# PROTON-CARBON EFFECTIVE ANALYZING POWER BETWEEN 95 AND 570 MeV 

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The p-C effective analyzing power has been measured with a good accuracy for laboratory scattering angles between $5^{\circ}$ and $20^{\circ}$ at 25 energies from 95 to 570 MeV . Carbon targets from 3 to 7 cm have been used. Measurements have been made at SIN with multiwire proportional chambers. A smooth angle and energy depending function has been fitted to the data. Reasonable agreement has been found with other available data.

## 1. Introduction

In urder to determine the spin dependence of the nuclear interaction, measurements of the polarization of particles in their final state is often required. At intermediate energies ( 100 MeV to 1 GeV ), the measured asymmetry resulting from a second scattering is widely used. The polarization of the particles can be extracted provided the analyzing power $A_{c}$ of the second scattering is known.

We present here the calibration measurement of a carbon polarimeter for protions used at SIN in the p-p elastic scattering program. These measurements have alluwed us to determine completely the scattering matrix [1,2] by the mean of the measurement of 2 - and 3 -spin observables $[2,3]$.

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## 2. Apparatus

The SIN PM1 proton beam line was used in its parasitic mode. This beam is produced by the elastic scattering of the 590 MeV unpolarized beam on a beryllium target at $8^{\circ}$ laboratory angle. The resulting beam polarization is $P_{B}=0.4165 \pm 0.0043$ as shown in ref. 4. A variable thickness copper degrader is used to lower the beam energy. Depolarization effects are negligible since particles are collected at zero degree after the degrader within a small solid angle and hecause the Coulomb interaction is dominant at these angles. Finally a superconducting solenoid allows a rotation of the beam polarization wathin the transwersal plane up to $180^{\circ}$.

Fig. 1. shows a scale drawing of the polarimeter. It consists of a variable length carbon target sandwiched by multiwire proportional chambers (MWPC) with ? mm wire spacing. The incoming and outgoing tracks are detected by two telescopes consisting of $\mathbf{3}$ (respectively 4) $x-y$ MWPCs. A scintillator (Z) placed into the first telescope is used to detect an incoming particle.


Fig. 1. Scale drawing of the apparatus.

The polarimeter is mounted on a turntable and was flaced direcily into the beam for the present measurement. Two beam counters A and B were also added. No coumters were used behind the carbon target.

## 1. Data acquisition system

A first level trigger was made with the coincidence between the A. B and $\mathbf{Z}$ counters. This opened the gate of the wire memories and initiated the MWPC coding system operations.

A second level hardware trigger, connected to the MWPC logic, was used to reduce the amount of data [5]. The slopes of the particle trajectories and the resulting scattering angle were then computed. Events without significant scattering were rejected. Configurations which were impossible to reconstruct in a further processing were also rejected. $90-95$ \% of the events were rejected. The duration of this decision was about $3.5 \mu \mathrm{~s}$.

Events having passed this filter were then transfered to DPNC $\$ 11$ minicomputers [6] under the control of a PDP $11 / 20$. The minicomputers fully reconstructed the events and accumulated event statistics and histogra is. Only sums were recorded onto magnetic tapes.

## 4. Measurements

The geometrical position of the MWPC's were measured on-line using unscattered events taken without the carbon target

Twent-five measurements were made at various enrege with a 3.5 or 7 cm thick carbon target. The 3 cm target was used mainly at low energies. For each measurement at least two datasets were taken with opposite bean polarization directions. Some of these were taken with 4 orientations: up. down. left and right.

During these measurements the operational conditions of the 2nd level trigger were changed in order to accept some unscattered data for a check of the geometry. These data were used to monitor possible residual misalignments and allowed an off-line correction of the asymmetries. An estimation of the angular resolution of the apparatus was also possible with these data.

## 5. Analysis

## $\therefore 1$ General formalism

Consider an incident proton beam propagating along the $\hat{k}$ direction with a transverse polarization vector $\boldsymbol{P}_{\mathrm{B}}$ described by
$\boldsymbol{P}_{\mathrm{B}}=P_{x} \cdot \hat{\boldsymbol{X}}+P_{y} \cdot \hat{\boldsymbol{Y}}$.

Let $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{k}}^{\prime}$ be unit vectors along the trajectories entering and leaving the carbon. For the scattering in a plane whose normal vector is given by
$\hat{n}=\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}^{\prime} /\left|\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}^{\prime}\right|$.
the polar $\theta$ and azimuthal $\phi$ scattering angles are defined by
$\sin \theta=\left|\hat{k} \times \hat{k}^{\prime}\right|, \quad \sin \phi=-\hat{X} \cdot \hat{n}, \quad \cos \phi=\hat{\boldsymbol{Y}} \cdot \hat{n}$.
The polarized cross section $I(\theta, \phi)$ is expressed by

$$
\begin{aligned}
I(\theta, \phi) & =I_{0}(\theta)\left[1+A_{\mathrm{c}}(\theta) P_{\mathrm{B}} \cdot \hat{n}\right] \\
& =I_{0}(\theta)\left[1+P_{y} A_{\mathrm{c}}(\theta) \cos \phi-P_{x} A_{\mathrm{c}}(\theta) \sin \phi\right] \\
& =I_{0}(\theta)\left[1+\epsilon_{n}(\theta) \cos \phi+\epsilon_{\mathrm{s}}(\theta) \sin \phi\right] .
\end{aligned}
$$

where $I_{0}(\theta)$ is the unpolarized cross section, $A_{c}(\theta)$ the carbon analyzing power and $\epsilon_{\mathrm{n}}, \epsilon_{\mathrm{s}}$ the asymmetries. These asymmetries can be obtained by a statistical analysis of the azimuthal distribution. Knowledge of $\boldsymbol{P}_{\mathrm{B}}$ allows extraction of $A_{\mathrm{C}}$.

A relative misalignment of the detectors produces errors in the reconstructed scattering angles $\theta$ and $\phi$ :
$\Delta \theta_{s}=\Delta(\operatorname{tg} \theta \cdot \sin \phi), \quad \Delta \theta_{y}=\Delta(\operatorname{tg} \theta \cdot \cos \phi)$.
As shown in ref. 7, the effects on the measured asymmetries are
$\Delta \epsilon_{n}=\Delta \theta_{\mathrm{x}} \frac{\mathrm{d}}{\mathrm{d} \theta}\left(\log I_{0}\right)$,
$\Delta \epsilon_{1}=\Delta \theta_{1} \frac{\mathrm{~d}}{\mathrm{~d} \theta}\left(\log I_{0}\right)$.
if the misalignments $\Delta \theta_{x}$ and $\Delta \theta_{1}$ are small.
These effects can hence be corrected if $I_{0}(\theta) . \Delta \theta$, and $\Delta \theta_{1}$ are known.

### 5.2. On-line analysis

Events accepted by the 2 nd level trigger were transferred to minicomputers for reconstruction. The raw coordinates were first corrected for the geometrical displacements of the MWPC's from their nominal positions. Events were then fully reconstructed and tested for the following requirements:

1) MWPC track alignment of the incoming and outgoing trajectories. The $S^{2}$ per degree of freedom was requested to be less than $4 \mathrm{~mm}^{2}$.
2) Projection of the incoming track into the last backward chamber in order to guarantee a $100 \%$ geometrical acceptance.
3) The distance of closest approach between incoming and outgoing tracks. This had to be less than 4 mm .
4) The reconstructed longitudinal position of the carbon vertex was required to fall within the target with an accuracy of $\pm 27 \mathrm{~mm}$.
5) Range of $\theta$ between $5^{\circ}$ and $20^{\circ}$.

Cuts (1) to (4) eliminated mainly events scattered on the counters or the MWPCs which were accepted by the

2nd level trigger. From 6 to 11 of the processed events were rejected mainly by cut (1), while 40 to $50 \%$ of the remaining events were rejected by cut ( 5 ) mainly because of a too small scattering angle. Accepted events were finally accumulated in a two dimensional histogram divided into 15 bins for $\theta$ and 16 bins for $\phi$.

Processing of the unscattered events differed slightly. No vertex was computed and a cut was applied on $\theta$ whose maximum value depends on energy and target length. Sums were accumulated in order to evaluate the mean and the width of the distribution of the projected angles $\operatorname{tg} \theta \cdot \cos \phi$ and $\operatorname{tg} \theta \cdot \sin \phi$.

### 5.3. Off-line analysis

The analysis was facilitated by the $100 \%$ geometrical acceptance. The asymmetries were extracted at each polar angle $\theta$ by a simple Fourier analysis of the $\phi$ distributions obtained from the two dimensional histograms. The effect of the bin width in $\phi$ was also taken into account. The angular dependence of the unpolarized inclusive cross section was also obtained from these histograms.

The residual misalignments were found to be less than $0.02^{\circ}$. The corresponding corrections have been applied on the asymmetries. They were of the order of 0.004 and smaller than the statistical errors.

The analyzing power $\boldsymbol{A}_{\mathrm{c}}$ was extracted for each beam polarization orientation. The data are very consistent as shown in table 1 . These have been consequently summed over the different orientations. This operation generally cancels the effects of the misalignment corrections.

## 6. Results

The experimental results for the effective proton carbon analyzing power $A_{c}$ are shown in table 1 and in fig. 2. $A_{\mathrm{c}}$ is maximum around 200 MeV and decreases rapidly at low energies. The inclusive differential cross section ( $\mathrm{d} \sigma / \mathrm{d} \Omega$ ) results can be seen in table 1 . The cross section values are in arbitrary units. Quoted errors are purely statistical. The $A_{c}$ results are subject to an overall normalization uncertainty of $1 \%$ coming from the beam polarization. The given kinetic beam energies are computed at the center of the target.

The target thickness was found to have no visible effect above 270 MeV . At lower energies and at larger angles however, the 7 cm data shows an higher analyzing power than the 3 cm data. This can be understood by considering the amount of inelastic events accepted by the apparatus. Since low energy particles cannot escape from a thick target, the length of the target acts as an inelasticity limiter. The 3 cm target allows a larger amount of inelasticities and the effective analyzing power is thus reduced.

At low energies $A$, drops rapidl to ere at imall angles. This is due to multupic Coulomh wallering which extends up to our measured angles. This effect is of course very dependent on the target length.

## 7. Energy dependent fits

A parametrization of $A_{\mathrm{c}}$ was necessary in: the pp elastic scattering data analysis [3] where the carbon analyzing power was needed for continuously variable kinematic conditions. We have performed an angle and energy dependent smoothing by adjusting the following empirical formula to our experimental data:

$$
\begin{aligned}
A_{\mathrm{c}}(\theta, T)= & D(\theta, T) \alpha(T) \\
& \times\left[\frac{\sin \theta}{1+\beta(T) \sin ^{2} \theta+}, T\right) \sin ^{4} \theta \\
& +\delta(T) \sin \theta]
\end{aligned}
$$

This expression is similar to the one suggested in ref. 8 but two extra terms $\delta(T)$ and $D(\theta, T)$ have been added. $\alpha, \beta, \gamma$ and $\delta$ are energy dependent poin nomish. of the form:
$\alpha(T)=\alpha_{0}+\alpha_{1} X+\alpha_{2} x^{2}+\alpha_{3} \cdot x^{3}+\alpha_{4} X^{4}$.
where $X=\left(T-T_{\text {entral }}\right) T_{\text {range }}$ is a dimensiontes energy variable depending on the region where the fit 1 . applied.
$D(\theta, T)$ is an empirical damping factor unced $w$ reproduce the sharp drop of $A$, touard small angle induced by multiple Coulomb swattering at low energes This can he expressed as:
$D(\theta, T)=-\frac{1}{1+C \exp \left[\theta^{2}, 2 \theta^{2}(T)\right]}$.
The term $\theta_{2}^{2}(T)=C_{0}+C_{1}(15 ; p \beta)^{2}$ is an attempt $\omega$ describe the angular resolution as the sum of the constant MWPC resolution plus a momentum dependent multiple scattering term. $C_{0}$ has been fixed to the measured value of the MWPCs resolution extrapolated $w$ $T \rightarrow \infty . C$ and $C_{1}$ are free parameters, the 15 $p \beta$ term being expressed in degrees.

Since the target length has a non negligible influence at low energies. it was not possible to fit all the data ower the catioc encrgy range. To facilitate the anathas of the pp experiment [3]. two separate fits were made
(1) A high energy fit (denoted by " $\mathrm{H}^{\prime}$ ) , alld fir ${ }^{-}$ cm target from 150 to 571 MeV . This contams all the present 7 cm data, 5 cm data above 270 MeV and 3 cm data above 300 MeV . Data from our previous experiment (SIN - DRAP. ref. 7). taken with the same beam and with a similar apparatus above 299 MeV . were also included.
Table I
Proton-carbon effective analyang power and differential crost-section. Cross-section data are in arbitrary units

| Energy | $151 \pm 3 \mathrm{MeV}$ | $179 \pm 3 \mathrm{McV}$ | $215 \pm 2 \mathrm{MeV}$ | $225+2 \mathrm{McV}$ | $269 \pm 2 \mathrm{MeV}$ | $300 \pm 2 \mathrm{MeV}$ | $345 \pm 2 \mathrm{MeV}$ | $384 \pm 2 \mathrm{MeV}$ | $427 \pm 2 \mathrm{McV}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 7 cm | 7 cm | 7 cm | 7 cm | 7 cm | 7 cm |  |  |  |
| Target length | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.46 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $1240 \mathrm{~g} / \mathrm{cm}^{2}$ |
| Angular resolution | $1.315^{\circ}$ |  | $1.043^{\circ}$ |  |  | $0.753^{\circ}$ | $0.684^{\circ}$ | 0.645 ${ }^{\circ}$ | 0.593 ${ }^{\circ}$ |
| Up-aown consistency $\left(x^{2} / d\right)$ | 1.740 | 1.092 | 0.929 | 1.383 | 1.332 | 1.087 | 0.869 | 1.688 | 1.152 |
| $\theta 1^{\circ} \mathrm{l}$ | $A_{c}$ | $A_{c}$ | $A_{\text {c }}$ | $A_{\text {c }}$ | $A_{\text {c }}$ | $A_{s}$ | $A_{1}$ | $A_{\text {c }}$ | $A_{\text {c }}$ |
| 5.5 | $0.156 \pm 0.011$ | $0.282 \pm 0.016$ | $0.355 \pm 0.013$ | $0.330 \pm 0.017$ | $0.385 \pm 0.015$ | $0.395 \pm 0.016$ | $0.365 \pm 0.010$ | $0.356 \pm 0.015$ | $0.326 \pm 0.011$ |
| 6.5 | $0.269 \pm 0.015$ | $0.414 \pm 0.017$ | $0.445 \pm 0.013$ | $0.444 \pm 0.017$ | $0.451 \pm 0.015$ | $0.450 \pm 0.015$ | $0.423 \pm 0.010$ | $0.415 \pm 0.015$ | $0.393 \pm 0.011$ |
| 7.5 | $0.368 \pm 0.015$ | $0.426 \pm 0.016$ | $0.493 \pm 0.012$ | $0.501 \pm 0.017$ | $0.487 \pm 0.015$ | $0.510 \pm 0.015$ | $0.466 \pm 0.010$ | $0.439 \pm 0.015$ | $0.399 \pm 0.011$ |
| 8.5 | $0.420 \pm 0.014$ | $0.464 \pm 0.016$ | $0.522 \pm 0.012$ | $0.522 \pm 0.016$ | $0.559 \pm 0.015$ | $0.548 \pm 0.016$ | $0.511 \pm 0.010$ | $0.447 \pm 0.016$ | $0.405 \pm 0.012$ |
| 9.5 | $0.425 \pm 0.014$ | $0.503 \pm 0.016$ | $0.575 \pm 0.012$ | $0.610 \pm 0.016$ | $0.570 \pm 0.015$ | $0.581 \pm 0.016$ | $0.484 \pm 0.011$ | $0.436 \pm 0.016$ | $0.391 \pm 0.013$ |
| 10.5 | $0.422 \pm 0.014$ | $0.537 \pm 0.016$ | $0.588 \pm 0.013$ | $0.627 \pm 0.017$ | $0.554 \pm 0.016$ | $0.564 \pm 0.017$ | $0.483 \pm 0.011$ | 0.435 $\pm 0.017$ | $0.384 \pm 0.013$ $0.355 \pm 0.014$ |
| 11.5 | $0.426 \pm 0.014$ | $0.547 \pm 0.016$ | $0.618 \pm 0.013$ | $0.600 \pm 0.017$ | $0.571+0.016$ | $0.533 \pm 0.017$ $0.524 \pm 0.018$ | $0.478 \pm 0.012$ $0.442 \pm 0.012$ | $0.407 \pm 0.018$ $0.391 \pm 0.019$ | $0.355 \pm 0.014$ $0.348 \pm 0.015$ |
| 12.5 | $0.461 \pm 0.014$ | $0.587 \pm 0.017$ | $0.625 \pm 0.013$ | $0.620 \pm 0.018$ | $0.584 \pm 0.017$ $0.528 \pm 0.018$ | $0.524 \pm 0.018$ $0.491 \pm 0.019$ | $0.442 \pm 0.012$ $0.387 \pm 0.013$ | $0.391 \pm 0.019$ $0.343 \pm 0.020$ | $0.348 \pm 0.015$ $0.284 \pm 0.016$ |
| 13.5 | $0.511 \pm 0.015$ | $0.590 \pm 0.017$ | 0.596 $\pm 0.014$ | $0.604 \pm 0.019$ | $0.528 \pm 0.018$ $0.476 \pm 0.018$ | $0.491 \pm 0.019$ $0.411 \pm 0.019$ | $0.387 \pm 0.013$ $0.348 \pm 0.013$ | $0.279 \pm 0.021$ | $0.271 \pm 0.016$ |
| 14.5 | $0.531 \pm 0.015$ | $0.563 \pm 0.018$ | $0.602 \pm 0.015$ $0.581 \pm 0.015$ | $0.597 \pm 0.020$ $0.567 \pm 0.020$ | $0.476 \pm 0.018$ $0.443 \pm 0.019$ | $0.411 \pm 0.019$ $0.383 \pm 0.020$ | $0.348 \pm 0.013$ $0.317 \pm 0.014$ | $0.279 \pm 0.021$ $0.260 \pm 0.021$ | $0.271 \pm 0.016$ $0.252 \pm 0.016$ |
| 15.5 | $0.484 \pm 0.016$ | $0.588 \pm 0.019$ | $0.581 \pm 0.015$ | $0.567 \pm 0.020$ $0.486 \pm 0.021$ | $0.443 \pm 0.019$ $0.391 \pm 0.019$ | $0.383 \pm 0.020$ $0.330 \pm 0.021$ | $0.317 \pm 0.014$ $0.281 \pm 0.014$ | 0.204 $\pm 0.021$ | $0.211 \pm 0.016$ |
| 16.5 | $0.524 \pm 0.017$ | $0.587 \pm 0.020$ | $0.539 \pm 0.016$ | $0.486 \pm 0.021$ | $0.391 \pm 0.019$ | $0.330 \pm 0.021$ | 0.289 +0.014 | 0.217 $\pm 0.022$ | $0.203 \pm 0.016$ |
| 17.5 | $0.537 \pm 0.017$ | $0.554 \pm 0.020$ | $0.494 \pm 0.016$ | $0.456 \pm 0.021$ | $0.330 \pm 0.020$ | $0.329 \pm 0.021$ | $0.239 \pm 0.014$ $0.224 \pm 0.014$ | $0.217 \pm 0.022$ $0.227 \pm 0.021$ | $0.208 \pm 0.016$ |
| 18.5 | $0.525 \pm 0.018$ | $0.533 \pm 0.021$ | $0.423 \pm 0.016$ | $0.421 \pm 0.022$ | $0.286 \pm 0.020$ | $0.260 \pm 0.021$ |  | $0.173 \pm 0.022$ |  |
| 19.5 | $0.512 \pm 0.019$ | $0.454 \pm 0.022$ | $0.376 \pm 0.017$ | $0.356 \pm 0.022$ | $0.254 \pm 0.020$ | $0.243 \pm 0.021$ | $0.238 \pm 0.014$ | $0.173 \pm 0.022$ | $0.192 \pm 0.016$ |
| $\delta_{\text {cip }}$ |  |  |  |  |  |  |  |  |  |
| Dsta/Fit (L) <br> Data/Fit (H) | $0.992 \pm 0.009$ | $1.002 \pm 0.009$ | $1.010 \pm 0.007$ | $1.012 \pm 0.009$ | $0.987 \pm 0.009$ | $1.020 \pm 0.010$ | $0.997 \pm 0.007$ | $0.992 \pm 0.012$ | $1.002 \pm 0.011$ |
| $\theta\left[{ }^{\circ}\right]$ | do/d! | $\mathrm{d} 0 / \mathrm{d} \boldsymbol{R}$ | do/d $\Omega$ | do, $4 \Omega$ | 4-is | do,d $\frac{1}{}$ | $\mathrm{d} \sigma / \mathrm{d} \Omega$ | do/g | de/o |
| 5.5 | $100.00 \pm 0.33$ | $100.00 \pm 0.47$ | $100.00 \pm 0.38$ | $100.00 \pm 0.49$ | $100.00 \pm 0.45$ | $100.00 \pm 0.46$ | :00.00 250.29 | $100.00 \pm 0.43$ | $100.00 \pm 0.32$ |
| 6.5 | $51.69 \pm 0.22$ | $75.72 \pm 0.37$ | $84.92 \pm 0.32$ | $86.52 \pm 0.42$ | $87.39 \pm 0.38$ | $87.07 \pm 0.39$ | $85.60 \pm 0.25$ | $84.61 \pm 0.37$ | $83.58 \pm 0.27$ |
| 7.5 | $4.02 \pm 0.19$ | $69.26 \pm 0.33$ | $77.15 \pm 0.28$ | $77.95 \pm 0.37$ | $75.91 \pm 0.33$ | $75.25 \pm 034$ | $72.41 \pm 0.21$ | $70.06 \pm 0.31$ | $68.20 \pm 0.23$ |
| 8.5 | $41.35 \pm 0.17$ | $63.88 \pm 0.30$ | $69.47 \pm 0.25$ | $69.39 \pm 0.33$ | $66.43 \pm 0.29$ | $63.73 \pm 0.29$ | $60.20 \pm 0.18$ | $57.28 \pm 0.26$ | $54.62 \pm 0.19$ |
| 9.5 | $38.49 \pm 0.16$ | $58.42 \pm 0.27$ | $61.74 \pm 0.23$ | $61.51 \pm 0.29$ | $56.66 \pm 0.26$ | $54.55 \pm 0.26$ | $50.18 \pm 0.16$ | $46.23 \pm 0.22$ | $44.00 \pm 0.16$ |
| 10.5 | $35.74 \pm 0.15$ | $51.60 \pm 0.24$ | $54.19 \pm 0.20$ | $53.32 \pm 0.26$ | $47.84 \pm 0.22$ | $45.74 \pm 0.22$ | $41.80 \pm 0.14$ | $38.22 \pm 0.19$ | $34.93 \pm 0.14$ |
| 11.5 | $32.20 \pm 0.13$ | $45.72 \pm 0.22$ | $46.62 \pm 0.18$ | $45.55 \pm 0.23$ | $40.80 \pm 0.20$ | $38.79 \pm 0.20$ | $34.46 \pm 0.12$ | $31.26 \pm 0.17$ | $28.43 \pm 0.12$ |
| 12.5 | $28.39 \pm 0.12$ | $39.53 \pm 0.20$ | $40.32 \pm 0.16$ | $39.02 \pm 0.20$ | $34.74 \pm 0.17$ | $32.71 \pm 0.17$ | $28.91 \pm 0.10$ | $25.98 \pm 0.15$ | $23.68 \pm 0.10$ |
| 13.5 | $24.66 \pm 0.11$ | $34.16 \pm 0.17$ | $34.58 \pm 0.14$ | $34.05 \pm 0.18$ | $29.86 \pm 0.16$ | $28.03 \pm 0.16$ | $24.53 \pm 0.09$ | $22.45 \pm 0.13$ | $20.19 \pm 0.09$ |
| 14.5 | $21.21 \pm 0.10$ | $29.26 \pm 0.16$ | $29.43 \pm 0.13$ | $28.90 \pm 0.16$ | $25.77 \pm 0.14$ | $24.35 \pm 0.14$ | $21.37 \pm 0.08$ | $19.23 \pm 0.12$ | $17.62 \pm 0.08$ |
| 15.5 | $18.19 \pm 0.09$ | $25.23 \pm 0.14$ | $25.84 \pm 0.12$ | $25.28 \pm 0.15$ | $23.01 \pm 0.13$ | $21.35 \pm 0.13$ | $19.21 \pm 0.08$ | $17.55 \pm 0.11$ | $16.13 \pm 0.08$ |
| 16.5 | $15.69 \pm 0.08$ | $21.90 \pm 0.13$ | $22.83 \pm 0.11$ | $22.57 \pm 0.13$ | $20.65 \pm 0.12$ | $19.39 \pm 0.12$ | $17.60 \pm 0.07$ | $16.04 \pm 0.10$ | $15.01 \pm 0.07$ |
| 17.5 | $13.30 \pm 0.07$ | $19.24 \pm 0.12$ | $20.40 \pm 0.10$ | $20.49 \pm 0.12$ | $18.84 \pm 0.11$ | $17.84 \pm 0.11$ | $16.27 \pm 0.07$ | $14.98 \pm 0.09$ | $14.10 \pm 0.07$ |
| 18.5 | $11.41 \pm 0.06$ | $16.95 \pm 0.11$ | $18.43 \pm 0.09$ | $18.48 \pm 0.12$ | $17.56 \pm 0.10$ | $16.90 \pm 0.10$ | $15.20 \pm 0.06$ | $14.38 \pm 0.09$ | $13.46 \pm 0.06$ |
| 19.5 | $9.78 \pm 0.06$ | $14.99 \pm 0.10$ | $16.96 \pm 0.08$ | $17.16 \pm 0.11$ | $16.21 \pm 0.10$ | $15.89 \pm 0.10$ | $14.68 \pm 0.06$ | $13.51 \pm 0.09$ | $12.77 \pm 0.06$ |
| No. of events | 782011 | 564131 | 903712 | 538507 | 60.4003 | 551495 | 1252522 | 534953 | 919093 |

Table 1 (continued)

| Energy | $434 \pm 2 \mathrm{MeV}$ | $475 \pm 2 \mathrm{MeV}$ | $520 \pm 2 \mathrm{MeV}$ | $568 \pm 2 \mathrm{McV}$ | $187 \pm 3 \mathrm{MeV}$ | $275 \pm 2 \mathrm{MeV}$ | $350 \pm 2 \mathrm{MeV}$ | $571 \pm 2 \mathrm{MeV}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Target | 7 cm | 7 cm | 7 cm | 7 cm | 5 cm | 5 cm | 5 cm |  |
| length | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $12.40 \mathrm{~g} / \mathrm{cm}^{2}$ | $8.85 \mathrm{~g} / \mathrm{cm}^{2}$ | $8.85 \mathrm{~g} / \mathrm{cm}^{2}$ | $8.85 \mathrm{~g} / \mathrm{cm}^{2}$ | $8.85 \mathrm{~g} / \mathrm{cm}^{2}$ |
| Angular |  |  |  |  |  |  |  |  |
| resolution | $0.597^{\circ}$ | $0.557^{\circ}$ | $0.526^{\circ}$ | $0.515^{\circ}$ |  | $0.701^{\circ}$ | $0.603^{\circ}$ | $0.476{ }^{\circ}$ |
| $\begin{aligned} & \text { Up-down } \\ & \text { consistency } \\ & {\left[x^{2} / d \eta\right]} \end{aligned}$ | 1.512 | 1.308 | 1.353 | 0.870 | 0.59 | 1.431 | 0.746 | 0.794 |
| $\theta 1^{\circ} \mathrm{O}$ | $A^{\text {c }}$ | $A_{c}$ | $A_{c}$ | $A_{c}$ | $A_{\text {c }}$ | $A_{c}$ | $A_{c}$ | $A_{c}$ |
| 5.5 |  | $0.283 \pm 0.011$ | $0.298 \pm 0.011$ | $0.295 \pm 0.014$ | $0.293 \pm 0.017$ | $0.372 \pm 0.016$ | $0.349 \pm 0.015$ | $0.299 \pm 0.014$ |
| 6.5 |  | $0.367 \pm 0.011$ | $0.330 \pm 0.012$ | $0.302 \pm 0.015$ | $0.431 \pm 0.017$ | $0.452 \pm 0.016$ | $0.419 \pm 0.015$ | $0.321 \pm 0.014$ |
| 7.5 | $0.383 \pm 0.014$ | $0.363 \pm 0.011$ | $0.323 \pm 0.012$ | $0.333 \pm 0.016$ | $0.435 \pm 0.017$ | $0.481 \pm 0.016$ | $0.489 \pm 0.015$ | $0.340 \pm 0.015$ |
| 8.5 | $0.394 \pm 0.015$ | $0.386 \pm 0.012$ | $0.339 \pm 0.013$ | $0.319 \pm 0.017$ | $0.498 \pm 0.016$ | $0.540 \pm 0.016$ | $0.472 \pm 0.016$ | $0.311 \pm 0.016$ |
| 9.5 | $0.403 \pm 0.016$ | $0.386 \pm 0.013$ | $0.339 \pm 0.014$ | $0.299 \pm 0.018$ | $0.506 \pm 0.016$ | $0.524 \pm 0.016$ | $0.486 \pm 0.016$ | $0.321 \pm 0.018$ |
| 10.5 | $0.342 \pm 0.017$ | $0.324 \pm 0.014$ | $0.313 \pm 0.015$ | $0.272 \pm 0.020$ | $0.527 \pm 0.016$ | $0.589 \pm 0.016$ | $0.489 \pm 0.017$ | $0.263 \pm 0.019$ |
| 11.5 | $0.353 \pm 0.018$ | $0.336 \pm 0.014$ | $0.241 \pm 0.016$ | $0.218 \pm 0.021$ | $0.575 \pm 0.017$ | $0.573 \pm 0.017$ | $0.450 \pm 0.018$ | $0.257 \pm 0.020$ |
| 12.5 | $0.307 \pm 0.019$ | $0.259 \pm 0.015$ | $0.238 \pm 0.017$ | $0.226 \pm 0.022$ | $0.591 \pm 0.017$ | $0.541 \pm 0.018$ | $0.394 \pm 0.019$ | $0.258 \pm 0.021$ |
| 13.5 | $0.278 \pm 0.020$ | $0.232 \pm 0.016$ | $0.212 \pm 0.618$ | $0.249 \pm 0.022$ | $0.619 \pm 0.018$ | $0.530 \pm 0.019$ | $0.359 \pm 0.020$ | $0.219 \pm 0.022$ |
| 14.5 | $0.239 \pm 0.020$ | $0.240 \pm 0.016$ | $0.212 \pm 0.018$ | $0.191 \pm 0.022$ | $0.579 \pm 0.018$ | $0.465 \pm 0.019$ | $0.338 \pm 0.020$ | $0.180 \pm 0.022$ |
| 15.5 | $0.241 \pm 0.021$ | $0.249 \pm 0.016$ | $0.227 \pm 0.018$ | $0.227 \pm 0.022$ | $0.567 \pm 0.019$ | $0.380 \pm 0.020$ | $0.275 \pm 0.021$ | $0.201 \pm 0.022$ |
| 16.9 | $0.228 \pm 0.021$ | $0.224 \pm 0.016$ | $0.208 \pm 0.018$ | $0.236 \pm 0.022$ | $0.568 \pm 0.020$ | $0.356 \pm 0.020$ | $0.283 \pm 0.021$ | $0.218 \pm 0.022$ |
| 17.5 | $0.184 \pm 0.021$ | $0.174 \pm 0.016$ | $0.219 \pm 0.018$ | $0.198 \pm 0.022$ | $0.524 \pm 0.021$ | $0.312 \pm 0.021$ | $0.255 \pm 0.021$ | $0.190 \pm 0.022$ |
| 18.9 | $0.204 \pm 0.021$ | $0.189 \pm 0.016$ | $0.189 \pm 0.018$ | $0.186 \pm 0.023$ | $0.454 \pm 0.021$ | $0.284 \pm 0.021$ | $0.217 \pm 0.021$ | $0.228 \pm 0.022$ |
| 19.5 | $0.215 \pm 0.021$ | $0.202 \pm 0.016$ | $0.190 \pm 0.018$ | $0.177 \pm 0.023$ | $0.442 \pm 0.022$ | $0.255 \pm 0.021$ | $0.218 \pm 0.021$ | $0.189 \pm 0.023$ |
| $\delta_{\text {exp }}$ Data/fit (L) |  |  |  |  |  |  |  |  |
| Data/fit (H) | $0.982 \pm 0.016$ | $1.012 \pm 0.012$ | $0.993 \pm 0.014$ | $0.993 \pm 0.018$ |  | $0.971 \pm 0.009$ | $0.982 \pm 0.012$ | $1.018 \pm 0.018$ |
| $\left.\theta{ }^{\circ} \mathrm{O}\right]$ | do/d? | do/d8 | $\mathrm{d} 0 / \mathrm{d} \Omega$ | $\mathrm{do} / \mathrm{d}$ / | da/d 8 | do/d $\Omega$ | $\mathrm{d} \sigma / \mathrm{d} \Omega$ | da/d, |
| 5.5 | $100.00 \pm 0.40$ | $100.00 \pm 0.31$ | $100.00 \pm 0.33$ | $100.00 \pm 0.41$ | $100.00 \pm 0.51$ | $100.00 \pm 0.47$ | $100.00 \pm 0.45$ | $100.00 \pm 0.40$ |
| 6.5 | $83.13 \pm 0.34$ | $81.96 \pm 0.26$ | $80.06 \pm 0.27$ | $78.03 \pm 0.33$ | $87.87 \pm 0.44$ | $87.21 \pm 0.40$ | $86.08 \pm 0.38$ | $77.28 \pm 0.32$ |
| 7.5 | $67.55 \pm 0.29$ | $64.20 \pm 0.21$ | $61.69 \pm 0.22$ | $58.47 \pm 0.27$ | $80.96 \pm 0.40$ | $76.57 \pm 0.35$ | $72.56 \pm 0.33$ | $58.62 \pm 0.26$ |
| 8.5 | $53.53 \pm 0.24$ | $50.44 \pm 0.18$ | $47.86 \pm 0.18$ | +4.74 $\pm 0.22$ | $73.89 \pm 0.35$ | $65.71 \pm 0.31$ | $60.15 \pm 0.28$ | $43.72 \pm 0.21$ |
| 9.5 | $43.00 \pm 0.20$ | $39.89 \pm 0.15$ | $36.66 \pm 0.15$ | $34.15 \pm 0.18$ | $67.05 \pm 0.32$ | $56.58 \pm 0.27$ | $50.31 \pm 0.24$ | $33.06 \pm 0.17$ |
| 109 | $34.16 \pm 0.17$ | $3129 \pm 0.13$ | $28.71 \pm 0.13$ | $26.57 \pm 0.15$ | $59.88 \pm 0.29$ | $48.18 \pm 0.24$ | $41.42 \pm 0.21$ | $25.70 \pm 0.15$ |
| 115 | $2773 \pm 0.15$ | $25.46 \pm 0.11$ | $23.05 \pm 0.11$ | $21.65+0.13$ | $52.56 \pm 0.26$ | $40.61 \pm 0.21$ | $34.41 \pm 0.18$ | $20.92 \pm 0.13$ |
| 125 | $23.09 \pm 0.13$ | $20.98 \pm 0.04$ | $19.46 \pm 0.10$ | $18.51 \pm 012$ | $45.37 \pm 0.23$ | $34.01 \pm 0.18$ | $28.84 \pm 0.16$ | $17.82 \pm 0.11$ |
| 13.5 | $1997 \pm 0.12$ | $18.19 \pm 0.08$ | $16.98 \pm 0.04$ | $16.57 \times 0.11$ | $39.50 \pm 0.21$ | $24.77 \pm 0.16$ | $24.33 \pm 0.14$ | $15.90 \pm 0.10$ |
| 14.5 | $17.62 \pm 0.11$ | $16.36 \pm 0.08$ | $15.34+0.018$ | $15.02+010$ | $33.59 \pm 0.18$ | $26.02 \pm 0.15$ | $21.26 \pm 0.13$ | $14.54 \pm 0.09$ |
| 159 | $16.03 \pm 010$ | $14.87 \pm 0.07$ | $1424 \times 0.07$ | $13.55+0.09$ | $24.52 \pm 017$ | $2283 \pm 013$ | $18.93 \pm 0.12$ | $13.59 \pm 0.09$ |
| 16.5 | 1505 $\pm 0.09$ | $13.95 \pm 0.17$ | $1335 \times 0.07$ | 1310:0.04 | $25.73 \pm 015$ | $20.69 \pm 0.12$ | $17.42 \pm 0.11$ | $12.69 \pm 0.08$ |
| 175 | $13.83+0.09$ | $1325 \pm 0.06$ | $12.52+0.07$ | $1231+008$ | $22.68 \pm 0.4$ | $1875+0.11$ | $16.25 \pm 0.10$ | $11.63 \pm 0.08$ |
| 18.5 | $1334 \pm 008$ | $12.46 \pm 0.06$ | $11.8 k+010$ | 1161:008 | $2024+013$ | 1772 ¢011 | $15.45 \pm 0.10$ | $11.08 \pm 0.07$ |
| 195 | $12.71 \pm 0.08$ | 1193 ;006 | 1121 +00\% | 11081:007 | 18.44:0.12 | $16.56+0.10$ | $14.56 \pm 0.09$ | $10.31 \pm 0.07$ |
| No of even's | 459671 | ${ }_{922523}$ | 771267 | 4k1401 | $31 \times 172$ | 547805 | 531424 | 502143 |

Table I (continued)

| Energy | $95 \pm 4 \mathrm{MeV}$ | $144 \pm 3 \mathrm{MeV}$ | $194 \pm 2 \mathrm{MeV}$ | $23 \mathrm{H}+2 \mathrm{MeV}$ | $281 \pm 2 \mathrm{MeV}$ | $111+2 \mathrm{MeV}$ | $347 \pm 2 \mathrm{MeV}$ | $3 \mathrm{mb} \pm 2 \mathrm{McV}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Turget | 3 cm | 3 cm |  | 3 cm | 3 cm | $3 \mathrm{~cm}$ | $3 \mathrm{~cm}$ | $3 \mathrm{~cm}$ |
| length | $5.31 \mathrm{~g} / \mathrm{cm}^{2}$ | $5.31 \mathrm{~g} / \mathrm{cm}^{2}$ | $5.31 \mathrm{~g} / \mathrm{cm}^{2}$ | $531 \mathrm{~g} / \mathrm{cm}^{2}$ | $3.31 \mathrm{~g} / \mathrm{cm}^{2}$ |  |  |  |
| Angular resolution | $1.355^{\circ}$ | $0.874^{\circ}$ | $0.734^{\circ}$ | $11651^{\circ}$ | $0.573^{\circ}$ | $0.551^{\circ}$ | $0.517^{\circ}$ | $0.468^{\circ}$ |
| $\begin{aligned} & \text { Up-down } \\ & \text { consistency } \end{aligned}$ | 0.629 | 1.431 | 0.849 | 1.080 | 1.129 | 0.715 | 1.074 | 0.751 |
| $\left.x^{2} / \mathrm{d}\right]$ |  |  |  |  |  |  |  |  |
| 91 ${ }^{\circ}$ | $A$ | $A_{5}$ | $A_{6}$ | 1. | $A_{\text {c }}$ | 1. | $A_{5}$ | 1. |
| 5.5 | $0.058 \pm 0.011$ | $0.239 \pm 0.015$ | $0.370 \pm 0.018$ | $0.393+0.020$ | $0.379 \pm 0.017$ | $0.400 \pm 0.018$ | $0.373 \pm 0.013$ | $0.391 \pm 0.015$ |
| 6.5 | $0.104 \pm 0.016$ | $0.315 \pm 0.015$ | $0.440 \pm 0.018$ | $0.438 \pm 0.019$ | $0.452 \pm 0.017$ | $0.448 \pm 0.018$ | $0.432 \pm 0.013$ | $0.423 \pm 0.015$ |
| 7.5 | $0.216 \pm 0.018$ | $0.349 \pm 0.014$ | $0.465 \pm 0.017$ | $0.471 \pm 0.019$ | $0.515 \pm 0.017$ | $0.500 \pm 0.018$ $0.539 \pm 0.018$ | $0.481 \pm 0.014$ $0.497 \pm 0.014$ | $0.426 \pm \pm 0.016$ |
| 8.5 | $0.192 \pm 0.018$ | $0.344 \pm 0.013$ | $0.522 \pm 0.017$ | $0.553 \pm 0.019$ | $0.530 \pm 0.017$ $0.565 \pm 0.017$ | $0.539 \pm 0.018$ $0.522 \pm 0.019$ | $0.497 \pm 0.014$ $0.508 \pm 0.014$ | $0.468 \pm 0.016$ $0.441 \pm 0.017$ |
| 9.5 | $0.210 \pm 0.017$ | $0.376 \pm 0.013$ | $0.531 \pm 0.017$ | $0.562 \pm 0.019$ $0.599 \pm 0.019$ | $0.565 \pm 0.017$ $0.578 \pm 0.018$ | $0.565 \pm 0.019$ | $0.489 \pm 0.015$ | $0.432 \pm 0.018$ |
| 10.5 | $0.236 \pm 0.017$ | $0.373 \pm 0.013$ | $0.569 \pm 0.017$ $0.587 \pm 0.017$ | $0.599 \pm 0.019$ $0.655 \pm 0.020$ | $0.578 \pm 0.018$ $0.574 \pm 0.018$ | $0.565 \pm 0.019$ $0.539 \pm 0.020$ | $0.479 \pm 0.016$ | $0.392 \pm 0.019$ |
| 11.5 | $0.202 \pm 0.016$ | $0.412 \pm 0.013$ | $0.587 \pm 0.017$ $0.578 \pm 0.018$ | $0.655 \pm 0.020$ $0.583 \pm 0.021$ | 0.557 $\pm 0.019$ | $0.470 \pm 0.021$ | $0.428 \pm 0.017$ | $0.409 \pm 0.020$ |
| 12.5 | $0.211 \pm 0.016$ | $0.423 \pm 0.013$ | $0.578 \pm 0.018$ $0.614 \pm 0.018$ | $0.583 \pm 0.021$ $0.593 \pm 0.021$ | $0.557 \pm 0.019$ $0.489 \pm 0.020$ | $0.471 \pm 0.022$ | $0.378 \pm 0.017$ | $0.318 \pm 0.021$ |
| 13.5 | $0.203 \pm 0.016$ | $0.424 \pm 0.013$ | $0.614 \pm 0.018$ $0.581 \pm 0.019$ | $0.593 \pm 0.021$ $0.574 \pm 0.022$ | $0.422 \pm 0.021$ | $0.394 \pm 0.023$ | $0.319 \pm 0.018$ | $0.284 \pm 0.021$ |
| 14.5 | $0.203 \pm 0.016$ | $0.448 \pm 0.014$ | $0.581 \pm 0.019$ $0.531 \pm 0.020$ | $0.574 \pm 0.022$ $0.498 \pm 0.023$ | $0.389 \pm 0.021$ | $0.407 \pm 0.024$ | $0.318 \pm 0.018$ | $0.222 \pm 0.022$ |
| 15.5 16.5 | $0.195 \pm 0.017$ | $0.473 \pm 0.015$ | $0.531 \pm 0.020$ $0.539 \pm 0.021$ | $0.399 \pm 0.024$ | $0.347 \pm 0.022$ | $0.281 \pm 0.024$ | $0.272 \pm 0.019$ | $0.285 \pm 0.022$ |
| 16.5 17.5 | $0.180 \pm 0.017$ $0.208 \pm 0.018$ | $0.476 \pm 0.015$ $0.455 \pm 0.016$ | $0.539 \pm 0.021$ $0.467 \pm 0.021$ | $0.382 \pm 0.024$ | $0.301 \pm 0.022$ | $0.270 \pm 0.024$ | $0.199 \pm 0.019$ | $0.244 \pm 0.022$ |
| 17.5 18.5 | $\begin{array}{r}0.22 i \\ \hline 0.0 .019\end{array}$ | $0.469 \pm 0.017$ | $0.416 \pm 0.022$ | $0.363 \pm 0.024$ | $0.324 \pm 0.022$ | $0.215 \pm 0.024$ | $0.226 \pm 0.019$ | $0.180 \pm 0.022$ |
| 19.5 | $0.239 \pm 0.020$ | $0.433 \pm 0.017$ | $0.402 \pm 0.022$ | $0.273 \pm 0.025$ | $0.205 \pm 0.023$ | $0.230 \pm 0.025$ | $0.220 \pm 0.019$ | $0.170 \pm 0.022$ |
|  |  |  |  | $0.998 \pm 0.011$ | $0.993 \pm 0.010$ | $1.010 \pm 0.012$ | $0.997+0.010$ | $1.001 \pm 0.013$ |
| $\begin{aligned} & \text { Data/fit (L) } \\ & \text { Data/fit (H) } \end{aligned}$ | $1.011 \pm 0.023$ | $0.995 \pm 0.009$ | $1.006 \pm 0.009$ | $0.98 \pm 0.01$ |  | $1.009 \pm 0.012$ | $1.004 \pm 0.010$ | $1.011 \pm 0.013$ |
| [ㅇ] | $\mathrm{do} / \mathrm{d} \Omega$ | $\mathrm{do} / \mathrm{d} \Omega$ | do/d $\Omega$ | do/d $\Omega$ | $\mathrm{d} 0 / \mathrm{d} \Omega$ | do/d $\Omega$ | $\mathrm{d} 0 / \mathrm{d} 8$ | do/d 8 |
| 5.5 | $100.00 \pm 9.31$ | $100.00 \pm 0.43$ | $160.00 \pm 0.54$ | $100.00 \pm 0.56$ | $100.00 \pm 0.50$ | $100.00 \pm 0.53$ | $100.00 \pm 0.40$ | $100.00 \pm 0.45$ |
| 6.5 | $35.19 \pm 0.17$ | $81.92 \pm 0.36$ | $90.74 \pm 0.48$ | $89.05 \pm 0.49$ | $88.56 \pm 0.44$ | $86.70 \pm 0.45$ | $85.92 \pm 0.34$ | $84.84 \pm 0.38$ |
| 7.5 | $23.94 \pm 0.13$ | $79.20 \pm 0.33$ | $84.16 \pm 0.43$ | $80.82 \pm 0.43$ | $77.00 \pm 0.38$ | $74.77 \pm 0.39$ | $71.93 \pm 0.29$ | $69.49 \pm 0.32$ |
| 8.5 | $22.24 \pm 0.12$ | $77.56 \pm 0.31$ | $77.05 \pm 0.38$ | $70.85 \pm 0.38$ | $65.75 \pm 0.33$ | $62.81 \pm 0.34$ | 60.41 50.56 | $57.18 \pm 0.27$ |
| 9.5 | $21.78 \pm 0.11$ | $73.59 \pm 0.28$ | $69.81 \pm 0.35$ | $62.06 \pm 0.34$ | $56.72 \pm 0.29$ | $52.95 \pm 0.29$ | $50.56 \pm 0.21$ | $46.52 \pm 0.23$ |
| 0.5 | $21.06 \pm 0.10$ | $67.53 \pm 0.26$ | $61.58 \pm 0.31$ | $53.12 \pm 0.30$ | $48.28 \pm 0.25$ | $44.57 \pm 0.26$ | $41.46 \pm 0.18$ $34.01 \pm 0.16$ | $38.14 \pm 0.20$ $31.05 \pm 0.17$ |
| 1.5 | $20.33 \pm 0.10$ | $61.74 \pm 0.23$ | $53.57 \pm 0.28$ | $45.81 \pm 0.26$ | $40.70 \pm 0.22$ | $37.23 \pm 0.22$ | $34.01 \pm 0.16$ |  |
| 2.5 | $18.85 \pm 0.09$ | $54.40 \pm 0.21$ | $46.32 \pm 0.25$ | $39.37 \pm 0.23$ | $34.39 \pm 0.20$ | $31.33 \pm 0.20$ | $28.44 \pm 0.14$ | $25.62 \pm 0.15$ $21.89 \pm 0.13$ |
| 13.5 | $17.49 \pm 0.08$ | $48.35 \pm 0.19$ | $40.11 \pm 0.22$ | $33.47 \pm 0.21$ | $29.77 \pm 0.18$ | $27.19 \pm 0.18$ | $24.41 \pm 0.13$ |  |
| 4.5 | $15.85 \pm 0.08$ | $41.62 \pm 0.17$ | $34.66 \pm 0.20$ | $29.34 \pm 0.19$ | $25.58 \pm 0.16$ | $23.51 \pm 0.16$ | $21.30 \pm 0.11$ $19.03 \pm 0.10$ | $19.17 \pm 0.12$ |
| 5.5 | $14.05 \pm 0.07$ | $36.27 \pm 0.16$ | $29.76 \pm 0.18$ | $25.79 \pm 0.17$ | $22.87 \pm 0.14$ | $20.85 \pm 0.14$ | $19.03 \pm 0.10$ |  |
| 6.5 | $12.59 \pm 0.06$ | $31.49 \pm 0.14$ | $26.68 \pm 0.16$ | $23.04 \pm 0.16$ | $21.02 \pm 0.13$ | $18.84 \pm 0.13$ | $17.42 \pm 0.10$ | 13.98 $\pm 0.10$ |
| 7.5 | $10.99 \pm 0.06$ | $26.76 \pm 0.13$ | $23.50 \pm 0.15$ | $21.07 \pm 0.14$ | $18.87 \pm 0.12$ | $17.41 \pm 0.12$ | $16.09 \pm 0.09$ |  |
| 18.5 | $9.60 \pm 0.05$ | $23.49 \pm 0.11$ | $21.42 \pm 0.14$ | $19.49 \pm 0.14$ | $17.68 \pm 0.12$ | $16.60 \pm 0.12$ $15.49 \pm 0.11$ | $15.33 \pm 0.09$ $14.44 \pm 0.08$ | $\begin{aligned} & 14.16 \pm 0.09 \\ & 13.61 \pm 0.09 \end{aligned}$ |
| 9.5 | $8.17 \pm 0.05$ | $20.41 \pm 0.10$ | $19.82 \pm 0.13$ | $1 \% 24 \pm 0.13$ | $16.81 \pm 0.11$ | $15.49 \pm 0.17$ |  |  |
| No of events | 660462 | 851752 | 493816 | $4 ، 9028$ | 473049 | 403172 | 682041 | 503088 |



Fig. 2A. The effective proton-carbon analyzing power as a function of the laboratory scattering angle $\theta$. The dot are our experimental points. The full lines show our 7 cm high energy fit (H).


Fig. 2B. The effective proton-carbon analyzing power as a function of the laboratory scattering angle $\theta$. The dots are our experimental points. The full lines show our 7 cm high energy fit (H). Our 3 cm low energy fit (L) is represented by the dotted lines.

Table 2
Results of our angle and energy dependent parametrization of the carbon analyzing power (see section 7).

|  | "H" fit 7 cm <br> $150-571 \mathrm{MeV}$ | "L" fit 3 cm <br> $90-386 \mathrm{MeV}$ |
| :--- | :---: | :---: |
|  | $X=\frac{T-400 \mathrm{MeV}}{200 \mathrm{MeV}}$ | $X=\frac{T-250 \mathrm{MeV}}{100 \mathrm{MeV}}$ |
| $\alpha_{0}$ | 3.3561 | 3.6991 |
| $\alpha_{1}$ | -0.91758 | 0.26957 |
| $\alpha_{2}$ | 0.38654 | -0.0012157 |
| $\alpha_{3}$ | 0.30807 | 0.17072 |
| $\beta_{0}$ | -7.9741 | -8.7225 |
| $\beta_{1}$ | 5.3176 | -3.7161 |
| $\beta_{2}$ | 12.532 | 12.869 |
| $\beta_{3}$ | -3.1091 | -2.6088 |
| $\beta_{4}$ | - | 1.6024 |
| $\gamma_{0}$ | 857.93 | 351.97 |
| $\gamma_{1}$ | 810.41 | 271.44 |
| $\gamma_{2}$ | -127.21 | -113.71 |
| $\gamma_{3}$ | -163.39 | -10.407 |
| $\gamma_{4}$ | - | 20.331 |
| $\delta_{0}$ | 0.079421 | - |
| $\delta_{1}$ | 0.12568 | - |
| $\delta_{2}$ | -0.082377 | - |
| $C$ | 58.361 | 75.383 |
| $C_{0}$ | $0.12\left[\mathrm{deg} .{ }^{2}\right]$ | $0.12\left[\right.$ deg. $\left.{ }^{2}\right]$ |
| $C_{1}$ | 0.38511 | 0.18472 |
| $x^{2} /$ d.f. | 1.14 | 1.17 |

(2) A low energy fit (denoted by "L") valid for 3 cm target from 90 to 386 MeV . This contains all the present 3 cm data. In addition, we have also included TRIUMF 3 cm data from the BASQUE group [8] between $5^{\circ}$ and $20^{\circ}$.


Fig. 3. Percentage uncertainty of our fits. All errors are included. At low energy, the effect of the uncertainty in the beam energy is dominant.

The 5 cm 187 MeV data have not been included in any fits since it was found that target length is signifi. cant at this energy.

Fitted parameters are given in table 2. Resulting curves are shown in fig. 2 where solid and dotted lines are used respectively for the " H " and " L " fits. The $\boldsymbol{x}^{2}$ per degree of freedom were 1.14 and 1.29 indicating that the empirical formula adequately describes the data in our angular range. The relative error of our fit (percentage uncertainty of $A_{c}$ ) is shown in fig. 3. Above 170 MeV , it is given mainly by statistical errors and hy beam polarization uncertainty (both of the order of $1 \%$ ). At low energy, the error is dominated by the uncertainty in the beam energy.

In order to check the goodness of our fit at each measured energy, we have computed a deviation factor ( $\delta=$ data/fit) between the measured data and the fitted formula. This represents the value by which the energy. dependent fit has to be multiplied to give the hest adjustment to the data at a particular energy, i.e..
$\delta_{\mathrm{exp}}=\frac{\sum_{\theta} w(\theta) \cdot A_{\mathrm{fit}}(\theta) \cdot A_{\mathrm{exp}}(\theta)}{\sum_{\theta} w(\theta) A_{\mathrm{fit}}^{2}(\theta)}$
where $\boldsymbol{\omega}(\theta)=1 / \sigma_{\theta}^{2}$ is the statistical weight of an experimental point. The error in $\delta_{\text {exp }}$ is given by
$\sigma_{\delta}^{2}=1 / \sum_{\theta} w(\theta) A_{f, 1}^{2}(\theta)$.
and gives the relative statistical error of the data. Value of $\delta_{\text {exp }}$ are given in table 1. Thev are generally well distributed around 1 within their error hars indicating that the relative systematic error introduced hy cur smoothing stays on the order of $1^{2}$ at each energy.

In fig. 4 we show the angle-averaged analyzing power $A_{c}$ as a function of the energy. This average was calculated between $5^{\circ}$ and $20^{\circ}$ giving equal weight to each angle. The curves are from our formula and the points represent the experimental data included in the fits. As can be seen, the effect of target length appears to be significant.

For the calculation of the average of experimental data we have fixed the angular shape of $A_{\mathrm{c}}$ as given bs our fit. The experimental averages are simply calculated by:

$$
\bar{A}_{e \backslash p}=\delta_{e \backslash p} \bar{A}_{\text {fit }}
$$

This way of computing $A_{\text {c }}$ differs shghtly from ref. 10 where the angular shape was adjusted to the data at each energy. Both methods generally give consistent results. Ours has the advantage of allowing comparison of data when the $5^{\circ}-20^{\circ}$ range is not completely cover•d or when the statistics are poor.


Fig. 4. Angle-averaged effective analyzing power. Curves show our fits. Points are the data included in the fits. Errors are statistical only

## 8. Compmison with other experiments

Data from different experiments are generally in very good graphical agreement. In order to compare various experiments at different energies, we have com-
puted deviation factors and averages as explained in section 7.

Data from our previous experiment at SIN (DRAP [7]) and BASQUE 3 cm data which were both included in our fits are shown in fig. 4. Other data are shown in


Fig. S. Angle-averaged effective analyzing power. Curves show the error corridors of our fits. The points come from previous measurements not included in these fits. The errors shown on the points are purely statistical. The angular range is between $5^{\circ}$ and $20^{\circ}$ (see section 8 for details).
fig. 5 where curves show the error corridors of our fits with all errors included. These data come from our previous experiment at the SC-CERN [9] as well as from TRIUMF (BASQUE 6 cm ) [8] and the LAMPF polarimeter [10]. Averages of the preliminary data from LAMPF-HRS were taken from ref. 10.

The data are generally in good agreement with perhaps a small discrepancy above 200 MeV where BASQUE 6 cm data in our angular range are 4\% higher than ours as already reported in ref. 10. Recent fits of BASQUE, SIN-DRAP and LAMPF data have also been presented in ref. 10. They are in reasonable agreement with our 7 cm fit.

## 9. Conclusion

We have presented new precise measurements of the proton-carbon effective analyzing power between 95 and 570 MeV . The data can be reproduced by an energy dependent fit with a relative accuracy of $1-2 \%$ above 170 MeV . Reasonable agreement with other available data has been found. At high energy, the effective analyzing power is insensitive to the thickness of the analyzer within statistical accuracy. At low energy however, it can be affected by the target length because of the influence of inelastic events.

A better accuracy would require a considerable effort on systematic effects, especially on the beam polarization calibration. In such a case, comparison between
different experiments at low energy would also require a careful study of angular resolution, target length and inelasticity effects as well as effects coming from the strong energy dependence of $A_{c}$ below 170 MeV .

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