ON THE PARTIALLY-FROZEN-SPIN METHOD

EDM Note, 21 March 2012

Yuri F. Orlov

Cornell University

In this Note I will show why the WM-YS method of edm measurement—in which the RFE and RFB fields oscillate in resonance with the g-2 frequency, ω_a, while the radial and vertical Lorentz forces are kept equal to zero—can be called (and actually is) a "partially-frozen-spin" method.

For simplicity, let us consider an azimuthally symmetric ring, in which all fields are distributed along a circular orbit:

$$B_V = B_0 + b_{V0}\cos\omega_a t \tag{1}$$

$$E_R = e_{R0}\cos\omega_a t \tag{2}$$

$$e_{R0} = \beta b_{V0}\,. \tag{3}$$

The planar spin equations are:

$$ds_L / dt = -\omega_a s_R + s_R \frac{eb_{V0}}{mc}\frac{1+a}{\gamma^2}\cos\omega_a t \tag{4}$$

$$ds_R / dt = \omega_a s_L - s_L \frac{eb_{V0}}{mc}\frac{1+a}{\gamma^2}\cos\omega_a t\ , \tag{5}$$

a=(g-2)/2. The solution of eqs. (4) and (5) is:

$$s_L = s_{L0}\cos\Phi, \quad s_R = s_{L0}\sin\Phi\ , \tag{6}$$

$$\Phi = \omega_a t - \left(\left(eb_{V0} / mc\right) / \omega_a\right) \frac{1+a}{\gamma^2} \sin \omega_a t \ . \tag{7}$$

It is easy to check (by using table integrals) that the average in time $\cos\Phi$ is not equal to zero:

$$\frac{1}{t}\int_0^t dt \cos\Phi \neq 0 \quad at\ t \to \infty \ . \tag{8}$$

This means that a part of the longitudinal spin component that is essential for edm measurements is constant in time, frozen. We can get this part, in the first approximation, directly from eq. (5):

$$s_L \approx s_{fr} + s_a \cos\omega_a t \ , \tag{9}$$

where the frozen mode is much smaller than the oscillating mode, so

$s_a \approx s_{L0} \sim 1,\ \ s_{fr} << s_{L0}$.

$$s_{fr} \approx \frac{1}{2}\left(\left(eb_{V0} / mc\right) / \omega_a\right) \frac{1+a}{\gamma^2} s_{L0},\ \ b_{V0} = e_{R0} / \beta \ . \tag{10}$$

This goes into the edm, Lorentz-free equation:

$$ds_V / dt = (e / 2mc)\eta s_{fr} \beta B_0 \ . \tag{11}$$

Let us now estimate the systematic error caused directly by some non-zero E_V and B_R components of the designed RF fields:

$$E_V = e_{V0} \cos\omega_a t \ , \tag{12}$$

$$B_R = b_{R0} \cos\omega_a t \ . \tag{13}$$

Assume that the Lorentz-free condition is satisfied not only for the radial but also for the vertical beam deviations, so

$$e_{V0} = -\beta b_{R0} \, . \tag{14}$$

In this case,

$$ds_V / dt = \frac{eb_{R0}}{2mc} \frac{1+a}{\gamma^2} s_{L0} + \frac{eb_{V0}}{2mc} \left((eB_0 / mc) / \omega_a \right) \frac{1}{2} \eta \beta \frac{1+a}{\gamma^2} s_{L0} \, . \tag{15}$$

To have the systematic error smaller than the edm signal, we need to saisfy the following condition for $b_{R0} / b_{V0} = e_{V0} / e_{R0}$:

$$b_{R0} / b_{V0} < 0.5 \eta \beta (eB / mc) / \omega_a \, . \tag{16}$$