Inner Working of the Spin Tracking Code Spink

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abstract

- *SPINK* is a spin tracking code for polarized spin 1/2 particles. The code tracks both trajectories in 3D and spin.

- It works using *MAD* or other tracking codes for orbits and thin element kicks based on the BMT equation to track spin.

- The code typically runs on a Linux platform, either sequentially or MPI-parallel.

Purpose

- Develop a code that tracks orbit and spin through the lattice of a real accelerator with all its bells and whistles
- A code fast enough to track millions to billions of turn of a synchrotron or storage ring in a manageable run time
- A code that is flexible enough to control and to hack if necessary for special purposes
- A code based on some widely used descriptors of a machine lattice (in our case it is *MAD*), but that can accept other descriptors if necessary or useful

Basics

- SPINK tracks 6 phase space coordinates for the orbit and 3 coordinate representing the Cartesian components of the unitary spin vector

- Orbit tracking is done by propagating the vector $\vec{r}$ through ray transfer maps. These maps are for "thick" elements (say, *MAD*), or "thin" elements, (say, *TEAPOT*)

- Spin tracking is done via "thin" elements spin rotation matrices

- *SPINK* can track several particle sequentially, or "trivially" in parallel using the *MPI* (Message Passing Interface) library, and averages the final results

- NOTE that in the cases we treat (RHIC etc.) the spin motion depends on the orbits, but the motion of the particles is not affected by the magnetic moment associated with the spin. *i.e.* Stern-Gerlach forces are negligible

Units and Definitions

- *SPINK* uses MKSA units. Canonical orbital phase space variables are

$$\mathbf{r} = \left(x,\, x' = \frac{p_x}{p},\, y,\, y' = \frac{p_y}{p},\, \Delta\phi = -c\Delta t,\, \frac{\Delta E}{pc} \right)$$

- Vector spin is treated as a 3-dimension real vector

$$\mathbf{S} = (S_x,\, S_y,\, S_z)$$

- *SPINK* has provision for tensor polarization

Conventions

- We follow the Frénet-Serret accelerator coordinate convention with $\widehat{x}$ radial, $\widehat{y}$ vertical, and $\widehat{z}$ longitudinal (in the spin literature the vertical is often $\widehat{z}$)

- Input and output of transverse phase space are in [m] and [mrad] for transverse coordinates, (x, x'), radial and (y, y'), vertical.

- The longitudinal coordinate $\Delta\phi = c\Delta t$ is in [m], and the canonical "energy" coordinate ΔE is in [GeV]. $\Delta\phi$ and ΔE are evaluated with respect to the phase ϕ_s and energy E_s of a reference synchronous particle

Dynamic Lattice

- The accelerator lattice used by *SPINK* can be static, *i.e.* read once from the lattice builder (say, *MAD*) and never changed, or dynamic *i.e.* continuously modified in the course of machine run. Two methods are implemented in the code to dynamically modify the lattice using tables

- (a) Reading from a table a list of descriptors for different lattices. In the first case, once a given energy is reached during acceleration the track initializer routine will read the next lattice

- (b) Reading tables of parameters of all quadrupoles, etc. and update the transport maps accordingly

Thomas-BMT Equation

- Spin propagation is calculated from the Thomas-BMT equation

$$\begin{cases} \dfrac{d\mathbf{S}}{dt} = \dfrac{q}{\gamma m}\ \mathbf{S} \times \mathbf{F} \\ \mathbf{F} = (1 + G\gamma)\mathbf{B}_{\perp} + (1 + G)\mathbf{B}_{\parallel} \end{cases}$$

  using the magnetic field components transverse and longitudinal to the velocity of the particle. G is the gyromagnetic ratio of the particle

- To provide spin kicks in thin elements, the BMT must be "flattened". This means that for spin tracking, instead of integrating the equation through a magnetic element of the machine, *SPINK* expresses the spin rotation in each machine element in matrix form

$$\mathbf{S} = \mathcal{M}\ \mathbf{S}_0.$$

  where $\mathcal{M}$ is a 3×3 matrix

Coordinate Transform (1)

- Use a coordinate system **e** that revolves around the accelerator

- The longitudinal axis $\widehat{z}$ is tangent to the reference orbit, $\widehat{x}$ and $\widehat{y}$ are the radial and vertical position respect to this orbit, respectively

- Call s the longitudinal coordinate along the orbit

- In **e** the derivative of a vector **a** is

$$\frac{d\mathbf{a}}{ds} = \frac{da_x}{ds}\widehat{x} + a_x\frac{d\widehat{x}}{ds} + \frac{da_y}{ds}\widehat{y} + a_y\frac{d\widehat{y}}{ds} + \frac{da_z}{ds}\widehat{z} + a_z\frac{d\widehat{z}}{ds}$$

- Use the local radius of curvature ρ of the reference orbit

$$\frac{d\widehat{x}}{ds} = 0, \quad \frac{d\widehat{y}}{ds} = -\frac{\widehat{z}}{\rho}, \quad \frac{d\widehat{z}}{ds} = \frac{\widehat{y}}{\rho}$$

Coordinate Transform (2)

- Transform the vector velocity **v** to the system **e**

- Transform the time derivative d/dt to space derivative along the ref. orbit d/ds

- Express the magnetic field components in the Thomas-BMT equation as

$$\mathbf{B}_\perp = \mathbf{B} \times \mathbf{u}, \qquad \mathbf{B}_\| = (\mathbf{B} \cdot \mathbf{u})\mathbf{u}, \qquad \mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_\|$$

with $\mathbf{u} = \mathbf{v}/|v|$

- Introduce in the BMT a vector

$$\mathbf{f} = \frac{q}{\gamma m}\mathbf{F} = \left(\frac{1}{ds/dt}\right)\frac{|v|}{B\rho}\left[(1+G\gamma)\mathbf{B} - G(\gamma-1)(\mathbf{B}\cdot\mathbf{u})\mathbf{u}\right]$$

Flat BMT

- Moving from time to space, the vectorial BMT differential equation is equivalent to 3 scalar diff.eqs.

$$\left\{\begin{array}{l} S'_x = f_z S_y - (f_y - \frac{1}{\rho}) S_z \\ S'_y = f_x S_z - f_z S_x \\ S'_z = (f_y - \frac{1}{\rho}) S_x - f_x S_y \end{array}\right.$$

- The system yields three 3.rd order formally identical linear equations for the three components of the spin

$$S''' + \omega^2 S = 0, \quad \text{with} \quad \omega^2 = f_x^2 + (f_y - \frac{1}{\rho})^2 + f_z^2.$$

- The general integral of each diff.eq. is

$$S = C_1 + C_2 \cos\mu + C_3 \sin\mu$$

with the angle of spin rotation

$$\delta\mu = \omega\ \delta s.$$

δs is the physical length of a (thin) machine element

Evolution of the spin vector around the accelerator

- Let us follow the path of a particle through the magnetic field of each element of the machine. This magnetic field is referred to a coordinate system that rotates around the (circular) accelerator, with $\hat{x}$ radial, $\hat{y}$ vertical, and $\hat{z}$ tangent to a reference particle trajectory

- The particle spin precesses on a cone around an axis directed along the total magnetic field seen by the particle.

- This axis will be mostly vertical in a bending magnet, mostly transversal to the trajectory in a quadrupole, mostly longitudinal in a solenoid

- In every machine element there will be a radius of curvature of the reference orbit that instantaneously moves on a plane perpendicular to the local total magnetic field

Spin Matrix

- The equation of spin motion can finally be expressed in natrix form. The spin transfer unitary matrix $\mathcal{M}$ is

$$\mathcal{M} = \begin{pmatrix} 1-(\mathcal{B}^2+\mathcal{C}^2)c & \mathcal{A}\mathcal{B}c+\mathcal{C}s & \mathcal{A}\mathcal{C}c-\mathcal{B}s \\ \mathcal{A}\mathcal{B}c-\mathcal{C}s & 1-(\mathcal{A}^2+\mathcal{C}^2)c & \mathcal{B}\mathcal{C}c+\mathcal{A}s \\ \mathcal{A}\mathcal{C}c+\mathcal{B}s & \mathcal{B}\mathcal{C}c-\mathcal{A}s & 1-(\mathcal{A}^2+\mathcal{B}^2)c \end{pmatrix}$$

- with

$$\left\{ \begin{array}{l} c = 1-\cos\delta\mu \\ s = \sin\delta\mu \end{array} \right. , \quad \mathcal{A} = \frac{1}{\omega} f_x, \quad \mathcal{B} = \frac{1}{\omega}(f_y - \frac{1}{\rho}), \quad \mathcal{C} = \frac{1}{\omega} f_z$$

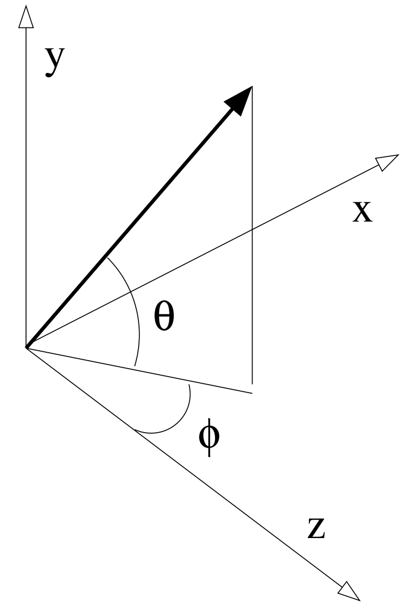
- It is $$det(\mathcal{M}) = 1$$

- NOTE that $\mathcal{M}$ represents a parametric linear transformation, whose elements are function of the values of the particle coordinates

Angles

- It is useful to look at $\mathcal{M}$ in another way: as a rotation of the spin vector by an angle $\delta\mu$ around an axis whose direction is defined through two angles, θ, latitude and ϕ, longitude

axis of spin rotation



- So, $\mathcal{M}$ is completely defined by three angles $\delta\mu,\ \theta,\ \phi$

Spin Tune

- Spin tune is defined as the number of spin complete revolutions per turn in an accelerator

- The spin tune is calculated from the eigenvalues of the one-turn spin matrix, obtained by multiplication of all the matrices visited by a particle in one turn

$$\nu_s = \frac{1}{2\pi} \arccos \left(\frac{Tr(\mathcal{M}) - 1}{2} \right)$$

  with $Tr(\mathcal{M})$ the trace of the spin matrix

- More exactly, the spin-one-turn matrix is such only when the particle reaches the same starting point in phase-space after several turns

Spin Matrix in Specific Devices

- Considering specific devices in *SPINK* is, in principle, not necessary, because it would suffice to consider the coefficients of field expansion at each location along the accelerators

- However, it may be convenient in practice

- Types of accelerator devices and insertions recognized by the code are:
  DRIFT, BEND, MULTIPOLE, SOLENOID, EDGE, SIBERIAN_SNAKE, SPIN_ ROTATOR, SPIN_FLIPPER, RF_DIPOLE and _SOLENOID, HELIX, KICKER ..etc.

- In the following we show simple examples of schematic devices, with matrix and spin kick and spin matrix to lowest order in the orbital quantities $x, x', y, ...$

Horizontal Bend Dipole

- According to the BMT equation the spin rotation must be proportional to $(1+G\gamma)$ in an element, as a bend dipole, with a magnetic field transverse to the beam, $\mathbf{B}_\perp$

- However, in a system of coordinates that revolves around a circular accelerator, by expansion in power of x, x', y, y', obtain to lowest order
$$\delta\mu \approx f_y - \frac{1}{\rho} \approx \frac{1+G\gamma}{\rho} - \frac{1}{\rho} = \frac{G\gamma}{\rho}$$
the "1" disappears and the spin rotation is simply $G\gamma\theta$, with θ the bend angle.

- To lowest order, the spin matrix for a bend is
$$\begin{pmatrix} \cos\delta\mu & 0 & \sin\delta\mu \\ 0 & 1 & 0 \\ -\sin\delta\mu & 0 & \cos\delta\mu \end{pmatrix}$$, a rotation around the vertical $\hat{y}$

Quadrupole

- Also in a quadrupole the field is normal to the beam, so we expect the spin rotation angle to remain proportional to $(1 + G\gamma)$

- By expansion in powers of x, x', y, y' obtain

$$\delta\mu \approx f \approx \frac{1 + G\gamma}{\rho} - \frac{\sqrt{x^2 + y^2}}{\rho_Q}$$

  The bend of the trajectory is contained in a plane that continuously changes from quad to quad

- The "1" can become smaller or larger than One, but on the average remains One, while the particle moves from a quad to the next in the lattice

- In a pure quadrupole the axis angles are

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}, \qquad \phi = \pm\frac{\pi}{2}$$

Horizontal Field RF Dipole

- In a RF dipole with horizontal oscillating field, $\mathbf{B}_{\perp}$, we expect that the spin angle kick should be proportional to $(1 + G\gamma)$

- Here it is

$$\delta\mu \approx f_x \approx \frac{1 + G\gamma}{\rho} - \frac{y}{\rho_D}$$

- The horizontal oscillating field also modulates the vertical betatron oscillation y with interesting consequences

- To lowest order, the spin matrix for RFDH is

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\delta\mu & \sin\delta\mu \\ 0 & -\sin\delta\mu & \cos\delta\mu \end{pmatrix}$, a rotation around the radial $\hat{x}$

Solenoid

- In a pure solenoid, the field in the BMT equation is $\mathbf{B}_{\parallel}$, parallel to the beam

- *SPINK* makes the spin rotation proportional to $(1+G)$, independent of the beam energy, as it should

- To lowest order, the spin matrix for a solenoid is

$$\begin{pmatrix} \cos\delta\mu & \sin\delta\mu & 0 \\ -\sin\delta\mu & \cos\delta\mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ a rotation around } \hat{z}$$

- In the solenoid fringe field, with longitudinal and transverse fields, the story is more complicated

Siberian Snakes and Spin Rotators

- Siberian snakes and Rotators rotate the spin by a given angle around a given axis

- Snakes break the condition of periodic spin kicks that produces spin resonances and polarization losses

- *SPINK* models snakes and rotators in three ways:
  (1) Synthetic, (2) Table driven, (3) Analytic

1. Synthetics are optically unit thin elements, "markers", where the angles of spin rotation $\delta\mu$ and axis θ, ϕ are given

2. Table driven are optically represented by a 6×6 matrix and by the three spin rotation angles dynamically read in a Table

3. Analytics are represented by analytic expressions for orbit and spin matrices

SPINK Flow

1. Run *MAD* to generate the lattice descriptor. *MAD* generates output files containing twiss parameters, transport matrices and errors

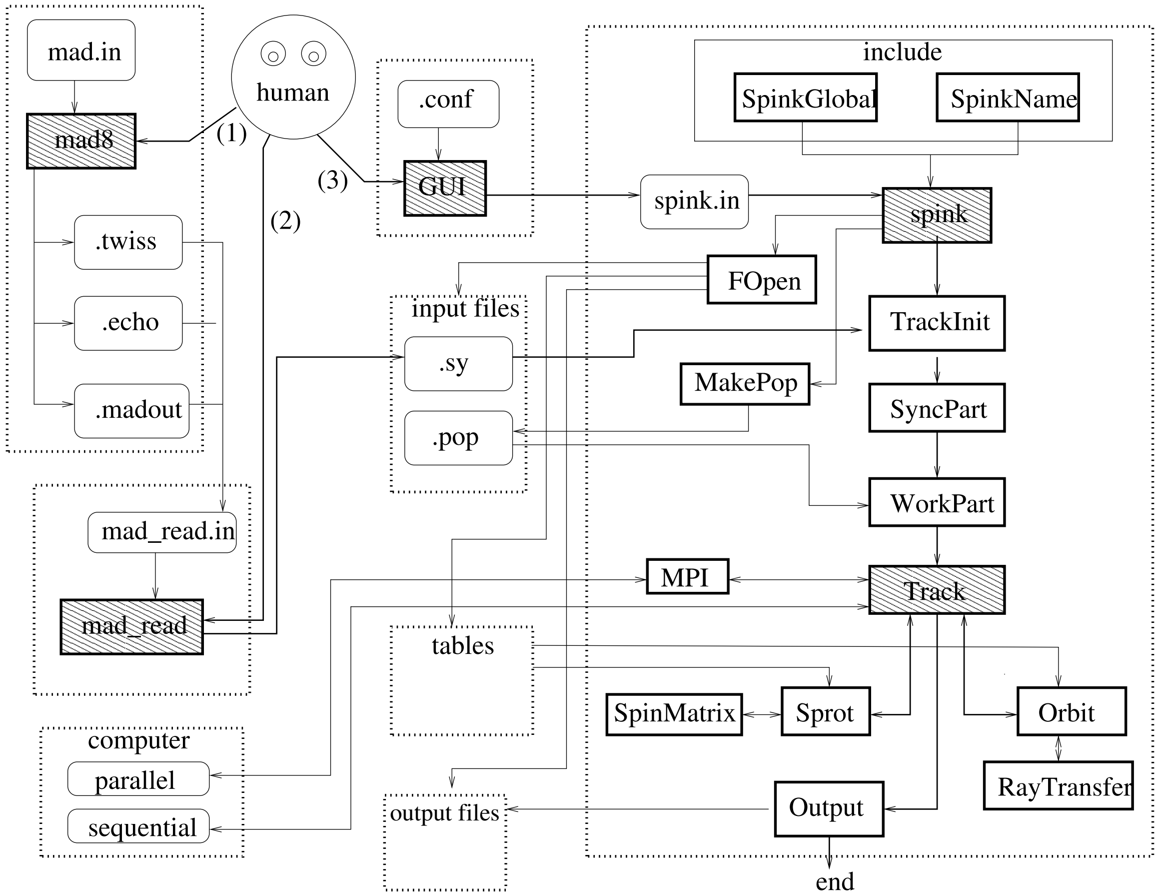
2. A pre-processor reads these files and generates a complete lattice descriptor

3. If a different orbit transport code is used, other than *MAD*, especially for non linear particle tracking, the transport maps or field coefficients are read in another way from *MAD* output

4. At the present we are implementing an *UAL-TEAPOT* orbit tracker for *SPINK*
   (see Fanglei Lin and Nikolay Malitrsky work for the EDM project)

5. Use the *GUI* to build an input configuration file and run *SPINK*. These operations are conveniently done through a Unix script

SPINK Flowchart



GUI

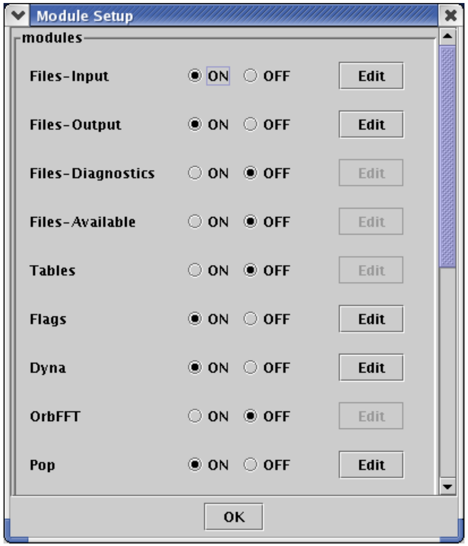
- *SPINK* is launched through a motif-window GUI.

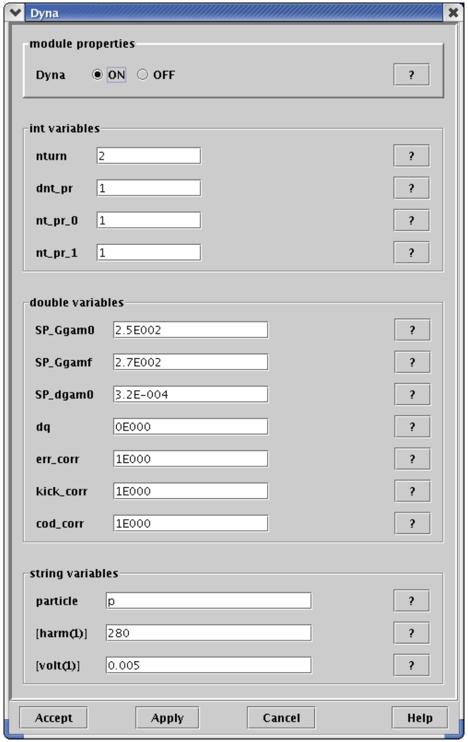
- The GUI is a convenient interface to set values for the variables in the program.

- A Menu window appears, that prompts to edit the configuration file, and leads to further windows for all modules of *SPINK*

- On each of the module editing windows there are help buttons $\boxed{?}$ explaining the meaning of each variable, string or Boolean parameter

3 out of the many GUI Windows



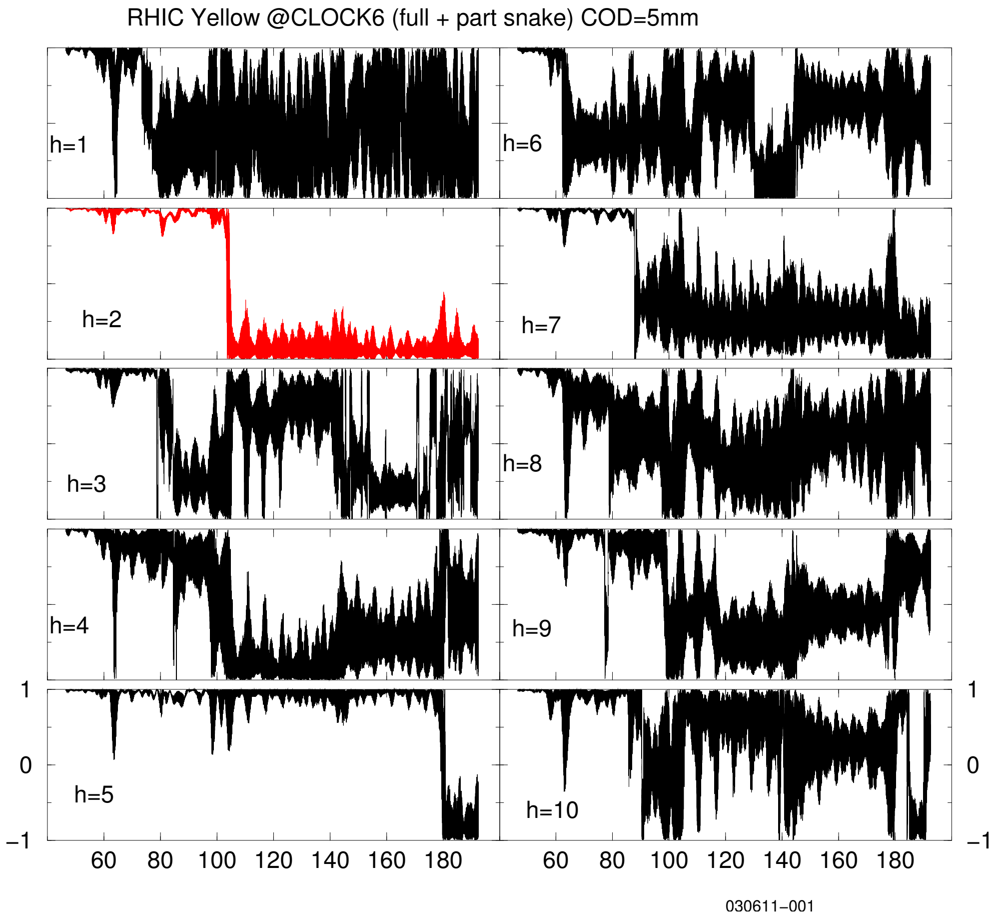


Main

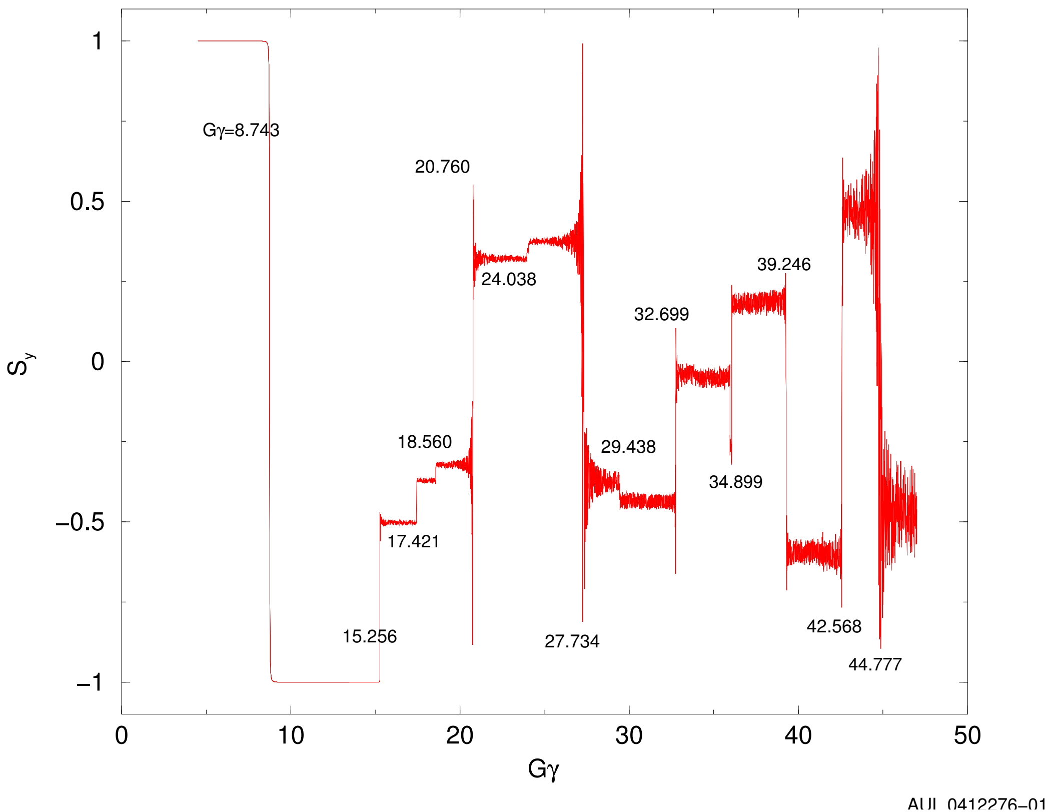
Dyna[mics]

Pop[ulation]

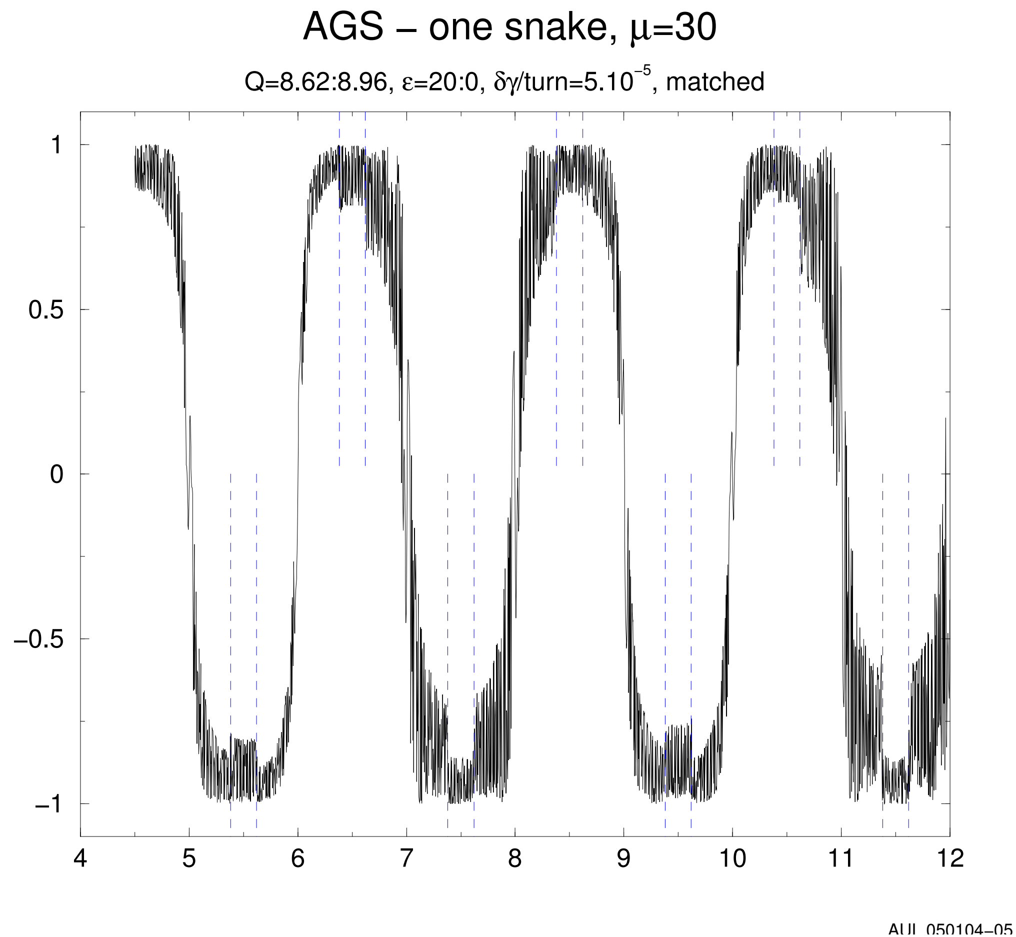
Example 1 - RHIC2003 - Broken Snake



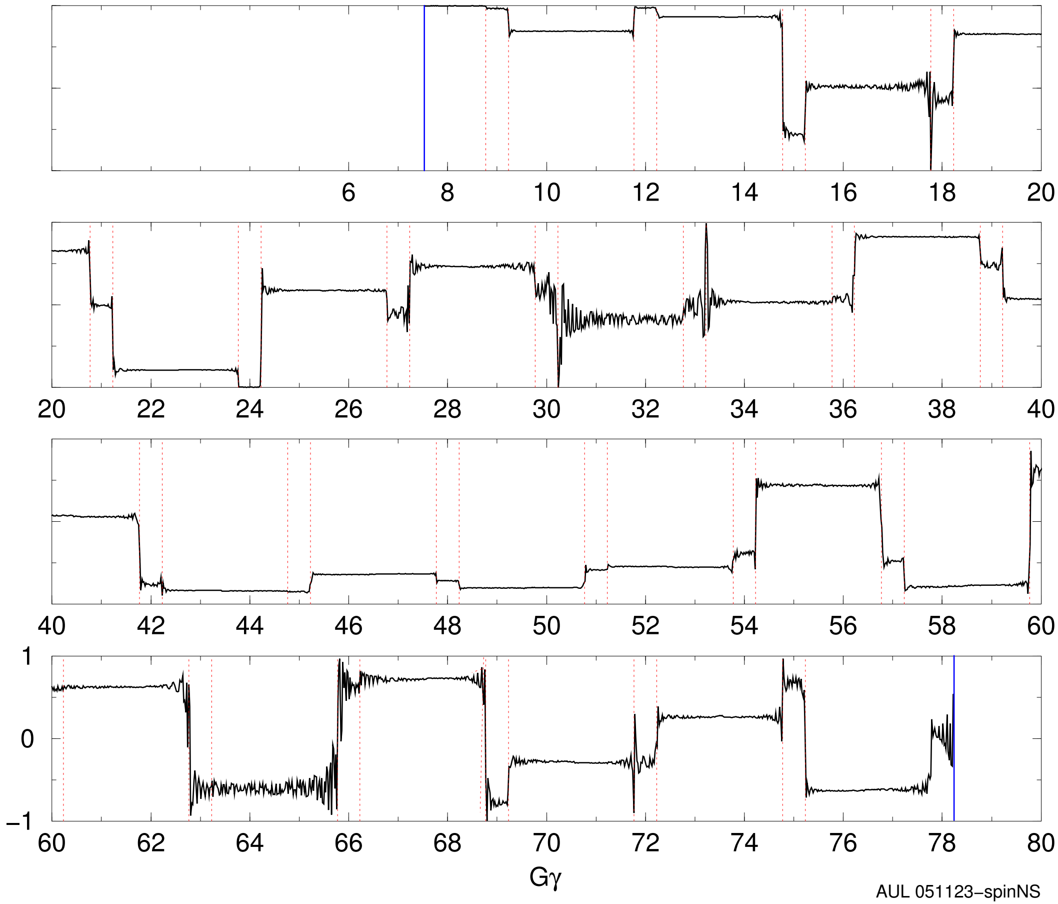
Example 2 - AGS - No Snakes



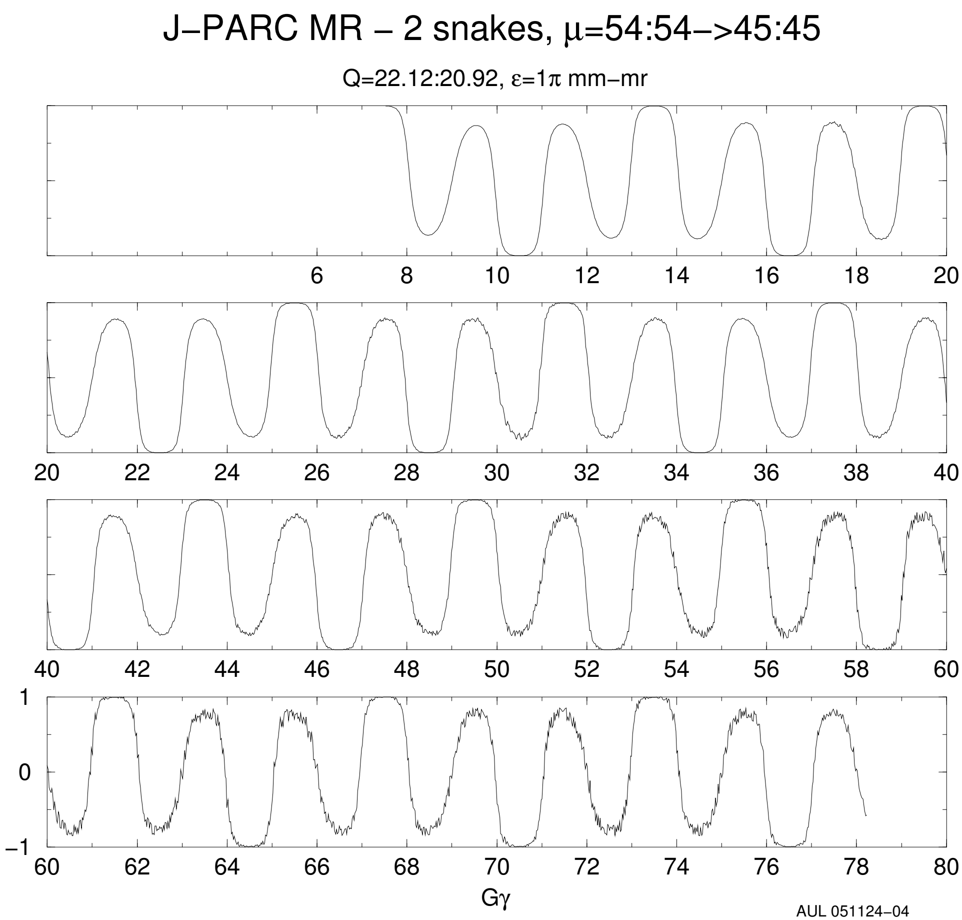
Exampke 3 - AGS - Hint of Horizontal Resonance



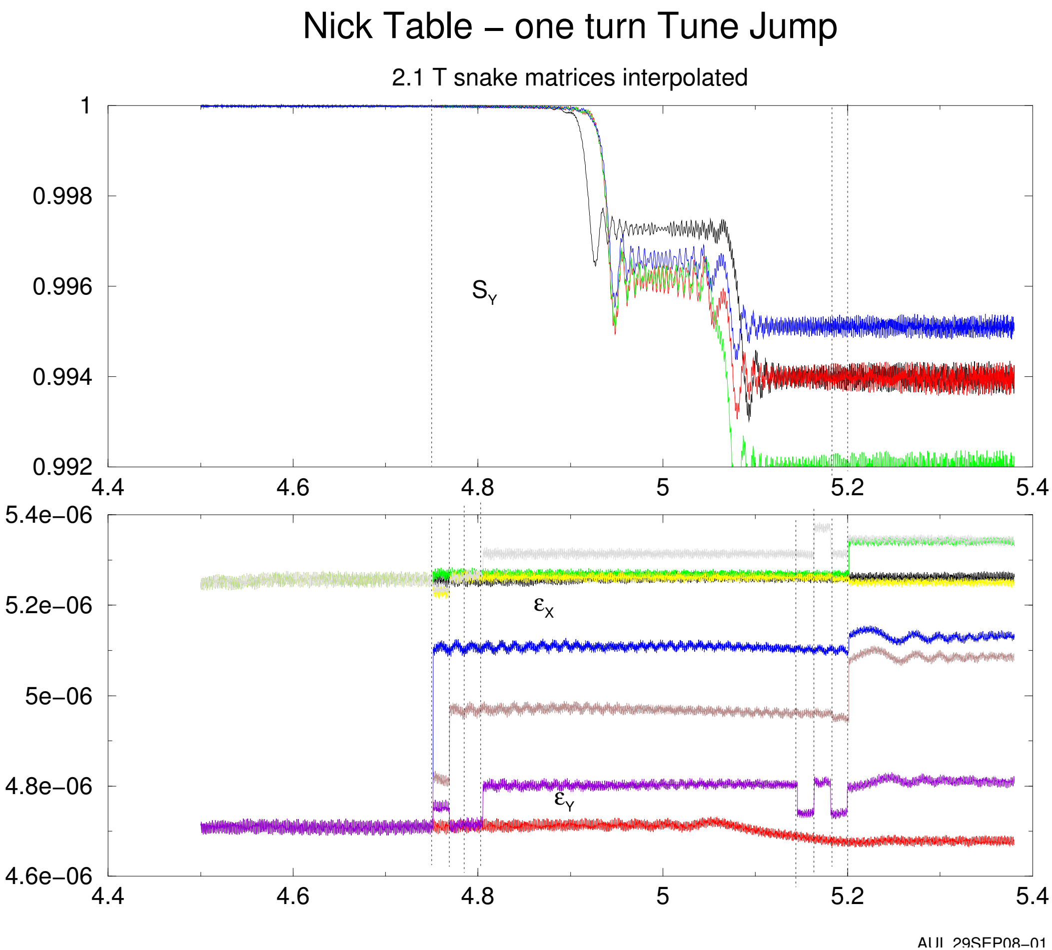
Example 4 - J-PARC - Main Ring - No Snakes



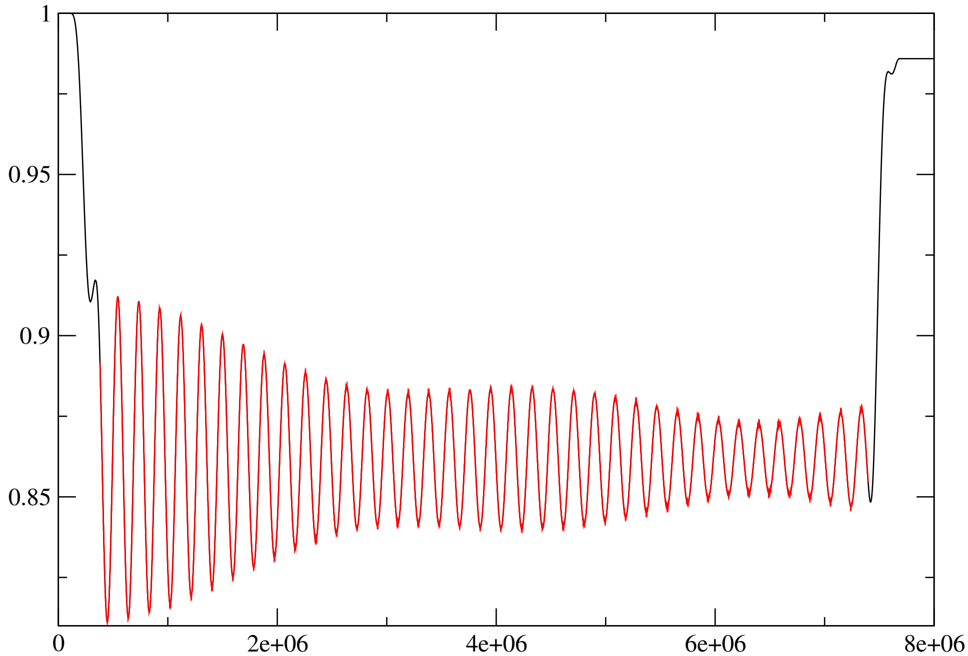
Example 5 - J-PARC - Main Ring with Partial Snakes



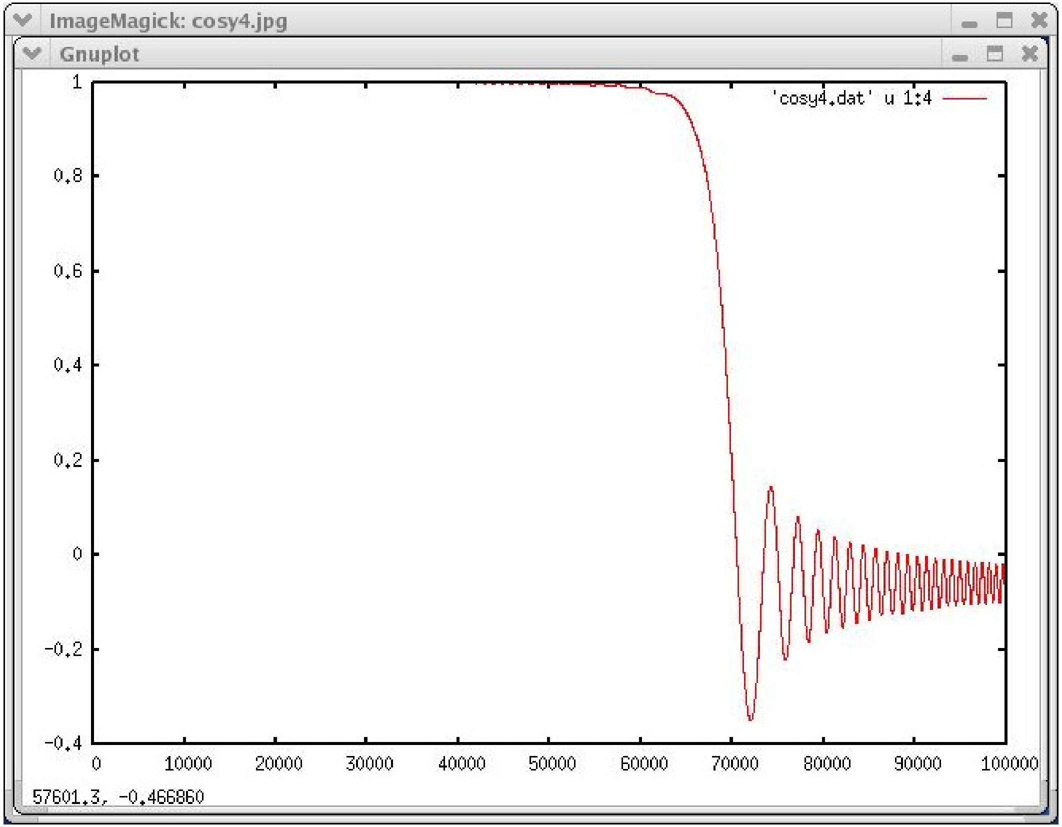
Example 6 - AGS - Tune Jump



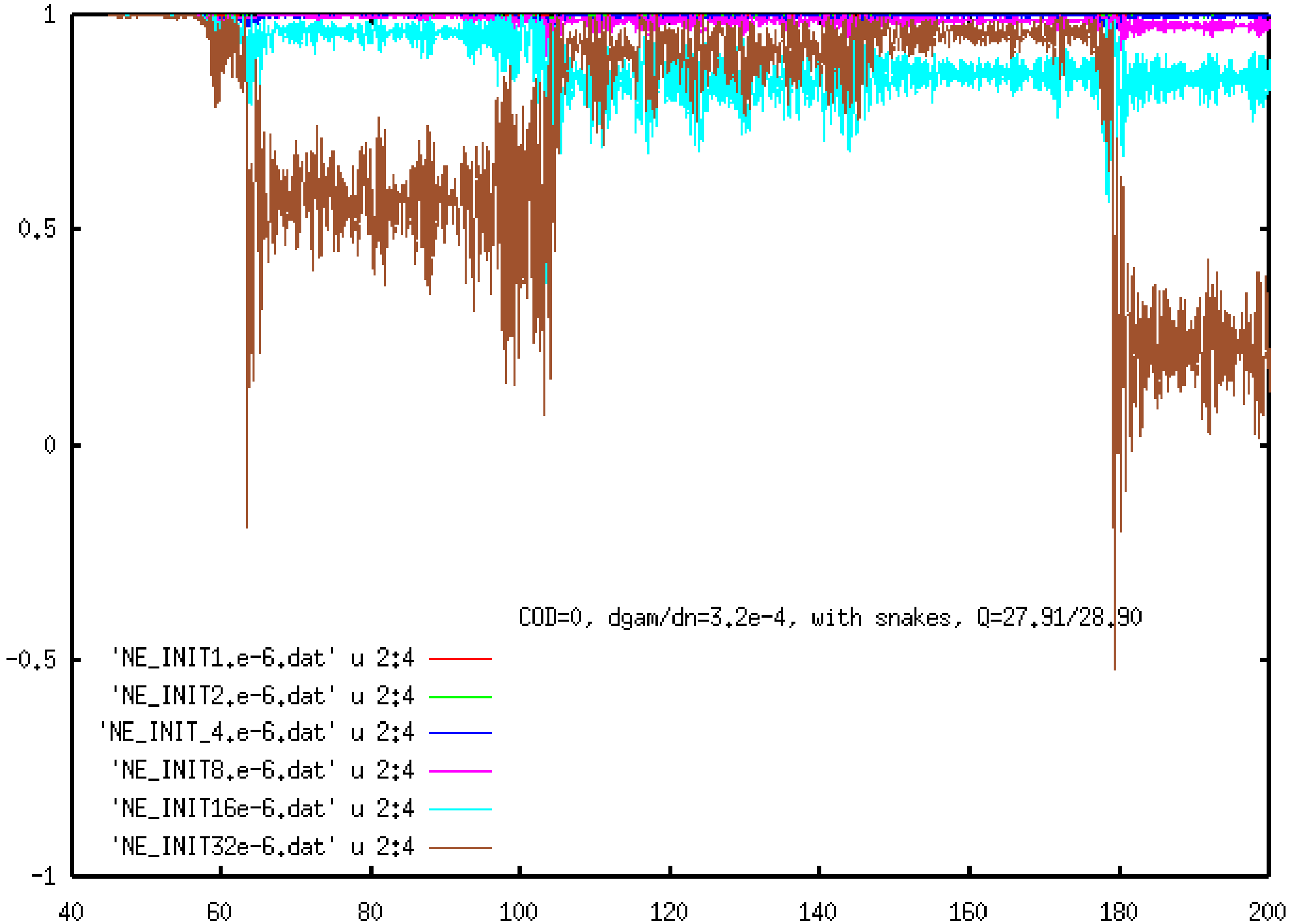
Example 7 - COSY - Deuterons, RF Solenoid spin flip



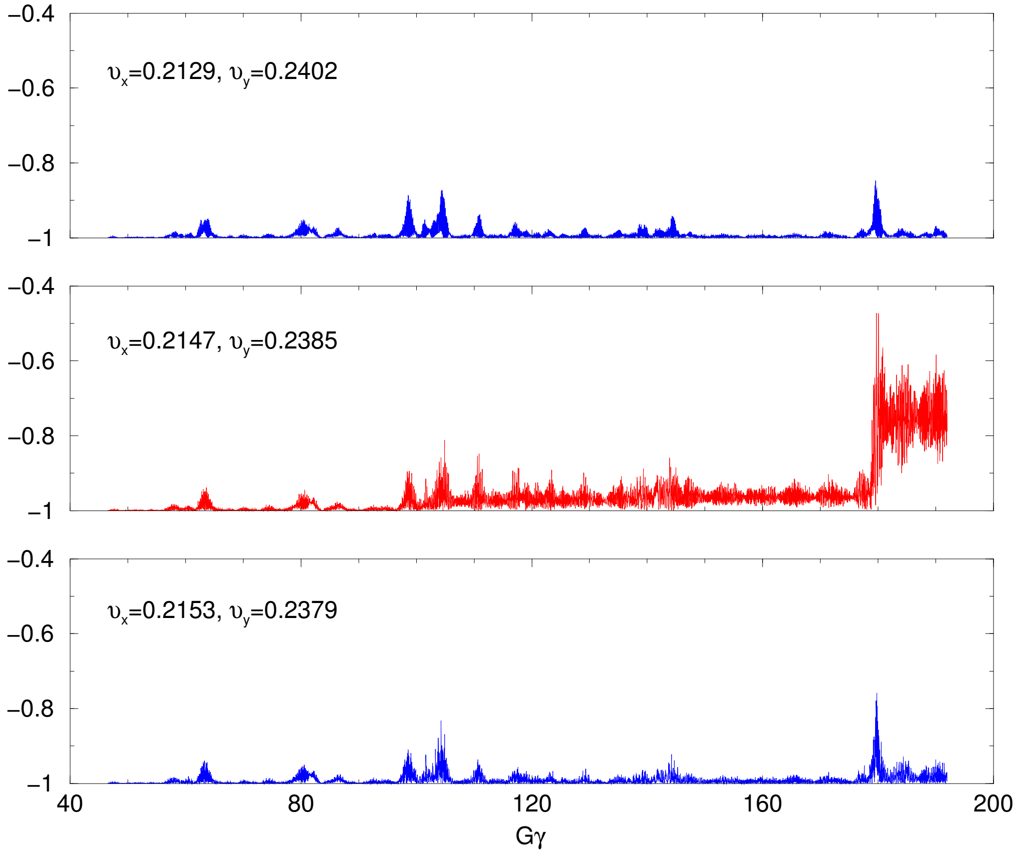
Example 8 - COSY - Deuterons, RF Dipole - Spin Flip



Example 9 - RHIC - Distorted COD



Example 10 - RHIC - Snake Resonance



Notes

- *SPINK* has been around for several years. We extensively use it in Brookhaven for polarized proton studies at the AGS and RHIC. The code has been exported and used in other laboratories, notably Jülich in Germany and KEK-JPARC in Japan.

- For circular accelerators other codes address some of the same issues such as
DEPOL by E.Courant, *SNAKE* by J. Buon, *SPRINT* by M.Vogt, *MAD* by Ch.Iselin, F.Schmidt et al, *ASPIRIN* by Y.Shatunov and V.Ptitsyn, *COSY-infinity* by M.Berz and G.C.Onderwater, PTC by E.Forest

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- Many have contributed to *SPINK*, notably Haixin Huang, Nick Tsoupas, Mei Bai, Fanglei Lin, Nikolai Malitski at Brookhaven, Andreas Lehrach at Jülich and Meiqin Xiao at RIKEN.
  Nick D'Imperio (BNL) contributed to the implementation of the GUI
  Constant interest and support by Thomas Roser has been important for the success of the code

- Vahid Ranjbar, Dan Abell at TechX have done much work on the code on a DOE grant, in order to consolidate its results, plus adding new features

Conclusions

- *SPINK* works very well in its limits and approximations
- It is in continuous development
- The code if available to everyone in the community
- There is also a User's Manual