

Two-Beam Instability in Electron Cooling

Alexey Burov

BNL Workshop, May 2006

Ion-electron interactions

- Electron cooling is a method to increase a phase space density of a hot (ion / pbar) beam by merging it with a cold electron beam.
- Cooling is caused by interactions of individual ions with individual electrons.
- Individual ions with collective electrons ⑨ incoherent beam-beam (weak – strong beam-beam).
- Ion beam with electron beam ⑨ two-beam instability (coherent beam-beam, strong-strong)

Any of the 3 channels may be damaging for ions.

Channel 1: More Cooled ⑨ Less Stable

- The more beam is cooled ⑨ the less stable it is.
 - $\Delta v_{SC} \gg |\xi - \eta n| \delta p / p$ ⑨ Landau damping $\rightarrow 0$
 - Beam stability parameter:
$$\frac{\Delta v_{SC}}{|\xi| (\delta p / p)} \propto D = \frac{N}{6 \varepsilon_{\perp} 4 \varepsilon_{\parallel}}$$
- Instabilities can be driven by:
 - Machine impedance;
 - Stochastic cooling system;
 - Damper;
 - Structure resonance of the tune-depressed envelope modes;
 - Interaction with residual ions and/or e-clouds;
 - Collective interaction with e-beam.
- For RR w/o damper, the stability limit $D=0.7-0.8$.
- With damper: $D=1.5$ (old kicker, measured)
- New kicker: $D=4$ (expected)

Channel 2: weak-strong beam-beam, etc.

- Individual ions with collective electrons: lifetime may degrade due to:
 - Nonlinear resonances excited by (static) e- space charge (DR);
 - Dipole e- oscillations (noise) at betatron sidebands;
 - Oscillations (noise) of e- current at $2\omega_b$ sidebands (PR)

Channel 3: two-beam instability

- Actually, the talk is beginning from here...

Two-Beam Instability in Electron Cooling

Some history

- Ion-electron instabilities, **IEI** were observed (or suspected) at several e-coolers (CELSIUS, IUCF, COSY, HIMAC, RR).
- A general idea was introduced by V. Parkhomchuk in ~1992. He posed a problem about this phenomenon.
- Transverse impedance of e-beam was calculated by myself in 1997 for $\Omega_{Le} \rightarrow \infty$: the impedance seemed too small for typical cases.

History, cont.

- In e-drift approximation, the problem was simultaneously treated by myself (1999) and P. Zenkevich & A. Bolshakov (1999)

- AB: The growth rate is proportional to a coupling parameter κ :

$$\Lambda = \alpha \kappa / (2T_0); \quad \alpha \propto I_e I_i - \text{interaction parameter}$$

- Z&B: “For realistic parameters, the instability is impossible in our model”.

Their model was essentially the same as mine.

- In 2000, V. Parkhomchuk & V. Reva applied the same ideas for an uncoupled revolution matrix with identical X and Y blocks and got

$$\Lambda = \alpha \beta_0 / (4T_0 l)$$

- All the 3 papers were in contradiction with each other, and every of them seemed to be arithmetically correct...

Incompressible e-beam

Actually, the theory of IEI starts from this slide...

The continuity equation $dn_e / dt = -n_e \nabla \cdot \mathbf{v}_e(\mathbf{r})$

plus the drift equation $\mathbf{v}_e(\mathbf{r}) = c\mathbf{E}(\mathbf{r}) \times \mathbf{B} / B^2$

leads to $dn_e / dt \propto n_e \nabla \cdot [\mathbf{E}(\mathbf{r}) \times \mathbf{B}] = n_e \mathbf{B} \cdot [\nabla \times \mathbf{E}(\mathbf{r})] = 0$

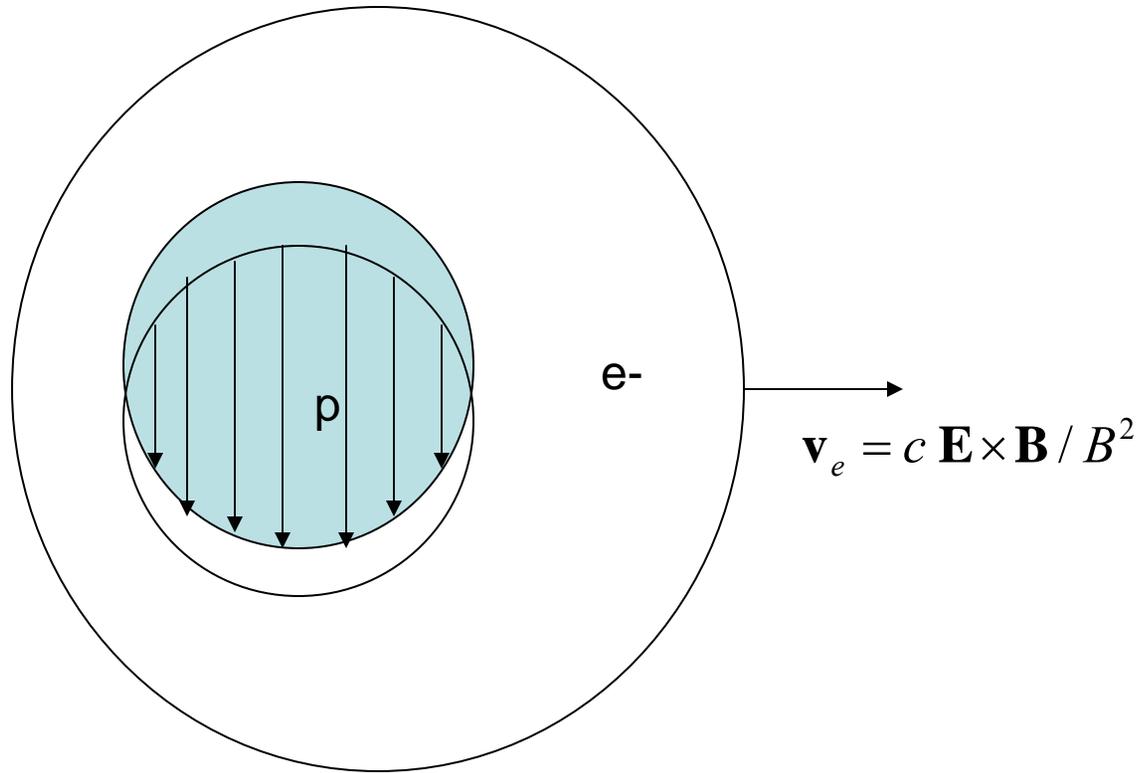
Thus, the drifting e-beam behaves as an incompressible liquid !

For a constant density e-beam

$$\frac{\partial n_e}{\partial t} = -\mathbf{v}_e \nabla \cdot n_e \propto \delta(r - a_e)$$

all its perturbation charges are on its surface.

Drift Response



Incompressible e-beam, consequences

- From here, 2 important consequences follow:
- The dipole ion-electron motion does not depend on the ions emittance as soon as the ions are mainly inside the e-beam; thus,

$$n_i \equiv \lambda_i / (\pi a_e^2)$$

- The quadrupole ion-electron growth rate relates to the dipole rate in the same way as for the conventional impedance:

$$\Lambda^{(2)} = 2(a_i / a_e)^2 \Lambda^{(1)}$$

Dipole motion in the cooler

Rotation symmetry in the cooler allows to use $\xi \equiv x + iy$:

$$\begin{aligned} \xi_i'' - k_{ie}^2 \xi_e + ik_{iL} \xi_i' &= 0 & \xi_i(0) &= \xi_{i0} ; \quad \xi_i'(0) = \xi_{i0}' \\ \xi_e' - ik_{ed} (\xi_i - \xi_e) &= 0 & \xi_e(0) &= 0 \end{aligned}$$

with $k_{ie}^2 = 2\pi n_e Z_i r_p / (\gamma^3 \beta^2 A_i)$; $k_{iL} = Z_i e B / (p_i c)$

$$k_{ed} = k_{ei}^2 / k_{eL} \propto Z_i n_i / B$$

From here, a matrix of the inner cooler \mathbf{S} is to be found; $\begin{pmatrix} \xi_i \\ \xi_i' \end{pmatrix} = \mathbf{S} \begin{pmatrix} \xi_{i0} \\ \xi_{i0}' \end{pmatrix}$

In practice, all the 3 phases (ie, iL, ed) are small, $\psi \equiv kl \ll 1$

So, a perturbation approach is proper.

Perturbed Solenoid Matrix

The interaction parameter: $\alpha = \psi_{ie}^2 \psi_{ed}$

Inner solenoid matrix: $\mathbf{S} = \mathbf{S}_0 + \alpha \mathbf{M}$

Entire solenoid matrix: $\mathbf{C} = \mathbf{S}_f^{-1} \cdot \mathbf{S} \cdot \mathbf{S}_f = \mathbf{C}_0 + \alpha \mathbf{S}_f^{-1} \cdot \mathbf{M} \cdot \mathbf{S}_f$

$$\mathbf{M} = \begin{pmatrix} \mathcal{M}_d & -\mathcal{M}_c \\ \mathcal{M}_c & \mathcal{M}_d \end{pmatrix} \quad \text{⑦ rotation invariance}$$

$$\text{coupling part: } \mathcal{M}_c = \frac{1}{2} \begin{pmatrix} 1/3 & l/12 \\ 1/l & 1/3 \end{pmatrix}$$

$$\text{diagonal part: } \mathcal{M}_d = \frac{\psi_{iL}}{6} \begin{pmatrix} 1/4 & l/10 \\ 1/l & 1/2 \end{pmatrix} + \frac{\psi_{ed}}{6} \begin{pmatrix} 1/4 & l/20 \\ 1/l & 1/2 \end{pmatrix}$$

Note: own / alien ~ small phases .

Normalized Perturbation

The entire revolution: $\mathbf{R} = (\mathbf{I} + \mathbf{P}) \cdot \mathbf{R}^{(0)}$

normalized perturbation: $\mathbf{P} = \alpha \mathbf{C}_0^{-1/2} \cdot \mathbf{S}_f^{-1} \cdot \mathbf{M} \cdot \mathbf{S}_f \cdot \mathbf{C}_0^{-1/2}$

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_d & -\mathbf{P}_c \\ \mathbf{P}_c & \mathbf{P}_d \end{pmatrix} = \mathbf{P}_d + \mathbf{P}_c$$

$$\mathbf{P}_d / \mathbf{P}_c \cong 0.1 \psi_{iL,ed} \sim 0.01 - 0.001$$

Note: The perturbation is coupling-dominated.

Step Aside: Perturbation Theory for 4D Optics

To get eigen-values for the perturbed revolution matrix, the perturbation formalism has to be developed for 4D optics.

In Lebedev-Bogacz 4D presentation, the 4 unperturbed eigenvectors are:

$$\mathbf{R}^{(0)} \mathbf{V}_m^{(0)} = \exp(-i\mu_m) \mathbf{V}_m^{(0)} \quad m = 1, 2, -1, -2 ;$$

$$\mathbf{V}_1^{(0)} = \left(\sqrt{\beta_{1x}}, \frac{i(u-1) - \alpha_{1x}}{\sqrt{\beta_{1x}}}, \sqrt{\beta_{1y}} e^{i\nu_1}, \frac{-iu - \alpha_{1y}}{\sqrt{\beta_{1y}}} e^{i\nu_1} \right)^T$$

$$\mathbf{V}_2^{(0)} = \left(\sqrt{\beta_{2x}} e^{i\nu_2}, \frac{-iu - \alpha_{2x}}{\sqrt{\beta_{2x}}} e^{i\nu_2}, \sqrt{\beta_{2y}}, \frac{i(u-1) - \alpha_{2y}}{\sqrt{\beta_{2y}}} \right)^T$$

$$\mathbf{V}_{-m}^{(0)} = \mathbf{V}_m^{(0)*}$$

Perturbation Theory, cont.

These basis vectors are orthogonal through the symplectic unit matrix \mathbf{U} :

$$\mathbf{V}_m^{(0)+} \cdot \mathbf{U} \cdot \mathbf{V}_n^{(0)} = -2i \delta_{mn} \operatorname{sgn}(m)$$

$$\mathbf{U} \equiv \begin{pmatrix} \mathbf{U}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_d \end{pmatrix} ; \quad \mathbf{U}_d \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The perturbation theory is constructed very similar to the Quantum Mechanics.

Result : as in QM, the tune shift is given by the diagonal matrix element.

The complex phase shifts: $\delta\mu_n = -\mathbf{V}_n^{(0)+} \cdot \mathbf{U} \cdot \mathbf{P} \cdot \mathbf{V}_n^{(0)} / 2$

The growth rates: $\Lambda_n = \operatorname{Im} \delta\mu_n / T_0 = -\operatorname{Im} \left(\mathbf{V}_n^{(0)+} \cdot \mathbf{U} \cdot \mathbf{P} \cdot \mathbf{V}_n^{(0)} \right) / (2T_0)$

Useful relation: $2T_0 (\Lambda_1 + \Lambda_2) = \det(\mathbf{R}) - 1 = \operatorname{tr}(\mathbf{P})$

Ion-Electron Growth Rates

The entire revolution: $\mathbf{R} = (\mathbf{I} + \mathbf{P}) \cdot \mathbf{R}^{(0)}$

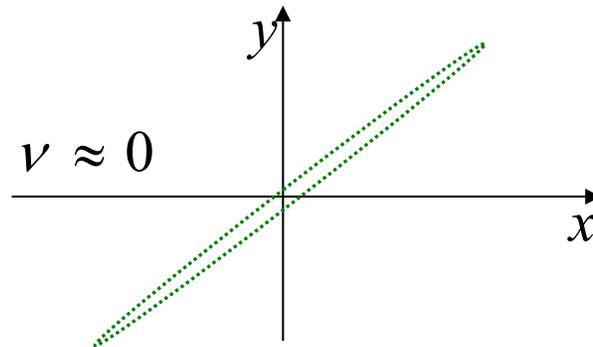
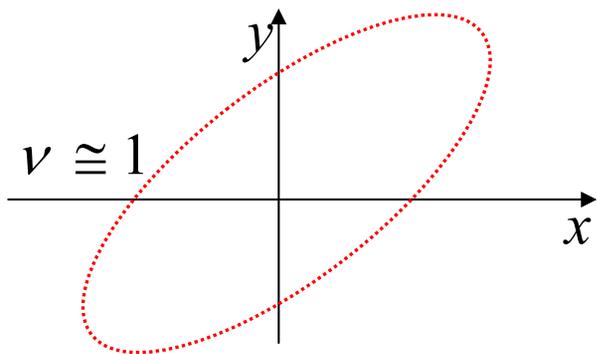
normalized perturbation: $\mathbf{P} = \alpha \mathbf{C}_0^{-1/2} \cdot \mathbf{S}_f^{-1} \cdot \mathbf{M} \cdot \mathbf{S}_f \cdot \mathbf{C}_0^{-1/2}$

The diagonal part is always relatively small, $\mathbf{P}_d / \mathbf{P}_c \sim 0.1 \psi_{ed, iL} \sim 0.01 - 0.001$

Neglecting the small diagonal part, the growth rate follows:

$$\Lambda_{1,2}^c = \pm \frac{\alpha \kappa_{xy}}{2T_0}$$

coupling parameter: $\kappa_{xy} = \sqrt{\beta_{1x} \beta_{1y} / l^2} \sin(\nu_1) = \sqrt{\beta_{2x} \beta_{2y} / l^2} \sin(\nu_2)$



Uncoupled Rates: Small

Without coupling, only the small diagonal part works, giving the growth rate:

$$\Lambda^d = \frac{\alpha}{2T_0} \left(\frac{\psi_{iL}}{24} - \frac{\psi_{ed}}{12} \right)$$

$$\frac{\Lambda^d}{\Lambda^c} \cong 0.1 \frac{\psi_{iL,ed}}{\kappa_{xy}} \cong (10^{-2} - 10^{-3}) / \kappa_{xy}$$

Why the coupling is important?

- Electrons drift in a direction, orthogonal to the ions offset. Thus, for planar (uncoupled) modes, the force, acting back on the ions, is orthogonal to the ions velocity. The resulted work is zero, and thus the rate is zero too (at the lowest order over the small phases).
- From formal point of view, for the planar modes, the diagonal matrix elements of the drift perturbation are zero (at lowest order).
- What's wrong is with Z&B (1999) conclusion (“instability is impossible”) ?
- What's wrong is with the P&R (PR2000) result ?

PR2000: $\Lambda = \alpha\beta_0 / (4T_0 l)$

Contradictions...

- Z&B based their conclusion on the determinant of the perturbation matrix only. But the main contribution to the rate goes from the alien part, which does not reveal itself in the determinant analysis.
- In 2001, P. Zenkevich abandoned his previous conclusion, and re-derived the PR2000 result

$$\Lambda = \alpha\beta_0 / (4T_0 l)$$

for a no-coupling revolution matrix

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_d \end{pmatrix}; \quad \mathbf{R}_d = \begin{pmatrix} \cos \mu & \beta_0 \sin \mu \\ -\sin \mu / \beta_0 & \cos \mu \end{pmatrix} .$$

This result seems contradicting my conclusion about coupling,
but none of them shows any mistake...

Contradiction Untangled

- Where the key is:
 - The unperturbed matrix of PR2000 is DEGENERATED .
 - For degenerated states, the perturbation formulae have to be used in a specific way (remember Quantum Mechanics!).
 - Any combination of the orthogonal degenerated states is an eigen-state as well. There are infinite possibilities for the eigenvectors, and every one of them gives its own result for the growth rate.
- The recipe (from the QM, again):
 - Correct eigen-vectors are those which make the perturbation diagonal.
 - Those are the circular modes (100% coupled !)

$$\mathbf{V}_1 = (\mathbf{V}_x + i\mathbf{V}_y) / \sqrt{2} ; \quad \mathbf{V}_2 = (i\mathbf{V}_x + \mathbf{V}_y) / \sqrt{2}$$

Substituted in my perturbation formula, they immediately give the PR result!

PR2000: correct and misleading altogether

- So, even without the machine coupling, there is an area around the coupling resonance, where even a small ion-electron interaction makes the optics 100% coupled.
- However, for practical cases, the width of this area is extremely small, $E-4 - E-5$, so practically the width is zero. **Without machine coupling, the growth rate is zero everywhere, apart of that separate punctured point.**
- PR2000 result follows from mathematically correct calculations for that punctured point. It was not realized, however, that even a tiny step out of that special point would suppress the rate by orders of magnitude if there is no machine coupling.

Z01

- A problem about rate dependence on the tune separation was formulated by P. Zenkevich in 2001 (Z01).
- He treated numerically the eigen-value problem for the following form of the revolution matrix:

$$\mathbf{R} = \mathbf{D}^{-1/2} \cdot \mathbf{R}_D \cdot \mathbf{D}^{-1/2} \cdot \mathbf{C}$$

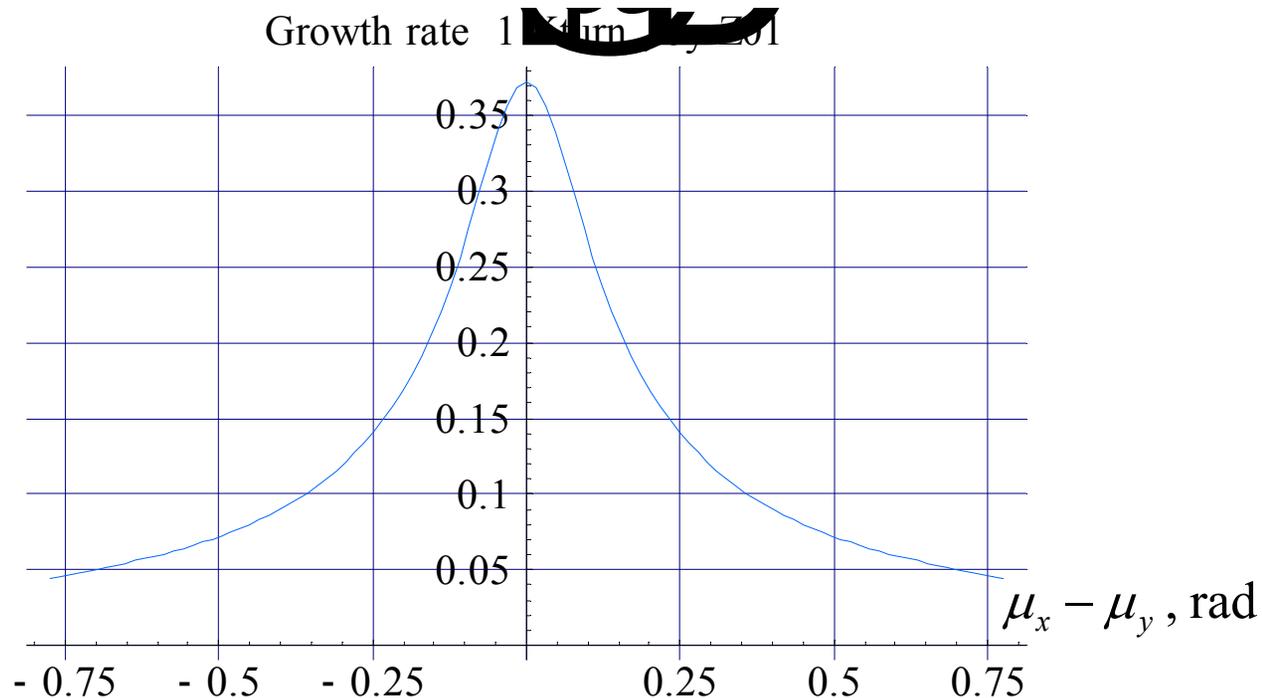
where $\mathbf{D}^{-1/2}$ is an inverse half-drift matrix, \mathbf{C} is the perturbed solenoid matrix, and

$$\mathbf{R}_D = \begin{pmatrix} \mathbf{R}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_y \end{pmatrix}; \quad \mathbf{R}_{x,y} = \begin{pmatrix} \cos \mu_{x,y} & \beta_0 \sin \mu_{x,y} \\ -\sin \mu_{x,y} / \beta_0 & \cos \mu_{x,y} \end{pmatrix} \cdot$$

intended to represent uncoupled machine revolution matrix.

Growth Rate as a Resonance Curve

For ACR (RIKEN) numbers, he calculated the following dependence of the growth rate on the tune separation:



The maximal peak value here is in exact agreement with PR2000.

What factor does determine the width? – this problem were not raised in Z01.

Width of the Resonance

Careful look at Z01 revolution matrix reveals that it DOES include a machine coupling, since it contains an uncompensated solenoid **C** :

$$\mathbf{D}^{-1} \cdot \mathbf{C} - \mathbf{I} \propto \psi_{iL}$$

So, it has to be that

the width is determined by the ion Larmor phase advance.

Taking the numbers, it looks pretty consistent with the figure above.

Working a bit more, the following formula can be checked to be an exact description of the Fig. Z01:

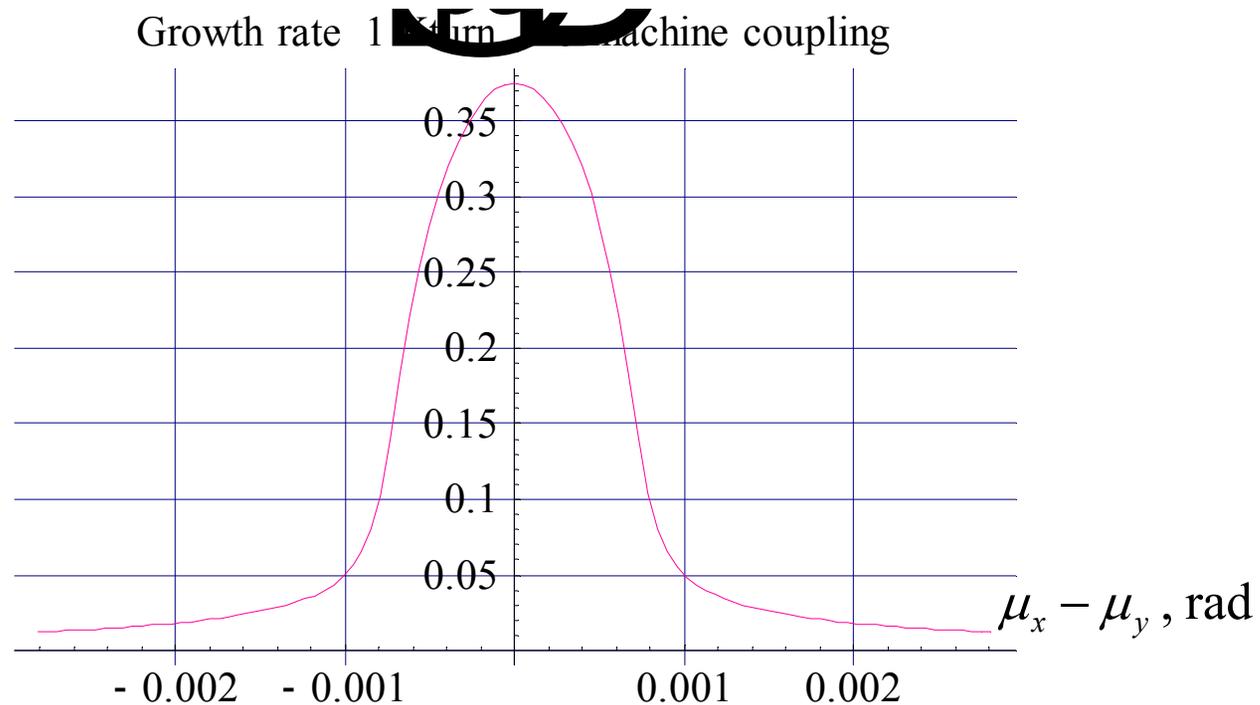
$$\Lambda = \frac{\alpha\beta_0}{4T_0l} \frac{1}{\sqrt{1 + (\mu_x - \mu_y)^2 / \psi_{iL}^2}}$$

Z01 - Uncoupled

If the solenoid is compensated, the revolution matrix is written as:

$$\mathbf{R} = \mathbf{C}_0^{-1/2} \cdot \mathbf{R}_D \cdot \mathbf{C}_0^{-1/2} \cdot \mathbf{C}$$

Numerical eigen-value calculation for the same parameters as Z01, but with this **compensated coupling**, reduces the width ~ 300 times! See Fig. below.



Can it really develop?

For the RR “old working point”, the coupling was about maximal. Calculation with M. Xiao – A. Valishev OptiM file gives the coupling factor $\kappa_{xy} = 0.3$.

For 300E10 antiprotons at 50% of the circumference, and 0.5 A electron beam,
The growth time is calculated as

$$\Lambda^{-1} = 1.5 \text{ sec} .$$

For “mining” states, this time could be ~ 10 times faster.

Could it be damped by

- Landau damping?
- Resistive impedance?
- Active damper?

Dipole Mode Damping

The Landau damping: $A_L(n) \propto f_{\parallel}(\delta p_* / p)$; $\delta p_* / p \cong \Delta v_{SC} / |\xi - \eta n|$
is vanishing for

$$n_* \cong \xi / \eta$$

If the chromaticity sign is correct, $n_* > 0$, the ring impedance causes damping for these modes (they are called “fast”).

For RR, at this 100 MHz frequency, and the same 300E10 pbars, the resistive impedance gives rise to 0.1 sec. of the growth / damping time.

If the dipole IEI is not damped by Landau + Impedance, it has to (and can) be damped by a feedback damper.

What about the quadrupole IEI ?

Quadrupole IEI

- The quadrupole ion-electron growth rate relates to the dipole rate in the same way as for the conventional impedance:

$$\Lambda^{(2)} = 2(a_i / a_e)^2 \Lambda^{(1)}$$

- Similar to the dipole mode, the Landau damping does vanishes at

$$n_* \cong \xi / \eta$$

- The resistive damping for these quadrupole fast modes is reduced much more, than the IEI rate. Thus, it might easily be insufficient to damp the quadrupole IEI motion.
- There are no such thing as a quadrupole damper (yet, at least).
- With $\eta \Delta p / p \ll \Delta v_{SC}$, the fast-modes quadrupole IEI has all the reasons to develop, if the coupling is significant.

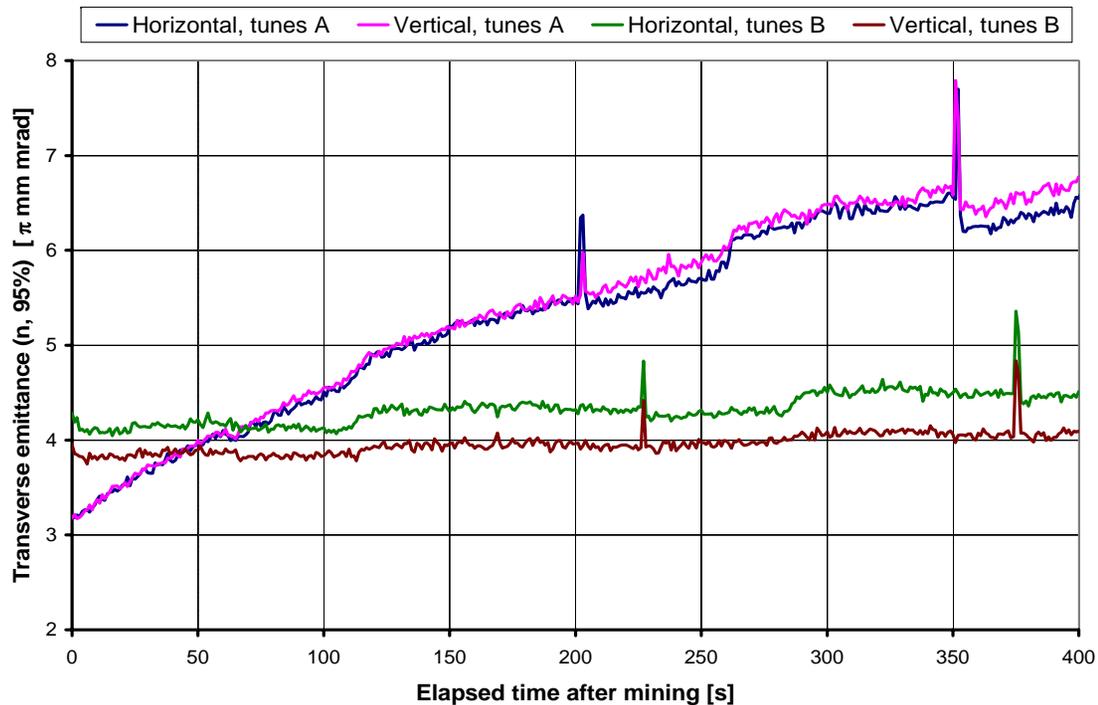
Quadrupole IEI: emittance growth, lifetime degradation

- If the coupling is significant, the quadrupole IEI easily develops.
- At some level, it is stopped by its own non-linearity, and stays forever.
- Due to vanished Landau-damping, transfer of this coherent motion into incoherent motion strongly suppressed.
- This suppressed energy transfer still goes, and gives slow emittance growth and lifetime degradation.
- To fix this problem, step a bit away from the coupling resonance, or / and reduce the coupling area.

RR experience

- Recycler (**RR**) used to stay at the coupling resonance, ~ 0.42 for the both tunes, **having $\sim 100\%$ coupling**.
- There was emittance growth, strongly correlated with the electron current and the pbar linear density.
- At “mining” state (max bunching), and 100 mA of e-beam, the typical rate was $\sim 30 \pi \text{ mm mrad/hr}$, or ~ 0.001 of the calculated quadrupole IEI rate for these parameters.
- The described theory pushed me to insist on more separation of the tunes.
- For the new WP, the coupling parameter reduced $\sim 10\text{-}20$ times .
- **The emittance growth reduced \sim by the same factor !**

Transverse emittance after mining for two operating points



Tunes A: 25.414, 24.418 (max coupling).

Tunes B: 25.451, 24.468. (suppressed coupling)

Pbars ~ 330 E10, the bunch length 6.1 ms, ~ 75 eV s, e- current 100 mA.

Stochastic cooling OFF.

Spikes are artifacts.

Growth rates dropped 10-20 times (consistent with this theory)!

Acknowledgements

- The author is grateful for observation details to
 - Cons Gattuso,
 - Martin Hu,
 - Sergei Nagaitsev,
 - Lionel Prost,
 - Stan Pruss
 - Sasha Shemyakin.
- My thanks to [A. Valishev](#) for his help with the RR optical file.
- My special thanks are to [Valeri Lebedev](#) for his multiple remarks related to various aspects of this paper.