Global phase diagram of a spin-orbital Kondo impurity model and the suppression of Fermi-liquid scale

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11/04/2019

Y. Wang *et al.* arXiv: 1910.13643

Elias Walter *et al*. arXiv:1908.04362

Center for Computational Material Spectroscopy and Design

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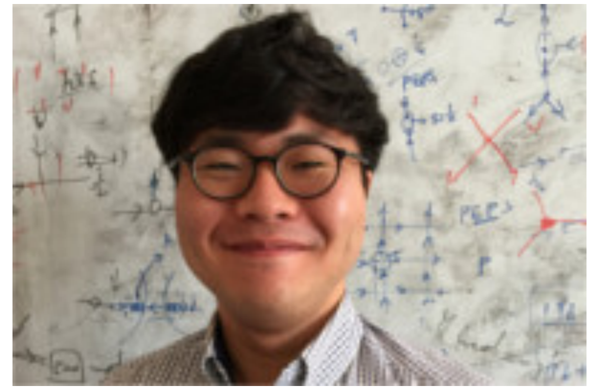

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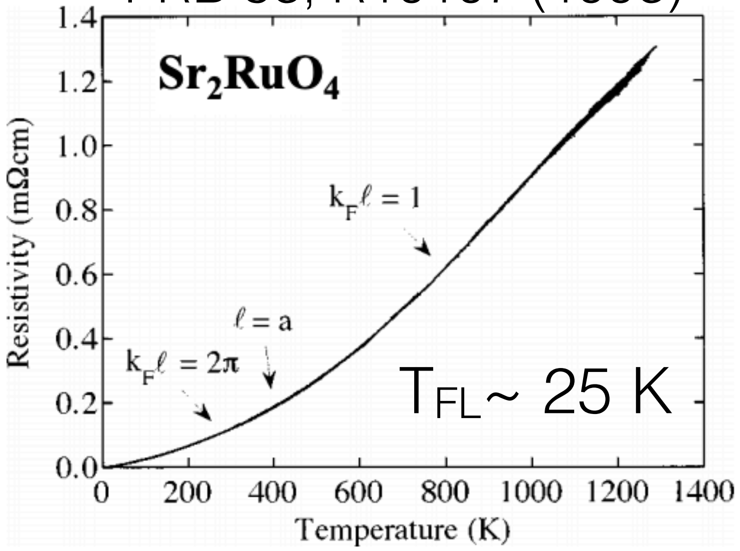
Motivation: very small T_{FL} in many correlated metallic materials

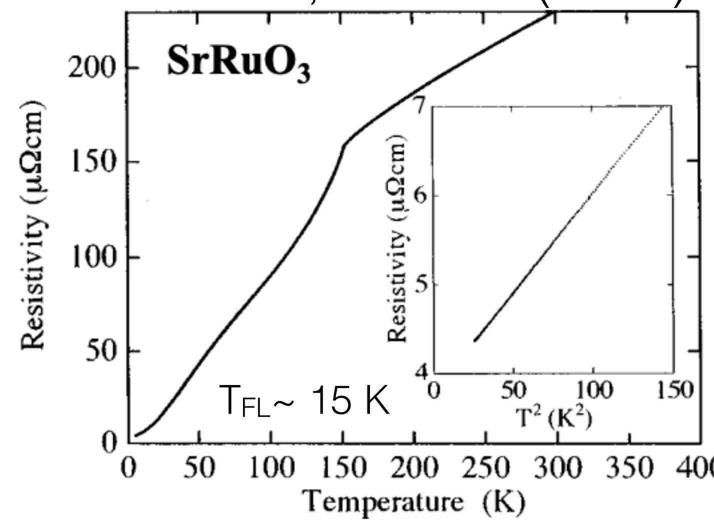
- Many correlated metallic materials can be described by the Landau Fermi-liquid theory at low energies
- but, the Landau Fermi-liquid coherence scale T_{FL} is found to be surprisingly small in many materials
- The Hund metals, including ruthenates, iron-based superconductors, are such class of materials
- For Landau FL theory, below T_{FL} , we would expect that the resistivity will show T^2 law and $\gamma(T)=C(T)/T$ ~constant

Small coherence scale in Hund metals

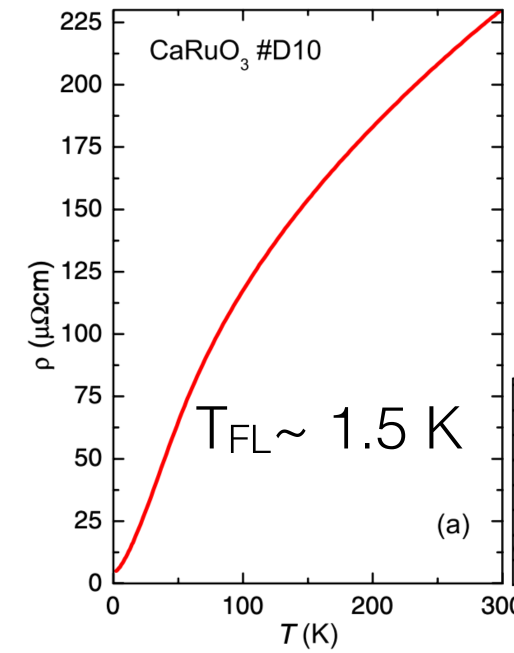
A.W. Tyler et al.
PRB 58, R10107 (1998)



A.P. Mackenzie et al.
PRB 58, R13318 (1998)

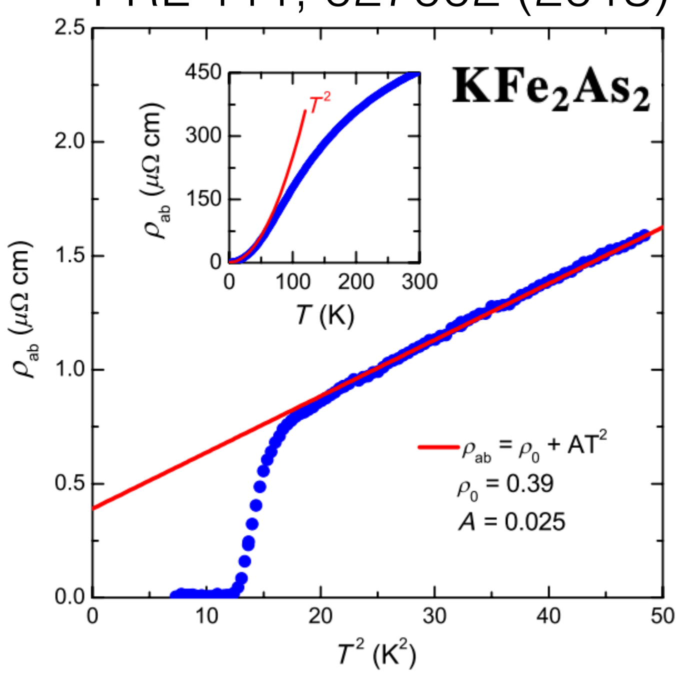


M. Schneider et al.
PRL 112, 206403 (2014)

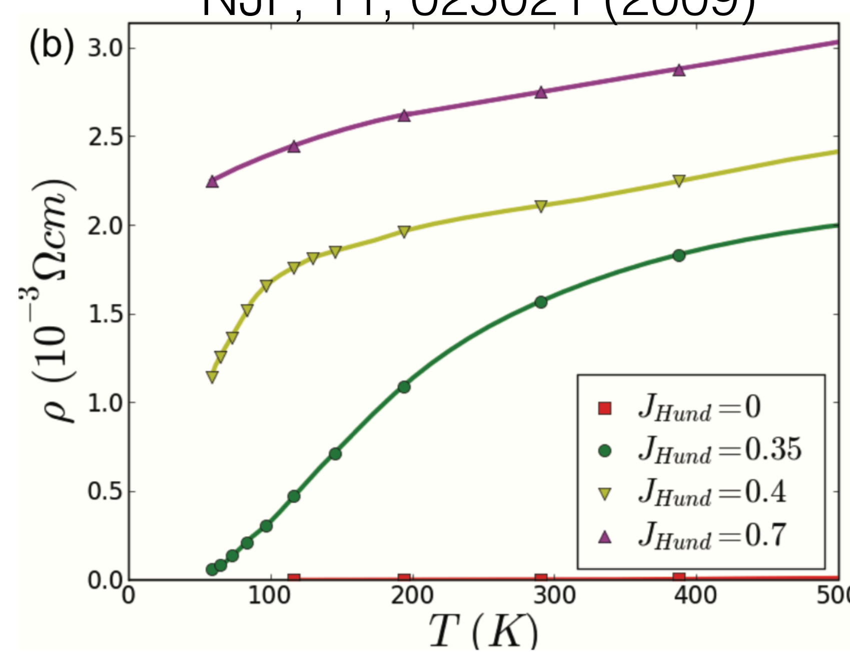


Compound	Magnetic order	γ/γ_{LDA}	$\rho \propto T^2$	Remarks
Sr_2RuO_4	PM	4	<25 K	Unconventional SC < 1.5 K
$SrRuO_3$	FM < 160 K	4	<15 K	$\sigma \propto \omega^{-0.5}$
$Sr_3Ru_2O_7$	PM	10	<10 K	Metamagnetic quantum-critical point and nematicity
$CaRuO_3$	PM	7	$T^{1.5}$ > 2 K	$\sigma \propto \omega^{-0.5}$, $\gamma = \gamma_{FL} + \log(T)$
Ca_2RuO_4	AF < 110 K	✗	✗	Insulator < 310 K

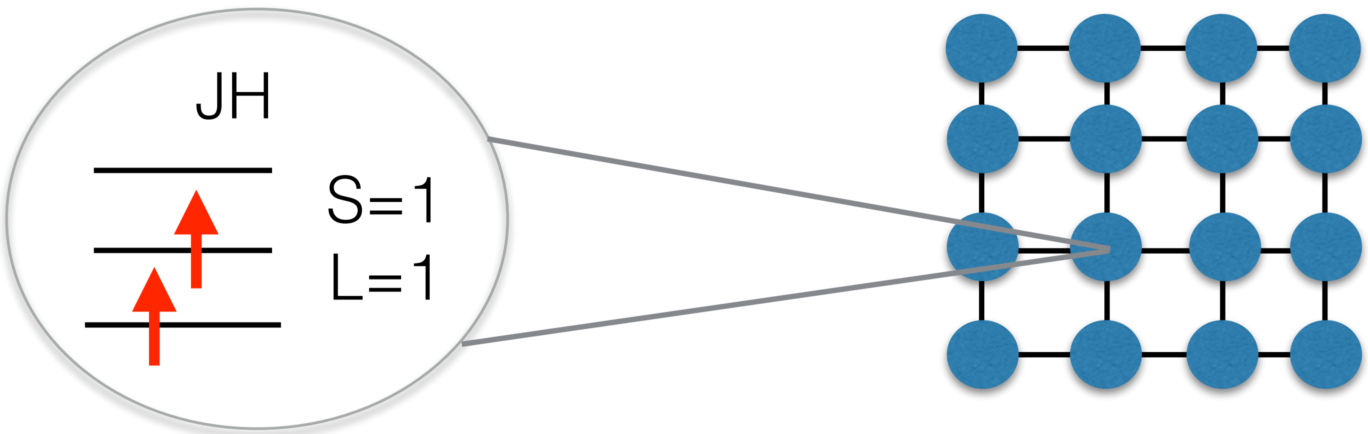
F. Hardy et al.
PRL 111, 027002 (2013)



K. Haule and G. Kotliar,
NJP, 11, 025021 (2009)



Hund metals: Strong correlation from Hund's coupling



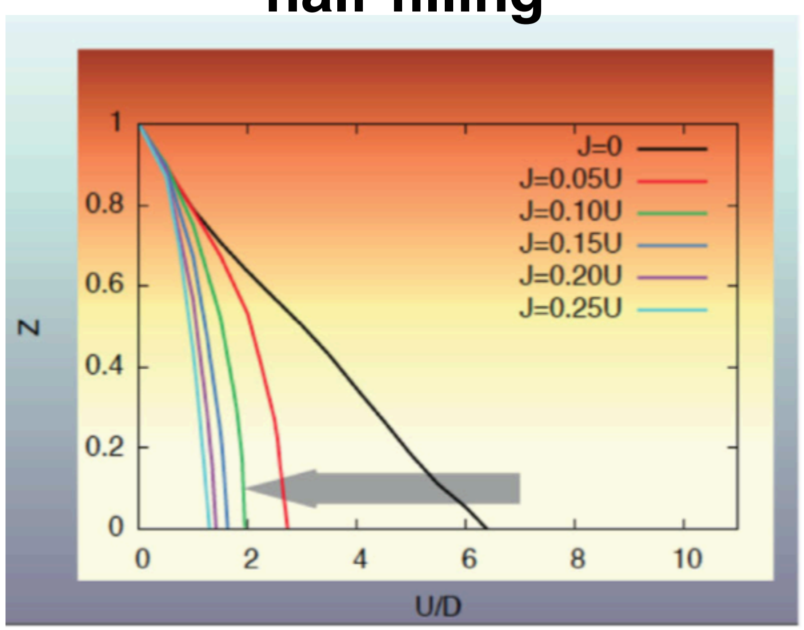
A. Georges *et. al*
Annu. Rev. Condens. Matter Phys. 4,137–78 (2013)

3-orbital Hubbard Model, DMFT

Luca de' Medici *et. al*, PRL 107, 256401 (2011)

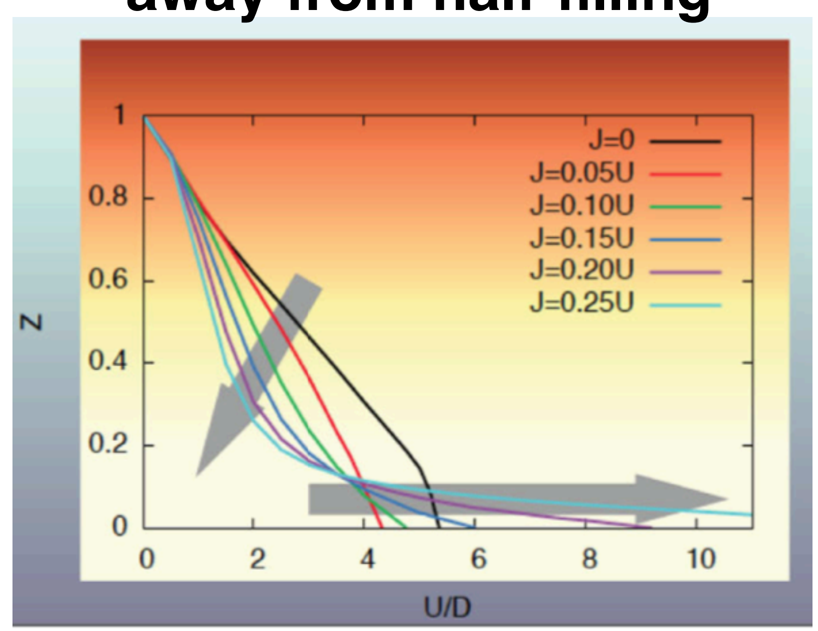
Youichi Yanase *et al.*
JPSJ 66, 3551 (1997)

half-filling

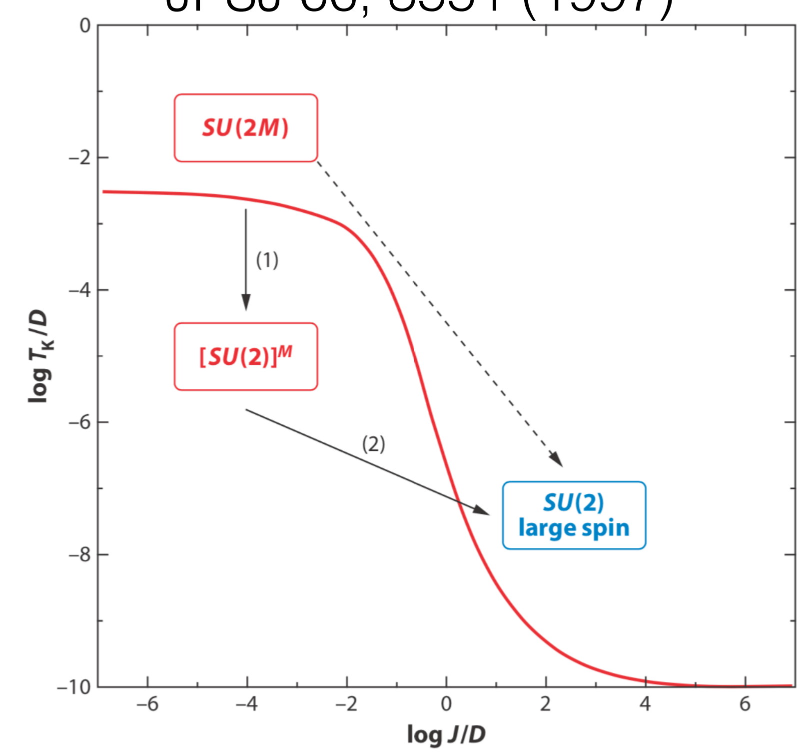


(c) $N = 3$

away from half-filling



(b) $N = 2$



Janus-faced influence of Hund couplings:

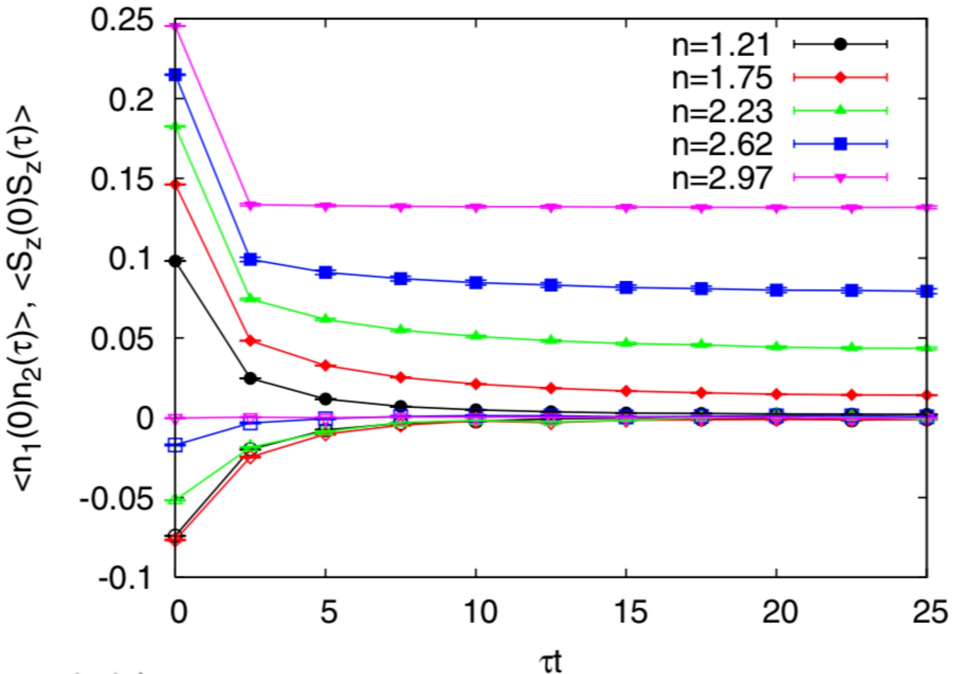
(1) High energy: It drives the system away from Mott transition
(2) Low energy: It largely suppresses the coherence scale T_{FL}, above T_{FL}, it shows NFL behavior

$J_K \propto 1/S$

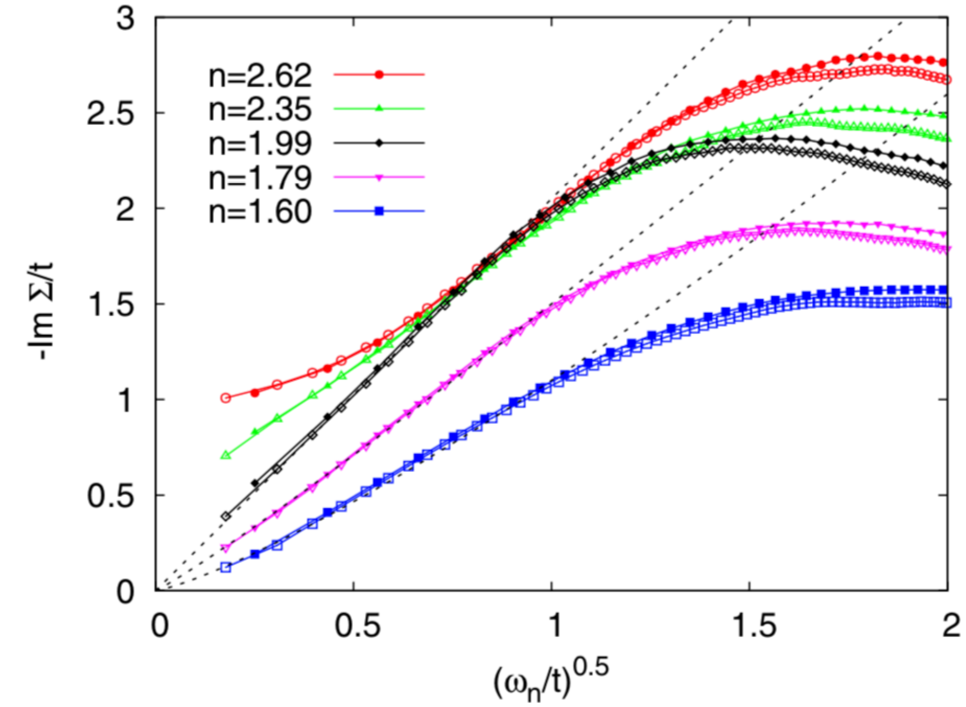
$T_K \propto \exp(-1/J_K)$

NFL: Spin-freezing and power-law behavior of self-energy above T_{FL}

Philipp Werner *et al*. PRL 101, 166405 (2008)

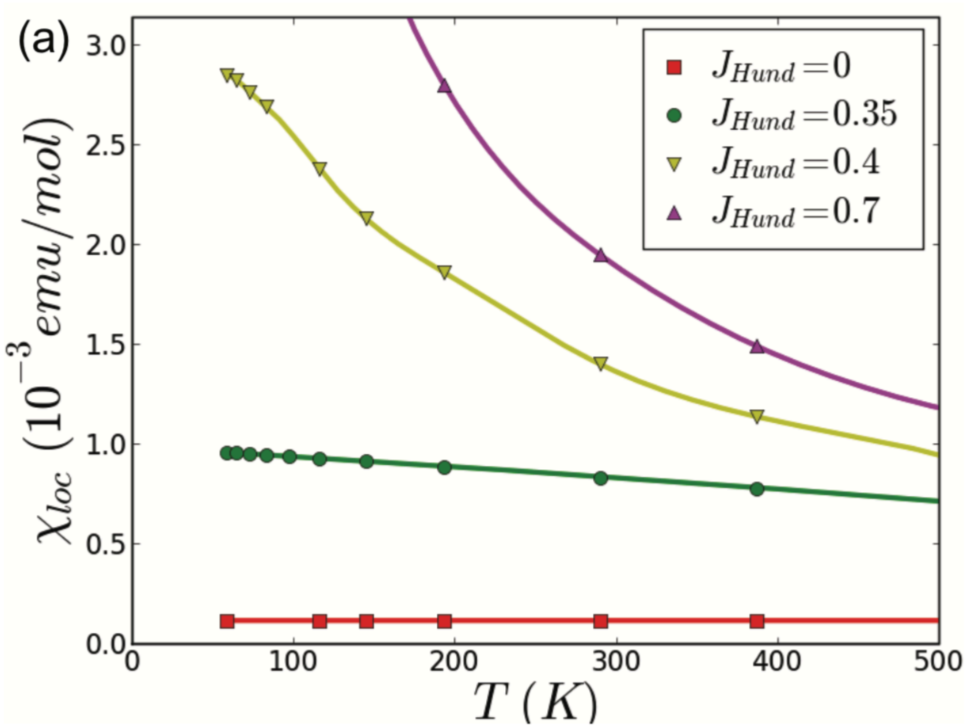


$\langle S_z(\tau)S_z(0)\rangle$ saturates to finite value at longer time, indicating frozen moments, "frozen-metal"

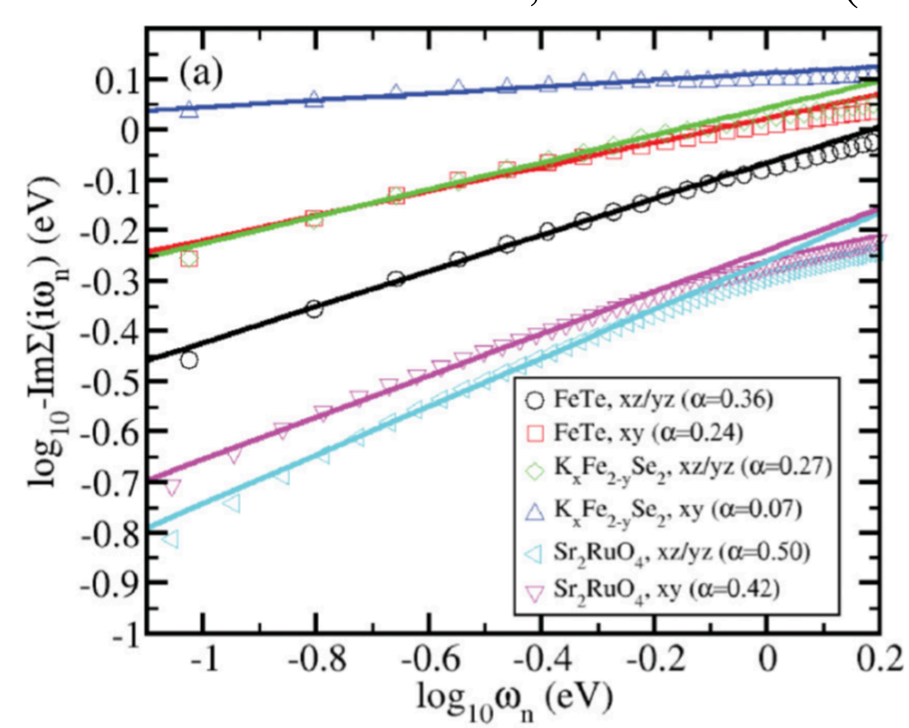


fractional power-law NFL self-energy

K. Haule and G. Kotliar, NJP, 11, 025021 (2009)



Z. P. Yin *et al*. PRB **86**, et al.195141 (2012)



Spin-Orbital-Separation (SOS) in Hund metals revealed by DMFT+NRG study

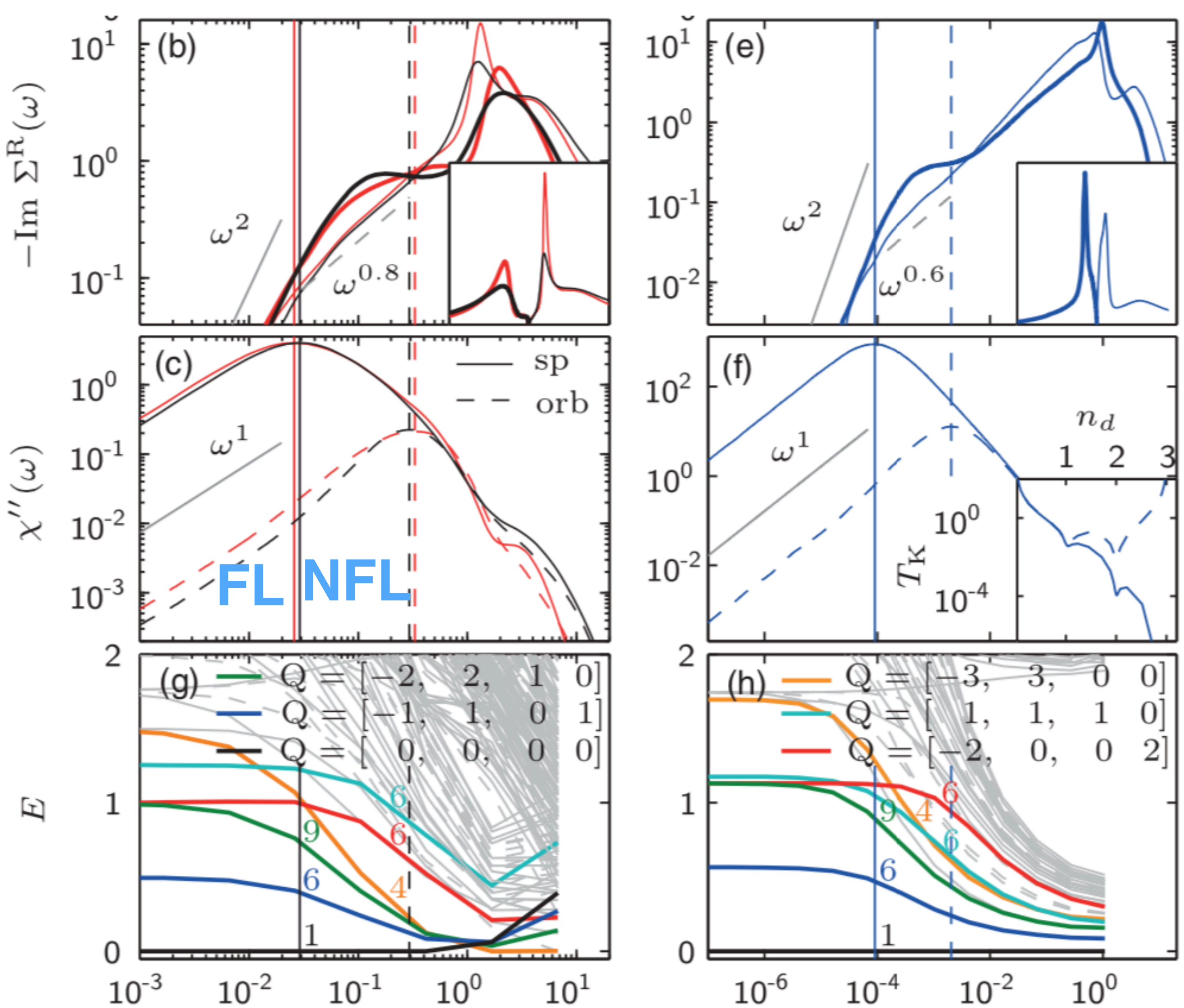
3-orbital Hubbard model   K. M. Stadler *et al*. PRL 115, 136401 (2015), Annals of Physics 405, 365 (2019)

$$\hat{H} = \sum_{i} (-\mu \hat{N}_i + \hat{H}_{\text{int}}[\hat{d}^{\dagger}_{i\nu}]) + \sum_{\langle ij \rangle \nu} t \hat{d}^{\dagger}_{i\nu} \hat{d}_{j\nu}, \qquad \text{(1a)}$$

$$\hat{H}_{\text{int}}[\hat{d}^{\dagger}_{i\nu}] = \frac{3}{4} J \hat{N}_i + \frac{1}{2}\left(U - \frac{1}{2}J\right) \hat{N}_i (\hat{N}_i - 1) - J \hat{\mathbf{S}}_i^2. \qquad \text{(1b)}$$

$\mathrm{U}(1)_{\text{ch}} \times \mathrm{SU}(2)_{\text{sp}} \times \mathrm{SU}(3)_{\text{orb}}$ symmetry

left: with DMFT self-consistency



right: without DMFT self-consistency, one-shot solution of Anderson model

Open questions

- The solution of the (LDA+) DMFT equations is a complex problem, which generically yields a non-zero FL scale

- How to reduce the FL scale to exactly zero ?

- How to characterize the ensuing anomalous NFL behavior above T_{FL} ?

Calculate the global phase diagram of a three channel spin-orbital Kondo (3soK) model using NRG method

Z.P. Yin et al. PRB86, 195141 (2012)
C. Aron and G. Kotliar PRB 91, 041110R (2015)

Schrieffer-Wolff transformation of the three orbital Anderson model

$$H_{\mathrm{K}} = H_{\mathrm{bath}} + H_{\mathrm{int}}$$

$\mathrm{U}(1)_{\mathrm{ch}} \times \mathrm{SU}(2)_{\mathrm{sp}} \times \mathrm{SU}(3)_{\mathrm{orb}}$ symmetry

$$H_{\mathrm{bath}} = \sum_{k,m,\sigma} \epsilon_k \, \psi^\dagger_{km\sigma} \psi_{km\sigma}$$

flat bath, with D=1

$$H_{\mathrm{int}} = J_0 \, S^\alpha \left(\psi^\dagger_{m\sigma} \frac{\sigma^\alpha_{\sigma\sigma'}}{2} \psi_{m\sigma'} \right)$$

spin spin interaction, impurity spin ***S***=1

$$+ K_0 T^a \left(\psi^\dagger_{m\sigma} \frac{\tau^a_{mm'}}{2} \psi_{m'\sigma} \right)$$

orbital orbital interaction, impurity orbital isospin ***T*** in the $\bar{3}$, i.e. (01) representation of SU(3) group [L=1 in SU(2)]

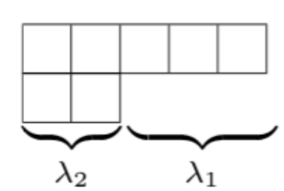
$$+ I_0 S^\alpha T^a \left(\psi^\dagger_{m\sigma} \frac{\sigma^\alpha_{\sigma\sigma'}}{2} \frac{\tau^a_{mm'}}{2} \psi_{m'\sigma'} \right)$$

spin-orbital spin-orbital interaction

J_0, K_0, I_0 are bare Kondo couplings, **positive (AFM)**, **negative (FM)**, $K_0=0.3$ throughout
They will be renormalized to effective values J, K, I during RG process

$$Q \equiv [q, S, (\lambda_1 \lambda_2)]$$

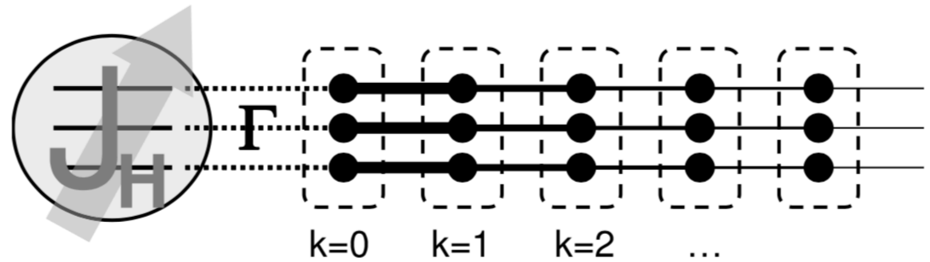
q — total charge of bath relative to half-filling of bath, q=0 for half-filling
S — total spin
$(\lambda_1 \lambda_2)$ — SU(3) representation



$$Q_{\mathrm{imp}} = [0, 1, (01)]$$

impurity site has no charge dynamics

NRG computational details



- The bath is discretized logarithmically and mapped to a semi-infinite "Wilson chain" with exponentially decaying hoppings, and the impurity coupled to chain site *k=0*.
- The chain is diagonalized iteratively (adding one more site each iteration), while discarding high-energy states, thereby zooming in on low-energy properties: the finite-size level spacing of a chain ending at site $k \geq 0$ is of order $\omega_k \propto \Lambda^{-k/2}$
- Here $\Lambda > 1$ is a discretization parameter chosen to be 4 in this work.
- The RG flow can be visualized by plotting the rescaled low-lying NRG eigenlevel spectra, $E = (\mathcal{E} - \mathcal{E}_{\mathrm{ref}})/\omega_k$ vs. ω_k , as increasing even or odd k

The imaginary part of dynamical susceptibilities at the impurity site or zeroth bath site at $T=10^{-16}$

$$\chi_{\mathrm{sp}}^{\mathrm{imp,bath}}(\omega) = -\frac{1}{3\pi}\mathrm{Im}\sum_{\alpha}\langle S^{\alpha}||S^{\alpha}\rangle_{\omega},$$

$$\chi_{\mathrm{orb}}^{\mathrm{imp,bath}}(\omega) = -\frac{1}{8\pi}\mathrm{Im}\sum_{a}\langle T^{a}||T^{a}\rangle_{\omega},$$

$$\chi_{\text{sp-orb}}^{\mathrm{imp,bath}}(\omega) = -\frac{1}{24\pi}\mathrm{Im}\sum_{\alpha,a}\langle S^{\alpha}T^{a}||S^{\alpha}T^{a}\rangle_{\omega},$$

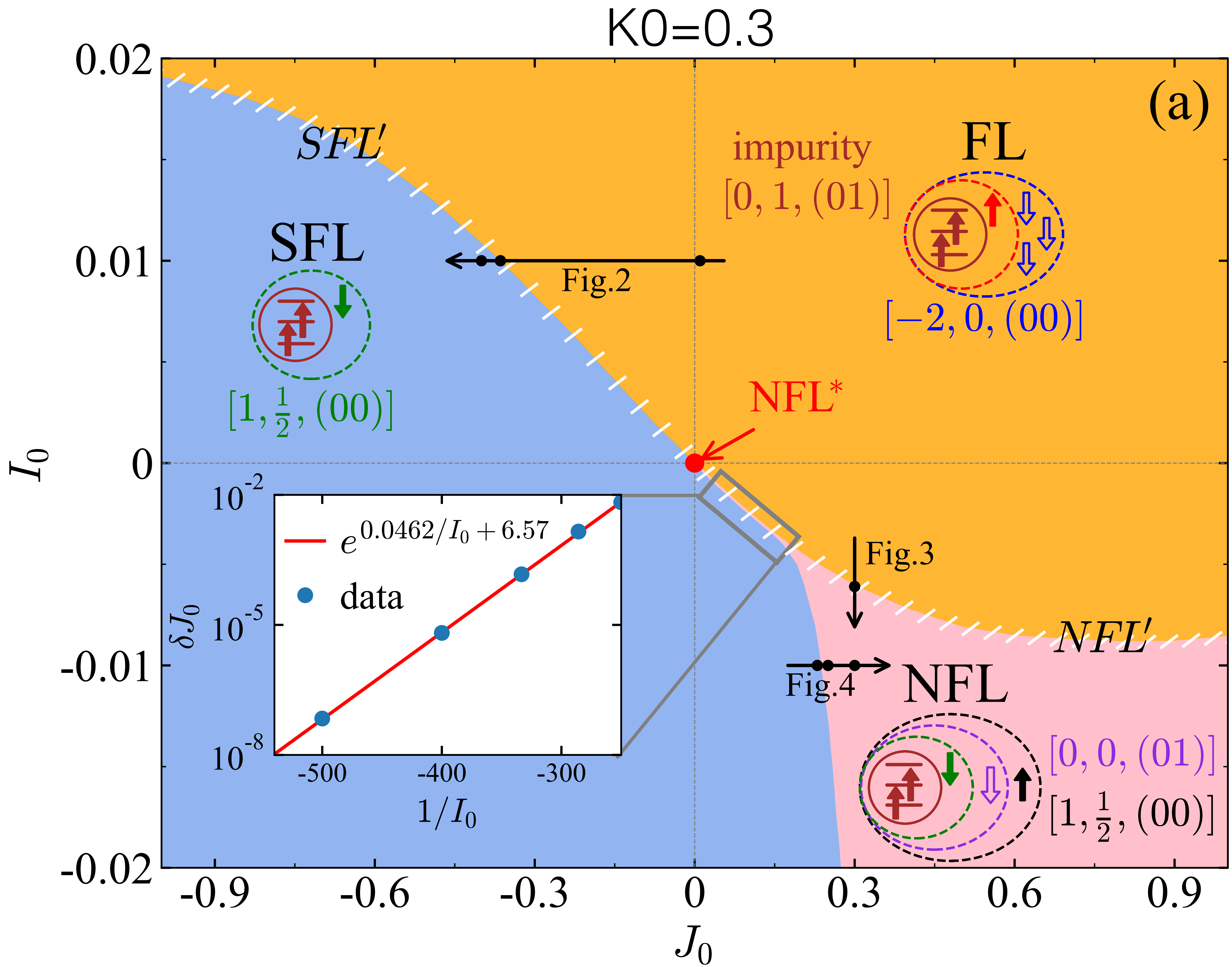
QSpace tensor library

Non-abelian symmetries in tensor networks: A quantum symmetry space approach

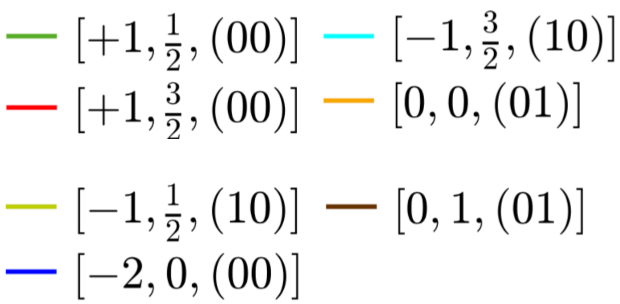
Andreas Weichselbaum

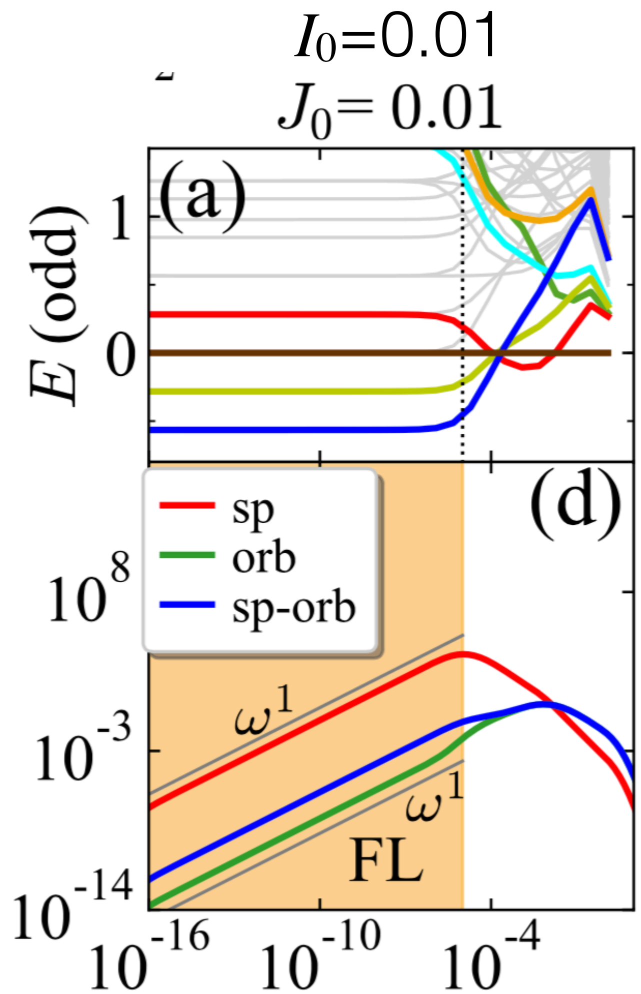
Annals of Physics 327 (2012) 2972–3047

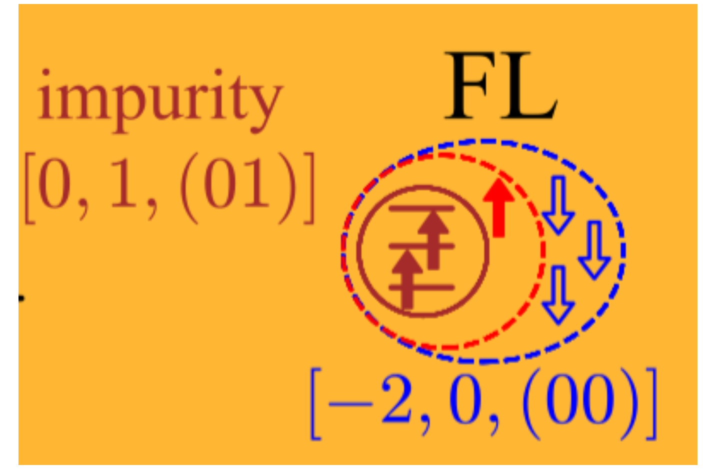
Global phase diagram vs. J_0 and I_0



FL Phase





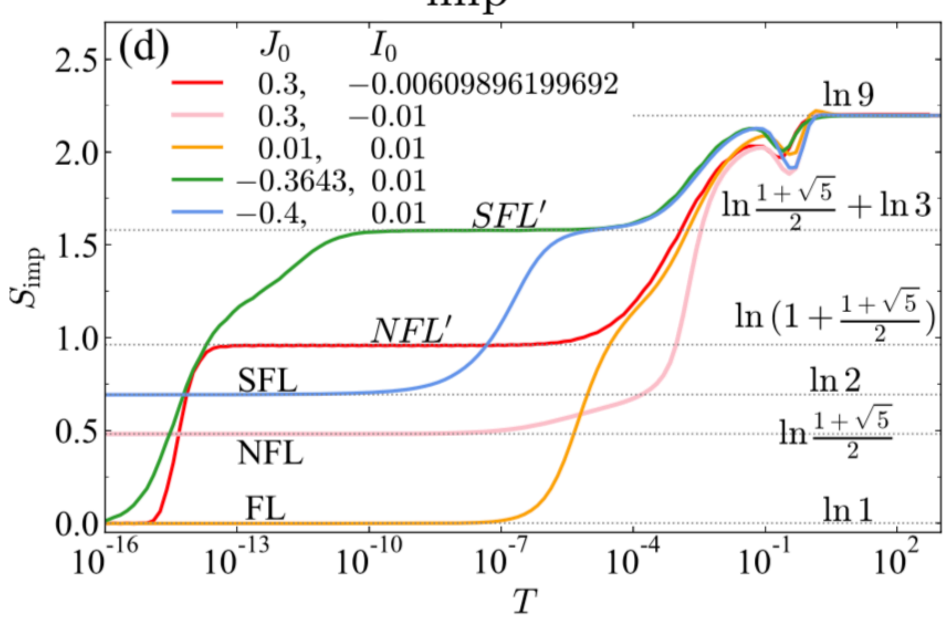


gs: [-2, 0, (00)]

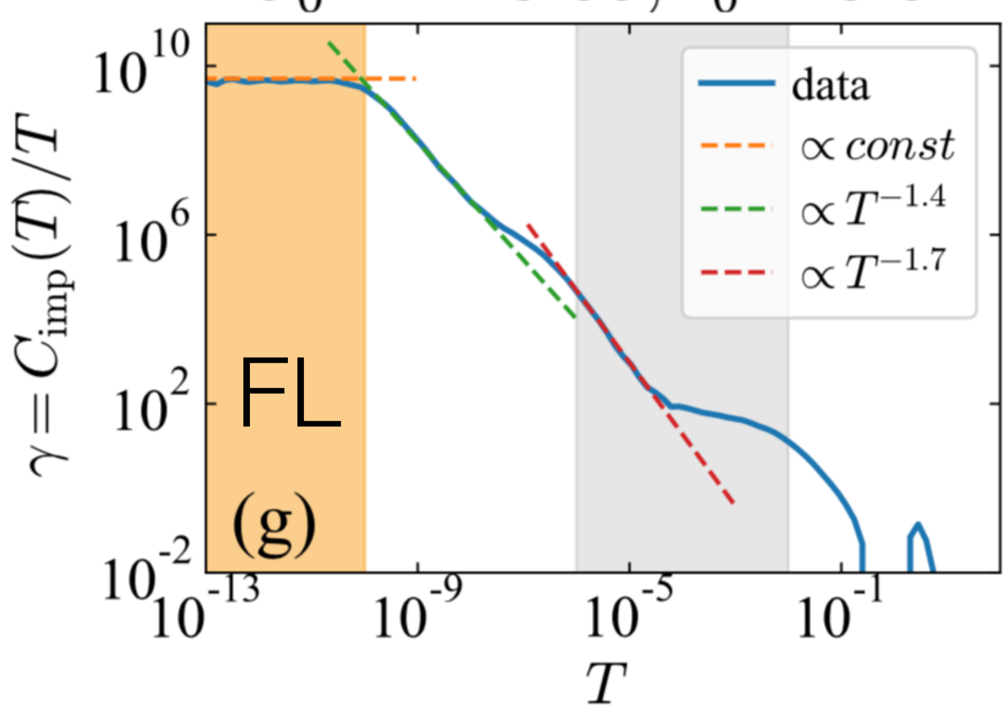
$$[0, 1, (01)] \otimes [+1, \tfrac{1}{2}, (10)] \rightarrow [+1, \tfrac{3}{2}, (00)] \oplus [+1, \tfrac{1}{2}, (00)]$$

$$[+1, \tfrac{3}{2}, (00)] \otimes [-3, \tfrac{3}{2}, (00)] \rightarrow [-2, 0, (00)]$$

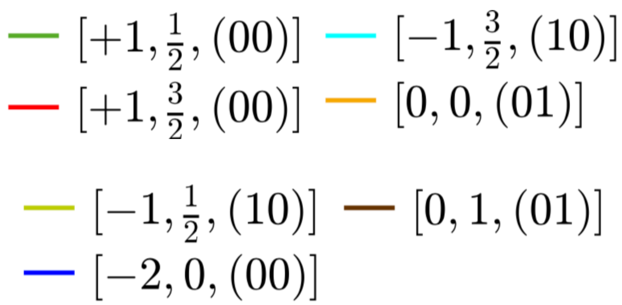
$S_{\mathrm{imp}}^{\mathrm{FL}} = \ln 1$



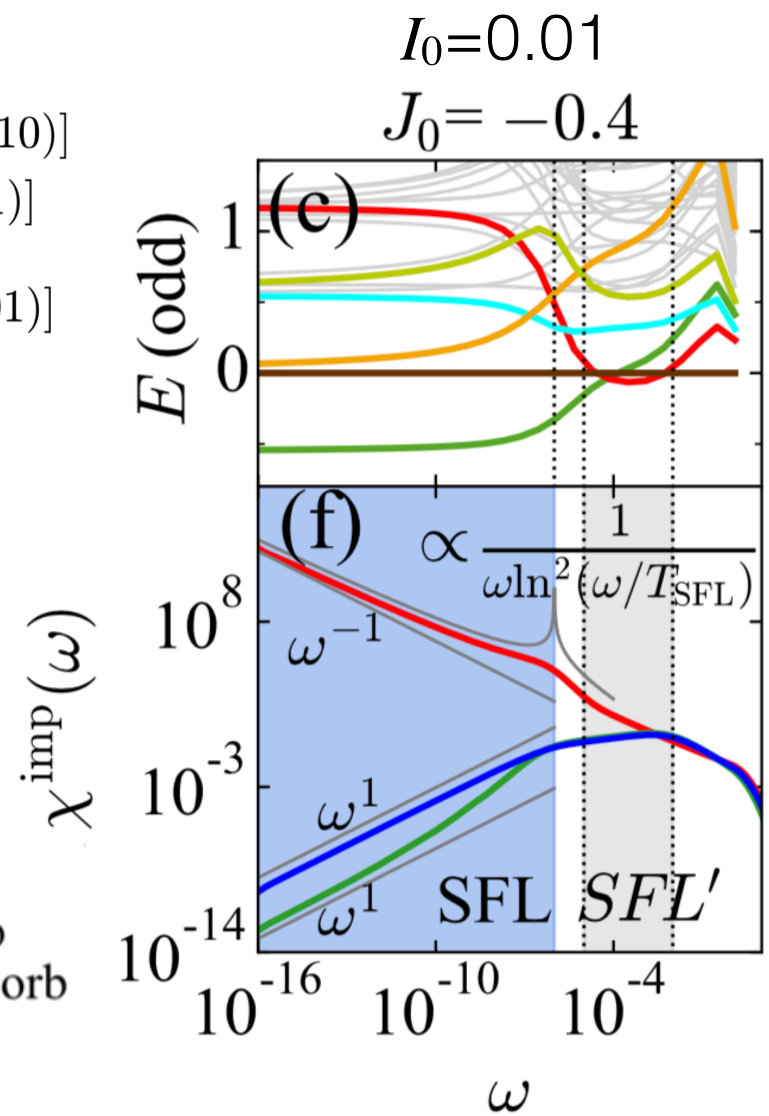
$J_0 = -0.35, I_0 = 0.01$

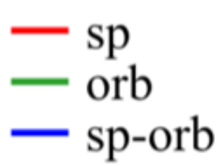


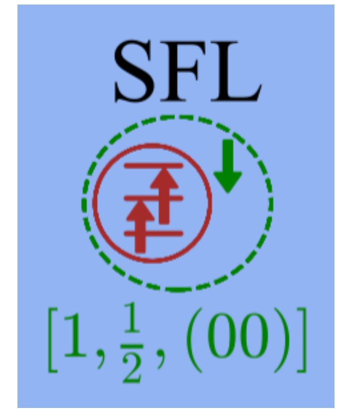
Singular-Fermi-liquid (SFL) Phase



gs: [+1, 1/2, (00)]







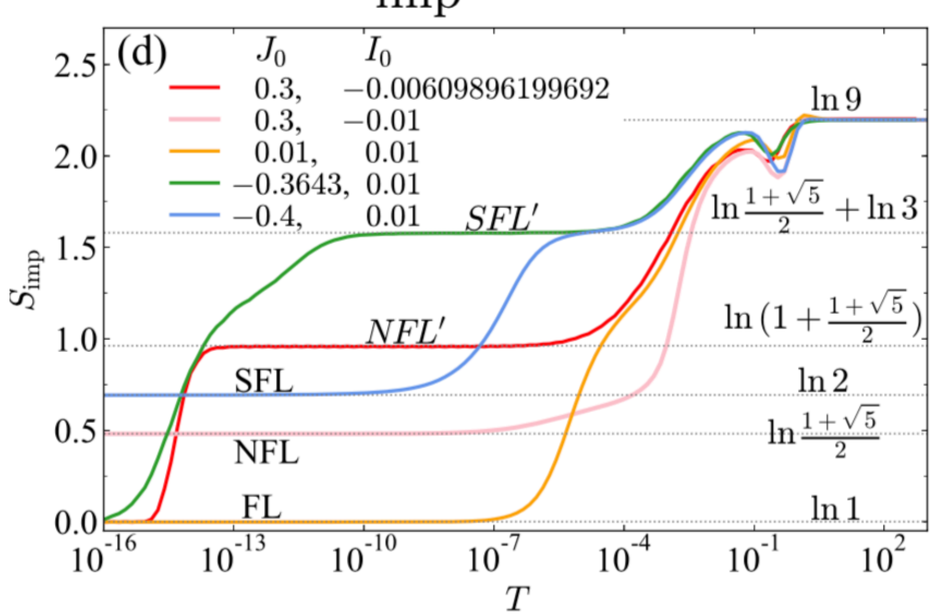
underscreened spin
fully screened orbitals

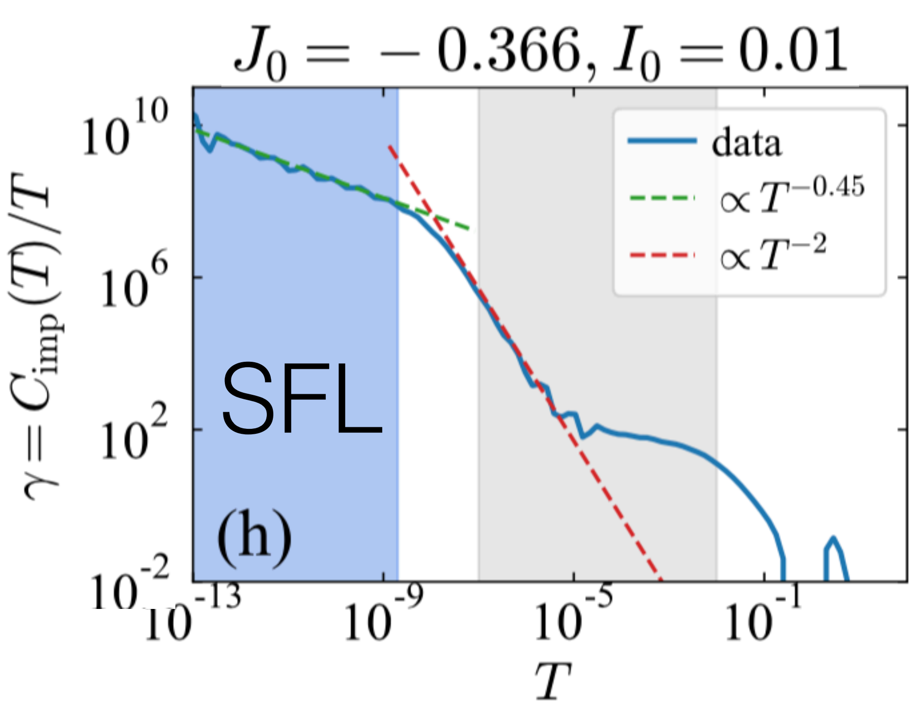
$$[0,1,(01)] \otimes [+1, \tfrac{1}{2}, (10)] \rightarrow [+1, \tfrac{3}{2}, (00)] \oplus [+1, \tfrac{1}{2}, (00)]$$

Tensor product of FL spectrum and impurity S=1/2, with residual effective FM Kondo couplings in spin sector at intermediate energies, showing singular behavior

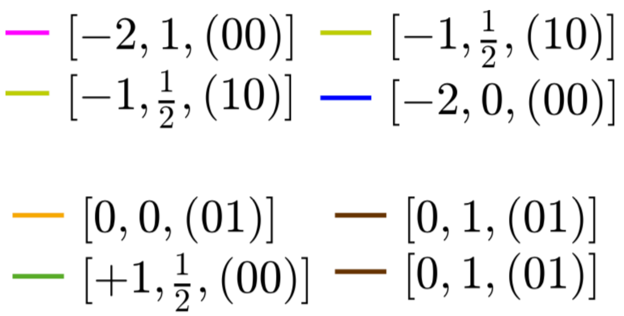
$$\chi_{\rm sp}^{\rm imp} \sim 1/(\omega \ln^2(\omega/T_{\rm SFL}))$$

$$S_{\rm imp}^{\rm SFL} = \ln 2$$





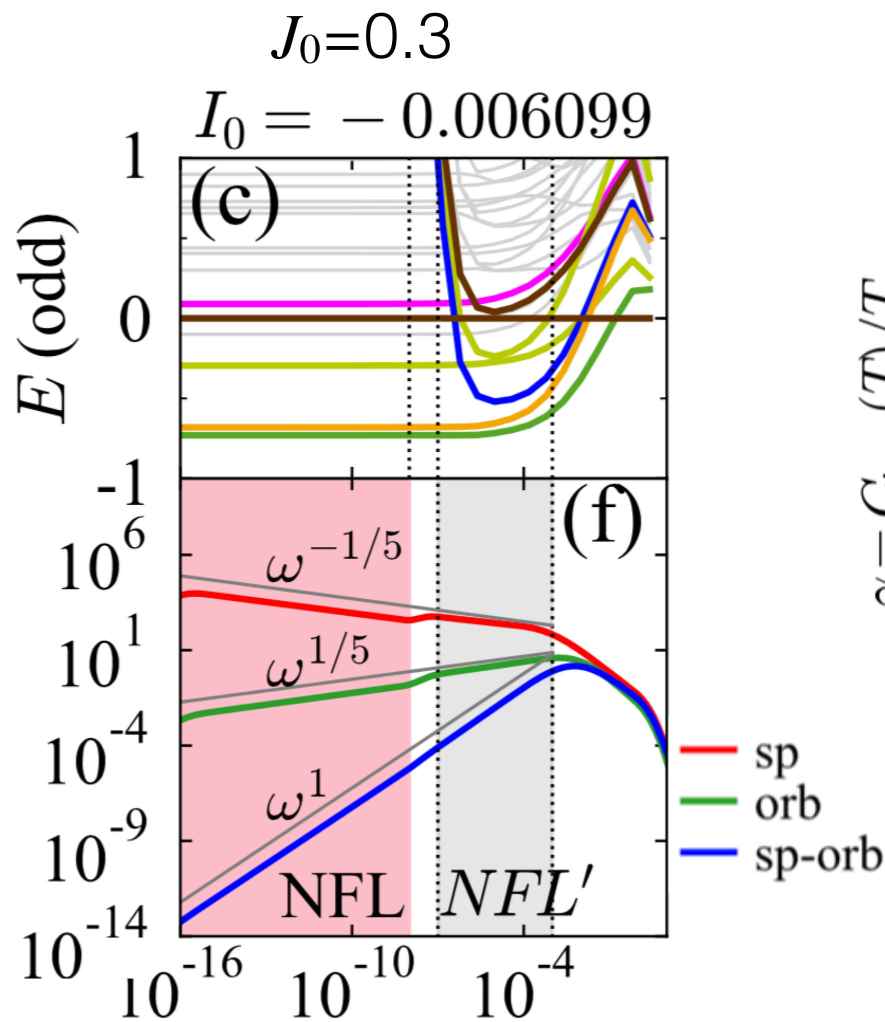
NFL Phase

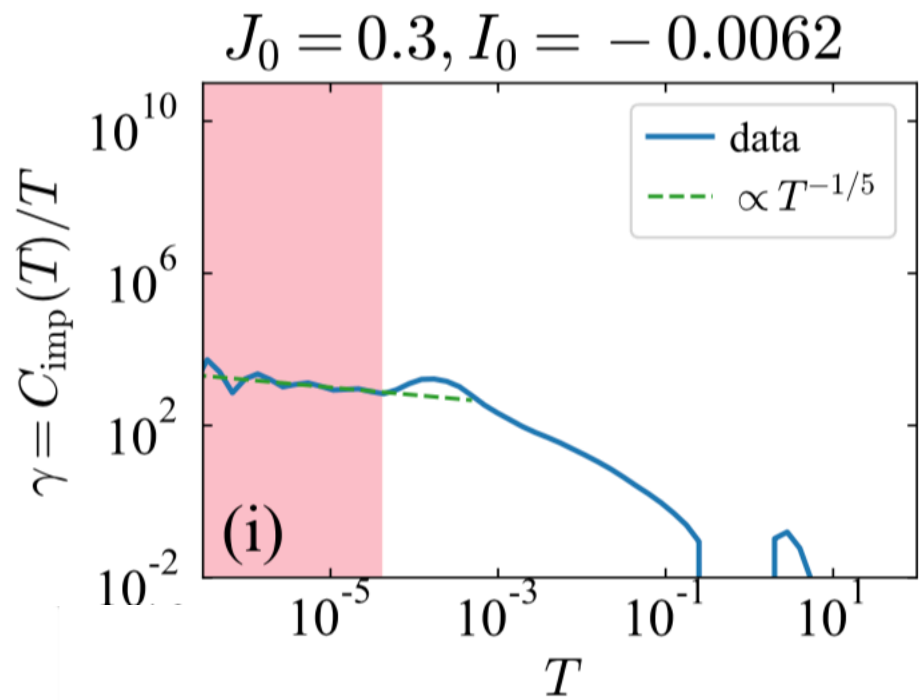


$[0, 0, (01)]$

gs: $[+1, \frac{1}{2}, (00)]$

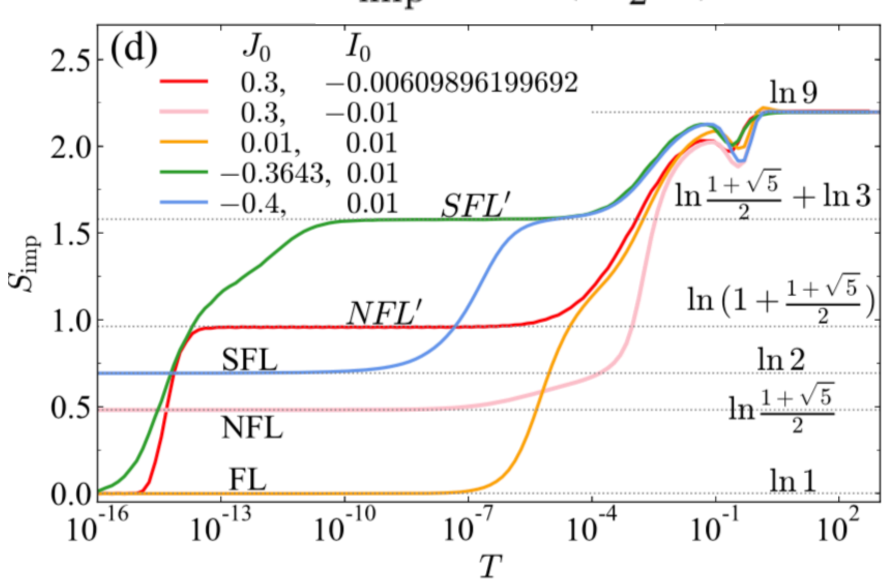
lowest and
close in energy





Olivier Parcollet *et al.*
PRB 58,3794 (1998)

$S_{\mathrm{imp}}^{\mathrm{NFL}} = \ln(\frac{1+\sqrt{5}}{2})$



general overscreened SU(N)$_K$ Kondo model

$$S_{\mathrm{imp}} = \ln \prod_{n=1}^{Q} \frac{\sin[\pi(N+1-n)/(N+K)]}{\sin[\pi n/(N+K)]}.$$

- N=3, K=2, Q=2, SU(3)$_2$ orbital-(01) Kondo model with overscreened fixed point
- N=2, K=3, Q=1, SU(2)$_3$ spin-1/2 Kondo model with overscreened fixed point

NFL Phase: Conformal Field Theory (CFT) arguments

Elias *et al*. arXiv:1908.04362

Ian Affleck, Nuclear Physics B 336, 517 (1990)
Ian Affleck and Andreas Ludwig, Nuclear Physics B 352, 849 (1991)
Ian Affleck and Andreas Ludwig, Nuclear Physics B 360, 641 (1991)
D.L. Cox and A. Zawadowski, Advances in Physics 47, 599 (1998)

$$H_{\text{bath}} = a_1 \mathcal{J}^\dagger_{\text{ch}} \mathcal{J}_{\text{ch}} + a_2 \mathcal{J}^\dagger_{\text{sp}} \mathcal{J}_{\text{sp}} + a_3 \mathcal{J}^\dagger_{\text{orb}} \mathcal{J}_{\text{orb}}$$

$$\mathrm{U}(1)_{\text{ch}} \times \mathrm{SU}(2)_{\text{sp}} \times \mathrm{SU}(3)_{\text{orb}} \text{ symmetry}$$

low-energy fixed point Hamiltonian

$$Q \equiv [q, S, (\lambda_1 \lambda_2)]$$

$$\tilde{\mathcal{J}}_{\text{sp}} = \mathcal{J}_{\text{sp}} + S_{\text{imp}} \qquad \tilde{\mathcal{J}}_{\text{orb}} = \mathcal{J}_{\text{orb}} + T_{\text{imp}}$$

$$H = H_{\text{bath}} + H_{\text{int}} = a_1 \tilde{\mathcal{J}}^\dagger_{\text{ch}} \tilde{\mathcal{J}}_{\text{ch}} + a_2 \tilde{\mathcal{J}}^\dagger_{\text{sp}} \tilde{\mathcal{J}}_{\text{sp}} + a_3 \tilde{\mathcal{J}}^\dagger_{\text{orb}} \tilde{\mathcal{J}}_{\text{orb}}$$

$$E(Q;\delta q) = \tfrac{1}{12}(q+\delta q)^2 + \tfrac{1}{5}\kappa_2(S) + \tfrac{1}{5}\kappa_3(\lambda_1,\lambda_2),$$
$$\kappa_2(S) = S(S+1),$$
$$\kappa_3(\lambda_1,\lambda_2) = \tfrac{1}{3}(\lambda_1^2+\lambda_2^2+\lambda_1\lambda_2+3\lambda_1+3\lambda_2),$$

Single-Fusion | **Double-Fusion**

free Fermion spectrum | effective impurity multiplet | NRG fixed-point spectrum | dual representation of effective impurity multiplet | CFT boundary operators $\hat{O}$

$$Q \equiv [q, S, (\lambda_1\lambda_2)] \quad \otimes \quad Q^{\text{eff}}_{\text{imp}} \quad \longrightarrow \quad Q' \equiv [q', S', (\lambda'_1\lambda'_2)] \quad \otimes \quad \bar{Q}^{\text{eff}}_{\text{imp}} \quad \longrightarrow \quad Q'' \equiv [q'', S'', (\lambda''_1\lambda''_2)]$$

$$E(Q;0) \qquad\qquad E(Q';\delta q) \qquad\qquad \Delta = E(Q'';\delta q)$$

Scheme I, SU(2)₃ fusion in spin sector with $Q^{\text{eff}}_{\text{imp}} = [+1, \tfrac{1}{2}, (00)]$ $\quad \bar{Q}^{\text{eff}}_{\text{imp}} = [-1, \tfrac{1}{2}, (00)]$

Scheme II, SU(3)₂ fusion in orbital sector with $Q^{\text{eff}}_{\text{imp}} = [0, 0, (01)]$ $\quad \bar{Q}^{\text{eff}}_{\text{imp}} = [0, 0, (10)]$

CFT results of NFL phase: SU(2)$_3$ fusion in spin sector

J0=0.3, I0=-0.01

free fermions					single fusion, with $Q^{\rm eff}_{\rm imp}=[+1,\frac{1}{2},(00)]$						NRG	double fusion, with $\bar{Q}^{\rm eff}_{\rm imp}=[-1,\frac{1}{2},(00)]$				
q	S	$(\lambda_1\lambda_2)$	d	E	q'	S'	$(\lambda'_1\lambda'_2)$	d'	E'	$\delta E'$	$E_{\rm NRG}$	q''	S''	$(\lambda''_1\lambda''_2)$	Δ	$\hat{O}$
0	0	(00)	1	0	+1	$\frac{1}{2}$	(00)	2	$\frac{7+5\delta q}{30}$	0	0	0	0	(00)	0	$\mathbb{1}$
												0	1	(00)	$\frac{2}{5}$ $(=\Delta_{\rm sp})$	$\Phi_{\rm sp}$
+1	$\frac{1}{2}$	(10)	6	$\frac{1}{2}$	+2	0	(10)	3	$\frac{9+5\delta q}{15}$	$\frac{11+5\delta q}{30}$ (0.374)	0.369	+1	$\frac{1}{2}$	(10)	$\frac{3+\delta q}{6}$	
					+2	1	(10)	9	$\frac{3+\delta q}{3}$	$\frac{23+5\delta q}{30}$ (0.774)	0.809	+1	$\frac{1}{2}$	(10)	$\frac{3+\delta q}{6}$	
												+1	$\frac{3}{2}$	(10)	$\frac{33+5\delta q}{30}$	
−1	$\frac{1}{2}$	(01)	6	$\frac{1}{2}$	0	0	(01)	3	$\frac{4}{15}$	$\frac{1-5\delta q}{30}$ (0.026)	0.026	−1	$\frac{1}{2}$	(01)	$\frac{3-\delta q}{6}$	
					0	1	(01)	9	$\frac{2}{3}$	$\frac{13-5\delta q}{30}$ (0.426)	0.422	−1	$\frac{1}{2}$	(01)	$\frac{3-\delta q}{6}$	
												−1	$\frac{3}{2}$	(01)	$\frac{33-5\delta q}{30}$	
0	1	(11)	24	1	+1	$\frac{1}{2}$	(11)	16	$\frac{5+\delta q}{6}$	$\frac{3}{5}$ (0.600)	0.600	0	0	(11)	$\frac{3}{5}$ $(=\Delta_{\rm orb})$	$\Phi_{\rm orb}$
												0	1	(11)	1 $(=\Delta_{\text{sp-orb}})$	$\Phi_{\text{sp-orb}}$
					+1	$\frac{3}{2}$	(11)	32	$\frac{43+5\delta q}{30}$	$\frac{6}{5}$ (1.200)	1.223	0	1	(11)	1	
+2	0	(20)	6	1	+3	$\frac{1}{2}$	(20)	12	$\frac{47+15\delta q}{30}$	$\frac{4+\delta q}{3}$ (1.348)	1.432	+2	0	(20)	$\frac{3+\delta q}{3}$	
												+2	1	(20)	$\frac{21+5\delta q}{15}$	
−2	0	(02)	6	1	−1	$\frac{1}{2}$	(02)	12	$\frac{27-5\delta q}{30}$	$\frac{2-\delta q}{3}$ (0.652)	0.655	−2	0	(02)	$\frac{3-\delta q}{3}$	
												−2	1	(02)	$\frac{21-5\delta q}{15}$	
+2	1	(01)	9	1	+3	$\frac{1}{2}$	(01)	6	$\frac{7+3\delta q}{6}$	$\frac{14+5\delta q}{15}$ (0.948)	0.954	+2	0	(01)	$\frac{9+5\delta q}{15}$	
												+2	1	(01)	$\frac{3+\delta q}{3}$	
					+3	$\frac{3}{2}$	(01)	12	$\frac{53+15\delta q}{30}$	$\frac{23+5\delta q}{15}$ (1.548)	1.599	+2	1	(01)	$\frac{3+\delta q}{3}$	
−2	1	(10)	9	1	−1	$\frac{1}{2}$	(10)	6	$\frac{3-\delta q}{6}$	$\frac{4-5\delta q}{15}$ (0.252)	0.248	−2	0	(10)	$\frac{9-5\delta q}{15}$	
												−2	1	(10)	$\frac{3-\delta q}{3}$	
					−1	$\frac{3}{2}$	(10)	12	$\frac{33-5\delta q}{30}$	$\frac{13-5\delta q}{15}$ (0.852)	0.844	−2	1	(10)	$\frac{3-\delta q}{3}$	

$E_{\rm NRG}$ have been shifted and rescaled such that the ground state is zero and the values of $E_{\rm NRG}$ and $\delta E'$ match for the multiplet $[+1,\frac{1}{2},(11)]$

δq is then determined by matching $E_{\rm NRG}$ and $\delta E'$ for the multiplet $[0,0,(01)]$

$$\delta q = 0.0433$$

$$E([0,0,(01)]) - E([+1,\tfrac{1}{2},(00)]) = \frac{1-5\delta q}{30}$$

power-laws from CFT

$$\chi^{\rm imp}_{\rm sp} \sim \omega^{2\Delta_{\rm sp}-1} = \omega^{-1/5},$$
$$\chi^{\rm imp}_{\rm orb} \sim \omega^{2\Delta_{\rm orb}-1} = \omega^{1/5},$$
$$\chi^{\rm imp}_{\text{sp-orb}} \sim \omega^{2\Delta_{\text{sp-orb}}-1} = \omega^{1}.$$

specific heat coefficient from CFT

$$\gamma(T) \propto T^{2\Delta_{\rm sp}-1} = T^{-1/5}$$

Olivier Parcollet *et al.*
PRB 58,3794 (1998)

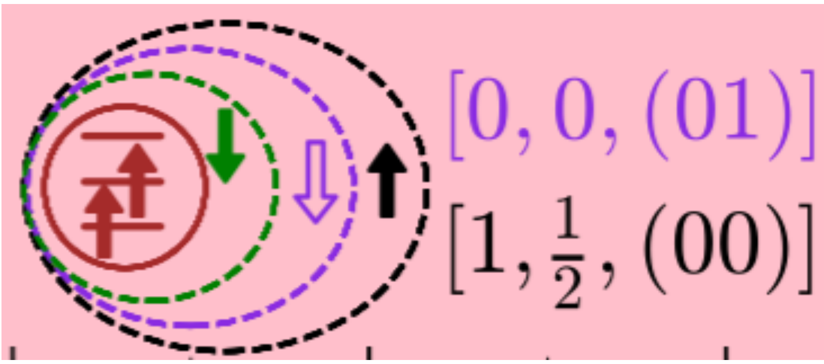
CFT results of NFL phase: $SU(3)_2$ fusion in orbital sector

J0=0.3, I0=-0.01

free fermions					single fusion, with $Q^{\text{eff}}_{\text{imp}} = [0, 0, (01)]$						NRG	double fusion, with $\bar{Q}^{\text{eff}}_{\text{imp}} = [0, 0, (10)]$				
q	S	$(\lambda_1\lambda_2)$	d	E	q'	S'	$(\lambda'_1\lambda'_2)$	d'	E'	$\delta E'$	E_{NRG}	q''	S''	$(\lambda''_1\lambda''_2)$	Δ	$\hat{O}$
0	0	(00)	1	0	0	0	(01)	3	$\frac{4}{15}$	$\frac{1-5\delta q}{30}$ (0.026)	0.026	0	0	(00)	0	$\mathbb{1}$
												0	0	(11)	$\frac{3}{5}$ $(=\Delta_{\text{orb}})$	Φ_{orb}
+1	$\frac{1}{2}$	(10)	6	$\frac{1}{2}$	+1	$\frac{1}{2}$	(00)	2	$\frac{7+5\delta q}{30}$	0	0	+1	$\frac{1}{2}$	(10)	$\frac{3+\delta q}{6}$	
					+1	$\frac{1}{2}$	(11)	16	$\frac{5+\delta q}{6}$	$\frac{3}{5}$ (0.600)	0.600	+1	$\frac{1}{2}$	(10)	$\frac{3+\delta q}{6}$	
												+1	$\frac{1}{2}$	(02)	$\frac{27+5\delta q}{30}$	
−1	$\frac{1}{2}$	(01)	6	$\frac{1}{2}$	−1	$\frac{1}{2}$	(10)	6	$\frac{3-\delta q}{6}$	$\frac{4-5\delta q}{15}$ (0.252)	0.248	−1	$\frac{1}{2}$	(01)	$\frac{3-\delta q}{6}$	
												−1	$\frac{1}{2}$	(20)	$\frac{27-5\delta q}{30}$	
					−1	$\frac{1}{2}$	(02)	12	$\frac{27-5\delta q}{30}$	$\frac{2-\delta q}{3}$ (0.652)	0.655	−1	$\frac{1}{2}$	(01)	$\frac{3-\delta q}{6}$	
0	1	(11)	24	1	0	1	(01)	9	$\frac{2}{3}$	$\frac{13-5\delta q}{30}$ (0.426)	0.422	0	1	(00)	$\frac{2}{5}$ $(=\Delta_{\text{sp}})$	Φ_{sp}
												0	1	(11)	1 $(=\Delta_{\text{sp-orb}})$	$\Phi_{\text{sp-orb}}$
					0	1	(20)	18	$\frac{16}{15}$	$\frac{5-\delta q}{6}$ (0.826)	0.825	0	1	(11)	1	
+2	0	(20)	6	1	+2	0	(10)	3	$\frac{9+5\delta q}{15}$	$\frac{11+5\delta q}{30}$ (0.374)	0.369	+2	0	(01)	$\frac{9+5\delta q}{15}$	
												+2	0	(20)	$\frac{3+\delta q}{3}$	
−2	0	(02)	6	1	−2	0	(11)	8	$\frac{14-5\delta q}{15}$	$\frac{7-5\delta q}{10}$ (0.678)	0.673	−2	0	(10)	$\frac{9-5\delta q}{15}$	
												−2	0	(02)	$\frac{3-\delta q}{3}$	
+2	1	(01)	9	1	+2	1	(10)	9	$\frac{3+\delta q}{3}$	$\frac{23+5\delta q}{30}$ (0.774)	0.809	+2	1	(01)	$\frac{3+\delta q}{3}$	
												+2	1	(20)	$\frac{21+5\delta q}{15}$	
					+2	1	(02)	18	$\frac{21+5\delta q}{15}$	$\frac{7+\delta q}{6}$ (1.174)	1.180	+2	1	(01)	$\frac{3+\delta q}{3}$	
−2	1	(10)	9	1	−2	1	(00)	3	$\frac{11-5\delta q}{15}$	$\frac{1-\delta q}{2}$ (0.478)	0.470	−2	1	(10)	$\frac{3-\delta q}{3}$	
					−2	1	(11)	24	$\frac{4-\delta q}{3}$	$\frac{11-5\delta q}{10}$ (1.078)	1.090	−2	1	(10)	$\frac{3-\delta q}{3}$	
												−2	1	(02)	$\frac{21-5\delta q}{15}$	

An effective $SU(2)_3$ spin Kondo model and an effective $SU(3)_2$ orbital Kondo model with overscreened fixed points are equivalent description of this NFL fixed point

Picture of NFL Phase: alternating overscreenings in spin and orbital sector



$$[0, 1, (01)] \otimes [+1, \tfrac{1}{2}, (10)] \to [+1, \tfrac{3}{2}, (00)] \oplus [+1, \tfrac{1}{2}, (00)]$$

contrast to FL, $[+1, \frac{1}{2}, (00)]$ has lower energy due to strong FM spin-orbital Kondo coupling I_0

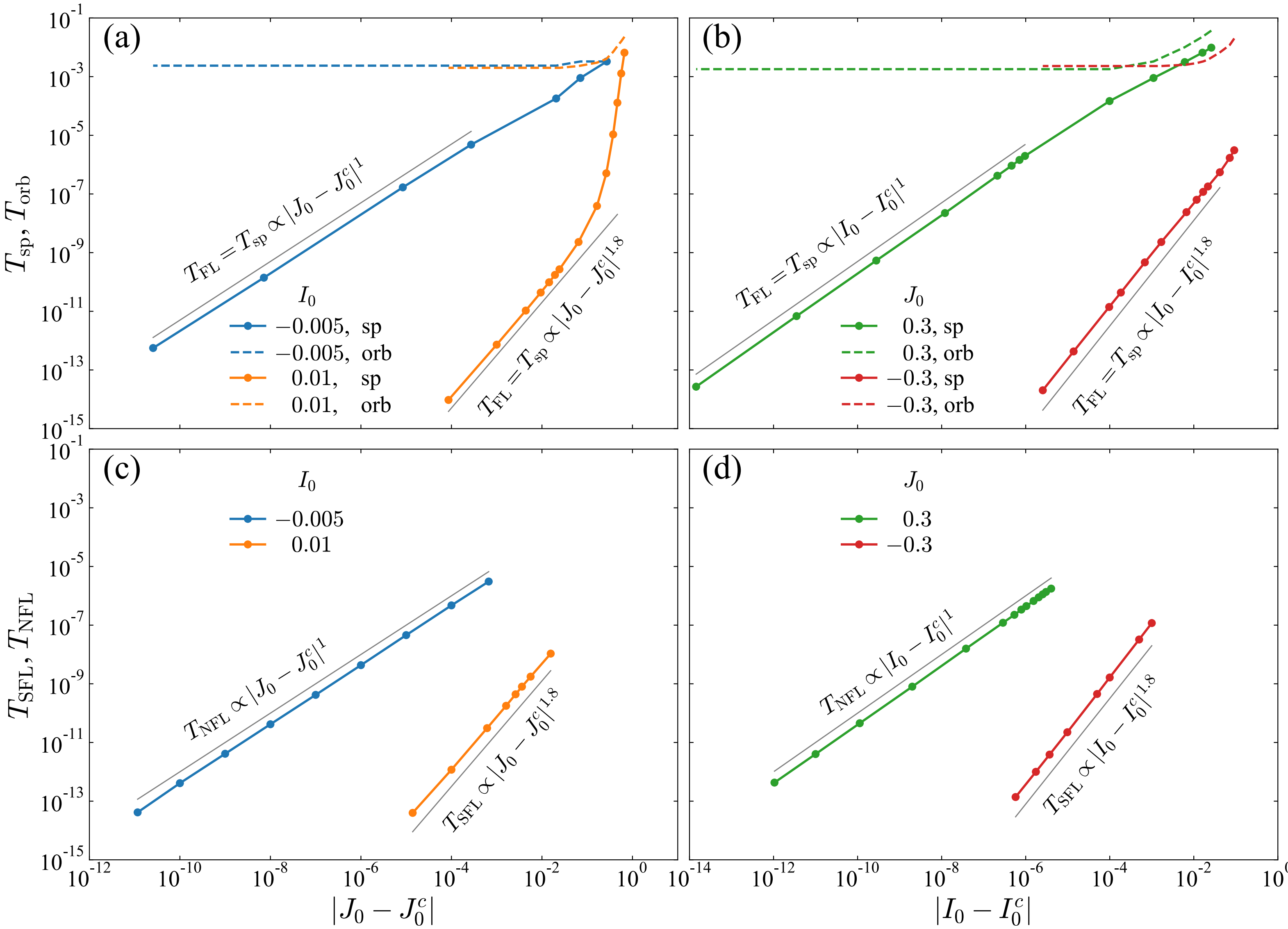
A: bind one hole $\quad [+1, \frac{1}{2}, (00)] \otimes [-1, \frac{1}{2}, (01)] \to [0, 0, (01)]$

B: bind one electron $\quad [0, 0, (01)] \otimes [+1, \frac{1}{2}, (10)] \to [+1, \frac{1}{2}, (00)]$

along the Wilson chain with odd-length

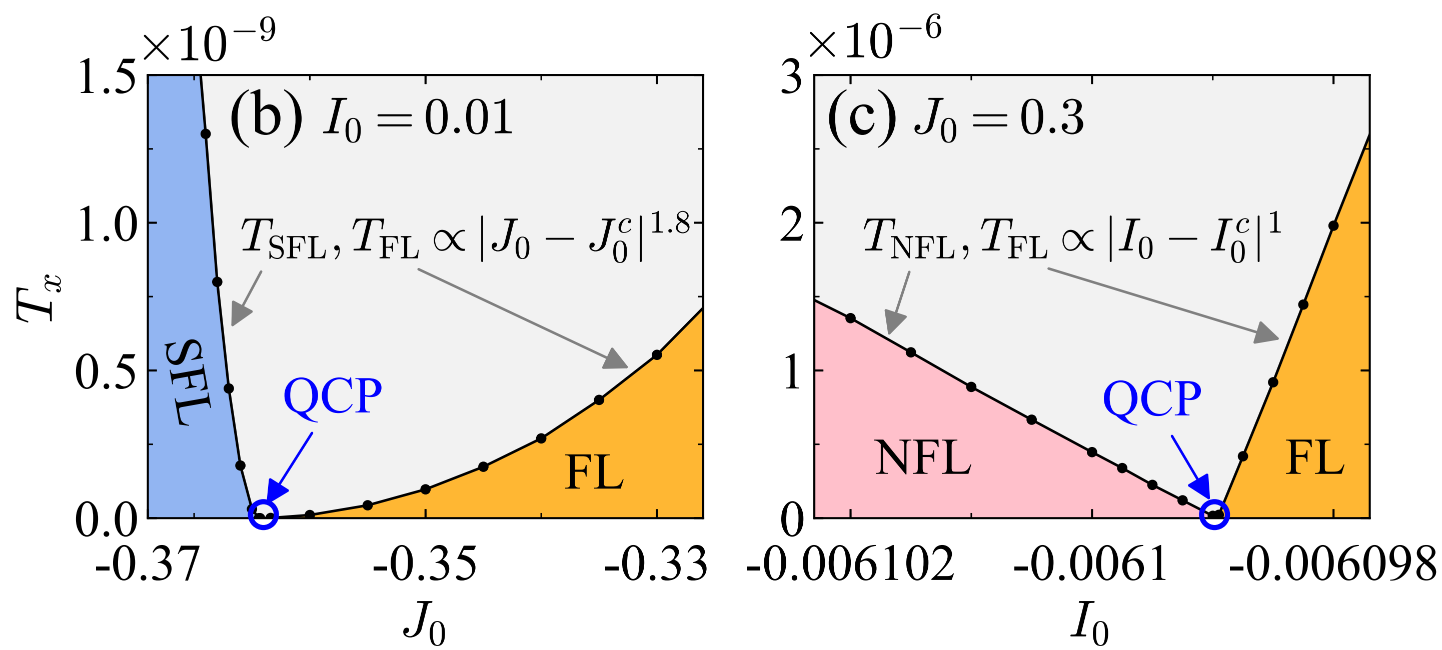
2k-1		2k+1		2k+3		2k+5	
$[0, 0, (01)]$	B	$[0, 0, (01)]$	B	$[0, 0, (01)]$	B	$[0, 0, (01)]$	B .....
$[+1, \frac{1}{2}, (00)]$	A	$[+1, \frac{1}{2}, (00)]$	A	$[+1, \frac{1}{2}, (00)]$	A	$[+1, \frac{1}{2}, (00)]$	A .....

Phase transition:
Suppression of FL scale and Quantum Critical Points (QCPs)



$$T_{\rm FL} = T_{\rm sp}$$

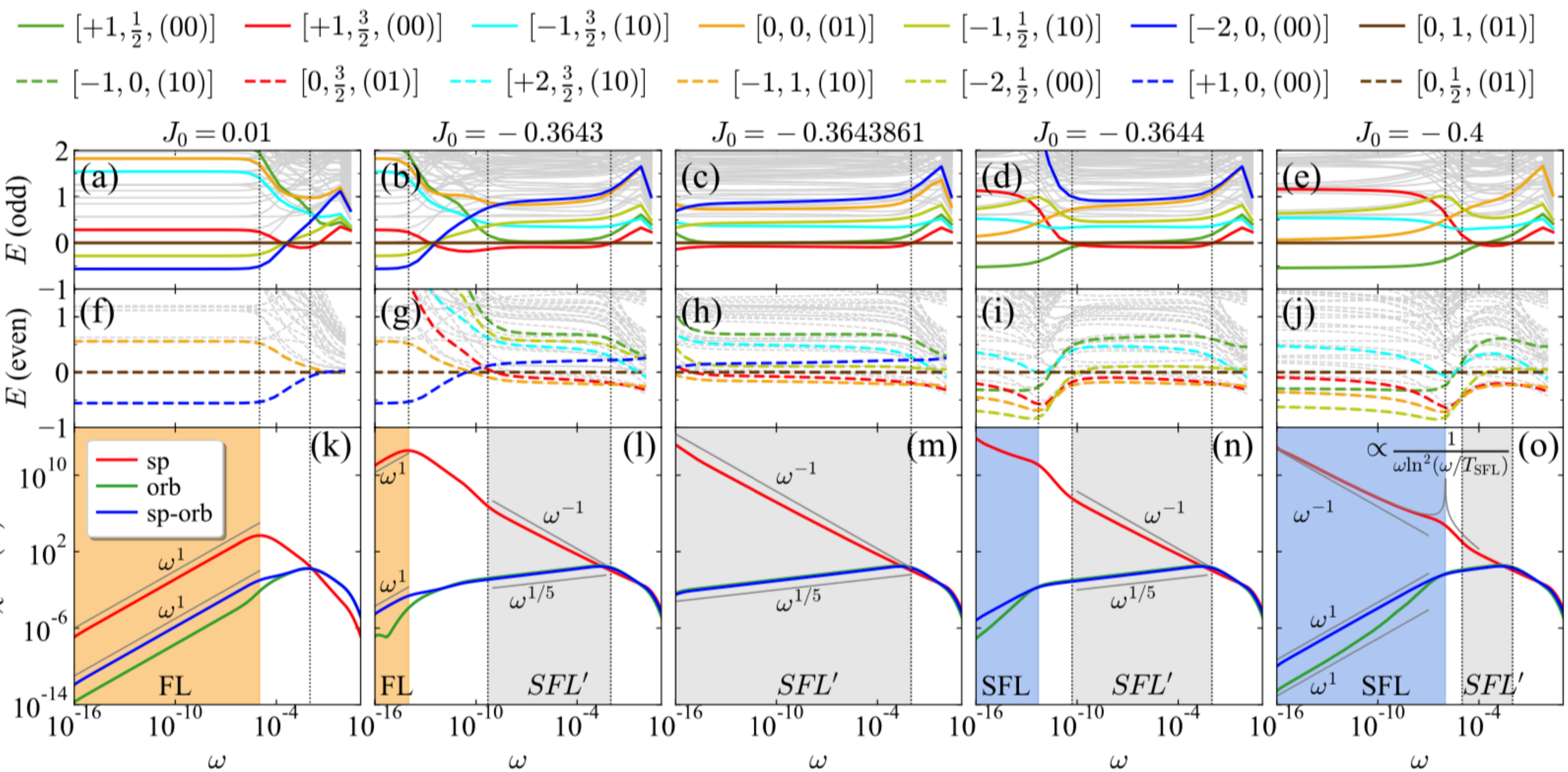
$$T_{\rm FL/SFL} \propto |J_0 - J_0^c|^{1.8},\ |I_0 - I_0^c|^{1.8},$$
$$T_{\rm FL/NFL} \propto |J_0 - J_0^c|^{1},\ \ |I_0 - I_0^c|^{1}.$$



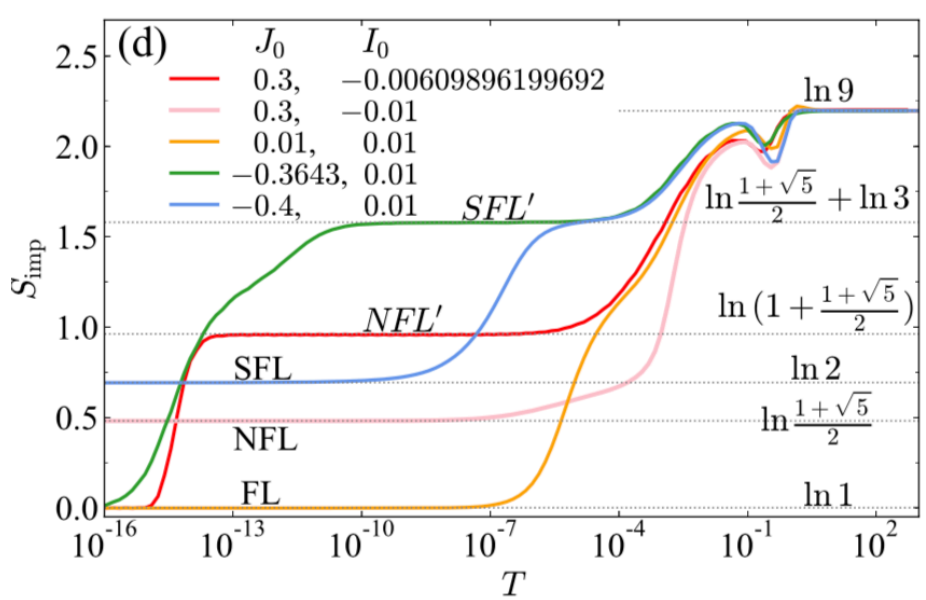
T_{FL} can become arbitrarily small close to these two QCPs

FL to SFL transition, crossover regime *SFL'* at intermediate energies

I_0=0.01



$$S_{\mathrm{imp}}^{SFL'} = \ln\frac{1+\sqrt{5}}{2} + \ln 3$$



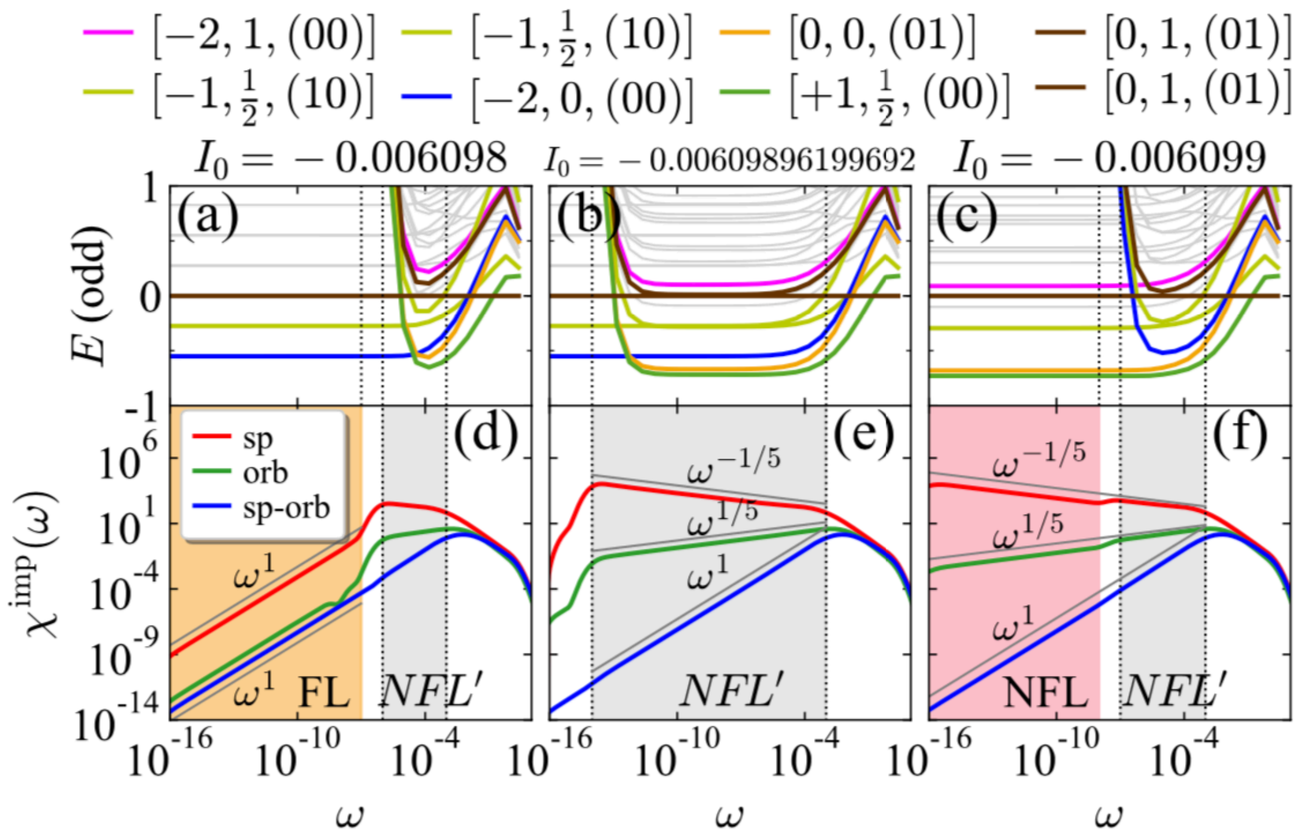
SU(3)$_2$ fixed point with overscreened orbitals and fluctuating spin moment of 1

Alen Horvat et al. arXiv:1907.07100

$$I_0 = 0 \quad J_0 \rightarrow 0^+$$

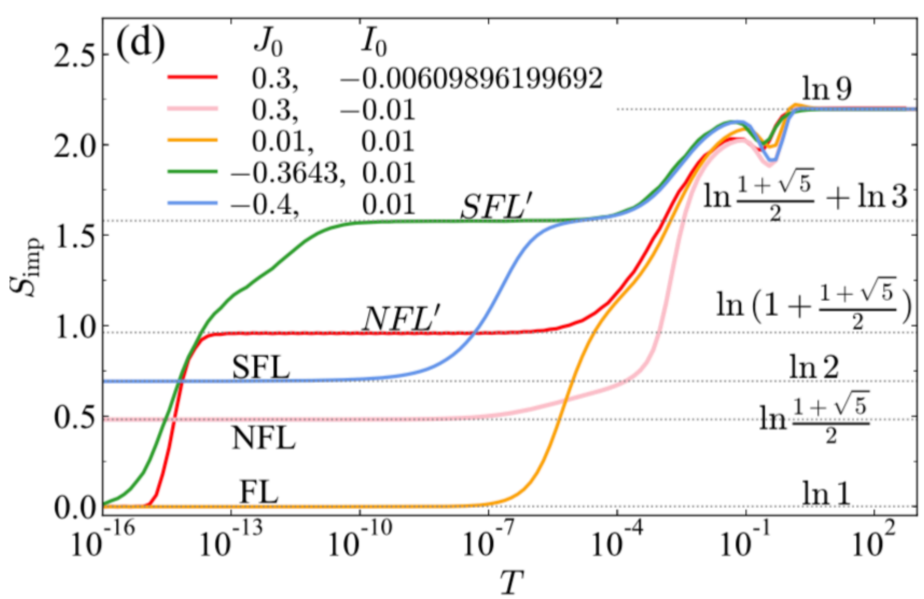
FL to NFL transition, crossover regime *NFL'* at intermediate energies

J_0=0.3



"level-crossing" picture

Multiplets				$E_{NFL'}$	$E_{\rm FL}$	$E_{\rm NFL}$
q	S	$(\lambda_1\lambda_2)$	d	$@I_0 = -0.00609896199692$	$@I_0 = -0.006098$	$@I_0 = -0.006099$
$+1$	$\frac{1}{2}$	(00)	2	-0.717930		-0.729529 $(= E_g)$
0	0	(01)	3	-0.668562		-0.680530
-2	0	(00)	1	-0.550738	-0.551028 $(= E_g)$	
-1	$\frac{1}{2}$	(10)	6	-0.282883	-0.275517 $(\simeq E_g + \epsilon_e)$	
-1	$\frac{1}{2}$	(10)	6	-0.275324		-0.295054
$+2$	0	(10)	3	-0.0880684		-0.100011
0	1	(01)	9	0	0 $(\simeq E_g + 2\epsilon_e)$	
0	0	(20)	6	0.000037	0 $(\simeq E_g + 2\epsilon_e)$	
0	1	(01)	9	0.011703		0
-2	1	(00)	3	0.102149		0.089692
$+1$	$\frac{3}{2}$	(00)	4	0.275407	0.275518 $(\simeq E_g + 3\epsilon_e)$	
$+1$	$\frac{1}{2}$	(11)	16	0.275410	0.275518 $(\simeq E_g + 3\epsilon_e)$	
$+1$	$\frac{1}{2}$	(11)	16	0.313029		0.300831
-1	$\frac{1}{2}$	(02)	12	0.416507		0.404921
-2	0	(11)	8	0.451411		0.439442
$+2$	1	(10)	9	0.550783	0.551033 $(\simeq E_g + 4\epsilon_e)$	
$+2$	0	(02)	6	0.550791	0.551033 $(\simeq E_g + 4\epsilon_e)$	



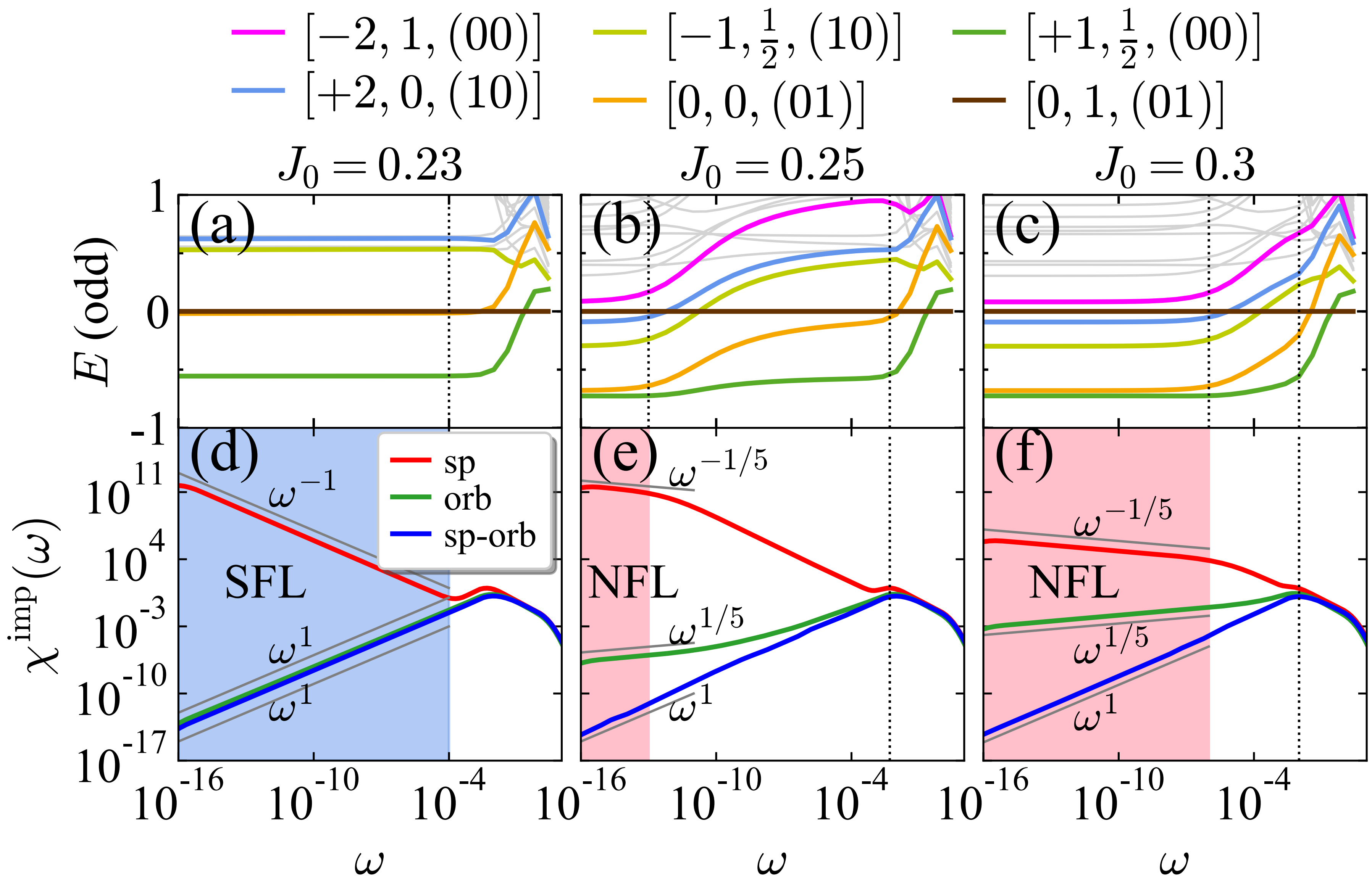
FL and NFL subspaces are orthogonal, both contribute to dynamical and thermodynamical properties, the effective impurity degrees of freedom in *NFL'* is the *sum* of those two sectors

$$S_{\rm imp}^{NFL'} = \ln(e^{S_{\rm imp}^{\rm FL}} + e^{S_{\rm imp}^{\rm NFL}}) = \ln(1 + \frac{1+\sqrt{5}}{2})$$

$$\text{not } \ln 1 + \ln\frac{1+\sqrt{5}}{2}$$

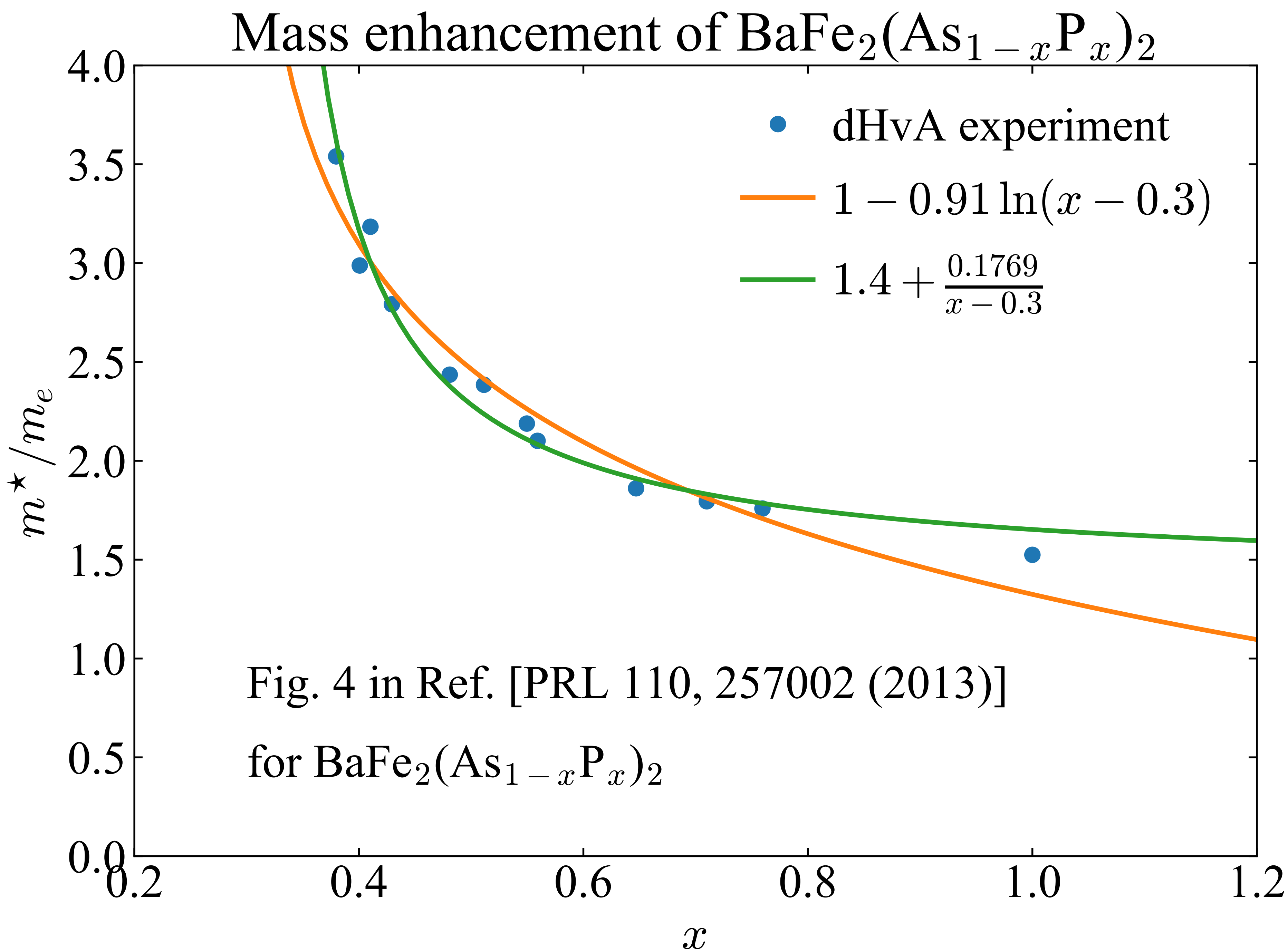
SFL to NFL transition

I_0=-0.01



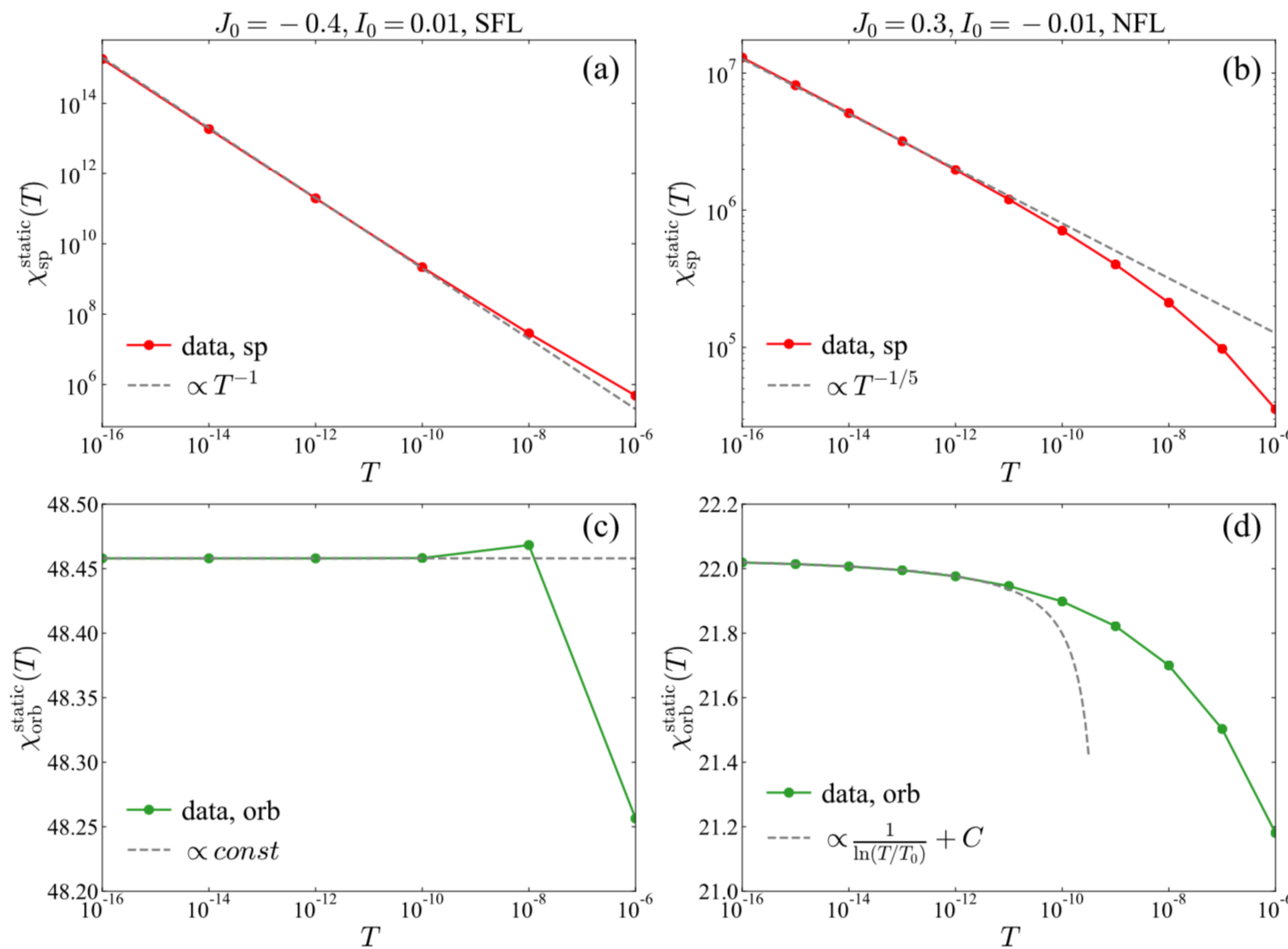
$$[+1, \tfrac{1}{2}, (00)] \otimes [-1, \tfrac{1}{2}, (01)] \to [0, 0, (01)]$$

Connection to the QCP in $BaFe_2(As_{1-x}P_x)_2$



mass enhancement ∝ **$1/T_{FL}$ ∝ $1/(x-x_c)$,**
suggests approaching to the NFL QCP

Connection to experiments: Static impurity susceptibilities as functions of temperature T



Summary

- We calculated a global phase diagram of the 3soK model relevant for Hund metal by NRG method, **two new QCPs are identified** by tuning spin and spin-orbital couplings into the ferromagnetic regimes

- ***T_{FL}* follows power-laws to approach zero energy close the QCPs, this allows us to follow the suppression of the coherence scale in Hund metals down to zero energy**

- We find quantum phase transitions to a SFL and a novel NFL phase. **The NFL phase shows interesting alternating overscreenings in spin and orbital sectors with universal power-laws in dynamical susceptibilities**, and we understand it by powerful CFT arguments. The NFL phase contains the essential ingredients to understand the incoherent behavior seen above ***T_{FL}***

- The NFL QCP presented in this work can be used to understand the mass divergence observed in iron pnictides doped with phosphorus

- The approach presented in this work can be generalized to study the unconventional quantum phase transitions observed in other heavy-fermion systems

Thank You !