Precise Measurements of Small Linear and Angular Displacements with Capacitance Methods

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• Motivation: EDM G-2 Upgrade (by Gerald Bennet)
• Types of Displacement and Angle Sensors
• Advantages of Capacitive Displacement Sensors
• Readout Methods
• Limits to Sensitivity: Electrical and Mechanical Noise
• MEMS/Microelectronics Applications: Accelerometers

Acknowledgment:

The EDM Collaboration, especially G. Bennet, R. Burns, W. Morse, Y. Semertzidis, L. Snydstrup

P. Rehak
Types of Displacement and Angle Sensors

1: Interferometric Sensor

- interferometric technique: optical wavelength-scale resolution
- Measures relative movements: laser light must be on all the time unless the reference point is lost
Types of Displacement and Angle Sensors

2: Interpolating Sensors (e.g. SONY MAGNESCALE)

- Uses spatial averaging: reduced differential non-linearity
- Large Dynamic range
- Integral non-linearity depends on accumulated error in N-S magnetization locations
Types of Displacement and Angle Sensors (cont)

LVDT: Linear Variable Differential Transformer

A ferromagnetic core moves with respect to two opposite windings, changing the coupled flux.

- S-shaped characteristics
- Bulky
- Slow response (uses low frequency)
- Sensitive to magnetic fields
- It is the most widely used displacement sensor
Types of Displacement and Angle Sensors (cont)

CAPACITIVE SENSORS

Piston type variable Capacitor

f~1 MHz

Differential Capacitors

(Noise < 100 μV RMS)
(Bandwidth ~3 kHz)
Advantages of Capacitive Sensors

- Excellent linearity over entire dynamic range when Area is changed (since stray electric fields are small)
- The system responds to average displacement of a large area of a moving electrode
- Freedom of electrode materials and geometry for demanding environments and applications
- Fractional change in capacitance can be made large
- Capacitive sensors can be made to respond to displacements in one direction only
- The forces exerted by the measuring apparatus are electrostatic, and usually small enough so that they can be disregarded
- Capacitors are noiseless: excellent S/N ratio can be obtained (or their dissipation factor D is large enough that the dominant noise sources are elsewhere)

\[
C = \varepsilon \cdot \frac{A}{d}
\]
Types of Readout

Two major categories:
1. Readout based on Resonance
2. Readout based on Bridge method

Readout based on Resonance

1. Measure frequency change of an oscillator built around the variable capacitor
2. Excite at resonance, measure amplitude change
3. Excite at resonance, measure phase change
4. Use feedback loop and VCO oscillator to track resonance change
Readout based on Resonance (cont)

Advantages
- Makes use of the high Q of a resonant circuit: does not require a low noise preamplifier
- Sensitive
- Simple and straightforward

Disadvantages
- All resonating elements are created equal: it cannot distinguish a change in L from a change in C.
  The overall stability depends on the stability of BOTH L and C, with different temperature coefficients and stray effects.
- Cannot take advantage of differential capacitance change
Readout Based on Resonance: Example
Rocking Beam Balancing in Atomic Force Microscope (G. L. Miller)
Types of Readout: AC Bridge

\[
C_1 - C_2 = \varepsilon A/(\delta x - x_0) - \varepsilon A/(\delta x + x_0) = 2 \ C_0 \ \delta x/x_0
\]

\[A=400 \ mm^2 \ x_0 = 75 \ \mu m \quad C_1 = C_2 = 50 \ \text{pF}\]

\[l = 30 \ mm \quad \delta \theta = 1 \ \text{nrad} \quad \delta x = \delta \theta \ l = 30 \ 10^{-12} \ \text{m}\]

\[\delta C = C_1 - C_2 = 40 \ \text{aF}\]

\[\text{Force} = \frac{1}{2} \ C_1 \cdot v_1^2\]

\[\text{F = 0 for a symmetric system}\]

\[i_D = V1 \ \omega (C_1 - C_2)\]
Types of Readout: AC Bridge
Readout Equivalent Circuit

For zero output from the capacitance balance detector must be:

\[
\frac{C_1}{C_2} = 1 + L_B (C_2 + C_B) \omega^2 - L_A (C_1 + C_A) \omega^2 + \frac{R_E}{R_A} - \frac{R_F}{R_B} + \frac{\delta V_1}{V_1}
\]

<table>
<thead>
<tr>
<th>L_A, L_B</th>
<th>0.4 mH</th>
<th>ratio transformer Leakage inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_E, R_F</td>
<td>&lt;1 Ω</td>
<td>transformer secondary DC resistance</td>
</tr>
<tr>
<td>R_A, R_B</td>
<td>&gt;10 MΩ</td>
<td>Dielectric losses</td>
</tr>
<tr>
<td>C_A, C_B</td>
<td>~ 1000 pF</td>
<td>Stray capacitance to “ground” (cables, interwinding C)</td>
</tr>
<tr>
<td>R_D, C_D</td>
<td></td>
<td>Impedance shunting the preamplifier. Does not affect balance</td>
</tr>
<tr>
<td>Z_D</td>
<td>low</td>
<td>Current preamplifier input impedance</td>
</tr>
</tbody>
</table>
Readout Circuit

EXCITATION

Instrumentation Amplifier

AD630

FILTER (LOW PASS)
AC Bridge: Noise

![AC Bridge Diagram]

The “equivalent noise capacitance” can be calculated as:

\[
\overline{\left(\frac{\overline{e^2_n}}{i_{D,n}}\right)} = \left(i_n^2 + \frac{\overline{e^2_n}}{1/\omega_0^2 C_T^2}\right)BW
\]

\[
C_T = C_1 + C_2 + C_D
\]

The “equivalent noise capacitance” can be calculated as:

\[
\left(i_n^2 + \frac{e^2_n}{1/\omega_0^2 C_T^2}\right)BW = \frac{V_1^2}{1/(\omega_0^2 C_n^2)}
\]

so that we have:

\[
\overline{C_n^2} = \frac{1}{V_1^2}\left(\overline{e^2_n C_T^2} + \frac{i_n^2}{\omega_0^2}\right)BW
\]
AC Bridge: Noise (cont)

To reduce noise:
  • Increase $V_1$
  • Decrease Bandwidth (i.e. increase averaging time)
  • Decrease $C_T$ (depends mostly on connection length, strays etc.)

Example: Tiltmeter

$$C_T = 1 \text{ nF}$$
$$e_n = 1 \text{ nV/Hz}^{1/2}$$
$$V_1 = 3 \text{ V}$$

$$C_n = 4 \times 10^{-19} \text{ F} = 400 \text{ zF}$$

Minimum Signal (for 1 nrad angular displacement) $= 40 \text{ aF} = 20 \times 10^{-18} \text{ F}$ $\Rightarrow S/N=100$

Example: Monolithic MEMS circuit

$$C_T = 1 \text{ pF}$$
$$e_n = 10 \text{ nV/Hz}^{1/2}$$
$$V_1 = 5 \text{ V}$$

$$C_n = 2 \times 10^{-21} \text{ F} = 2 \text{ zF}$$
Mechanical Noise

The mechanical rms fluctuation can be computed by means of the fluctuation dissipation theorem. The rms fluctuation of the displacement of a suspended mass m is:

$$\delta x^2 \equiv \frac{k_B T}{m \omega_0^2 \tau S}$$

Where:

- $k_B = 1.38 \times 10^{23}$ J/K
- $m \sim 20$ g
- $\omega_0 = \text{mechanical resonant frequency} \sim 10^5 \text{ rad/sec}$
- $\tau = \text{damping time constant} \sim 5-10 \text{ sec}$
- $S = \text{Averaging period} \sim 1 \text{ sec}$

So that $(\delta x^2)^{1/2} \sim 10^{-17}$ m giving a “noise” too small to be detected.

Absolute Limits:

Since the damping could be increased (e.g. using more dense fluid to increase friction) there is no fundamental limit to the mechanically generated noise.
Other Sources of Errors

- Temperature variations
  
  Thermal expansion => cancelled in a symmetrical design
  Effects on the readout electronics: Gain Variation
  
  temperature dependence of dielectric constant (2 $10^{-6}/^\circ C$ for dry air at STP;
  
  $700 \times 10^{-6}$ for moist air

- Humidity variations
  
  at $20^\circ C$ a change in humidity from 40 to 90% changes the dielectric constant by 200 ppm

- Pressure changes
  
  a pressure change of 1 atm at $20^\circ C$ changes the dielectric constant by 200 ppm
  
  causes dimensional changes (a brass cube of 1 cm contracts by 3 $\mu$m for a 1 atm change)

- Oxidation of surfaces (rhodium plating recommended)

- Stability of materials (70-30 brass gives good results)

- Creep of materials

- Relaxation of screw tension

- Microseismicity (about 2 $\mu$m peak to peak displacement, period 3-8 s)
MEMS Accelerometers

It consists of multiple fingers on each side of a movable center member.

They constitute the center plates of a paralleled set of differential capacitors. Pairs of fixed fingers attached to the substrate interleave with the beam fingers to form the outer capacitor plates. The beam is supported by tethers which serve as mechanical spring.

“Force” fingers are used for calibration

mass = 0.5 μg

SIZE: 0.5 mm x 0.4 mm, 2 μm thick,

- Requirement: Avoid “stiction” => rigid cantilevered beam
Sensor operation; ADI’s implementation

- Folded tethers have more consistent spring constants, leading to better part to part consistency.
Self test operation

- Extra fixed outer plates may be added which when exited, force the proof mass to move. So you can electronically test the accelerometer.

Additional fixed outer plates are electrically excited to induce movement of the proof mass. Acceleration is measured by the standard fixed plates as usual.
Interesting facts

- 0.1µgrams Proof Mass
- 0.1pF per Side for the Differential Capacitor
- 20aF (10\(^{-18}\)f) Smallest Detectable Capacitance Change
- Total Capacitance Change for Full-scale is 10fF
- 1.3µm Gaps Between Capacitor Plates
- 0.2Å Minimum Detectable Beam Deflection (one tenth of an Atomic diameter)
- 1.6 µm Between the Suspended Beam and Substrate
- 10 to 22kHz Resonant Frequency of Beam
MEMS Accelerometers:

**Readout**

![Readout circuit diagram]

**Noise**

<table>
<thead>
<tr>
<th>Specifications</th>
<th>ADXL105</th>
<th>ADXL202</th>
<th>ADXL05</th>
<th>ADXL50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>±5 g</td>
<td>±2 g</td>
<td>±5 g</td>
<td>±50 g</td>
</tr>
<tr>
<td>Noise (μg/√Hz)</td>
<td>225</td>
<td>500</td>
<td>500</td>
<td>6500</td>
</tr>
<tr>
<td>Bandwidth (kHz)</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Supply Current (mA)</td>
<td>2</td>
<td>0.6</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Number of Axes</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Output Type</td>
<td>Analog</td>
<td>Analog/ Digital</td>
<td>Analog</td>
<td>Analog</td>
</tr>
</tbody>
</table>

**ADXL105:**

C = 150 fF

Sensitivity = 100 aF for 1g acceleration

Noise = 225 g/Hz\(^{1/2}\)

“capacitance” noise = \(22.5 \times 10^{-21} \text{ F/Hz}^{1/2}\) = 22.5 zF/Hz\(^{1/2}\)
Bibliography

1. G.L. Miller “Sensor and actuators for small motions”, unpublished report available from S. Rescia
7. A good tutorial on random noise in mechanical system as applied to gravitational wave antennas is in P. S. Saulton “Physics of gravitational wave detection: resonant and interferometric detectors” available at “http://www.astro.psu.edu/users/steinn/Astro597/saulson.pdf”