A new method of measuring electric dipole moments in storage rings

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A new method of looking for electric dipole moments of charged particles in storage rings is described. The major systematic errors inherent in the method are addressed and ways to minimize them are suggested. It seems possible to measure the muon EDM to levels that test speculative theories beyond the standard model.

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The existence of an electric dipole moment (EDM) for an elementary particle would violate parity (P) and time reversal symmetry (T) [1]. In the standard model, the electron EDM is $< 10^{-38}\mathrm{e}\cdot\mathrm{cm}$ [2] with the muon EDM scaled up by the mass ratio m_μ/m_e, a factor of 206, but some new theories predict much larger values [3–7]. For example, ref. [4] predicts the muon EDM could be as large as $5\times10^{-23}\mathrm{e}\cdot\mathrm{cm}$, while the electron EDM is predicted to be $\sim 10^{-28}\mathrm{e}\cdot\mathrm{cm}$, an order of magnitude below the present limit [8]. The present 95% confidence limit for the muon EDM is $10^{-18}\mathrm{e}\cdot\mathrm{cm}$ [9]. This paper discusses a new way of measuring the EDM of the muon, which may also be applied to other particles.

To measure the EDM experimentally, the particle should be in an electric field which exerts a torque on the dipole and induces an observable precession. If the particle is charged this electric field inevitably accelerates the particle; it will move to a region where the field is zero or leave the scene according to Schiff's theorem [10]. There are many known exceptions to Schiff's theorem when weak and strong nuclear forces, weak electron-nucleon forces and relativistic forces are included. This theorem does not apply to particles in a storage ring, particularly to the method described here, where motional fields are employed, because it is not possible to factorize particle velocity and electric field, which constituted the basis of the theorem [11].

In particular, when muons of velocity $\vec{\beta} = \vec{v}/c$ and relativistic mass factor $\gamma = (1-\beta^2)^{-1/2}$ are circulating in a vertical magnetic field $\vec{B}$, the electric field in the muon rest frame is $\vec{E}^* = \gamma c\vec{\beta}\times\vec{B}$ and can be much larger than any electric field realizable in the laboratory. In this situation, the muon spin precesses relative to the momentum vector about a vertical axis because of its magnetic anomaly $a = (g-2)/2$, at angular frequency $\vec{\omega}_a = -a(e\vec{B}/m)$. If there is an EDM of magnitude $d = \eta e\hbar/4mc \simeq \eta\times 4.7\times10^{-14}\mathrm{e}\cdot\mathrm{cm}$ there will be, in addition, a precession angular frequency

$$\vec{\omega}_e = -\frac{\eta}{2}\frac{e}{m}\,\vec{\beta}\times\vec{B} \tag{1}$$

about the direction of $\vec{E}^*$, that is radial with respect to the orbit [12]. The vector combination of $\vec{\omega}_a$ (vertical) and $\vec{\omega}_e$(radial) tilts the precession plane sideways leading to a small vertical component of muon spin, oscillating at angular frequency ω_a. This produces an oscillating vertical asymmetry in the number of decay electrons emitted in muon decay. A search for this asymmetry during the CERN $(g-2)$ experiment [13] led to the current limit [9] which corresponds to $\omega_e/\omega_a \leq 10^{-2}$.

This method is being used during the current muon $(g-2)$ experiment at Brookhaven [14], but is limited by serious systematic effects. First, the EDM can in effect only act for about one quarter of the $(g-2)$ period. Also the two extremes of the vertical oscillation occur when the muon spin is aligned radially inwards and outwards. In these two extremes the decay electrons, whose up/down asymmetries are to be compared, follow rather different tracks through the magnetic field. In one case the majority are emitted radially inwards and take a short path to the detectors; while in the other case they are emitted predominantly outwards and reach the detectors after a longer track with more opportunity to spread vertically or to be bent by stray radial magnetic fields. The horizontal $(g-2)$ precession thus interferes with attempts to observe the vertical precession due to the EDM.

The new technique is to cancel the $(g-2)$ precession $\vec{\omega}_a$ so that $\vec{\omega}_e$ can operate by itself, causing the muon spin to precess slowly and continuously about a radial axis, gradually building up an observable vertical asymmetry. This can be achieved by applying a strong radial electric field $\vec{E}$ to the orbit. The equation for the $(g-2)$ precession in a vertical magnetic field $\vec{B}$ and a radial electric field $\vec{E}$ with $\vec{\beta}\cdot\vec{E} = \vec{\beta}\cdot\vec{B} = 0$ is

$$\vec{\omega}_a = -\frac{e}{m}\left[a\vec{B} + \left(\frac{1}{\beta^2\gamma^2} - a\right)\vec{\beta}\times\vec{E}/c\right] \tag{2}$$

If $1/(\beta^2\gamma^2) \gg a$ and the electric field is adjusted to

$$E = E_0 = aBc\beta\gamma^2 \qquad (3)$$

ω_a can be reduced to zero. The correct value can be set in the laboratory by monitoring the gradual cancellation of the $(g-2)$ precession with electron detectors on the inside of the ring. Then $\vec{\omega}_e$ in equation (1) will have its full effect, moving the spin steadily out of the horizontal plane. The vertical asymmetry can be observed with detectors, located above and below the orbit, to measure the EDM without the systematic errors mentioned above.

To obtain the best accuracy it is desirable to use a high magnetic field and high energy muons which live longer and so have more time to precess vertically under the action of a hypothetical EDM. But equation (3) shows that this would require impractically large electric fields. The parameters of a possible experiment are shown in Table 1.

Table 1

E	Aperture	B	p	γ	τ	R
2 MV/m	0.1 m	0.25 T	0.5 GeV/c	5	2.2μs	7 m

The uncertainty in η is

$$\sigma_\eta = \frac{\sqrt{2}}{\gamma\tau(e/m)\beta BAP\sqrt{N}}, \qquad (4)$$

where A is the vertical asymmetry of the detected electrons for 100% muon beam polarizarion and P is the actual muon beam polarization. N is the total number of detected electrons.

For example, to reach the sensitivity of 10^{-24}e $\cdot$ cm in the EDM corresponding to $\omega_e/\omega_a = 10^{-8}$, the vertical spin angle to be measured after 3 lifetimes (33μs) would be $\sim$ 50 nR, generating a counting asymmetry 10^{-8} and requiring about 4×10^{16} registered events, assuming $A = 0.3$ and $P = 0.5$, i.e. $NP^2 = 10^{16}$. In contrast, the present $(g-2)$ measurement at Brookhaven [14] has recorded a total of order 10^{10} events which, in principle, would give an EDM sensitivity of $\sim 10^{-21}$e $\cdot$ cm. To reach 10^{-24}e$\cdot$cm would require a high intensity muon source plus a storage ring of large acceptance. The muon EDM collaboration has submitted a letter of intent to J-PARC [15] which satisfies these requirements.

In practice detectors, called $(g-2)$ detectors, would be set up to monitor the $(g-2)$ precession (horizontal spin motion). Other detectors, called EDM detectors, would be set up to measure the vertical spin motion. With the electric field set to some value below E_0 one can observe the $(g-2)$ precession and adjust the EDM detectors so that they are insensitive to horizontal spin components. As E approaches E_0 the $(g-2)$ frequency will gradually decrease but the amplitude of the $(g-2)$ signal should remain the same. At the same time the EDM signal should have the same period as $(g-2)$ but its amplitude should grow. When the $(g-2)$ motion is cancelled the EDM signal should grow linearly with time.

A number of imperfections in the magnetic or electric fields would make the spin move out of the plane of the orbit even though the EDM is zero, giving rise to a false EDM signal. In the discussion "the horizontal plane" means the plane of the orbit, while "vertical" and subscripts "V" refer to components normal to the plane of the orbit. The following imperfections have been considered:

1. Vertical corrugations of the orbit due to a radial magnetic field B_r.
2. The plane of the radial electric field does not coincide with the plane defined by the magnetic field (called the "magnetic plane"), that is $E_{\rm v} \neq 0$ although the electric field is perfectly in a single plane so $\langle E_{\rm v}\rangle = 0$ with the brackets $\langle\ \rangle$ indicating the average over the orbit.
3. The electric field is not in one plane, $\langle E_{\rm v}\rangle \neq 0$.
4. Local orbit distortions near the detectors simulate detector rotation around the beam direction, so small residual $(g-2)$ precession ("horizontal") has a component in the "vertical" direction looking like a false EDM.
5. Change of up detector response relative to down detector response during the muon storage time.
6. Azimuthal components B_θ of the magnetic field parallel to the momentum vector $\vec{p}$. Although $\langle B_\theta\rangle = 0$ if there is no electric current through the orbit, higher harmonics of the azimuthal B-field could be significant.

We will discuss these effects in turn. In any ring structure with magnetic and/or electric fields for each particle momentum, there exists a closed orbit; the particle repeats this track perfectly from turn to turn. Other particles, starting at different transverse positions or transverse angles, oscillate about the closed orbit. To define the plane of the closed orbit, split it up into many small equal sections, each with its local angular velocity vector $\vec{\omega}$ and find the average value $\langle\vec{\omega}\rangle$. The orbit plane is defined as the plane perpendicular to $\langle\vec{\omega}\rangle$; on average the momentum vector rotates in this plane but may oscillate above and below it. With only a magnetic field, the orbit plane will therefore be defined by the average direction of $\vec{B}$. The radial electric field may not lie exactly in this plane. In this case the orbit plane will change when the electric field is applied.

Since we are interested in the spin direction relative to the momentum vector, we consider the electric and magnetic field components $\vec{E}^*$ and $\vec{B}^*$ in the rest frame of the particle circulating in the orbit plane. For the closed orbit to be stable vertically, the mean vertical force in the lab frame

$$\langle E_\mathrm{v} + \beta B_r \rangle \;=\; 0. \tag{5}$$

Transforming to the rest frame one finds $\langle E^*_\mathrm{v}\rangle = 0$, not unexpectedly because in the rest frame it is $\vec{E}^*$ that moves the orbit while B^* generates no force.

B^*_r rotates the spin out of the orbit plane when the EDM is zero. Using Eq. (5)

$$\langle B^*_r\rangle = \langle \gamma B + \beta\gamma E_\mathrm{v}\rangle = -\langle E_\mathrm{v}/\beta\gamma\rangle. \tag{6}$$

It follows that with no electric field there is no false EDM whatever the shape of the orbit (error 1). If the radial electric field is exactly in one plane so that $\langle E_\mathrm{v}\rangle = 0$ then $\langle B^*_r\rangle$ is zero and there is again no false EDM, (error 2).

A further effect of electric field misalignment is that $\vec{\beta}\times\vec{E}$ is not parallel to $\vec{B}$ so that when (3) is satisfied there is a net horizontal angular velocity $\vec{\omega}_r$ acting on the spin. However, if this is radially inwards on one side of the ring, it will be radially outwards on the other, generating a small vertical spin oscillation which does not accumulate from turn to turn as long as the $(g-2)$ precession is zero: no false EDM. If there is a residual $(g-2)$ precession and a radial electric field is present, it is possible that a radial spin component can be transformed into a vertical component. This is the case when the radial electric field is not exactly orthogonal to the magnetic field. A single detector at a specific azimuthal location will observe a small EDM like signal that has the opposite sign at a detector located 180° apart. The effect is proportional to the misalignment of the electric and magnetic fields from orthogonality and it goes to zero when the detector signals from all azimuthal locations are summed.

If the radial electric field is not precisely in a plane (error 3) there will be a net vertical electric field $\langle E_\mathrm{v}\rangle \neq 0$. This will move the orbit until $\langle B^*_r\rangle$ satisfies Eq. 6 and this will precess the spin out of the plane generating a false EDM. Every precaution must be taken to minimize this effect but fortunately it is cancelled by injecting the particles clockwise (CW) and counter-clockwise (CCW). This requires discussion of the signs of the real and false EDM signals for μ^+ and μ^- in each case. The following equations indicate the signs (not magnitudes) of the real and false EDM angular velocities $\vec{\omega}_e$ and $\vec{\omega}_F$:

$$\vec{\omega}_e \;=\; \vec{\sigma}\times\left[\vec{d}\times\left(\vec{p}\times\vec{B}\right)\right] \tag{7}$$

$$\vec{\omega}_F \;=\; \vec{\sigma}\times\left[\vec{\mu}\times\left(\vec{p}\times\vec{E}_\mathrm{v}\right)\right] \tag{8}$$

If there is a finite EDM $\vec{d}$, the CPT theorem requires $\vec{d}\cdot\vec{\sigma}$ to change sign going from μ^+ to μ^-. In Table 2 we show the truth table for the four different configurations μ^+ / μ^- combined with the orbit directions CW/CCW listing the variables in the order $\vec{p},\ \vec{\sigma},\ \vec{\mu},\ \vec{d},\ \vec{B},\ \vec{E}_\mathrm{v}$. We are displaying the situation at a fixed point in the ring, assuming that the muons come from pion decay in the backward direction and we arbitrarily make all variables positive for the reference case (μ^+, CW).

Table 2

	CW	CCW
Particle	$\vec{p},\ \vec{\sigma},\ \vec{\mu},\ \vec{d},\ \vec{B},\ \vec{E}_\mathrm{v}$	$\vec{p},\ \vec{\sigma},\ \vec{\mu},\ \vec{d},\ \vec{B},\ \vec{E}_\mathrm{v}$
μ^+	$+,\ +,\ +,\ +,\ +,\ +$	$-,\ -,\ -,\ -,\ -,\ +$
μ^-	$+,\ -,\ +,\ +,\ -,\ -$	$-,\ +,\ -,\ -,\ +,\ -$

If the muons are emitted backward in the decay of pions in flight, the majority of decay electrons initially go forward (in the direction of $\vec{p}$), so the observed asymmetry obeys:

$$\vec{A}_{e,F} \;=\; \vec{p}\times\vec{\omega}_{e,F}. \tag{9}$$

Applying equations 7 , 8 and 9 the results for real EDM asymmetry A_e and false EDM asymmetry A_F are listed in Table 3.

Table 3

	CW	CCW
Particle	$A_e,\ A_F$	$A_e,\ A_F$
μ^+	$+,\ +$	$-,\ +$
μ^-	$+,\ +$	$-,\ +$

We see that the real EDM signal changes sign when the direction of rotation in the ring is reversed, while the false EDM due to the out-of-plane electric field remains the same. So this error may be cancelled by changing from CW to CCW if all other factors can be held the same.

If the electric field is misaligned but in a plane (error 2) the orbit plane will change when the electric field is applied, so the detectors, set to respond only to vertical spin components will include small contribution from horizontal spin. This will however be opposite on opposite sides of the ring so will largely cancel. Error 4 has a similar effect.

The response of upper and lower detectors may change with time (error 5), so that a false asymmetry develops during each muon storage cycle. Such effects can be caused by unequal detector responses to the changing counting rates or transients in the system triggered at injection time. This effect should remain the same when muons are injected CW and CCW while the real EDM asymmetry changes sign.

It might be supposed that the electric and magnetic fields could be applied to separate sections of the orbit, with the result that the spin makes small to and fro movements about the vertical axis but the net $(g-2)$ precession is zero over one turn. While this would fulfill the main requirement, some misalignment errors would not be perfectly cancelled. For example, a harmonic of the azimuthal field B_θ (error 6) would cause the spin to oscillate about the horizontal axis parallel to $\vec{p}$. Because rotations do not commute, the combination with the $(g-2)$ oscillation would generate a net rotation about the radial axis, leading to a false EDM. This is an example of Berry's phase [16].

Similarly, a misalignment of the electric field (error 2) would generate an oscillating radial angular velocity ω_r as explained above. Combined with the $(g-2)$ oscillation this would give rise to a false EDM. To avoid these effects, the electric and magnetic fields must be located at the same place and (3) should be adequately satisfied at every point along the orbit.

Further tests can be made by injecting muons with the opposite longitudinal polarization coming from pion decay in the forward direction. In this case the maximum decay electron intensity is directed backwards and all asymmetries are reversed. But a false asymmetry due to detector effects (error 5) should remain the same.

Therefore, all false signals, unlike the EDM signal, will be cancelled by CW and CCW beam injection and by summing up the counts of all the detectors.

The method can be applied to other particles or atoms provided [17] that the gyromagnetic ratio g is not too far from 2 so the $(g-2)$ precession can be cancelled by an accessible electric field, for example the deuteron. Since the deuteron is stable, another scheme must be utilized to track the deuteron spin. This would most likely involve the use of an internal target in the ring. One possible target is hydrogen gas as elastic d+p scattering is sensitive to all of the polarization moments of the deuteron beam [18].

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