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**ELECTRIC DIPOLE
MOMENTS FROM SOFT
SUSY PHASES And The
MUON $g - 2$**

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- **Supersymmetry** models contain new sources of **CP** violation phases.
- These **phases** can contribute to the **electric dipole moments** of **electron, neutron** etc. We have strict experimental limits on electron and neutron EDMs:
 $|d_e| < 4.3 \times 10^{-27}$ ecm and $|d_n| < 6.3 \times 10^{-26}$ ecm. [The SM contribution is very small.]
- Phase of **O(1)** size can predict large EDM, which can be larger than the experimental limits.

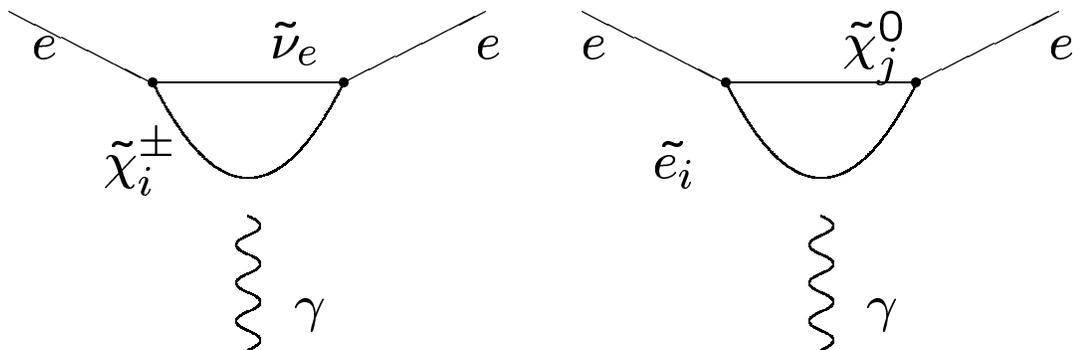
Conventionally,

- The phases are assumed to be small **O(10⁻²–10⁻³)**.
- The masses are very heavy (several **TeV**).

Recent proposal:

The **EDM** constraint can be satisfied by tuning the **cancellation** among different Feynman diagrams which contribute to the calculation of **EDM** [Ibrahim, Nath '98, Falk, Olive '98, Brhlik, Good, Kane '99]. For example,

Electron EDM:



- These diagrams can **cancel** each other and satisfy the **EDM** constraint.
- We can have **large** SUSY phases in the theory even in the regions of **smaller** sparticle masses.

In this talk:

- I will discuss the **cancellation** mechanism for **EDM** in the mSUGRA model with a generalized phase structure. I will show that the cancellation is **fine tuned** at the GUT scale for certain regions of parameter space.
- I will discuss the **fine tuning** problem in the parameter space allowed by the following constraints:
 - (i) The recent result of $a_\mu (\equiv (g - 2)/2)$ of muon: This has restricted the SUSY parameter space. The calculation of a_μ^{mSUGRA} and **EDM** involve the same set of diagrams. The former however is related to the real part.

(ii) **Cosmological constraint** : The SUSY model, I discuss, conserves R parity and thus the lightest supersymmetric particle can be a candidate for cold dark matter. The parameter space is restricted by the **relic density constraint** .

(iii) **Recent accelerator and rare decay bounds**: The latest bound on Higgs mass and the CLEO bound on $b \rightarrow s\gamma$ put important constraint in the model space.

Model Parameters:

Supergravity GUT models with universal soft breaking of supersymmetry, mSUGRA, depend upon **five** parameters at the GUT scale:

$m_{1/2}$ (the universal gaugino mass),

A_0 (the cubic soft breaking mass),

B_0 (the quadratic soft breaking mass),

μ_0 (the Higgs mixing parameter) and

m_0 (the universal squark and slepton mass).

- Electroweak symmetry is broken radiatively.
- Magnitudes of μ and B are determined from the electroweak symmetry breaking conditions.

We assume a general phase structure at the GUT scale i.e.

- $\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, A_0, B_0$ and μ are complex at the GUT scale.
- Among these phases, we rotate away one of the gaugino phases.
- Finally, we have the following phases at the GUT scale:
 - $|\tilde{m}_1|e^{i\phi_1}, |\tilde{m}_3|e^{i\phi_3}, |A_0|e^{i\phi_{0A}},$
 $|B_0|e^{i\theta_{0B}}, |\mu_0|e^{i\theta_\mu}.$

- The RGEs relate these GUT scale parameters to the weak scale values.

For example:

$$\tilde{m}_i = \frac{\alpha_i}{\alpha_G} \tilde{m}_{1/2}$$

In the low and intermediate $\tan\beta$ region, we can solve the RGEs to obtain:

$$A_t(t) = D_0 A_0 + \sum \Phi_i |m_{1/2i}| e^{i\phi_i}$$

the Φ_i are real and $O(1)$, $D_0 \simeq 0.25$,

$$B = B_0 - \frac{1}{2}(1 - D_0)A_0 - \sum \Phi'_i |m_{1/2i}| e^{i\phi_i}$$

[Accomando, Arnowitt, Dutta'99]

- Superpartner masses: The **chargino** and the **neutralino** mass matrices are:

$$M_{\chi^\pm} = \begin{pmatrix} \tilde{m}_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & |\mu|e^{i\theta} \end{pmatrix}$$

$$M_{\chi^0} = \begin{pmatrix} |\tilde{m}_1|e^{i\phi_1} & 0 & a & b \\ 0 & \tilde{m}_2 & c & d \\ a & c & 0 & -|\mu|e^{i\theta} \\ b & d & -|\mu|e^{i\theta} & 0 \end{pmatrix}$$

where $a = -M_Z \sin\theta_W \cos\beta$, $b = M_Z \sin\theta_W \sin\beta$, $c = -\cot\theta_W a$, $d = -\cot\theta_W b$, $\tan\beta = v_2/v_1$ ($v_{1,2} = |\langle H_{1,2} \rangle|$) and θ_W is the weak mixing angle.

- The phase θ is given by

$$\theta = \epsilon_1 + \epsilon_2 + \theta_\mu$$

where at the electroweak scale, $\langle H_{1,2} \rangle = v_{1,2}e^{i\epsilon_{1,2}}$, and $\mu = |\mu|e^{i\theta_\mu}$.

- $\tan\beta$ is a free parameter.

- The slepton mass matrix can be written as

$$\tilde{m}_l^2 = \begin{pmatrix} m_{l_{LL}^*}^2 & m_{l_{LR}}^2 \\ m_{l_{RL}}^2 & m_{l_{RR}}^2 \end{pmatrix}$$

$$m_{l_{LR}}^2 = m_l (A_l e^{-i\phi_{A_l}} - |\mu| e^{i\theta} \tan \beta)$$

where m_l is the lepton mass.

$$m_{l_{LL}}^2 = m_L^2 + m_l^2 - 1/2(2\cos^2\theta_W - 1)M_Z^2\cos 2\beta$$

$$m_{l_{RR}}^2 = m_R^2 + m_l^2 - \sin^2\theta_W M_Z^2\cos 2\beta$$

Electroweak symmetry breaking and relation among phases:

- The condition for electroweak symmetry breaking is obtained by minimizing the effective potential V_{eff} with respect to v_1 , ϵ_1 , v_2 and ϵ_2 . The Higgs sector of V_{eff} is

$$V_{eff} = m_1^2 v_1^2 + m_2^2 v_2^2 - 2|B\mu|\cos(\theta + \theta_B)v_1 v_2 + \frac{g_2^2}{8}(v_1^2 + v_2^2)^2 + \frac{g_1^2}{8}(v_2^2 - v_1^2)^2 + V_1$$

where V_1 is the one loop contribution, $m_i^2 = \mu^2 + m_{H_i}^2$ and $m_{H_{1,2}}^2$ are the $H_{1,2}$ running masses.

We now minimize V_{eff} in order to determine the Higgs VEVs i.e. $v_1, v_2, \epsilon_1, \epsilon_2$.

- In the tree approximation, the extrema equations

$\partial V_{eff}/\partial \epsilon_i = 0$ yield $2|B\mu|\sin(\theta + \theta_B) = 0$.
Hence the minimum of V_{eff} requires

$$\theta = -\theta_B$$

At the one loop level, one gets a correction of the form

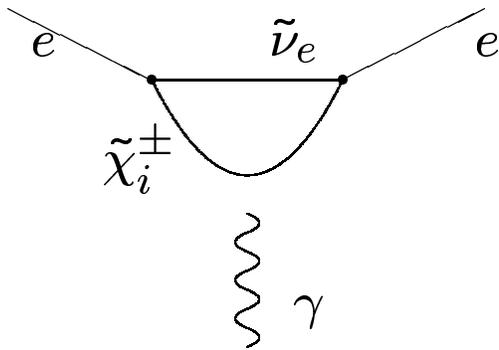
$$\theta = -\theta_B + f_1(-\theta_B + \phi_{A_q}, -\theta_B + \phi_{A_l})$$

where f_1 is the one loop correction with θ approximated by its tree value [Demir'99, Pilaftsis, Wagner'99].

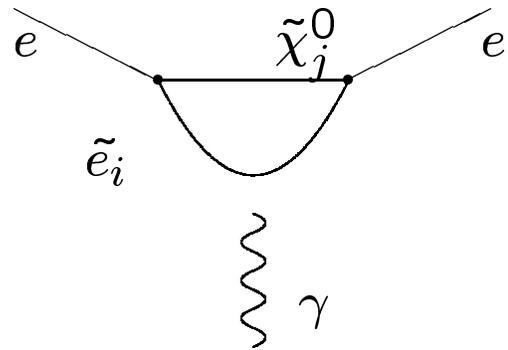
- This correction can become significant for large $\tan\beta$.

In our analysis we use $\theta = \theta_\mu$.

EDM calculation:



$$\propto \sin \theta_\mu$$



$$\propto \sin(\theta_\mu + \phi_1)$$

$$\propto \sin(\phi_A + \phi_1)$$

$$\propto \sin(\theta_\mu + \phi_A)$$

- The above diagrams cancel with each other and hence large phases can be allowed.

- The condition we get (assuming $\phi_A = 0$):

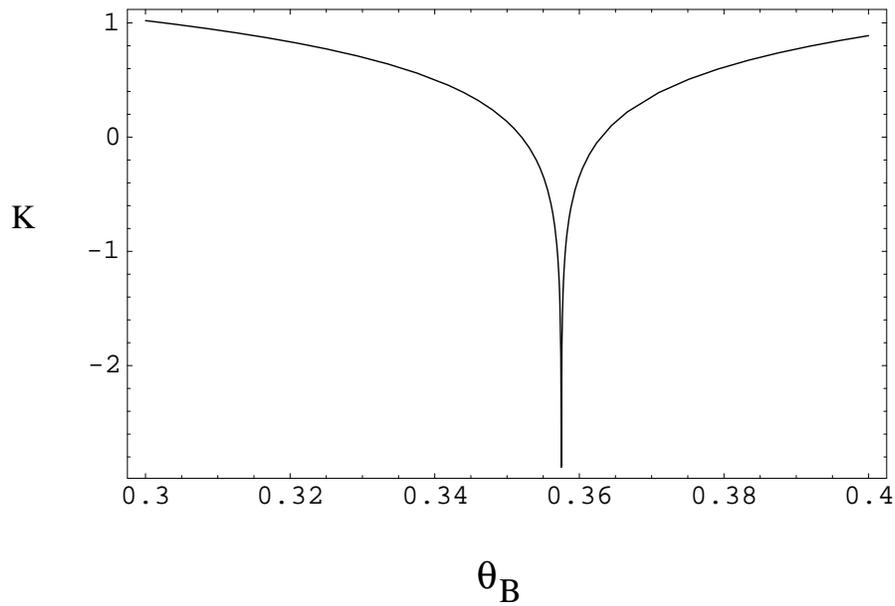
$$d_{\tilde{\chi}_1^0} \sin(\theta_\mu + \phi_1) + d_{\tilde{\chi}_1^0 + \tilde{\chi}_1^\pm} \sin(\theta_\mu) = 0$$

$$\Rightarrow \tan \theta_B = \frac{\epsilon \sin \phi_1}{1 + \epsilon \cos \phi_1}$$

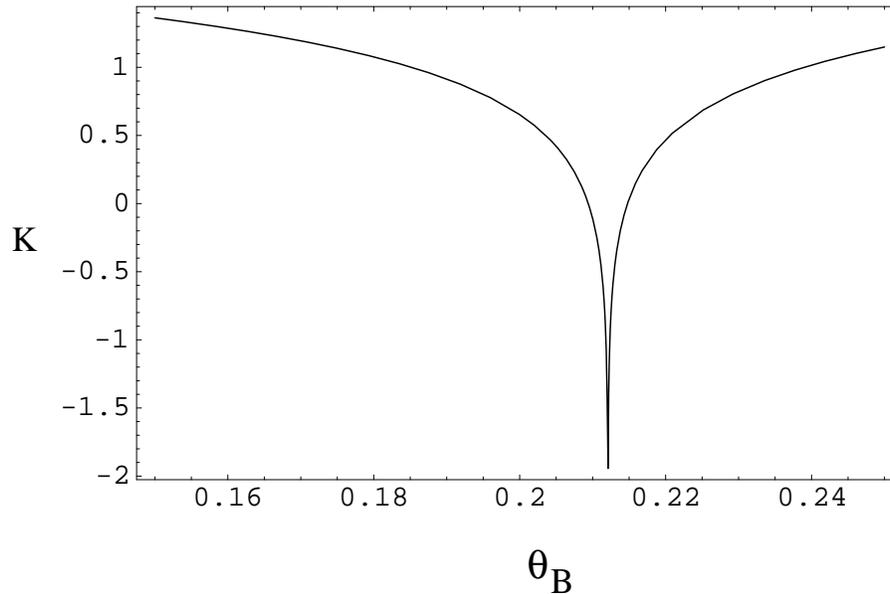
where $\epsilon = \frac{d_{\tilde{\chi}_1^0}}{d_{\tilde{\chi}_1^0 + \tilde{\chi}_1^\pm}}$.

$d_{\tilde{\chi}_1^0 + \tilde{\chi}_1^\pm}$ contains diagrams involving chargino and neutralino and $d_{\tilde{\chi}_1^0}$ contains diagrams involving neutralino only. Typically $\epsilon \simeq 0.3$.

- This is the situation where we have exact cancellation and electron EDM is 0. But the experimental upper bound generates a spread of θ_B which depends on the experimental bound on EDM.



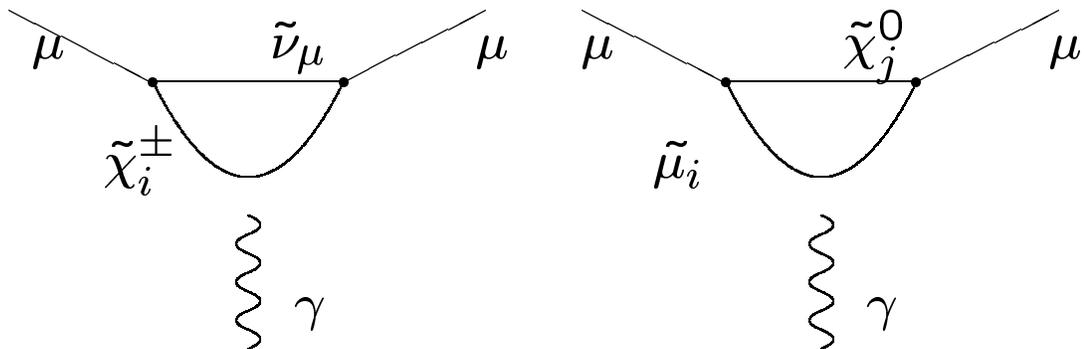
$K = \log_{10} \left| \frac{d_e}{(d_e)_{exp}} \right|$ vs θ_B for $\tan \beta = 15$,
 $m_0 = 100$ GeV, $m_{1/2} = 400$ GeV, $\phi_1 = 0.9$
and $A_0 = 0$.



$K = \log_{10} \left| \frac{d_e}{(d_e)_{exp}} \right|$ vs θ_B for $\tan \beta = 40$,
 $m_0 = 210$ GeV, $m_{1/2} = 400$ GeV, $\phi_1 = 0.9$
 and $A_0 = 0$.

- The magnitude of m_0 being different compared to $\tan \beta = 15$ case is because of dark matter constraint.

Calculation of a_μ :



- The real part of the diagrams contribute in this case.
- BNL 821 experiment gives a **2.6 σ deviation** from standard model[Brown, hep-ex/0102017]

$$a_\mu^{EXP} - a_\mu^{SM} = 43(16) \times 10^{-10}$$

- This deviation can be explained in the framework of **SUGRA GUT** model. [Kosower, Krauss, Sakai'83, Yuan, Arnowitt, Chamseddine, Nath '84]

- In mSUGRA model, the dominant contribution is given by:

$$a_\mu \simeq \frac{\alpha \tan \beta}{4\pi \sin^2 \theta_W} \frac{m_\mu^2}{|\tilde{m}_2| |\mu|} A [\cos \theta_\mu + \epsilon \cos(\theta_\mu + \phi_1)]$$

A is expressed in terms of the ratio of masses and is $O(1)$.

- Using $\tan \theta_B = \frac{\epsilon \sin \phi_1}{1 + \epsilon \cos \phi_1}$ (where EDM is 0), we find

$$a_\mu \simeq \frac{\alpha \tan \beta}{4\pi \sin^2 \theta_W} \frac{m_\mu^2}{|\tilde{m}_2| |\mu|} A \sqrt{(1 + 2\epsilon \cos \phi_1 + \epsilon^2)}$$

for $\phi_1 = 0$, we have $a_\mu \propto (1 + \epsilon)$,

for $\phi_1 = \pi$, we have $a_\mu \propto (1 - \epsilon)$.

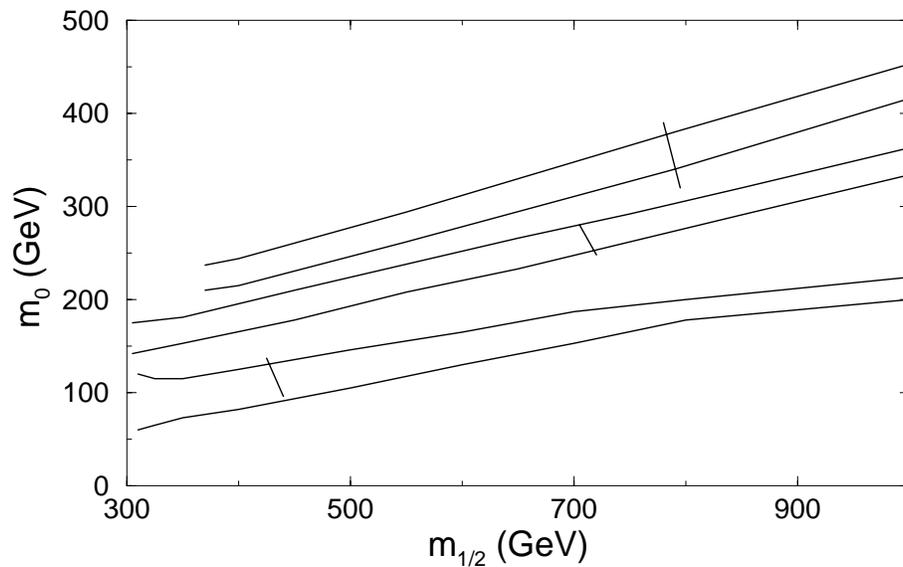
Calculational details:

- We use the Higgs mass constraint : $m_h > 111$ GeV.
This bound puts constraint on $m_{1/2}$ e.g. for $\tan\beta = 10$, $m_{1/2} > 300$ GeV.
- We use the $b \rightarrow s\gamma$ constraint.
We use NLO contributions to SUSY and charged Higgs diagrams [Degrassi et al.'00, Ciuchini et al.'00]
- We use SUSY one loop correction to b -mass. This correction is important for large $\tan\beta$. [Rattazzi, Sarid' 94; Carena, Wagner, Pokorski'94].
We do not demand $b - \tau$ unification
- In our calculation $m_{\tilde{\chi}_1^0}$ is the lightest SUSY particle. We use the dark matter constraint in the parameter space, i.e. $0.02 < \Omega_{\tilde{\chi}_1^0} h^2 < 0.25$

Dark matter constraint (in detail)

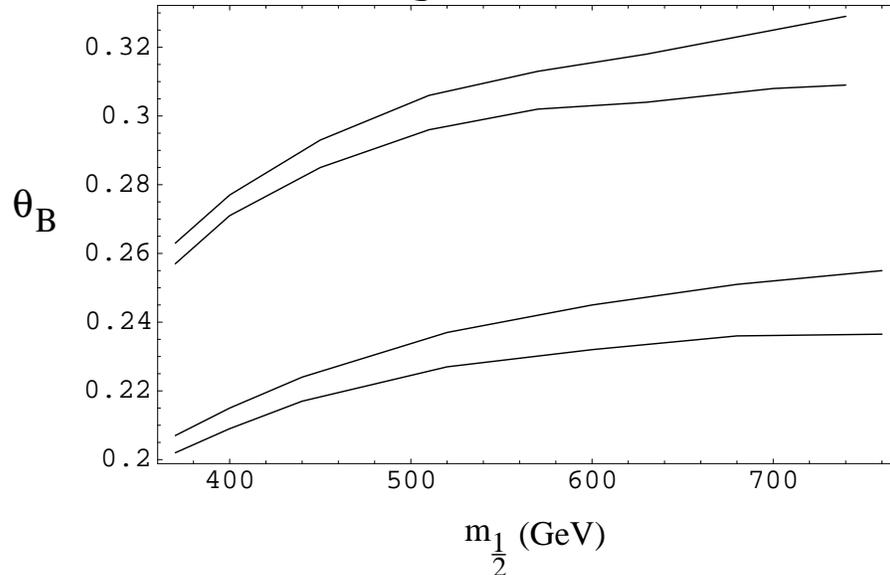
- $\Omega_{\chi_1^0} h^2 \propto \frac{1}{\langle \sigma v \rangle}$
- As m_0 and $m_{1/2}$ increase σ decreases and $\Omega_{\chi_1^0} h^2$ increases. The new LEP limit on the Higgs mass and the $b \rightarrow s\gamma$ constraint require larger values of $m_{1/2}$.
- However, it is possible to decrease $\Omega_{\chi_1^0} h^2$ by choosing the mass of one of the scalar particles to be close to the neutralino and thereby giving rise to **coannihilation** in the early universe.
- In mSUGRA model the lighter stau mass comes close to the **neutralino** mass naturally. When this happens we find the **relic density** in the desired range. [Arnowitt, Dutta, Santoso'01; Falk, Ellis, Olive, Srednicki'01]

- In the coannihilation corridors m_0 gets fixed within a narrow window for a given value of $m_{1/2}$.



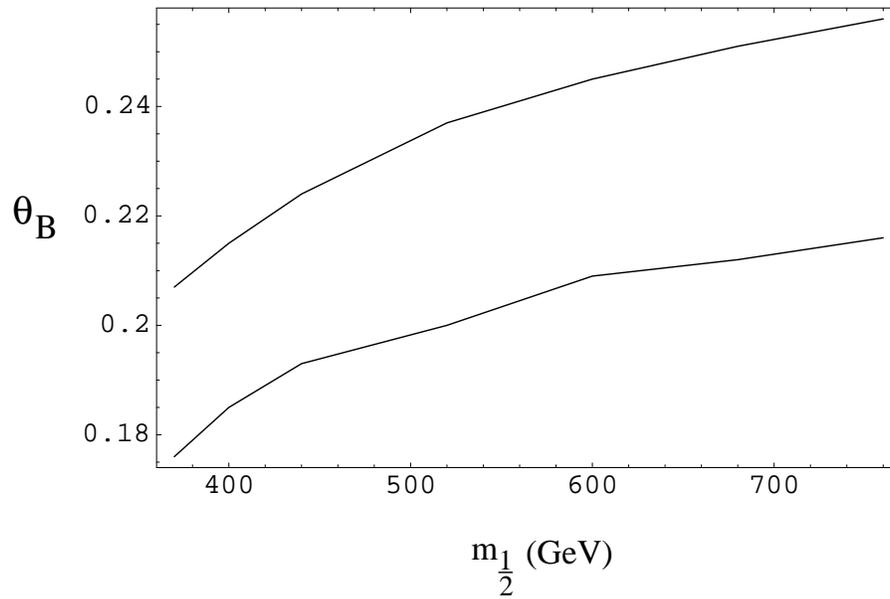
Corridors in the $m_0 - m_{1/2}$ plane allowed by the relic density constraint for $\tan \beta = 10, 30$ and 40 (from bottom to top), $m_h > 111$ GeV, $\mu > 0$ for $A_0 = 0$. The slanted lines indicate the bound from a_μ .

Numerical results using different constraints

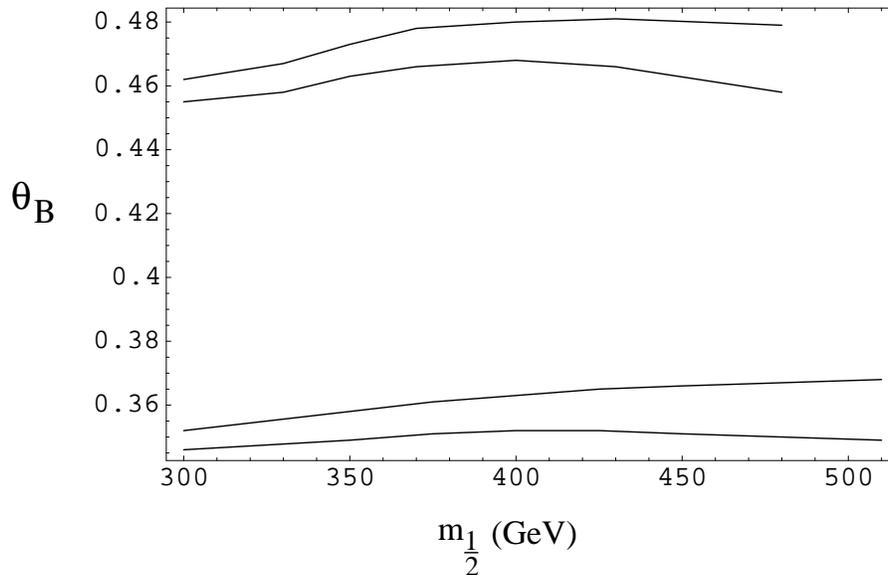


θ_B vs $m_{1/2}$ for $\tan\beta = 40$ and $A_0 = 0$.
 The upper region is for $\phi_1 = 1.2$ and the lower region is for $\phi_1 = 0.9$. The corridors appear due to the EDM constraint.

- Since we are in the coannihilation region, roughly $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0} \leq 25$ GeV. We choose m_0 for a fixed $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ value.
- The regions terminate at low $m_{1/2}$ is due to $b \rightarrow s\gamma$ bound and the termination at high $m_{1/2}$ is due to the lower bound on a_μ .



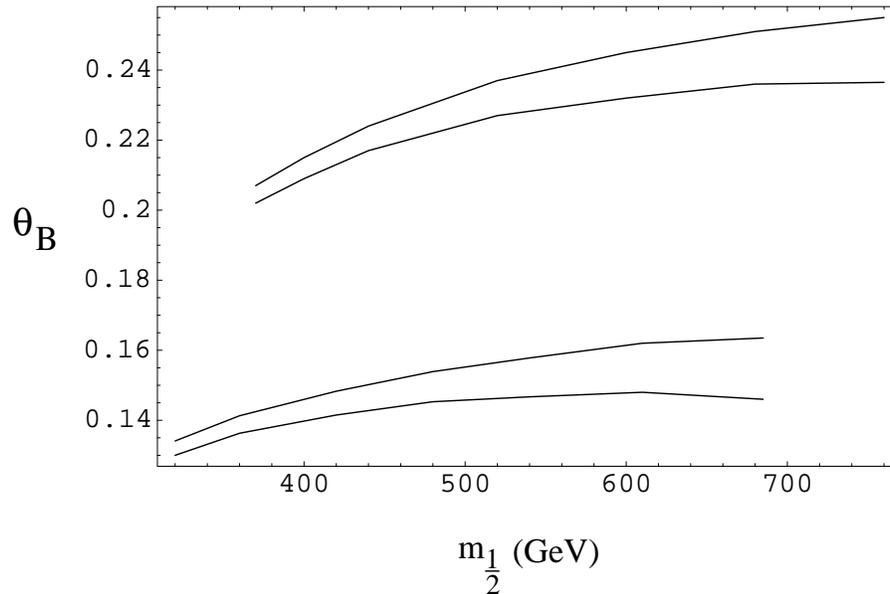
θ_B vs $m_{1/2}$ for $\tan\beta = 40$, $A_0 = 0$ and $\phi_1 = 0.9$. The maximum and the minimum values of phases correspond to the entire range of $\Omega_{\tilde{\chi}_1^0} h^2$ i.e. $0.02 - 0.25$.



θ_B vs $m_{1/2}$ for $\tan \beta = 15$ and $A_0 = 0$.

The upper region is for $\phi_1 = 1.2$ and the lower region is for $\phi_1 = 0.9$.

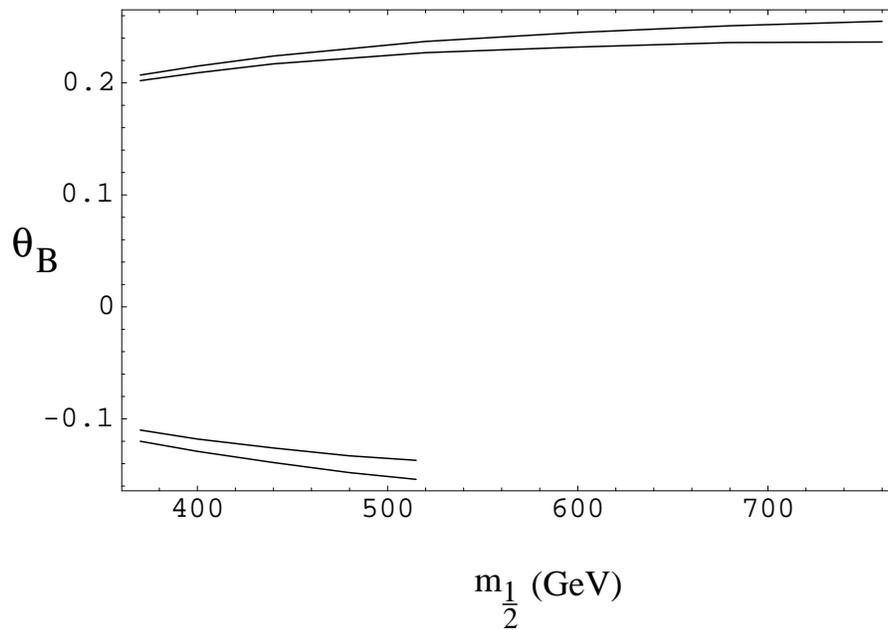
- The regions terminate at low $m_{1/2}$ is due to the Higgs mass constraint.
- θ_B is larger in this case compared to the $\tan \beta = 40$ case.



θ_B vs $m_{1/2}$ for $\tan \beta = 40$.

The upper region is for $A_0 = 0$ and the lower region is for $A_0 = 2m_{1/2}$ and $\phi_{0A} = 0.5$.

- $A_0 = 2m_{1/2}$ region requires larger m_0 to satisfy the dark matter constraint.



θ_B vs $m_{1/2}$ for $\tan \beta = 40$ and $A_0 = 0$.
The upper region is for $\phi_1 = 0.9$ and the
lower region is for $\phi_1 = 3.4$.

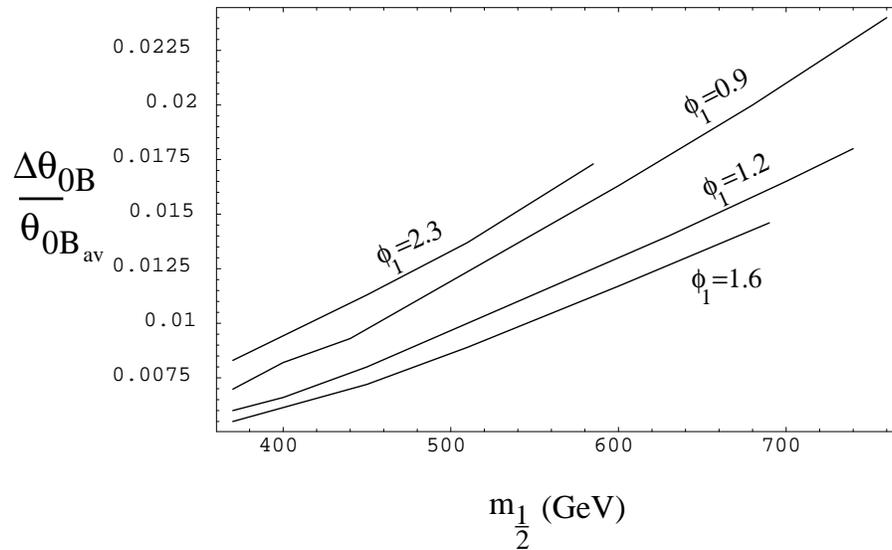
Fine tuning of phases at the GUT scale:

- $\frac{\Delta\theta_\mu}{\theta_{\mu av}} > 1\%$ for most of the parameter space.
[where $\Delta\theta_\mu$ is the regions allowed in the pervious figures by EDM]
- However, $\frac{\Delta\theta_{0B}}{\theta_{0B av}} < 1\%$ for larger values of ϕ_1 and lower values of $m_{1/2}$.

$\Delta\theta_{0B}$ is the allowed range of θ_{0B} at the GUT scale and is smaller compared to the $\Delta\theta_B$.

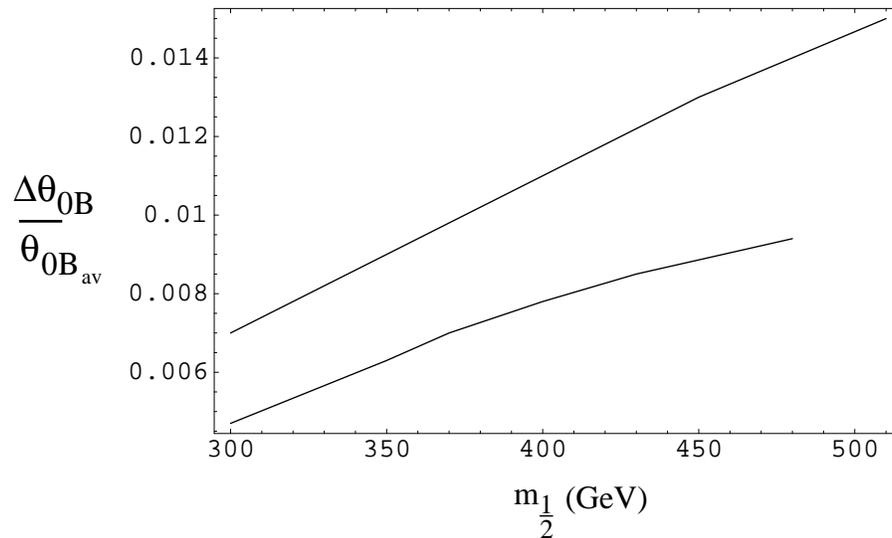
This is because $\Delta\theta_{0B} \simeq \frac{|B|}{|B_0|} \Delta\theta_B$ and $|B_0| > |B|$.

- $\frac{\Delta\theta_{\phi_1}}{\phi_{1 av}} > 1\%$ for most of the parameter space.



$\frac{\Delta\theta_{0B}}{\theta_{0B_{av}}}$ vs $m_{1/2}$ for $\tan\beta = 40$, $A_0 = 0$.

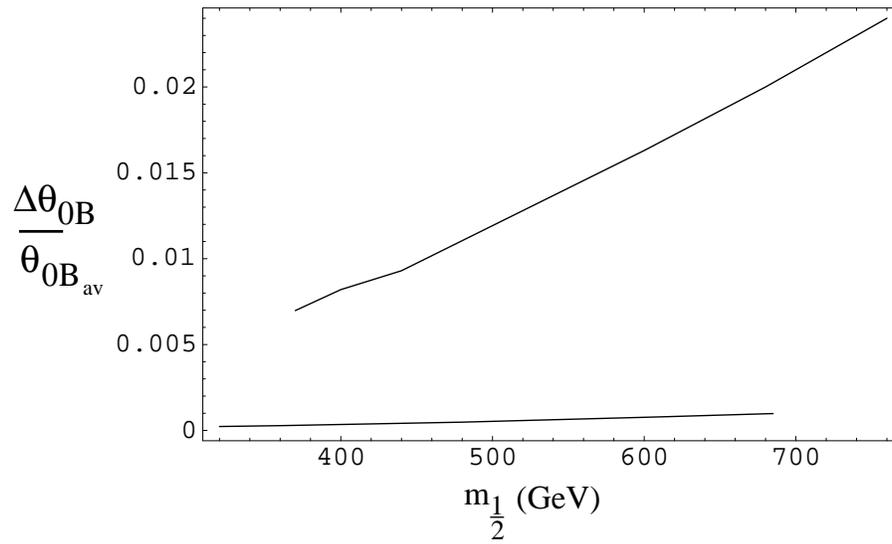
- If we require $\frac{\Delta\theta_{0B}}{\theta_{0B_{av}}} > 1\%$, the lower values of $m_{1/2}$ are disfavored.



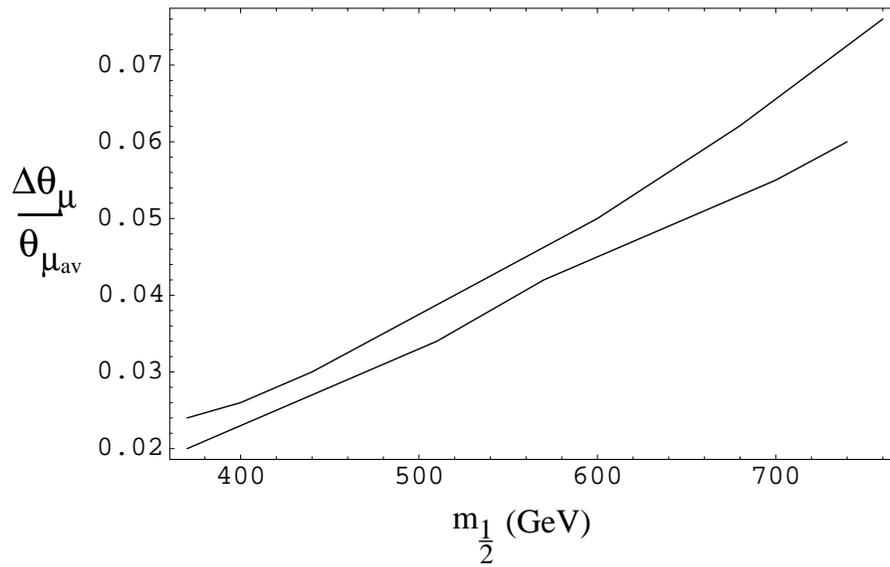
$\frac{\Delta\theta_{0B}}{\theta_{0B_{av}}}$ vs $m_{1/2}$ for $\tan\beta = 15$, $A_0 = 0$.

The upper line is for $\phi_1 = 0.9$ and the lower line is for $\phi_1 = 1.2$.

- If we require $\frac{\Delta\theta_{0B}}{\theta_{0B_{av}}} > 1\%$, $\phi_1 > 1.2$ is disfavored.

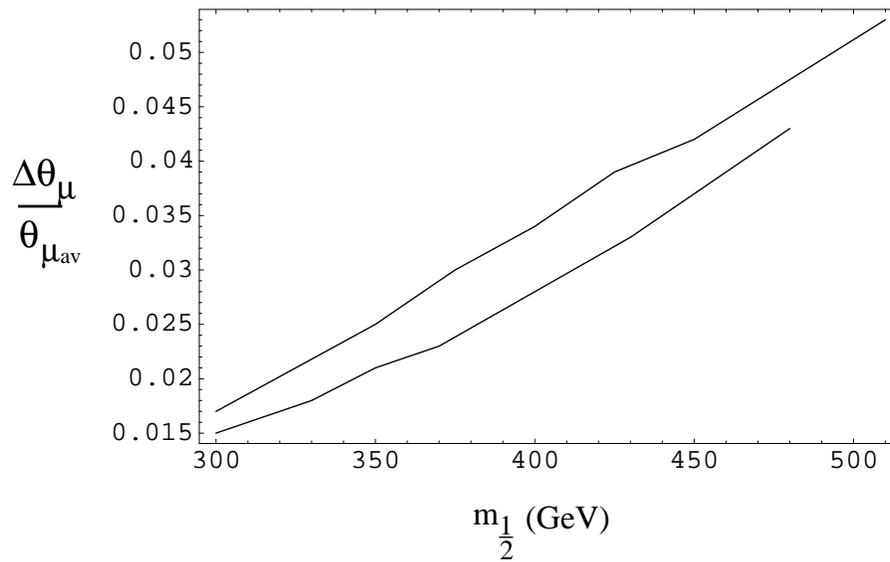


$\frac{\Delta\theta_{0B}}{\theta_{0B_{av}}}$ vs $m_{1/2}$ for $\tan\beta = 40$ and $\phi_1 = 0.9$.
 The upper line is for $A_0 = 0$ and the lower line is for $A_0 = 2m_{1/2}$ and $\phi_{0A} = 0.5$.



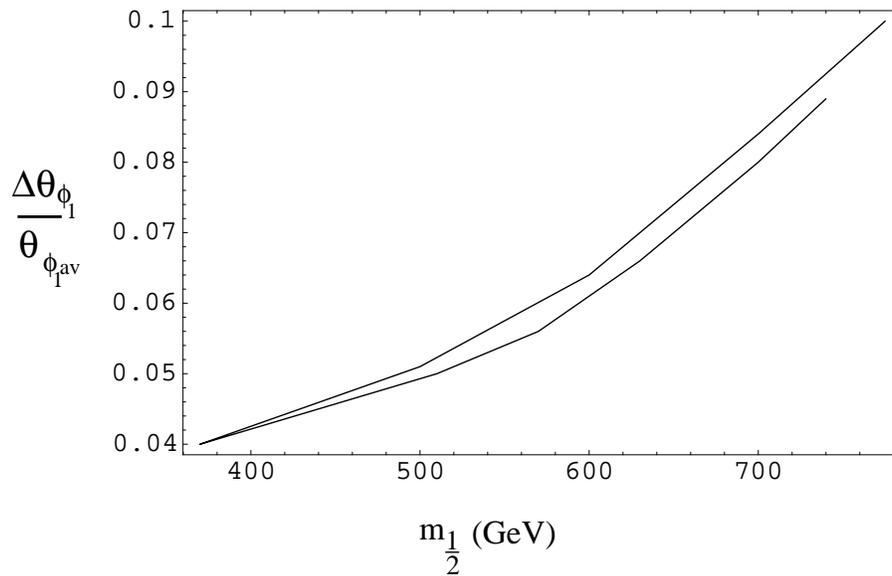
$\frac{\Delta\theta_\mu}{\theta_{\mu_{av}}}$ vs $m_{1/2}$ for $\tan\beta = 40$, $A_0 = 0$.

The upper line is for $\phi_1 = 0.9$ and the lower line is for $\phi_1 = 1.2$.



$\frac{\Delta\theta_\mu}{\theta_{\mu_{av}}}$ vs $m_{1/2}$ for $\tan\beta = 15$, $A_0 = 0$.

The upper line is for $\phi_1 = 0.9$ and the lower line is for $\phi_1 = 1.2$.



$\frac{\Delta\phi_1}{\phi_{1av}}$ vs $m_{1/2}$ for $\tan\beta = 40$, $A_0 = 0$.

The upper line is for $\theta_\mu = 0.2$ and the lower line is for $\theta_\mu = 0.3$.

Conclusion:

- The **phases** allowed by **EDM** of electron are not necessarily very **small**. This happens due to some cancellation among the contributing diagrams.
- This cancellation is necessary for the entire region of the parameter space allowed by a_μ in order to allow large CP violating phases.
- However, the phase of B can become fine tuned at the GUT scale in certain regions of the parameter space.

If fine tuning less than **1%** is excluded then for $\tan\beta = 15$, $\phi_1 \leq 1.2$ in the parameter space allowed by a_μ bound.

For the allowed values of ϕ_1 , the lower values of $m_{1/2}$ are disfavored. For example, for $\phi_1 = 0.9$, $m_{1/2} < 380 \text{ GeV}$ is disfavored.

The large $\tan \beta$ scenarios are better since a_μ allows higher values of m_0 and $m_{1/2}$. The lower values of $m_{1/2}$ are again disfavored and depend on ϕ_1 , for $\phi_1 = 0.9$, $m_{1/2} < 470 \text{ GeV}$ is disfavored.

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J. Feng and K. Matchev, Phys.Rev.Lett.86, 3480, 2001; U. Chattopadhyay and P. Nath, hep-ph/0102157; S. Komine, T. Moroi and M. Yamaguchi, Phys.Lett.B506, 93,2001; hep-ph/0103182; T. Ibrahim, U. Chattopadhyay and P. Nath, hep-ph/0102324; J. Ellis, D.V. Nanopoulos, K. A. Olive, hep-ph/0102331; R. Arnowitt, B. Dutta, B. Hu and Y. Santoso, Phys.Lett.B505, 177, 2001; H. Baer, C. Balazs, J. Ferrandis and X. Tata, hep-ph/0103280; F. Richard, hep-ph/0104106