Comparison of IBS rates with BetaCool

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Various approaches for IBS calculation

1. IBS based on rms rates

1.a) BetaCool (standard version) – rms beam rates

1.b) SimCool – individual particles kicks based on rms rates –

this approach is now also implemented in BetaCool (working version)

1.c) Rms based rate – for changing distribution (bi-Gaussian)

2. Detailed rates (dependence on individual particle actions)

2.a) Burov's results – implemented in BetaCool – to be reported

2.b) Burov's integrals – numeric evaluation – being implemented

2.3) Direct particle interaction: Molecular Dynamics – being implemented

Comparison of available rms beam rates

Goals:

1. We wanted to see difference between various standard formulas.

   ↓

   Choose the most realistic one in cooling calculations.

2. We want to see how lattice-dependent IBS calculations agree with lattice-averaged formulas and thus check accuracy of IBS formula used in SimCool, based on diffusion coefficient directly.

IBS rates for rms beam parameters

1. Martini's model – dependence on lattice functions – complex integrals:

   1.1) Numerical evaluation of integrals

   1.2) Bjorken approach

   1.3) Series expansion

2. Jie Wei approximation – replace integrals by analytic function – approximation of a FODO lattice – coupled/averaged.
3. Jie Wei scaling approximation above transition – keeping dependence on dispersion function
4. Jie Wei scaling above transition – averaged dispersion – no lattice dependence

1. Martini's model (Parzen/Wei notation):

$$\begin{bmatrix} \dfrac{1}{\sigma_p}\dfrac{d\sigma_p}{dt} \\ \dfrac{1}{\sigma_x}\dfrac{d\sigma_x}{dt} \\ \dfrac{1}{\sigma_z}\dfrac{d\sigma_z}{dt} \end{bmatrix} = \frac{A_0}{2}\int e^{-Dz}\ln\left(1+C^4z^2\right)\begin{bmatrix} n_b(1-d^2)g_1 \\ a^2g_2+(d^2+\bar{d}^2)g_1 \\ b^2g_3 \end{bmatrix}\sin\theta d\theta d\varphi dz$$

$$A_0 = \frac{cr_p^2 NZ^4\beta_x\beta_z}{32\pi A^2\sigma_x^2\sigma_z^2\sigma_p\sigma_s\beta^3\gamma^4} \qquad d = \frac{D_p\sigma_p}{\sqrt{\sigma_x^2+D_p^2\sigma_p^2}}$$

Integrals can be taken in various ways:

1.1- numerically – needs many times steps – converging result is close to 1.2

1.2- Bjorken – we use this method in subsequent studies

1.3 - approximations

2. Jie Wei approximation – coupled/averaged:

- the quantity $\ln(1 + C^4 z^2)$ can be substituted by a constant $2L_C$ which is about 20
- for accelerator consisting of regular cells the variation in is small along the ring circumference, thus the terms including and can be neglected
- for simplification of the integration over θ and φ the sin2φ and cos2φ are replaced by their average value of 1/2.

$$\begin{bmatrix} \dfrac{1}{\sigma_p}\dfrac{d\sigma_p}{dt} \\ \dfrac{1}{\sigma_{x,z}}\dfrac{d\sigma_{x,z}}{dt} \end{bmatrix} = 4\pi A_0 L_C F(\chi) \begin{bmatrix} n_b (1 - d^2) \\ \left(-\chi + d^2\right)/2 \end{bmatrix}$$

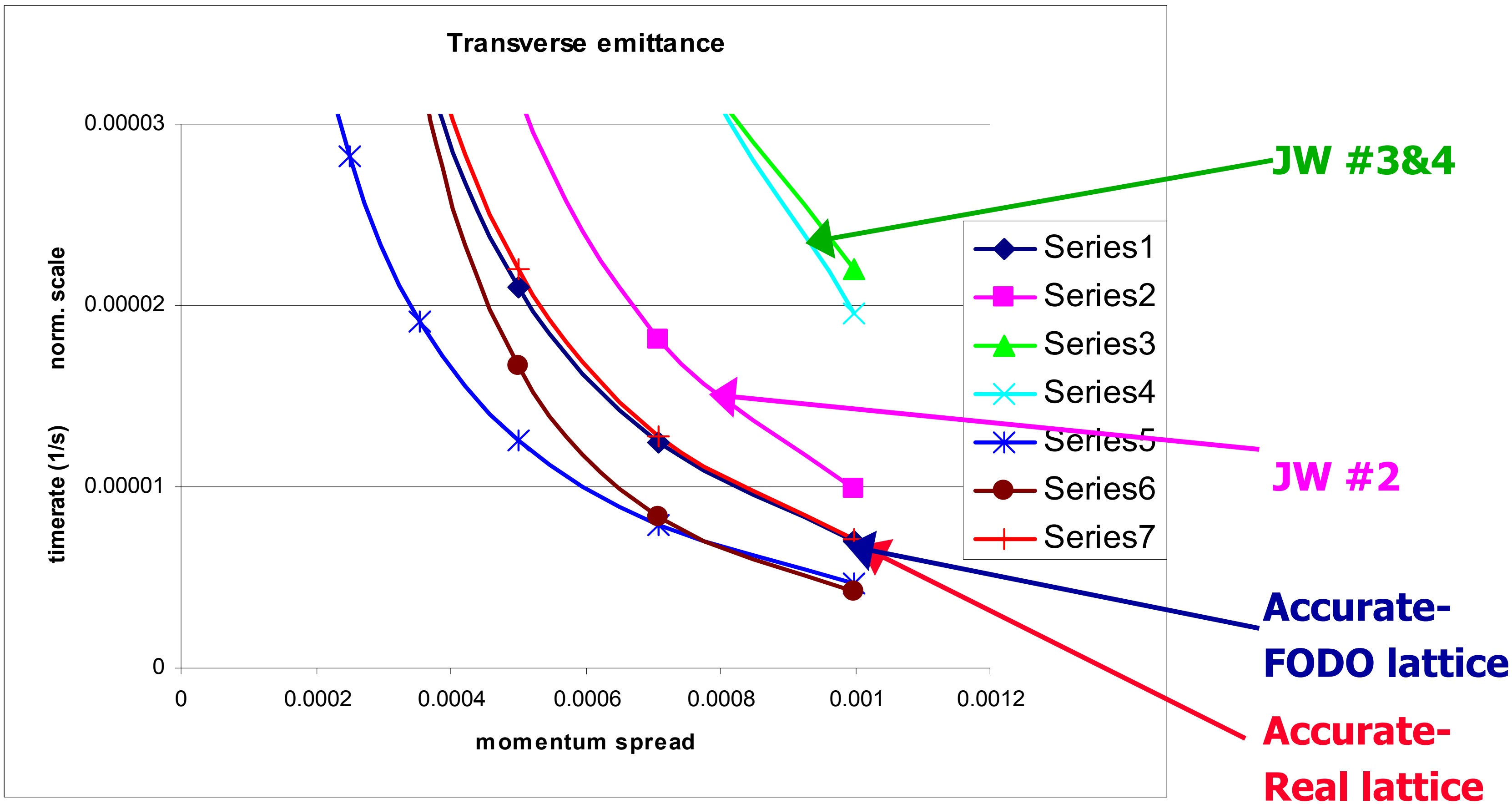
3. Jie Wei – scaling approximation above transition

$$\frac{1}{\tau}=\frac{\pi}{16}\frac{Nr_0^2 mc^2\Lambda_{ibs}}{\gamma_t \varepsilon_x \varepsilon_y S} f(D)$$

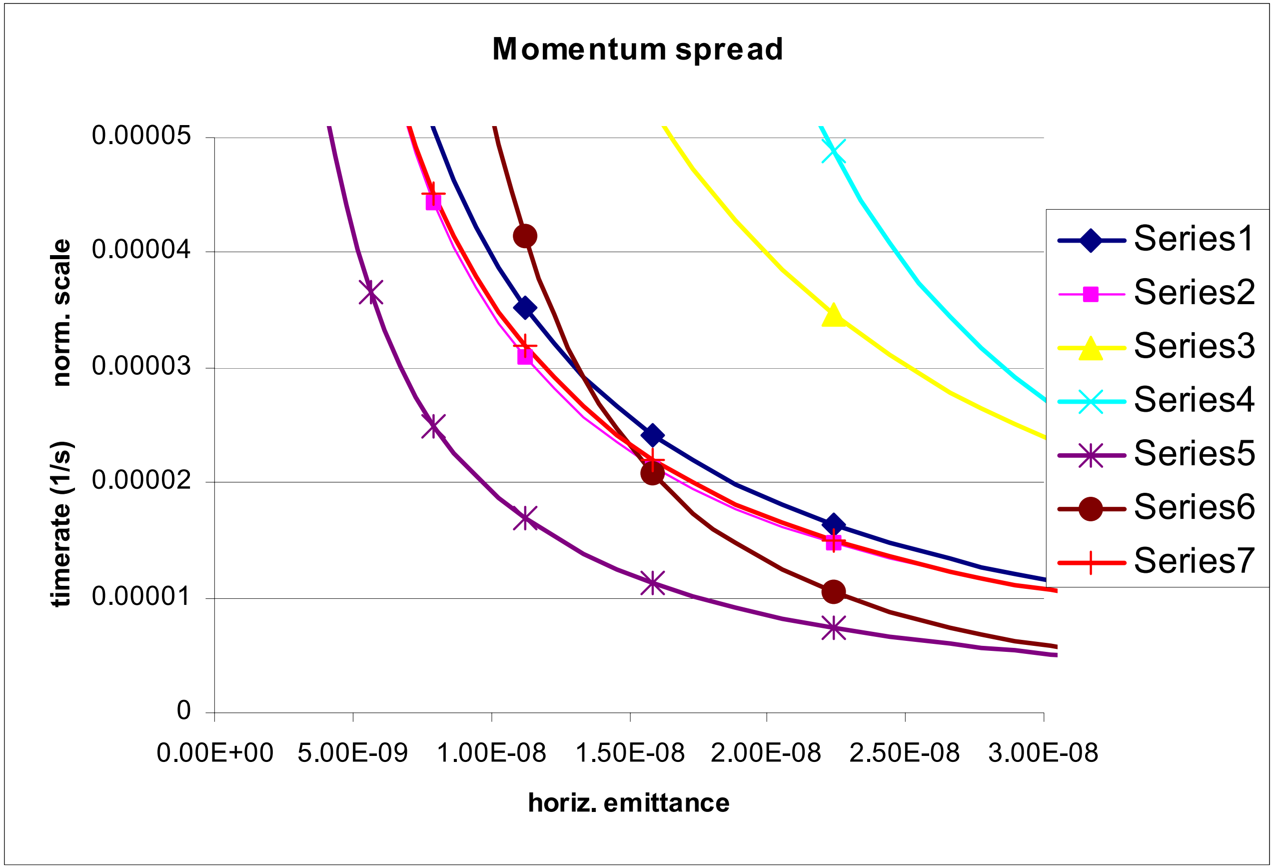
4. Scaling approximation + averaged dispersion

$$\frac{1}{\tau}=\frac{\pi}{16}\frac{Nr_0^2 mc^2\Lambda_{ibs}}{\gamma_t \varepsilon_x \varepsilon_y S} f$$

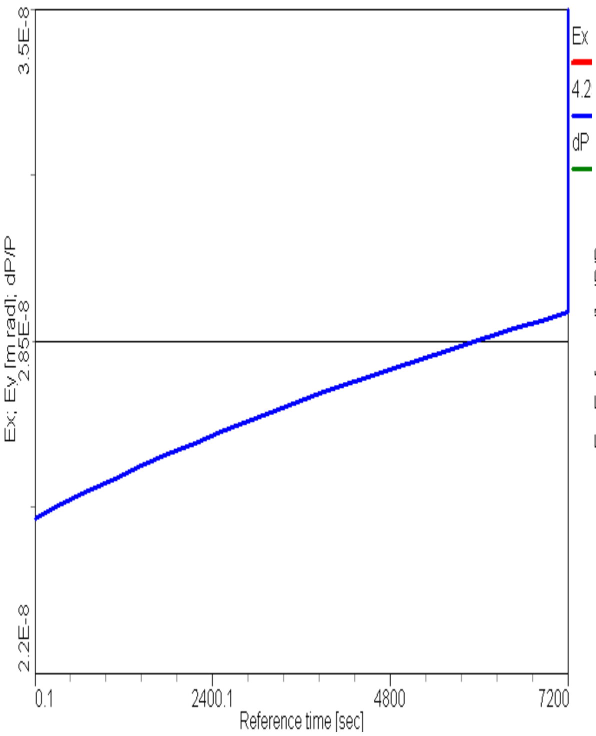
Transverse rates



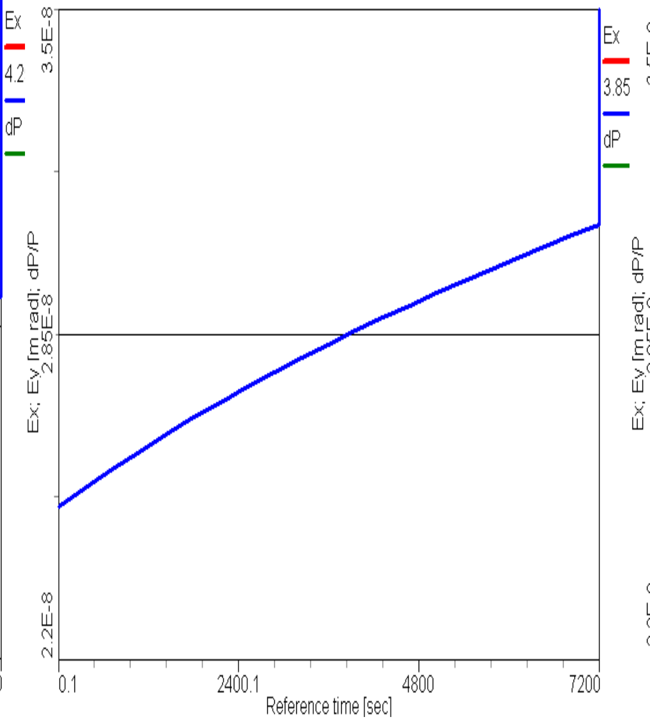
Longitudinal rates



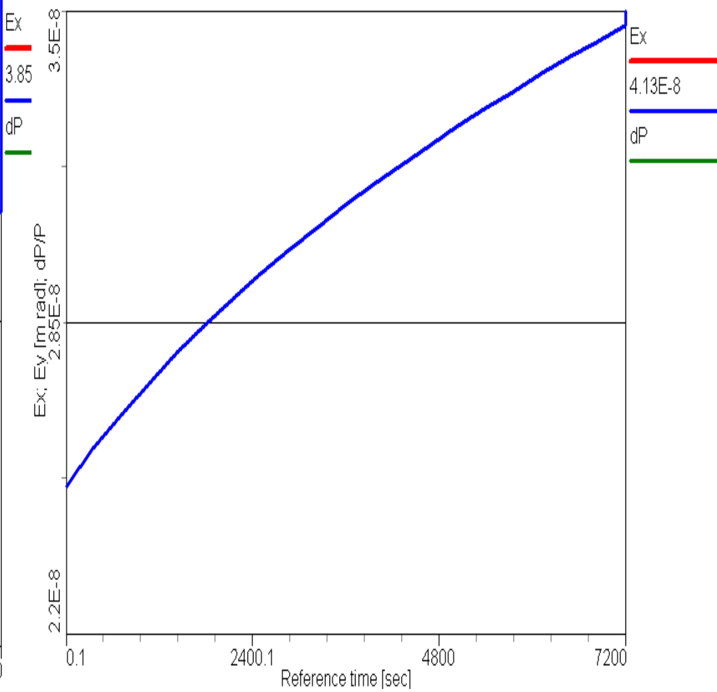
Emittance growth at store



Bjorken

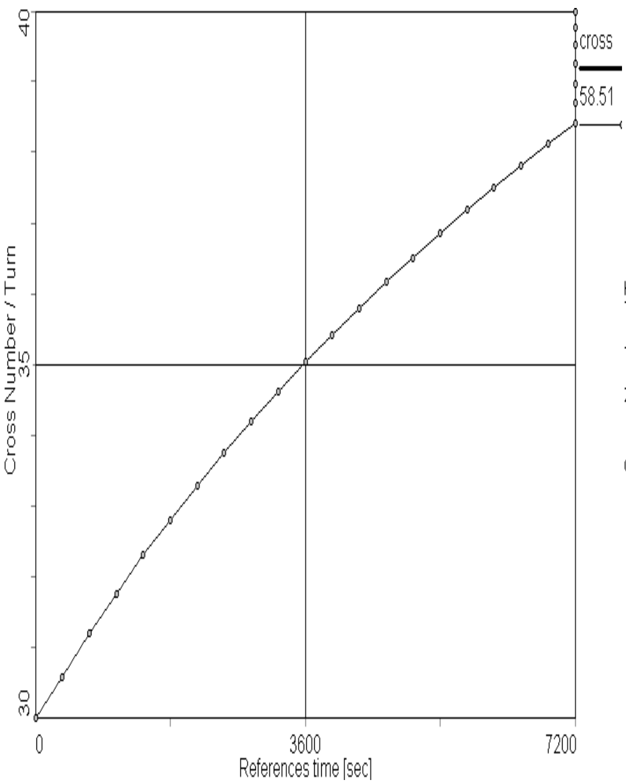


JW #2

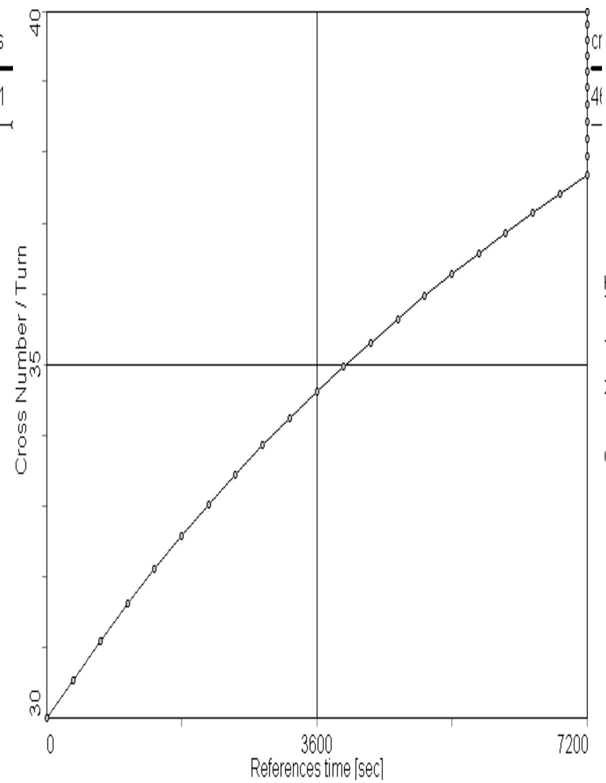


JW scaling 3&4

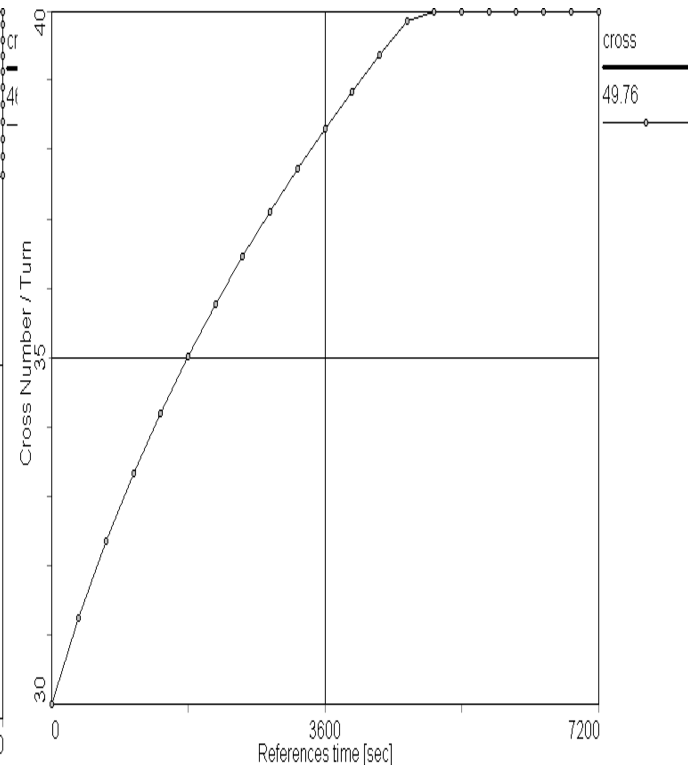
Bunch lengthening at store



Bjorken

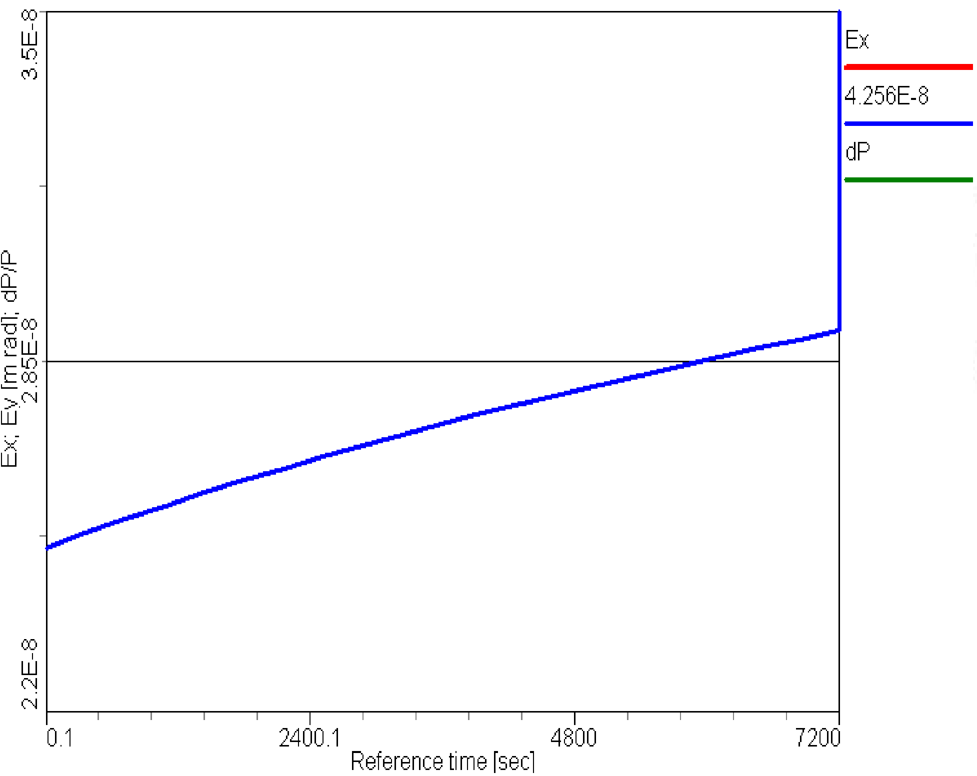


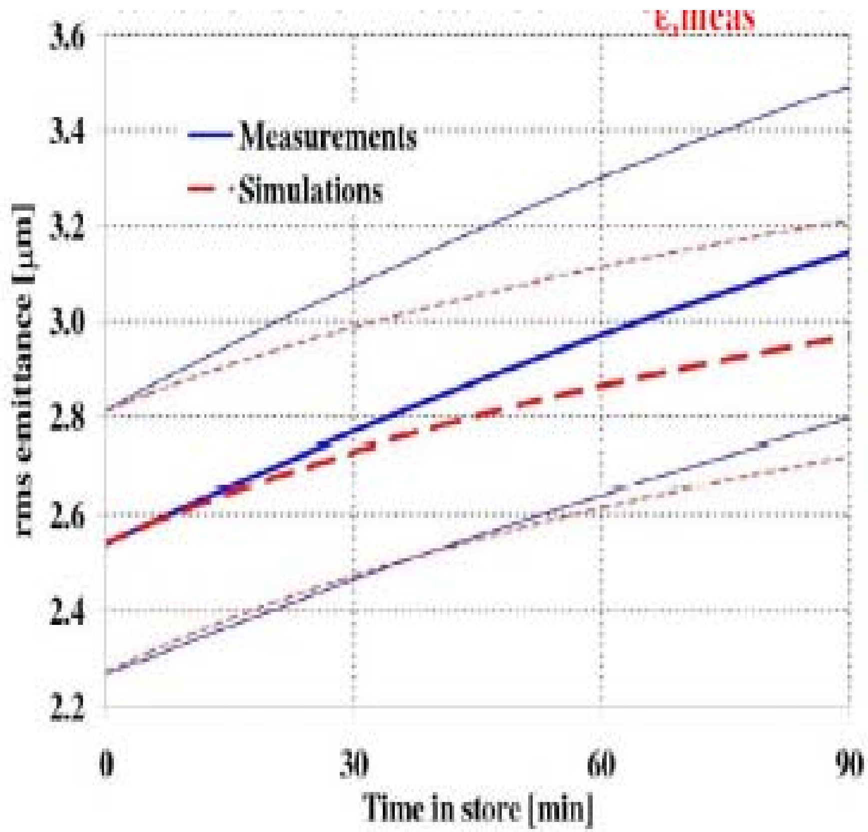
JW # 2



JW # 3 & 4

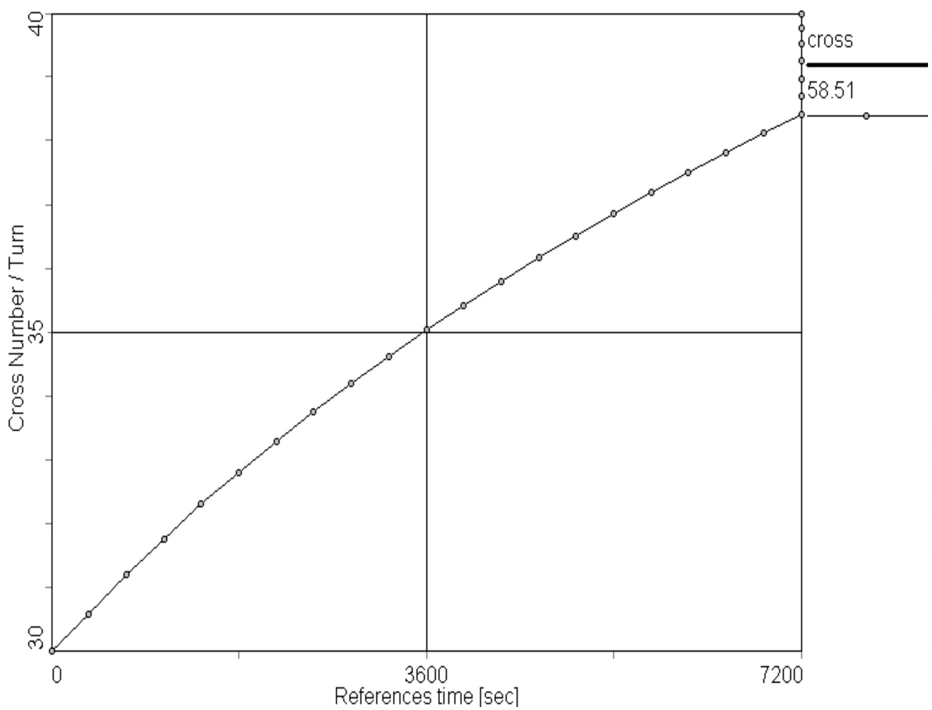
Emittance – comparison with experiment

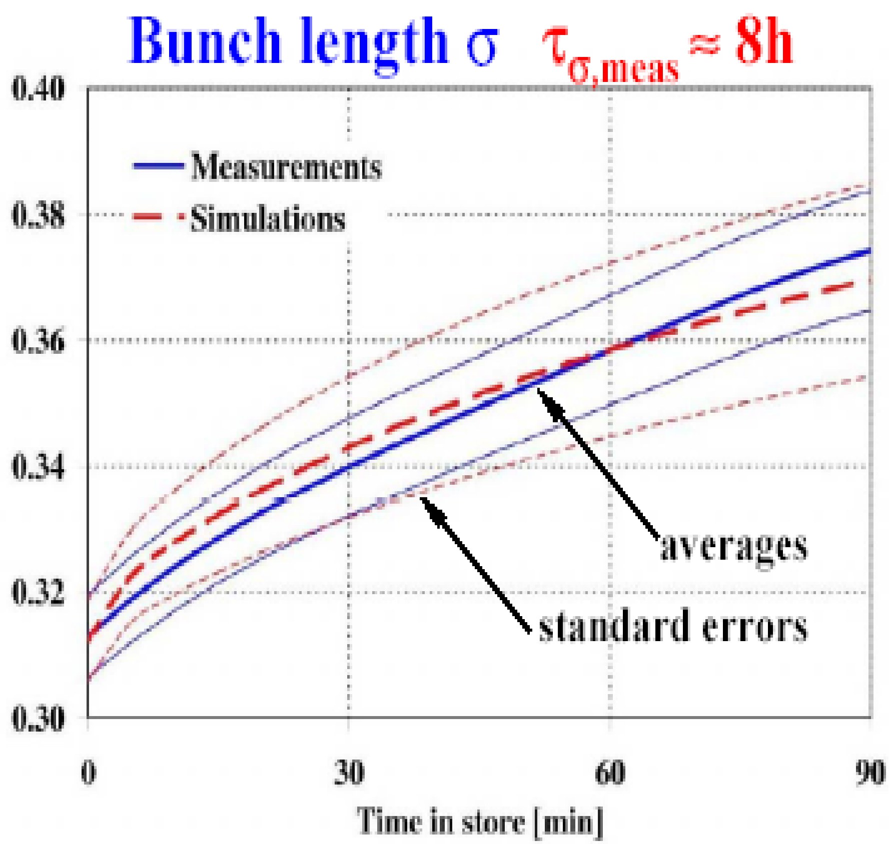




Bjorken formula **W. Fischer et el.**

Bunch length comparison with experiment





Bjorken

Some conclusion

- **Martini – numerical – converges to Bjorken.**
- **Jie Wei approximation (# 2) of FODO lattice is pretty close to a more accurate treatment (Bjorken).**
- **Jie Wei scaling formulas (# 2 & 3)– for high energy (further simplification of #2 result) overestimate growth rate.**

Jie Wie (#2) & Bjorken – in good agreement with experiment – one better fits longitudinal plane while the other – transverse.

- It was found that "scaling approximation" 3&4 overestimate growth rate.
- Why are we interested in such scaling formulas, anyway?

These formulas do not have lattice dependence – in fact, they are very similar to a formula which can be derived using standard plasma analysis

Such gas-relaxation formula is used in SimCool.

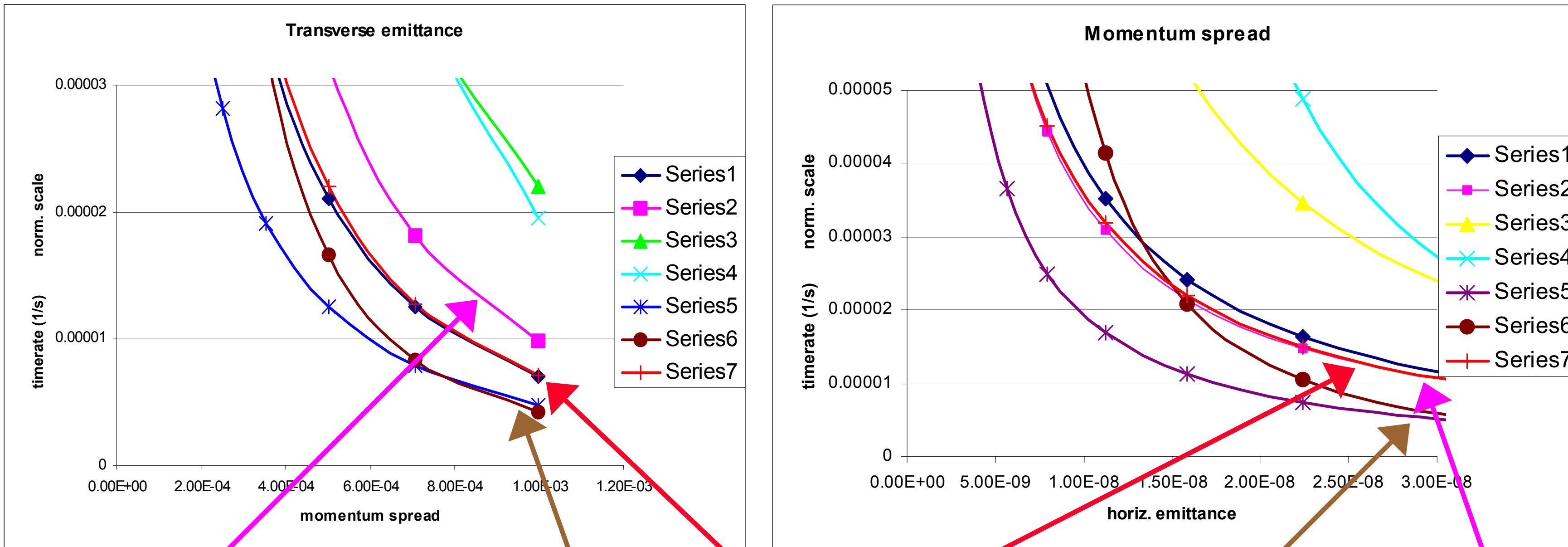
Immediate questions for SimCool formula:

1. How accurate it is since it does not contain lattice dependence?
2. Do we miss additional large factor due to gamma?

$$\frac{1}{\tau_{JW}} = \frac{\pi}{16} \frac{N r_0^2 m c^2 \Lambda_{ibs}}{\gamma_t \varepsilon_x \varepsilon_y S} f \qquad \frac{1}{\tau_{simcooltest}} = \frac{\pi}{16} \frac{N r_0^2 m c^2 \Lambda_{ibs}}{\gamma \varepsilon_x \varepsilon_y S} f$$

caution: not in the code

Rate comparsion for γ=100



JW- sligtly overestimates

accurate

JW – agrees well

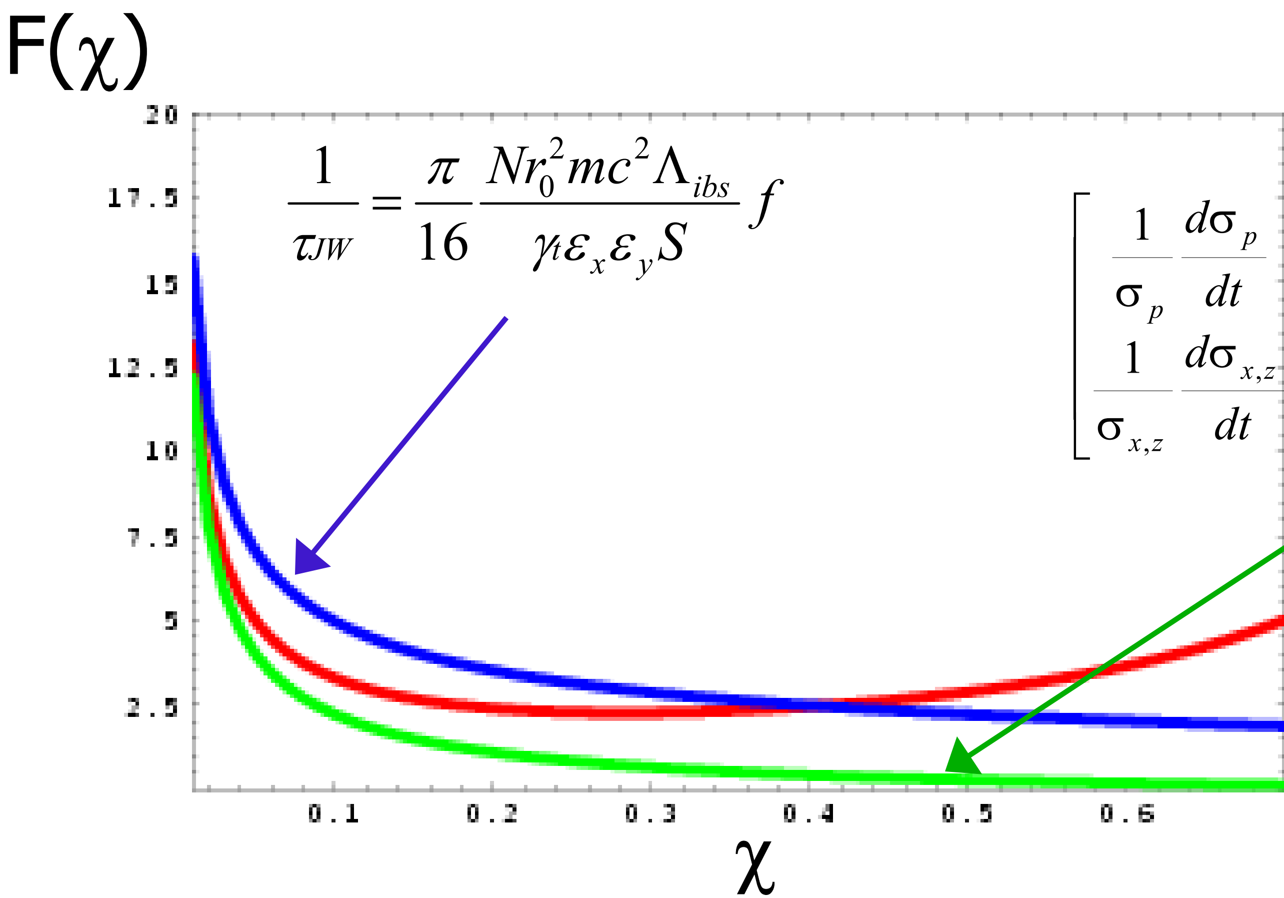
"simcooltest" formula- slightly underestimate

Summary/Application

1. We can use Bjorken or JW#2 formulas for the rms rates in BetaCool.

2. Plasma formula, used in SimCool (for this γ "simcooltest" formula gave similar result)– reasonable approximation (only slightly underestimates a more accurate treatment)

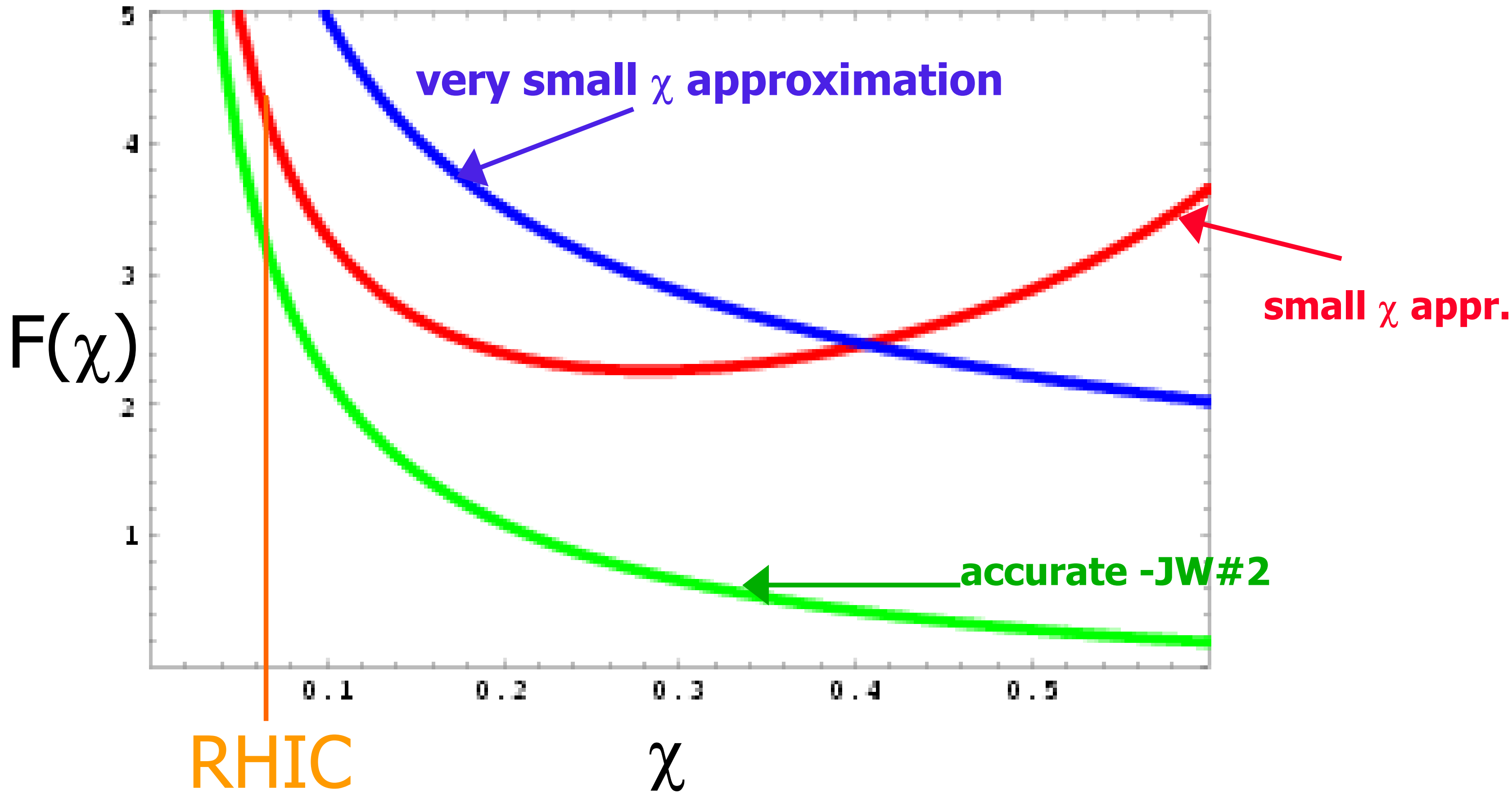
Appendix 1: JW's scaling



$$\frac{1}{\tau_{JW}} = \frac{\pi}{16} \frac{N r_0^2 m c^2 \Lambda_{ibs}}{\gamma_t \varepsilon_x \varepsilon_y S} f$$

$$\begin{bmatrix} \dfrac{1}{\sigma_p} \dfrac{d\sigma_p}{dt} \\ \dfrac{1}{\sigma_{x,z}} \dfrac{d\sigma_{x,z}}{dt} \end{bmatrix} = 4\pi A_0 L_C F(\chi) \begin{bmatrix} n_b (1 - d^2) \\ \left(-\chi + d^2\right)/2 \end{bmatrix}$$

A1: JW's scaling



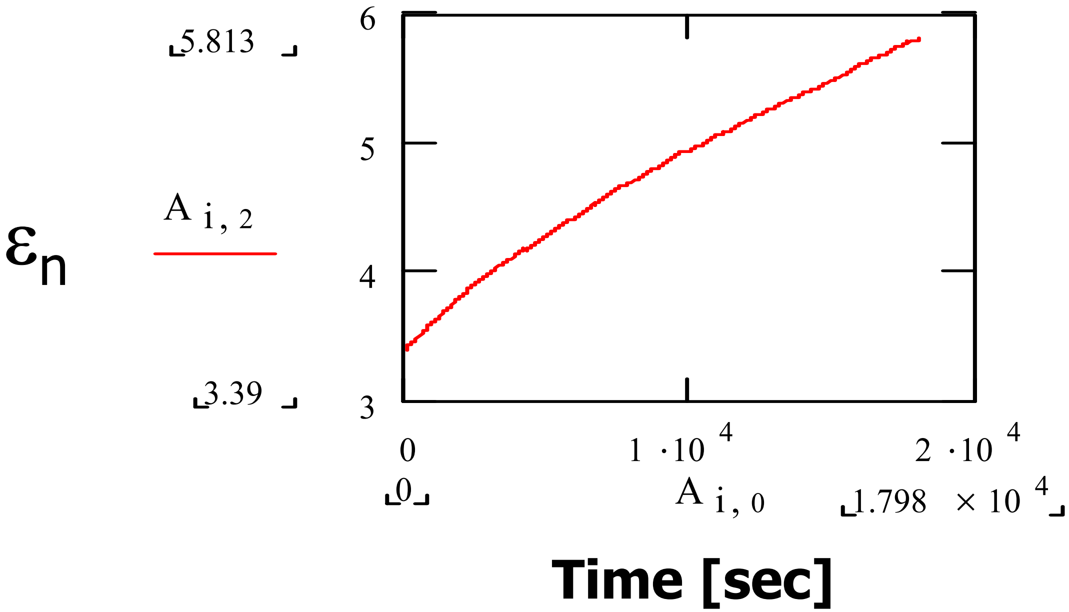
Tests of IBS treatment in SimCool

γ dependence of IBS rates

A1: IBS rates for various γ

γ	Bjorken $[\tau^{-1}]$	accurate JW (with function F(χ))	JW approximation for high gamma
30	6*10^-4 8.6*10^-4	1.4*10^-4 6*10^-4	3*10^-3 6*10^-3
100	3*10^-5 8*10^-5	4*10^-5 7*10^-5	9*10^-5 1.6*10^-4
1000	6*10^-8 1.5*10^-7	7.5*10^-8 1.2*10^-7	9*10^-8 1.7*10^-7

SimCool - IBS growth



$\gamma=100$

- **Reasonable growth for $\gamma=100$**
- **Dependence on γ, bunch length, etc. – to be studied**

1. JW above transition – if γ_t -> γ – which we thought is equivalent to

SimCool's diffusion kick converted into

rms rate ("simcooltest" formula) –
overestimate by factor of 10 at low gamma close to transition.

2. Direct calculation with the SimCool code both for low and very high gamma give reasonable rms growth rates.

Dependence of IBS rates in SimCool on bunch length, sigma_p, etc.

and benchmarking with BetaCool is in progress.