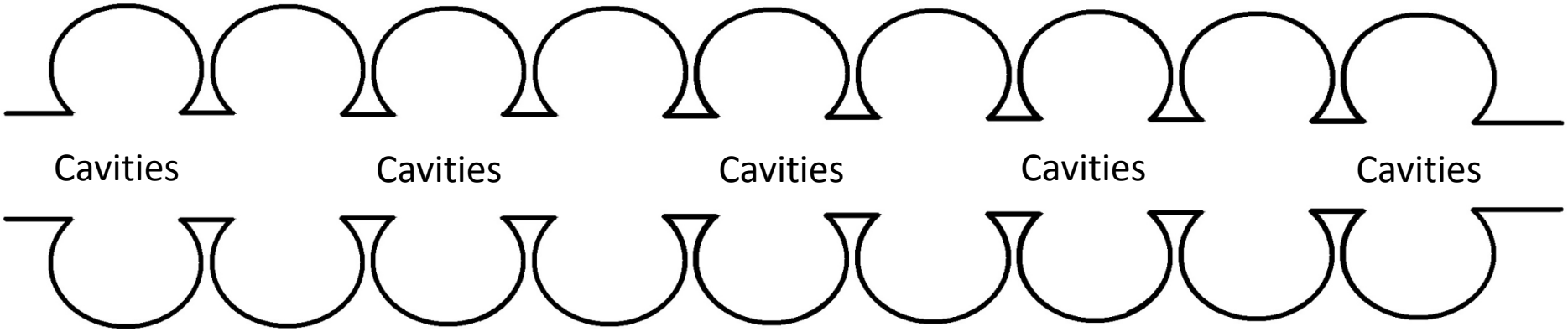
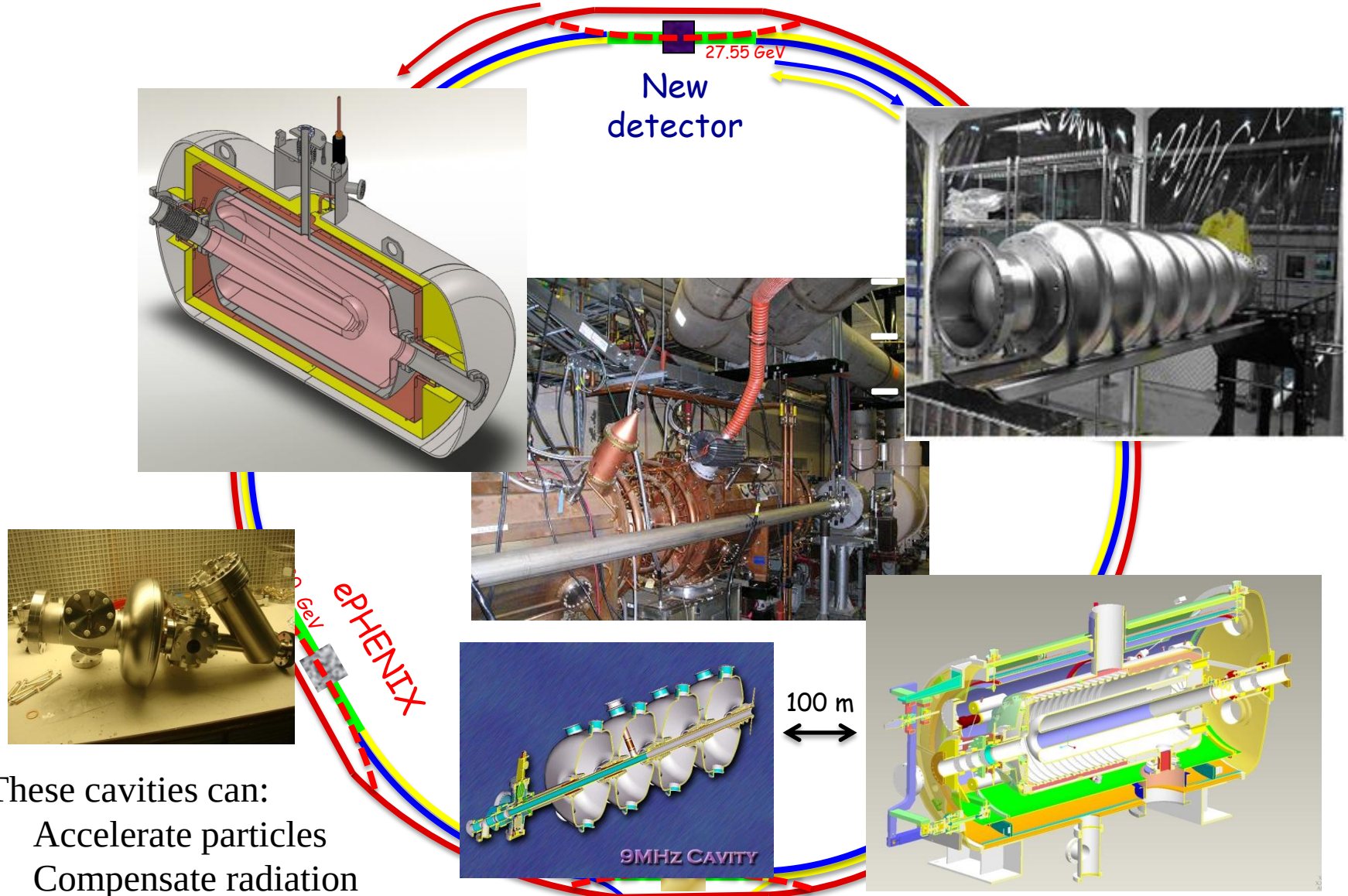


Qiong Wu
Sergey Belomestnykh
Wencan Xu
April 19/21, 2011

Part I

- Accelerating the particles
- The pillbox – where it all begins
- From coaxial line to resonators

Accelerating the particles

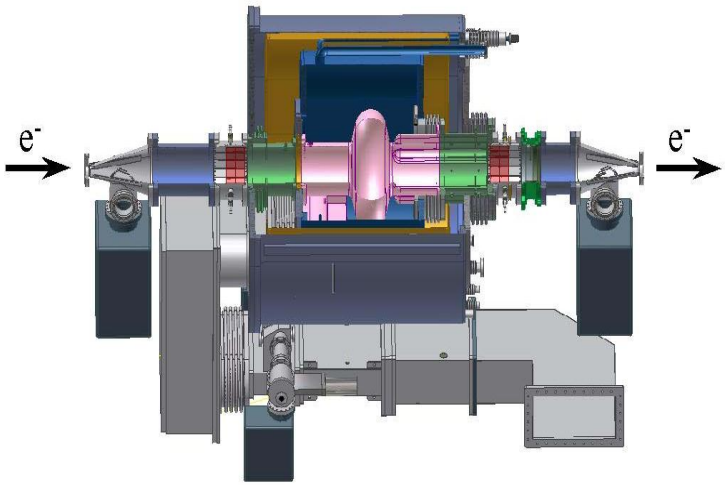


These cavities can:

- Accelerate particles
- Compensate radiation
- Increase colliding possibility

Overview

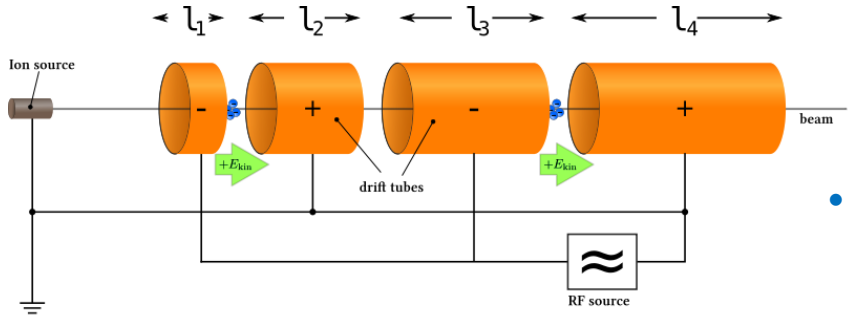
- The main purpose of using RF cavities in accelerators is to provide **energy exchange** to charged-particle beams at a fast acceleration rate.
- The highest achievable gradient, however, is NOT always optimal for an accelerator. There are other factors (both machine-dependent and technology-dependent) that determine operating gradient of RF cavities and influence the cavity design, such as accelerator cost optimization, maximum power through an input coupler, necessity to extract HOM power, etc.
- Moreover, although the cavity is the heart, the central part of an accelerating module, it is only one of many parts and its design cannot be easily decoupled from the design of the whole system.
- In many cases requirements are competing.



Taiwan Light Source cryomodule

The B.C. (Before Cavity) era:

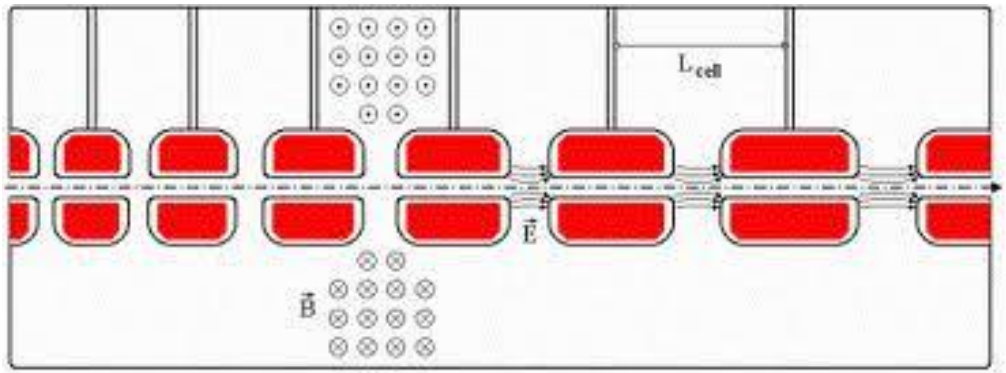
Wideröe Linac (1927)



- the Wideröe linac was only efficient for low-velocity particles (low-energy heavy ions),
- higher frequencies (> 10 MHz) were not practical, because then the drift tubes would act more like antennas and radiate energy instead of using it for acceleration,
- when using low frequencies, the length of the drift tubes becomes prohibitive for high-energy protons (for 20MeV protons, the length will reach 3m)

Alvarez Linac (1946)

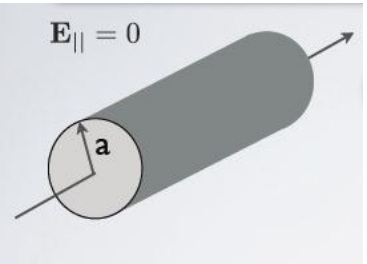
Enclosed RF system is made possible by the development of high power & high frequency RF sources after WWII

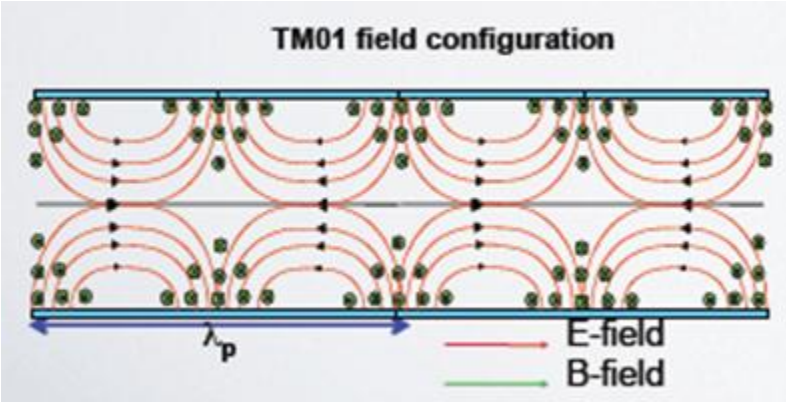


Operated @ 200MHz

Alternatives? EM field in waveguide?

Q: Why RF?
A: Higher fields can be obtained at higher frequency
Q: Why resonators?
A: resonators are used because their response can amplify an external excitation.





EM wave propagates through a cylindrical waveguide have a frequency of:

$$\omega = kc = \sqrt{\omega^2 - {\omega_c}^2}$$

The phase velocity of the wave is

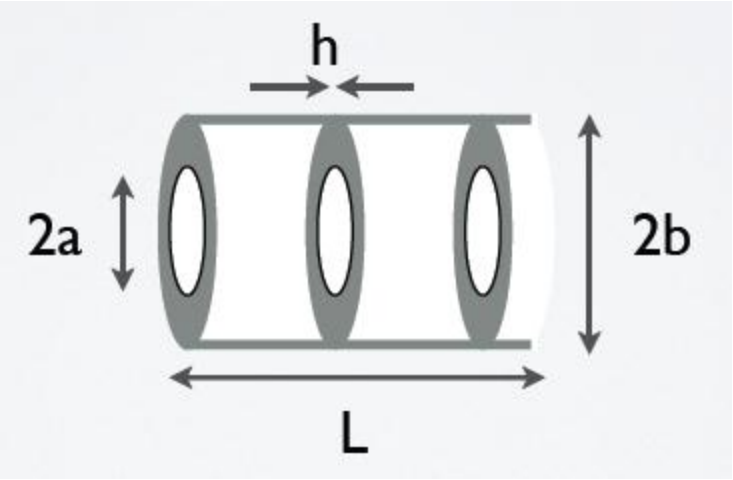
$$v_{ph} = \frac{\omega}{k} = \frac{\omega c}{\sqrt{\omega^2 - {\omega_c}^2}} > c$$

synchronism with RF (necessary for acceleration) is impossible because a particle would have to travel at $v=v_{ph}>c$!

We need to slow down the phase velocity!

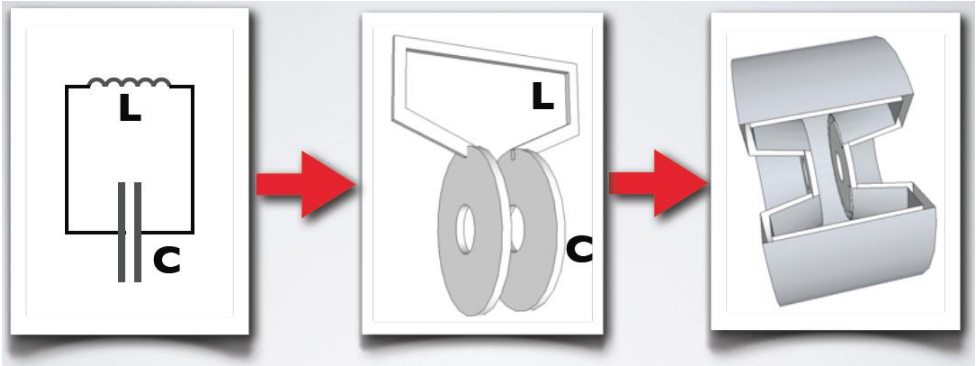
Slow down the phase velocity

Put obstacles into the waveguide:



The pillbox – where it all begins

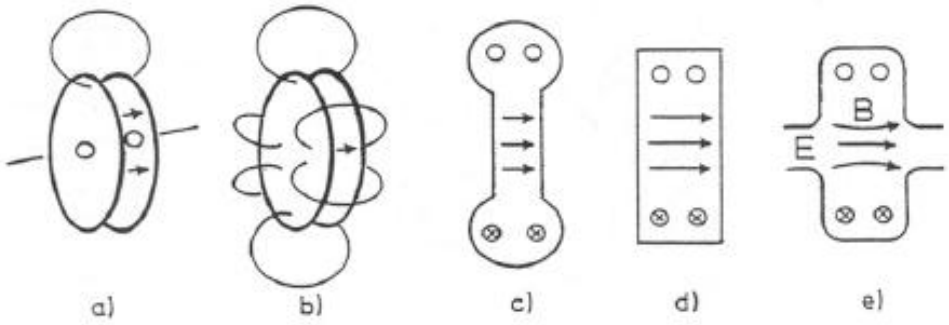
RLC circuit to Pillbox cavity



The following aspects are shared in common by RF cavities and RLC circuits

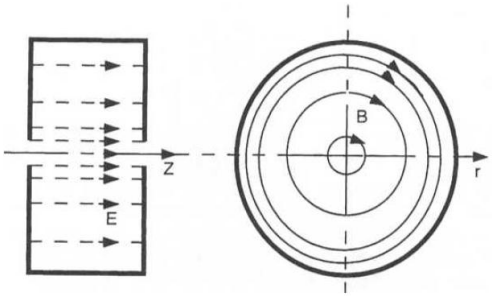
- Energy is stored via electric field and magnetic field in the system
- Energy is exchanged between electric field and magnetic field
- Without any external input, the stored power will eventually all turn into heat dissipation
- The system always has a resonant frequency. When the external drive is of this frequency, the EM field in the system is excited to extreme

LC circuit and RF cavities

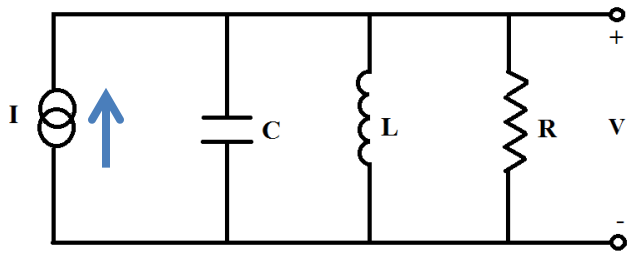


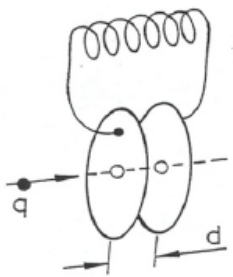
Metamorphosis of the LC circuit into an accelerating cavity:

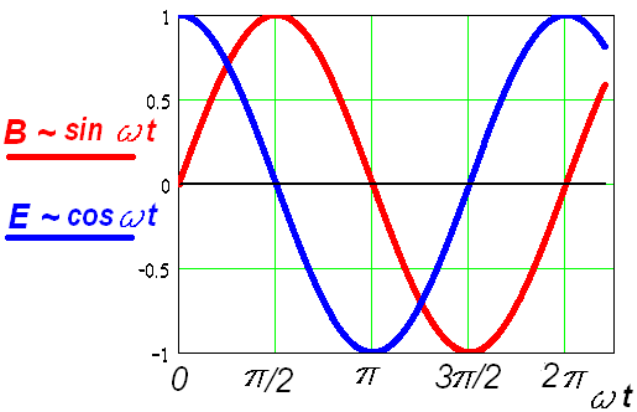
1. **Increase resonant frequency by lowering L, eventually have a solid wall.**
2. **Further frequency increase by lowering C → arriving at cylindrical, or "pillbox" cavity geometry, which can be solved analytically.**
3. **Add beam tubes to let particle pass through.**

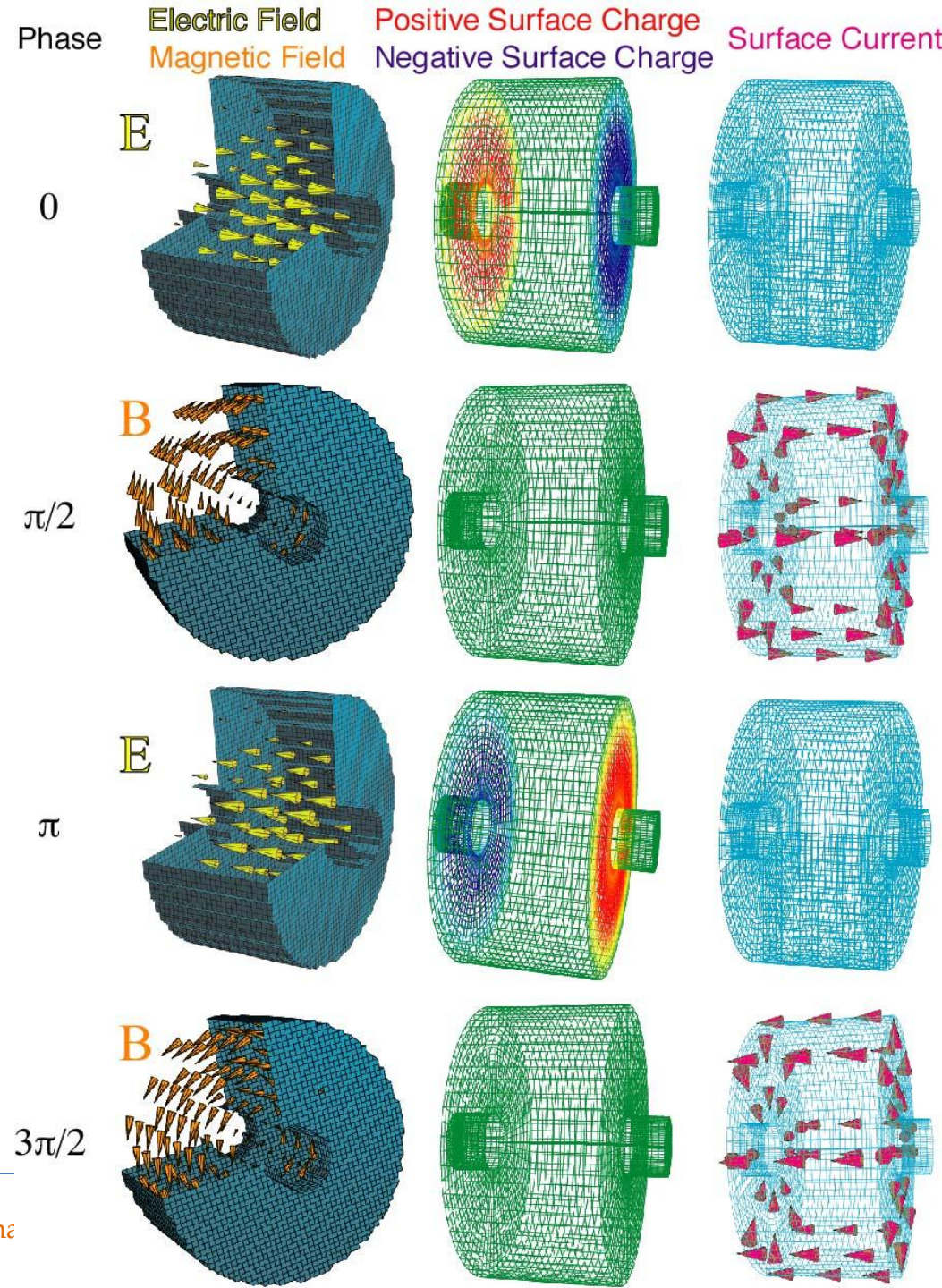


- **Magnetic field is concentrated at the cylindrical wall, responsible for RF losses.**
- **Electric field is concentrated near axis, responsible for acceleration.**


I
C
L
R
V
+
-


q
d


B ~ sin ωt
E ~ cos ωt
1
0.5
0
-0.5
-1
0
π/2
π
3π/2
2π
ωt

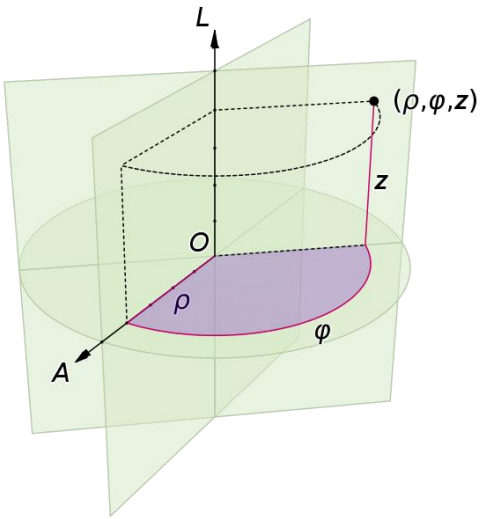

Phase
Electric Field
Magnetic Field
Positive Surface Charge
Negative Surface Charge
Surface Current
E
0
B
π/2
E
π
B
3π/2

Modes in Pillbox Cavity

- TM_{010}
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Frequency depends only on radius, independent on length
- TM_{0np}
  - Monopoles modes that can couple to the beam and exchange energy

- TM_{1np}
  - Dipole modes that can deflect the beam

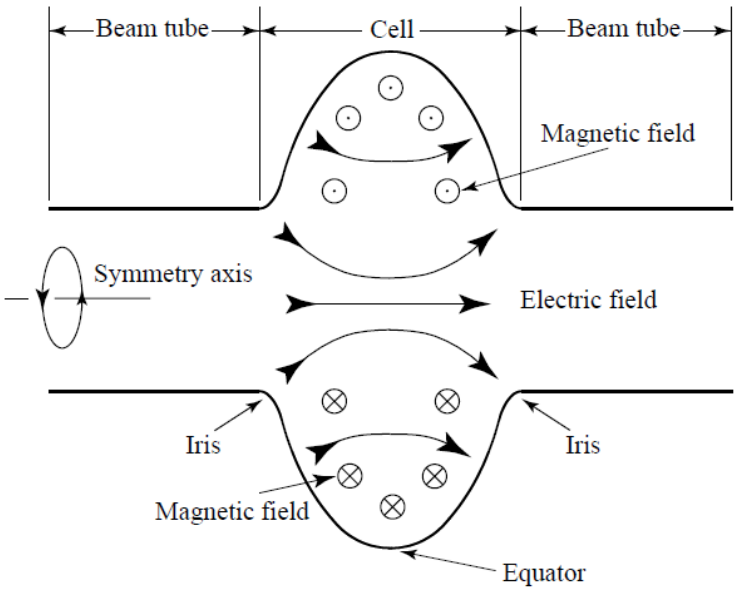
- TE modes
  - No longitudinal E field
  - Cannot couple to the beam

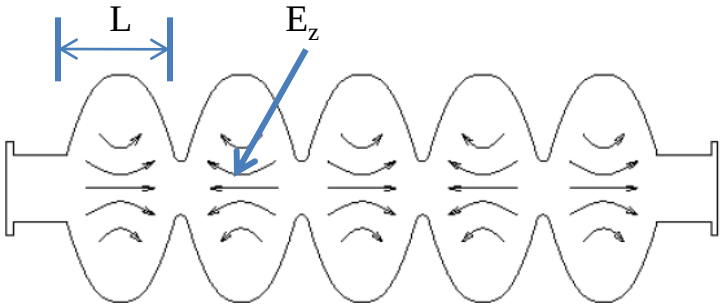


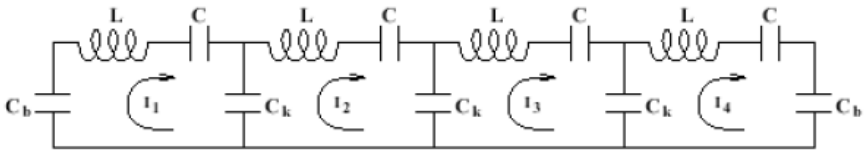
$$TM_{mnp}$$

The integer indices m, n, and p are measures of the number of sign changes E_z undergoes in the ϕ, ρ, and z directions, respectively.

Let's go one step further - Elliptical Cavity







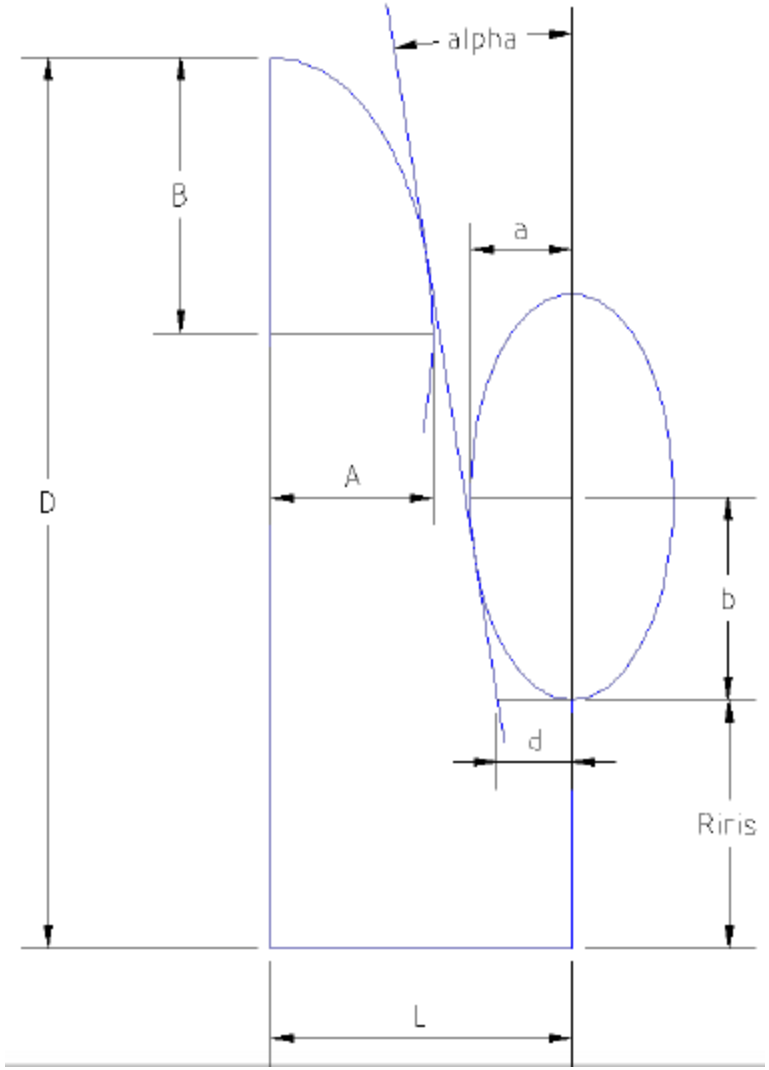
What we gain from pillbox to elliptical:

- Better manipulation of cavity characteristics
- Higher acceleration field
- Less unwanted effects, e.g. multi-pacting

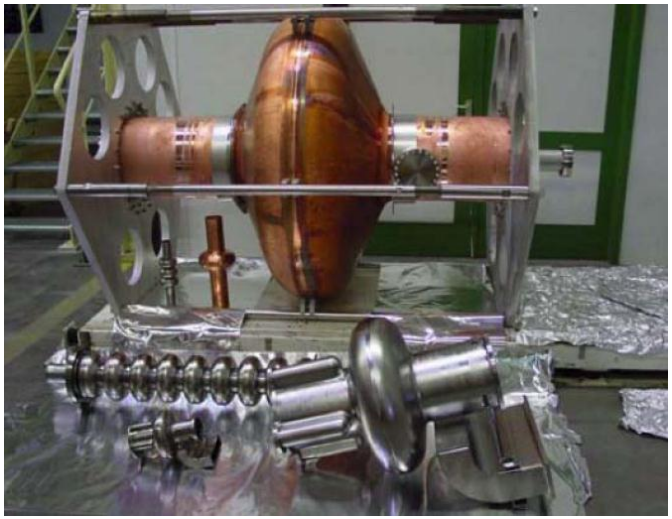
Energy gain $\Delta W = E_0 \frac{\lambda}{\pi} sin \frac{\pi L}{\lambda}$

Accelerating gradient $E_{acc} = \frac{\Delta W}{\lambda/2} = E_0 \frac{2}{\pi} sin \frac{\pi L}{\lambda}$

Design of an elliptical cavity:



Behavior of all EM and mechanical properties has been found as a function of the parameters on the left

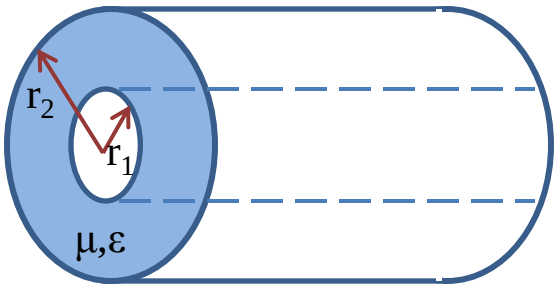


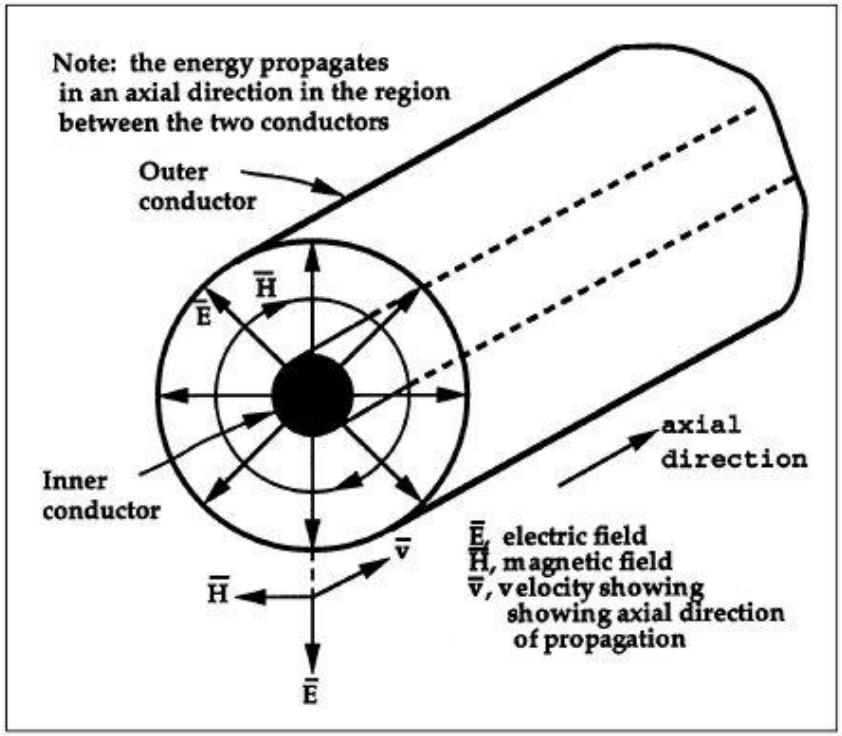
From coaxial line to resonators

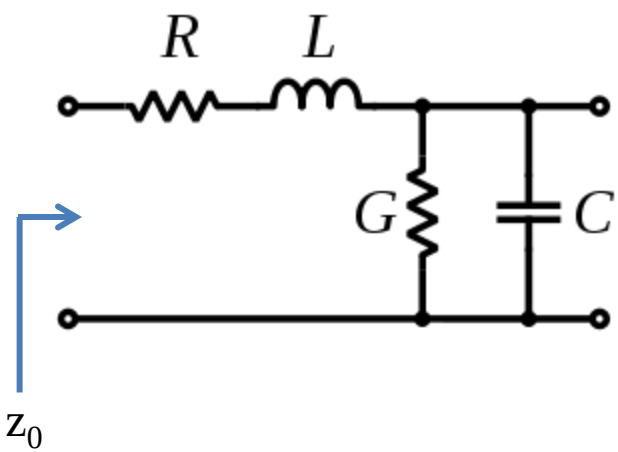
--Road to Compact size + Low Frequency

Inside a Coaxial Line

Coaxial line - TEM wave transmission





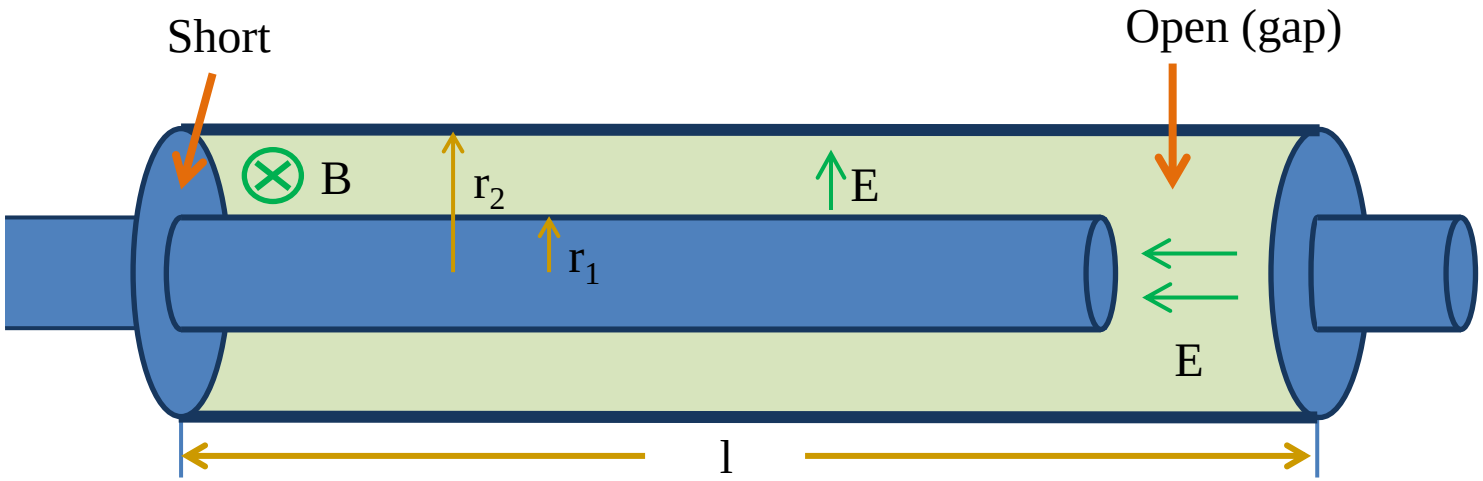


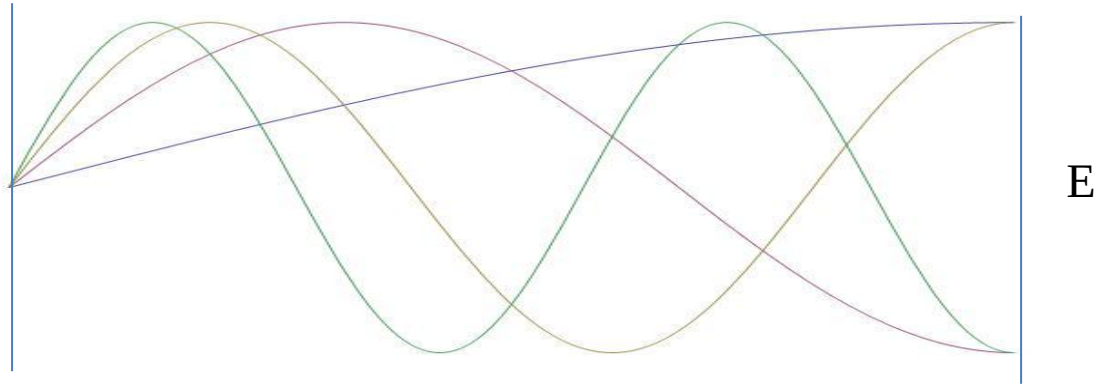
$$z_0 = \sqrt{L/C} = \frac{\sqrt{\mu/\epsilon}}{2\pi}\log(\frac{r_2}{r_1})$$

Characteristics of a coaxial line:

- Both E&B fields are transverse
- No cutoff frequency for TEM modes
- What if we have boundary condition?

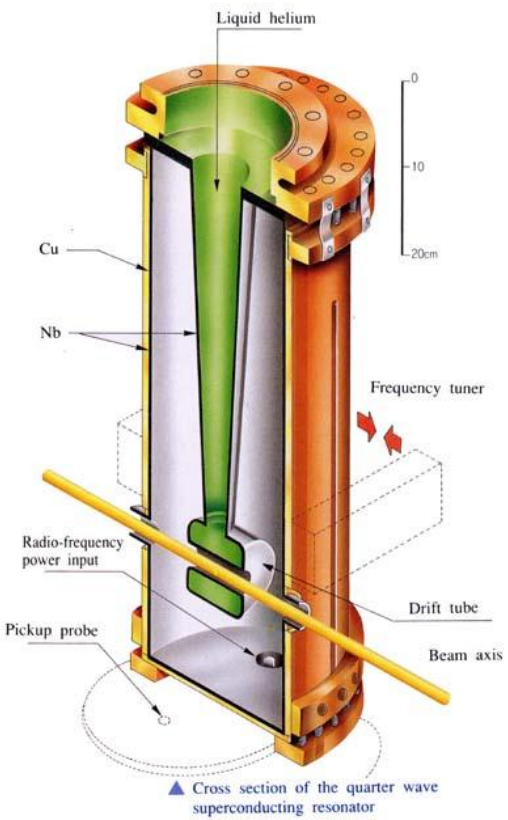
Quarter Wave Resonator (QWR)





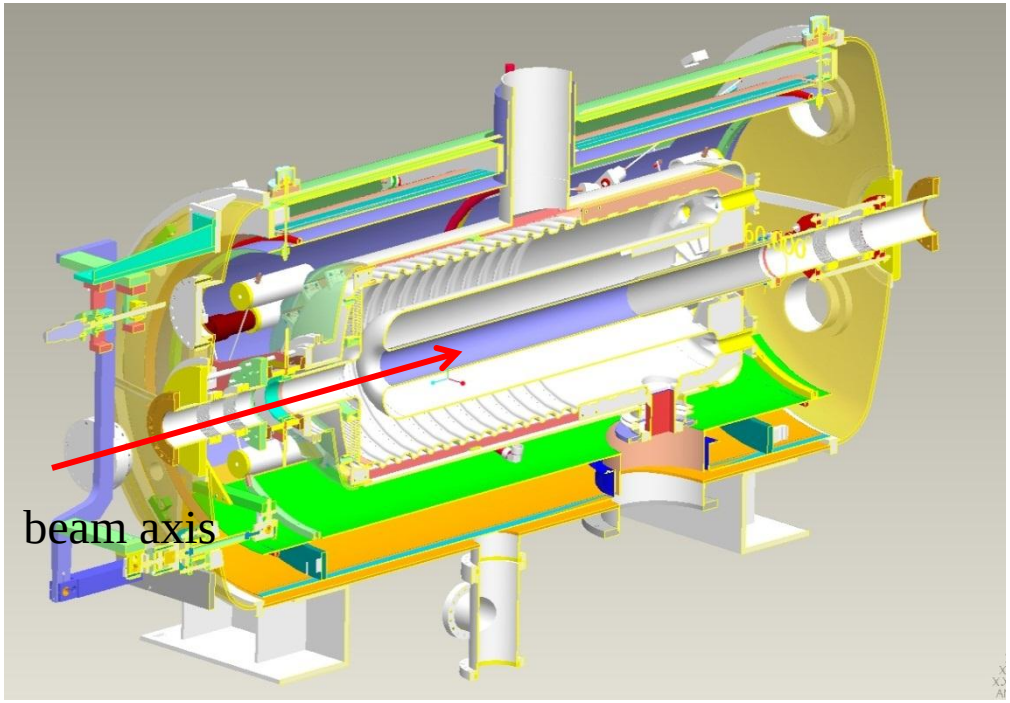
- E field must be normal to the metal surface
- The short end does not have E field in a coaxial line
- The lowest resonance frequency in the above structure is $\omega = 2\pi c/(4l)$. (The gap is usually very small compared to the total length)
- There are also higher order modes with $\omega_n = (2n+1)2\pi c/(4l)$.

Orientation: Do we have a choice?



Quarter wave resonator developed by JAERI

vs



56MHz SRF cavity for RHIC

Application: Higher accelerating voltage can be obtained with double gap, but only good for low β particles in order to get acceleration in both gaps.

Application: Single gap makes this scheme capable for high β particles, but can not provide as high voltage as the vertical setup.

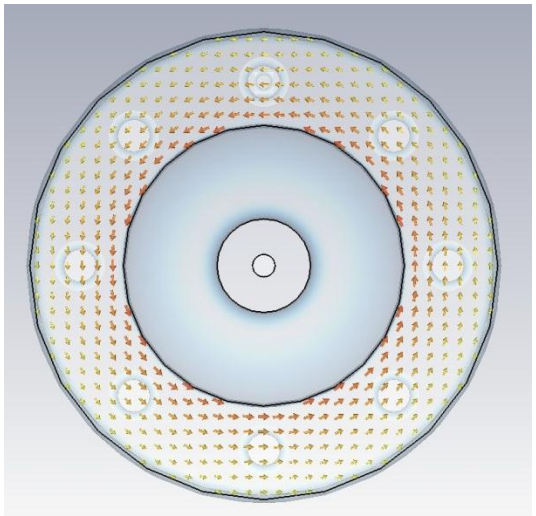
Inside a QWR

Fundamental Mode (Lowest resonance frequency)

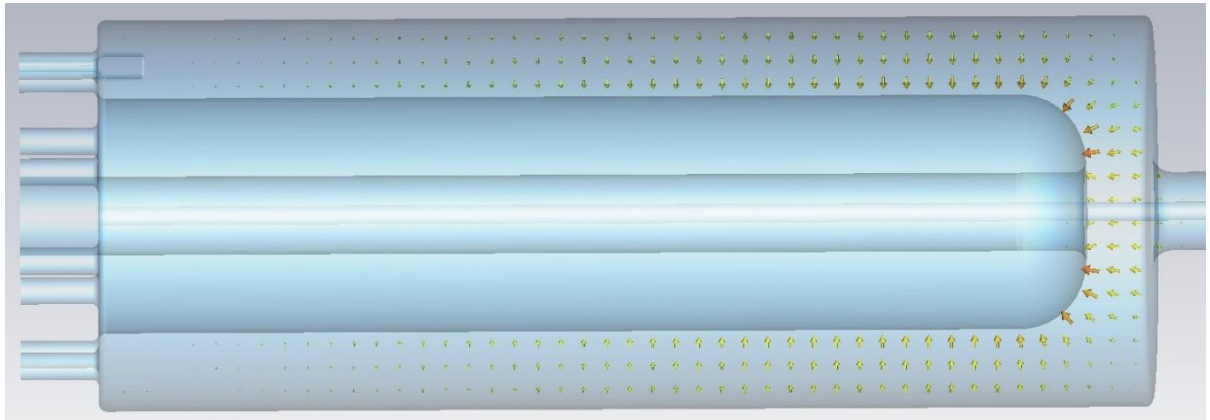
- The E field automatically peaks at the gap
- The B field maximized at the short end

Higher Order Modes

- Can be excited by the beam and act back on it
- Harmful for the beam (instabilities)
- Can be easily extracted from the shorted end

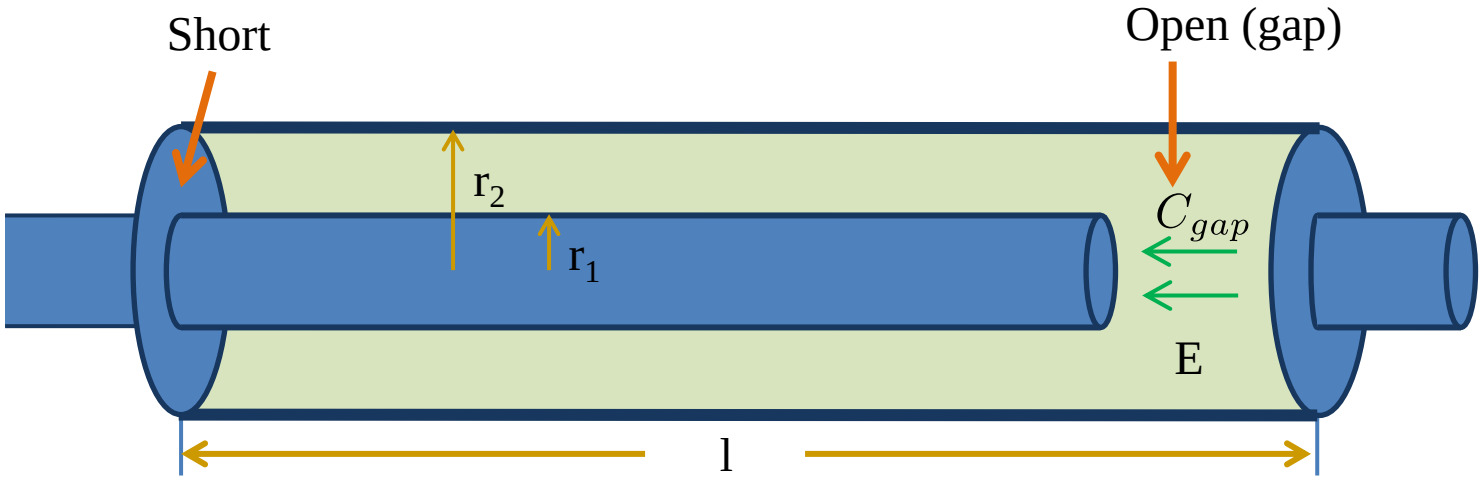


B field of lowest resonance mode in QWR



E field of lowest resonance mode in QWR

Deeper Look



The resonance condition (more accurate) $z_0 \tan(\omega l/c) = 1/(\omega C_{gap})$

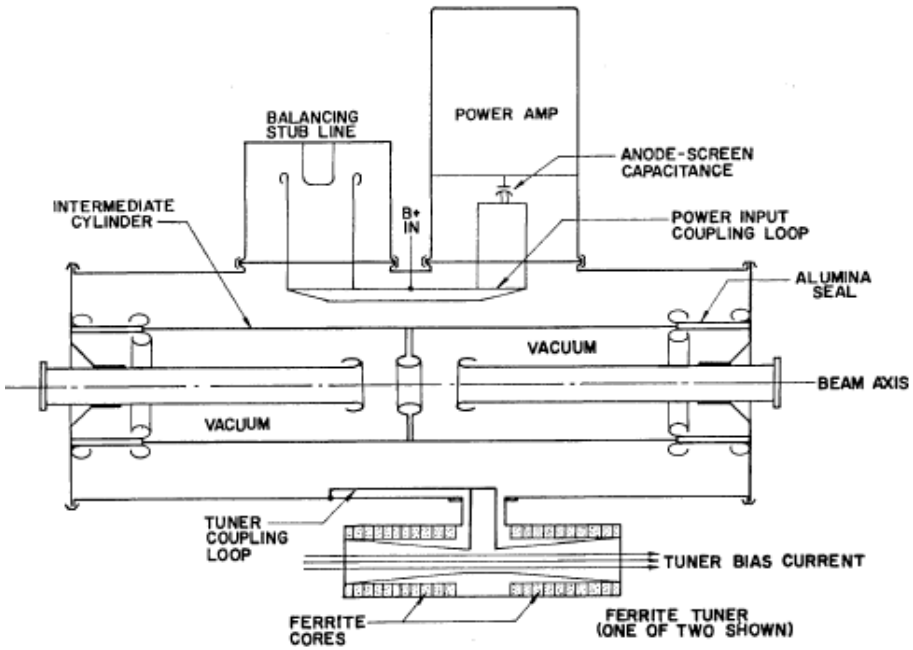
The condition regain its simple relation when $C_{gap} \to 0$

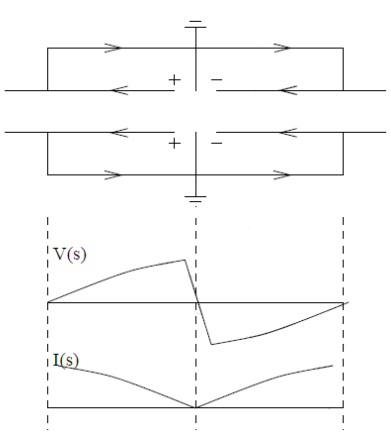
Resistance of unit length (R_s is the surface resistivity): $R_c = \frac{R_s}{2\pi}[\frac{1}{r_1} + \frac{1}{r_2}]$

Quality factor Q: $Q_n \sim \omega_n (L/R_c)$

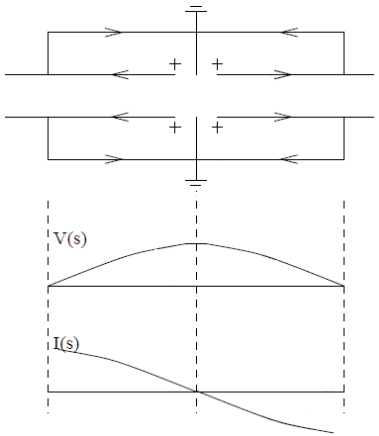
Shunt resistance $R_{shunt} = V^2/2W = 8z_0^2/(R_c\lambda)$

Higher voltage? -- Half Wave Resonator (HWR)





odd fundamental mode (good)



even fundamental mode (bad)

- (x1) frequency, quality factor (Q)
- (x2) length, gap voltage, stored energy, ohmic power loss, shunt impedance
- (x2) All modes

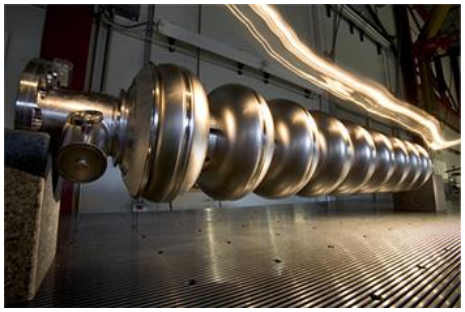
Summary of Part I

- RF resonators/cavities are one of the most essential and delicate component of the accelerator, and they play various roles in the machine.
- The application of pillbox(elliptical) cavities is a big step forward to high-frequency high-gradient particle acceleration. They can be solved analytically.
- QWR/HWR is morphed from coaxial lines. Their special characteristics make them to be very efficient in applications that are not so suitable for elliptical cavities.

The families of cavities

Via operating temperature:

vs

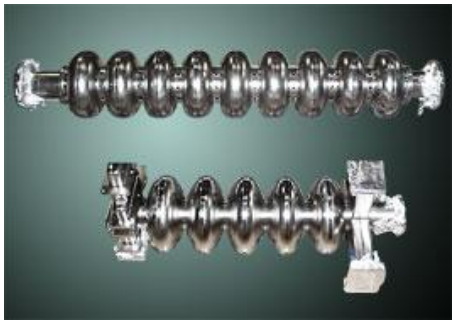


Normal conducting

Superconducting

Via geometry:

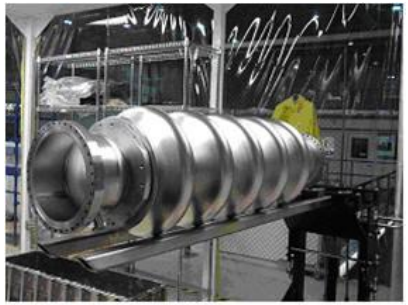
vs



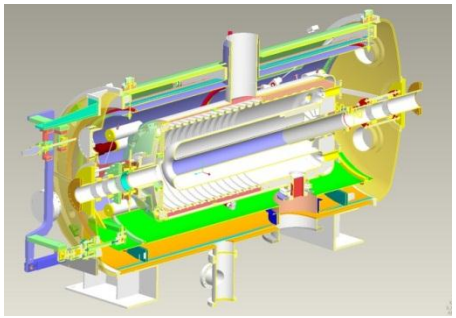
QWR/HWR

Elliptical

Via purpose:



vs



Acceleration

Better beam quality

Part II

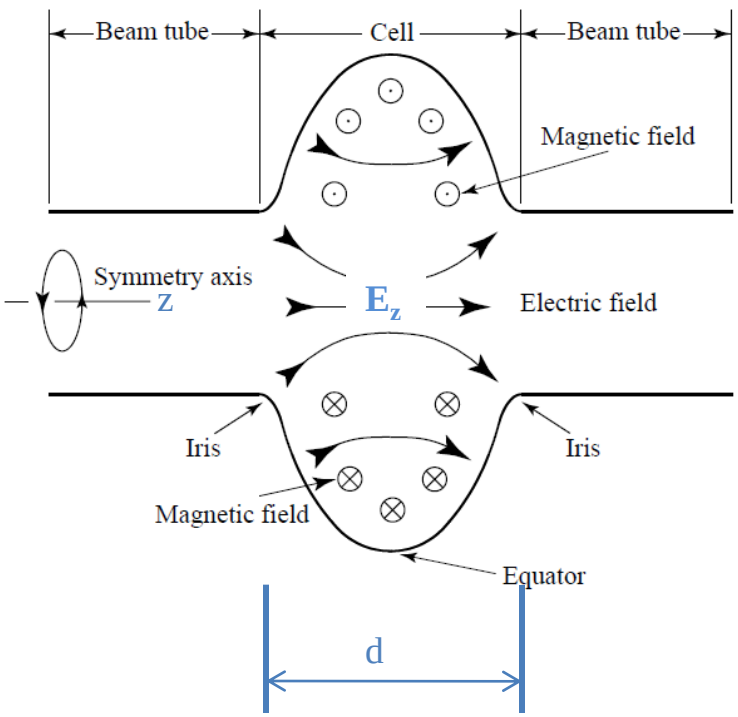
- Figures of Merit
- Cavity ↔ RLC circuit

Figures of Merit

To describe a RF cavity, we will need to know:

- Accelerating voltage
- Shunt impedance
- Dissipated power
- Transit time factor
- Surface impedance
- Stored energy
- Quality factor (Q)
- Geometry factor
- R/Q

Together, they tell the whole story:



For pillbox cavity:

$$E_z = E_0 J_0\left(\frac{2.405r}{R}\right)e^{i\omega t}$$

$$V_c = E_0 d \cdot T$$

$$E_{acc} = {2E_0}/{\pi}$$

$$T\big|_{\max acceleration} = {2}/{\pi}$$

- The wanted (accelerating) mode is excited at the good frequency and position from a RF power supply through a power coupler
- The phase of the electric field is adjusted to accelerate the beam
- Acceleration field E_z
- Acceleration voltage $V_c = \int \hat{E}_z(z) \cdot dz$
- Average Accelerating field $E_{acc} = \dfrac{V_c}{d}$
- The maximum energy that can be gained by a particle in the cavity $\Delta U_{\max} = qV_c \cdot T$
- The difference between the particle velocity and the phase velocity of the accelerating field, leads to an efficiency drop of the acceleration. The transit time factor T characterizes the actual efficiency.

$$T = \frac{1}{V_c}\int \hat{E}_z(z) \cdot e^{j\phi(z)} \cdot dz$$

The story cont'd

- Because of the sinusoidal time dependence and 90° phase shift, the energy oscillates back and forth between the electric and magnetic field. The stored energy in a cavity is given by

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 dv = \frac{1}{2}\varepsilon_0 \int_V |\mathbf{E}|^2 dv$$

- An important figure of merit is the quality factor, which for any resonant system is

$$Q_0 \equiv \frac{\omega_0 \cdot (\text{stored energy})}{\text{average power loss}} = \frac{\omega_0 U}{P_c} = 2\pi \frac{1}{T_0}\frac{U}{P_c} = \omega_0 \tau_0 = \frac{\omega_0}{\Delta\omega_0}$$

$$Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds}$$

roughly 2π times the number of RF cycles it takes to dissipate the energy stored in the cavity. It is determined by both the material properties and cavity geometry and ~10^4 for NC cavities and ~10^{10} for SC cavities at 2K.

The story cont'd

- One can see that the ration of two integrals in the last equation determined only by cavity geometry. Thus we can re-write it as $Q_0 = \frac{G}{R_s}$

  with the parameter G known as the geometry factor or geometry constant

- The geometry factor depends only on the cavity shape and electromagnetic mode, but not its size. Hence it is very useful for comparing different cavity shapes. G = 257 Ohm for the pillbox cavity.

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$$

- Surface impedance $Z_s = \frac{1+i}{\sigma\delta} = R_s + iX_s$ σ: electrical conductivity δ: skin depth

- The real part of the surface impedance is called surface resistivity and is responsible for losses. The losses per unit area are simply

$$P'_{diss} = \frac{1}{2} R_s H_0^2$$

The story cont'd

- Dissipated power P_d is the mean power dissipated in the cavity over one RF period

- Shunt impedance $R_{sh} = V_c^2 / P_d \quad \Rightarrow \quad P_d = V_c^2 / R_{sh}$

- A related quantity is the ratio of the shunt impedance to the quality factor, which is independent of the surface resistivity and the cavity size:

$$\frac{R_{sh}}{Q_0} = \frac{V_c^2}{\omega_0 U}$$

- This parameter is frequently used as a figure of merit and useful in determining the level of mode excitation by bunches of charged particles passing through the cavity. *R/Q* = 196 Ohm for the pillbox cavity. Sometimes it is called geometric shunt impedance.

The story cont'd

- Now we take another look at the power loss in the cavity walls is

$$P_d = \frac{V_c^2}{R_{sh}} = \frac{V_c^2}{Q_0 \cdot (R_{sh}/Q_0)} = \frac{V_c^2}{(R_s \cdot Q_0)(R_{sh}/Q_0)/R_s} = \frac{V_c^2 \cdot R_s}{G \cdot (R_{sh}/Q_0)}$$

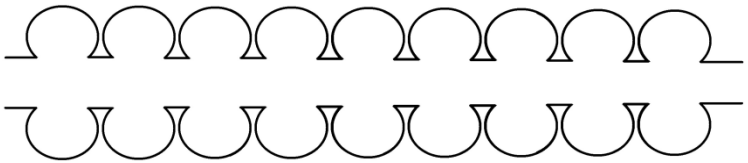
- To minimize the losses one needs to maximize the denominator. By modifying the formula, one can make the denominator material-independent: $G \cdot R/Q$ – this new parameter can be used during cavity shape optimization.

Some sense of the parameters:

Figures	Unit	Typical Value
V_c	Volts	~MV
$E_{0\text{-limit}}$	MV/m	~100 for SC
$B_{0\text{-limit}}$	mT	180 for SC
T	N/A	0<T<1
Q_0	N/A	10^4(NC) 10^{10}(SC)
R_{sh}/Q_0	Ohm	~100
P_d	W	~10W(SC)
G	Ohm	~100

Cavity ↔ RLC circuit

1



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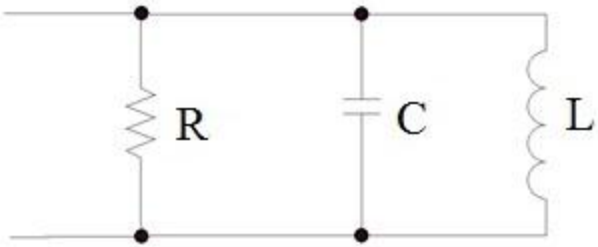

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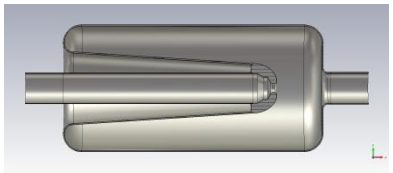
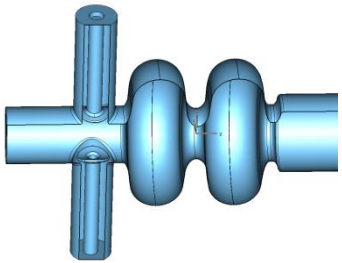
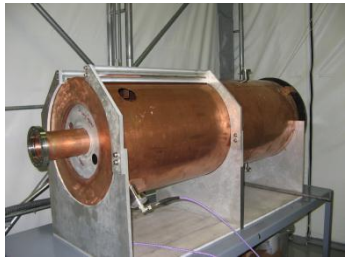
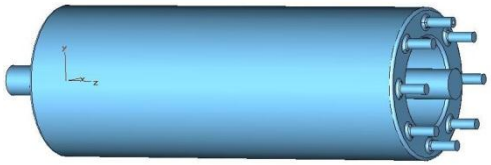
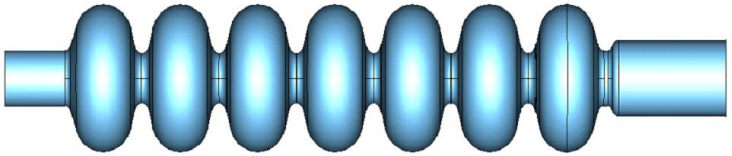
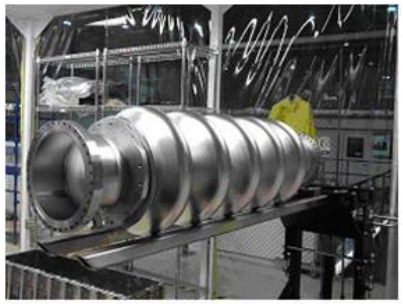


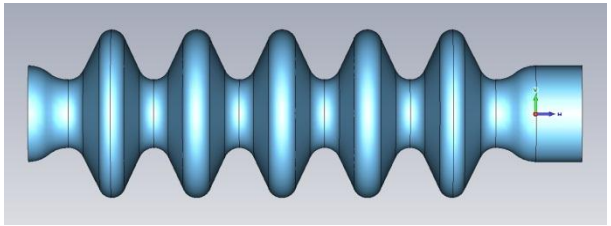


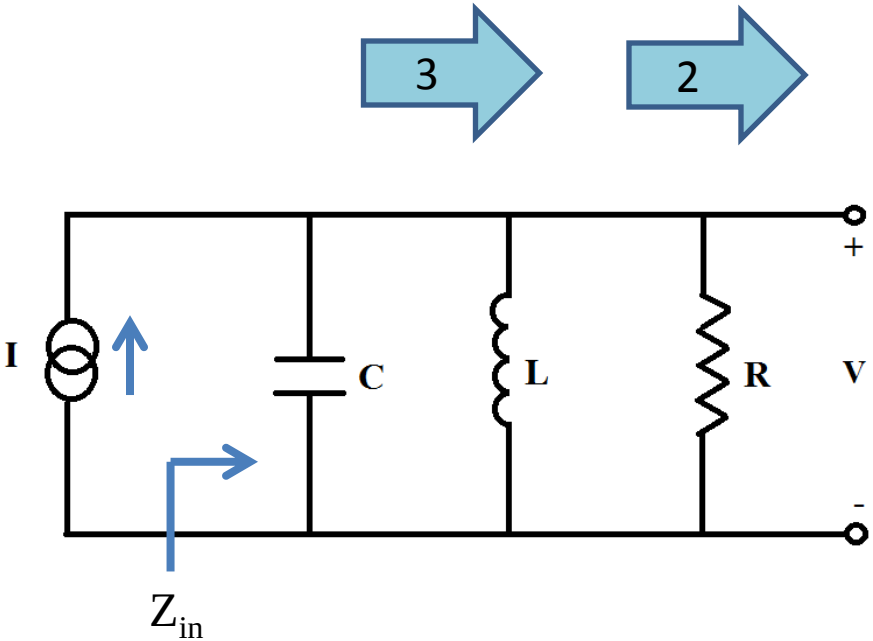












$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} Z_{in} |I|^2 = \frac{1}{2} |V|^2 \frac{1}{Z_{in}}$$
$$= \frac{1}{2} |V|^2 \left(\frac{1}{R} + \frac{j}{\omega L} - j\omega C\right)$$

$$P_{loss} = \frac{1}{2} \frac{|V|^2}{R}$$

$$P_M = \frac{1}{4} |V|^2 \frac{1}{\omega^2 L}$$

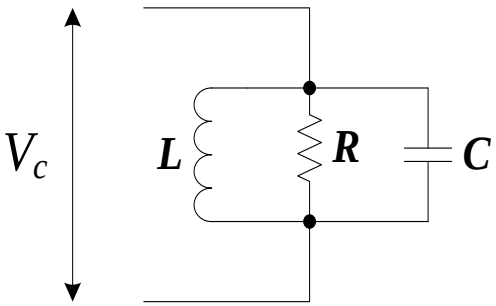
$$P_E = \frac{1}{4} |V|^2 C$$

$$P_{in} = P_{loss} + 2j\omega(P_M - P_E)$$

A resonant cavity can be modeled as a series of parallel circuits representing the cavity eigenmodes:

dissipated power $$P_c = \frac{V_c^2}{2R}$$

shunt impedance $$R_{sh} = 2R$$



quality factor $$Q_0 = \omega_0 CR = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}$$

impedance $$Z = \frac{R}{1 + iQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \approx \frac{R}{1 + 2iQ\left(\frac{\omega - \omega_0}{\omega_0}\right)}$$

Summary of Part II

- Figures of Merit give quantitatively characterized the cavity and its performance
- Equivalent RLC circuit is a very helpful way of analyze a RF cavity