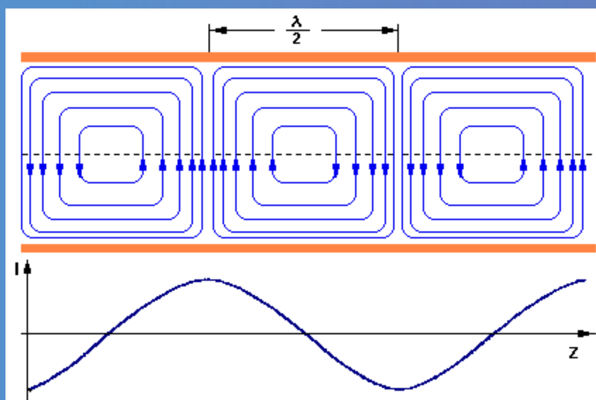
RF fundamentals for mechanical engineers

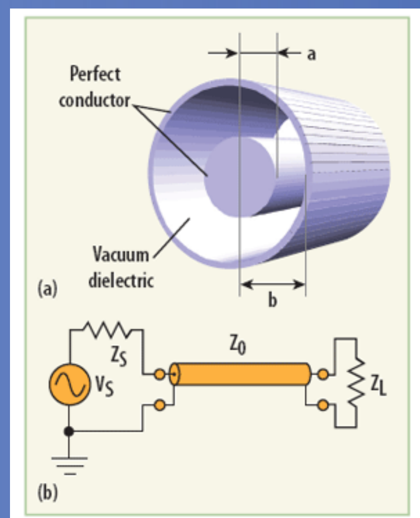
Lecture 1: Transmission lines and S-parameters

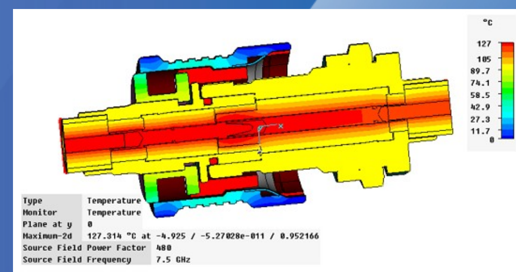


Sergey Belomestnykh

BNL

April 19, 2011





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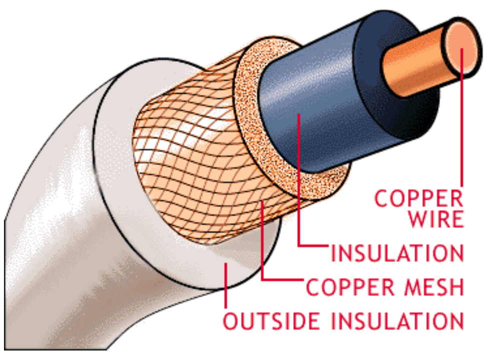
Introduction to lectures

- The goal of these lectures is to provide C-AD mechanical engineers some background in RF.
- The formulae for the most part will not be derived, but rather given with (hopefully) sufficient explanations.
- We are not in a position to cover all topics, but just a few directly related to developing accelerating structures. RF circuits, power generation, LLRF, etc. will not be covered. Most of the topics came from the original request of Steve Bellavia.
- The material is split into three lectures that will be delivered during two one-hour sessions on April 19th (3:00 pm to 4:00 pm) and April 21st (10:00 am to 11:00 am).

- ☐ Lecture 1 – S. Belomestnykh – April 19: ***Transmission lines and S-parameters***.
- ☐ Lecture 2 – Q. Wu – April 19 & 21: ***Resonant cavities (pill-box, elliptical cavities, quarter-wave and half-wave resonators), figures of merits***.
- ☐ Lecture 3 – W. Xu – April 21: ***Input couplers, multipacting, HOM couplers.***

Outline

- ☐ What is an RF transmission line?
- ☐ Transmission line parameters.
- ☐ Reflection.
- ☐ Impedance transformation in transmission lines.
- ☐ S-parameters.
- ☐ Common types of transmission lines used in accelerators: Coaxial line and Rectangular waveguide.

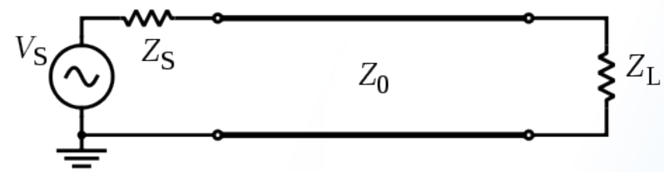


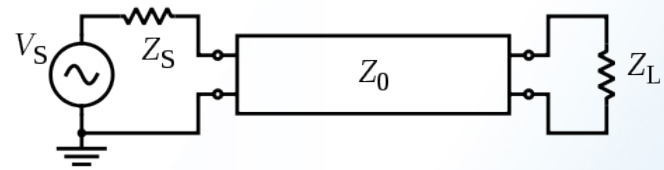
What is an RF transmission line?

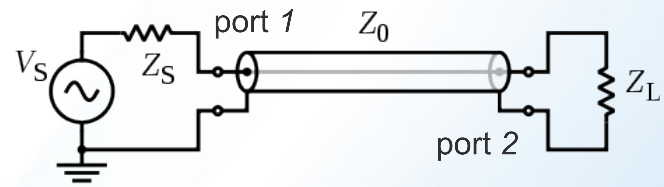
- ☐ In general, a transmission line is a medium or structure that forms a path to direct energy from one place to another.
- ☐ In RF engineering this term has a more specific meaning: transmission lines are specialized cables and waveguides designed to carry alternating current (AC) and electromagnetic waves of radio frequency ➔ the currents with frequency high enough that its wave nature has to be taken onto account.
- ☐ RF transmission lines “enclose” electromagnetic waves, preventing them from being radiated off the line, which would cause power loss.
- ☐ RF currents also tend to reflect from discontinuities such as connectors, and travel back down the line toward the source. These reflections act as bottlenecks, preventing the power from reaching the destination.
- ☐ Transmission lines use specialized construction such as precise conductor dimensions and spacing, and impedance matching, to carry electromagnetic signals with minimal reflections and power losses.

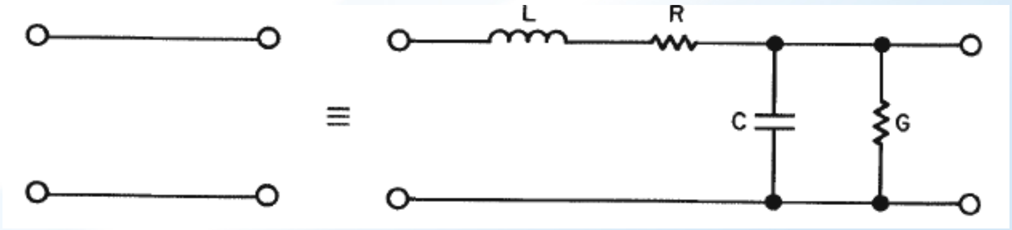
Properties of transmission lines

- A transmission line can be modeled as a two-port network:
- The network is assumed to be linear for simple lines, and the two ports are interchangeable.
- If the line is uniform along its length, then its behavior is largely described by a single parameter – the characteristic impedance Z_0, ratio of the complex voltage of a given wave to the complex current of the same wave at any point of the line.
- When sending power down a transmission line, it is usually desirable that as much power as possible will be absorbed by the load and as little as possible will be reflected back to the source. This can be ensured by making the load impedance equal to Z_0, in which case the transmission line is said to be matched.
- A small section of a transmission line can be analyzed using lumped-circuit analogy. A unit-length-long piece of the line is represented by a series inductor L with resistance R and a parallel capacitor C with conductance G:









Properties of transmission lines (2)

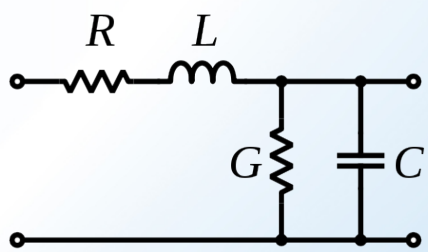
- The inductor L represents a current-carrying conductor, forming a magnetic field around itself that delays the voltage. The resistance R represents ohmic or resistive losses in the conductor. These two parameters form the impedance:

$$Z = R + j\omega L$$

- The capacitor C is due to finite distance between two conductors. Finally, the conductance G represents the dielectric losses in a dielectric medium that fills the space between two conductors. These two parameters form the admittance:

$$Y = G + j\omega C$$

- One can imagine now that a transmission line is built of an infinite number of infinitely short length of this type of two-port network cascaded one after another.



- Two parameters can be derived using the impedance and admittance:
  1. The propagation constant

$$\gamma = \sqrt{Z \cdot Y} = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

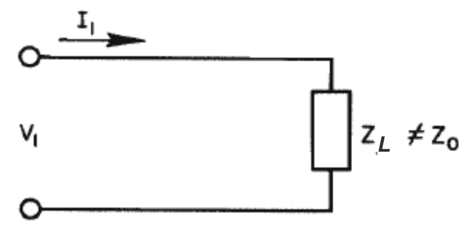
here α is the attenuation constant, $\beta = 2\pi/\lambda$ is the phase constant (wave number).

  2. The characteristic impedance

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}}$$

Reflection

- Let us excite a transmission line with: $V_1 = V_{1p}e^{j\omega t},\ \ I_1 = I_{1p}e^{j\omega t},\ \ \frac{V_1}{I_1} = Z_0$



- A wave will proceed along the line: $V = V_1 e^{\gamma l},\ \ I = I_1 e^{\gamma l}$

- If the line is terminated with an impedance Z_L not equal to the line characteristic impedance, not all energy propagated down the line will be absorbed. Part of the incident wave will be reflected back because the line is mismatched:

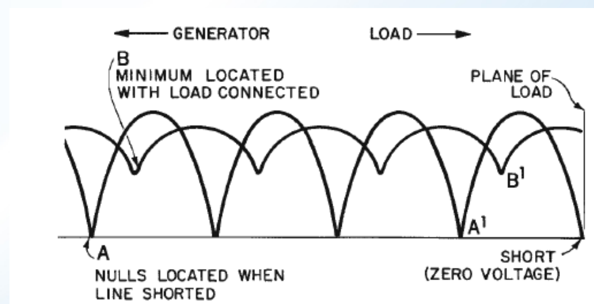
$$V = V_1 e^{\gamma l} + V_2 e^{-\gamma l},\ \ I = I_1 e^{\gamma l} - I_2 e^{-\gamma l}$$

- The voltage and current across the load (at l = 0)

$$V_L = V_1 + V_2,\ \ I_L = I_1 - I_2 \quad \text{have to satisfy} \quad Z_L = \frac{V_L}{I_L}$$

- From here we can get the reflection coefficient:

$$\Gamma = \frac{V_2}{V_1} = \frac{Z_L - Z_0}{Z_L + Z_0}$$



- The voltage standing wave ratio is defined as

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

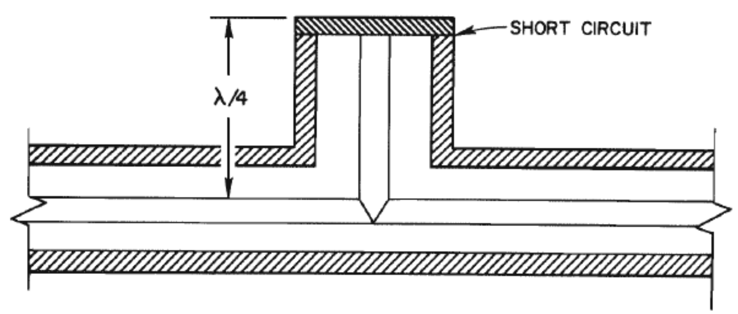
Impedance transformation

- It can be shown that the impedance measured at a given position l from the load impedance is

$$Z_{in}(l) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \approx Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

- Special cases:
  - half wavelength, $\beta l = n\pi$: $Z_{in} = Z_L$
  - quarter wavelength, $\beta l = (2n+1)\pi/2$: $Z_{in} = \frac{Z_0^2}{Z_L}$
  - matched load: $Z_{in} = Z_L = Z_0$

- This property of the transmission line is used to design impedance matching networks or to provide a very rigid support to the center conductor:



Sources of attenuation

- Attenuation can be contributed by many factors:
  - Resistive losses
  - Dielectric losses
  - Mismatch losses
  - Losses due to radiation
- The first two are absorptive by nature – they dissipate energy. The last two reflect the energy and guide it away from the transmission line, respectively.
- When considering resistive losses, one needs to take into account the skin effect. As frequency increases, skin effect becomes more critical. At high frequencies the current is restricted to travel only in the surface layer of the conductor. The skin depth is the thickness of the layer where the current density drops to $1/e$ the value on the surface:

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}},$$

  here s is the specific conductivity of the conductor. Warning: this formula is not applicable to very good conductors (e.g. copper) at low temperatures. One has to take into account the anomalous skin effect.
- Most dielectric materials have losses, which are taken into account by the imaginary part of the permittivity:

$$\varepsilon = \varepsilon' + j\varepsilon'', \qquad \tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

Sources of attenuation (2)

- Mismatch losses occur when not all available power reaches the load, but part of it reflected back. It describes how many decibels less than incident power available is being absorbed by the termination:

$$\text{mismatch loss [dB]} = -10\log_{10}\left(1-|\Gamma|^2\right)$$

- Return loss tells us how far down the reflected wave is from the incident wave:

$$\text{return loss [dB]} = -20\log_{10}|\Gamma|$$

- Openings on the transmission line (e.g. poorly fitted connectors and flanges, copper mesh outer conductor) allow the electromagnetic wave to radiate out of the line and cause losses. At high frequencies some cables are double- and even triple-shielded to alleviate losses due to radiation.

S-parameters

- Scattering parameters or S-parameters are coefficients (elements) of a scattering matrix. They are used to describe behavior of linear microwave networks.
- S-parameters are members of a family of similar parameters: Y-parameters (admittance), Z-parameters (impedance), T-parameters (transmission), and ABCD-parameters (cascade or transmission line).
- S-parameters are widely used in RF/microwave measurements as they formulate the transformation properties in terms of incident and reflected waves and use matched loads. S-parameters can be directly measured with network analyzers.
- It is common to normalize incident and reflected waves as

$$a_n = \frac{V_{n+}}{\sqrt{Z_{0n}}}, \quad b_n = \frac{V_{n-}}{\sqrt{Z_{0n}}}$$

- And then for a two-port network the S-parameters are given by

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \Rightarrow \begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

- With the source applied to port 1 and the output matched so that $a_2 = 0$, $\mathbf{S_{11}}$ is just the input reflection coefficient (magnitude and phase), and $\mathbf{S_{21}}$ is the transmission coefficient.

T-parameters

- For cascaded networks it is more useful to use transmission or T-parameters

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} \Rightarrow \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} 1/S_{11} & -S_{22}/S_{12} \\ S_{11}/S_{12} & \left(S_{12}^2 - S_{11}S_{22}\right)/S_{12} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix}$$

- Then for the cascaded network

$$\left(T_{total}\right) = \left(T_1\right)\left(T_2\right)\cdots\left(T_n\right)$$

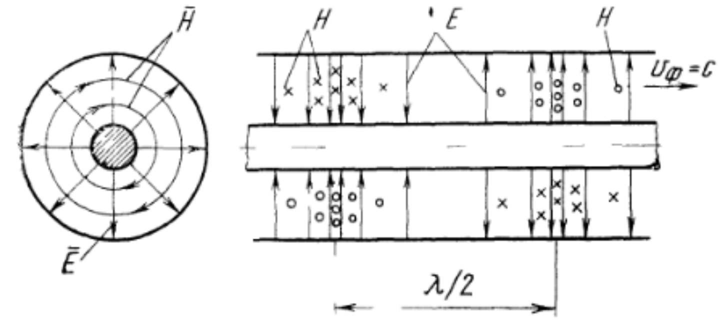
Transmission lines: coaxial line

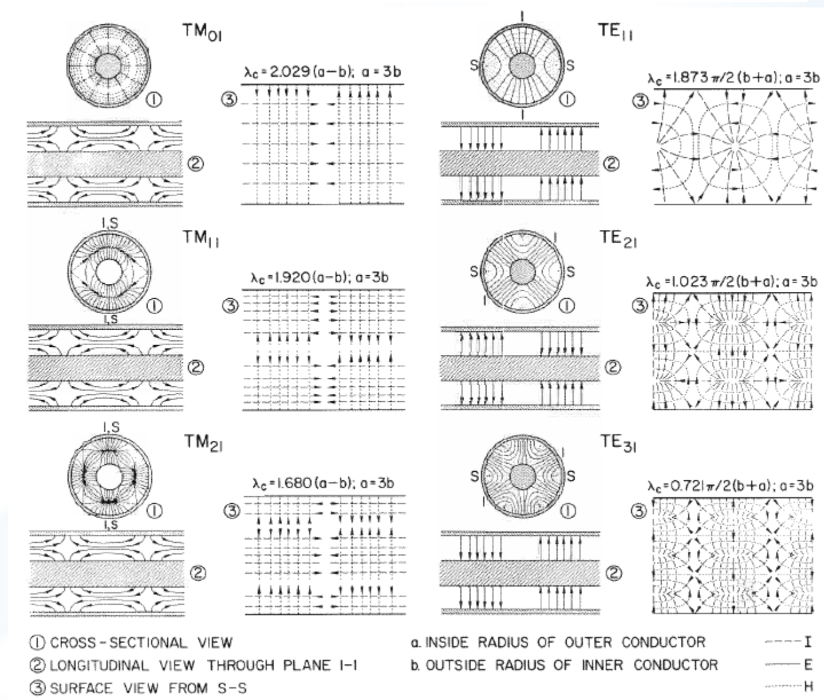
- Two types of transmission lines are commonly used in accelerator RF systems: a coaxial line and a rectangular waveguide.
- The coaxial line has two conductors, center and outer, and therefore can support TEM mode (as well as waveguide modes).
- The bandwidth of a coaxial line is theoretically infinite, however in practice the maximum frequency is limited to the cutoff of the lowest waveguide mode → the line dimensions become smaller at high frequencies.
- The characteristic impedance of the coaxial line is

$$Z_0 = \frac{\eta_0}{2\pi}\ln\left(\frac{R_o}{R_i}\right) \approx 60\sqrt{\frac{\mu_r}{\varepsilon_r}}\ln\left(\frac{R_o}{R_i}\right), \quad \eta_0 = \sqrt{\mu/\varepsilon}$$

- If the losses are small

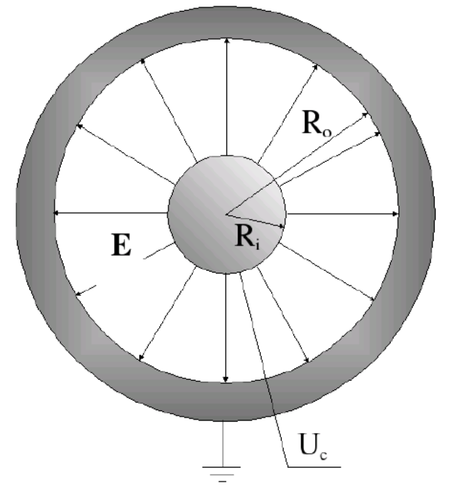
$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}, \quad \lambda_l = \frac{\lambda}{\sqrt{\mu_r \varepsilon_r}}$$





Losses in coaxial lines

- The equivalent circuit parameters for the coaxial line are:



$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{R_o}{R_i}\right)} \qquad L = \frac{\mu}{2\pi}\ln\left(\frac{R_o}{R_i}\right)$$

$$G = \frac{2\pi\omega\varepsilon''}{\ln\left(\frac{R_o}{R_i}\right)} \qquad R = \frac{R_s}{2\pi}\left(\frac{1}{R_o}+\frac{1}{R_i}\right)$$

here $R_s = 1/\sigma\delta$ is the surface resistivity of conductor.

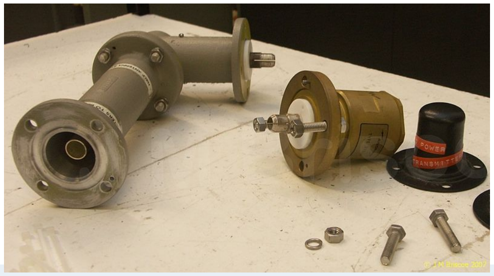
- From these parameters we can get the attenuation due to conductor losses and due to dielectric losses

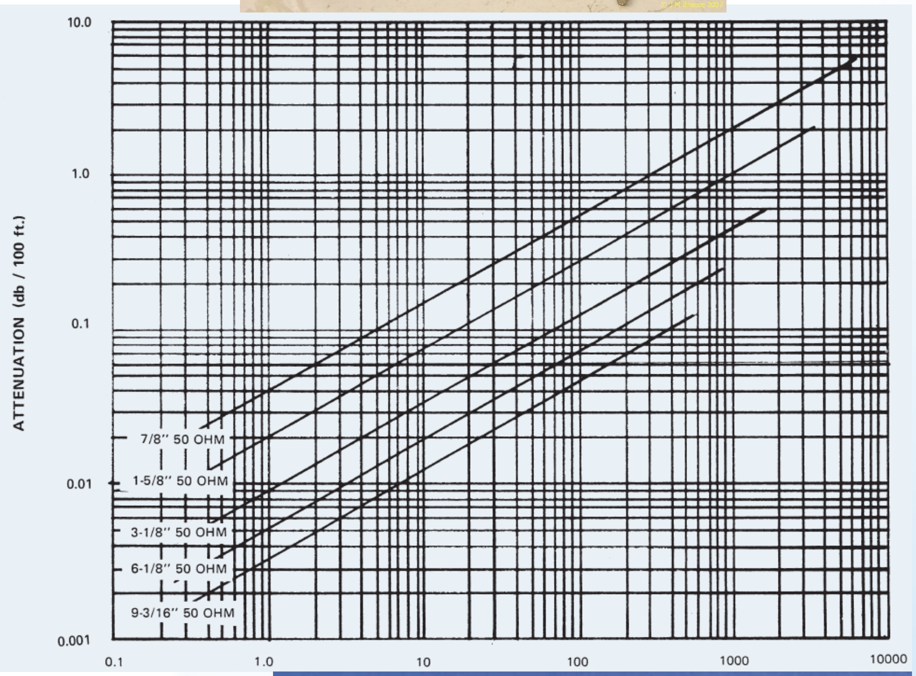
$$\alpha = \alpha_{cond} + \alpha_{diel} = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) = \frac{1}{2}\left[\frac{R_s}{\eta\ln\left(R_o/R_i\right)}\left(\frac{1}{R_o}+\frac{1}{R_i}\right) + \omega\eta\varepsilon''\right] \text{ Np/m}$$

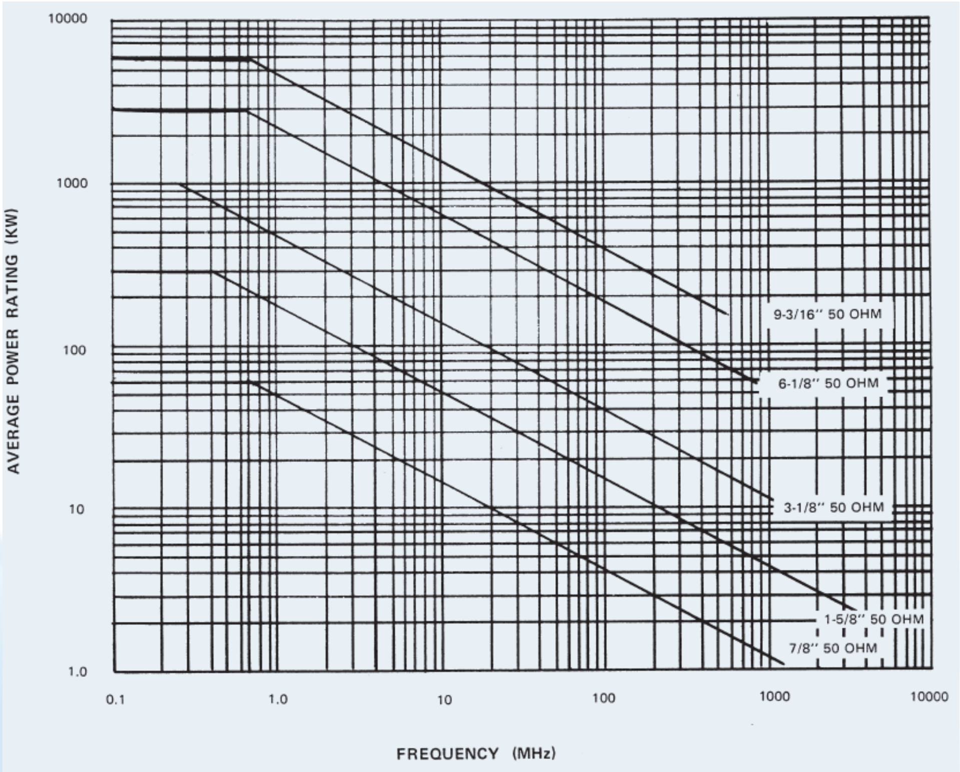
- Attenuation in decibels per meter is 8.686 times the attenuation in nepers per meter.
- Losses at lower frequencies are usually dominated by the conductor losses and increase as $\sqrt{f}$ due to skin effect.
- For fixed $\boldsymbol{R_o}$, minimal attenuation is achieved at $R_o/R_i = 3.6$ (77 Ohm), maximum power (limited by the breakdown electric field at the inner conductor) can be carried by a line with $R_o/R_i = 1.65$ (30 Ohm), maximum voltage between the conductors can be maintained at $R_o/R_i = 2.72$ (60 Ohm).

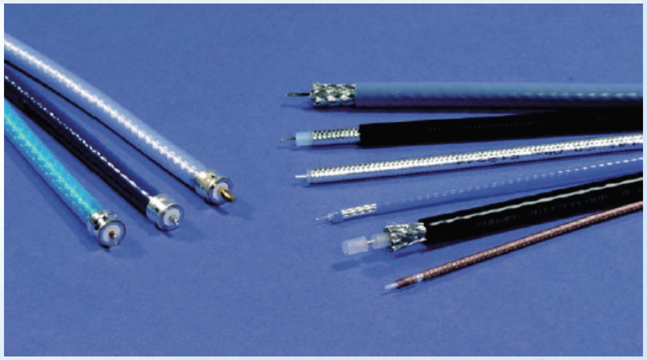
Commercial coaxial lines

- Coaxial lines are fabricated as cables (flexible, semi-flexible, semi-rigid) and rigid lines.
- Rigid lines are specified by their impedance and outer connector OD.





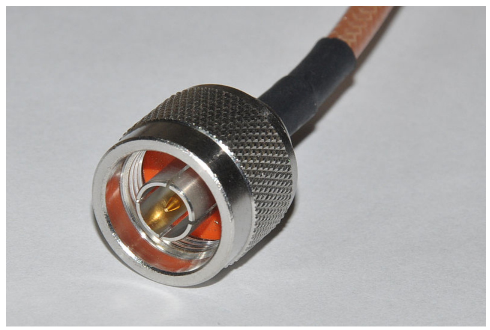




Connectors for coaxial lines

- There is a wide variety of coaxial connectors available for use at different frequencies and power levels.
- EIA (Electronic Industries Alliance) standard connectors are used for rigid lines.
- The table lists some of them and their maximum frequency.

SMA



Type N

7-16 DIN

7/8 EIA

Connector Type	Maximum Frequency (GHz)
2.4mm	50
2.92mm/K	40
3.5mm	34
SMA precision	26.5
BNC	4
TNC	18
Type N	11
Type N precision	18
Type C	12
7-16 DIN	7.5
7/8 EIA	7.5
1 5/8 EIA	3

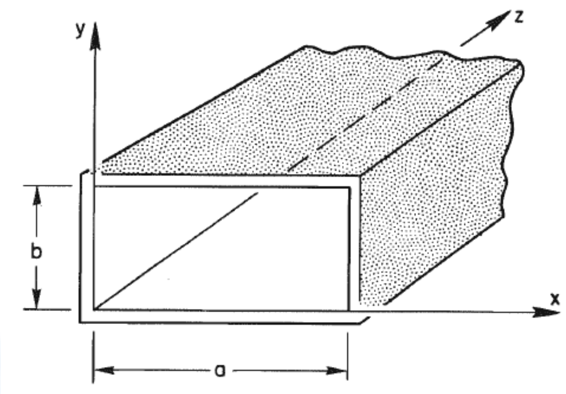
Transmission lines: waveguide

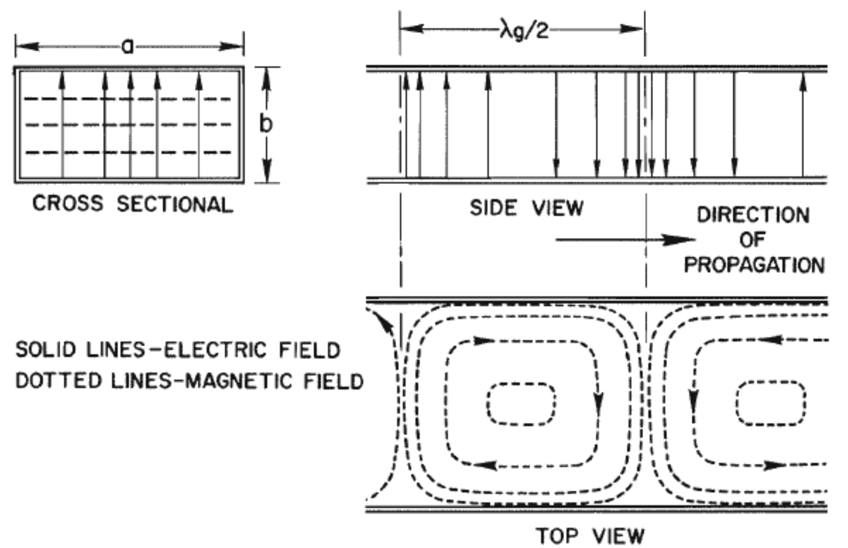
- Waveguides can support only TE and TM modes. Usually the lowest mode, TE_{10}, mode is used and the bandwidth is limited by the cutoff frequencies if this and the next lowest modes. For TE_{10}

$$\lambda_c = 2a, \qquad \lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{\lambda}{\sqrt{1-(\lambda/2a)^2}}$$

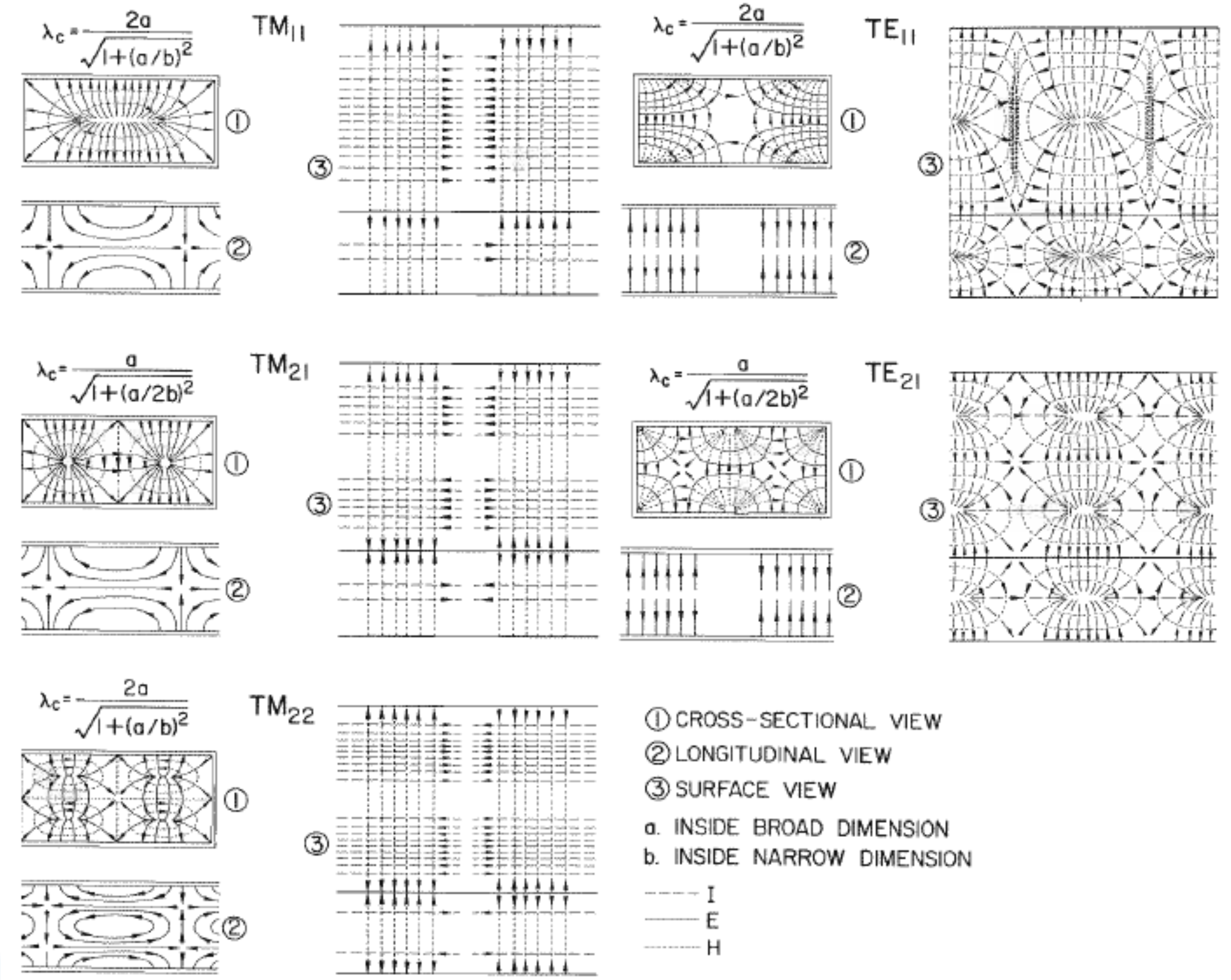
- A rectangular waveguides is the most common type.
- Waveguides are usually less lossy than coaxial lines due to bigger dimensions and absence of inner conductor.
- Losses increase as $\sim f^{3/2}$ as in addition to the skin depth decrease one has to use smaller and smaller size waveguides.

$$\alpha_{cond} = \frac{R_s}{b\eta\sqrt{1-(\lambda/2a)^2}}\left[1+\frac{2b}{a}\left(\frac{\lambda}{2a}\right)^2\right] \text{ Np/m}$$



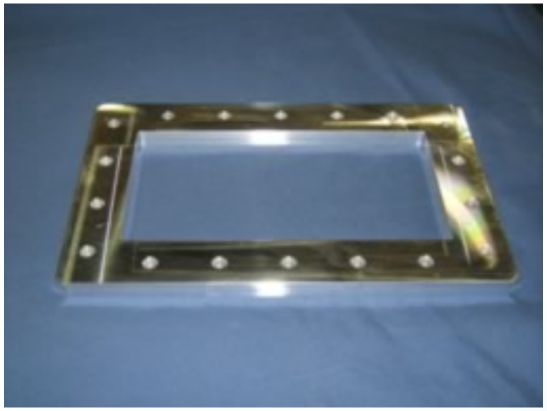


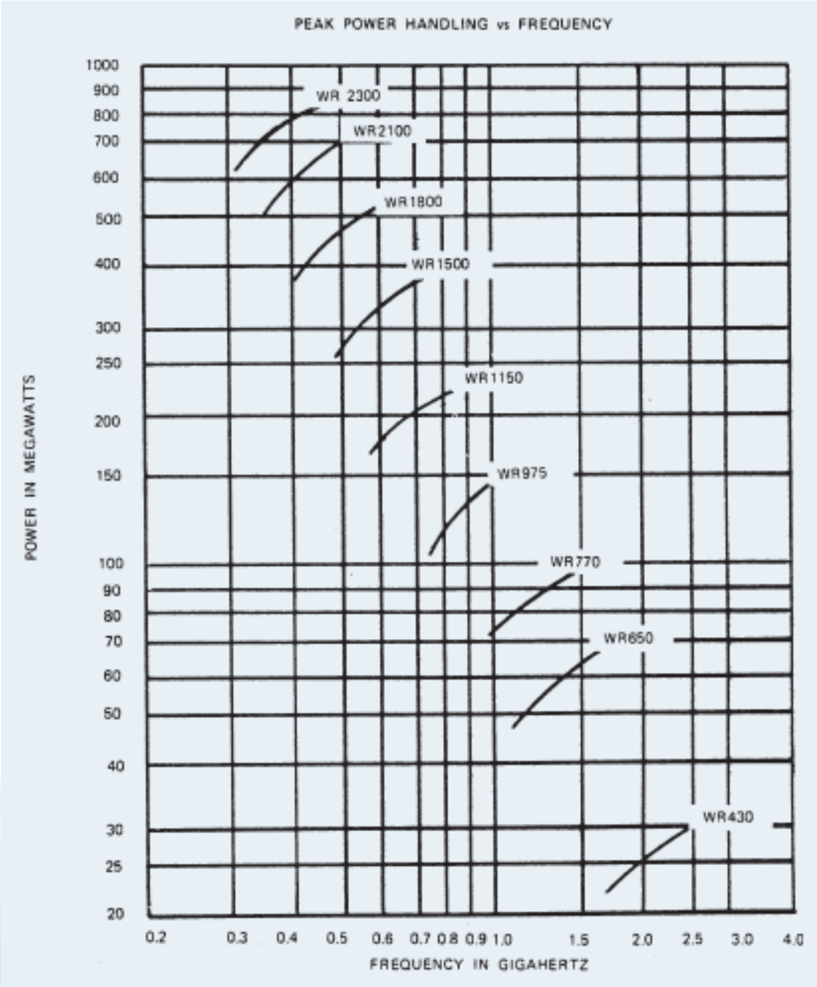
Modes in rectangular waveguide

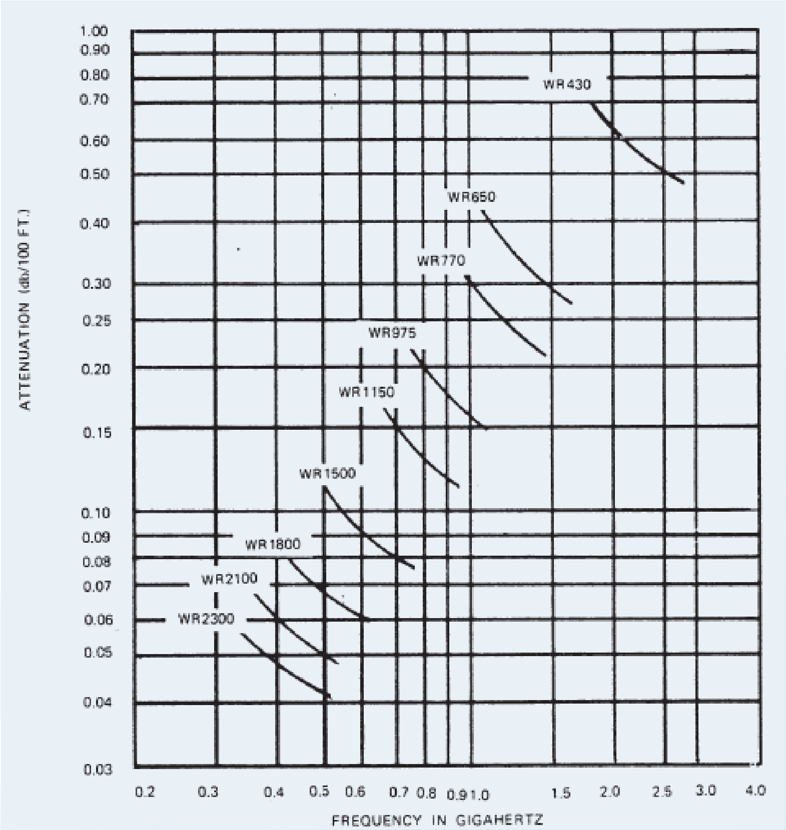


Commercial waveguides

- WR waveguides are commercially produced.
- The number after WR is the width (*a*) of the waveguide in inches×100.







Rectangular waveguide data

Table 2.3-1. Reference table of rigid rectangular waveguide data and fittings.†

EIA WG WR ( )	MDL Band	Recommended Operating Range for TE_{10} Mode: Frequency (kmc/sec)	Recommended Operating Range for TE_{10} Mode: Wavelength (cm)	Cut-off for TE_{10} Mode: Frequency (kmc/sec)	Cut-off for TE_{10} Mode: Wavelength (cm)	Range in $\frac{2\lambda}{\lambda c}$	Range in $\frac{\lambda g}{\lambda}$	Theoretical cw power rating lowest to highest frequency (mw)	Theoretical attenuation lowest to highest frequency (db/100 ft.)	EIA WG WR ( )	JAN WG RG ( )	Material Alloy	JAN FLANGE: Choke UG ( )/U	JAN FLANGE: Cover UG ( )/U	DIMENSIONS (Inches): Inside	Tol. (±)	DIMENSIONS (Inches): Outside	Tol. (±)	Wall Thickness (nom.)
2300		0.32-0.49	93.68-61.18	0.256	116.84	1.60-1.05	1.68-1.17	153.0-212.0	.051-.031	2300		Alum.			23.000-11.500	0.020	23.376-11.876	.020	0.188
2100		0.35-0.53	85.65-56.56	0.281	106.68	1.62-1.06	1.68-1.18	120.0-173.0	.054-.034	2100		Alum.			21.000-10.500	0.020	21.376-10.876	.020	0.188
1800		0.41-0.625	73.11-47.96	0.328	91.44	1.60-1.05	1.67-1.18	93.4-131.9	.056-.038	1800	201	Alum.			18.000-9.000	0.020	18.250-9.250	.020	0.125
1500		0.49-0.75	61.18-39.97	0.393	76.20	1.61-1.05	1.62-1.17	67.6-93.3	.069-.050	1500	202	Alum.			15.000-7.500	0.015	15.250-7.750	.015	0.125
1150		0.64-0.96	46.84-31.23	0.513	58.42	1.60-1.07	1.82-1.18	35.0-53.8	.128-.075	1150	203	Alum.			11.500-5.750	0.015	11.750-6.000	.015	0.125
975		0.75-1.12	39.95-26.76	0.605	49.53	1.61-1.08	1.70-1.19	27.0-38.5	.137-.095	975	204	Alum.			9.750-4.875	0.010	10.000-5.125	.010	0.125
770		0.96-1.45	31.23-20.67	0.766	39.12	1.60-1.06	1.66-1.18	17.2-24.1	.201-.136	770	205	Alum.			7.700-3.850	0.010	7.950-4.100	.010	0.125
650	L	1.12-1.70	26.76-17.63	0.908	33.02	1.62-1.07	1.70-1.18	11.9-17.2	.317-.312 .269-.178	650	69 103	Brass Alum.		417A* 418A*	6.500-3.250	0.010	6.660-3.410	.010	0.080
510		1.45-2.20	20.67-13.62	1.157	25.91	1.60-1.05	1.67-1.18	7.5-10.7		510					5.100-2.550	0.010	5.260-2.710	.010	0.080
430	W	1.70-2.60	17.63-11.53	1.372	21.84	1.61-1.06	1.70-1.18	5.2-7.5	.588-.385 .501-.330	430	104 105	Brass Alum.		435A* 437A*	4.300-2.150	0.008	4.460-2.310	.008	0.080
340		2.20-3.30	13.63-9.08	1.736	17.27	1.58-1.05	1.78-1.22	3.1-4.5	.877-.572 .751-.492	340	112 113	Brass Alum.		553* 554*	3.400-1.700	0.005	3.560-1.860	.005	0.080
284	S	2.60-3.95	11.53-7.59	2.078	14.43	1.60-1.05	1.67-1.17	2.2-3.2	1.102-.752 .940-.641	284	48 75	Brass Alum.	54B 585A	53 584	2.840-1.340	0.005	3.000-1.500	.005	0.080
229		3.30-4.90	9.08-6.12	2.577	11.63	1.56-1.05	1.62-1.17	1.6-2.2		229					2.290-1.145	0.005	2.418-1.273	.005	0.064
187	C	3.95-5.85	7.59-5.12	3.152	9.510	1.60-1.08	1.67-1.19	1.4-2.0	2.08-1.44 1.77-1.12	187	49 95	Brass Alum.	148C 406B	149A 407	1.872-0.872	0.005	2.000-1.000	.005	0.064
159		4.90-7.05	6.12-4.25	3.711	8.078	1.51-1.05	1.52-1.19	0.79-1.0		159					1.590-0.795	0.004	1.718-0.923	.004	0.064