Electron Beam Cooling Calculations Based on Plasma Relaxation Formalism; Is Single Pass Cooling Possible?

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Prevailing plasma physics relaxation time formalism is applied to electron beam cooling, with higher density electrons than in presently used electron beams, in contrast to currently used electron beam cooling computations. As an example electron beam cooling is examined, **using the test particle model**, as an option to reduce momentum of gold ions exiting the EBIS LINAC before injection into the booster. Electron beam parameters are based on experimental data (obtained at BNL) of electron beams extracted from a plasma cathode. Preliminary calculations indicate that **single pass cooling might be feasible**; momentum spread could be reduced by an order of magnitude in about one meter. The two approaches are not contradictory, since each is applicable to different plasma regimes.

Due to seemingly still debatable issue of the Booster acceptance and due to the need for further needed electron gun R&D, the presentation is focused on differences and applicability of different models to cooling computations, and validation of a previously used approach.

Although the subject matter might still be academic at present, “in-principle” feasibility must be ascertained before proceeding further.

Friction Calculations Models (Analogies? Shielding is Missing)

Friction force due to multiple small-angle binary collisions = ball moving through a field of billiard balls.

Slowing down (dynamic friction) under test-particle plasma relaxation might be analogous to a ball moving through and interacting with many billiard balls simultaneously (Vlasov is wave-particle interaction).

Fokker-Planck equation is basically a model for treating collisional relaxation due to binary collisions. With it changes in plasma distribution are calculated due to binary collisions in terms of the cross section for two-body scattering.

But in dense plasma, a particle is not seen as bare. It carries its shielding cloud when undergoing scattering. The test particle model includes shielding in collision calculations.

Basic difference: Fokker-Planck type collisional relaxation due to binary collisions describes a process that occurs in vacuum, while collisional relaxation based on the test particle model are essentially collisions that occur in a dielectric medium.

Ion Beam Parameters

Expected ion beam parameters at the exit of the EBIS-RFQ-LINAC system are: energy 2 MeV/u, momentum spread $\Delta p_{\parallel}/p = 10^{-3}$, and $\Delta p_{\perp}/p = 5x10^{-3}$, beam diameter 1 cm, and gold ion charge state Au^{+32}, with ion density[2] at the LINAC exit n_i=8x10^7 cm^{-3}. For electrons to match ion velocity, their energy U must be about 1 KeV. At these energies, ion and electron velocities are about 2x10^7 meter/second, hence β = 0.0667 and γ = 1.0022.

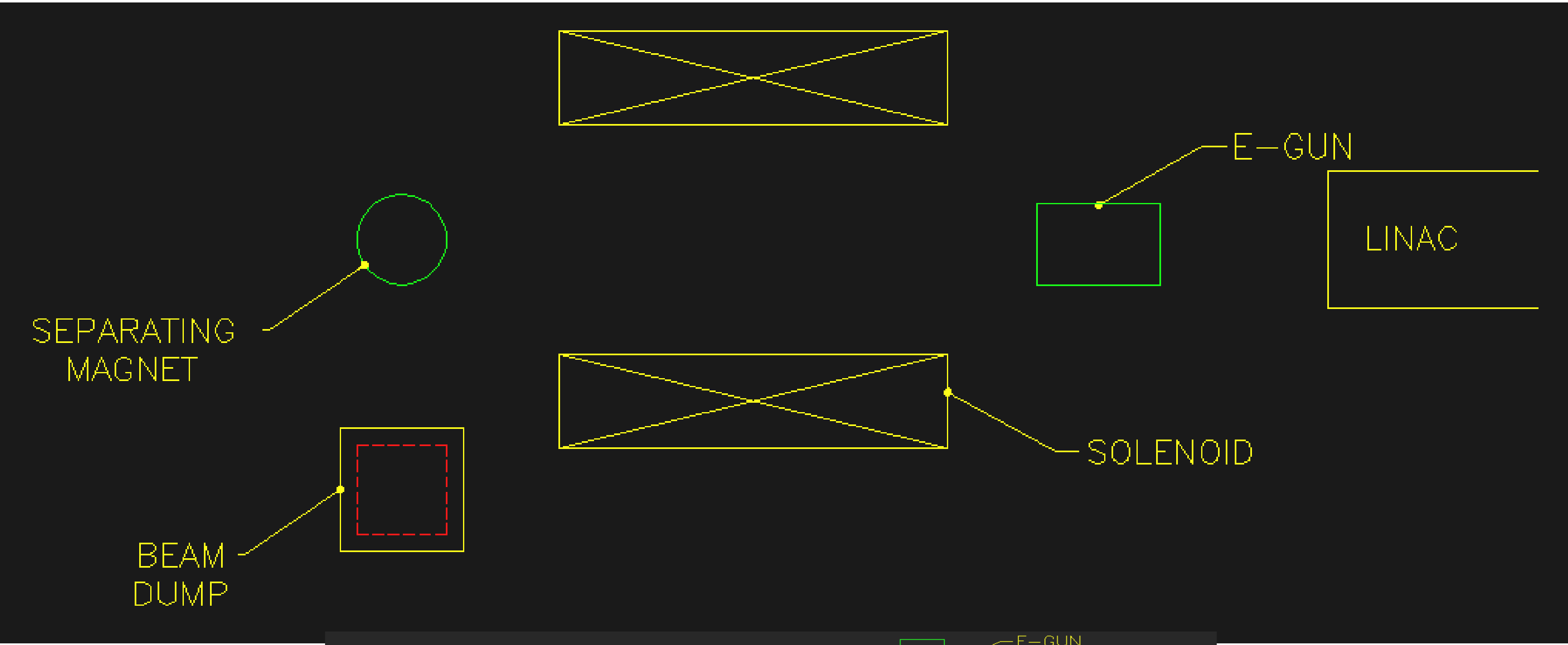
Electron Beam Parameters

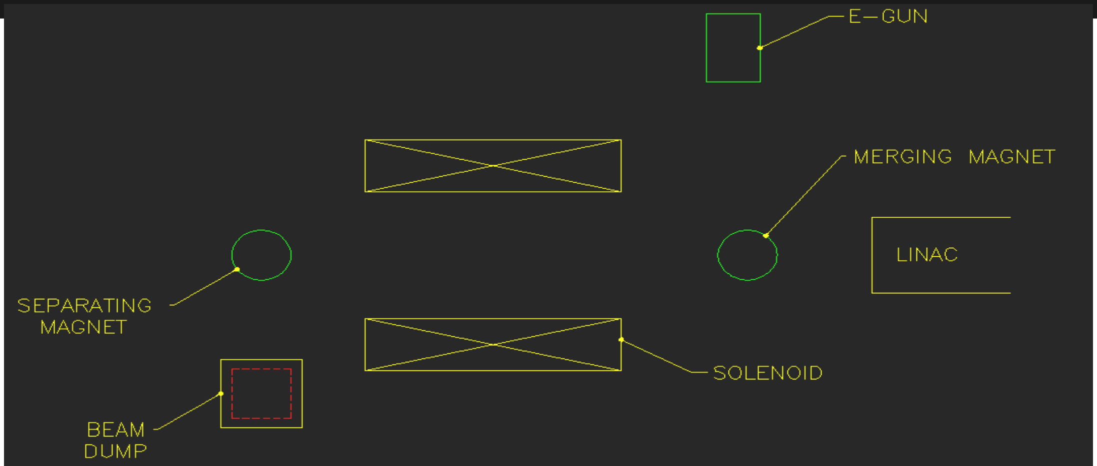
Previously developed electron gun with plasma cathode, from which 9 A were extracted at 1 KeV through a 6 mm aperture, is considered. Based on these parameters the electron density n can be computed from $n = {}^{I}\!/_{Aev}$, to be about $n \approx 10^{11}\, cm^{-3}$. Balk electrons energy spread before extraction was about 0.1 eV. Due to kinematic compression, energy spread of the accelerated electrons T_e is $T_e = T * 0.5\left({}^{T}\!/_{U}\right)^{1/2}$, where T is thermal spread of unaccelerated electrons. For T = 0.1 eV and U = 1 KeV, T_e = 5×10^{-4} eV.

Presentation Layout

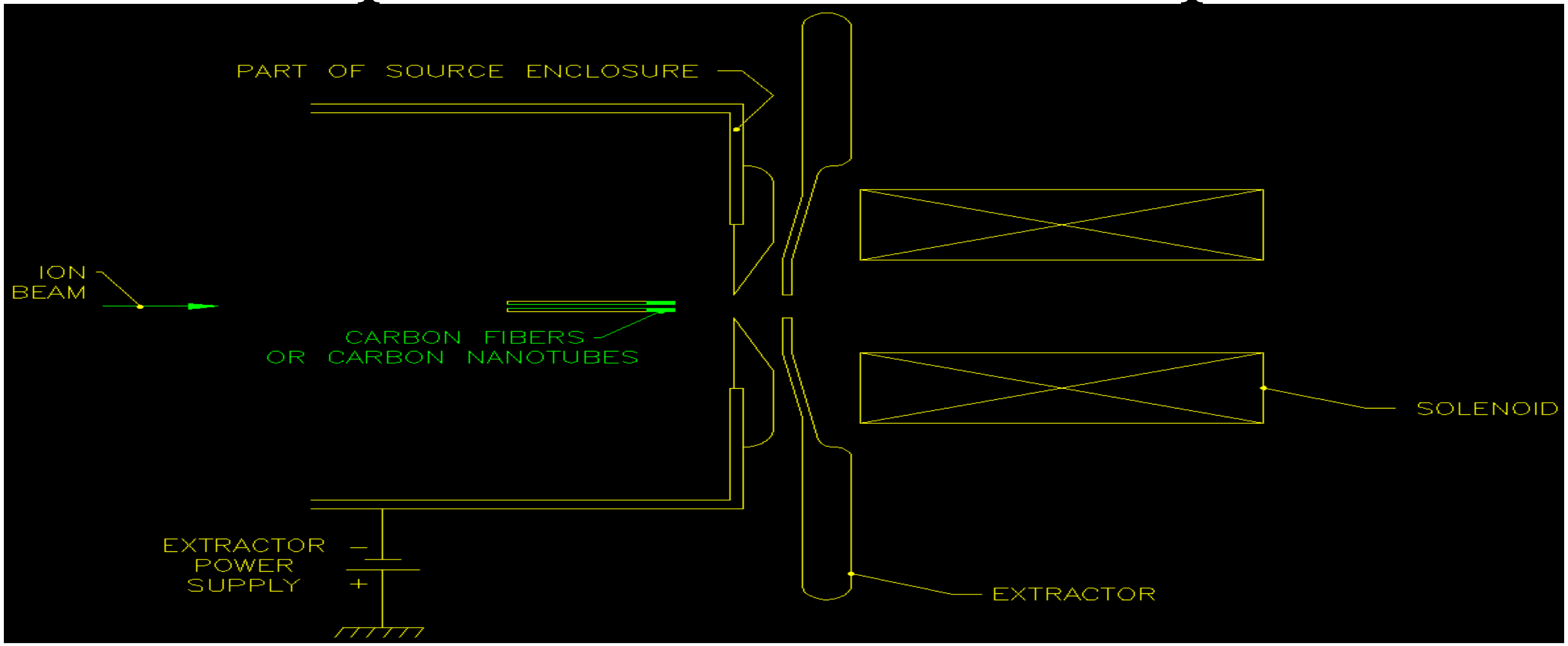
- Ion and E-beam parameter; embodiments.
- Conventional cooling formulas & their origin (very Short).
- Cooling based on Parkhomchuk's empirical formula.
  Parkhomchuk claims that an empirical formula is in much better agreement with computer simulations of fully magnetized cooling for magnetic fields of up to 4kG. Additionally, experiments and computer simulations showed reasonable agreement with Parkhomchuk.
- Test Particle Model: derivation and cooling/heating calculations. Derivation of Parkhomchuk's empirical formula. Heating calculations. *New issue to be resolved.*
- Other pertinent physics issues
- Justification for using the Test Particle Model: when to use which model

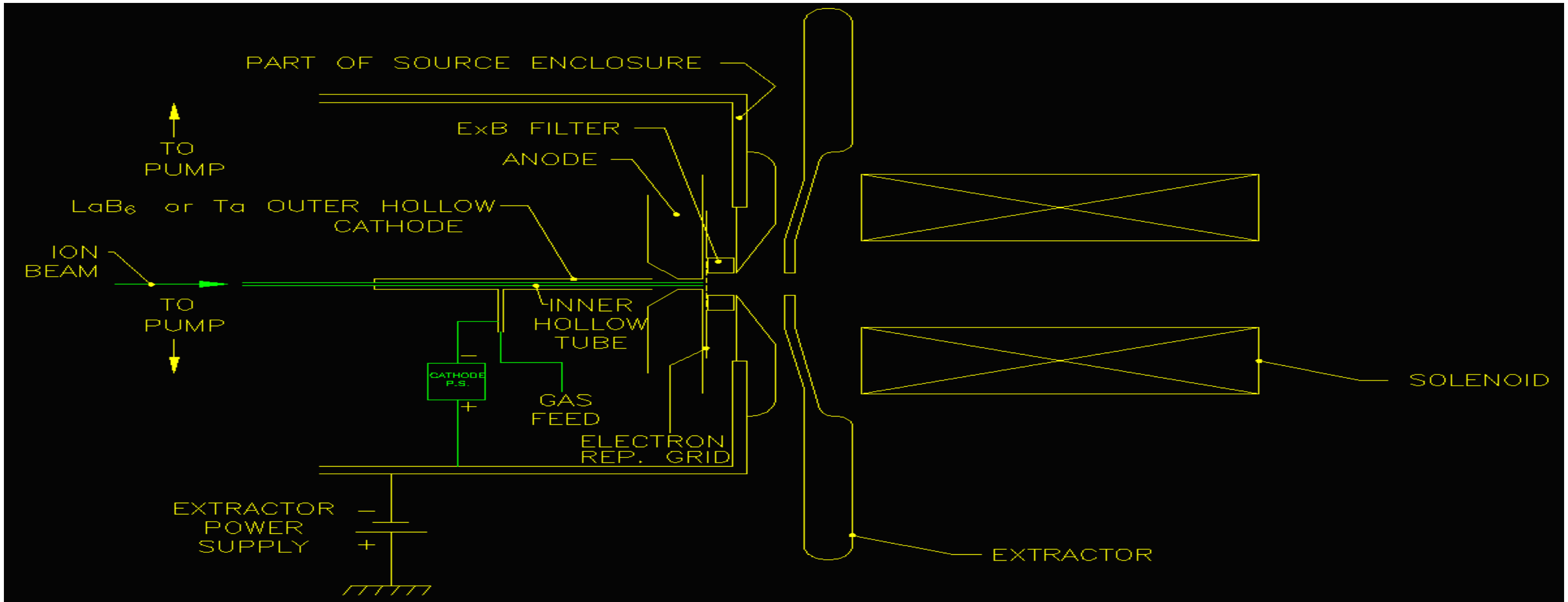
Possible Embodiments





A Couple of Electron Gun Options





Origin of "Conventional" Cooling Formulas

Boltzmann works well in gases: short-range forces. Fokker-Planck equation was originally derived to treat Brownian motion (short-range forces). Later was used to evaluate collision terms in the Boltzmann equation for cases of multiple small angles binary collisions due to long-range forces: Chadrasekhar multiple long-range binary collisions between stars, and Spitzer's long-range binary Coulomb collisions. Finally there is the elegant mathematical treatment by Rosenbluth, MacDonald, and Judd for multiple long-range binary Coulomb collisions in magnetized plasma and deriving the Fokker-Planck coefficients like friction and diffusion.

Belyaev and Budker derived energy transfer time, due to friction force for multiple small angle binary Coulomb collisions,. Based on that formalism, Budker proceeds to evaluate electron beam cooling for protons and antiprotons.

Finally, theory for computing magnetized friction forces & cooling times was derived by Derbenev and Skrinsky

$$F_{\|} = -\frac{2\pi e^4 Z^2 n\lambda}{M} \bullet \frac{3v_{i\perp}^2}{v_i^5} v_{i\|} \qquad F_{\perp} = \frac{2\pi e^4 Z^2 n\lambda}{M} \bullet \frac{2v_{i\perp}^2 - 2v_{i\|}^2}{v_i^5} v_{i\perp}$$

Cooling rate, which is proportional to ion energy E loss rate to electrons, is defined as $\frac{dE}{dt} = F \bullet V$. Hence, $\tau = \frac{Mv_i}{F}$

Consequence of "Conventional" Cooling Formulas

For Maxwellian distribution ion cooling due to the friction force is larger than ion heating due to velocity space diffusion by a factor of 2 only!

Based on Derbenev and Skrinsky, Particle Accelerators, 8, 235 (1978).

And unpublished note by Alexei Fedotov January 8, 2004.

Preceding Theory is Characterized by **Particle Discreteness** (binary collisions)

Parkhomchuk's Empirical Formula

Parkhomchuk claims that an empirical formula is in much better agreement with computer simulations of fully magnetized cooling for magnetic fields of up to 4kG. Additionally, experiments and computer simulations showed reasonable agreement with Parkhomchuk. (2.4 Tesla is used in this case; based on booklet)

Magnetized friction force and cooling time based on the empirical formula in cgs units, are (for the case of hot ions and cold electron) $F = v_i \dfrac{4Z^2e^4 n\lambda}{m v_{thi}^3}$ and cooling time

$$\tau_c = \frac{Mv_i}{F} = \frac{Mv_i m v_{thi}^3}{4Z^2e^4 n\lambda v_i} \qquad \lambda = \ln\left(\frac{\left[\frac{v_{thi}}{\omega_{pe}}\right]}{\rho_e}\right)$$

$\lambda = 3.4$; $\tau_c = 6\text{x}10^{-8}$ sec and a cooling length of 1.2 meter is needed.

Based on Derbenev and Skrinsky, Parkhomchuk, as well as a booklet by Dikansky, Kudelainen, Lebedev, Meshkov, Parkhomchuk, Sery, Skrinsky, and Sukhina, "*Ultimate Possibilities of Electron Cooling*," INP, Novosibirsk, USSR Report, Preprint 88-61 (1988), resultant cooling, electron energy spread, ion loss, needed magnetic field (**except for ion heating due to velocity space diffusion**) etc. are computed

Thermal equilibrium $T_{i\perp}$ is given by $T_{i\perp} = 5Ze^2 n^{1/3}\left(\sqrt{4\pi}\,\frac{\tau_0\Omega_e^2}{\omega_{pe}}\right)^{1/3}$ in cgs units, where τ_0 is time an ion spends in the electron beam, ω_{pe} and Ω_e are electron plasma and cyclotron frequencies respectively. For a magnetic field of 2.4 Tesla, and τ_0 of $6x10^{-8}$ sec, the perpendicular ion total velocity spread is reduced to $7.9x10^3$ m/s, and the transverse momentum (full) spread to $\Delta p_\perp / p \approx 3.9x10^{-4}$, & the parallel momentum spread is reduced by an order of magnitude as well. Electron beam thermal spread due to the electrostatic space charge potential $e^2 n^{1/3}$ in cgs units yields an energy spread of $6.68x10^{-4}$ eV, which is orders of magnitude lower than the ion beam thermal spread. Inelastic interactions are: electron capture and ionization. But due to the relative low energy differential, the only ion **loss** mechanism is due to **recombination**,

$$\alpha = 3.02x10^{-13}\frac{Z^2}{\sqrt{T_{e\perp}}}\left[\ln\left(\frac{11.32Z}{\sqrt{T_{e\perp}}}\right) + 0.14\left(\frac{T_{e\perp}}{Z^2}\right)^{1/3}\right]$$

Rate coefficient for recombination $\tau_{rec} = \frac{\gamma}{n\alpha}$ in cgs units except for T, which is in eV. For our parameters **τ_{rec} is about 1.5 msec**, which **is orders of magnitude longer than any computed cooling time**. Additionally, electron capture is suppressed in such a large magnetic field. Hence, electron recombination is not an issue in this process.

Test Particle Model

Liouville equation (based on Liouville theorem conservation of probability in phase space) for phase space density $D(X_1, X_2, X_3, \ldots X_N; t)$

$$\left\{\frac{\partial}{\partial t} + \sum_{i=1}^{N}\left[\vec{v}_i \bullet \frac{\partial}{\partial X_i} - \frac{q}{m}(\vec{E}(X_i, t) + \vec{v}_i \times \vec{B}) \bullet \frac{\partial}{\partial \vec{v}_i}\right]\right\} D = 0$$

where B is constant magnetic field applied externally. Here $X_i = (\vec{x}_i, \vec{v}_i)$ is the 6 dimensional phase space (position and velocity) coordinate of the i^{th} particle. Only Coulomb forces are considered, hence

$\vec{E}(\vec{x}_i, t) = q\sum_{j=1}^{N'} \frac{\partial}{\partial \vec{x}_i} \frac{1}{|\vec{x}_i - \vec{x}_j|}$,the prime indicates that the term i=j is omitted.

By taking moments of the Liouville equation, a hierarchy similar to the BBGKY (Bogoliubov-Boron-Green-Kirkwood-Yvon) hierarchy was obtained by Rostoker and Rosenbluth. Basically, Rostoker and Rosenbluth integrated out the coordinates of all particles but one, but two, etc. of the Liouville equation to obtain a chain of equations for one-body, two-body, etc., distribution functions.

Next reduced probability distributions are defined as f_s/V^s, where V^s is the configuration space volume (f_s a.k.a. s-body function), $f_s(X_1, X_2, \ldots X_s; t) \equiv V^s \int D dX_{s+1} \ldots dX_N$;and moments of the Liouville equation are taken. After that cluster expansion, similar to Mayer cluster expansion [$f_1(X_1) = f_1(X_1)$; $f_2(X_1X_2) = f_1(X_1)f_1(X_2) + P(X_1X_2)$; $f_3(X_1X_2X_3) = f_1(X_1)f_1(X_2)f_1(X_3) + f_1(X_1)P(X_2X_3) + f_1(X_2)P(X_3X_1) + f_1(X_3)P(X_2X_1) + T(X_1X_2X_3)$, where P and T are correlation functions] is performed. But, Rostoker and Rosenbluth performed the expansion to order of the discreteness parameter, which is proportional to the plasma parameter $g = (n\lambda_D^3)^{-1}$. In dense plasma g is a very small number, there Rostoker and Rosenbluth treated T as higher than first order, P as first order and f_1 as zeroth order. To move away from (decreasing) particle discreteness, the limits, where $q \rightarrow 0, m \rightarrow 0, n \rightarrow \infty$ such that q/m and qn remain constant are taken. These are also the limits taken to derive the Vlasov equation. The difference between the use of the Vlasov equation and the test particle model is that in the first interactions are via waves, while in the latter shielded particles are interacting.

Test Particle Model (continued)

After lots of math, Rostoker and Rosenbluth were able to derive a Fokker-Planck like equation with expressions for (dynamical) friction force, diffusion, etc. on a test particle. However, since the shield cloud of the test particle had a very complex form, it was very difficult to even determine the maximum impact parameter. Rostoker continued to develop the shielding aspect of the theory ("**dressed test particles**"), which behave like statistically independent quasiparticles. However, applying the resultant equations to experimental setups is rather challenging even for 21st century computation capabilities (let alone the early 1960's). Finally exact relaxation rates expressions for a Maxwellian field particle distribution were derived. Commonly used in fusion!

Pertinent (to this case) relaxation rates $\nu^{i/e}$ in $\sec^{-1}$ (ion test particle in a background of field electrons) are given in the following equations (without relativistic corrections!)

Test Particle Model (continued)

$$\frac{d}{dt}\overrightarrow{v_i} = -\nu_s^{i/e}\overrightarrow{v_i} \qquad (1)$$

$$\frac{d}{dt}(\overrightarrow{v_i} - \overline{v_i})_\perp^2 = \nu_\perp^{i/e}\overrightarrow{v_i^2} \qquad (2)$$

$$\frac{d}{dt}(\overrightarrow{v_i} - \overline{v_i})_\parallel^2 = \nu_\parallel^{i/e}\overrightarrow{v_i^2} \qquad (3)$$

Velocities are denoted by v while rates are indicated by ν. Subscripts $(s, \perp, \& \parallel)$ denote slowing down, transverse diffusion in velocity space and parallel diffusion in velocity space respectively. Averages are performed over an ensemble of test particle distributions for a Maxwellian field particle distribution. Exact formulas exist for relaxation rates that can be written as,

$$\nu_s^{i/e} = \left(1 + \frac{M}{m}\right)\psi(x)\nu_0^{i/e}; \;\; \nu_\perp^{i/e} = 2[(1 - \frac{1}{2}x)\psi(x) + \psi'(x)]\nu_0^{i/e}; \nu_\parallel^{i/e} = [\psi(x)/x]\nu_0^{i/e}$$

where $\nu_0^{i/e} = 4\pi Z^2 e^4 \lambda n / M^2 v_i^3$; x is essentially the ratio of the test particle (ion) energy to the field particle (electron) temperature. Z is ion charge state, e elementary charge and λ is the Coulomb logarithm.

$\psi(x) = \frac{2}{\sqrt{\pi}}\int_0^x t^{1/2}e^{-t}dt$ and $\psi'(x) = \frac{d\psi}{dx}$, (here e is not the elementary charge)

In cases where x>>1 or x<<1, (i.e. for very fast or very slow test particles) simpler limiting forms of the relaxation rates exit. These equations are utilized in the next subsections for ion cooling and ion heating computations. Comparing equations 2 and 3 to equation 1, it is obvious that on a long time scale ion cooling dominates, since ion velocity slows-down with time t, while ion heating develops as $\sqrt{t}$, as is the case in multi-pass cooling.

Test Particle Model (continued)

Experimental Verification

The test particle model has been experimentally verified: J. Bowles, R. McWilliams, and N. Rynn, Phys. Rev. Letters **68**, 1144 (1992); J. Bowles, R. McWilliams, and N. Rynn, Physics of Plasmas **1**, 3418 (1994); J.J. Curry, F. Skiff, M. Sarfaty, and T.N. Good, Phys. Rev. Letters **74**, 1767 (1995).

Performed in Q machines by optical tagging and laser induced fluorescence (LIF): Ba, Ar ions are pumped from ground state to a long lived metastable state (tagged ions). LIF leads to photon emissions from tagged ions, from which velocity space distributions and their evolutions are determined (measurements in quiescent plasmas only without turbulence).

Test Particle Model (continued)

Fully justified to apply the **test particle model plasma physics formalism** to this case: λ_D=7.43x10^{-4} cm⇒**1346 Debye lengths in a beam diameter & 182 electrons in a Debye sphere**. Electron gyro-radius is 3.13x10^{-5} cm⇒ **almost 32,000 electron gyro-radii in a beam diameter**. Electron gyro-frequency is 6.72x10^{10} Hz. **During an interaction time** of τ = 6x10^{-8} sec, an **electron completes 4032 gyrations**. Ion gyro-period over a factor of 5 larger than longest cooling time⇒ions are not magnetized. Ion inter-particle distance is 2.3x10^{-3} cm, i.e. larger than 3 Debye lengths⇒ions are totally shielded; relaxation rates of a single ion ("**dressed**" test particle), whose energy equals ion beam thermal spread, streaming through cold electrons (field particles), is a reasonable representation for the ion beam velocity space relaxation rates.

Ion Cooling

Computations are performed in the beam rest frame, since $\gamma = 1.0022$, corrections to time dilations are minuscule. Pertinent relaxation rates $\nu^{i/e}$ in $\sec^{-1}$ (ion test particle slowing down in a background of field electrons) are:

$$\nu_s^{i/e} = 1.7x10^{-4}\mu^{1/2}nZ^2\lambda\varepsilon^{-3/2} \quad (4)$$

$$\nu_\perp^{i/e} = 1.8x10^{-7}\mu^{-1/2}nZ^2\lambda\varepsilon^{-3/2} \quad (5)$$

$$\nu_\parallel^{i/e} = 1.7x10^{-4}\mu^{1/2}nZ^2\lambda T_e\varepsilon^{-5/2} \quad (6)$$

Units are cgs and eV. T_e is the electron, μ ion to proton mass ratio. Since Maxwellian distribution is assumed, and the spread in the perpendicular direction is much larger

$$\varepsilon_\perp = \frac{1}{2}Mv_{thi\perp}^2$$

It is the energy of "test" representative ion.

Electron are magnetized⇒the Coulomb logarithm (b is the smallest impact parameter) is

$$\lambda = \ln\left(\frac{\rho_e}{b}\right) \approx 3.2$$

From equation 4 cooling time

$$\tau_c = \tau_s \approx 1\Big/\nu_s^{i/e} \approx 5.35x10^{-8} \ \sec, \quad (7)$$

which implies a cooling length of 1.07 meter! The difference between this value and that obtained from Parkhomchuk's empirical formula is about 12%.

Ion Heating (velocity space diffusion)

Slowing down (cooling) transverse velocity space diffusion ration,

$$\frac{\nu_s^{i/e}}{\nu_\perp^{i/e}} = \frac{1.7x10^{-4}\mu^{1/2}nZ^2\lambda\varepsilon^{-3/2}}{1.8x10^{-7}\mu^{-1/2}nZ^2\lambda\varepsilon^{-3/2}} \cong 1.7x10^5$$

$\tau_\perp \approx 1.7x10^5 \bullet 6x10^{-8} \approx 1x10^{-2}$ sec, i.e., about 10 msec

Slowing down (cooling) parallel velocity space diffusion ration,

$$\frac{\nu_s^{i/e}}{\nu_\parallel^{i/e}} = \frac{1.7x10^{-4}\mu^{1/2}nZ^2\lambda\varepsilon^{-3/2}}{1.7x10^{-4}\mu^{1/2}nZ^2\lambda T_e\varepsilon^{-5/2}} = \frac{\varepsilon}{T_e} \cong 1.87x10^5$$

$\tau_\parallel \approx 2x10^5 \bullet 6x10^{-8} \approx 1.1x10^{-2}$ sec, i.e. about 11 msec

Thus, velocity space diffusion relaxation times are 5 orders of magnitude longer than the computed cooling times.

Ion Heating (mobility due to electric field)

Interesting to note that as $\varepsilon \rightarrow T_e$, $\nu_s^{i/e} \rightarrow \nu_\parallel^{i/e}$

In case of an electric field (external, due to charge separation, or space charge imbalance) there is also a mobility term, which can result in modification of both velocity and configuration space distribution functions. In all computations so far spatially homogeneous plasmas are assumed; not necessarily correct.

Modification (perturbation) to the distribution function

$$\vec{v} \bullet \vec{\nabla}_r f_0 + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \bullet \vec{\nabla}_v f_0 = -\nu_m \tilde{f}$$

Unresolved issue; can lead to radial ion current (slight radial reduction); have not had a chance to evaluate

Comparison of Parkhomchuk's empirical formula to Test Particle Model Plasma Physics Formalism

Cooling time computed from Parkhomchuk's empirical formula,

$$\tau_c = \frac{Mv_i}{F} = \frac{Mv_i m v_{thi}^3}{4Z^2 e^4 n\lambda v_i}$$

Substituting numerical values for the constants m and e

$$\tau_c = 4.28x10^9 \frac{Mv_{thi}^3}{Z^2 n\lambda} \qquad (8)$$

Test Particle Model Plasma Physics formalism cooling time τ_c can be written as

$\tau_c = 5.88x10^3 \dfrac{\varepsilon^{3/2}}{Z^2 n\lambda\mu^{1/2}}$ in mixed (fusion plasma; mostly cgs and eV) units. But $\mu \equiv \dfrac{M}{m_p}$ and $\varepsilon \equiv \dfrac{1}{2} M v_{thi}^2$. Hence,

$$\tau_c = 5.88x10^3 \frac{\left(\frac{1}{2}\right)^{3/2} m_p^{1/2} M^{3/2} v_{thi}^3}{Z^2 n\lambda M^{1/2}} \left(\sqrt{2}\right)^3 \left(1.6x10^{-12} \frac{erg}{eV}\right)^{-3/2}$$

$$\tau_c = 3.76x10^9 \frac{Mv_{thi}^2}{Z^2 n\lambda} \qquad (9)$$

Comparing equations 8 & 9 yields very good agreement between cooling time computed from Parkhomchuk's empirical formula and cooling time computed from plasma relaxation based on the test particle model.

Crux-of-the-Matter

Dressed ions with shielding clouds (test particle model) versus bare ions in "conventional cooling theories".

According to "conventional theory" rates for ion cooling and ion heating are comparable. But, cooling evolves as t while ion heating develops as $\sqrt{t}$. Need long cooling time!

In plasma relaxation theories based on the test particle model, ion dynamic friction (cooling, slowing-down) rate is much faster than ion velocity space diffusion (heating).

Other Pertinent Physics Issues

Electron beam stability and adverse effects the electron beam (or electron gun) might have on the gold ions.

Magnetic Field Required for Electron Beam Equilibrium and Stability: to prevent electron beam expansion. Square electron density profile, electric field E_e at the outer beam radius R is given in mks units by $E_e = -I/2\pi\varepsilon_0 vR$

needed magnetic field $0.5\varepsilon_0 E_e^2 = \frac{B^2}{2\mu_0}$; $B^2 = E_e^2/c^2$ E_e=2.7x10^6 V/m. B = 90 Gauss. A more stringent magnetic field requirement is imposed by plasma stability $\frac{\omega_{pe}^2}{\Omega_e^2} \leq 1$ which necessitates a magnetic 2x10^3 Gauss or 0.2 Tesla. These magnetic fields are small compared to the 2.4 Tesla magnetic field, which maximizes the cooling decrement.

Other channels of recombination like three body collisional recombination or dielectronic recombination have extremely low probability Cross section for the latter is usually of the order of 10^{-19} cm^2 or less. The low cross sections combined with 10's nsec interaction time and a mean free path of 10^8 cm render these processes unimportant.

Other Pertinent Physics Issues (continued)

Charge Exchange

Pressure in HC plasma cathode electron gun 10^{-5} Torr of argon gas. High charge exchange cross sections 10^{-14} cm^2: mean free path is 336 cm. Inside the hollow cathode the pressure is about 10^{-2} Torr: MFP < 0.3 cm. But with concentric HC system ion beam injection through hollow cathode might be an option. In the extractor pressure is under 10^{-5} Torr; outside the extractor pressure is 10^{-7} Torr, where charge exchange is no longer an issue.
Electron guns with carbon fiber cathodes have generated close to 1 MA of electron current. With this cathode currents of up to 2 kA at 2 kV were obtained in microseconds long pulses. Depending on the current generated, pressure during the electron beam pulse can be between 10^{-3} to 10^{-6} Torr (or even lower where large pumping capability is available). Since the needed electron beam currents are well below 100 A, pressures below 10^{-6} Torr, where charge exchange is not an issue, are expected.

Other Plasma Instabilities

The electron beam should be stable for an axial magnetic field of 2.4 Tesla. If the electron beam is stable, there should, in principle, be no other instabilities. The only possible plasma instability might be to the ions (like a rotating two stream instability). Like all beam instabilities, it has a density threshold. Since the ion density is more than three orders of magnitude lower than the electron density, there should be no beam instabilities.

Personal Chronology

- 1994 – suggested using intense electron diode as final stage (single pass) cooling of muons for the muon collider.
- 1996 – results included in major muon collider documents; Palmer,…Hershcovitch… et al, "Muon Collider Design", Nuclear Physics B <u>51A</u>, 61 (1996).
- 1997- PAC97 was told wrong equations, should use: N.S. Dikansky, V.I. Kudelainen, V.A. Lebedev, I.N. Meshkov, V.V. Parkhomchuk, A.A. Sery, A.N. Skrinsky, and B.N. Sukhina, "*Ultimate Possibilities of Electron Cooling*," Institute of Nuclear Physics, Novosibirsk, USSR Report, Preprint 88-61 (1988).
- Shortly after told muon collider colleague: re-evaluate previous results.
- 2008 – old approach is probably valid $n\lambda_D^3 \approx 10^9$

Discussion: justification & limitations

Plasma parameter , $n\lambda_D^3$ which is inversely proportional to the discreteness parameter must be a very large number. Here it is 182. In hollow cathode arcs it is over 10^6; in the Plasma Window it is over 10^8; and, in fusion grade plasma (Tokamak) the plasma parameter is over 10^9! Nevertheless, ion are well shielded from each other, since intra ion distance is more than three Debye lengths.

Additionally simplified (useful) equations assume Maxwellian distributions.

As ions cool off the asymptotic expressions lose validity.

Discussion: space charge affect on ions

In quasi-neutral dense plasmas, interactions can be separated into two classes: strong close-range (binary or multi) and collective long-range through average electric and magnetic fields, which in absence of turbulence, are much weaker.

Here the plasma in non-neutral! Radial electric field effect on peripheral ion in the presence of a strong magnetic field has yet to be analyzed including centrifugal forces.

Work-in-progress

If space charge is a problem, explore space-charge mitigation (like variation on plasma flood guns [2.777×10^{-6} Torr CX MFP over 12 m] or proton plasma); must be done below instability density threshold or using stabilizing mechanisms…haven’t thought it through yet