Methods under development in BETACOOL

1. Momentum variation

1.1. Electron cooling. Artificial diffusion

1.2. IBS

1.2.1. Mean rates

1.2.2. Detailed rates

2. Action variation

Problem of large rates

1.1. Electron cooling. Artificial diffusion

Thin lens approximation:

co-ordinate is not changed,

friction force is calculated at the entrance of the cooling section

$$\Delta p = \left.\frac{dp}{ds}\right|_o L_{cool}$$

$$I_i = \frac{1+\alpha_i^2}{\beta_i} i_\beta^2 + 2\alpha_i i_\beta i'_\beta + \beta_i i'^2_\beta$$

$$\delta I_z = 2\left(\beta_z z'_\beta + \alpha_x z_\beta\right)\frac{\delta p_z}{p} + \beta_z\left(\frac{\delta p_z}{p}\right)^2$$

$$\frac{\delta p_z}{p} = \delta z'_\beta$$

$$\delta I_z = \beta_z\left(2z'_\beta + \frac{\delta p_z}{p}\right)\frac{\delta p_z}{p}$$

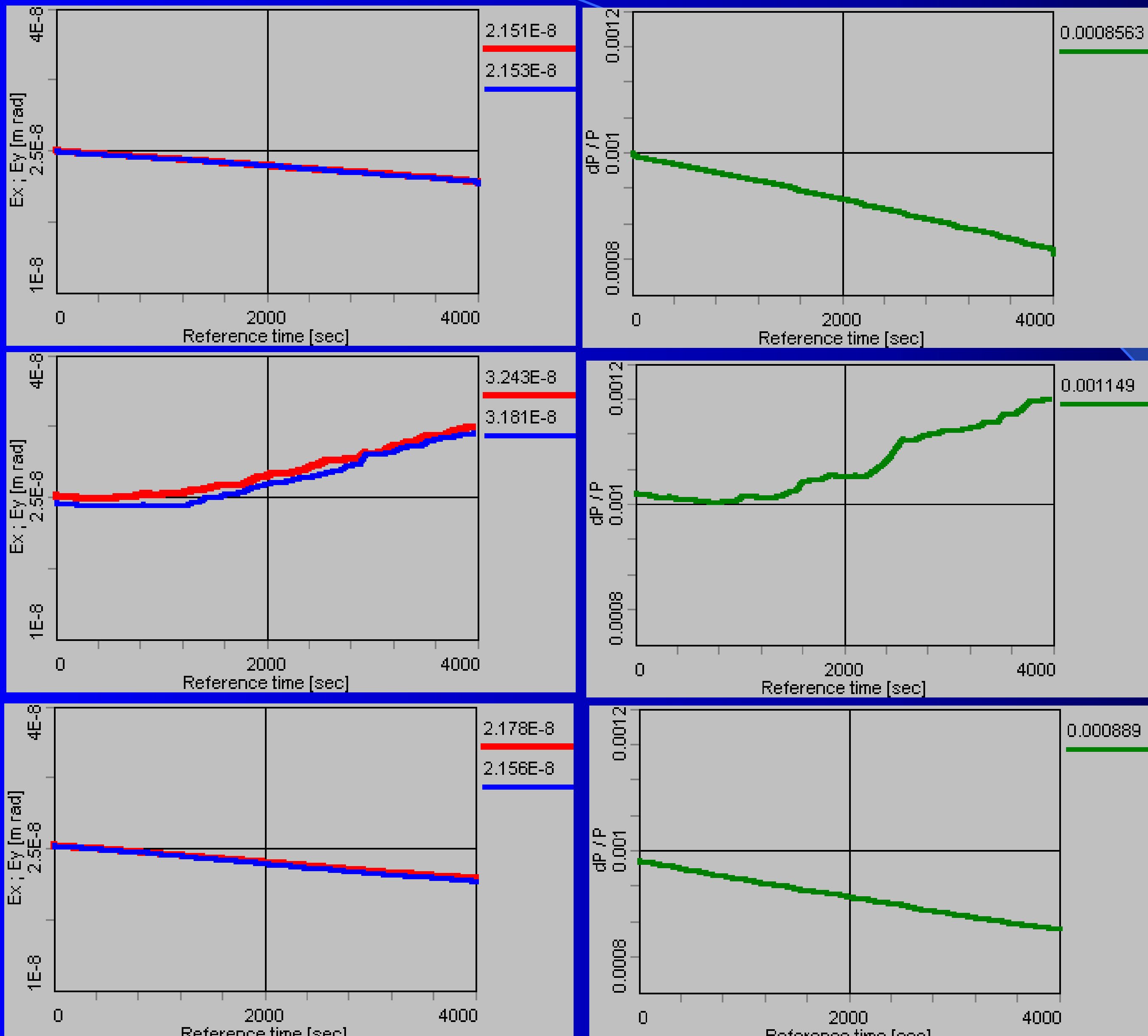
$$\left|\frac{\delta p_z}{p}\right| << \left|z'_\beta\right| \qquad \delta I_z = 2z'_\beta \beta_z \frac{\delta p_z}{p} < 0$$

$$\left|\frac{\delta p_z}{p}\right| >> \left|z'_\beta\right| \qquad \delta I_z = \beta_z\left(\frac{\delta p_z}{p}\right)^2 > 0$$

Equilibrium

$$z'_\beta \sim 0.5\left|\frac{\delta p_z}{p}\right|_{\max}$$

ECOOL (2000 particles, 10^{11} electrons, bunch length 3 cm, step $5*10^6$ turns)

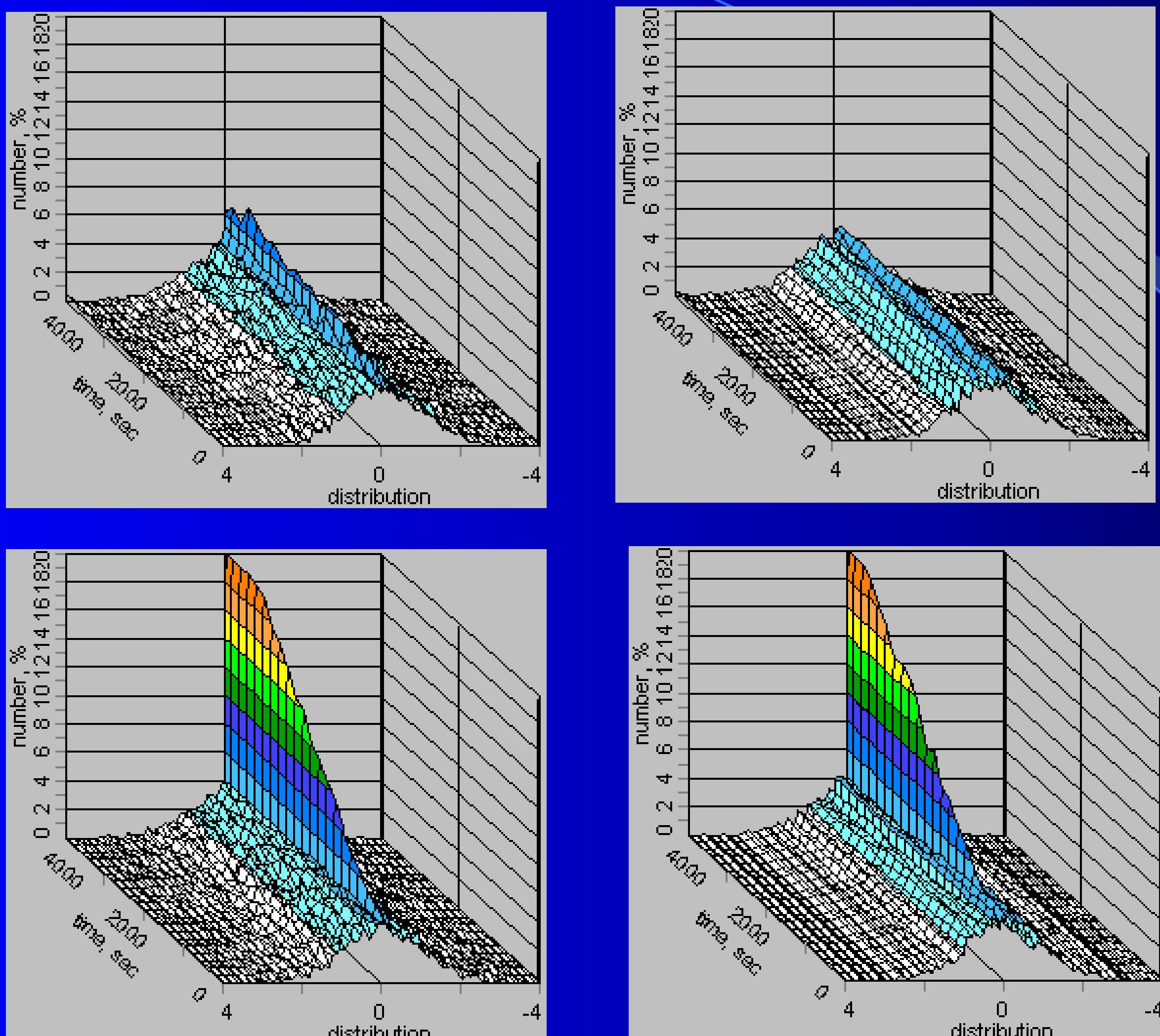


RMS dynamics

Diffusion

Diffusion off

Evolution of distribution function



Diffusion on

Diffusion off

Momentum spread

Horizontal profile

1.2. IBS kick calculation

Array of particles

Beam emittances for chosen lattices (current, or average)

current r.m.s. parameters

Calculation of the IBS rates τ accordingly to the defined model and for real lattices

Cycle over particles

$$1. < \theta^2 >= 2\frac{\varepsilon}{\beta}\frac{\Delta T}{\tau}$$

$$2. \Delta\theta = \sqrt{< \theta^2 >} \cdot Gaussian\ value$$

$$3. \theta = \theta_0 + \Delta\theta$$

Calculation of the scattering angle through emittance growth rate

Dispersion and alpha-function is zero

$$\varepsilon_x = \sqrt{\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle \left\langle \left(x' - \langle x' \rangle\right)^2 \right\rangle}$$

After the scattering

$$\varepsilon_{x,new} = \sqrt{\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle \left\langle \left(x' - \langle x' \rangle + \theta\right)^2 \right\rangle}$$

θ is random Gaussian value

$$\varepsilon_{x,new} = \varepsilon_{x,0} \sqrt{1 + \frac{\left\langle \theta^2 \right\rangle}{\left\langle \left(x' - \langle x' \rangle\right)^2 \right\rangle}}$$

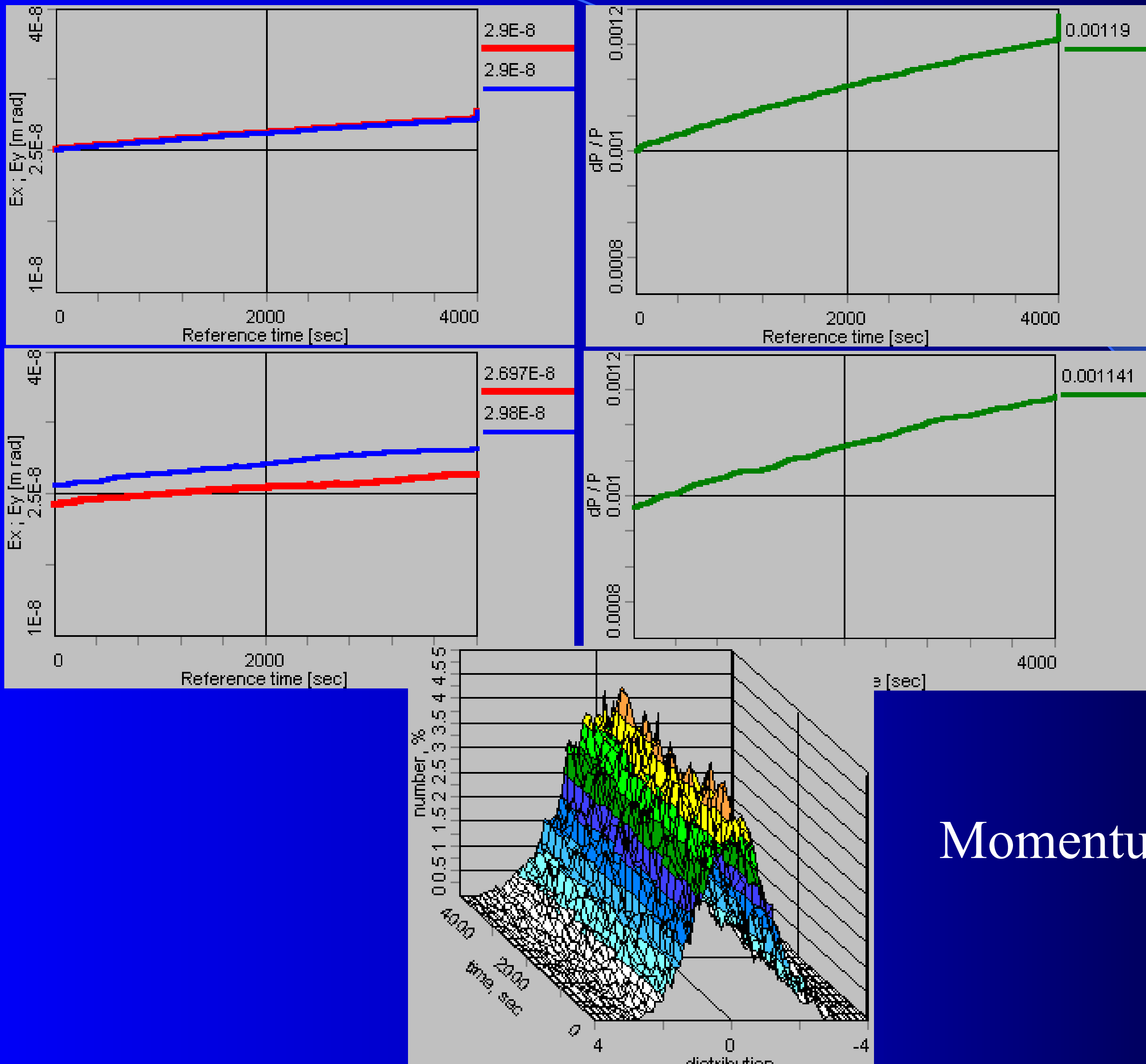
If the scattering angle is less than r.m.s. angular spread one can to expand in series with accuracy to the first term

$$\left\langle \theta^2 \right\rangle = 2 \frac{\Delta \varepsilon_x}{\varepsilon_{x,0} \beta_x}$$

Using definition of rate

$$\left\langle \theta^2 \right\rangle = 2 \frac{\varepsilon}{\beta} \frac{T_{rev}}{\tau}$$

Mean IBS rates, 10^9 ions, Bjorken (2000 particles, step $5*10^6$ turns)

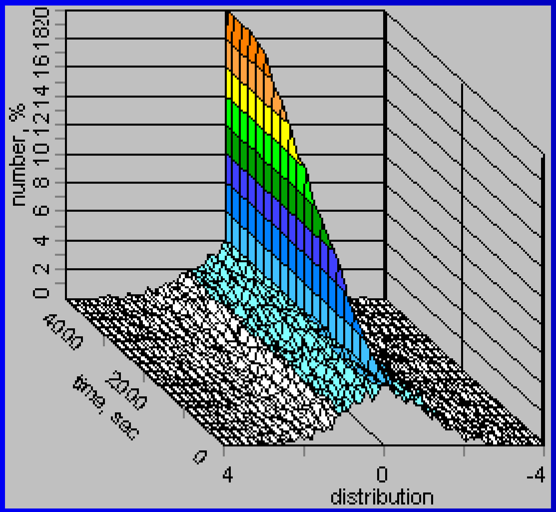


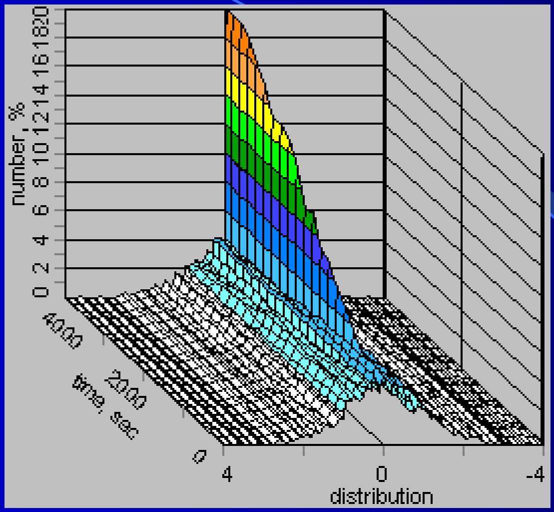
RMS dynamics

Multi particle

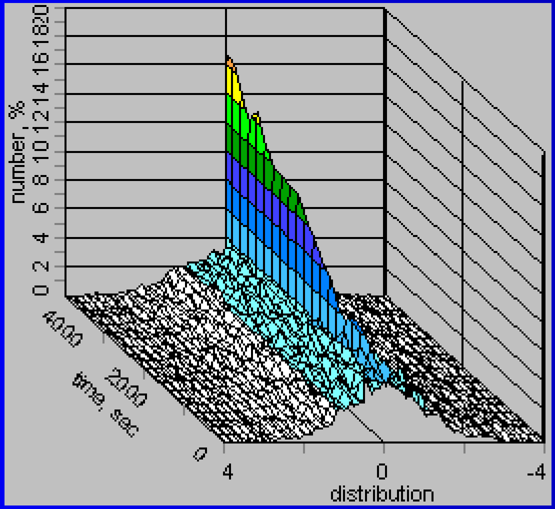
Momentum spread

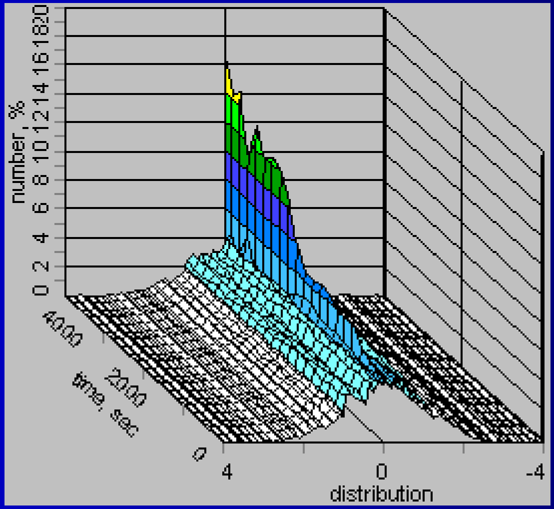
ECOOL + IBS





ECOOL

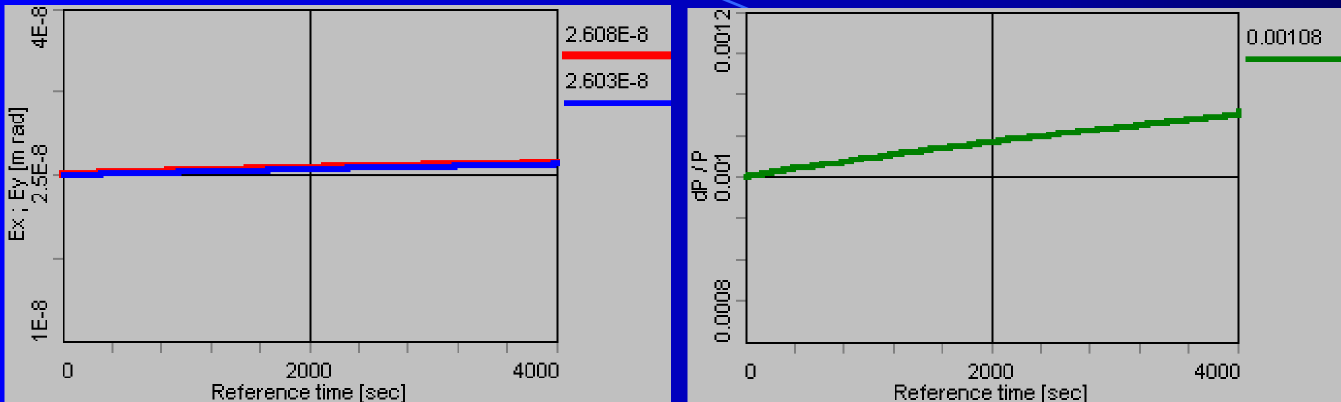




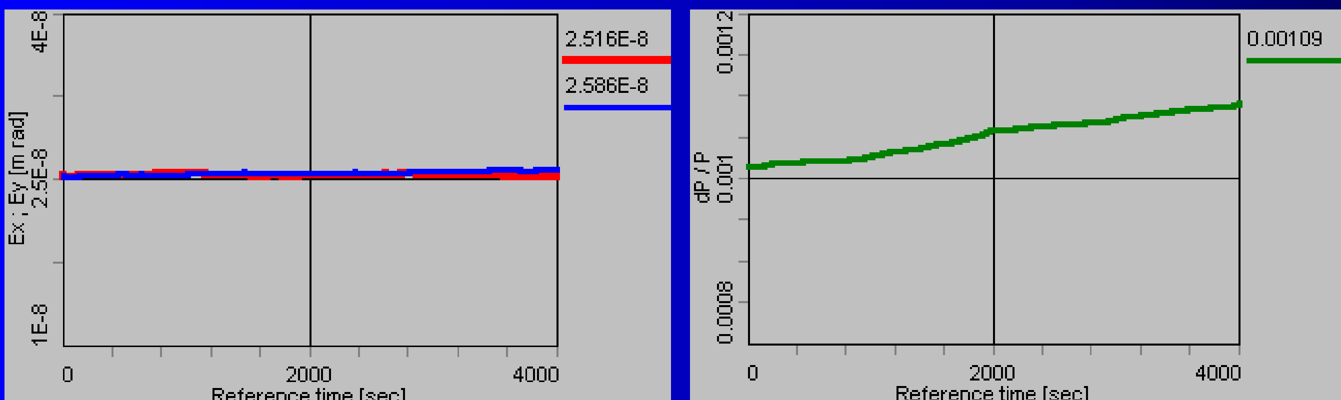
ECOOL + IBS

Momentum spread

Horizontal profile



RMS dynamics



Multi particle

2. Action variation

For each particle one calculates

$$\frac{1}{\tau} = \frac{1}{I}\frac{\delta I}{T_{rev}}$$

And at $\Delta T / \tau$ <<1

$$I_{i+1} = I_i\left(1 + \frac{\Delta T}{\tau}\right)$$

Near maximum of the friction force $\frac{dI}{dt} = \frac{I}{\tau}$ and $I_{i+1} = I_i \exp\left(\frac{\Delta T}{\tau}\right)$

The heating time in the case of diffusion $\tau = \frac{2I^2}{D}$

$$I_{i+1} = I_i\sqrt{1 + \frac{2\Delta T}{\tau}}$$