Quantum-Assisted Telescope Arrays

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Astronomical Interferometry

Still blurry...
Visible light increases resolution and reveals new science

P. Stee et al., arXiv: 1703.02395
Quantum Networks

Massive investment and rudimentary versions are already online

What are they good for?
- Quantum key distribution
- ...
- Connecting telescopes (metrology)
Classical Interferometer

Telescope Arrays

We can determine the angle $\phi$ of a point source by measuring the local phase $e^{i\theta}$.
Resolution scales with $d$ and $\lambda^{-1}$.

At small $\lambda$, light is weak and thermal:

$$\rho_{AB} = (1 - \epsilon)\rho_{\text{vac}} + \epsilon\rho^{(1)} + O(\epsilon^2)$$

$$\left|0, 0\right> + \left(|0, 1\right> \pm e^{i\theta} \left|1, 0\right>) / \sqrt{2}$$

To optimally estimate $e^{i\theta}$, need nonlocal measurement.

Why Quantum?

Classically, can bring together the modes
But then photon losses limit $d$ ...

Solution: Connect with entanglement!

Problem: Don’t know when photon arrives, so very wasteful in EPR pairs

Solution

Capture photon (in a quantum memory), then do the interference (using entanglement). Repeat

Compressing the arrival time in binary works well:

\[ |00001000000000000000\rangle \rightarrow |1010\rangle \]

Tradeoff between memory and entanglement

Fortunately we can do the operations efficiently
Protocol

**Code:** Arrival rate $\epsilon \ll 1$, so store one photon with time index $\sim \frac{1}{\epsilon}$.

$$|\tilde{0}\rangle \equiv |0 \ldots 0\rangle \quad |\tilde{1}_m\rangle \equiv |m_2\rangle$$

$m \in [M]$ has binary representation $m_2$

Memory: $\log_2(M + 1)$ qubits. $M \sim \frac{1}{\epsilon}$
Coding Details

Encoding: **Local $Z$-teleportation** from photon to logical memory qubit
Need to track phase correction

Decoding: **Nonlocal parity checks** between parallel registers, using Bell pairs that are then measured in the $X$ basis
Projects onto the photon $\rho_{AB}^{1/\epsilon} \rightarrow \rho^{(1)}$
and fixes its arrival time $(-+\,-)_2 \rightarrow (101)_2 \rightarrow (5)_{10}$
Can then extract $e^{i\theta}$
Readout of Phase $g \equiv |g|e^{i\theta}$

Up to the correction, the memories are in a known entangled state
$$\frac{(1 \pm |g|)}{2} |\psi^+_{\theta}\rangle \langle \psi^+_{\theta}| + \frac{(1 \mp |g|)}{2} |\psi^-_{\theta}\rangle \langle \psi^-_{\theta}|$$

where, e.g., $|\psi^\pm_{\theta}\rangle = (|00, 11\rangle \pm e^{i\theta} |11, 00\rangle)/\sqrt{2}$

Apply interferometric phase $U_\delta = |0\rangle \langle 0| + e^{i\delta} |1\rangle \langle 1|$ to some qubit

Measure all the qubits in the $X$ basis, obtaining a string $c \in \mathbb{F}_2^m$ of $n$ measurement outcomes with probability
$$P(\langle c, c\rangle |g) = (1 + (-1)^{\langle c, c\rangle} \text{Re}(ge^{-i\delta}))/2$$

$\langle c, c\rangle$ counts the number of $|\perp\rangle$ outcomes, mod 2 (parity)

Repeat, varying $\delta$ to estimate $g$, as is standard in interferometry
Parameter Estimation

The Fisher information matrix $F$ describes the sensitivity of a measurement (POVM) $E(y)$ to the parameters $g_i$ of a quantum state $\rho$

$$F = \sum_y \frac{D(y|g)}{P(y|g)}, \quad D(y|g) = \left( \frac{\partial P(y|g)}{\partial g_1} \right)^2 \frac{\partial P(y|g)}{\partial g_2} - \left( \frac{\partial P(y|g)}{\partial g_2} \right)^2 \frac{\partial P(y|g)}{\partial g_1}, \quad P(y|g) = \text{tr}(E(y)\rho)$$

Operationally, it bounds the covariance matrix $\Sigma$ of $g_i$

Cramér-Rao bound: $\Sigma \geq 1/F$

Taking the trace norm $\|F\|$ quantifies the total obtainable information. $\|F\| \geq M\epsilon$ for nonlocal schemes and $\|F\| \leq M\epsilon^2$ for LOCC schemes

**Signal-to-noise (S/N) ratio** $\sim 1 \leftarrow \|F\| \geq 1$

Resources

\[ \log \frac{1}{\epsilon} \text{ entangled and classical bits.} \]
Near information-theoretic limit

\[ 2 \log \frac{1}{\epsilon} \text{ memory qubits. Compare:} \]

- ebits: \[ \frac{2}{\epsilon} \sim \log \frac{1}{\epsilon} \]
- qubits: \[ \frac{2}{\epsilon} \]

Memoryless \hspace{1cm} General local

Due to postselection, errors also have logarithmic accumulation

Good for near-term networks!

\[ \frac{1}{\epsilon} S(A|B)_\rho \approx \frac{1}{\epsilon} S(B)_\rho \xrightarrow{\epsilon \ll 1} \frac{1}{2} \log \frac{1}{\epsilon} \]

\[ \frac{1}{\epsilon} I(A; R)_\varphi = \frac{1}{\epsilon} S(AB)_\rho \xrightarrow{\epsilon \ll 1} \log \frac{1}{\epsilon} \]

97% gate or 80% entanglement fidelity in depolarizing error model.
Multiple Nodes

Encode into memory as before
Decode with promoted Bell states:

a) Collapse to 2 nodes with $W$
b) Maintain network coherence with $GHZ$

Phases $\leftrightarrow$ intensity distribution

Coherent processing with QFT
- Flat source: $\sqrt{N}$ extra improvement in S/N
- Point source: No noise!
Physical Realizations

Encoding set by chosen detector bandwidth (10 GHz)
Decoding at photon arrival rate (1 kHz)

<table>
<thead>
<tr>
<th>Experimental platform</th>
<th>One-qubit gate</th>
<th>Two-qubit gate</th>
<th>Atom-photon gate</th>
<th>Memory</th>
<th>Readout</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$</td>
<td>$\tau$</td>
<td>$f$</td>
<td>$\tau$</td>
<td>$f$</td>
</tr>
<tr>
<td>Rb atoms in cavity [11–13]</td>
<td>95%</td>
<td>7.5 $\mu$s</td>
<td>92%</td>
<td>0.9 $\mu$s</td>
<td>87%</td>
</tr>
<tr>
<td>Quantum dots &amp; nanophotonics [10, 14–16]</td>
<td>98%</td>
<td>3 ps</td>
<td>80%</td>
<td>150 ps</td>
<td>78% (99%)(^1)</td>
</tr>
<tr>
<td>SiV in diamond nanocavity [17, 18]</td>
<td>99%</td>
<td>10 ns</td>
<td>92%(^3)</td>
<td>10 ns</td>
<td>87%(^3)</td>
</tr>
</tbody>
</table>

Light-Matter Interface

**Cavities** enable natural photonic entangling gates
Broadband light

Parallelization improves bandwidth and gate time

For $M$ time bins and $R$ frequency bands:

- $O(\log_2(MR))$ entanglement
- $O(R \log_2 M)$ memory
Realistic Error Budget

<table>
<thead>
<tr>
<th>Error</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric turbulence</td>
<td>0.1 μm</td>
</tr>
<tr>
<td>Tip-tilt</td>
<td>0.1 μm</td>
</tr>
<tr>
<td>Stellar orientation</td>
<td>0.1 μm</td>
</tr>
<tr>
<td>Baseline</td>
<td>1 cm</td>
</tr>
<tr>
<td>Quantum operations</td>
<td>3%</td>
</tr>
<tr>
<td>Entanglement</td>
<td>20%</td>
</tr>
</tbody>
</table>

Classical, any length scale
Interferometric stability relaxed
Errors can be corrected
The Technology Exists

Memory-enhanced quantum communication

Here, applied to quantum key distribution

Experimentally demonstrated high-fidelity quantum network node:
- Quantum memory
- Gate operations
- Measurements

M. Bhaskar et al., arXiv: 1909.01323
Outcomes

Nonlocal measurement is necessary for probing weak thermal light. Use entanglement to connect sites!
Introducing memories enables efficient coding and processing
The scheme generalizes well to handle realistic interferometry

Quantum information: Provable advantage using modest resources
10 GHz $\rightarrow$ 200 kHz entanglement rate, 20 noisy memory qubits
$\Rightarrow$ near-term development and deployment of NISQ devices!
Astronomy: $330 \text{ m} \rightarrow 10^{2-4} \text{ km} \Rightarrow \text{mas} \rightarrow \mu\text{as} - 10 \text{ nas resolution}$!
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Collaborators: