

BETACOOOL: new feature & developments

JINR electron cooling group

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1. Status on summer 2003

- Evolution of r.m.s. parameters of the ion distribution function
- Simplest model of the ion beam - the beam is represented with three r.m.s. emittances and the particle number.

Advantages:

- High calculation speed.
- Possibility to include into calculations many effects which can be described in the terms of characteristic times - electron and stochastic cooling, intrabeam scattering (IBS), scattering with residual gas, interaction with internal target, particle loss in collision point and so on.
- The program is object oriented and its structure permits to use for calculations:
- Different formulae for friction force calculation, a few models of electron beam, considering errors in the cooler geometry and so on.
- Different analytical models for IBS growth rates calculation (For this aim an output MAD file is used).
- Platform independent initial code in standard of C++ language.
- interface part for Windows providing a service for input files preparation and for data post processing.

The goals of the BETACOOOL development:

- **Simulation of the ion distribution function evolution.**
- **Usage results of numerical calculation of the friction force for the cooling simulation.**

General problem: long time scale of investigated processes
of the order of 1 hour $\sim 10^8$ revolutions in the ring.

Turn by turn beam dynamics simulation is technically impossible.

Two basic algorithms:

Momentum variation (SimCool) algorithm

Action variation (Monte Carlo) algorithm

2. Program structure

Kit of the algorithms:

- r.m.s. dynamics simulation,
- SimCool algorithm -
analytical model of IBS process simulation (calculating mean rates or detailed rates for each particle),
- MC algorithm using analytical models of IBS process,
- MC based on MD technique.
- MC with macro particles

Library of the Effects:
ECOOL,
IBS,
Rest gas scattering,
Collisions

Set of the ring models:

- mean lattice parameters,
- lattice parameter variations along the ring,
- set of optic elements from the optic library,
- module structure

Set of the beam models:

- set of Twiss parameters,
- set of the r.m.s. emittances and particle number,
- real particle array,
- array of macro particles.

3. Algorithms of multi particle tracking

Momentum variation (SimCool)

Step in dynamics simulation corresponds to large number of the turns in the ring ($\sim 10^5 - 10^6$).

Each effect acting on the ion beam distribution function is represented by the variation of the particle momentum components after revolution in the ring. These momentum variations are amplified by the turn number used as a step for dynamics simulation.

Action variation (Monte Carlo)

The beam dynamics is simulated accurately during one revolution in the ring. For each particle deviations of its actions (two Courant-Snyder invariants for the transverse degrees of freedom and square of the maximum relative momentum deviation for the longitudinal one) are calculated. At given step of the dynamics simulation for each particle new value of the actions are calculated in accordance with the corresponding action deviations. New phases of the particle betatron and synchrotron oscillations are generated randomly.

SimCool block- scheme

Generation of the particle array in the selected point of the Ring

Matching with the Ring Lattice and RF system

Cycle
over
dynamics
simulation

IBS:

- Calculation of the rms parameters
- Calculation of IBS growth rates
- Calculation of the square rms scattering angle amplified by the turn number

cycle
over
particles

Random change of the particle momentum
in accordance to the scattering angle

ECOOOL:

cycle
over
particles

Change of the particle momentum
in accordance with the friction force
value and the turn number

Bunch rotation using the transformation matrix

SimCool algorithm

Advantages:

- Simplicity – fast debug process, minimum required inf. about Ring structure
- Required particle number is determined by the accuracy of the luminosity calculation mainly and can be of the order of a few thousand – high calc. speed
- Results are slightly depend on initial particle distribution

disadvantage:

- IBS process can be simulated only at some analytical assumption about the distribution function shape (for instance Gaussian).

As a result the problem solution is not self-conjugated.

Independent development of the algorithm possible as in the frame of SimCool program as BETACOOOL program:

- Debug of the algorithm by comparison of results obtaining by independent programs based on the same numerical model
- This algorithm included into the BETACOOOL program gives automatically a possibility to use all the models for IBS and ECOOL calculation developed for r.m.s. evolution simulation.
- In the frame of BETACOOOL the algorithm can be tested by comparison with the results of r.m.s. dynamics simulation. It requires a unification of data post processing, development of standard output format for different algorithms.

Monte Carlo simulation

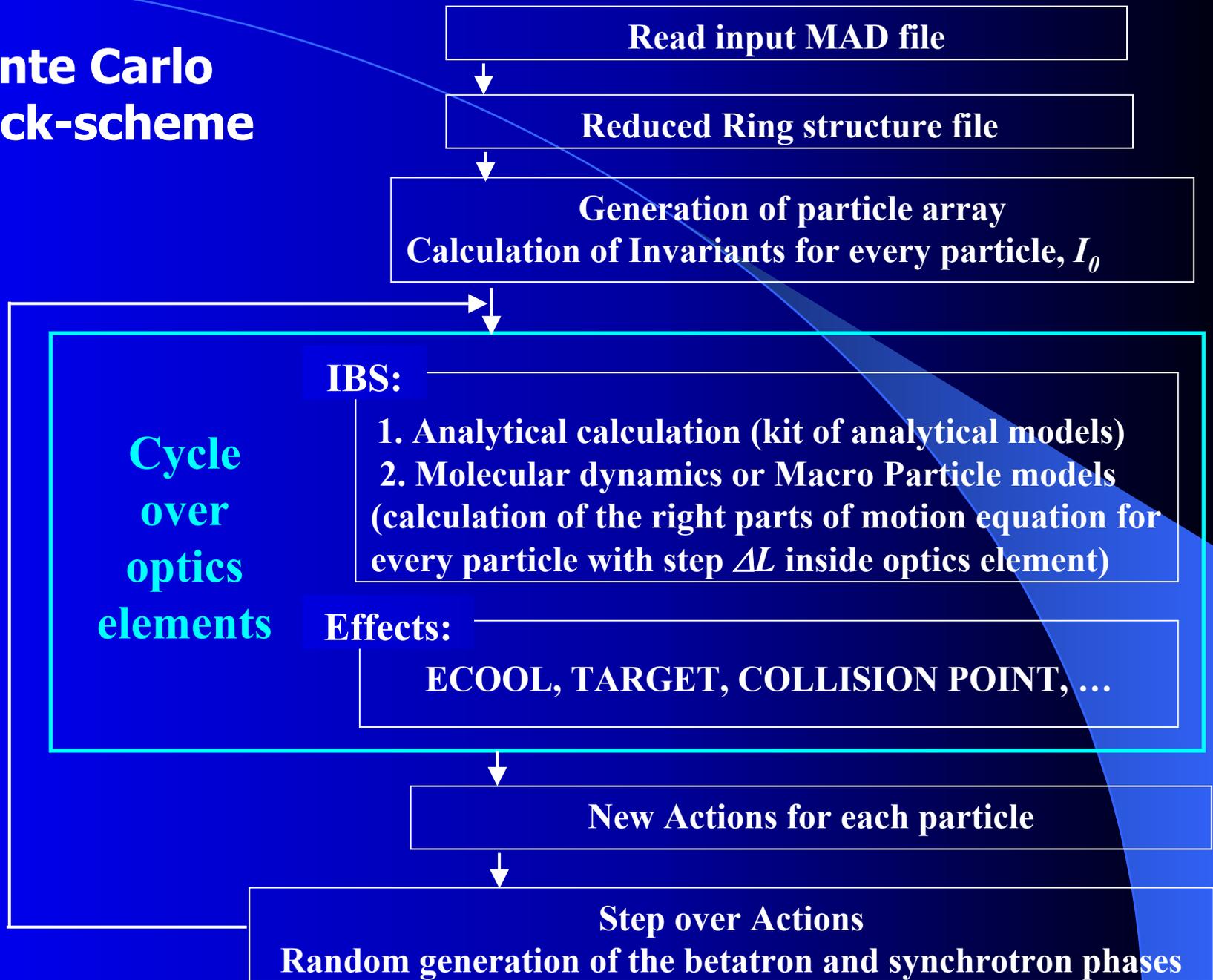
Two stages of the algorithm development:

- 1. Analytical models for IBS calculation**
- 2. Numerical solution of the particle motion equation in the real optic structure tacking into account interactions between particles.**

Two possibilities to reduce required particle number:

- molecular dynamics,**
- macro particle simulation.**

Monte Carlo block-scheme



4. IBS simulations

Detailed calculation of the IBS effect

- generation of particle array accordingly to the lattices (current point or averaged over ring)
- calculation of the current beam rms parameters;
- **calculation of the IBS rate for every particle of the beam**
- changing the particle angles due to the IBS kick

Two concepts:

1. Semi-analytical calculation of the rates (Burov's model)
2. Numerical calculation of the rates – next step

IBS kick calculation. Detailed algorithm

Array of particles

current r.m.s. parameters
(ε , σ , in accordance with mean lattices)

Calculation of the IBS rates τ using Burov's or
Numerical model, for mean Ring lattices

**Cycle
over
particles**

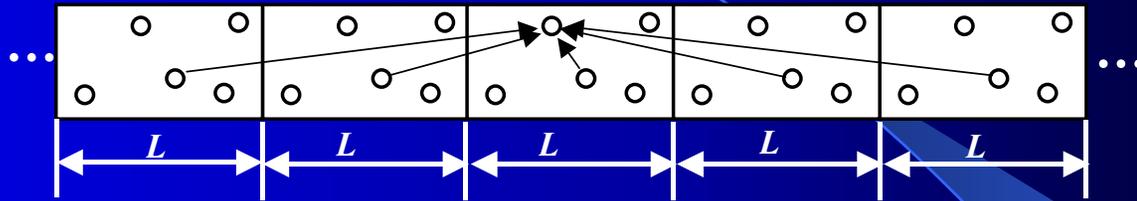
$$1. \langle \theta_i^2 \rangle = \frac{\varepsilon \Delta T}{\beta \tau_i}$$

$$2. \Delta \theta_i = \sqrt{\langle \theta^2 \rangle_i} \cdot \text{Gaussian value}$$

$$3. \theta_i = \theta_{i0} + \Delta \theta_i$$

Molecular dynamics

Usage of a periodic boundary condition, or Molecular Dynamics (MD) technique.



The interaction between particles is calculated in accordance with:

$$F_{MD} = (0, F_{px}, 0, F_{py}, 0, F_{pz}) .$$

Here

$$F_{p,X} = \frac{1}{4\pi\epsilon_0} \frac{q^2 (X - X_i)}{m_0 c^2 \gamma_0^2 \beta_0^2} \left(\frac{1}{a^3} - \frac{2I_1}{rL^2} \right), \quad X: x \text{ or } y$$

$$F_{pz} = \frac{1}{4\pi\epsilon_0} \frac{q^2 (z - z_i)}{m_0 c^2 \gamma_0^2 \beta_0^2} \left(\frac{1}{a^3} - \frac{2I_0}{sL^2} \right)$$

a - distance between particles, $s = z - z_i$, r – radial distance, L – MD cell size, Integrals I_1 and I_0 were numerically calculated and are used in the program as table values.

To calculate IBS process using MD technique one needs to determine the particle number using in the calculations N_{cell} in accordance with the required accuracy. Then the cell length is calculated in accordance with:

$$L = \frac{N_{cell}}{N} C$$

C - ring circumference, N - ion number in the ring.

The initial distribution of the particles inside the cell is generated randomly in accordance to the beam emittance and ring lattice parameters in the initial position. Coordinates:

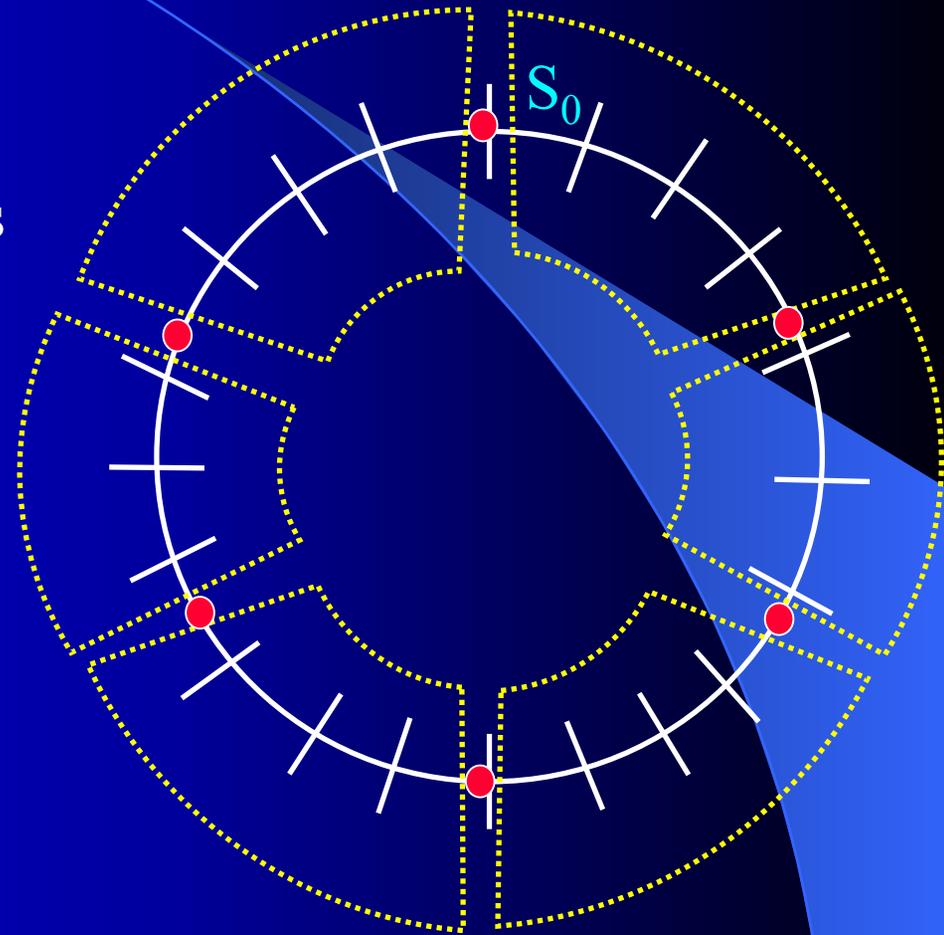
$X, Y, dP/P \rightarrow$ Gaussian law,

$S \rightarrow$ uniformly inside the cell.

Thereafter the particles in the sell are propagated through the ring: element by element in accordance to the ring structure described in the input file.

5. Storage ring model. Module structure.

1. With ring transformation matrix calculate lattice at S_0 point
2. Twiss tracking
3. Reducing of optics structure
4. Calculation of sector's matrices
5. Step by step tracking of all particles through selected points:
 - IBS in every point
 - other effects in point is exist



Initial data files

For beam tracking the following types of data files are developed:

- file with real lattice structure (for RHIC – a file about of 1.6 MB, 10130 rows, ~5800 optics elements)
- file with reduced lattice structure (has to be made by user in easy way). Here user selects points (optics elements with parameters and coordinates). Transformation matrices are recalculated at those points in accordance with entire optics structure. Useful for preliminary, very fast calculation.
- file with selected points inside the ring where necessary effects are to be calculated (optics elements and effects with parameters and coordinates are listed).

Examples of initial data files

file with reduced lattice structure (*.red) :

```
0 ,  
39 ,  
80 ,  
119 , QUAD  
160 , BEND  
199 ,  
240 ,  
280 ,  
321 , QUAD  
360 ,  
402 , BEND  
441 ,
```

*.ibs file structure :

```
0, s = 0, L = 15 : ECOOL  
10, s = 9.5, L = 2 : SBEND  
20, s = 20, L = 1 :  
30, s = 29.5, L = 1 :  
40, s = 40.5, L = 0.5 : QUADRUPOLE  
50, s = 50, L = 0.5 : QUADRUPOLE  
60, s = 60.5, L = 1 :  
70, s = 70, L = 1 :  
80, s = 79.5, L = 2 : SBEND  
90, s = 90, L = 1 :  
100, s = 99.5, L = 1 :  
110, s = 110.5, L = 0.5 : QUADRUPOLE  
120, s = 120, L = 0.5 : QUADRUPOLE  
130, s = 130.5, L = 1 :  
140, s = 224.7333638, L = 0 : TARGET
```

6. Current status and plans

- simplest versions of the basic algorithms are developed,
- procedures supporting format of input files for ring structure are developed and tested,
- procedures supporting different beam models are developed and tested,
- preparation of the UNIX version is in progress

Plans:

- benchmarking of the algorithms,
- development of algorithms for particle losses calculation,
- numerical calculation of the luminosity,
- usage of the friction force calculated numerically.