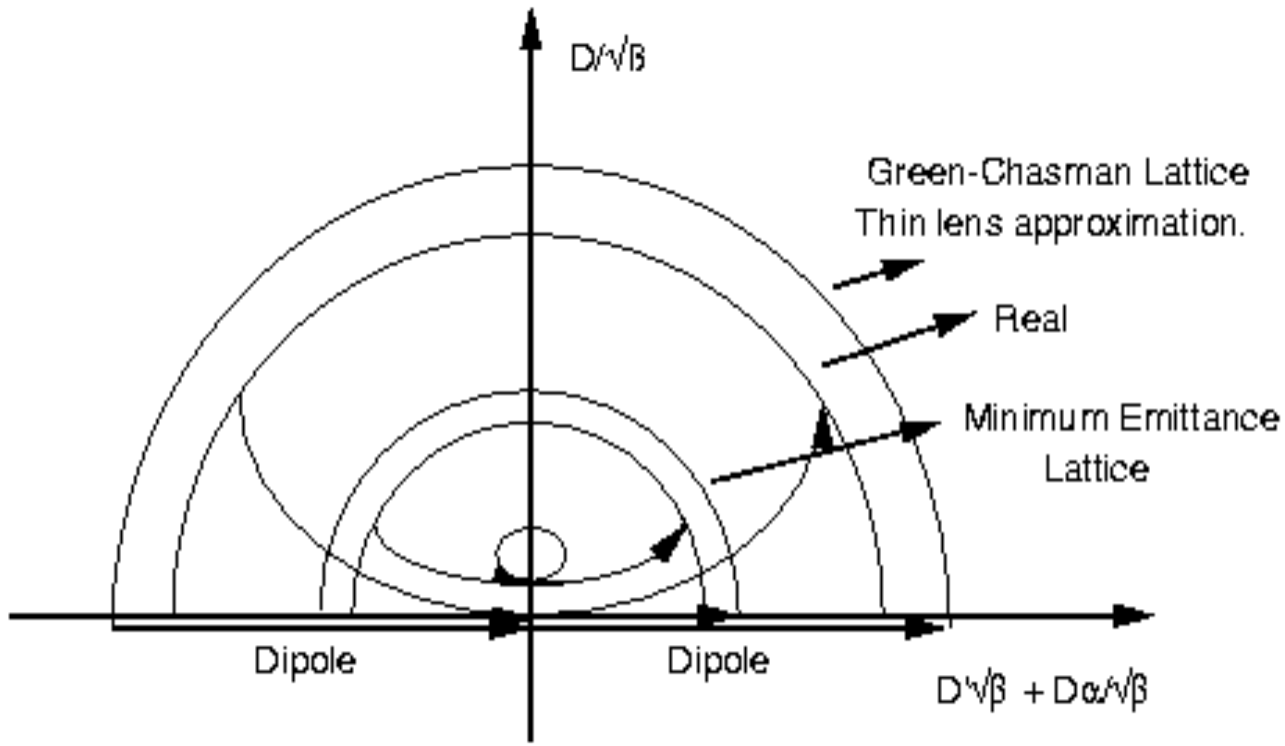
$$D_{xmin} = \theta * Ld/24$$

# NSLS present lattice in normalized dispersion space:

Fig.4 Low Emittance Lattices in the Normalized Dispersion
VUV-NSLS and the Lowest Emittance Lattice



# Green-Chasman lattice in normalized dispersion space:



# Non-scaling FFAG concept

- Orbit offsets are proportional to the dispersion function:

$$\Delta x = D_x * \delta p/p$$

- To reduce the orbit offsets to ±4 cm range, for momentum range of $\delta p/p \sim \pm 50\ \%$ the dispersion function $D_x$ has to be of the order of:

$$D_x \sim 4\ cm\ /\ 0.5 = 8\ cm$$

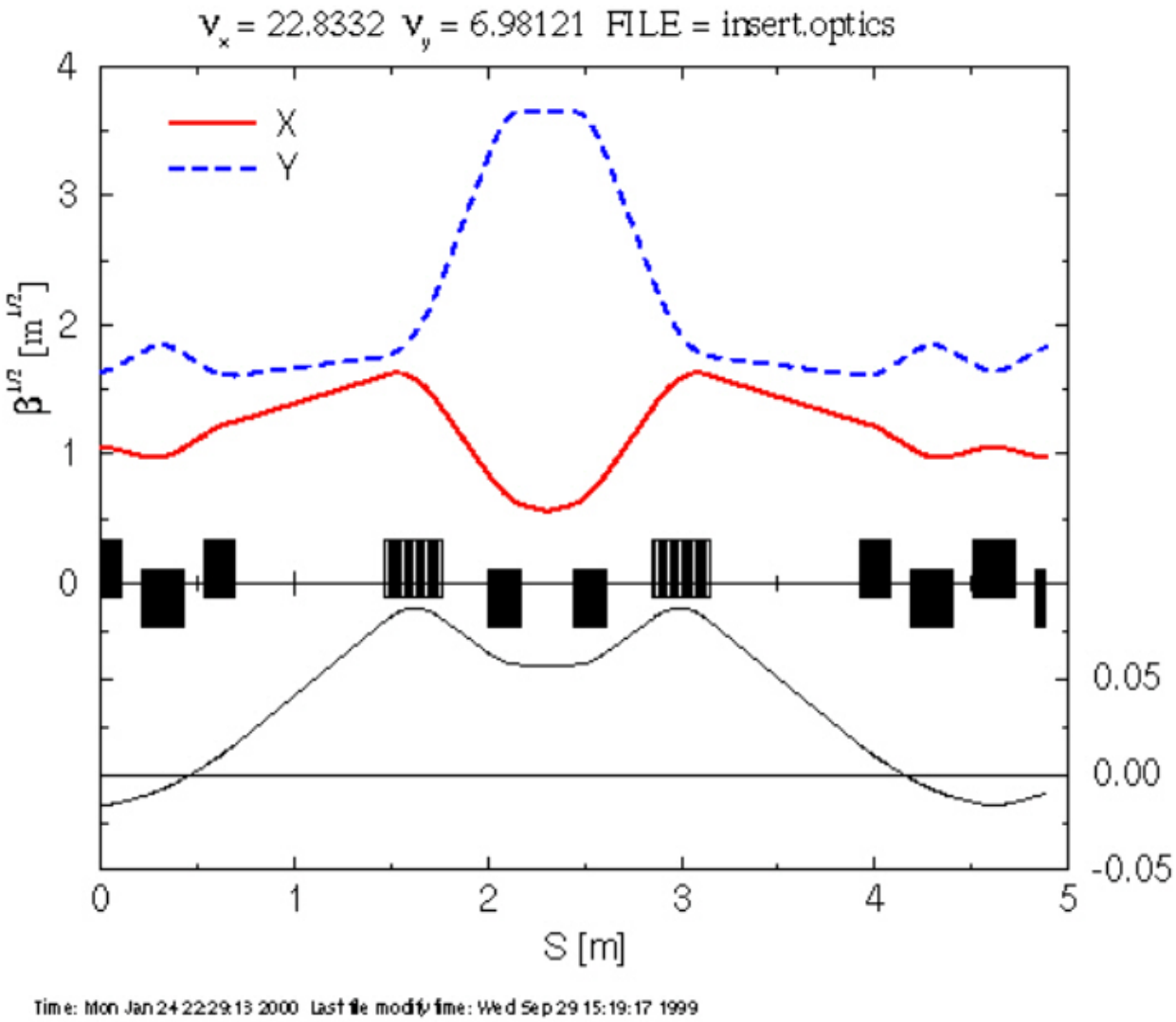
- The size and dependence of the dispersion function is best presented in the normalized space and by the *H* function:

$$\boxed{W} = D_x / \boxed{W}_x \quad \text{and} \quad \boxed{W} = D'_x \boxed{W}_x + \boxed{W}_x D_x \boxed{W}_x$$

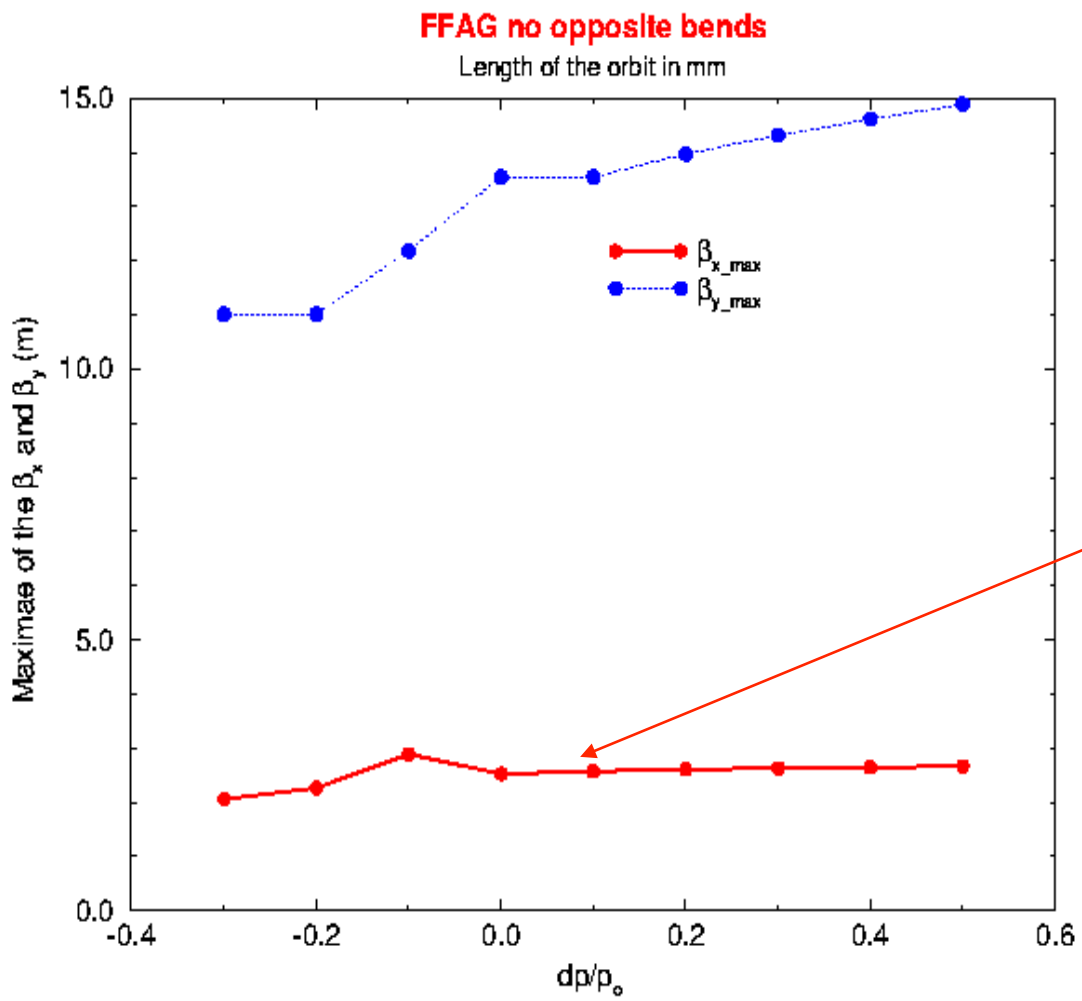
with *H:*

$$H = \boxed{W}^2 + \boxed{W}^2$$

# Our first attempt to make a non-scaling FFAG (1999)

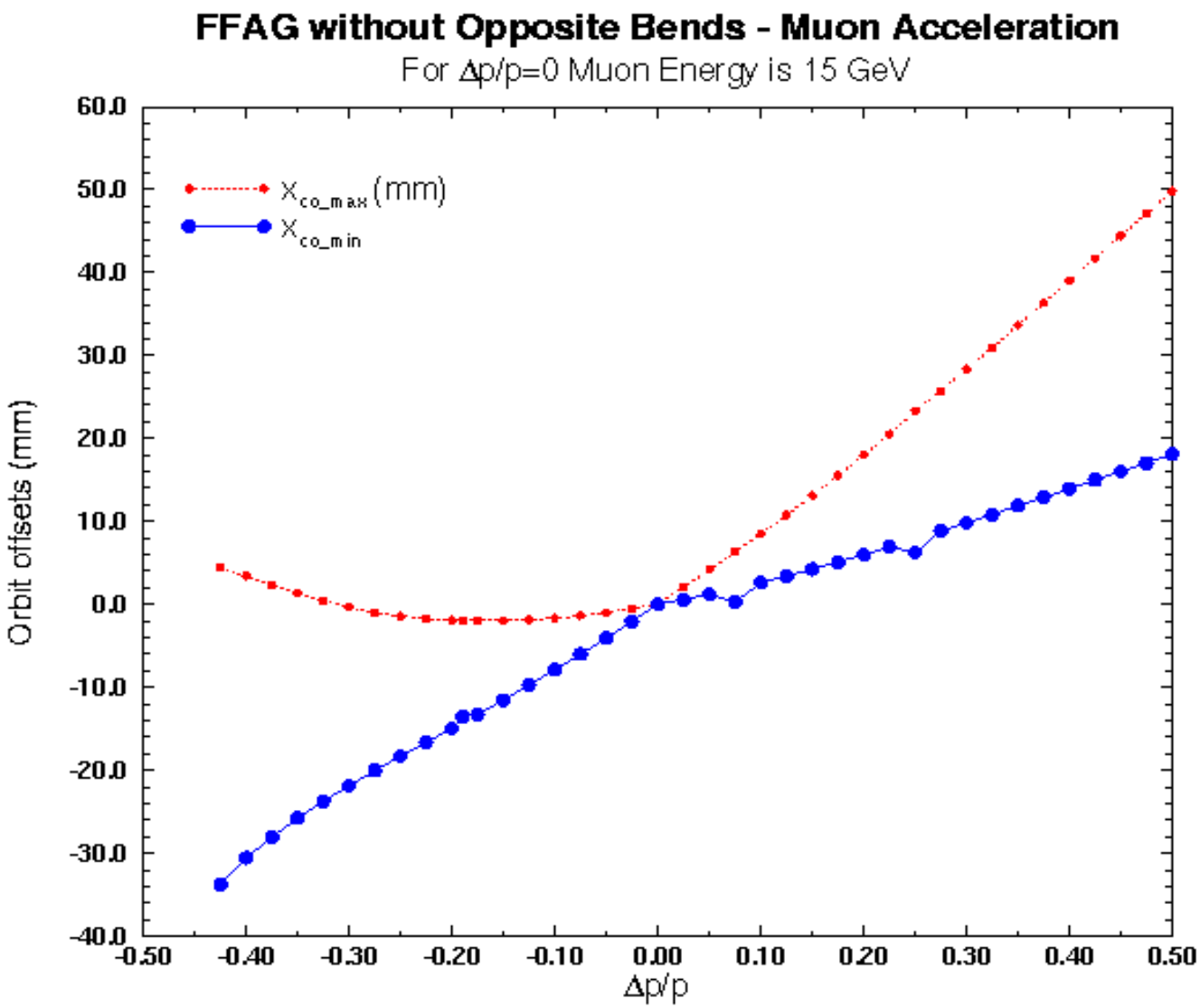


# Betatron functions dependence on momentum (Montauk 1999 example)

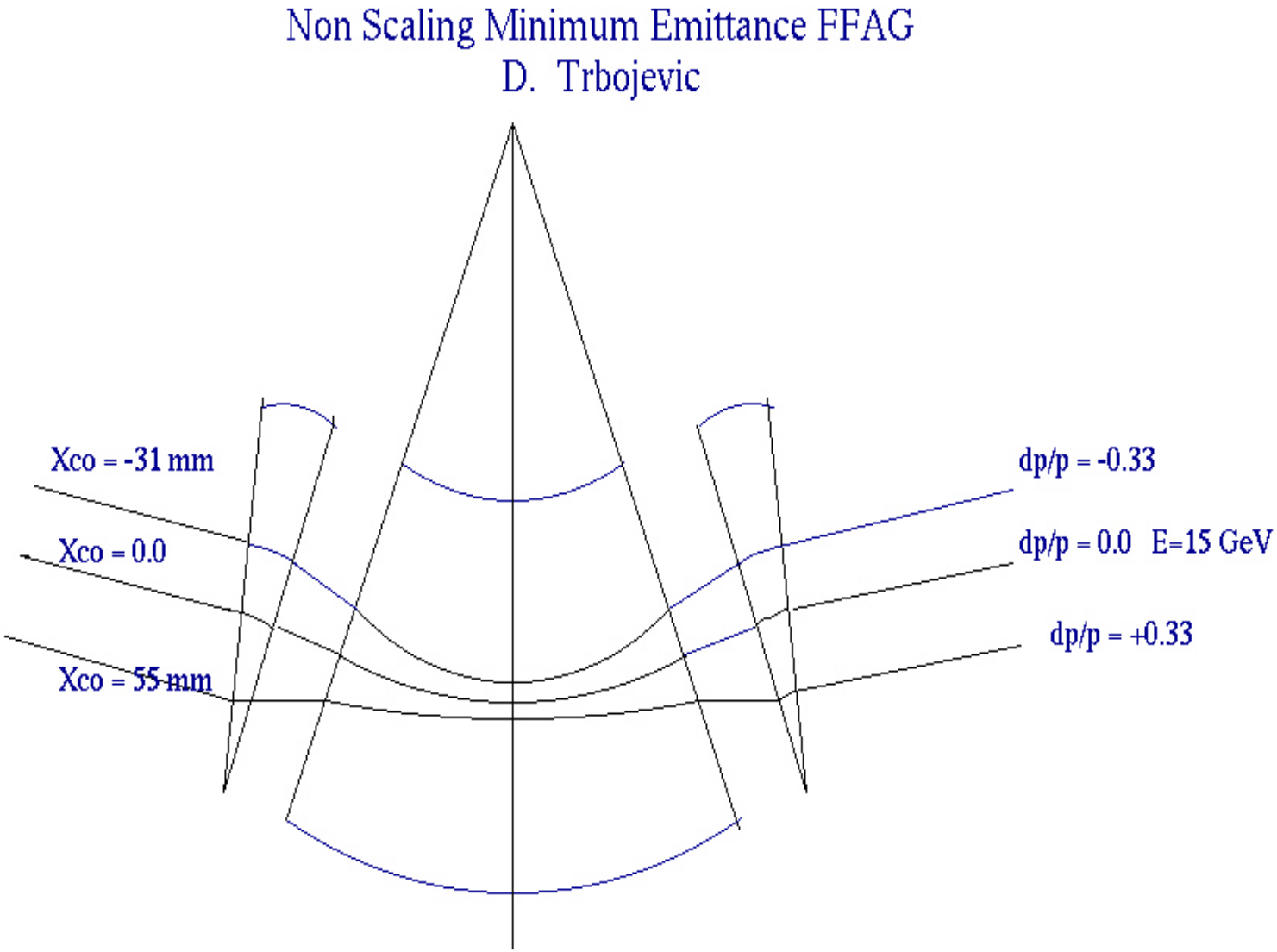



Extreme focusing
$\beta_x$=2.5 m!!!!

# Maximum momentum offsets in the Montauk example:



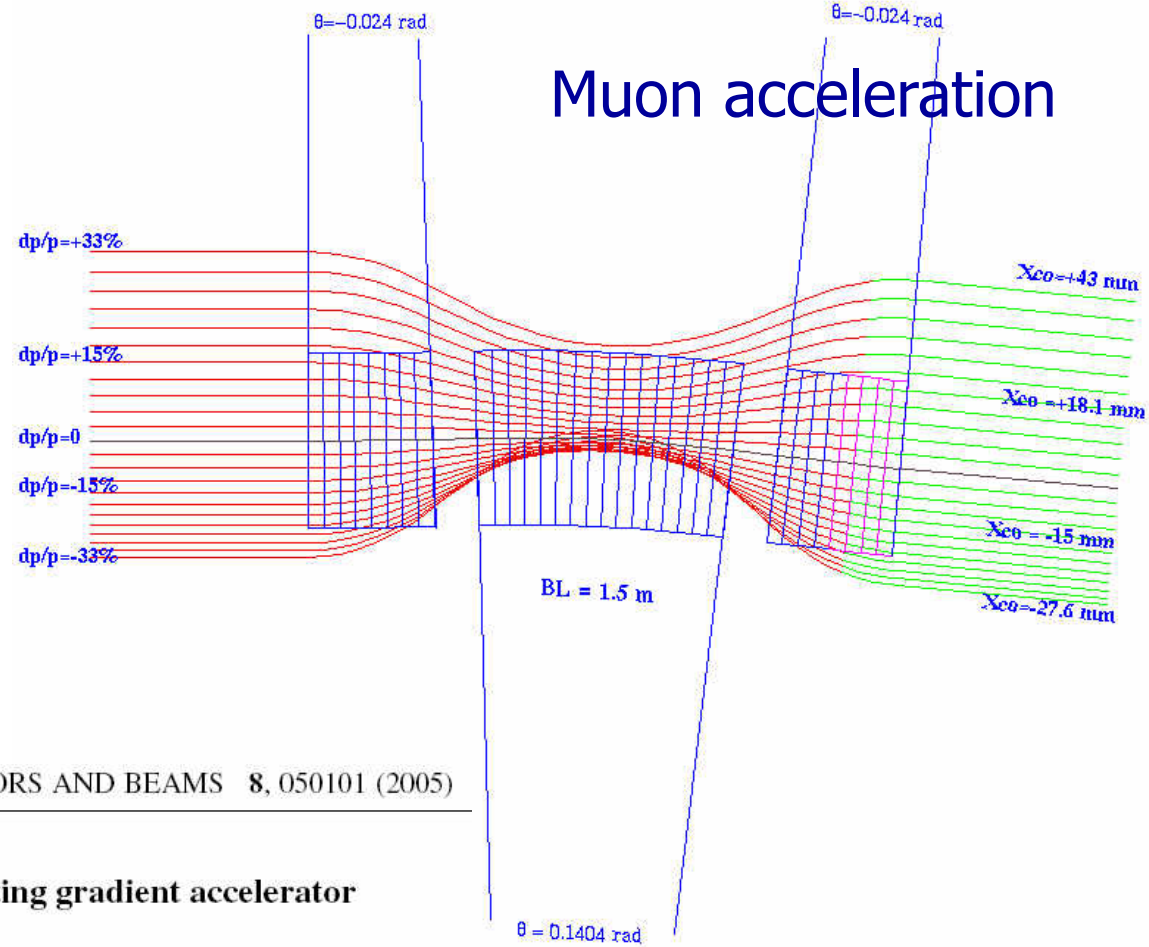
# Lattice got simplified with smaller number of magnets: just two kinds



# Basic Properties of the non-scaling FFAG

- Concept introduced 1999 at Montauk meeting –Trbojevic, Courant, Garren) using the light source lattice with small emittance minimized *H* function

  - **Extremely strong focusing with small dispersion function.**
  - large energy acceptance.
  - tunes variation
  - very small orbit offsets
  - small magnets
  - linear magnetic field



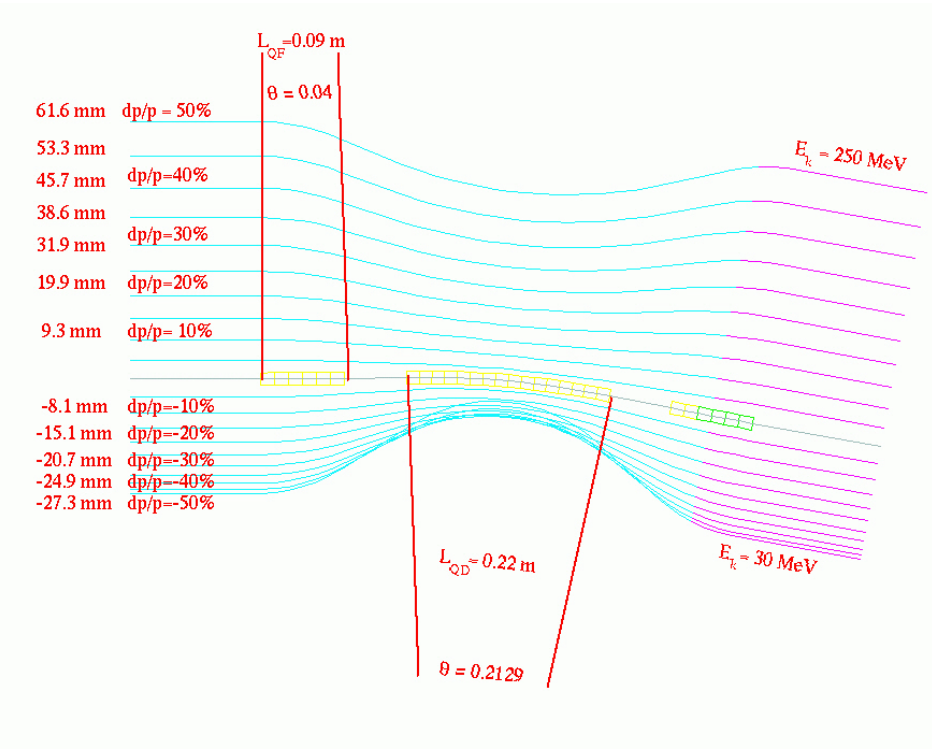

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **8**, 050101 (2005)

**Design of a nonscaling fixed field alternating gradient accelerator**

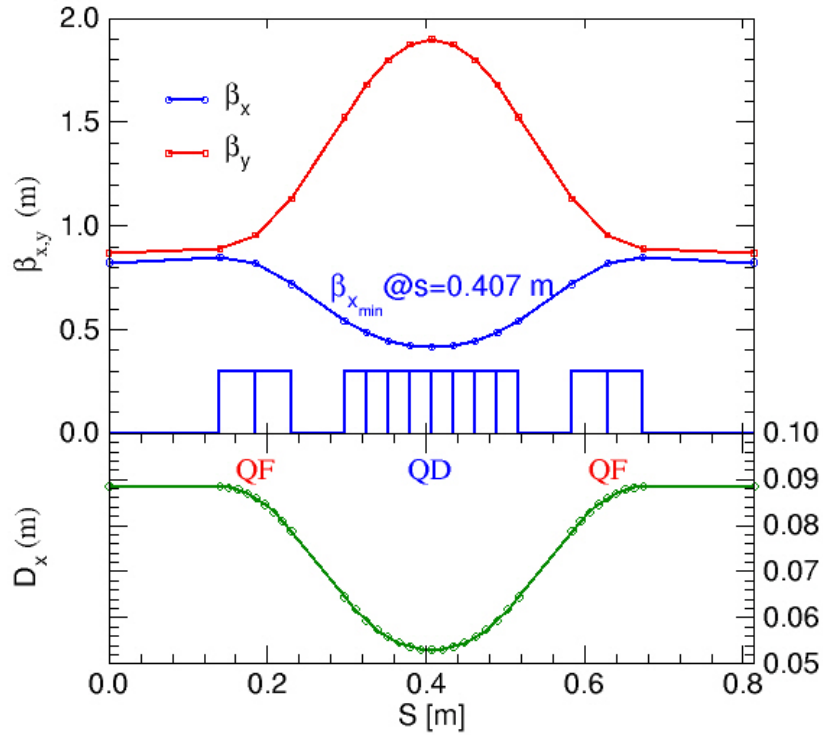
D. Trbojevic,* E. D. Courant, and M. Blaskiewicz
*BNL, Upton, New York 11973, USA*

# Basic Properties of the Non-Scaling FFAG

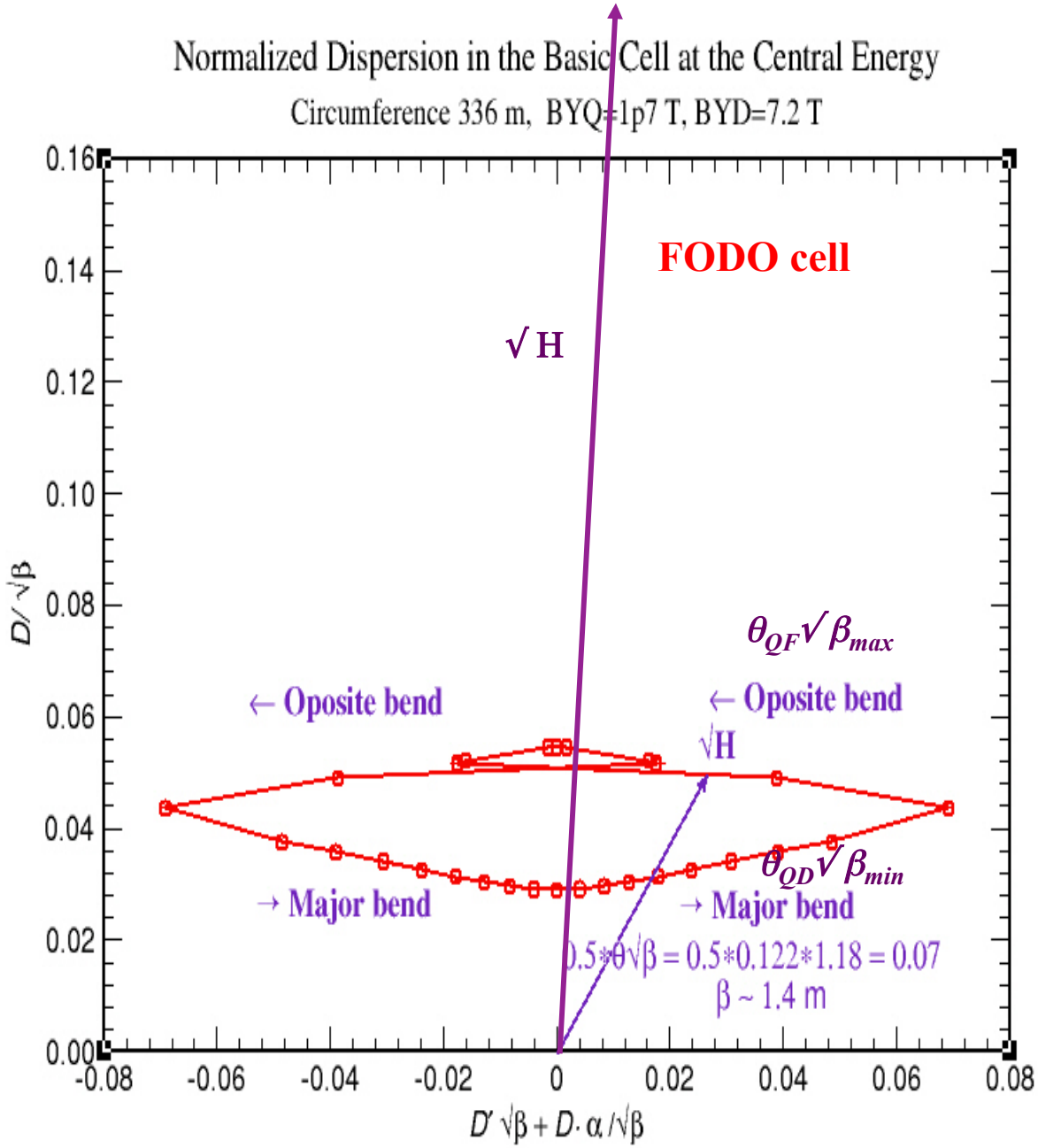
**A . Particle orbits**

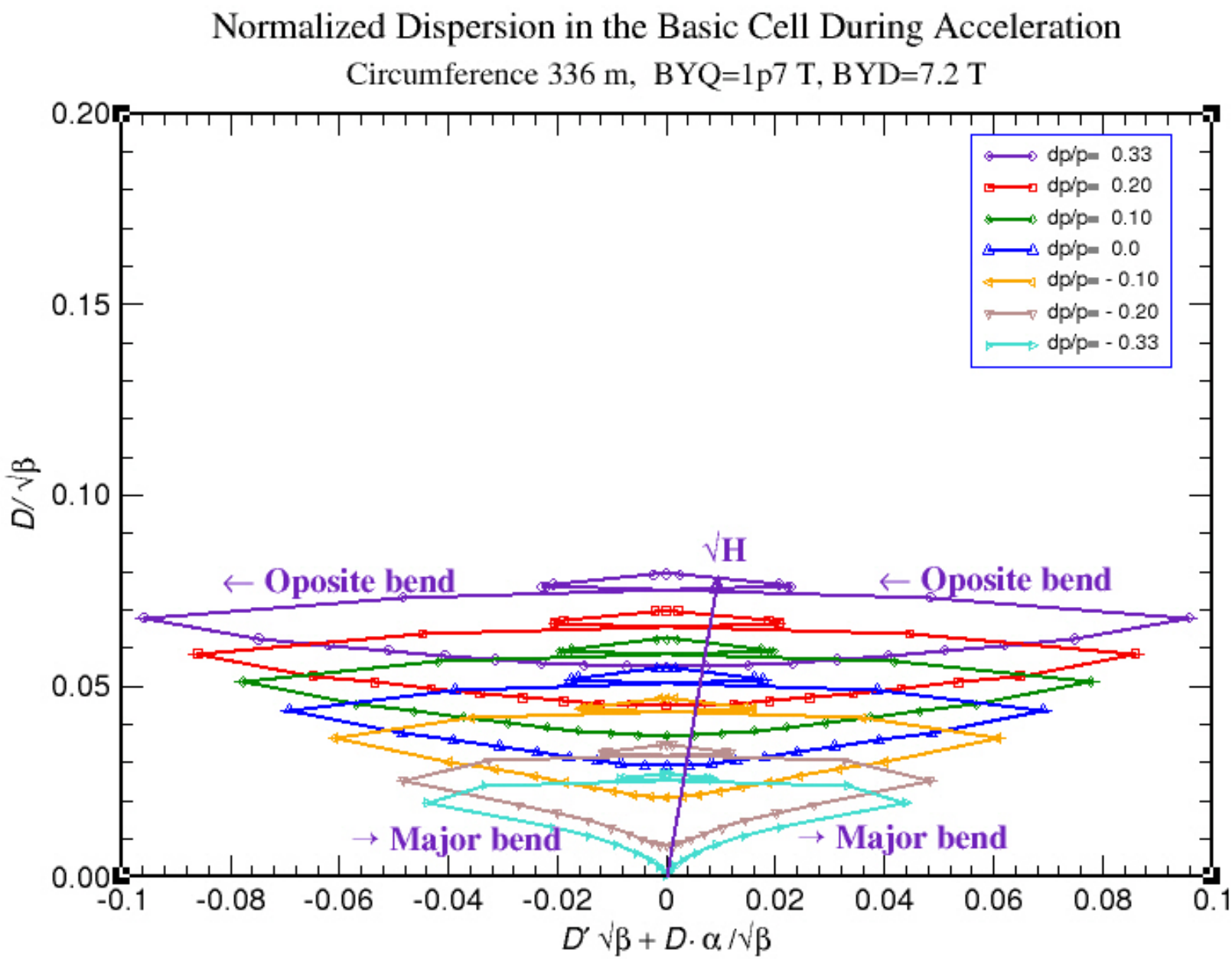



**B. Lattice**



*Minimization of the H function applied for the FFAG design*




Normalized Dispersion in the Basic Cell During Acceleration
Circumference 336 m, BYQ=1p7 T, BYD=7.2 T
dp/p= 0.33
dp/p= 0.20
dp/p= 0.10
dp/p= 0.0
dp/p= - 0.10
dp/p= - 0.20
dp/p= - 0.33
√H
← Oposite bend
← Oposite bend
→ Major bend
→ Major bend
D/√β
D′ √β + D· α /√β
0.20
0.15
0.10
0.05
0.00
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1

# Hamiltonian for the sector bend (exact ):

This is taken from the PTC "Manual": the Hamiltonian for the body of the TEAPOT magnet:

$$H = \underbrace{-\left(1+\frac{x}{\rho_d}\right)\sqrt{(1+\delta)^2 - p_x^2 + p_y^2}}_{T_1} + \underbrace{V(x,y;\rho_y^{-1})}_{T_2}$$

δ=∂p/p

Talman splits:

$$= \underbrace{-\left(1+\frac{x}{\rho_d}\right)\sqrt{(1+\delta)^2 - p_x^2 + p_y^2} + b_1\left(x+\frac{x^2}{2\rho_d}\right)}_{H_1} + \underbrace{V(x,y;\rho_y^{-1}) - b_1\left(x+\frac{x^2}{2\rho_d}\right)}_{H_2}$$

$$\left[(\rho_d + x)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial}{\partial x}\right) - \frac{\partial}{\partial x}\right] V = 0$$

$$b_x = -\frac{1}{\rho_d + x}\frac{\partial}{\partial x} V$$

$$b_y = \frac{1}{\rho_d + x}\frac{\partial}{\partial y} V$$

$$b_s = 0$$

$$b_x^{y=0} = \sum_{n=1}^{Sector_Nmul} a_n x^{n-1}$$

$$b_y^{y=0} = \sum_{n=1}^{Sector_Nmul} b_n x^{n-1}$$

$$\text{the full map in PTC for the sector bend:}$$

$$M(L) = R_{xy}^{-1} \circ F_1^{out} \circ F_2^{out} \circ B(L) \circ F_2^{in} \circ F_1^{in} \circ R_{xy}$$

TEAPOT : Richard Talman
Correct solution!
COSY: Martin Berz
uses Taylor expansion
MAD8: Christoph Iselin
correct to the second order
SYNCH: Ernest Courant and
All Garren combined function
magnet exact in any dp.
PTC: Etienne Forest - EXACT

# Tools test: Make an artificial cyclotron with 5 combined function magnets:

$$\frac{\partial^2 x}{\partial s^2} = -\frac{x}{\rho^2} + \frac{B_y - B_0(s)}{B\rho} \quad (1)$$

$$\frac{\partial^2 y}{\partial s^2} = -\frac{B_x}{B\rho} \quad (2)$$

Field in combined function magnet:

$$B_x = B_0 \frac{n_0\, y}{\rho_0} \quad (3)$$

$$B_y = B_0\left(1 - \frac{n_0\, x}{\rho_0}\right) = B_0 + G\,x \quad (4)$$

$\rho = \rho_0 + x_0$ ←off momentum:

$$e\left(B_0 + G\,x_0\right)\left(\rho_0 + x_0\right) = p_0\left(1+\delta\right) \quad (5)$$

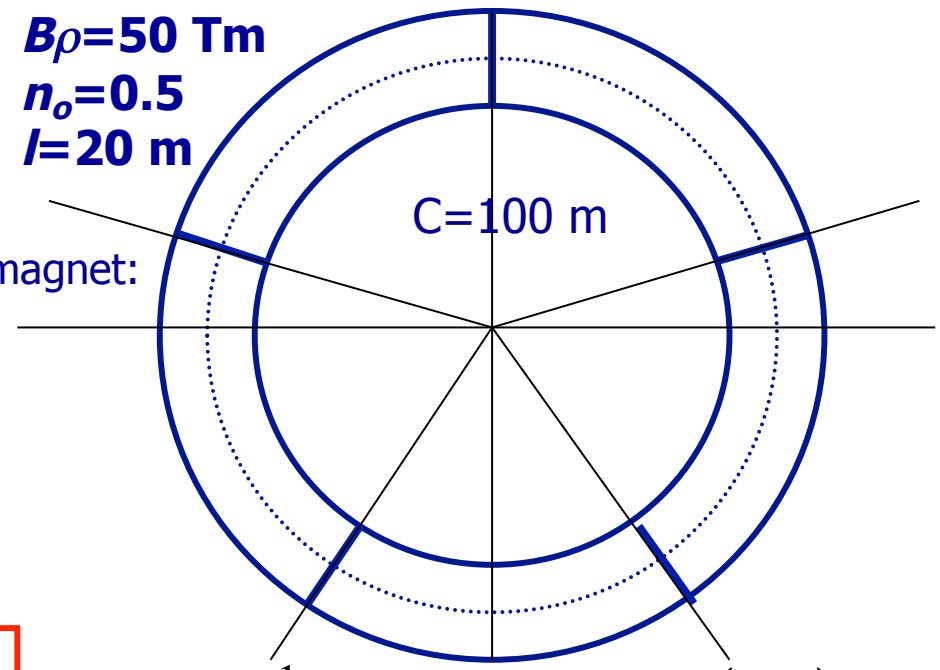
substitutes: ↙ $u = x_0/\rho_0$

$$n_0 u^2 - \left(1 - n_0\right)u + \delta = 0 \quad (5)$$

Two solutions: ↙ $\lambda = (1 - n_0)/(2n_0)$

$$u_{1,2} = \lambda \pm \sqrt{\lambda^2 - \frac{\delta}{n_0}}$$

**$B\rho$=50 Tm**
**$n_o$=0.5**
**$l$=20 m**




$$n = \frac{1+u}{1-n_0 u} n_0 \rightarrow \frac{\partial^2 x}{\partial s^2} = -\frac{(1-n)}{\rho^2} x \quad (1)$$

$$\frac{\partial^2 y}{\partial s^2} = -\frac{n}{\rho^2} y \quad (2)$$

tunes:

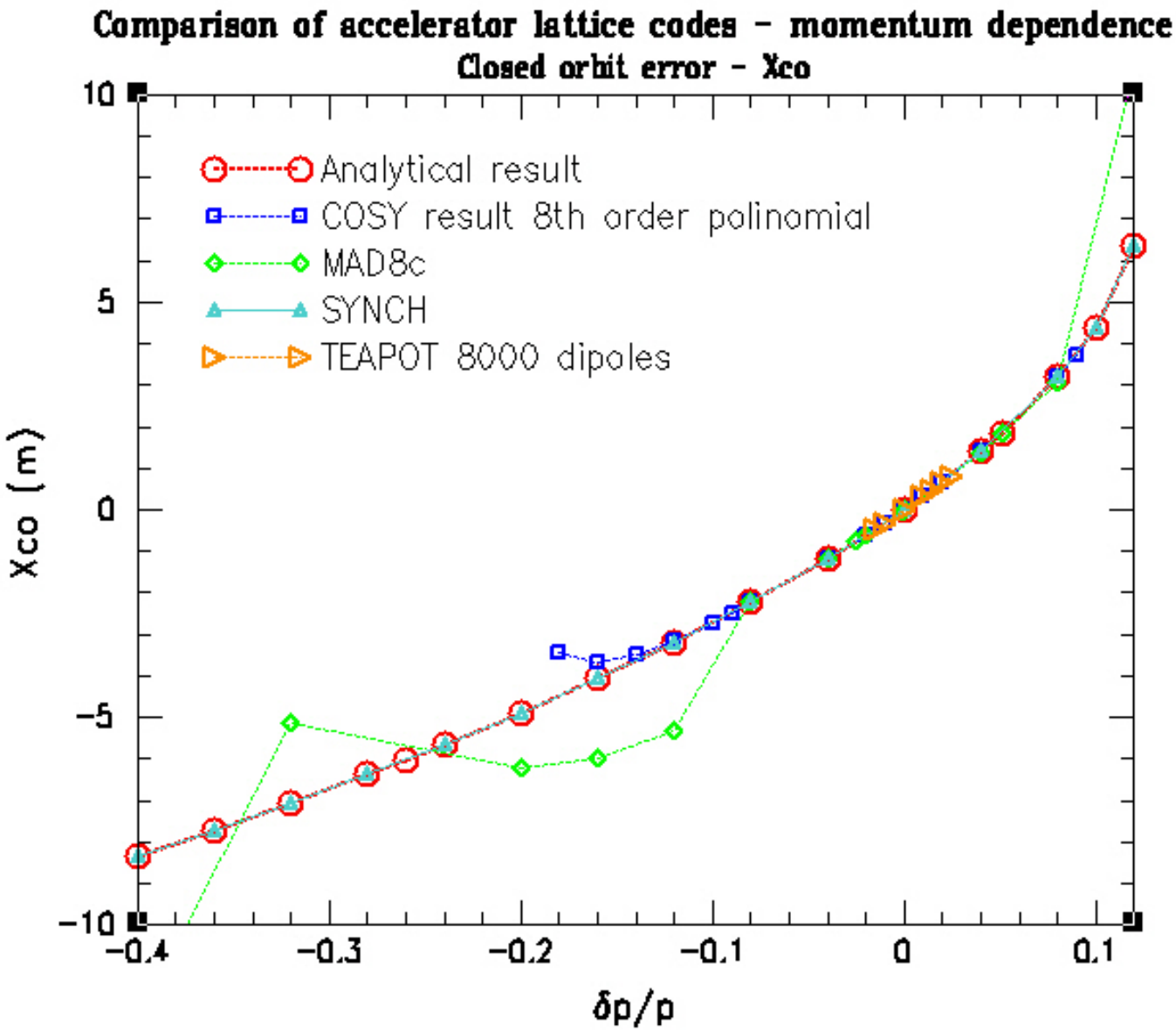
$$\nu_x = \sqrt{1-n},\ \nu_y = \sqrt{n},$$

beta's: $\beta_x = \frac{\rho}{\sqrt{1-n}},\ \beta_y = \frac{\rho}{\sqrt{n}}$

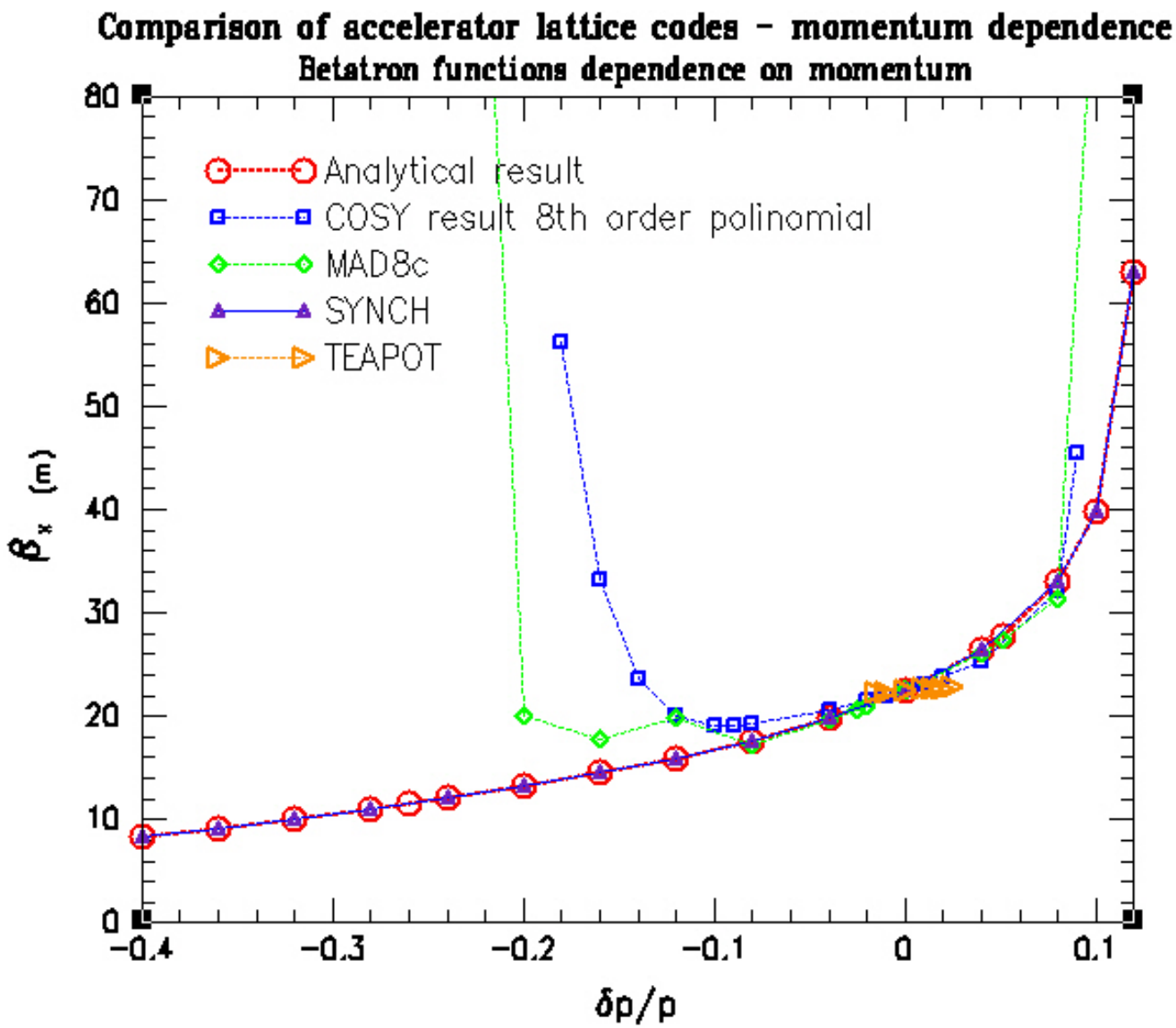
dispersion and momentum compaction:

$D_x = \partial x_0/\partial\delta = \rho/(1-n)$ and $\alpha_c = D_x/\rho = 1/(1-n)$

# The whole ring with two straight sections - linacs



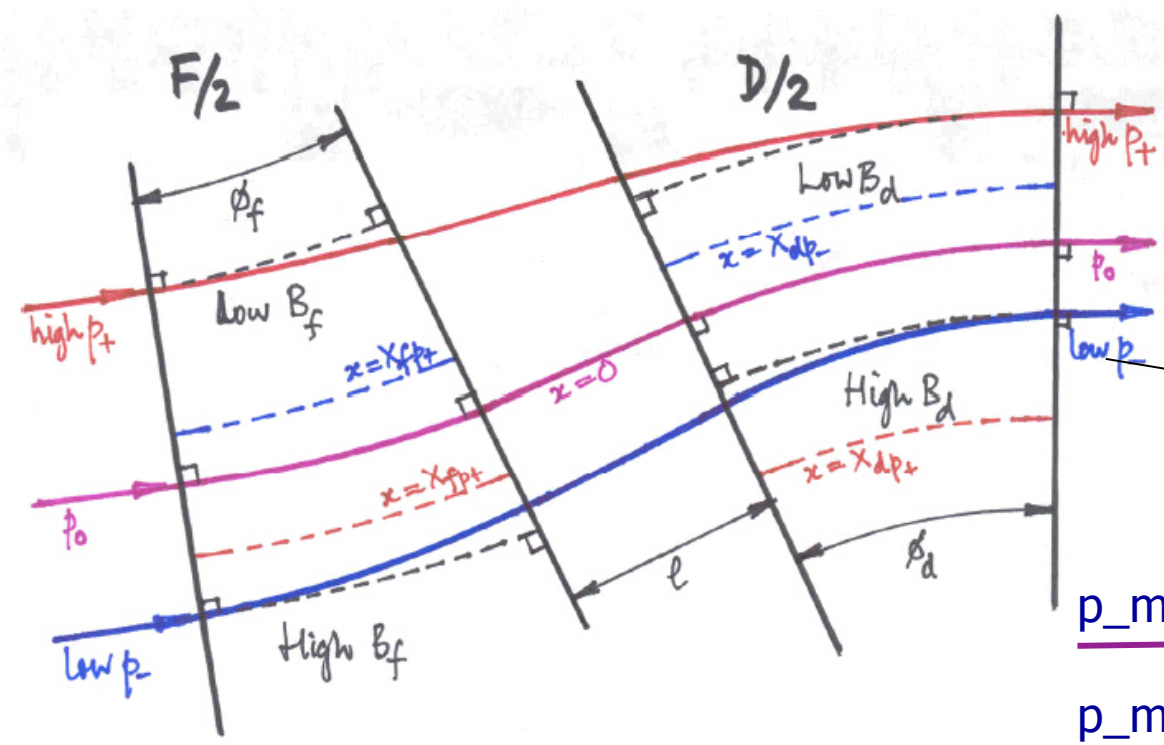
# The whole ring with two straight sections - linacs



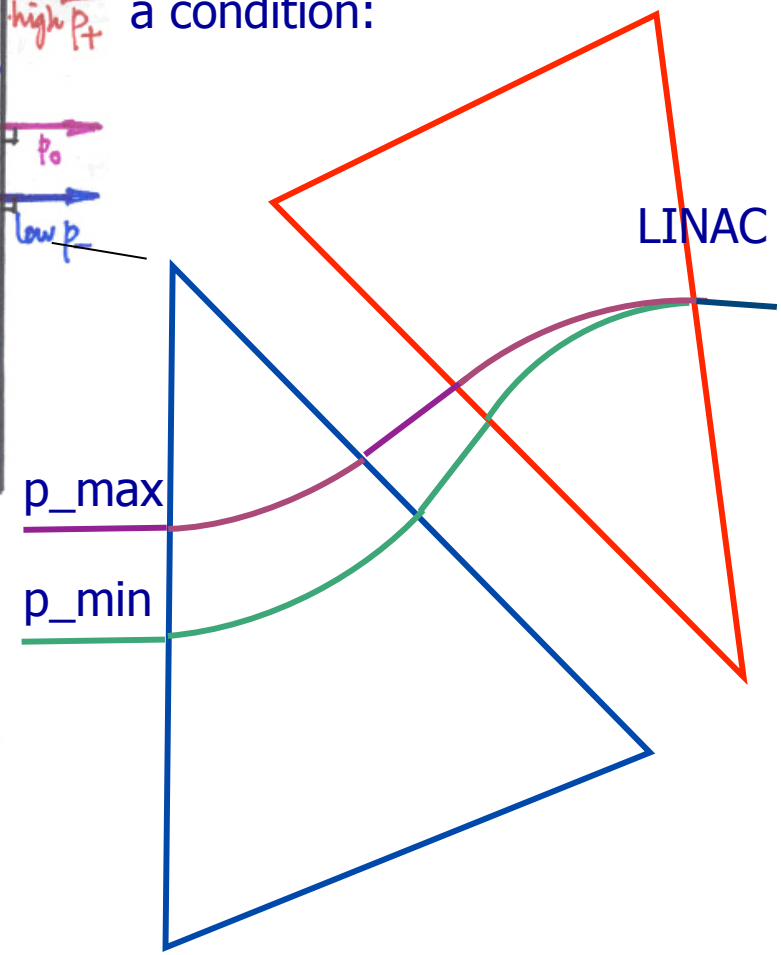
# Analytical formulae from the combined function magnet:

## ORBITS IN FDF SECTOR GRADIENT MAGNETS

The figure shows a half cell:




This is what I am presently designing for eRHIC linac and trying to analyticaly figure out a condition:




Each magnet has an equilibrium orbit for each momentum $p$

- a circular arc (radius $\rho_{fp}$, $\rho_{dp}$) with constant field $B_{fp}$, $B_{dp}$
- offset by $X_{fp}$, $X_{dp}$ w.r.t. the reference orbit $x = 0$:

$$X_p = \frac{r_0}{2n_0}\left\{(1-n_0)+\sqrt{(1-n_0)^2-4n_0(\Delta p/p_0)}\right\}$$

# Analytical formulae from the combined function magnet:

## MATCHING CONDITIONS

At the ends of the short drift between F and D,
the betatron displacements and divergences are:

$$x_{f/} - X_{fp} = A_f \cos\alpha_f \qquad x_{d/} - X_{dp} = A_d \cosh\alpha_d$$

$$\tan\chi_f \cong \frac{A_f}{\rho_{f0}}\sqrt{1-n_f}\,\sin\alpha_f \qquad \tan\chi_d \cong \frac{A_d}{\rho_{d0}}\sqrt{n_d-1}\,\sinh\alpha_d$$

where $\alpha_f \equiv \sqrt{1-n_f}\,\phi_f \qquad \alpha_d \equiv \sqrt{n_d-1}\,\phi_d \qquad \tan\chi = \frac{1}{r}\frac{dr}{d\phi}$

and $n_f$, $n_d$ are evaluated at $X_{fp}$, $X_{dp}$.

Matching the divergences, we find $\frac{A_f}{A_d} \cong \sqrt{\frac{n_d-1}{1-n_f}}\,\frac{\rho_{f0}\sinh\alpha_d}{\rho_{d0}\sin\alpha_f}$ .

Matching the displacements, so that: $x_{f/} - x_{d/} = \ell\tan\chi_d$

we get:

$$A_d = \frac{X_{fp} - X_{dp}}{\cosh\alpha_d + \frac{\sqrt{n_d-1}}{\rho_{d0}}\sinh\alpha_d\left[\ell - \frac{\rho_{f0}}{\sqrt{1-n_f}\,\tan\alpha_f}\right]}$$

and hence $A_f$ too, enabling us to:

- compute the offsets $x_p(\phi)$ for any azimuthal angle $\phi$
- but giving no insight into the momentum dependence,
  - since $X_{fp}$, $X_{dp}$, $n_f$, $n_d$, $\alpha_f$, and $\alpha_d$ all individually vary with $p$.
- though Shane has gone much further, using Mathematica to extract the $p$-dependence, $\beta$ and $\nu$ values.....

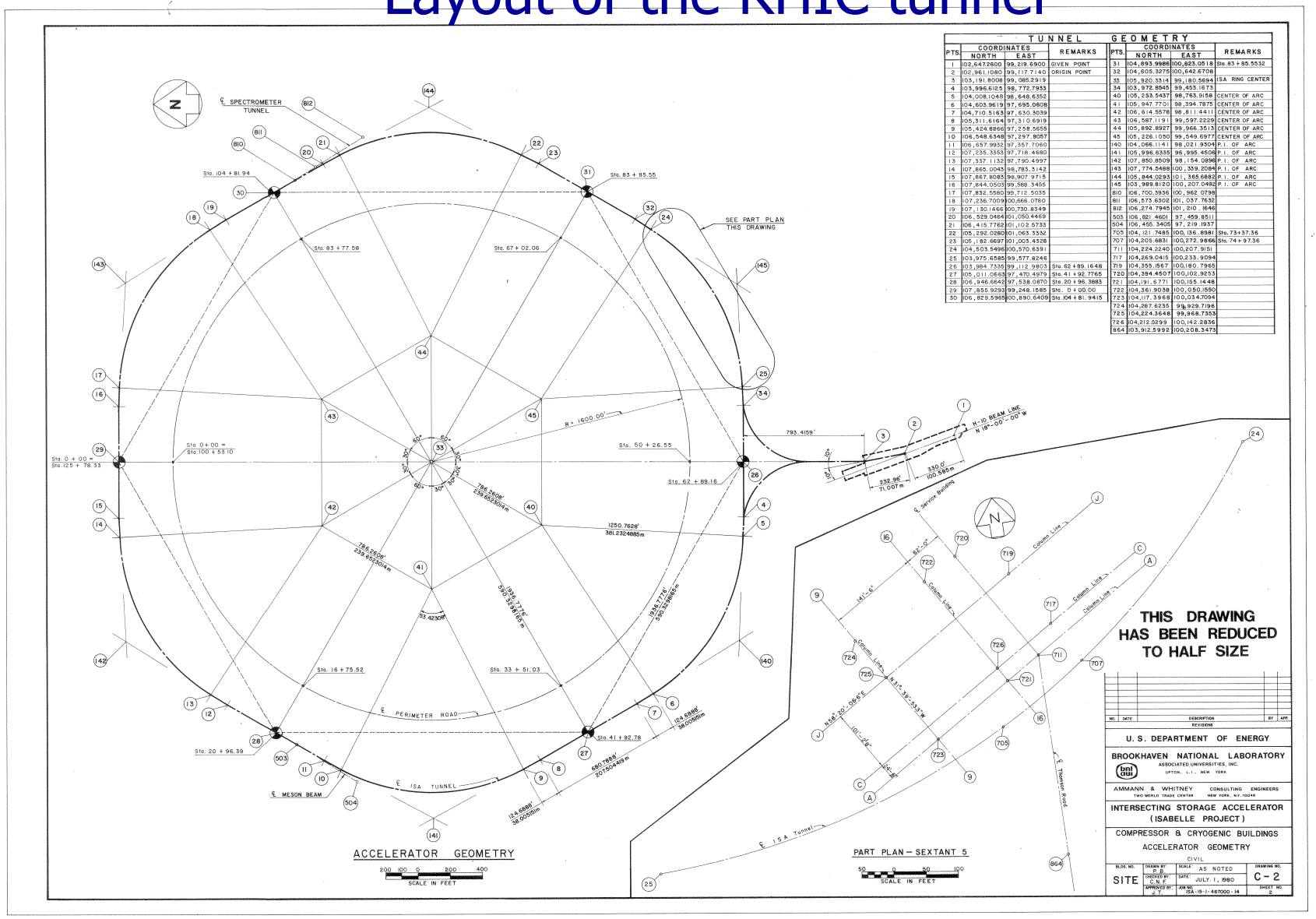
Mike Craddock took one of our examples designed by the PTC and compared the results with approximations:

| 10 GeV/c | Dejan FDF Triplets | | |
|---|---|---|---|
| Circumference $C$ (m) | 323 | 328 | 348 |
| E.O. offset $X_{d10}$ (m) for D | 0.0480 | 0.0398 | 0.0335 |
| Osc'n amplitude $A_d$ (m) | 0.0458 | 0.0387 | 0.0330 |
| Offset $x_{d\perp}$ (m) - formula | **0.0022** | **0.0011** | **0.0005** |
| " " " " " - lattice code | **0.0024** | **0.0010** | **0.0005** |
| " " " " " - difference | -0.0002 | 0.0000 | 0.0000 |
| E.O. offset $X_{f10}$ (m) in F | 0.0141 | 0.0107 | 0.0089 |
| Osc'n amplitude $A_f$ (m) | 0.0436 | 0.0361 | 0.0306 |
| Offset $x_{f\perp}$ (m) - formula | **-0.0295** | **-0.0255** | **-0.0217** |
| " " " " " - lattice code | **-0.0290** | **-0.0250** | **-0.0213** |
| " " " " " - difference | -0.0006 | -0.0005 | -0.0004 |
| Extra path $\Delta C$ (m) - thin lens | 0.2393 | 0.1974 | 0.1675 |
| " " " " " " " - formula | **0.1339** | **0.1050** | **0.0873** |
| " " " " " " - lattice code | **0.1434** | **0.1165** | **0.0992** |
| " " " " " " - difference | -0.0095 | -0.0115 | -0.0119 |
| 20 GeV/c | | | |
| E.O. offset $X_{d20}$ (m) for D | -0.0485 | -0.0402 | -0.0338 |
| Osc'n amplitude $A_d$ (m) | 0.0713 | 0.0598 | 0.0507 |
| Offset $x_{d\perp}$ (m) - formula | **0.0228** | **0.0196** | **0.0168** |
| " " " " " - lattice code | **0.0232** | **0.0197** | **0.0169** |
| " " " " " - difference | -0.0004 | -0.0002 | -0.0001 |
| E.O. offset $X_{f20}$ (m) in F | -0.0140 | -0.0107 | -0.0089 |
| Osc'n amplitude $A_f$ (m) | 0.0597 | 0.0493 | 0.0416 |
| Offset $x_{f\perp}$ (m) - formula | **0.0456** | **0.0386** | **0.0327** |
| " " " " " - lattice code | **0.0465** | **0.0388** | **0.0328** |
| " " " " " - difference | -0.0008 | -0.0002 | -0.0001 |
| Extra path $\Delta C$ (m) - thin lens | 0.2393 | 0.1974 | 0.1675 |
| " " " " " " " - formula | **0.1360** | **0.1154** | **0.0981** |
| " " " " " " - lattice code | **0.1409** | **0.1184** | **0.0994** |
| " " " " " " - difference | -0.0049 | -0.0030 | -0.0014 |

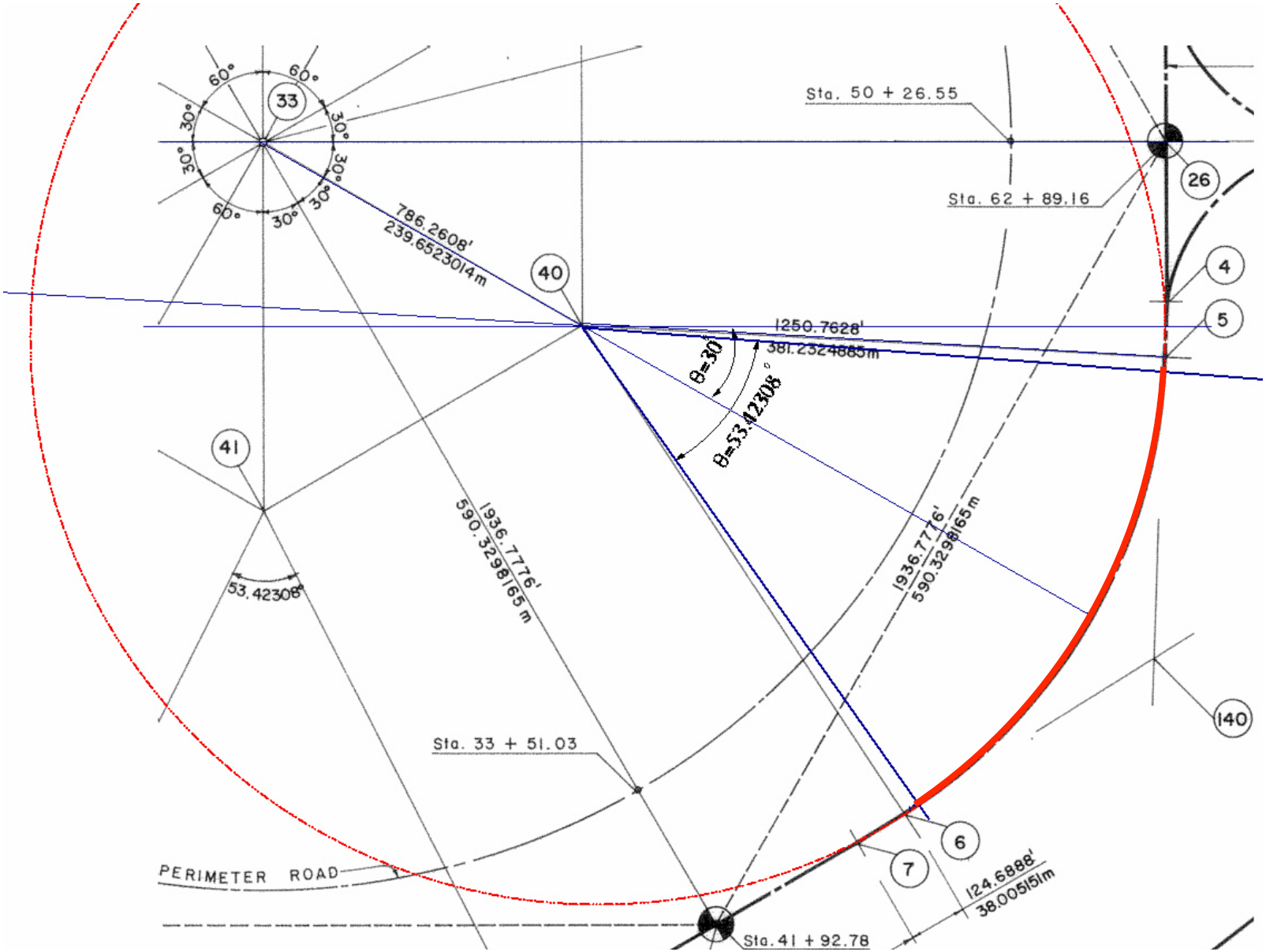
Few more examples just from the press:

- eRHIC application
- Muon RLA acceleration
- Small proton therapy non-scaling FFAG
- Patent on a carbon/proton gantry

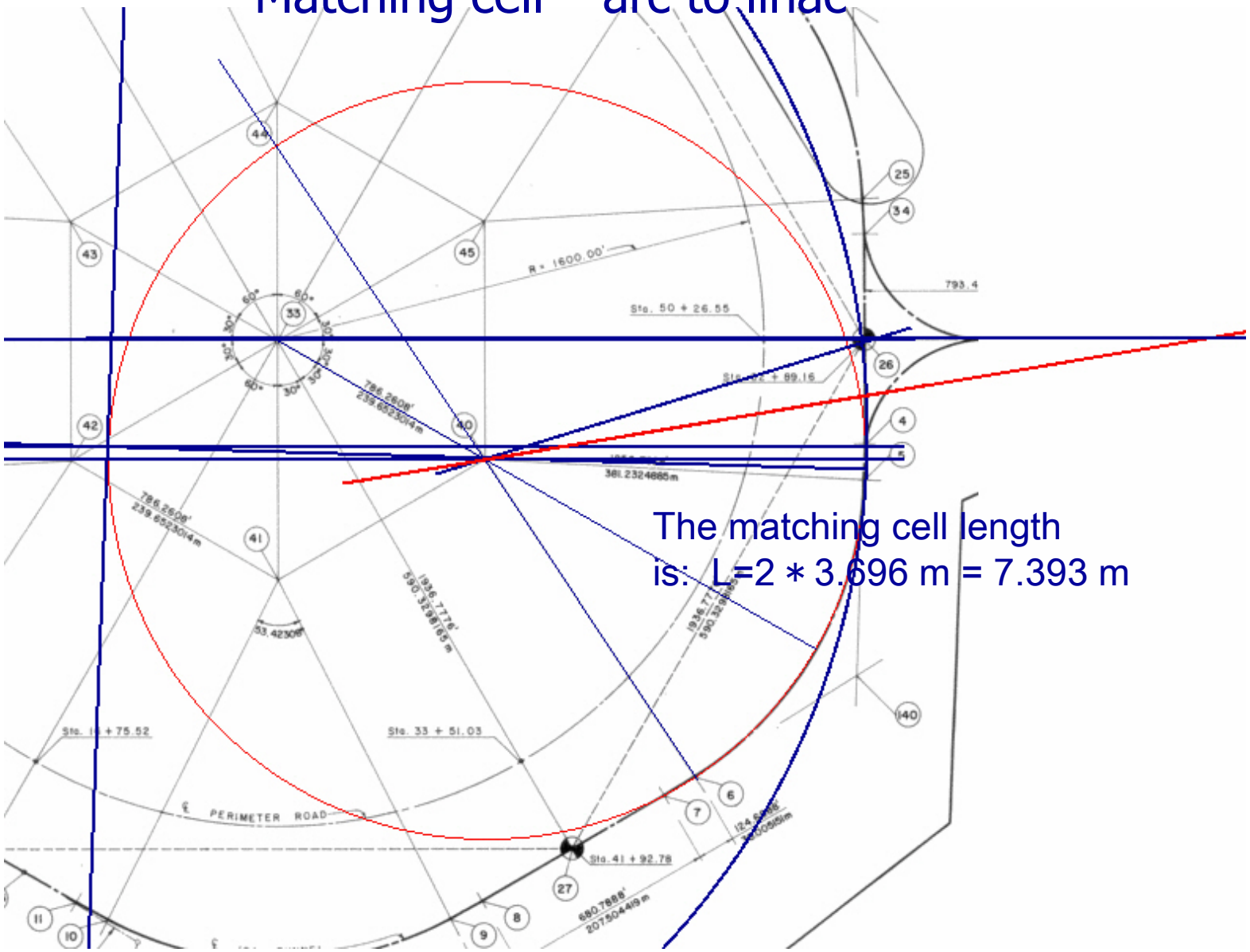
# Layout of the RHIC tunnel



# Layout of the Arc in RHIC



# Matching cell – arc to linac




The matching cell length is: L=2 ∗ 3.696 m = 7.393 m

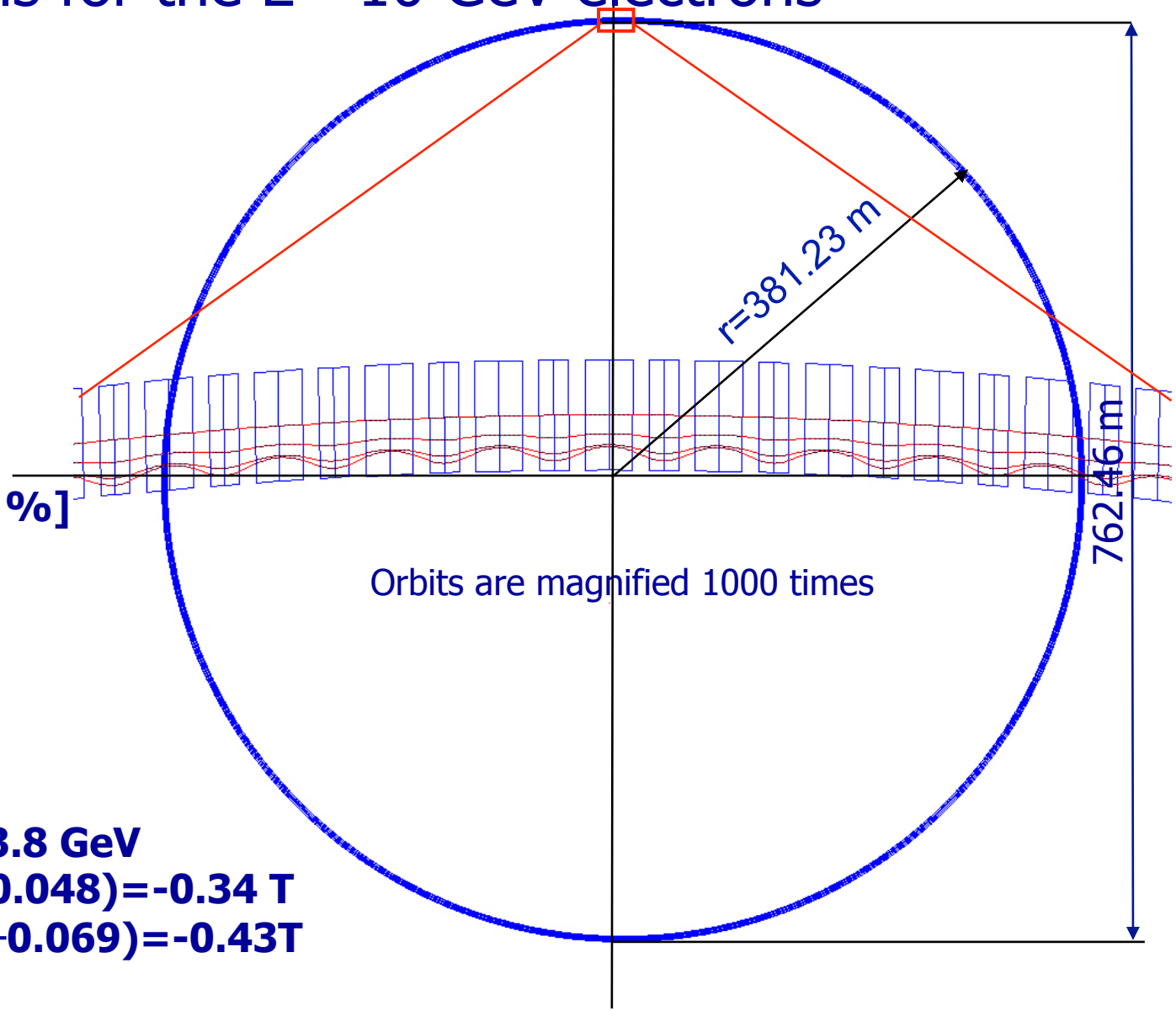
# Design of the arcs – from the densely populated FODO cells for the 2 - 10 GeV electrons

**N=648 cells**
**L=3.6965 m**
**R1=381.23249 m**
**$L_{BD}$=1.6 m**
**$L_{QF}$=1.0 m**

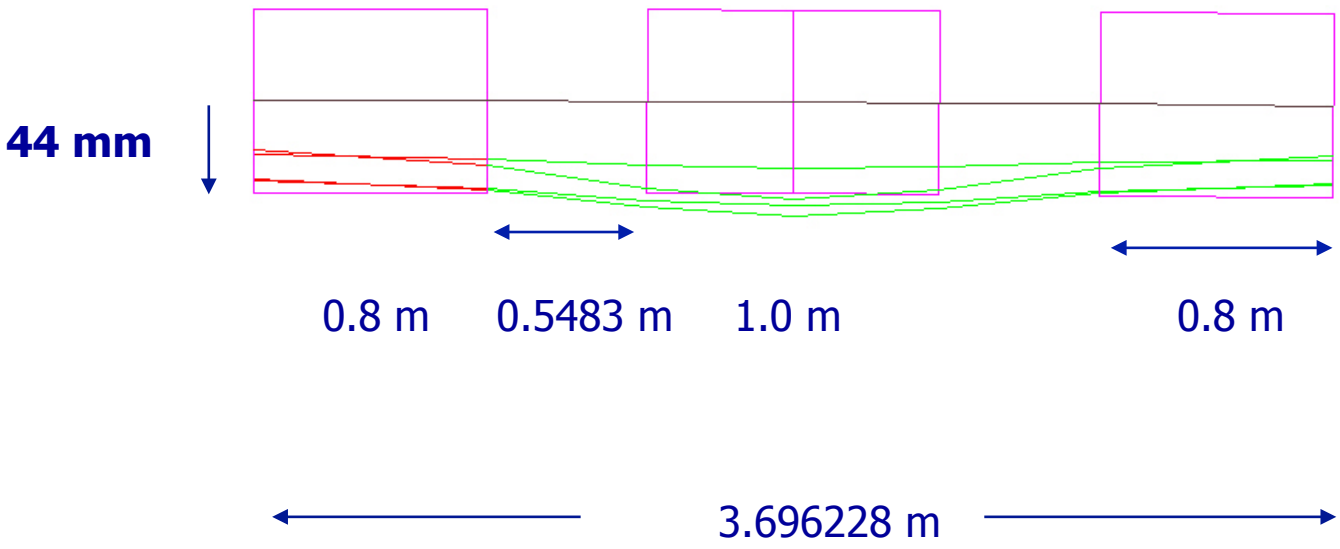
**For the $\delta p/p$=[0,-80 %]**
**$B_{BD}$ = + 0.078 T**
**$B_{QF}$ = + 0.199 T**

**GF= 9.15 T/m**
**GD= - 5.4 T/m**

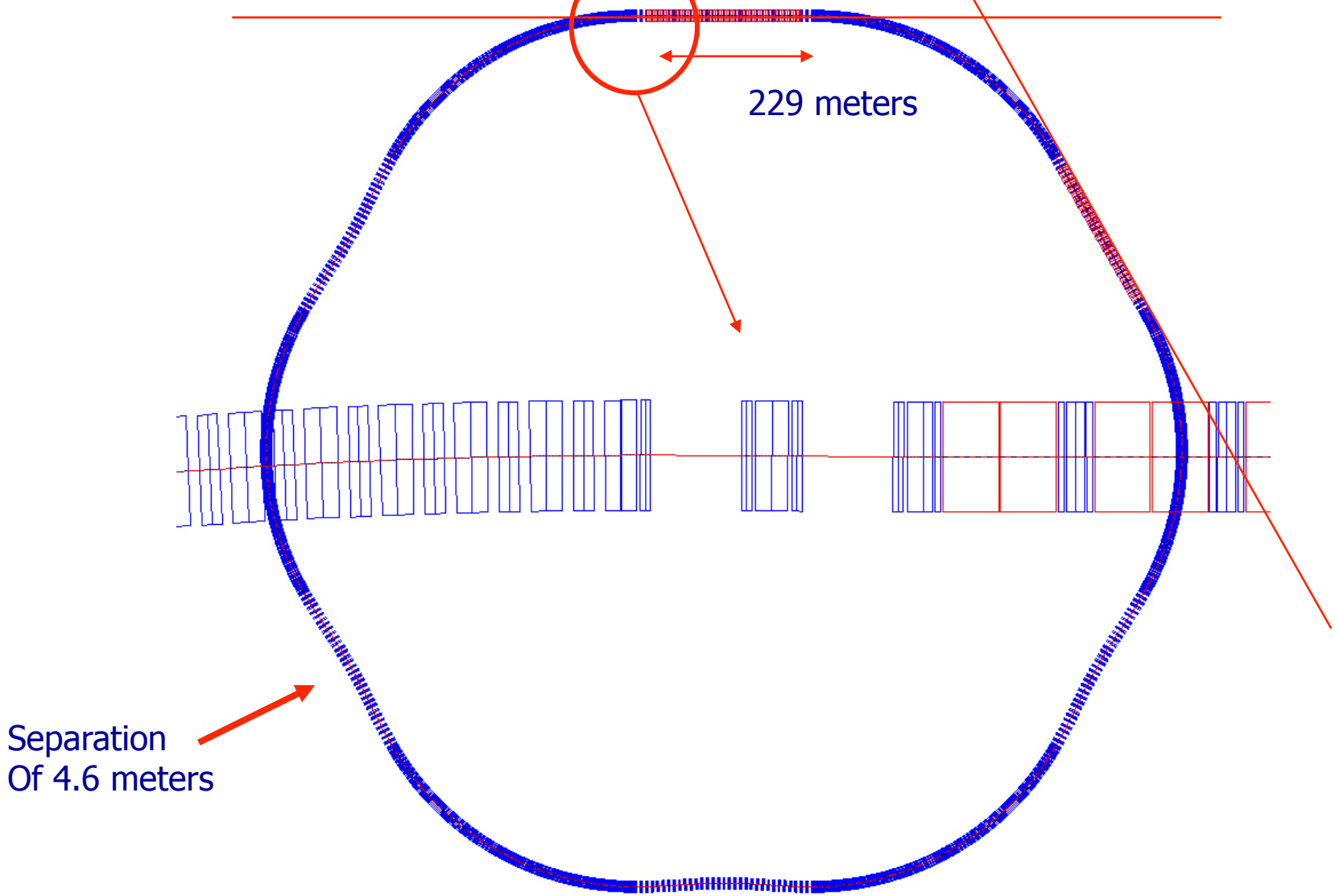
**@ electron energy=3.8 GeV**
**$B_{max}$=0.078 - 5.4 *(-0.048)=-0.34 T**
**$B_{max}$=0.199 +9.15*(−0.069)=-0.43T**



# Design of the arcs – from the densely populated FODO cells for the energy range of 2.4 -> 10 GeV electrons

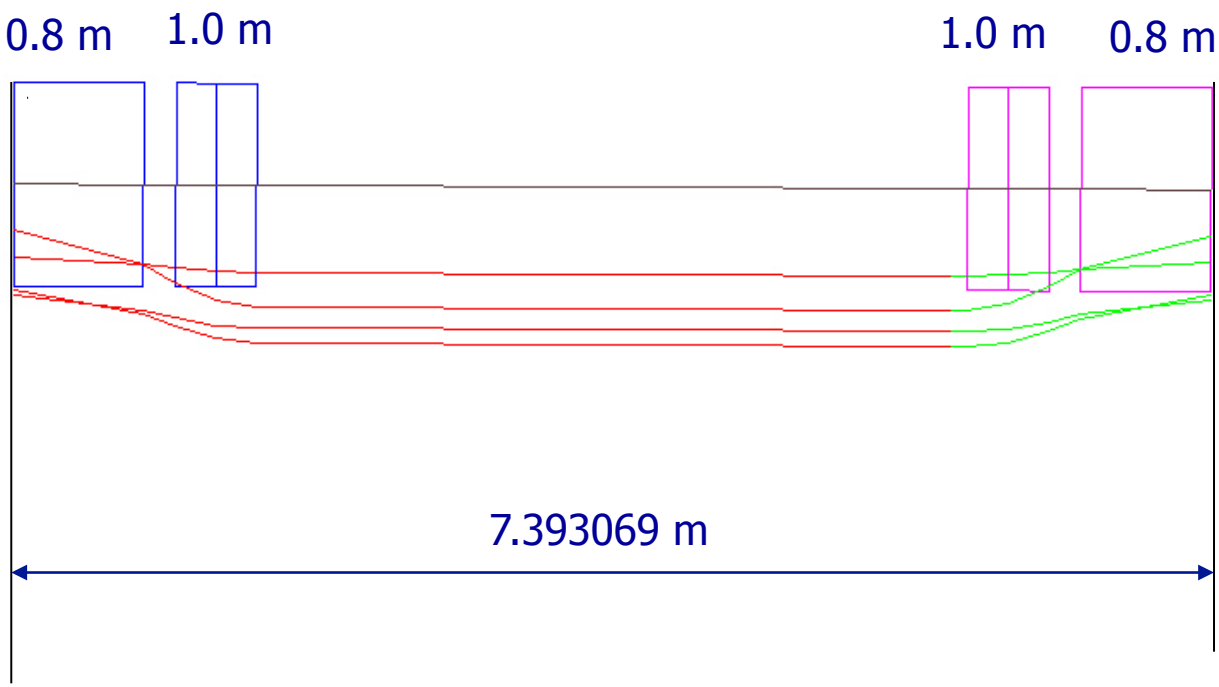


# The whole ring with two straight sections - linacs

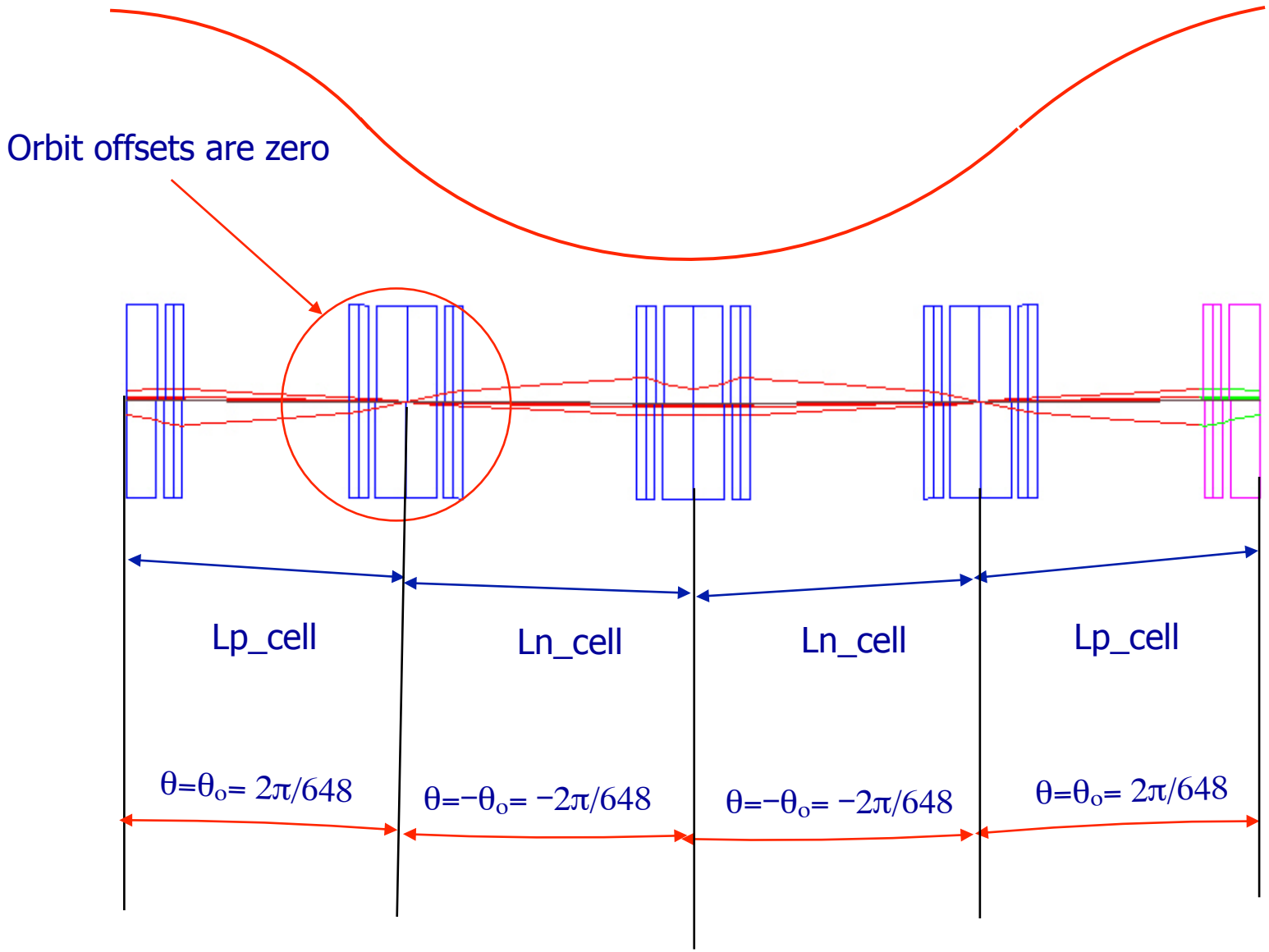


# Design of the large cells – match to the linac

**Orbit offset are identical to the arc orbit offsets**

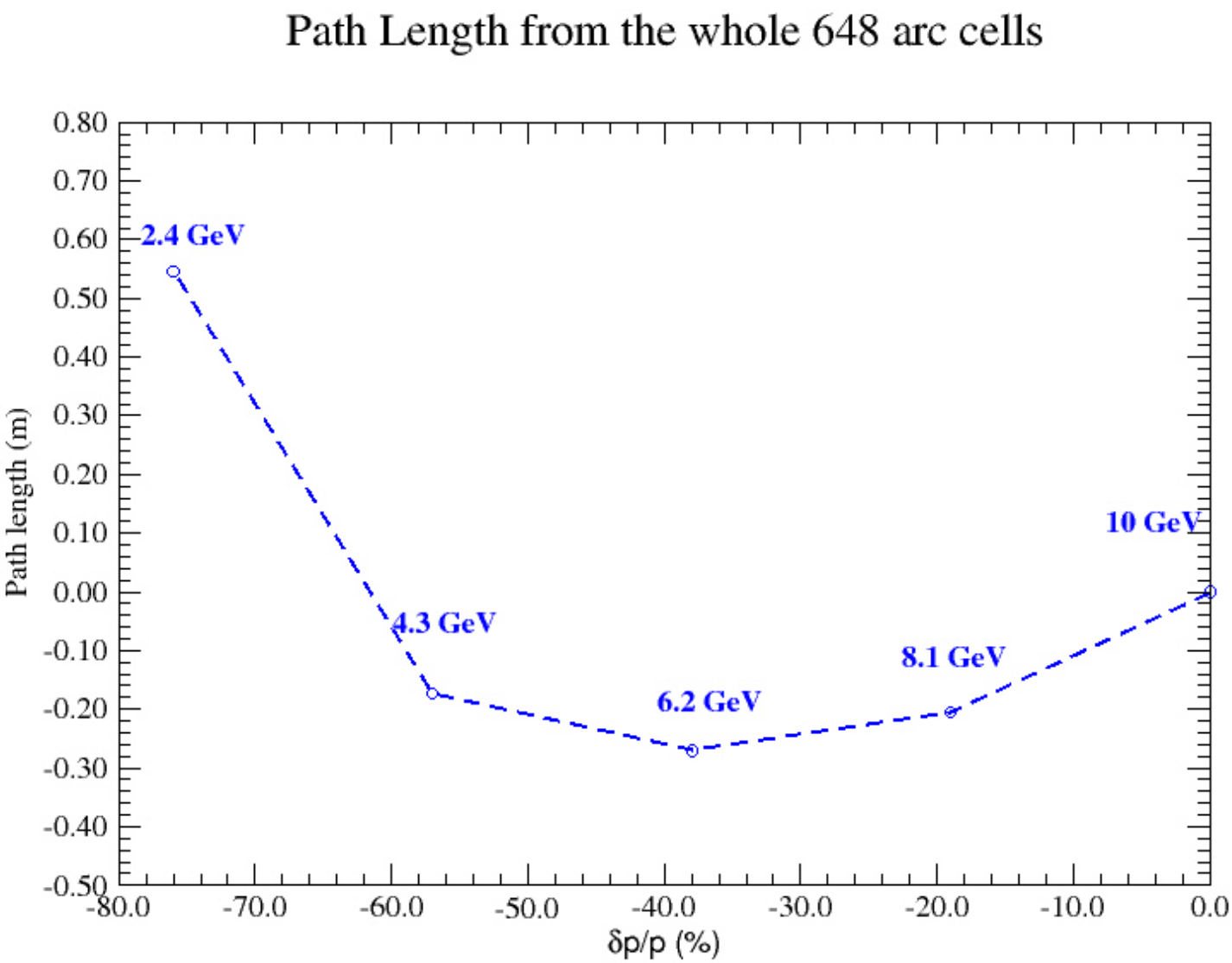


# Larger cells to match to linac from arcs



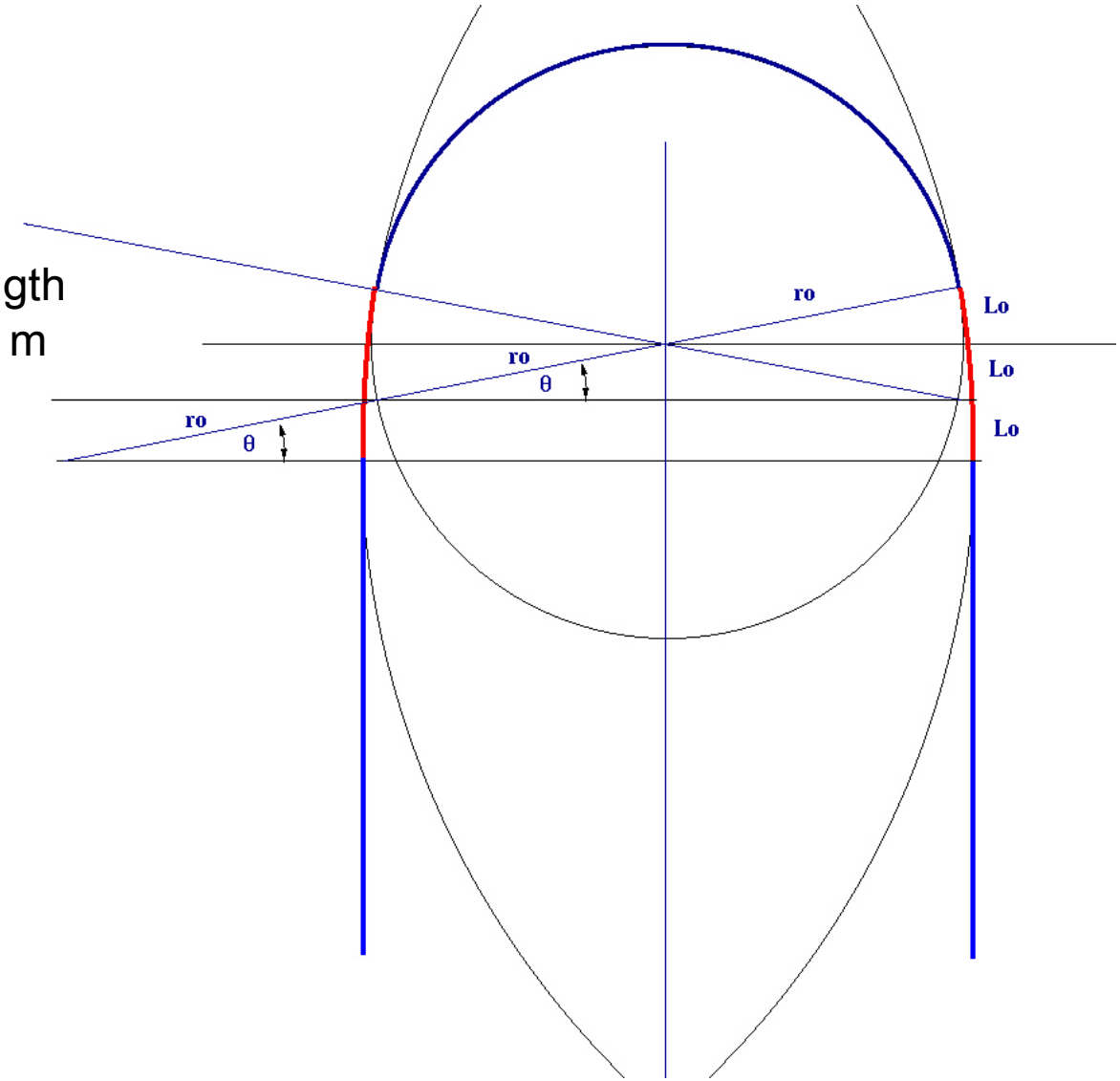
# The whole ring with two straight sections - linacs

# Time of flight – Path length dependence on energy



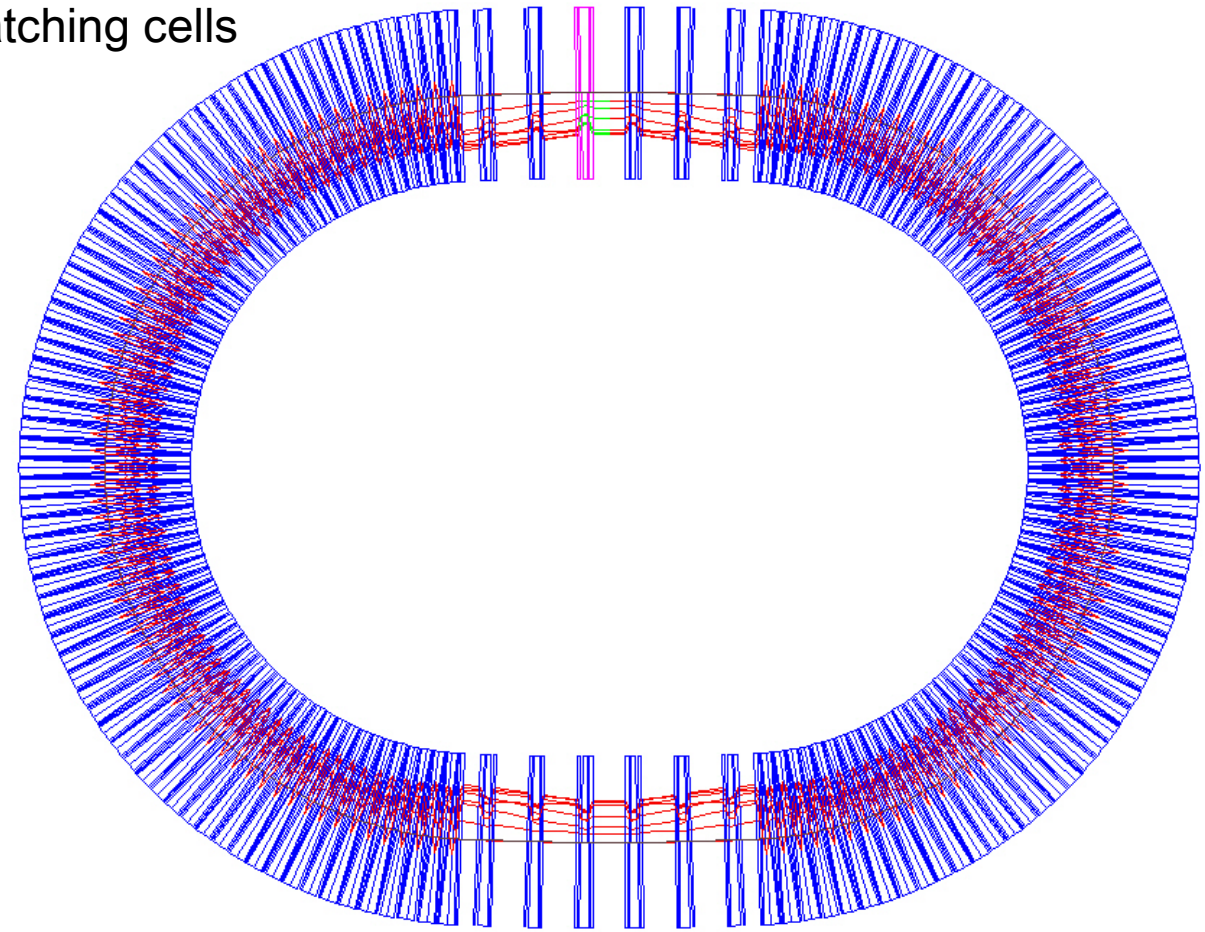
# RLA muon acceleration matching cell – arc to linac

The matching cell length is: L=3 ∗ 1.9 m = 5.7 m



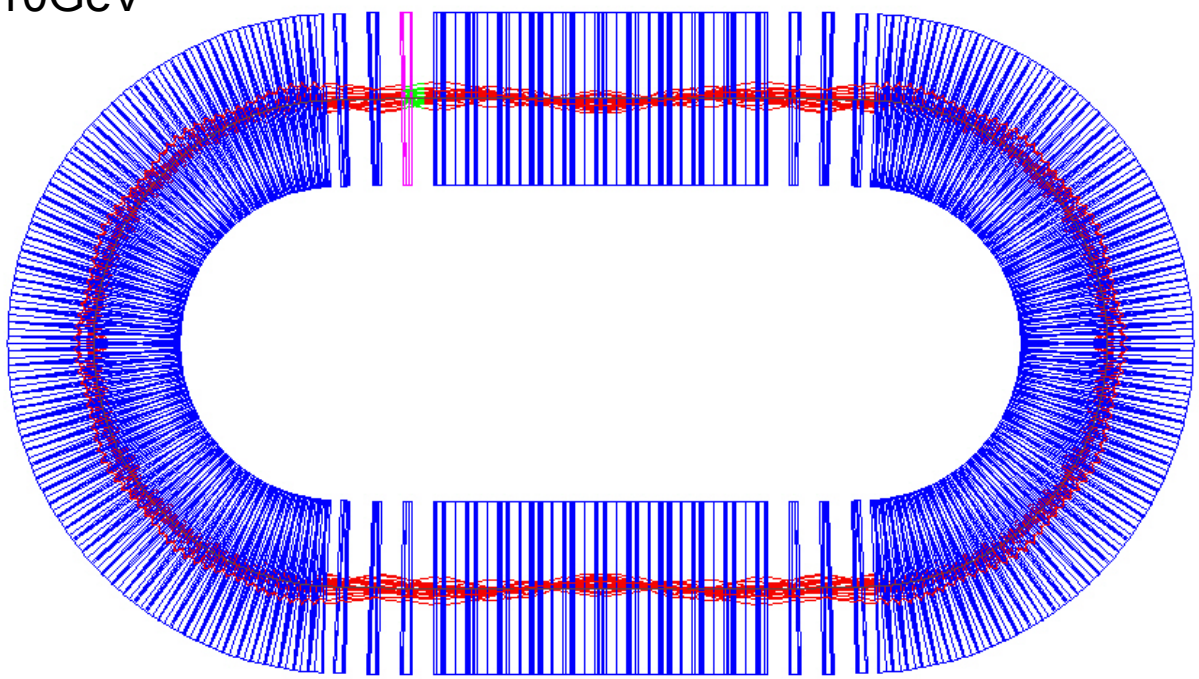
# Matching cell with arcs without linac

Orbits from 2.5 – 10 GeV
Through the matching cells
and arcs:

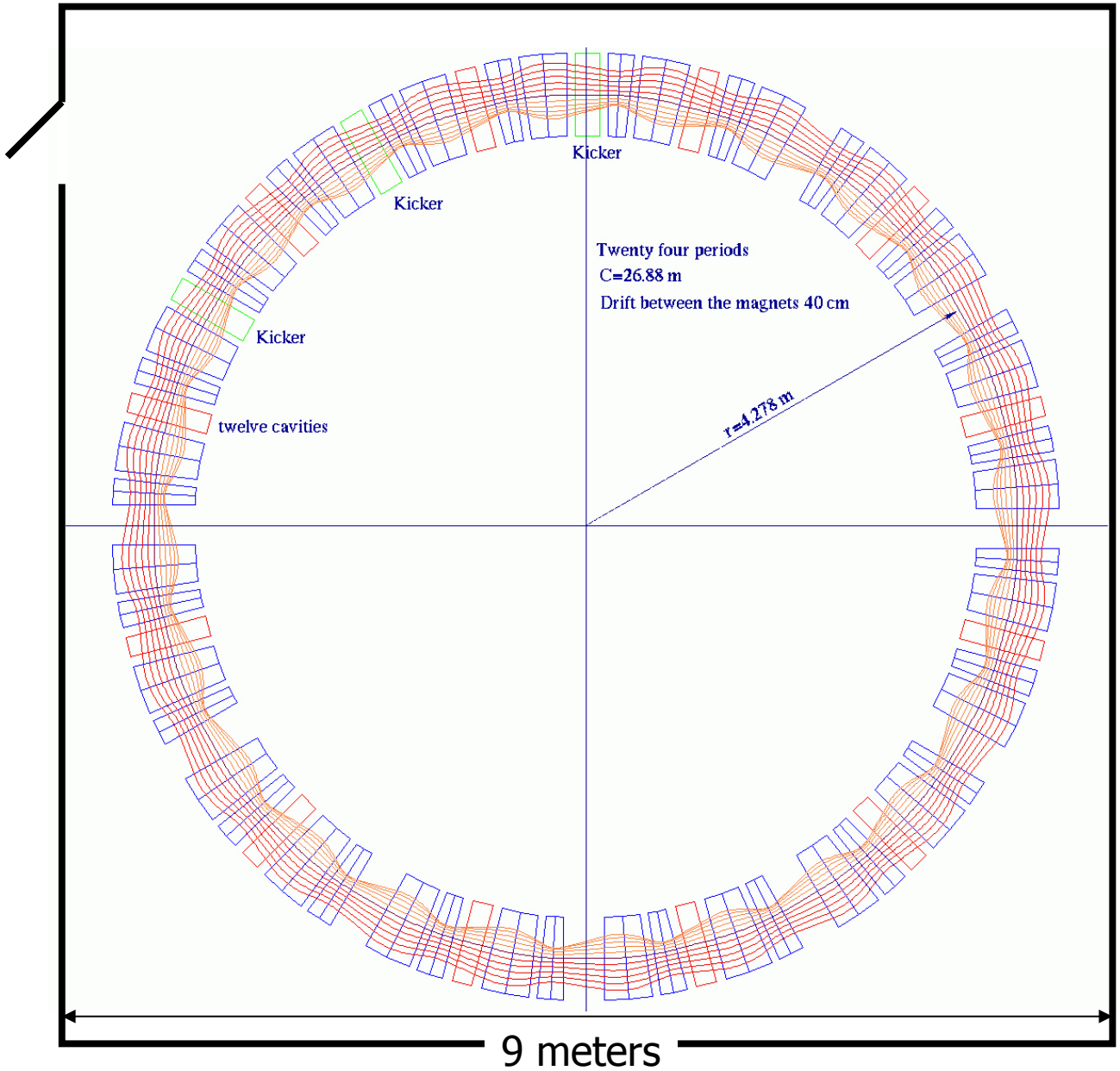


# RLA to accelerate muons 2.5-10 GeV

Orbits magnified 100 times
From 2.5 GeV- 10GeV



# Small proton therapy machine

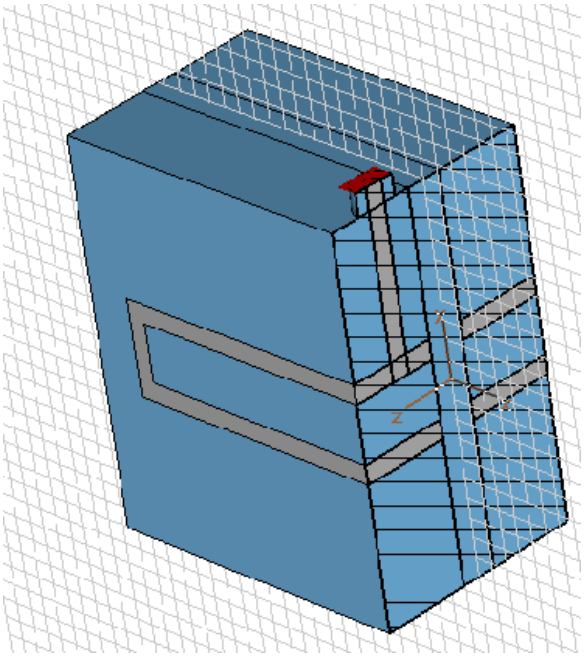


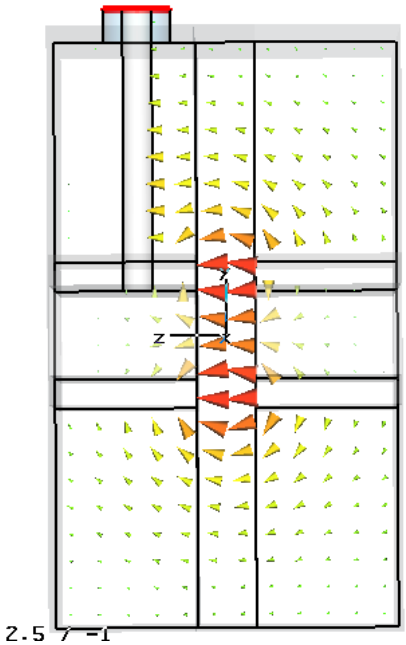
# Accelerating cavity – Mike Blaskiewicz:

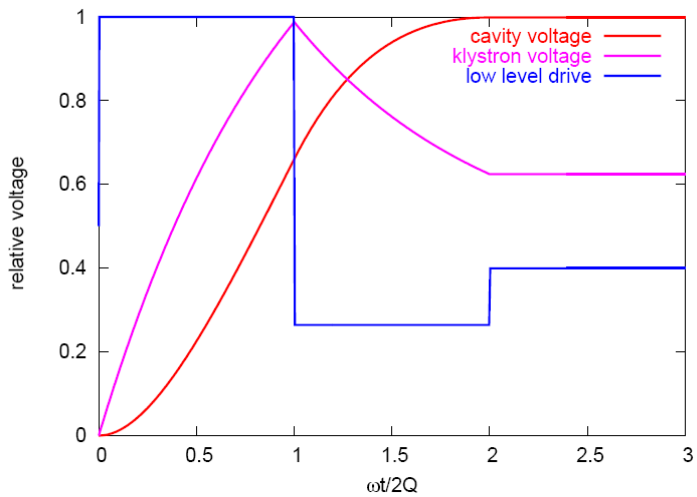
The voltage scales with beam velocity as

$$V(\beta) = E_0 \int \frac{\cos(2\pi z/\beta\lambda)}{\cosh(z/a)} dz = E_0 \frac{a\pi}{\cosh\left(\frac{a\pi^2}{\beta\lambda}\right)}$$

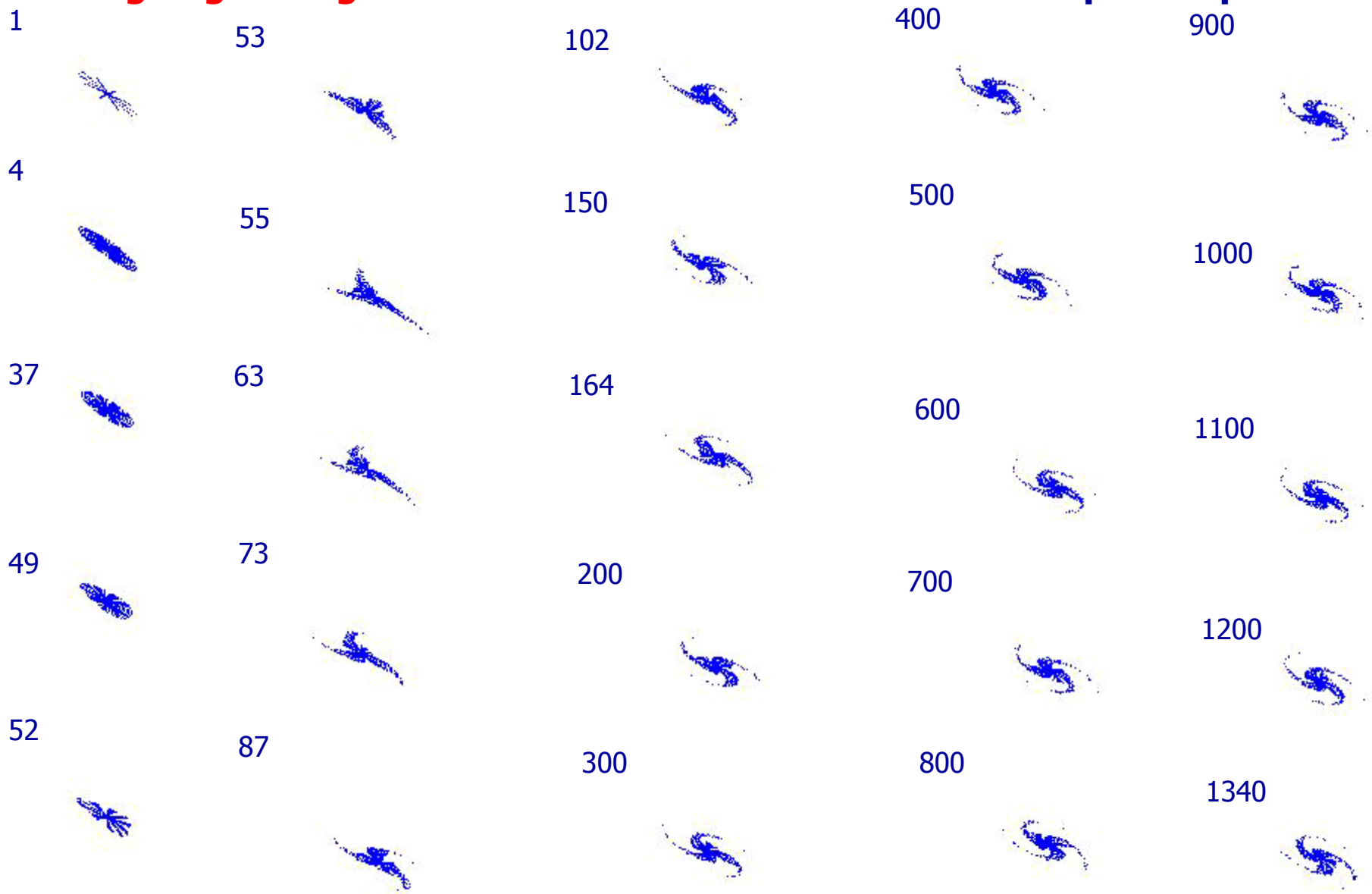
8



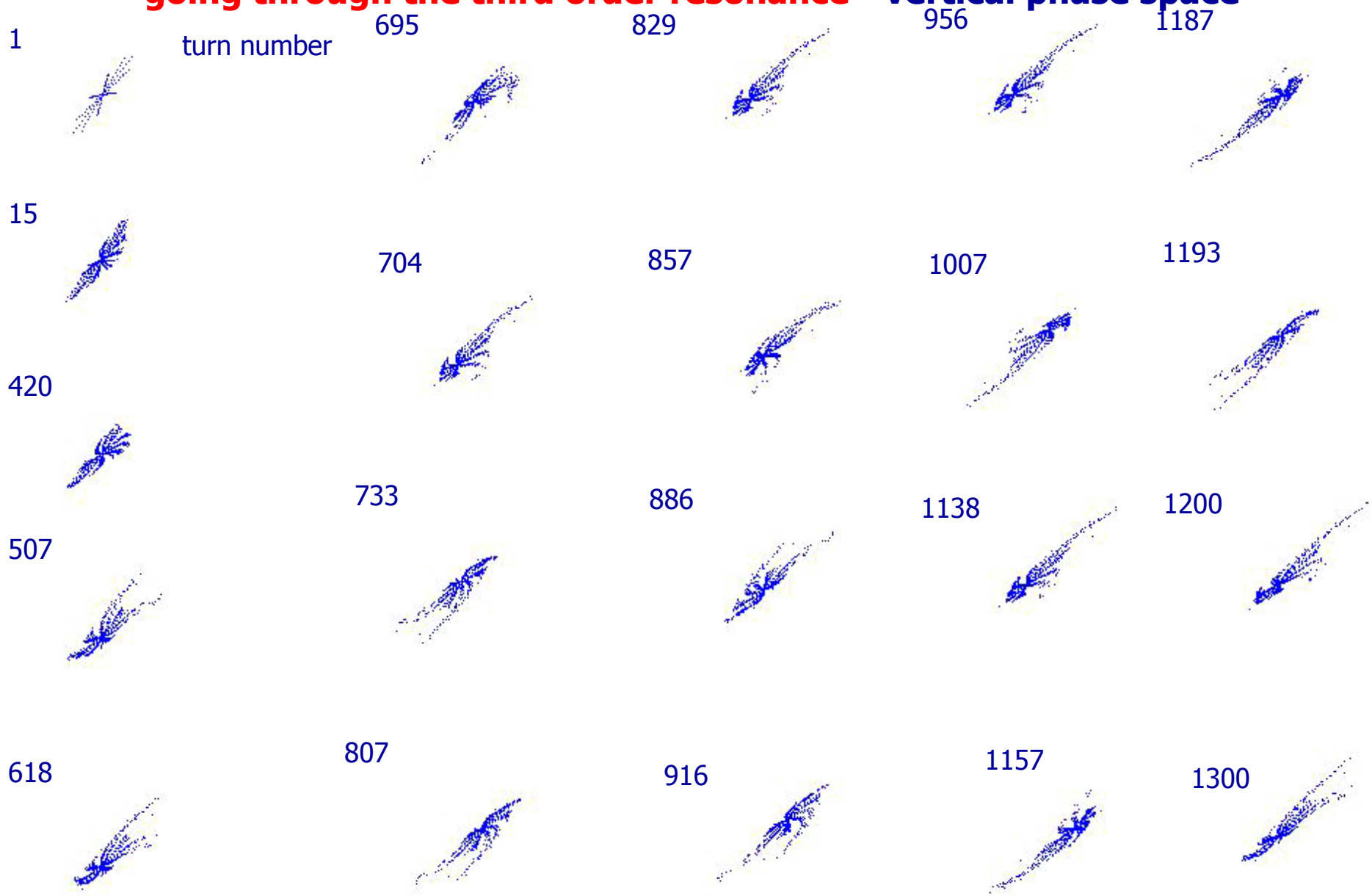




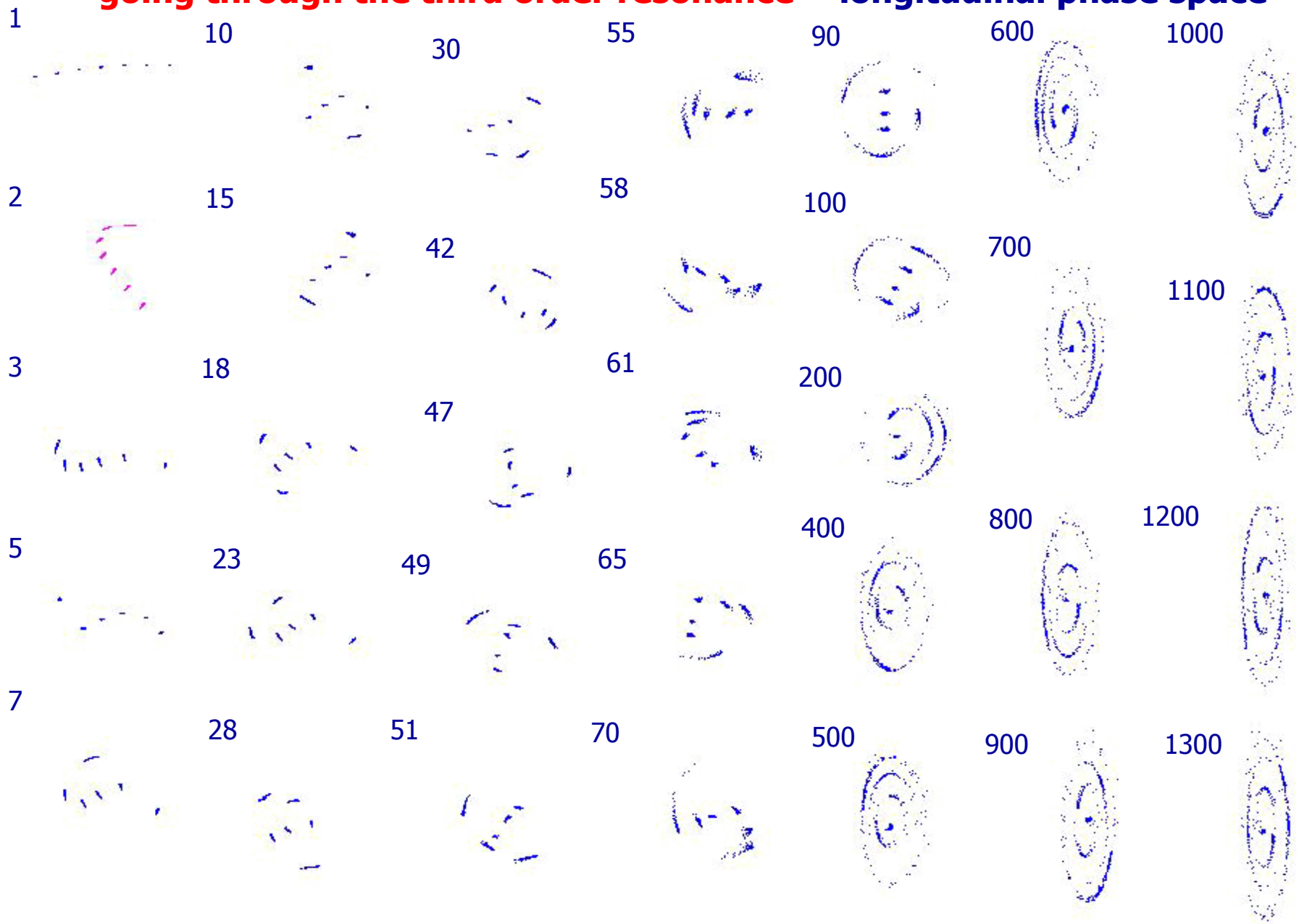
## 24 cells – twelve cavities 30 kV per cavity ~ 1300 turns:
## going through the third order resonance - horizontal phase space



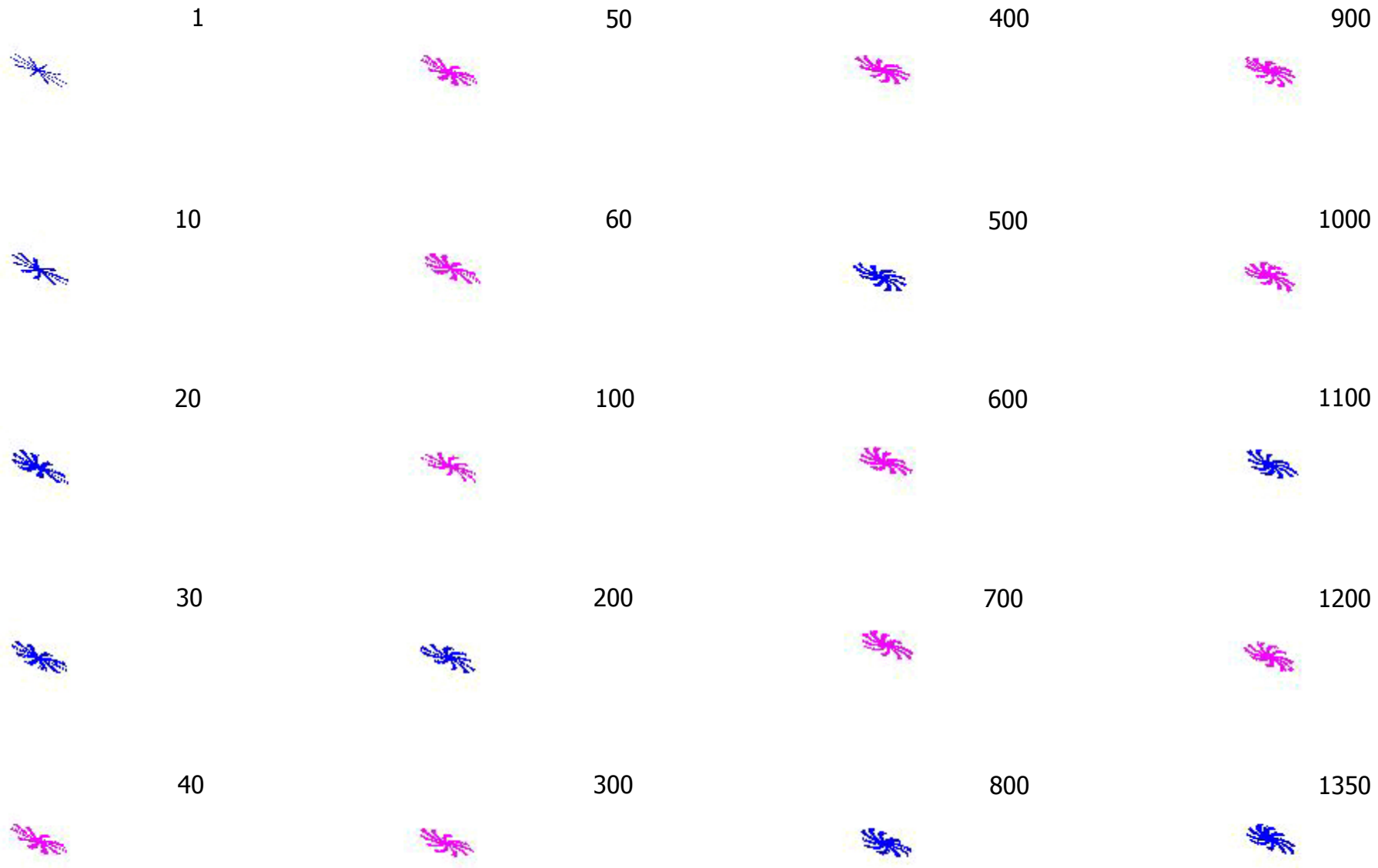
# 24 cells – twelve cavities 30 kV per cavity ~ 1300 turns:
# going through the third order resonance - vertical phase space



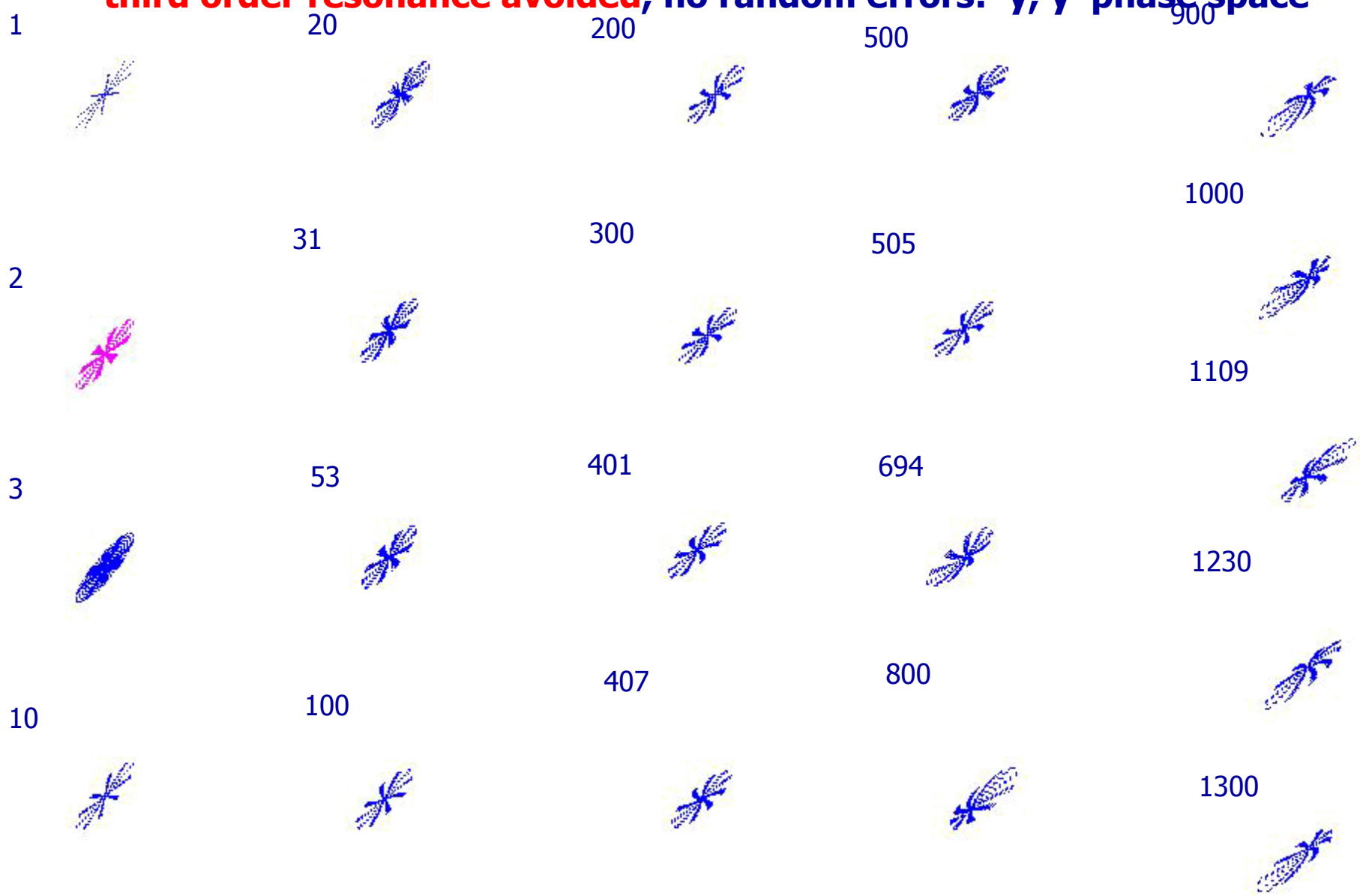
## 24 cells – twelve cavities 30 kV per cavity ~ 1300 turns:
## going through the third order resonance - longitudinal phase space



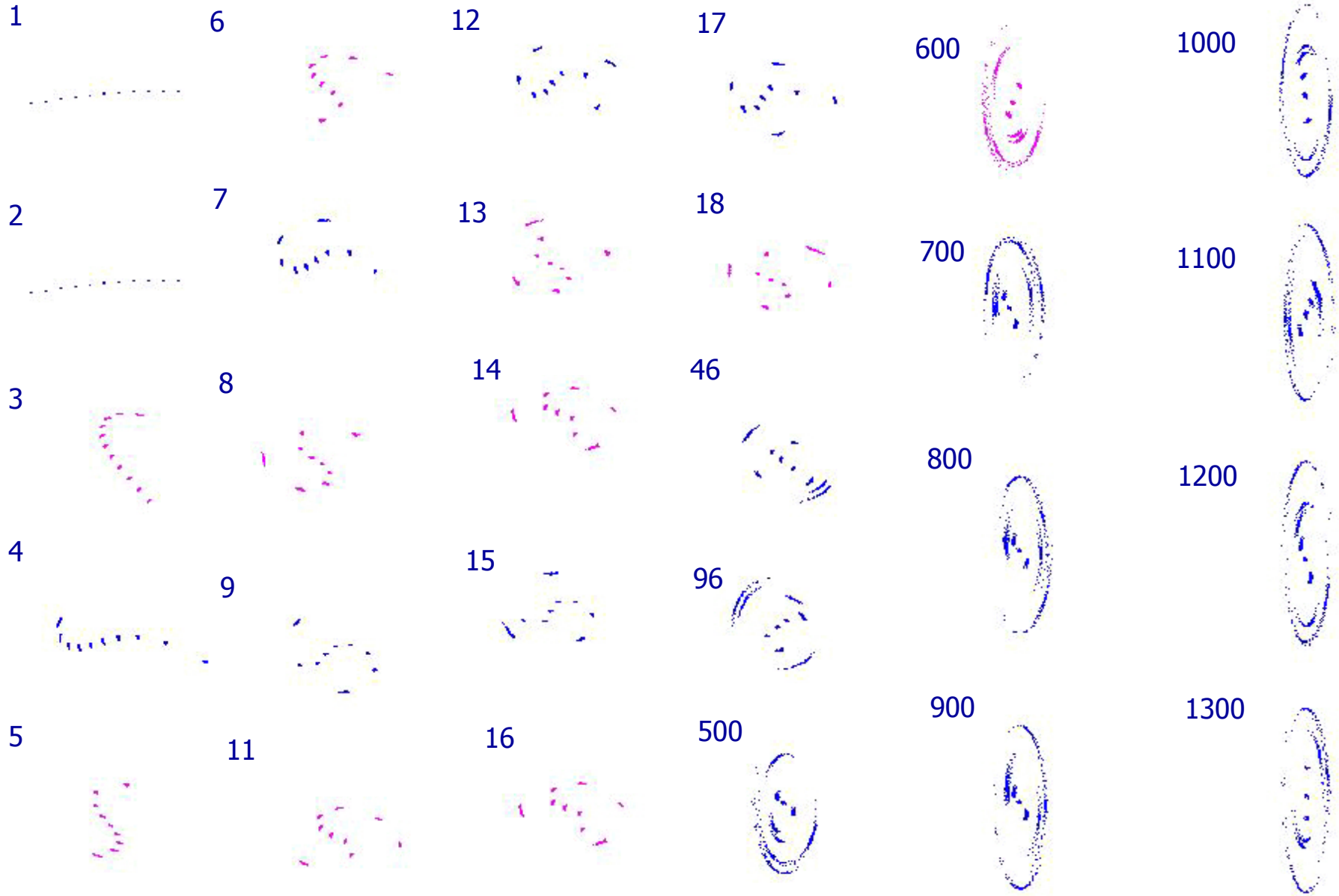
## 24 cells – twelve cavities 30 kV per cavity ~ 1300 turns: third order resonance avoided, no random errors: x, x' phase space



## 24 cells – twelve cavities 30 kV per cavity ~ 1300 turns: third order resonance avoided, no random errors: y, y' phase space



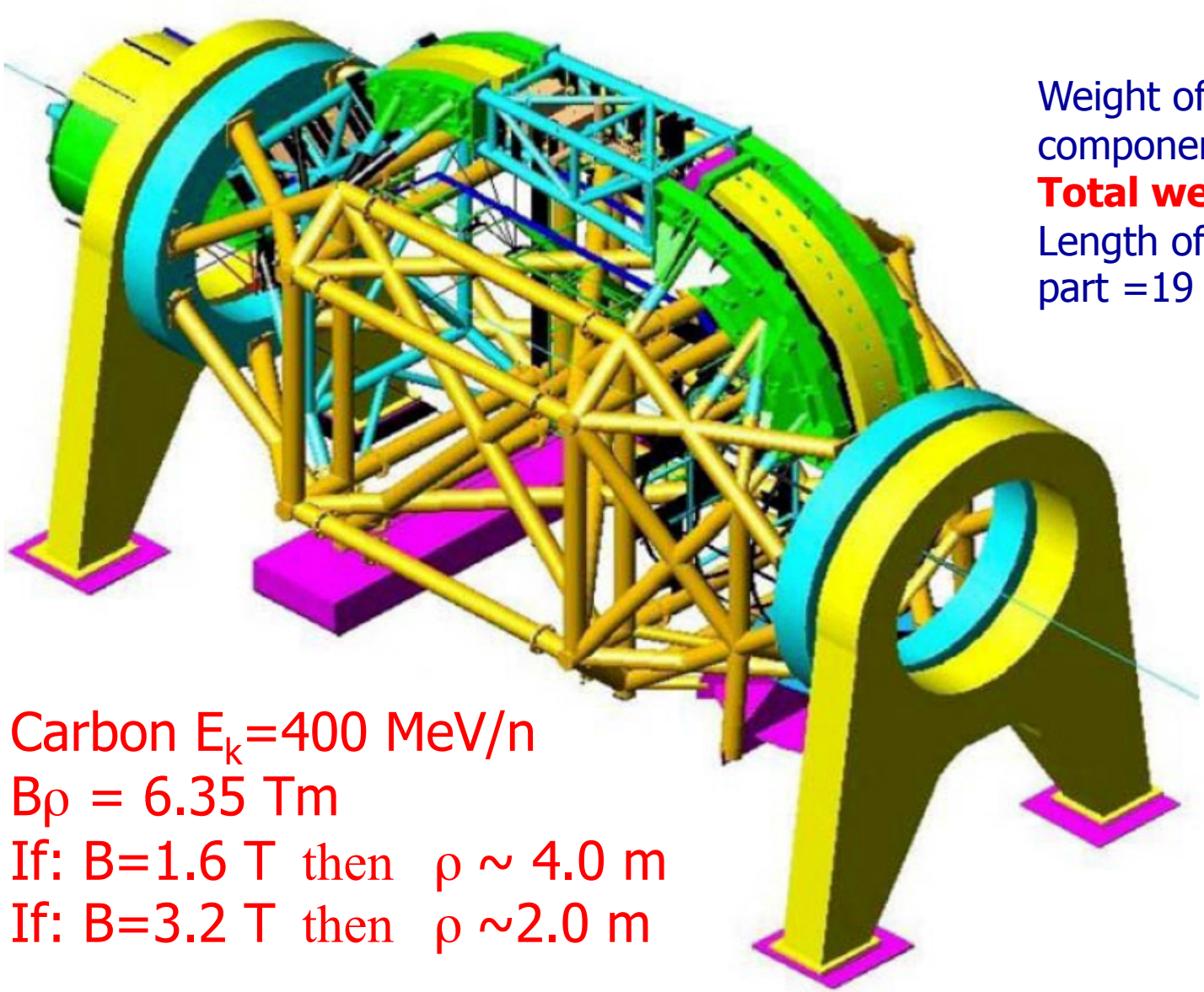
## 24 cells – twelve cavities 30 kV per cavity ~ 1300 turns:
## Third order resonance avoided, no random errors - longitudinal phase space



# MOTIVATION: large weight of the present gantries

- Large $B\rho=6.35$ Tm for carbon ions of $E_k$=400 MeV/n requires large magnetic fields.
- Presently the beam scanning requires very large magnet at the end of the gantry to accommodate parallel beams to the patient.
- Results are: very large magnets and large weight of the transfer line and the whole support (630 /tons). The carbon/proton cancer therapy facilities constraints are very difficult to fulfill with the warm temperature magnets.
- This leads us to a new concept – non-scaling light (1.5 tons) small superconducting gantry.

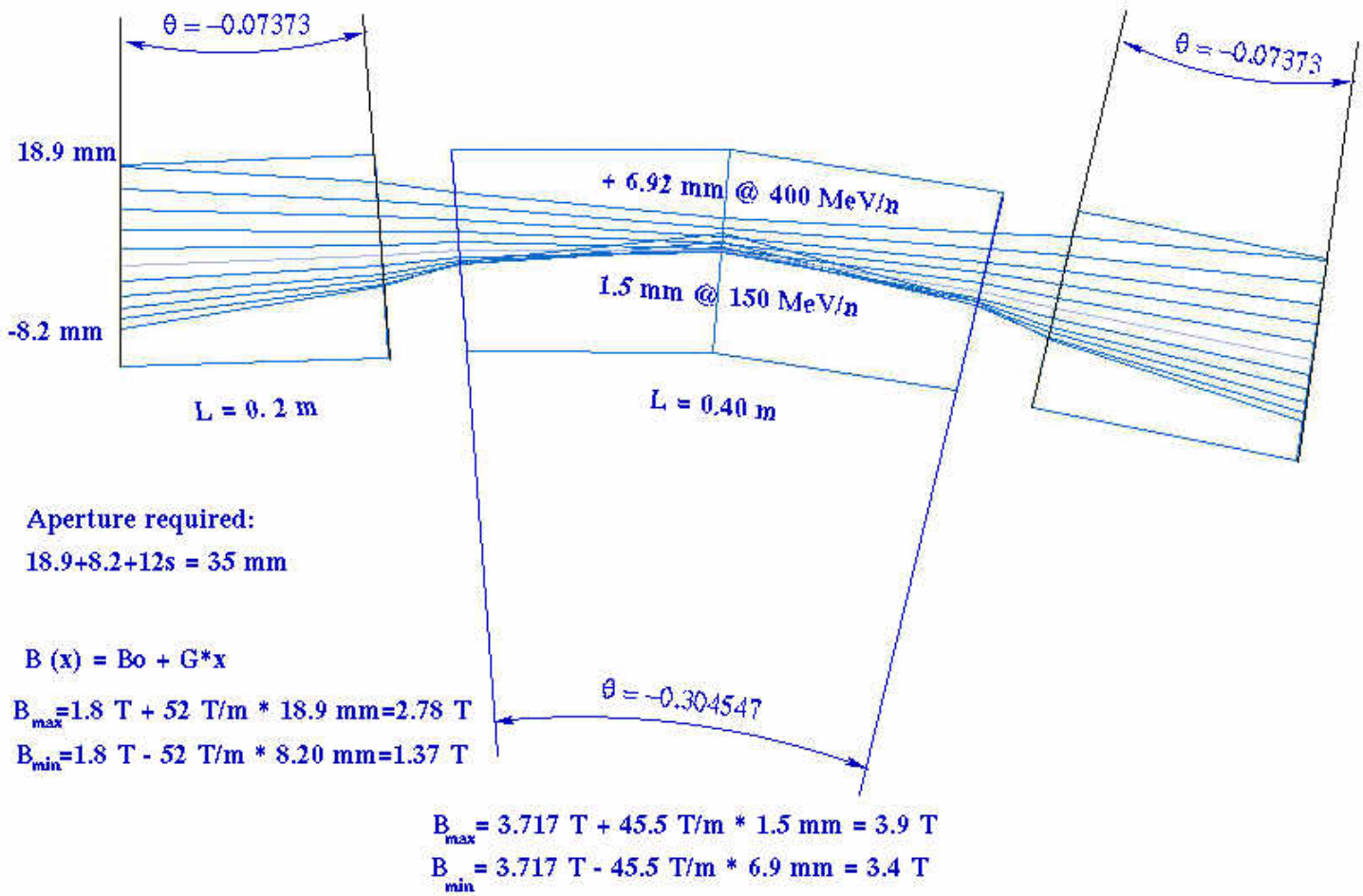
# Carbon Gantry in Heidelberg



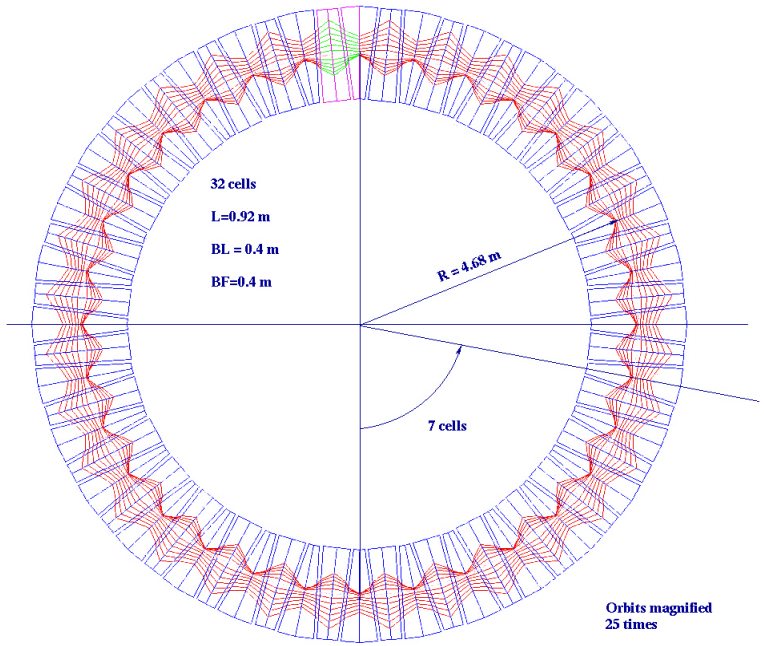
Weight of the transport components – **135 tons**
**Total weight = 630 tons**
Length of the rotating part =19 m

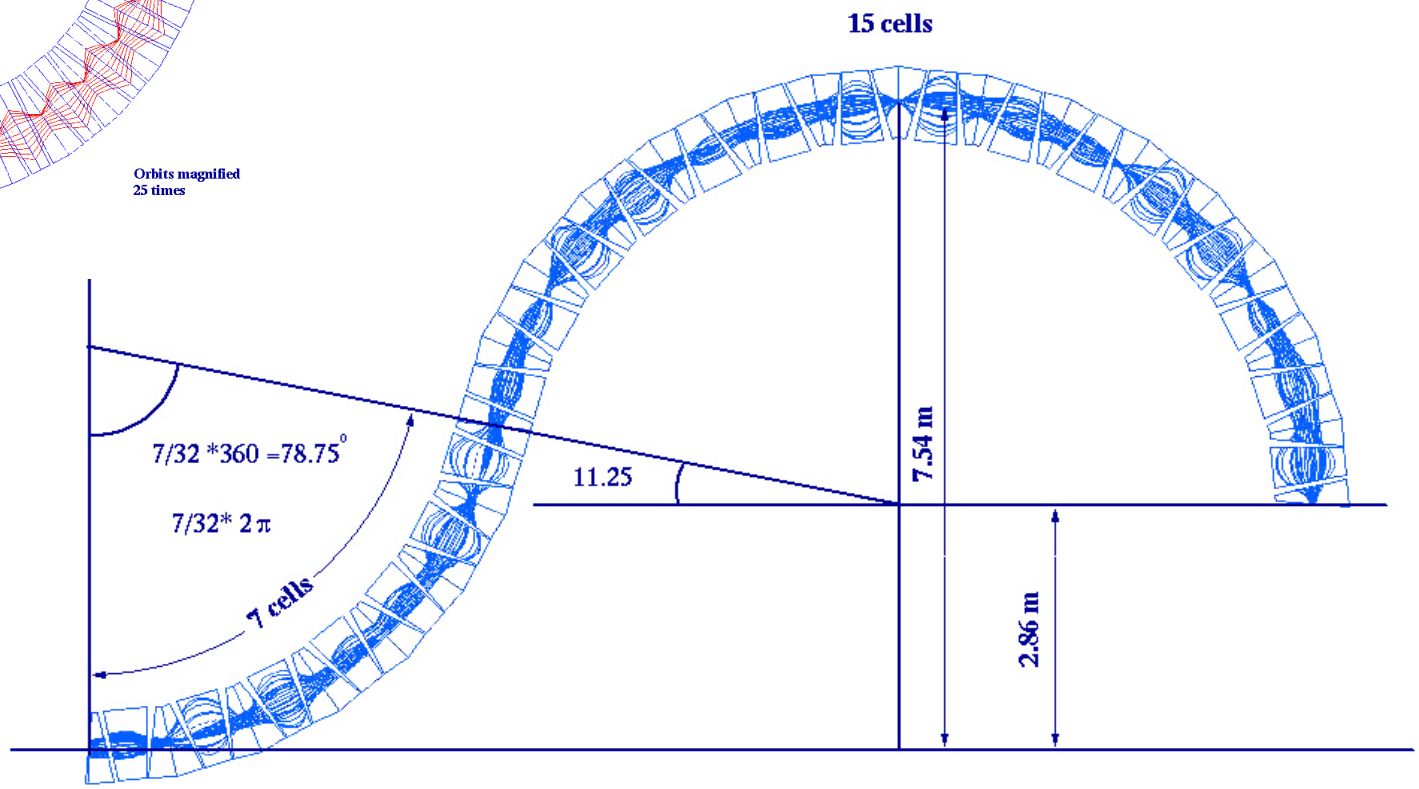
Carbon $E_k$=400 MeV/n
$B\rho$ = 6.35 Tm
If: B=1.6 T then $\rho \sim 4.0$ m
If: B=3.2 T then $\rho \sim 2.0$ m

# Example of the non-scaling FFAG carbon gantry:

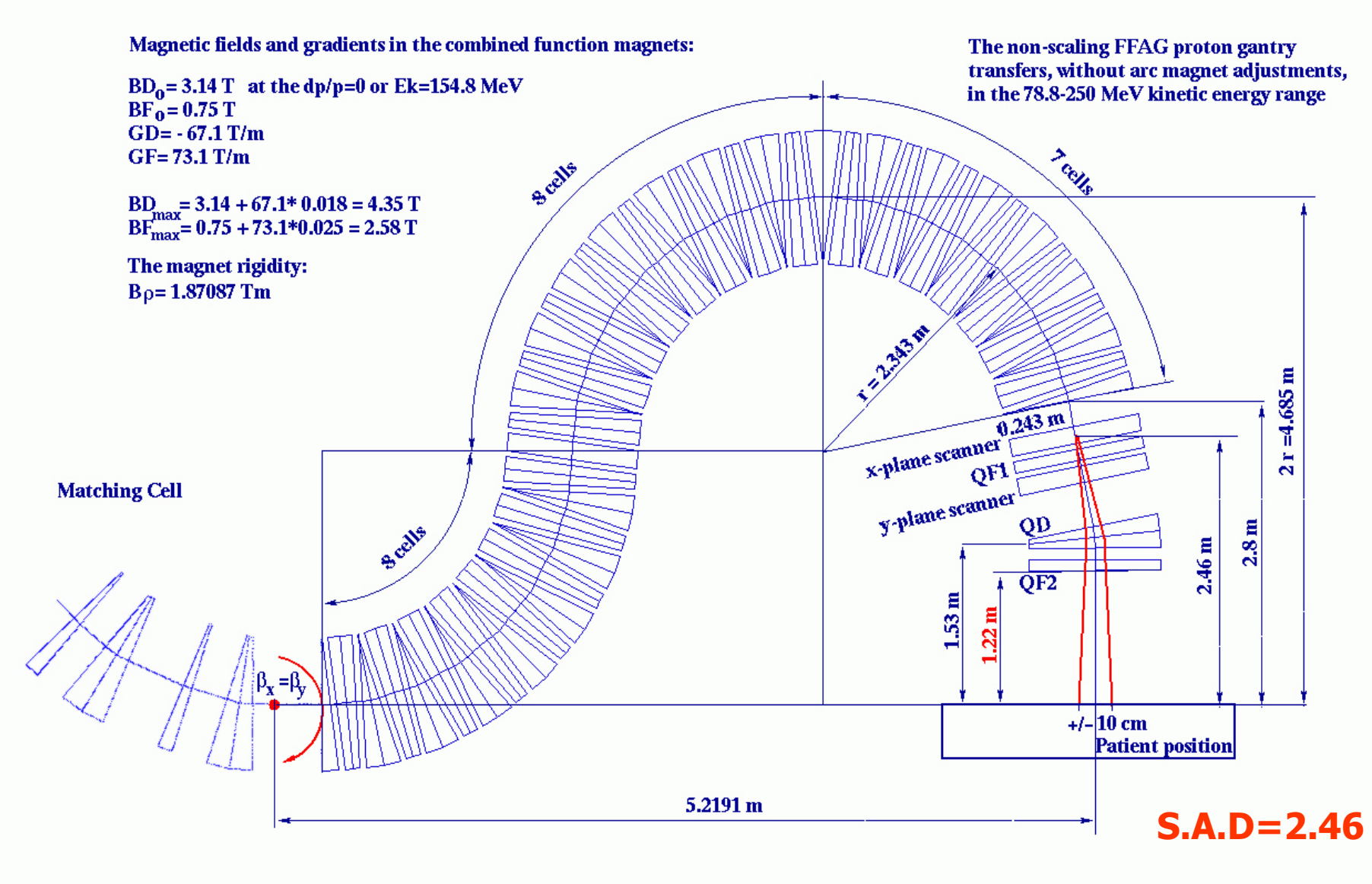


# Carbon gantry design



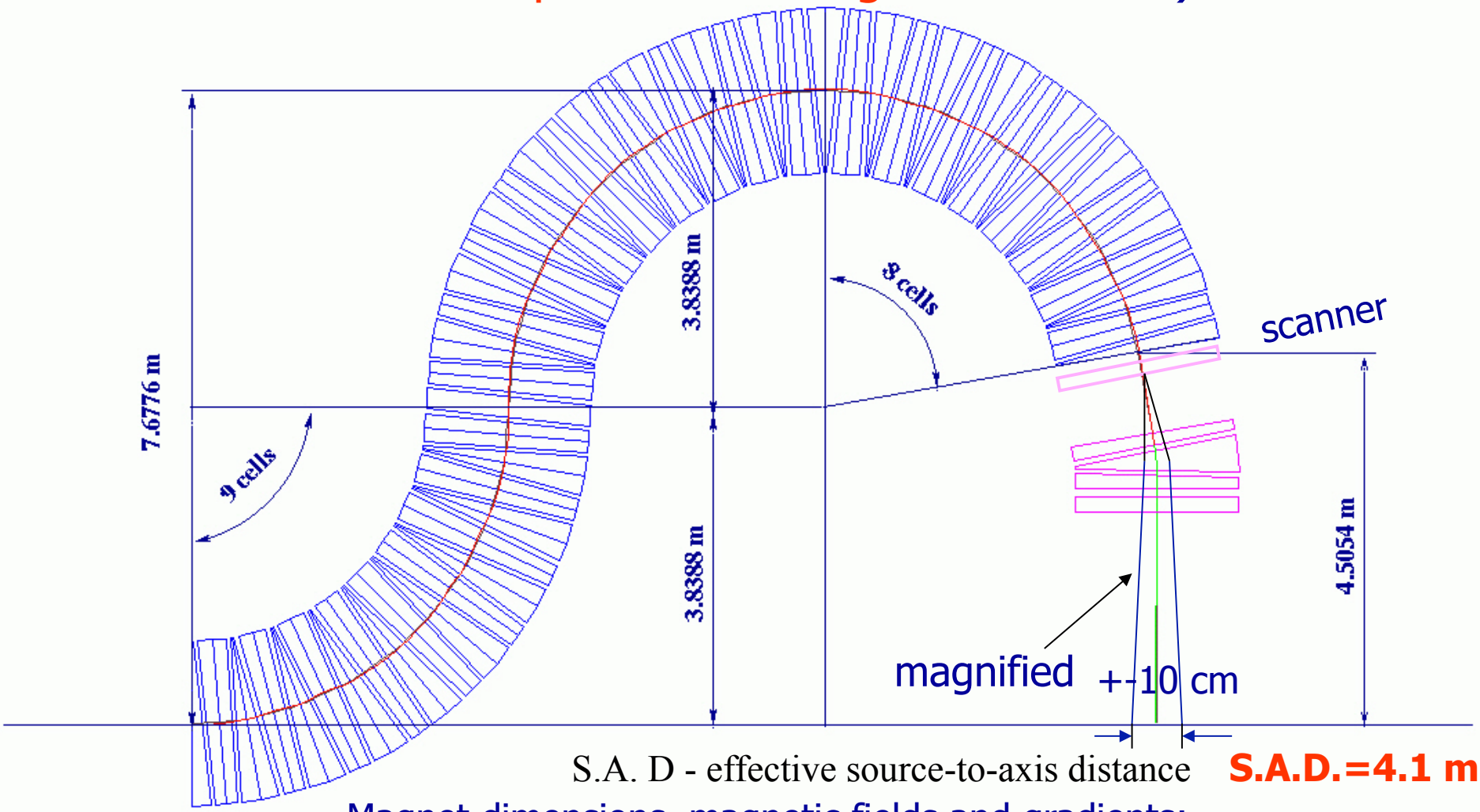



# Smaller non-scaling FFAG proton gantry – height 4.7 m




**S.A.D=2.46 m**

# Proton Gantry with triplet and scanning magnets(it could be built with small permanent magnets Φ=2 cm)



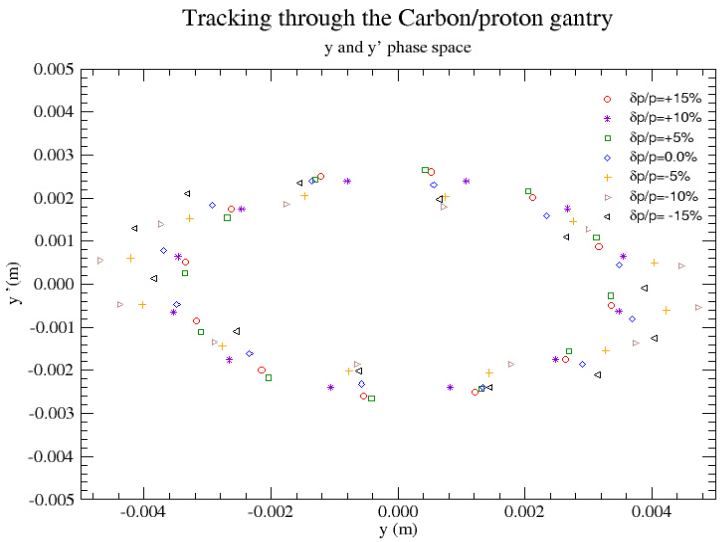

S.A. D - effective source-to-axis distance **S.A.D.=4.1 m**

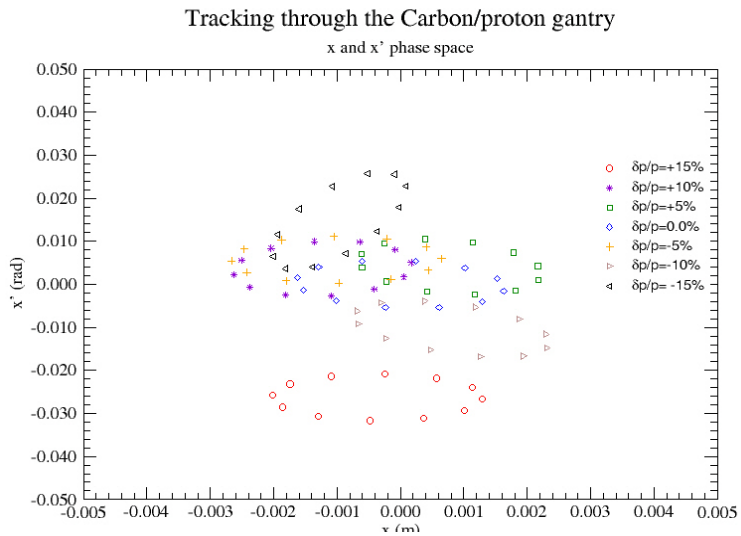
Magnet dimensions, magnetic fields and gradients:

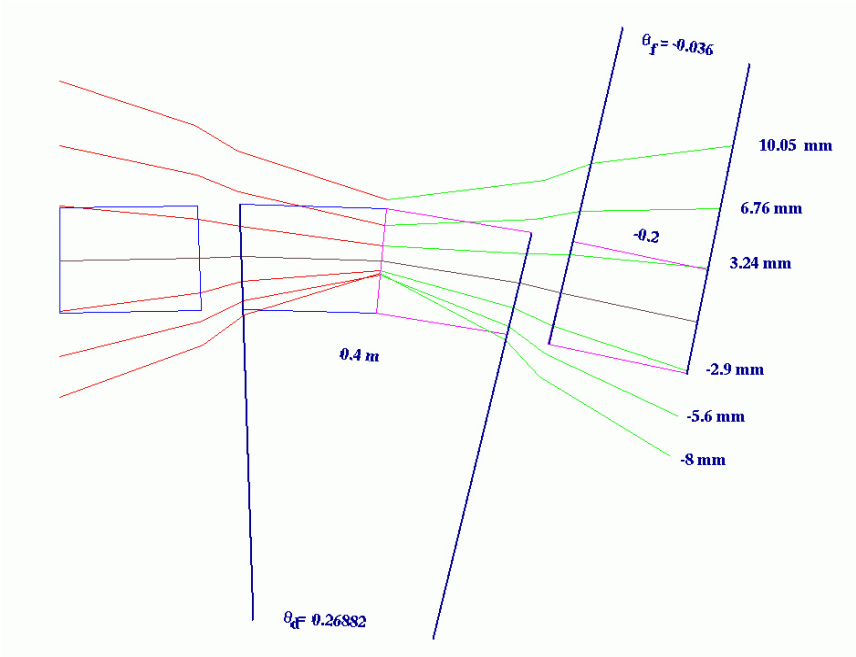
$L_{_BD}$ = 25 cm, GD =-33.7 T/m, $B_{max}$= 1.5 T + 33.7 T/m*0.012 mm = 1.9 T

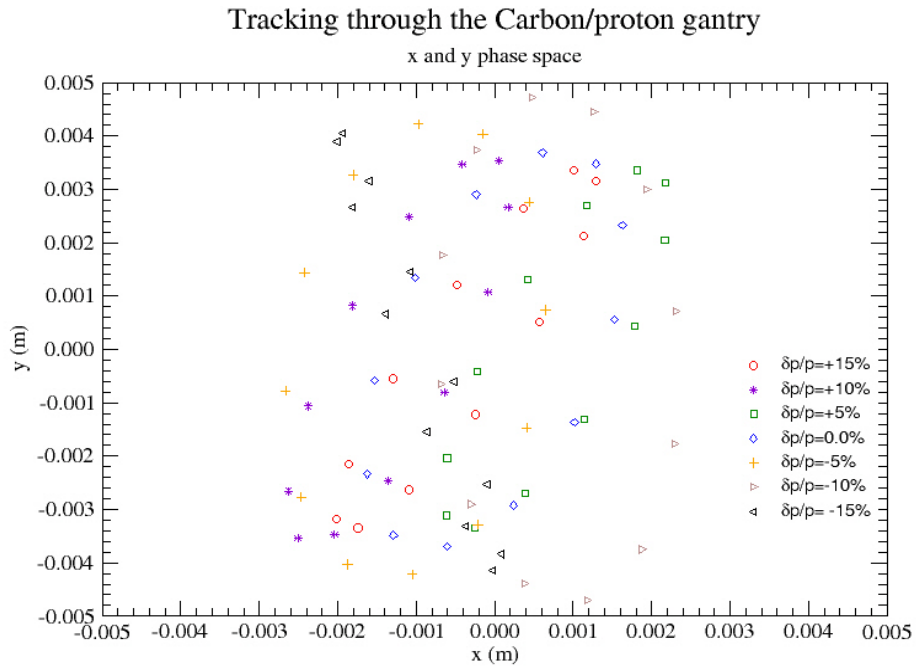
$L_{_BF}$ = 30 cm, GF =+35.5T/m, $B_{max}$=-0.25 T -+ 35.5 m*0.028 mm = 1.2 T

# Carbon gantry presented at PAC07



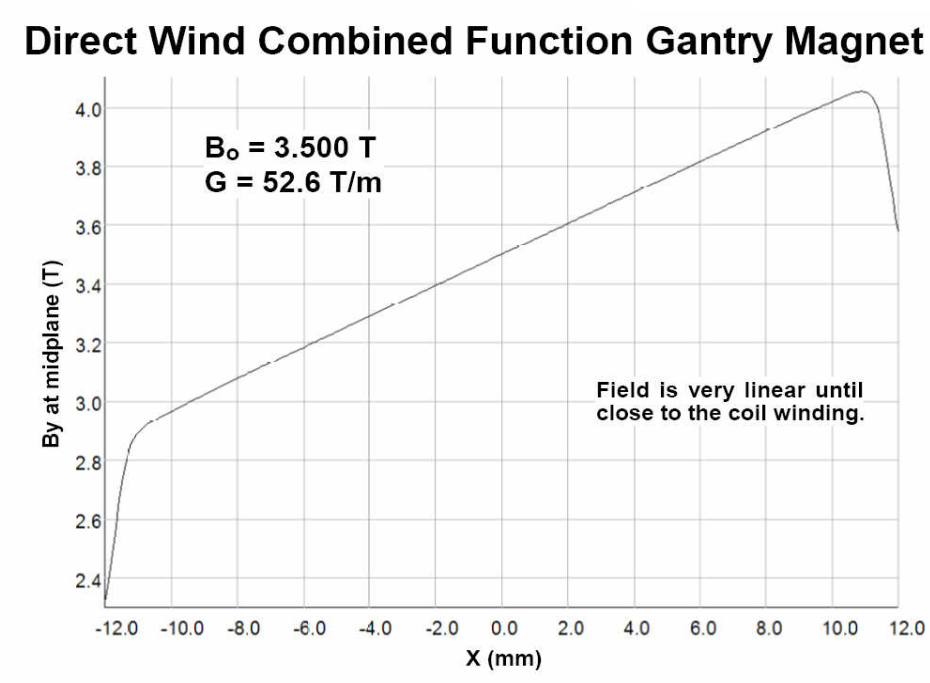



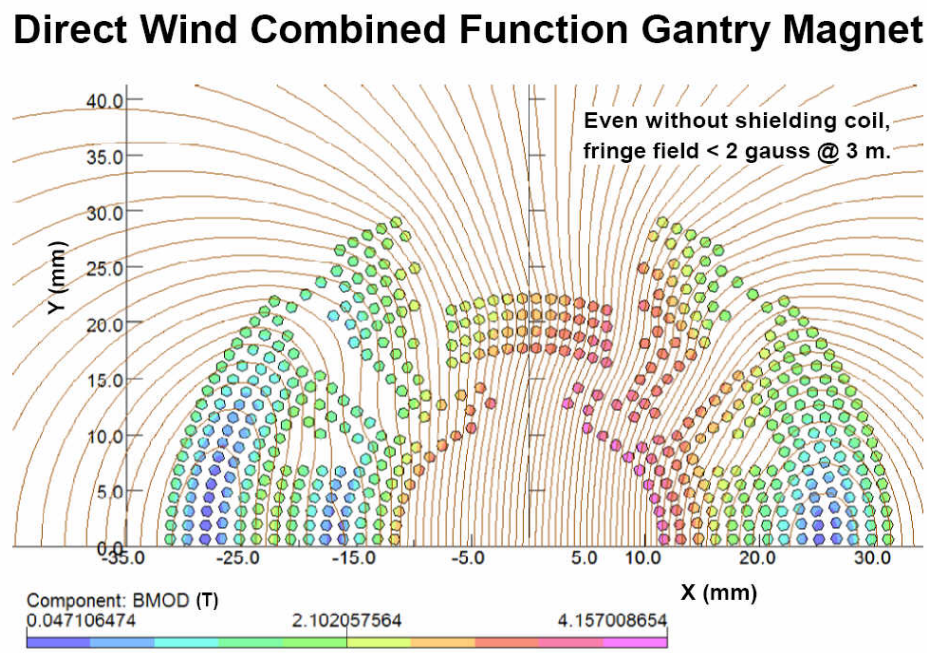

**-5 mm** **+ 5mm**

# Combined Function magnet for the Carbon Gantry (Brett Parker design)

Table 1: Magnet properties

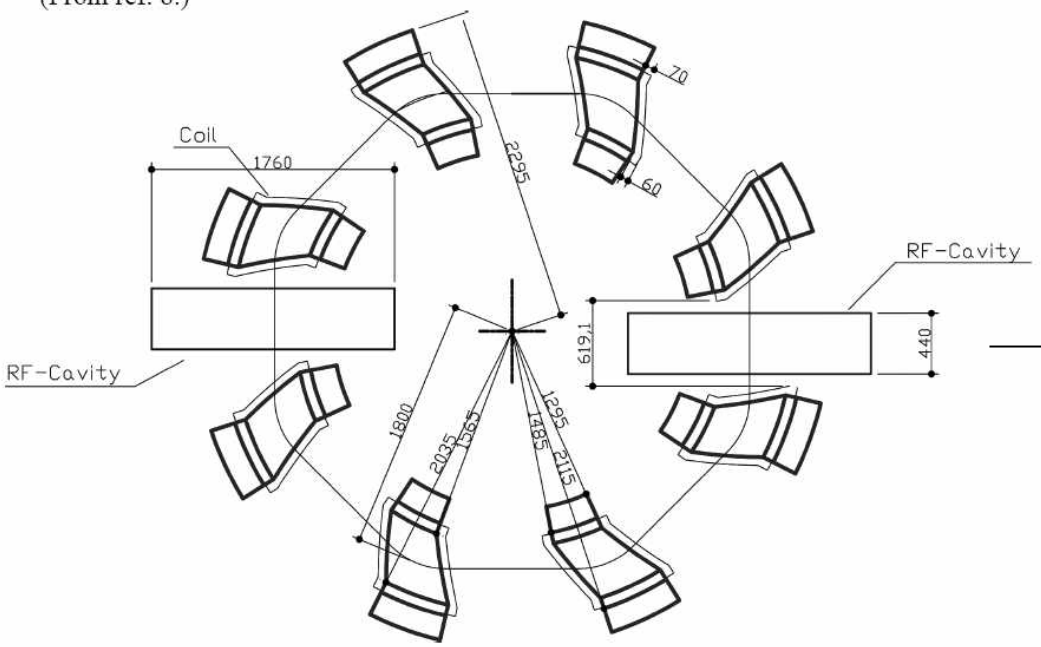
| | L(m) | B(T) | G(T/m) | $A_p$(m) | $B_{max}$(T) |
|---|---|---|---|---|---|
| BD | 0.40 | 3.7-4.3 | -68.5 | ± .008 | 4.24 |
| BF | 0.40 | 1.00 | 71 | ± .010 | 1.8 |

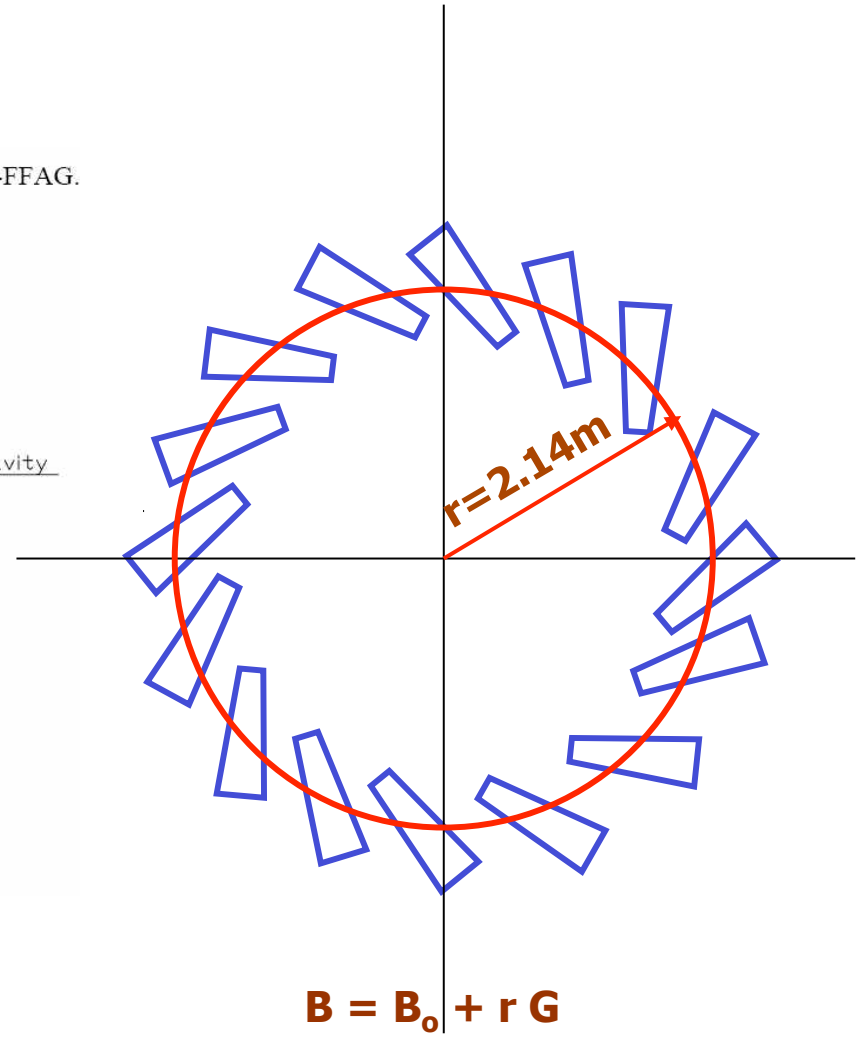


# Spiral scaling FFAG magnets

Figure 3. Design view of a spiral-sector storage storage ring considered for ERIT-FFAG. (From ref. 8.)



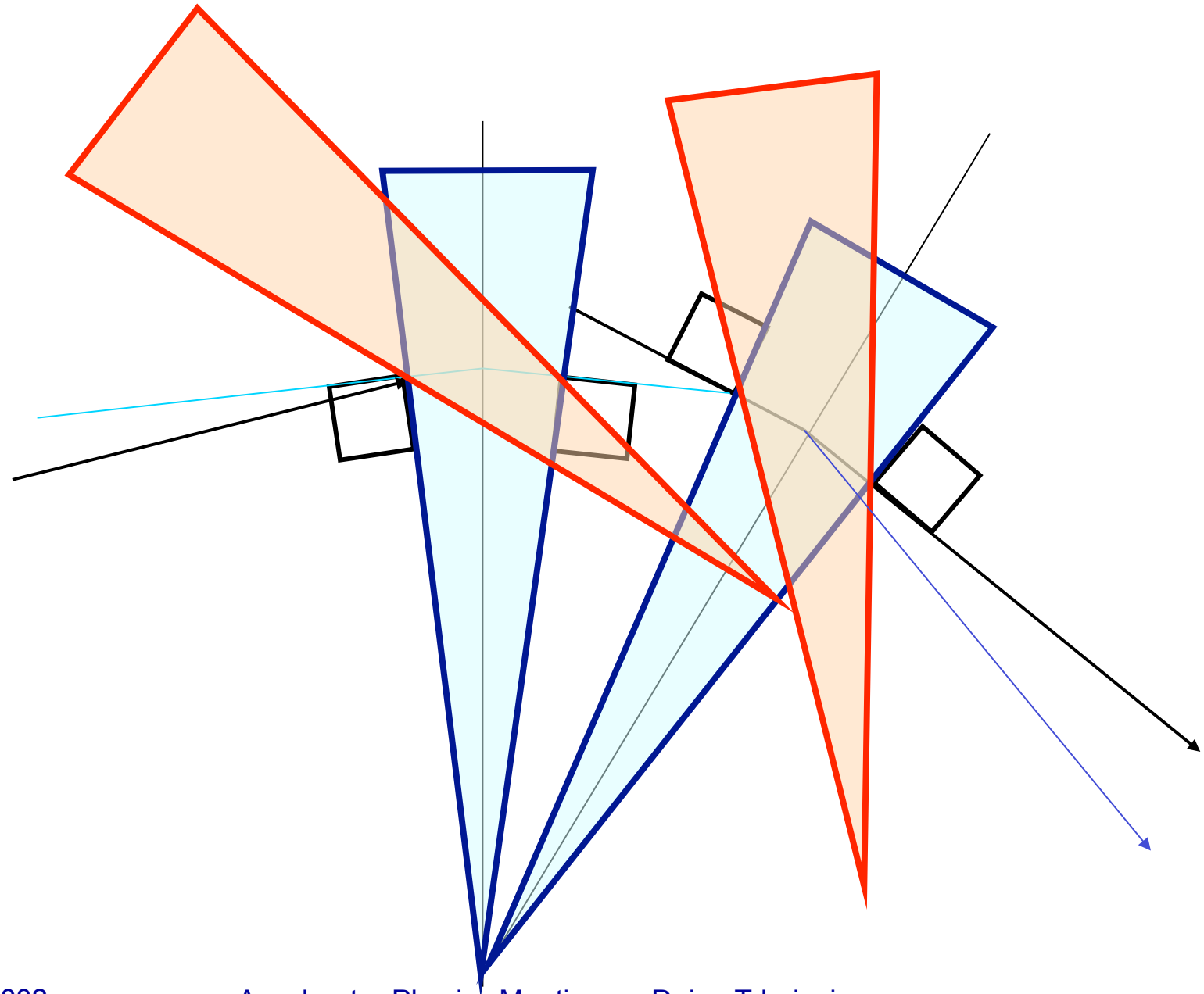

$B = B_o + r\,G$

$-20\% < \delta p/p < +50\%$

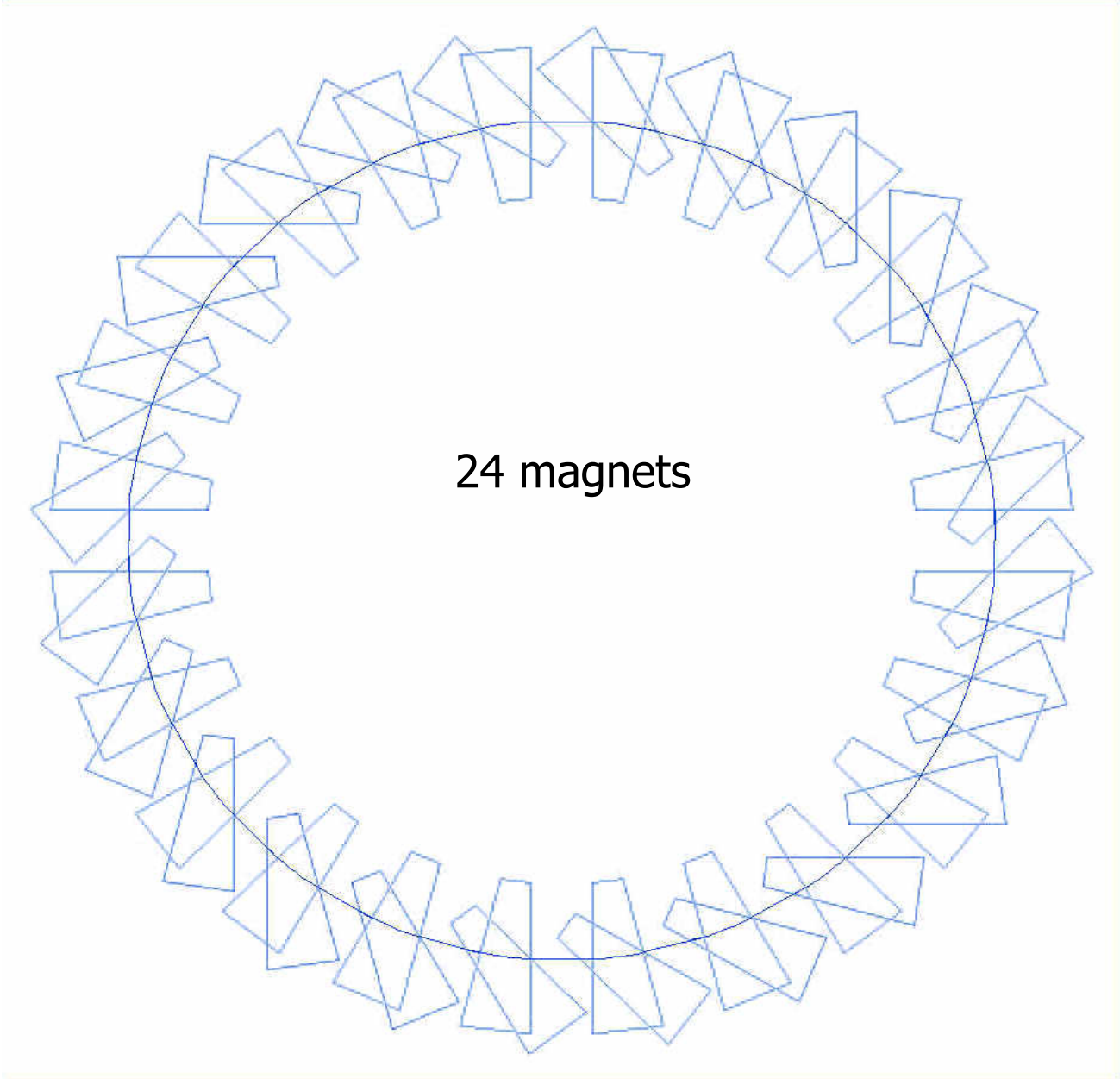
$E_{k_min} < E_{kc} < E_{k_max}$

$77.5 < 118.4 < 250$ MeV

# Rotated Linear Sector Magnet – using the edge fields



# Non-scaling FFAG single magnets with linear fields



# Summary

- Presented the way how a new concept of the non-scaling FFAG had developed.
- Layed out problems and advantages: correct Hamiltonian, third order resonance in the case of multi-turn applications, end of magnetic field in the combined function dipole, showed limitations of the concept ($\delta p/p = \pm 60\%$) as well as advantages: fixed magentic field, extreme focusing, saving on linac length, …
- Tried in a very short time to summarize work in a recent non-scaling FFAG area – found it to be very difficult.