# Can Multipole Magnetic Fields Play a Useful Role in Transporting, Polarizing and Focusing Neutron Beams on Small Samples?


**V. J. Ghosh**

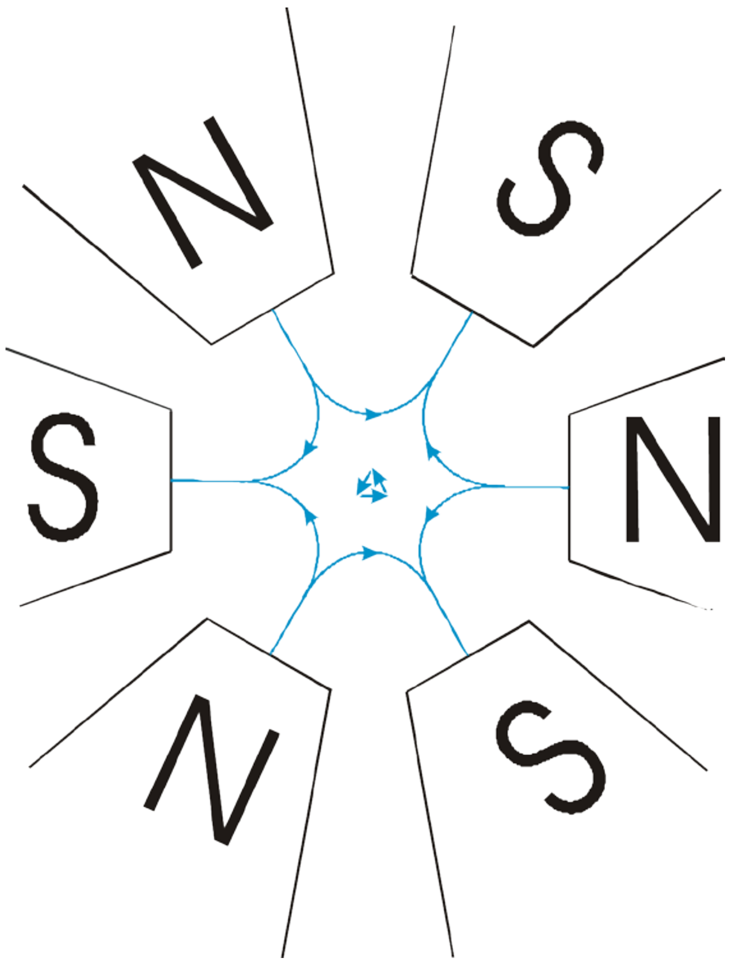
**A.U. Luccio**

**L. Passell**

**Brookhaven National Laboratory**

**Upton, NY 11973**

# Sextapole Magnetic Field




**Force on a Neutron in a Sextapole Magnetic Field**

$$F = \pm\mu \, \mathrm{grad} B$$

$$U = cr^n \cos n\varphi$$

$$B(r) = (B_r{}^2 + B_\varphi{}^2)^{1/2} = cnr^{n-1}$$

$$B(r) = B_0(r/r_w)^{n-1}$$

**Sextapole field (n=3)**

$$F_r = \pm 2\mu(B_0/r_w)(r/r_w)$$

# The Trajectory of a Neutron in a Sextapole Magnetic Field

$$d^2r/dt^2=F_r/m_n=\pm[(2\mu B_0/r_w)(r/r_w)]/m_n=\pm(k/m_n)r$$

$m_n \equiv$ the mass of the neutron.
$r_w \equiv$ the radius of the beam tube wall
$B_0 \equiv$ the value of the field at $r_w$

**If $(S\uparrow B)$**

$$r_{\perp}(t) = (v\sin\theta/\omega)\sin\omega t.$$

$v \equiv$ the neutron velocity
$\theta \equiv$the angle of the incident neutron trajectory with respect to the beam tube axis
$\omega \equiv (k/m_n)^{1/2} = [2\mu B_0/m_n r_w{}^2]^{1/2}$

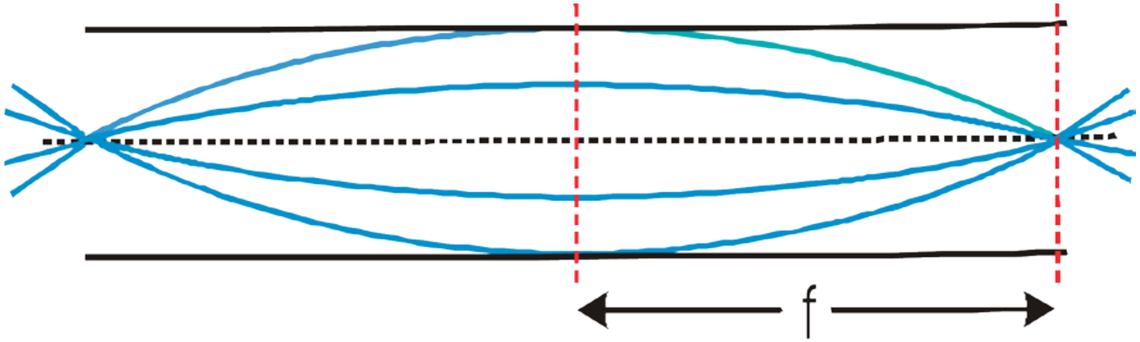
$$z(t)=v(\cos\theta)t.$$

**If $(S\downarrow B)$**

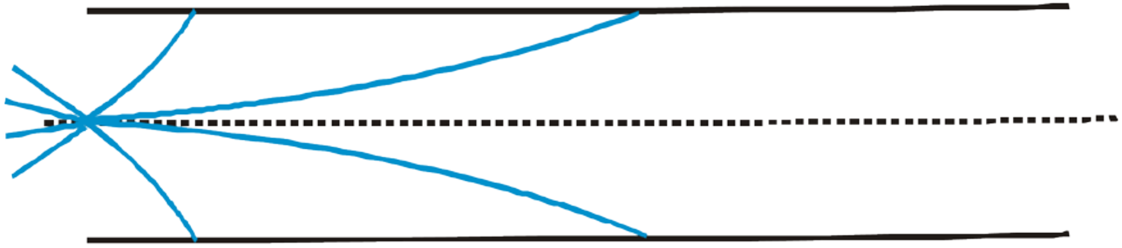
$$\ln\{(r\omega/v\sin\theta)+[(r\omega/v\sin\theta)^2+1]^{1/2}\}=\omega t$$

$$z(t)=v(\cos\theta)t.$$

# Neutron Trajectories in a Sextapole Magnetic Field




spins parallel to the magnetic field



spins anti-parallel to the magnetic field

# Transmission of Parallel Spin ($S_n \uparrow B$) Neutrons

**For neutron confinement amplitude of oscillation:**

$$v\sin\theta/\omega \leq r_w$$

$$v\sin\theta \leq \omega r_w = (2\mu B_0/m_n)^{1/2}.$$

**If we assign $B_0$ an upper limit value of, say, 2.5 T then**

**$\theta$<0.001358 radians/Å= 0.07780 degrees/Å**

**A sextapole beam transport system with $B_0$=2.5 T would thus be equivalent to an**

**m = 0.78 guide**

## Focusing of Parallel Spin ($S_n$↑B) Neutrons

**The focal length f is velocity dependent.**

$$f = \pi v_a / 2\omega.$$

**$v_a \equiv v\cos\theta$ is the axial component of the incident neutron velocity.**

**If $B_0$= 2.5 T and $r_w$ =1.5 cm**

**ω=357 radians/sec (57 cycles/sec)**

**For 5 meV neutrons (v=978 m/sec, λ=4Å) the primary focal length is**

**f = 4.3 m**

**Angular divergence = 0.312 degrees.fwhm**

**Subsequent focii at 12.9 m, 21.5 m, . . . .**

## Angular Distribution on the Focal Plane

**Central spot composed of neutrons that cross the entrance plane parallel to the axis. .**

**Neutrons entering at larger angles of incidence intersect the focal plane at progressively larger radii.**

**Density on the focal plane varies as $1/r_f$.**

# Beam Polarization

**How efficient would a sextapole field be as a polarizer of a Be-filtered beam if the beam tube had absorbing walls?**

**Sextapole field of length 12.9 m**
**$B_0$=2.5T**
**$r_w$=1.5 cm**

**0.1 percent of the 5 meV anti-parallel spin neutrons from a point source at the object position on the axis would reach the focal plane.**

**The longer wavelengths (for which the focal plane represents progressively higher order focii i.e.$\lambda$=6.67,9.33,12, 14.67,17.33, 24 Å) would be even more efficiently polarized. Thus in this (highly) idealized model**

**P>0.999**

# Getting Real

- **Do the neutron moments always maintain their alignment with respect to the sextapole magnetic field?.**

- **What about neutron trajectories that are not in planes containing the beam tube axis?**

- **Sextapole-produced neutron polarization is not uni-directional. Can the beam be brought out of the field without significant loss of polarization?**

- **Small angular acceptance (a 3 cm diameter polarizer equivalent to a guide with m=1 requires a field of about 4 T). Are fields this large realistic?**

- **Only one spin state is both transmitted and focused.**

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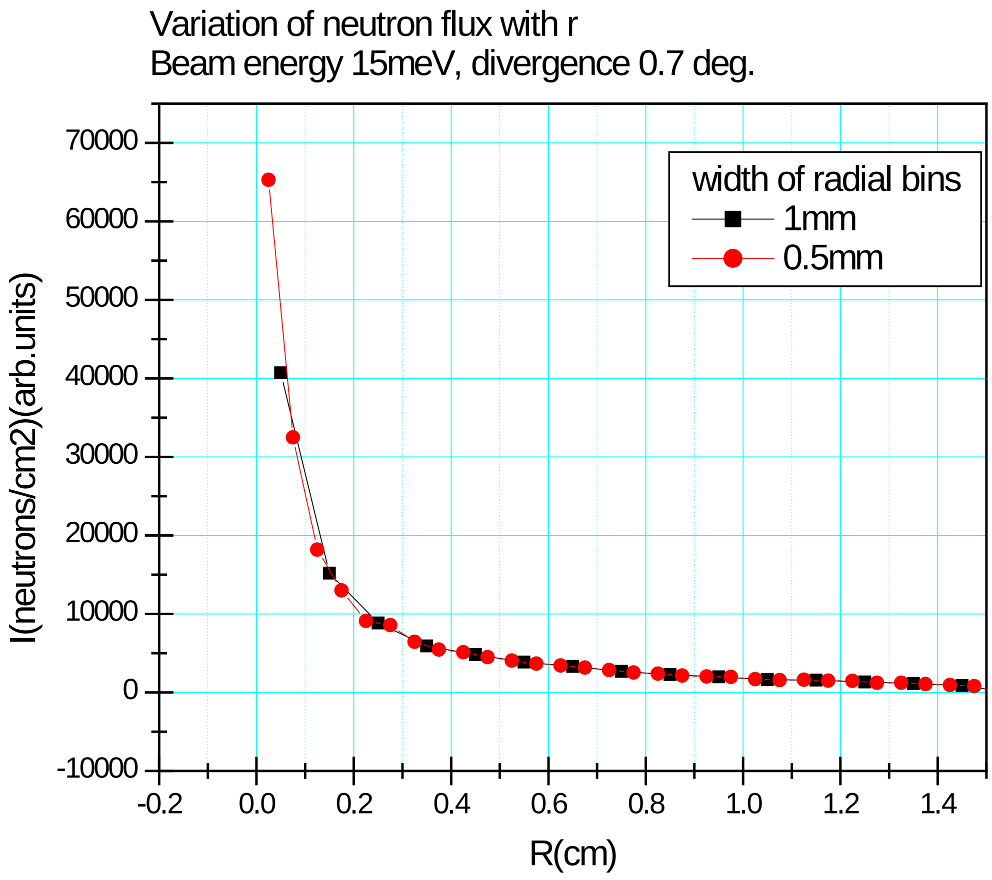
- **Practical applications require electromagnets that produce large and (in some cases) rapidly varying fields. Are such magnets realistic?**

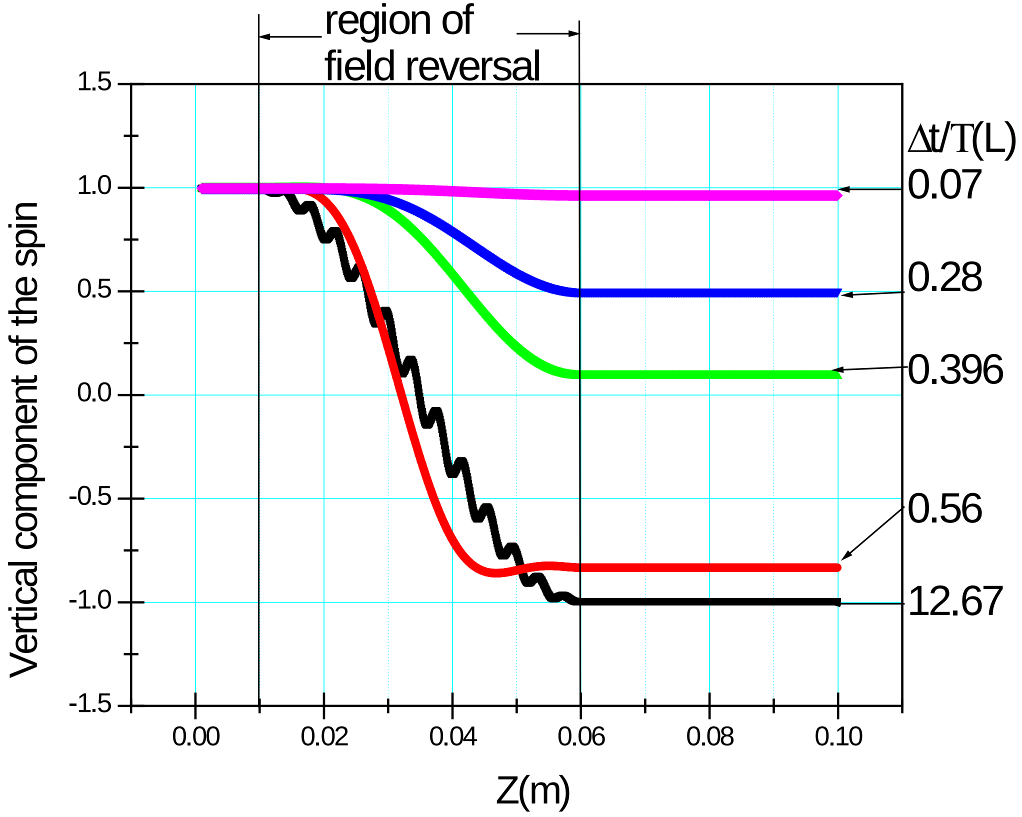
# Computer-Based Monte-Carlo Simulation of Neutron Spin Orientations and Trajectories in Magnetic Fields

- Force **F**=grad( $\boldsymbol{\mu}_\mathbf{n}$.**B**)
- Torque d**S**/dt=2π($\boldsymbol{\mu}_\mathbf{n}$x**B**)/h
- Either analytic representations of fields or computer-generated or experimentally-measured field maps can be used
- Fields can be super-imposed
- Beam tube walls can be either absorbing, transparent or super-mirror coated

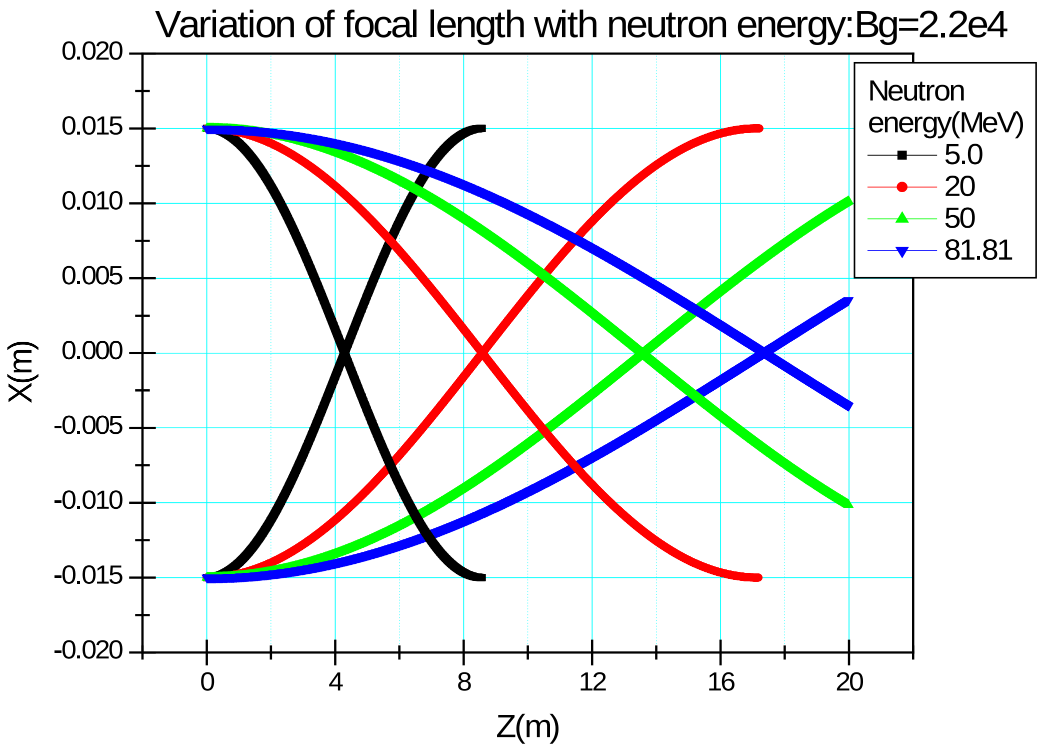
# Density Profile on the Focal Plane



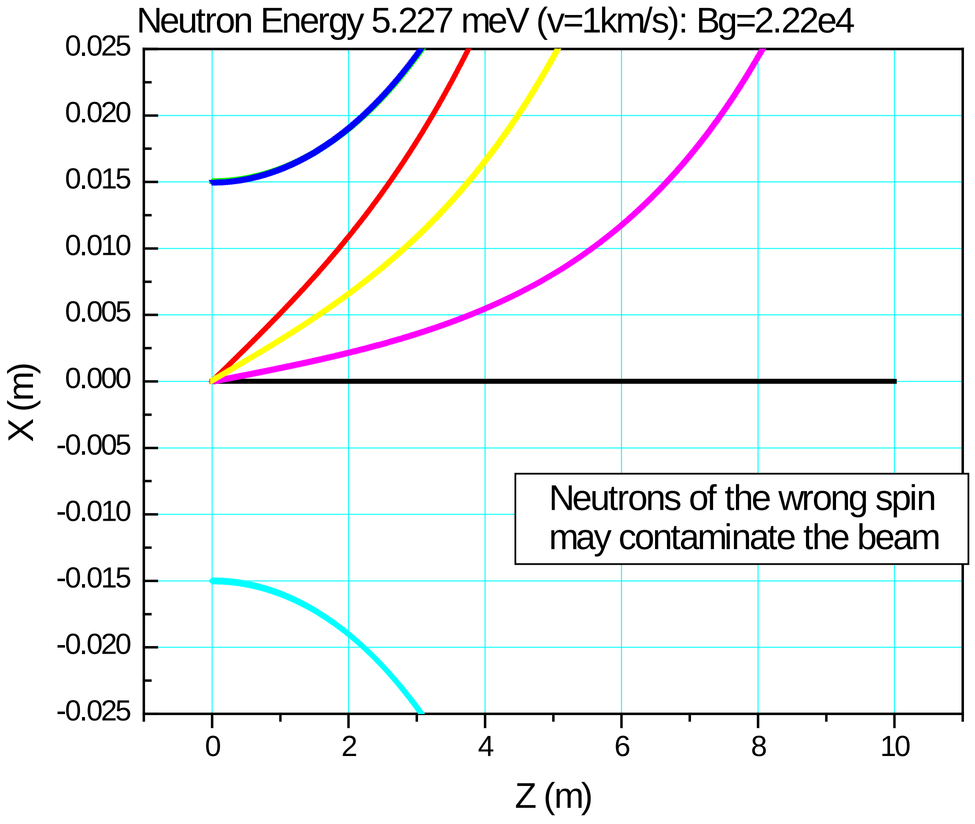
# Spin Progression in a Region of Rapidly Varying Magnetic Field



# Focusing at Different Neutron Energies



# Trajectories of Anti-Parallel Spin Neutrons



# Focusing with Different Field Strengths