NUCLEAR INSTRUMENTS AND METHODS 109 (1973) 41–59; © NORTH-HOLLAND PUBLISHING CO.

TECHNIQUES FOR MEASUREMENT OF SPIN-$\frac{1}{2}$ AND SPIN-1 POLARIZATION ANALYZING TENSORS*

GERALD G. OHLSEN and P. W. KEATON, Jr.

Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544, U.S.A.

Received 8 January 1973

Techniques for measurement of spin-$\frac{1}{2}$ and spin-1 analyzing tensors with a view to the control or elimination of first-order systematic errors which may arise from misalignments of various types are discussed. The concept of a "proper flip" and a "nonproper flip" is introduced for both spin $\frac{1}{2}$ and spin 1. Several methods for measurement of spin-1 analyzing tensors are described and their relative merits are evaluated.

1. Introduction

Polarization analyzing tensors in nuclear physics may always be measured by means of ratios of one type or another. For this reason, it is intrinsically easier to obtain accurate absolute information about these quantities than is the case for differential cross sections. Typical ratios in this context might be "left–right asymmetries," or polarized-beam to unpolarized-beam yield ratios.

Many of the observables give rise to an azimuthal angular variation in the analyzing reaction cross section; for measurements of these "false asymmetries," which can arise from imperfect apparatus construction or alignment, are a major source of error. Other observables can only be determined from ratios of yields obtained with different values of the beam polarization. The spin-1 longitudinal second-rank analyzing power, A_{zz}, is of this character. The precision with which observables of this type can be measured depends not only on apparatus alignment, but also on the accuracy of beam intensity monitoring, target thickness control, etc.

In the present paper we will restrict ourselves to cases where the incident polarized beam is produced by an ion source; that is, we assume that the beam polarization possesses a symmetry axis. The absolute magnitude of the spin polarization will be regarded as known. The nominal direction of the spin polarization will be assumed to be under the experimenter's control, but the possibility of small deviations from the desired direction will be allowed for.

Previous papers which discuss experimental problems of the type which form our subject have been cited in an annotated bibliography by Waymire and Jarmie[1]); see especially the paper by Hanna[2]). Also, an elementary treatment of this subject is being prepared as an informal report in which double scattering techniques are included[3]).

2. Coordinate system and specification of beam polarization

In accordance with the usual conventions, we assume a Cartesian coordinate system with z along the incident beam momentum, $\boldsymbol{k}_{\rm in}$, y along $\boldsymbol{k}_{\rm in} \times \boldsymbol{k}_{\rm out}$, where $\boldsymbol{k}_{\rm out}$ is the scattered particle momentum, and x such as to define a right-handed coordinate system. Unit vectors along the x, y and z coordinate axes are represented by $\boldsymbol{l}$, $\boldsymbol{n}$ and $\boldsymbol{k}$, respectively. The unit vector pointing along the spin quantization axis is denoted by $\boldsymbol{s}$; its direction is defined in terms of the angles β, ϕ as shown in fig. 1. That is, β measures the angle between the quantization axis and the beam direction, so that $\cos\beta = \boldsymbol{k}\cdot\boldsymbol{s}$, while ϕ measures the angle between the projection of $\boldsymbol{s}$ in the

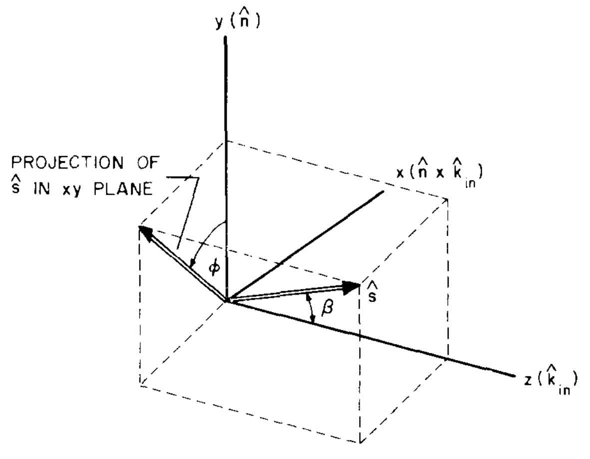

Fig. 1. Definition of the spin angles β, ϕ. The scattering is in the x, z plane.

* Work performed under the auspices of the U.S. Atomic Energy Commission.

x, y plane and the y axis. The sense of ϕ is important and is positive as illustrated in fig. 1.

We also use a Cartesian description of the beam polarization. The first- and second-rank polarizations relative to the quantization axis, $\boldsymbol{s}$, are referred to as p_Z and p_{ZZ}, respectively. The vector polarization, p_Z, has a range of possible values between $+1$ and -1 for either a spin-$\frac{1}{2}$ or a spin-1 beam, while the tensor polarization, p_{ZZ}, may vary between $+1$ and -2 for a spin-1 beam and is, of course, zero for a spin-$\frac{1}{2}$ beam.

With these definitions, the momentum vector of the scattered particle always lies in the x, z half-plane with positive x. Throughout this paper, we will let the direction of the transverse component of the beam quantization axis $\boldsymbol{s}_\perp[\equiv \boldsymbol{s}-(\boldsymbol{s}\cdot\boldsymbol{k})\boldsymbol{k}]$ *define* the direction we call "up." If the y axis is along $\boldsymbol{s}_\perp(\phi=0)$, the scattering is to the left (according to an observer who is looking along the beam direction and who is "aligned" with the direction $\boldsymbol{s}_\perp$). For right, up, and down scattering we have $\phi=180°$, $\phi=270°$ and $\phi=90°$, respectively (see fig. 1).

3. Measurement of spin-$\frac{1}{2}$ analyzing power

3.1. Geometric mean concept

For our spin-$\frac{1}{2}$ discussion, we will deal exclusively with a symmetric two-detector analyzer system such as is shown schematically in fig. 2. The cross section for a polarized spin-$\frac{1}{2}$ beam may be written

$$I(\theta,\phi) = I_0(\theta)\left[1+p_y A_y(\theta)\right], \qquad (1)$$

where $I_0(\theta)$ is the cross section for scattering an unpolarized beam into the scattering angle θ, $A_y(\theta)$ is the "analyzing power" or "efficiency tensor" (of rank one) of the reaction, and p_y is the y component of the beam polarization. In terms of the magnitude of the beam polarization, $\boldsymbol{p}$, and the normal unit vector, $\boldsymbol{n}$,

$$p_y = \boldsymbol{p}\cdot\boldsymbol{n} \equiv p\sin\beta\cos\phi \equiv p_\perp\cos\phi, \qquad (2)$$

where $p_\perp$ is the component of beam polarization perpendicular to its direction of motion. For the remainder of our spin-$\frac{1}{2}$ discussion, we will assume $\beta \approx 90°$, so that $p_\perp$ differs from p at most by a second-order term. Since only first-order errors are of present concern, this difference will be neglected and we set $p_\perp \equiv p$.

The actual number of counts recorded in a detector, $N(\theta,\phi)$, may be written, for solid target geometry, as follows:

$$N(\theta,\phi) = nN_A\Delta\Omega E I(\theta,\phi). \qquad (3)$$

Here n is the number of particles incident on the target, N_A is the number of target nuclei per square centimeter, $\Delta\Omega$ is the solid angle subtended by the detector, and E is the efficiency for detection. For gas target geometry, the corresponding (first-order) expression is[4])

$$N(\theta,\phi) = nN_V G_{00} E I(\theta,\phi), \qquad (4)$$

where n and E are as in eq. (3), N_V is the number of target nuclei per cubic centimeter, and G_{00} is a purely geometric factor. For vertical front slits, and in terms of the notation of fig. 3,

$$G_{00} = \frac{2b_1 a}{rh\sin\theta}, \qquad (5)$$

where a is the area of the rear aperture. ($a=2b_2 l$ for the rectangular aperture shown.) As will be shown, solid and gas target geometry lead to somewhat different "false asymmetry" problems.

We will use the symbol Ω to denote either G_{00} or $\Delta\Omega$ and N to denote either N_A or N_V, and, for the moment, we will assume a perfectly aligned analyzer. However, we allow the solid angle factor which corresponds to detector 1 (Ω_1) and its efficiency (E_1) to be different from the corresponding quantities for detector 2 (Ω_2 and E_2). The efficiency differences could come

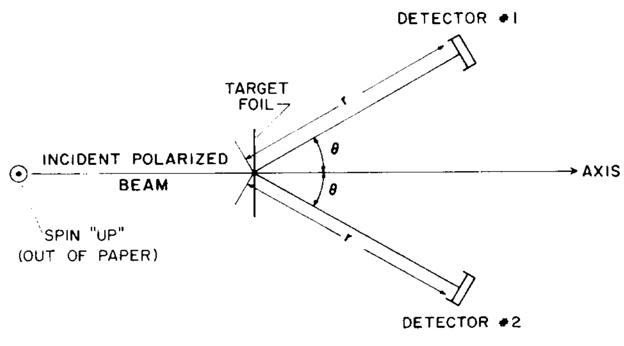

Fig. 2. Idealized symmetric two-detector arrangement for spin-$\frac{1}{2}$ analyzing power measurements.

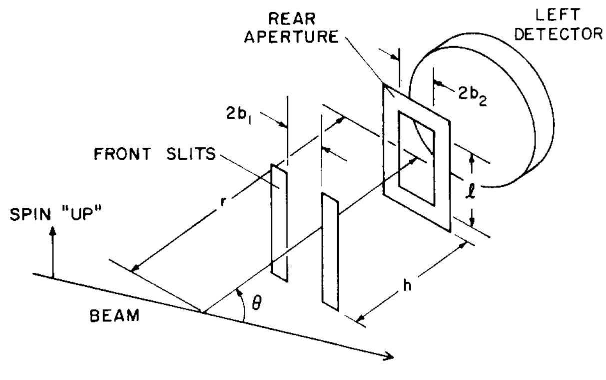

Fig. 3. Gas target geometry.

about, for example, if dissimilar detectors are used or if there are slight differences in the pulse height requirements which are set for the two detection channels. Defining $N_1(\theta,\phi)$ and $N_2(\theta,\phi)$ as the value of $N(\theta,\phi)$ for detectors 1 and 2, respectively, we have

$$
\begin{aligned}
N_1(\theta,0) &\equiv L_1 = nN\Omega_1 E_1 I_0(\theta)\left[1+pA_y(\theta)\right], \\
N_2(\theta,\pi) &\equiv R_2 = nN\Omega_2 E_2 I_0(\theta)\left[1-pA_y(\theta)\right].
\end{aligned} \tag{6}
$$

If we now "flip" the polarization, i.e., let $\boldsymbol{p}\rightarrow -\boldsymbol{p}$,

$$
\begin{aligned}
N_1(\theta,\pi) &\equiv R_1 = n'N'\Omega_1 E_1 I_0(\theta)\left[1-pA_y(\theta)\right], \\
N_2(\theta,0) &\equiv L_2 = n'N'\Omega_2 E_2 I_0(\theta)\left[1+pA_y(\theta)\right],
\end{aligned} \tag{7}
$$

where the primes are used to indicate that the integrated charge and the effective target thickness may not be the same for the two runs. If we form the geometric means

$$
\begin{aligned}
L &\equiv \sqrt{(L_1 L_2)} = \left[nn'NN'\Omega_1\Omega_2 E_1 E_2\right]^{\frac{1}{2}} I_0(1+pA_y), \\
R &\equiv \sqrt{(R_1 R_2)} = \left[nn'NN'\Omega_1\Omega_2 E_1 E_2\right]^{\frac{1}{2}} I_0(1-pA_y),
\end{aligned} \tag{8}
$$

and solve for pA_y we find the left–right asymmetry, ε, to be

$$
\varepsilon = \frac{L-R}{L+R} = pA_y(\theta), \tag{9}
$$

which is independent of relative detector efficiencies and solid angles, of relative integrated charge, and of target thickness variations. Those quantities common to the two channels, i.e., n and N, are averaged over the data accumulation period so that time fluctuations in the beam current or target density are of no consequence. On the other hand, those quantities which are different in the two channels, E and Ω, must not vary with time. Dead time in the counting equipment may be either common to the two channels or not, depending on the equipment used. If it is not common, a correction is required.

If we define the geometric mean of the number of particles detected by detector 1 (2) in the two intervals as N_1 (N_2), we have

$$
\begin{aligned}
N_1 &\equiv \sqrt{(L_1 R_1)} = \left\{NN'nn'\left[1-(pA_y)^2\right]\right\}^{\frac{1}{2}}\Omega_1 E_1 I_0, \\
N_2 &\equiv \sqrt{(L_2 R_2)} = \left\{NN'nn'\left[1-(pA_y)^2\right]\right\}^{\frac{1}{2}}\Omega_2 E_2 I_0.
\end{aligned} \tag{10}
$$

The ratio of these is

$$
\frac{N_1}{N_2} = \frac{\Omega_1 E_1}{\Omega_2 E_2}. \tag{11}
$$

That is, monitoring this ratio provides a check on the performance of the apparatus; it is just the quantity which is required to be constant in time if the asymmetry determination is to be accurate.

For the present case of perfect alignment, instead of reversing the beam polarization (that is, letting $\boldsymbol{p}\rightarrow -\boldsymbol{p}$), it is equivalent to "flip" the analyzer, i.e., to rotate the analyzer 180° around the beam direction so that detector 1 occupies the exact position of detector 2 and vice versa. When misalignments are considered, one can distinguish two ways which such a physical rotation could be carried out; this is considered in detail in sec. 3.3.

3.2. Statistical error and figure of merit for a polarized spin-½ beam

The statistical error associated with a measurement of the asymmetry ε is given by

$$
\Delta\varepsilon = \frac{1}{L+R}\sqrt{\left[(1-\varepsilon)^2(\Delta L)^2+(1+\varepsilon)^2(\Delta R)^2\right]}. \tag{12}
$$

If $\Delta L=\sqrt{L}$ and $\Delta R=\sqrt{R}$, as for a single counting interval, the expression of eq. (12) reduces to

$$
\Delta\varepsilon = \sqrt{\left(\frac{1-\varepsilon^2}{L+R}\right)}. \tag{13}
$$

For the geometric mean $L=\sqrt{(L_1 L_2)}$, the error is given by

$$
\Delta L = \tfrac{1}{2}L\sqrt{\left(\frac{1}{L_1}+\frac{1}{L_2}\right)},
$$

and for the geometric mean $R=\sqrt{(R_1 R_2)}$, the error is given by

$$
\Delta R = \tfrac{1}{2}R\sqrt{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}.
$$

Thus, the more general error expression of eq. (12) is required if geometric means are to be used.

We define a figure of merit to be inversely proportional to the counting time, t, required to measure A_y to a given statistical accuracy ΔA_y. Since, for $\varepsilon\approx 0$,

$$
\Delta A_y \rightarrow \frac{1}{p}\frac{1}{\sqrt{(L+R)}} \propto \frac{1}{p\sqrt{(it)}}, \tag{14}
$$

where i is the beam current and t is the measurement time, we see that $p^2 i$ is the relevant figure of merit. Notice that, because the factor $\sqrt{(1-\varepsilon^2)}$ is neglected, this figure markedly underestimates the advantage of a large beam polarization for cases where A_y is also large.

3.3. Effect of misalignment on spin-$\frac{1}{2}$ analyzing power measurements

3.3.1. *General*

We may distinguish three distinct types of geometrical errors. They are:

a) False asymmetries. We reserve this term to mean that asymmetry which would still be observed in the apparatus if the beam polarization were to vanish. This type of error can be eliminated exactly by a "proper flip" (see sec. 3.3.2).

b) Polarization-dependent misalignment effects. This term refers to errors which enter because of geometrical imperfections but which are absent for an unpolarized beam. These effects are of second order for our spin-$\frac{1}{2}$ discussion and can be reduced to second order in the spin-1 case by proper technique.

c) Errors induced by spin angle uncertainty. For spin-$\frac{1}{2}$ measurements, where $\beta = 90°$ is invariably chosen, this effect is of second order. However, for certain of the spin-1 methods to be described, such effects can be of first order. Some of the methods we will discuss eliminate these errors and some do not.

Suppose the beam is displaced an amount x_0 and rotated an amount k_x with respect to the symmetry axis of the detector system. This is shown for solid and gas target geometry, respectively, in figs. 4 and 5. It is clear that these displacements result in (1) a change in the effective scattering angle for each detector, and (2) a change in the effective distance between the interaction volume and each detector. To first order, we find, for the left detector,

$$\Delta r = -x_0 \sin\theta, \qquad \Delta\theta = -\frac{x_0}{R}\cos\theta - k_x, \qquad \text{solid target geometry}, \tag{15}$$

and

$$\Delta r = -\frac{x_0}{\sin\theta}, \qquad \Delta\theta = -k_x, \qquad \text{gas target geometry}. \tag{16}$$

All four of these quantities reverse in sign for the right detector. (Additional but nonreversing first-order terms can arise in the solid target case if the target is displaced along the beam direction; see sec. 3.4.)

These results can now be used to calculate the false asymmetries that would arise from beam misalignment by two different "flip" methods. In one type of flip, which we will designate as a "proper" flip, we assume that the polarization of the beam is reversed, or, what amounts to the same thing, that the analyzer is rotated through 180° in such a way that the beam direction and position are held invariant with respect to each of the detectors. Experimentally, such a flip can be conveniently carried out either by reversing the guide fields

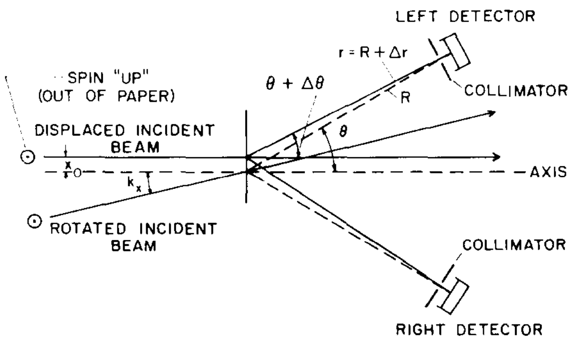

Fig. 4. Misalignment in solid target geometry.

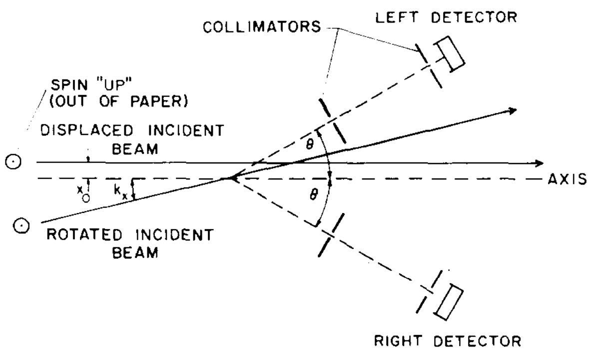

Fig. 5. Misalignment in gas target geometry.

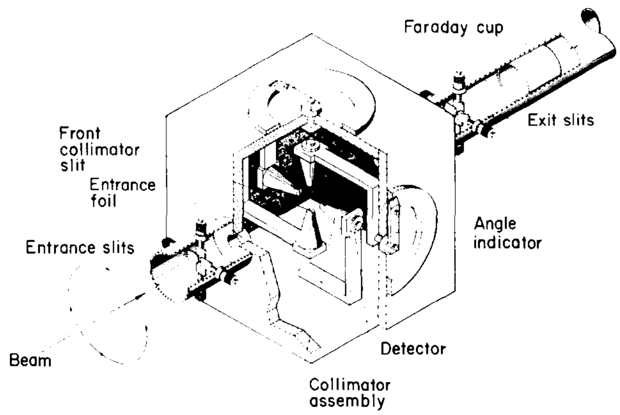

Fig. 6. Schematic diagram of a rotatable "cube" scattering chamber. Note that the entrance and exit slits rotate with the chamber.

in a polarized ion source or by actual rotation of an apparatus about the beam direction, as indicated in fig. 6. The important feature for the mechanical rotation method is that slits at the entrance and exit of the apparatus rotate with it, and that the experimenter requires the proportion of the beam current intercepted by each slit to be the same before and after the rotation. The particular apparatus shown, which has detectors in the left, right, up, and down positions, is also useful for spin-1 measurements (see sec. 4). The up-down detectors play no important role for spin-$\frac{1}{2}$ experiments. Notice that the mechanical precision with which the rotation is accomplished is not important *provided* the above mentioned slit current condition is maintained.

In the "nonproper" type of flip, which we describe in detail in sec. 3.3.3, the analyzer is rotated about its own axis while the beam is allowed to remain fixed in space. As will be seen, the proper flip method results in the exact elimination of false asymmetry effects, whereas the nonproper flip results in first-order false asymmetry effects and should therefore be avoided.

3.3.2. *Proper flips*

If detector 1 is at the left and detector 2 is at the right, the yields may be written

$$
\begin{aligned}
L_1 &\equiv N_1(\theta+\Delta\theta_1, 0) \\
&= nN E_1 \Omega_1(\Delta r_1, \Delta\theta_1) I_0(\theta+\Delta\theta_1)[1+pA_y(\theta+\Delta\theta_1)], \\
R_2 &\equiv N_2(\theta+\Delta\theta_2, \pi) \\
&= nN E_2 \Omega_2(\Delta r_2, \Delta\theta_2) I_0(\theta+\Delta\theta_2)[1-pA_y(\theta+\Delta\theta_2)],
\end{aligned}
\tag{17}
$$

where $\Delta\theta_1$, $\Delta\theta_2$, Δr_1 and Δr_2 denote total error quantities. If a proper flip is executed, yields in detectors 1 and 2 become

$$
\begin{aligned}
R_1 &\equiv N_1(\theta+\Delta\theta_1, \pi) \\
&= n' N' E_1 \Omega_1(\Delta r_1, \Delta\theta_1) I_0(\theta+\Delta\theta_1)[1-pA_y(\theta+\Delta\theta_1)], \\
L_2 &\equiv N_2(\theta+\Delta\theta_2, 0) \\
&= n' N' E_2 \Omega_2(\Delta r_2, \Delta\theta_2) I_0(\theta+\Delta\theta_2)[1+pA_y(\theta+\Delta\theta_2)].
\end{aligned}
\tag{18}
$$

The crux of the present assumption is that the errors in r, θ are correlated with the physical detectors, and not with their position in space. Forming the appropriate geometric means, we have

$$
\begin{aligned}
L \equiv \sqrt{(L_1 L_2)} = \{&nn'NN'E_1E_2\Omega_1(\Delta r_1, \Delta\theta_1)\times \\
&\times \Omega_2(\Delta r_2, \Delta\theta_2) I_0(\theta_1+\Delta\theta_1)\times \\
&\times I_0(\theta_2+\Delta\theta_2)[1+pA_y(\theta+\Delta\theta_1)]\times \\
&\times [1+pA_y(\theta+\Delta\theta_2)]\}^{\frac{1}{2}}, \\
R \equiv \sqrt{(R_1 R_2)} = \{&nn'NN'E_1E_2\Omega_1(\Delta r_1, \Delta\theta_1)\times \\
&\times \Omega_2(\Delta r_2, \Delta\theta_2) I_0(\theta_1+\Delta\theta_1)\times \\
&\times I_0(\theta_2+\Delta\theta_2)[1-pA_y(\theta+\Delta\theta_1)]\times \\
&\times [1-pA_y(\theta+\Delta\theta_2)]\}^{\frac{1}{2}}.
\end{aligned}
\tag{19}
$$

If we assume for the moment that A_y is *independent of scattering angle* in the range $\theta+\Delta\theta_1$ to $\theta+\Delta\theta_2$, we have

$$
\varepsilon = pA_y = \frac{L-R}{L+R}; \tag{20}
$$

i.e., there is *exact* cancellation of false asymmetry effects. Since the form of Ω was not specified in the argument, it applies to both solid and to gas target geometries. Since $\Delta\theta_1$, $\Delta\theta_2$, Δr_1 and Δr_2 were each arbitrary, one does not even require symmetry in the analyzer. If $A_y(\theta)$ depends on θ, the determination of the effective scattering angle $[=\theta+\frac{1}{2}(\Delta\theta_1+\Delta\theta_2)]$ becomes an important experimental task. Note that $\Delta\theta_2 \approx -\Delta\theta_1$ for the first-order errors of eqs. (15) and (16). A discussion of the removal of first-order errors in the angle determination is presented in sec. 3.4.

3.3.3. *Nonproper flips*

We now turn to the nonproper flip where we interchange the detectors while the beam remains fixed in space. We assume, therefore, that the errors $\Delta\theta$ and Δr are associated with the left and right positions rather than with the detector number as above. For this calculation we further assume that $\Delta\theta$ and Δr for the right detector are equal in magnitude but opposite in sign to the corresponding quantities for the left detector [as is true in the first-order expressions of eqs. (15) and (16)]. We have, for the first orientation of the detectors,

$$
\begin{aligned}
L_1 &\equiv N_1(\theta+\Delta\theta_L, 0) = nNE_1\Omega_1(\Delta r, \Delta\theta)\times \\
&\quad \times I_0(\theta+\Delta\theta)[1+pA_y(\theta+\Delta\theta)], \\
R_2 &\equiv N_2(\theta+\Delta\theta_R, \pi) = nNE_2\Omega_2(-\Delta r, -\Delta\theta)\times \\
&\quad \times I_0(\theta-\Delta\theta)[1-pA_y(\theta-\Delta\theta)].
\end{aligned}
\tag{21}
$$

For the second (flipped) orientation, we have

$$R_1 \equiv N_1(\theta+\Delta\theta_R, \pi) = n'N'E_1\Omega_1(-\Delta r, -\Delta\theta) \times \times I_0(\theta-\Delta\theta)\,[1-pA_y(\theta-\Delta\theta)],$$

$$L_2 \equiv N_2(\theta+\Delta\theta_L, 0) = n'N'E_2\Omega_2(\Delta r, \Delta\theta) \times \times I_0(\theta+\Delta\theta)\,[1+pA_y(\theta+\Delta\theta)]. \quad (22)$$

The geometric means then take the form

$$L = K\Omega_L I_0(\theta+\Delta\theta)\,[1+pA_y(\theta+\Delta\theta)],$$

$$R = K\Omega_R I_0(\theta-\Delta\theta)\,[1-pA_y(\theta-\Delta\theta)], \quad (23)$$

where

$$K = \sqrt{(nn'NN'E_1E_2)},\ \Omega_L = \sqrt{[\Omega_1(\Delta r, \Delta\theta)\Omega_2(\Delta r, \Delta\theta)]}$$

and

$$\Omega_R = \sqrt{[\Omega_1(-\Delta r, -\Delta\theta)\Omega_2(-\Delta r, -\Delta\theta)]}.$$

At this point, it is necessary to consider gas target and solid target geometry separately. For solid target geometry, we expand the solid angle factors to first order:

$$\Omega_L \propto 1 - 2\Delta r/R,$$

$$\Omega_R \propto 1 + 2\Delta r/R. \quad (24)$$

The cross section may also be expanded

$$I(\theta\pm\Delta\theta) = I_0(\theta)\,[1\pm G\Delta\theta], \quad (25)$$

where G is the logarithmic derivative of I_0:

$$G = \frac{1}{I_0}\frac{\partial I_0}{\partial\theta}. \quad (26)$$

First let us assume that the analyzing power, $A_y(\theta)$, vanishes. The resulting value of ε, which is what we define as the false asymmetry and will denote by ε', is

$$\varepsilon' = G\Delta\theta - 2\Delta r/R, \qquad \text{(solid target geometry)}, \quad (27)$$

where G is expressed in rad^{-1}. Inserting eq. (15), this becomes

$$\varepsilon' = (2\sin\theta - G\cos\theta)\,(x_0/R) - Gk_x, \qquad \text{(solid target geometry)}. \quad (28)$$

For gas target geometry, the corresponding expressions are

$$\varepsilon' = G\Delta\theta - \Delta r/R, \quad (29)$$

and

$$\varepsilon' = \cos\theta\,(x_0/R) + (\cot\theta - G)\,k_x. \quad (30)$$

There are three comments we wish to make about these results:

a) One should always use a proper flip if possible. (If the polarized beam arises from a nuclear reaction, for example, it may not be possible.) Even when a proper flip is attempted, however, there may be some (hopefully small) component of nonproper flip. If x_0, x_0' and k_x, k_x' represent the spatial and angular displacements in the two configurations, the quantities x_0-x_0' and k_x-k_x' are the relevant ones for application of eq. (28) or (30), and will define the lower limit of error from false asymmetries.

b) Gas target geometry is considerably less sensitive to spatial displacement, but more sensitive to angular displacement, than is solid target geometry. Since angular errors are much easier to control than spatial displacements, gas geometry usually possesses a considerable practical advantage.

c) If the analyzing power, $A_y(\theta)$, is nonzero, the observed asymmetry takes the form

$$\varepsilon = \frac{pA_y+\varepsilon'}{1+\varepsilon' pA_y} \approx pA_y + \varepsilon'(1-p^2A_y^2). \quad (31)$$

Thus, for large values of pA_y, the effect of false asymmetries is greatly reduced. For example, if $pA_y = 0.9$ and $\varepsilon' = 0.05$, the observed asymmetry, ε, is ≈ 0.91. That is, in this particular example, a false asymmetry of 0.05 leads to an error of only 0.01.

3.4. Determination of the scattering angle

Before a measured analyzing power can be regarded as free of first-order errors, the angle at which the measurement is made must be known to a suitable accuracy.

We consider first the gas target case. As previously noted, the use of a symmetric left-right detector system cancels the first-order angle errors associated with the misalignment parameters k_x and x_0. Two errors could remain, however. They are (1) an error in the "zero" angle setting of each detector, and (2) an error arising from the failure of the plane defined by the telescope slit system to intersect the axis about which the telescope is rotated when changes in θ are made. If the left and right detectors are interchanged, both of these possible effects are removed. Thus, a suitable experimental procedure would be to measure the asymmetry with the #1 (left) and #2 (right) detectors, set in their "normal" positions, i.e., at θ_1 and $-\theta_2$, respectively, followed by a measurement with their roles interchanged; i.e., with #1 set at $-\theta_1$ and with #2 set at θ_2. The average of the asymmetries so obtained would be free of first-order errors and would correspond to the angle $\frac{1}{2}(\theta_1+\theta_2)$. The indicated angles θ_1 and θ_2 would, of course, normally be chosen to be the same.

In the solid target case, there are again two errors not automatically removed by a symmetric detector system. The first is, as in the gas target case, a possible error in the "zero" angle, and this can be handled as before. The second arises from the difficulty of placing the point at which the beam intersects the target foil exactly at the axis of rotation of the detector arms. The displacement in the x direction is removed by the symmetric system, but the z displacement is not. To remove this effect, one may make still another asymmetry measurement with the target foil rotated through the angle $\pi-2\theta_{\mathrm{targ}}$, where θ_{targ} is the angle of inclination of the target foil (see fig. 7). This procedure changes the sign of the z displacement, and results in a mean angle of θ_1 for the #1 detector and a mean angle of θ_2 for the #2 detector. Thus, the mean asymmetry determined by these two measurements corresponds to a scattering angle of $\frac{1}{2}(\theta_1+\theta_2)$. Notice that the axis about which the target is rotated is assumed to coincide with the axis about which the detectors are rotated. If this is not the case, a "permanent" z displacement error in the angle will be present. This could, of course, be calibrated and the angle readings appropriately corrected.

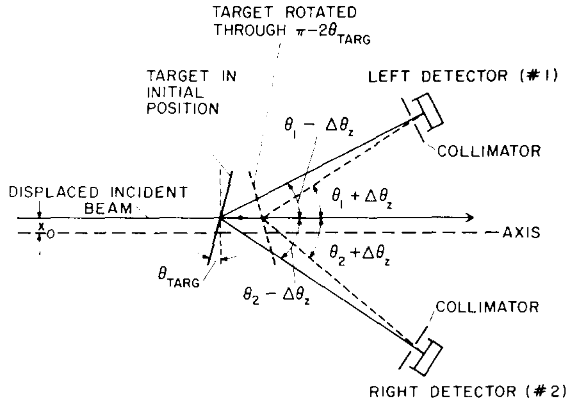

Fig. 7. Diagram showing the two positions of the target foil which can be used to eliminate the first-order error in the scattering angle which arises from an uncertainty in the foil position.

4. Measurement of spin-1 analyzing powers

4.1. Observable asymmetries and ratios

In the coordinate system of fig. 1, the most general form for the cross section induced by spin-1 particles may be written

$$I(\theta,\phi) = I_0(\theta)\,[1+\tfrac{3}{2}p_y A_y(\theta)+\tfrac{2}{3}p_{xz}A_{xz}(\theta)+ + \tfrac{1}{3}p_{xx}A_{xx}(\theta)+\tfrac{1}{3}p_{yy}A_{yy}(\theta)+\tfrac{1}{3}p_{zz}A_{zz}(\theta)]\,. \tag{32}$$

The small p's represent the polarization of the incident beam and the A's represent the analyzing power of the reaction. In general, an incident beam may have vector polarization components p_x, p_y and p_z, but because of parity conservation, the reaction is sensitive only to the component normal to the scattering plane. Similarly, although an incident beam may contain all six tensor polarization components $p_{xy}, p_{yz}, p_{xz}, p_{xx}, p_{yy}$ and p_{zz}, the reaction is sensitive, again because of parity, only to those indicated in eq. (32). The quantities defined are normalized so that the vector quantities (p_x, p_y, p_z and A_y) may vary between $+1$ and -1, the tensor quantities (p_{xy}, p_{yz}, p_{xz} and A_{xz}) may vary between $+\frac{3}{2}$ and $-\frac{3}{2}$, and the tensor quantities (p_{xx}, $p_{yy}, p_{zz}, A_{xx}, A_{yy}$ and A_{zz} may vary between $+1$ and -2.

The second-rank beam polarization components, p_{xx}, p_{yy} and p_{zz}, satisfy the identity

$$p_{xx}+p_{yy}+p_{zz} = 0\,, \tag{33}$$

and similarly, the second-rank analyzing tensors, $A_{xx}(\theta)$, $A_{yy}(\theta)$ and $A_{zz}(\theta)$, satisfy

$$A_{xx}(\theta)+A_{yy}(\theta)+A_{zz}(\theta) = 0\,. \tag{34}$$

Thus, only four of the five analyzing tensors which appear in eq. (32) are independent. These identities allow us to write the last three terms in the cross section expression in several equivalent forms,

$$\tfrac{1}{3}p_{xx}A_{xx}+\tfrac{1}{3}p_{yy}A_{yy}+\tfrac{1}{3}p_{zz}A_{zz} = \tfrac{1}{6}(p_{ll}-p_{mm})\times \times(A_{ll}-A_{mm})+\tfrac{1}{2}p_{nn}A_{nn}\,, \tag{35}$$

where l, m, n are x, y, z in any order. This relation allows us to write, for example,

$$I = I_0(1+\tfrac{3}{2}p_y A_y+\tfrac{1}{2}p_{yy}A_{yy}) \tag{36}$$

for the cross section if the beam polarization symmetry axis is along the y axis (so that $p_{xx}=p_{zz}=-\frac{1}{2}p_{yy}$).

For a polarized beam produced by an ion source, in the coordinate system of fig. 1, the polarization components are

$$\begin{aligned} p_x &= -\,p_Z\sin\beta\sin\phi\,,\\ p_y &= p_Z\sin\beta\cos\phi\,,\\ p_z &= p_Z\cos\beta\,,\\ p_{xy} &= -\,\tfrac{3}{2}p_{ZZ}\sin^2\beta\cos\phi\sin\phi\,, \end{aligned}$$

$$
\begin{aligned}
p_{yz} &= \tfrac{3}{2} p_{ZZ} \sin\beta \cos\beta \cos\phi\,, \\
p_{xz} &= -\tfrac{3}{2} p_{ZZ} \sin\beta \cos\beta \sin\phi\,, \\
p_{xx} &= \tfrac{1}{2} p_{ZZ} (3 \sin^2\beta \sin^2\phi - 1)\,, \\
p_{yy} &= \tfrac{1}{2} p_{ZZ} (3 \sin^2\beta \cos^2\phi - 1)\,, \\
p_{zz} &= \tfrac{1}{2} p_{ZZ} (3 \cos^2\beta - 1)\,,
\end{aligned} \tag{37}
$$

and the often used difference, $\frac{1}{2}(p_{xx} - p_{yy})$, is given by

$$\tfrac{1}{2}(p_{xx} - p_{yy}) = -\tfrac{3}{4} p_{ZZ} \sin^2\beta \cos 2\phi\,. \tag{38}$$

Inserting eq. (37) into eq. (32), the cross section for scattering in the directions left, right, up and down ($\phi = 0°$, $180°$, $270°$, $90°$) can be written

$$
\begin{aligned}
I_{\mathrm{L}} &= I_0[1 + \tfrac{3}{2} p_Z \sin\beta A_y + \tfrac{1}{2} p_{ZZ} \times \\
&\qquad \times (\sin^2\beta A_{yy} + \cos^2\beta A_{zz})]\,, \\
I_{\mathrm{R}} &= I_0[1 - \tfrac{3}{2} p_Z \sin\beta A_y + \tfrac{1}{2} p_{ZZ} \times \\
&\qquad \times (\sin^2\beta A_{yy} + \cos^2\beta A_{zz})]\,, \\
I_{\mathrm{U}} &= I_0[1 + p_{ZZ} \sin\beta \cos\beta A_{xz} + \tfrac{1}{2} p_{ZZ} \times \\
&\qquad \times (\sin^2\beta A_{xx} + \cos\ \beta A_{zz})]\,, \\
I_{\mathrm{D}} &= I_0[1 - p_{ZZ} \sin\beta \cos\beta A_{xz} + \tfrac{1}{2} p_{ZZ} \times \\
&\qquad \times (\sin^2\beta A_{xx} + \cos^2\beta A_{zz})]\,.
\end{aligned} \tag{39}
$$

Assuming for a moment that detector geometries and efficiencies are identical, and denoting the counts observed in the various detectors by L, R, U and D, we can define five useful asymmetries:

$$
\begin{aligned}
\varepsilon_1 &= \frac{L-R}{L+R} = \frac{\tfrac{3}{2} p_Z \sin\beta A_y}{1 + \tfrac{1}{2} p_{ZZ}[\sin^2\beta A_{yy} + \cos^2\beta A_{zz}]}\,, \\
\varepsilon_2 &= \frac{U-D}{U+D} = \frac{p_{ZZ} \sin\beta \cos\beta A_{xz}}{1 + \tfrac{1}{2} p_{ZZ}[\sin^2\beta A_{xx} + \cos^2\beta A_{zz}]}\,, \\
\varepsilon_3 &= \frac{2(L-R)}{L+R+U+D} = \frac{\tfrac{3}{2} p_Z \sin\beta A_y}{1 + \tfrac{1}{4} p_{ZZ}(3\cos^2\beta - 1) A_{zz}}\,, \\
\varepsilon_4 &= \frac{2(U-D)}{L+R+U+D} = \frac{p_{ZZ} \sin\beta \cos\beta A_{xz}}{1 + \tfrac{1}{4} p_{ZZ}(3\cos^2\beta - 1) A_{zz}}\,, \\
\varepsilon_5 &= \frac{(L+R) - (U+D)}{L+R+U+D} = \frac{-\tfrac{1}{4} p_{ZZ} \sin^2\beta (A_{xx} - A_{yy})}{1 + \tfrac{1}{4} p_{ZZ}(3\cos^2\beta - 1) A_{zz}}\,.
\end{aligned} \tag{40}
$$

We emphasize that "up" is *defined* to be the direction pointing along the transverse component of the unit vector which described the direction of the beam polarization, $\boldsymbol{s}$, and may or may not have anything to do with real space.

From eq. (40) one sees that the quantities A_y, A_{xz} and $A_{xx} - A_{yy}$ each give rise to characteristic asymmetries. To obtain A_{zz}, a yield ratio with two different values of p_{ZZ} is required, since the A_{zz} term has no azimuthal dependence. More generally, we may consider the left+right yield, the up+down yield, or the total yield in all four detectors:

$$
\begin{aligned}
T_1 &\equiv \tfrac{1}{2}(L+R) \\
&= nN\Omega E I_0[1 + \tfrac{1}{2} p_{ZZ}(\sin^2\beta A_{yy} + \cos^2\beta A_{zz})]\,, \\
T_2 &\equiv \tfrac{1}{2}(U+D) \\
&= nN\Omega E I_0[1 + \tfrac{1}{2} p_{ZZ}(\sin^2\beta A_{xx} + \cos^2\beta A_{zz})]\,, \\
T_3 &\equiv \tfrac{1}{4}(L+R+U+D) \\
&= nN\Omega E I_0[1 + \tfrac{1}{2} p_{ZZ}(\tfrac{1}{2}(3\cos^2\beta - 1) A_{zz})]\,.
\end{aligned} \tag{41}
$$

All of these expressions are of the same form:

$$T = nN\Omega E I_0(1 + \tfrac{1}{2} p_{ZZ} B)\,. \tag{42}$$

If the yields T, T' are observed for the beam polarization p_{ZZ}, p'_{ZZ}, respectively, assuming the factor nN to be the same for the two cases, we find

$$B = \frac{2(T - T')}{p_{ZZ} T' - p'_{ZZ} T}\,. \tag{43}$$

These expressions are useful mainly for $\beta = 0°$, where T_1, T_2 and T_3 each leads to $B = A_{zz}$, or for $\beta = 90°$, where T_1 leads to $B_1 = A_{yy}$, T_2 leads to $B_2 = A_{xx}$ and T_3 leads to $B_3 = -\frac{1}{2} A_{zz}$.

4.2. Statistical error and figure of merit for a polarized spin-1 beam

In table 1, column 2, we give the statistical error expressions associated with each of the asymmetries ε_1–ε_5 and the ratios B_1–B_3, in terms of the errors ΔL, ΔR, ΔU and ΔD, respectively, in L, R, U, D. In the special case $\Delta L = \sqrt{L}$, $\Delta U = \sqrt{U}$, etc., the expressions listed in column 3 are obtained. (For the geometric means of later interest, the more general error expressions must be used.)

A figure of merit, $p^2 i$, for a polarized spin-$\frac{1}{2}$ beam was discussed in sec. 3.2. We now wish to define three analogous quantities for a spin-1 beam[5]). Since the

TABLE 1

Observable ratios and asymmetries.[a]

Measured quantity	Statistical uncertainty (general case)	Statistical uncertainty (special case)	Combination of observables determined
1. $\varepsilon_1 = \dfrac{L-R}{L+R}$	$\Delta\varepsilon_1 = \dfrac{1}{L+R}\sqrt{[(1-\varepsilon_1)^2(\Delta L)^2+(1+\varepsilon_1)^2(\Delta R)^2]}$	$\sqrt{\left(\dfrac{1-\varepsilon_1^2}{L+R}\right)}$	$\dfrac{\frac{3}{2}p_Z \sin\beta A_y}{1+\frac{1}{2}p_{ZZ}(\sin^2\beta A_{yy}+\cos^2\beta A_{zz})}$
2. $\varepsilon_2 = \dfrac{U-D}{U+D}$	$\Delta\varepsilon_2 = \dfrac{1}{U+D}\sqrt{[(1-\varepsilon_2)^2(\Delta U)^2+(1+\varepsilon_2)^2(\Delta D)^2]}$	$\sqrt{\left(\dfrac{1-\varepsilon_2^2}{U+D}\right)}$	$\dfrac{p_{ZZ}\sin\beta\cos\beta A_{xz}}{1+\frac{1}{2}p_{ZZ}(\sin^2\beta A_{xx}+\cos^2\beta A_{zz})}$
3. $\varepsilon_3 = \dfrac{2(L-R)}{T_3}$	$\Delta\varepsilon_3 = \dfrac{2}{T_3}\sqrt{\{(1-\frac{1}{2}\varepsilon_3)^2(\Delta L)^2+(1+\frac{1}{2}\varepsilon_3)^2(\Delta R)^2+(\frac{1}{2}\varepsilon_3)^2[(\Delta U)^2+(\Delta D)^2]\}}$	$\sqrt{\left\{\dfrac{[4(L+R)]/T_3-\varepsilon_3^2}{T_3}\right\}}$	$\dfrac{\frac{3}{2}p_Z\sin\beta A_y}{1+\frac{1}{4}p_{ZZ}(3\cos^2\beta-1)A_{zz}}$
4. $\varepsilon_4 = \dfrac{2(U-D)}{T_3}$	$\Delta\varepsilon_4 = \dfrac{2}{T_3}\sqrt{\{(1-\frac{1}{2}\varepsilon_4)^2(\Delta U)^2+(1+\frac{1}{2}\varepsilon_4)^2(\Delta D)^2+(\frac{1}{2}\varepsilon_4)^2[(\Delta L)^2+(\Delta R)^2]\}}$	$\sqrt{\left\{\dfrac{[4(U+D)]/T_3-\varepsilon_4^2}{T_3}\right\}}$	$\dfrac{p_{ZZ}\sin\beta\cos\beta A_{xz}}{1+\frac{1}{4}p_{ZZ}(3\cos^2\beta-1)A_{zz}}$
5. $\varepsilon_5 = \dfrac{(L+R)-(U+D)}{T_3}$	$\Delta\varepsilon_5 = \dfrac{1}{T_3}\sqrt{\{(1-\varepsilon_5)^2[(\Delta L)^2+(\Delta R)^2]+(1+\varepsilon_5)^2[(\Delta U)^2+(\Delta D)^2]\}}$	$\sqrt{\left(\dfrac{1-\varepsilon_5^2}{T_3}\right)}$	$\dfrac{-\frac{1}{4}p_{ZZ}\sin^2\beta(A_{xx}-A_{yy})}{1+\frac{1}{4}p_{ZZ}(3\cos^2\beta-1)A_{zz}}$
6. $B_1 = \dfrac{2(T_1-T_1')}{p_{ZZ}T_1'-p_{ZZ}'T_1}$	$\Delta B_1 = \dfrac{2(p_{ZZ}-p_{ZZ}')}{(p_{ZZ}T_1'-p_{ZZ}'T_1)^2}\sqrt{[(T_1')^2(\Delta T_1)^2+(T_1)^2(\Delta T_1')^2]}$	$\dfrac{2(p_{ZZ}-p_{ZZ}')}{(p_{ZZ}T_1'-p_{ZZ}'T_1)^2}\sqrt{[(T_1')^2T_1+(T_1)^2T_1']}$	$\sin^2\beta A_{yy}+\cos^2\beta A_{zz}$
7. $B_2 = \dfrac{2(T_2-T_2')}{p_{ZZ}T_2'-p_{ZZ}'T_2}$	$\Delta B_2 = \dfrac{2(p_{ZZ}-p_{ZZ}')}{(p_{ZZ}T_2'-p_{ZZ}'T_2)^2}\sqrt{[(T_2')^2(\Delta T_2)^2+(T_2)^2(\Delta T_2')^2]}$	$\dfrac{2(p_{ZZ}-p_{ZZ}')}{(p_{ZZ}T_2'-p_{ZZ}'T_2)^2}\sqrt{[(T_2')^2T_2+(T_2)^2T_2']}$	$\sin^2\beta A_{xx}+\cos^2\beta A_{zz}$
8. $B_3 = \dfrac{2(T_3-T_3')}{p_{ZZ}T_3'-p_{ZZ}'T_3}$	$\Delta B_3 = \dfrac{2(p_{ZZ}-p_{ZZ}')}{(p_{ZZ}T_3'-p_{ZZ}'T_3)^2}\sqrt{[(T_3')^2(\Delta T_3)^2+(T_3)^2(\Delta T_3')^2]}$	$\dfrac{2(p_{ZZ}-p_{ZZ}')}{(p_{ZZ}T_3'-p_{ZZ}'T_3)^2}\sqrt{[(T_3')^2T_3+(T_3)^2T_3']}$	$\frac{1}{2}(3\cos^2\beta-1)A_{zz}$

[a] $T_1 = \frac{1}{2}(L+R)$, $T_2 = \frac{1}{2}(U+D)$, $T_3 = \frac{1}{4}(L+R+U+D)$.

observable quantities are, except in special cases, not related linearly to the beam polarization and efficiency tensors, the situation is somewhat more complicated than in the spin-$\frac{1}{2}$ case.

First consider B_1, B_2 and B_3. Using B_3 as a typical one of these, for small analyzing tensors and no background, we find from table 1

$$\Delta B_3 \rightarrow \frac{1}{(p_{ZZ}-p'_{ZZ})\sqrt{T_3}}. \tag{44}$$

The figure of merit relevant for measuring B_3 is thus given by $(\Delta p_{ZZ})^2 i$, where i is the beam current and where Δp_{ZZ} is the change in p_{ZZ} obtainable between the two modes of operation for the ion source. The same figure of merit applies to B_1 and B_2.

Returning to the asymmetries ε_1 through ε_3, we see that if the denominator factor has already been determined, the figure of merit becomes $p_Z^2 i$ for ε_1 and ε_3, and $p_{ZZ}^2 i$ for ε_2, ε_4 and ε_5.

In summary, spin-1 beams may be described in terms of three figures of merit: $p_Z^2 i$ for A_y measurements, $p_{ZZ}^2 i$ for $(A_{xx}-A_{yy})$ and A_{xz} measurements, and $(\Delta p_{ZZ})^2 i$ for A_{xx}, A_{yy} and A_{zz} measurements of the ratio type.

4.3. Some practical spin-1 measurement systems

Before discussing the geometrical errors associated with spin-1 analyzing tensor determinations, we discuss four measurement schemes which have been used at LASL. It is the authors' belief that presentation of these particular examples, restricted as they are to a particular experimental configuration, will nevertheless serve to illuminate all of the important principles in a clearer way than would a more general treatment. A characteristic of spin-1 experiments is that there are a very large number of measurement schemes possible. We will make a few comments on the way that the various schemes might be modified to be useful under other experimental conditions.

The LASL polarized source produces "pure" spin states mixed with an unpolarized background beam. The polarized part of the beam has the following vector and tensor polarizations for the usual operating conditions[6]):

State selected :	p_Z:	p_{ZZ}:
$m_I = 1$,	1,	1,
"$m_I = 0$",	0.012,	−1.966,
"$m_I = -1$",	−0.984,	0.952.

The quotation marks denote that the states, except for $m_I = 1$, are not quite pure, as is indicated by the values of p_Z and p_{ZZ}*. The quoted admixtures are characteristic of a metastable deuterium atom when it is ionized in a 60 G field.

The fraction of the beam which is polarized is represented by p_Q. p_Q is usually obtained by the "quench ratio" method[7]). For example, for $m_I = 1$ selection, the vector and tensor polarizations would be given by $p_Z = p_{ZZ} = p_Q$, and for $m_I = 0$ selection by $p_Z = 0.012 p_Q$ and $p_{ZZ} = -1.966\, p_Q$. We refer loosely to such beams as $m_I = 1$ and $m_I = 0$ beams, respectively.

On the LASL source-accelerator system, the spin quantization axis can be oriented in any direction. However, we most frequently place the quantization axis in the horizontal plane because of the greater ease of setting arbitrary β angles; $\beta = 0°$, 45°, 54.7° and 90° will be of interest in the methods to be discussed. For $\beta = 90°$, we sometimes align the quantization axis in a vertical direction. In any event, as mentioned earlier, in the discussion to follow the half-plane containing the spin quantization axis will be referred to as "up", regardless of its actual orientation in real space.

In the following, it will be assumed that detector efficiency and solid angle factors have been removed by an appropriate proper flip and geometric mean calculation. For schemes which simultaneously involve detectors in all four half-planes, we define a flip to be a series of four counting periods during which each of the detectors 1–4 occupies each of the positions left, right, up and down, with the beam held in rigid alignment with respect to each of the four physical detector systems. This operation is not readily accomplished at the source by polarized ion source systems because of the different treatment of the horizontal and vertical spin components by a typical accelerator and beam transport system. Thus, we are thinking specifically of physical rotation of a four-detector analyzer with entrance and exit collimators rigidly attached to the analyzer. We define the four geometric means

$$\begin{aligned}
L &= [N_1(\theta,0)\,N_2(\theta,0)\,N_3(\theta,0)\,N_4(\theta,0)]^{\frac{1}{4}},\\
R &= [N_1(\theta,\pi)\,N_2(\theta,\pi)\,N_3(\theta,\pi)\,N_4(\theta,\pi)]^{\frac{1}{4}},\\
U &= [N_1(\theta,\tfrac{3}{2}\pi)\,N_2(\theta,\tfrac{3}{2}\pi)\,N_3(\theta,\tfrac{3}{2}\pi)\,N_4(\theta,\tfrac{3}{2}\pi)]^{\frac{1}{4}},\\
D &= [N_1(\theta,\tfrac{1}{2}\pi)\,N_2(\theta,\tfrac{1}{2}\pi)\,N_3(\theta,\tfrac{1}{2}\pi)\,N_4(\theta,\tfrac{1}{2}\pi)]^{\frac{1}{4}}.
\end{aligned} \tag{45}$$

For the moment we assume perfect beam and spin alignment. Misalignment effects will be considered in sec. 4.4.

* The $m_I = 1$ state with the ion source fields reversed is often used instead of the $m_I = -1$ state. For this choice, $p_Z = -1$ and $p_{ZZ} = 1$.

We may also form the geometric mean of the counts observed by a given detector in each of its four positions. For example, for detector 1,

$$\begin{aligned} N_1 &= [N_1(\theta, 0)\, N_1(\theta, \pi)\, N_1(\theta, \tfrac{3}{2}\pi)\, N_1(\theta, \tfrac{1}{2}\pi)]^{\frac{1}{4}} \\ &= k\Omega_1 E_1, \end{aligned} \tag{46}$$

where k is a constant. Thus, the quantities N_1, N_2, N_3 and N_4 are proportional to the respective detector efficiency and solid angle factors. Monitoring these quantities provides a check on the overall performance of the apparatus; these are exactly the quantities which must be constant in time if the asymmetry determination is to be accurate.

4.3.1. *Rapid method*

In this method[8]), the spin angle $\beta = 54.7°$ is chosen for the measurement of A_y, A_{xz} and $\frac{1}{2}(A_{xx} - A_{yy})$. From eq. (40), we obtain

$$\begin{aligned} \varepsilon_3 &= [2(L-R)]/T = \sqrt{\tfrac{3}{2}}\, p_Z A_y, \\ \varepsilon_4 &= [2(U-D)]/T = \tfrac{1}{3}\sqrt{2}\; p_{ZZ} A_{xz}, \\ \varepsilon_5 &= [(L+R)-(U+D)]/T = -\tfrac{1}{6} p_{ZZ}(A_{xx} - A_{yy}), \end{aligned} \tag{47}$$

where L, R, U and D are the geometric means of eq. (45). Thus, A_y, A_{xz} and $\frac{1}{2}(A_{xx} - A_{yy})$ are determined in a current-integrator free manner. An $m_I = 1$ beam is used. There is a first-order dependence of the results on the actual spin angles β and ϕ in this method. The most severe limitation arises from the "cross talk" between the L–R and U–D asymmetries if the ϕ angles deviate from the ideal 0°, 90°, 180° and 270° values. Because of the numerical factors $\sqrt{\frac{3}{2}}$ vs $\frac{1}{3}\sqrt{2}$, a large A_y can contaminate the A_{xz} measurement 27/4 times more severely than a large A_{xz} can contaminate the A_y measurement. If a beam with pure tensor polarization is used, such as an $m_I = 0$ beam, the A_{xz} measurement is greatly improved but, of course, no A_y measurement is simultaneously obtained, and the method begins to lose the "rapid" character.

To obtain A_{zz}, the spin angle $\beta = 0°$ and an $m_I = 1$ to $m_I = 0$ ratio is used. The expressions for B_1, B_2 or B_3 of table 1 each yield A_{zz}. In this case, the measurement depends on the relative current integration, dead time correction, and the like, and the beam polarization in each of the two states must be separately known. As will be discussed in sec. 4.4, a symmetric pair of detectors together with a twofold flip is the minimum configuration which eliminates first-order geometrical errors for A_{zz} by this method.

4.3.2. *Ratio method*

Except for the A_{xz} tensor, this method avoids the requirement of an accurate knowledge of the spin angle, β, but all variables are dependent on current integration, dead time factors, etc. To measure A_y, A_{yy} and A_{xx}, a spin angle of 90°, a twofold flip for $m_I = 1$, and a twofold flip for $m_I = 0$ is used. A_{yy} and A_{xx} are calculated from the ratios B_1 and B_2, respectively (see table 1). These may be rewritten for the special $m_I = 1$, $m_I = 0$ case:

$$\begin{aligned} A_{xx} &= \frac{2[(U_{(1)} + D_{(1)}) - (U_{(0)} + D_{(0)})]}{p_Q[1.966(U_{(1)} + D_{(1)}) + (U_{(0)} + D_{(0)})]}, \\ A_{yy} &= \frac{2[(L_{(1)} + R_{(1)}) - (L_{(0)} + R_{(0)})]}{p_Q[1.966(L_{(1)} + R_{(1)}) + (L_{(0)} + R_{(0)})]}, \end{aligned} \tag{48}$$

where the subscripts indicate whether the yield corresponds to the substate $m_I = 1$ or $m_I = 0$.

To obtain A_y, we note from eq. (1) of table 1 that, for $\beta = 90°$ and for the $m_I = 1$ run of the above sequence,

$$\varepsilon_1 = \frac{L_{(1)} - R_{(1)}}{L_{(1)} + R_{(1)}} = \frac{\frac{3}{2} p_Z^{(1)} A_y}{1 + \frac{1}{2} p_{ZZ}^{(1)} A_{yy}} \equiv \frac{\frac{3}{2} p_Q A_y}{1 + \frac{1}{2} p_Q A_{yy}}. \tag{49}$$

If one substitutes the expression for A_{yy} of eq. (48) into eq. (49) and solves for A_y, one obtains

$$A_y = \frac{2.966(L_{(1)} - R_{(1)})}{p_Q[1.966(L_{(1)} + R_{(1)}) + (L_{(0)} + R_{(0)})]}. \tag{50}$$

This quantity also depends on relative current integration, but not as strongly as do A_{yy} and A_{xx}.

The remaining tensor, A_{xz}, is most economically obtained with $\beta = 45°$, $m_I = 0$, and a twofold flip, using the up-down detectors only. Then

$$\varepsilon_2 = \frac{U_{(0)} - D_{(0)}}{U_{(0)} + D_{(0)}} = \frac{\frac{1}{2} p_{ZZ}^{(0)} A_{xz}}{1 - \frac{1}{4} p_{ZZ}^{(0)} A_{yy}} \equiv \frac{-0.983\, p_Q A_{xz}}{1 + 0.492\, p_Q A_{yy}}, \tag{51}$$

since $p_{ZZ}^0 = -1.966\, p_Q$. If A_{yy} is known from previous measurement, this determines A_{xz}.

Notice that in this method the left–right and up–down planes are not inter-related. Thus, quantities like $L+R+U+D$ cannot be considered. On the other hand, this method can be used with a two-detector system, provided that either the scattering chamber or the spin direction can be rotated 90° around the beam direction.

4.3.3. *Monitor free method*

A four-detector method which is independent of beam monitoring for all four analyzing powers can be constructed. This method is essentially a combination of the two methods given above. First the quantities $\frac{1}{2}(A_{xx}-A_{yy})$ and A_{xz} are determined from appropriate asymmetries with $\beta = 54.7°$, as in sect. 4.3.1. $m_I = 0$ is preferred for greater accuracy on both quantities. Turning to the $\beta = 90°$ part of the method of sec. 4.3.2, but assuming the (unknown) ratio of charge for the $m_I = 1$ and $m_I = 0$ runs to be given by k, we can write

$$A_{xx} = \frac{2[k(U_{(1)}+D_{(1)}) - (U_{(0)}+D_{(0)})]}{p_Q[1.966\,k(U_{(1)}+D_{(1)}) + (U_{(0)}+D_{(0)})]},$$

$$A_{yy} = \frac{2[k(L_{(1)}+R_{(1)}) - (L_{(0)}+R_{(0)})]}{p_Q[1.966\,k(L_{(1)}+R_{(1)}) + (L_{(0)}+R_{(0)})]},$$

$$A_y = \frac{2.966\,k(L_{(1)}-R_{(1)})}{p_Q[1.966\,k(L_{(1)}+R_{(1)}) + (L_{(0)}+R_{(0)})]}. \quad (52)$$

If one half the difference of the first and second of eq. (52) is set equal to the value of $\frac{1}{2}(A_{xx}-A_{yy})$ determined from the third of eq. (47), the resulting expression can be solved for k. The value of k thus determined can then be used to calculate A_y, A_{yy} and A_{xx} from eq. (52). In this method all four tensors depend in first order on the accuracy with which the spin angle, β, is known for the $\beta = 54.7°$ part of the procedure. However, for $\beta = 54.7°$, the sum of the geometric mean yields in the left, right, up and down detectors is independent of polarization. That is

$$L+R+U+D = I_0[1 + \tfrac{1}{4}p_{ZZ}(3\cos^2\beta - 1)A_{zz}] \to I_0. \quad (53)$$

Thus, an $m_I = 1$ to $m_I = 0$ ratio of this quantity is a sensitive measurement of β, provided A_{zz} is suitably large. This permits the spin angle to be set to perhaps $\pm\frac{1}{4}°$ accuracy. However, since the spin angle is affected slightly by all beam transport elements[9]), this should be checked frequently until such a time as adequate stability is demonstrated.

4.3.4. *Three spin state method (no rotation of the apparatus)*

In this method, counts are observed in each of the detectors with three different beam polarizations, e.g., for an $m_I = 1$, $m_I = 0$, and $m_I = -1$ beam. First $\beta = 90°$ is used to ascertain A_y, A_{yy} and A_{xx} as follows. The yields in the left detector are

$$L_{(1)} = n^{(1)}N^{(1)}\Omega_L E_L I_0[1 + \tfrac{3}{2}p_Z^{(1)}A_y + \tfrac{1}{2}p_{ZZ}^{(1)}A_{yy}],$$
$$L_{(0)} = n^{(0)}N^{(0)}\Omega_L E_L I_0[1 + \tfrac{3}{2}p_Z^{(0)}A_y + \tfrac{1}{2}p_{ZZ}^{(0)}A_{yy}],$$
$$L_{(-1)} = n^{(-1)}N^{(-1)}\Omega_L E_L I_0[1 + \tfrac{3}{2}p_Z^{(-1)}A_y + \tfrac{1}{2}p_{ZZ}^{(-1)}A_{yy}], \quad (54)$$

in an obvious notation. The ratios $l_{(1)}$ and $l_{(-1)}$ can be defined

$$l_{(1)} = \frac{L_{(1)}}{L_{(0)}}\frac{n^{(0)}N^{(0)}}{n^{(1)}N^{(1)}} = \frac{1 + \frac{3}{2}p_Z^{(1)}A_y + \frac{1}{2}p_{ZZ}^{(1)}A_{yy}}{1 + \frac{3}{2}p_Z^{(0)}A_y + \frac{1}{2}p_{ZZ}^{(0)}A_{yy}},$$

$$l_{(-1)} = \frac{L_{(-1)}}{L_{(0)}}\frac{n^{(0)}N^{(0)}}{n^{(-1)}N^{(-1)}} = \frac{1 + \frac{3}{2}p_Z^{(-1)}A_y + \frac{1}{2}p_{ZZ}^{(-1)}A_{yy}}{1 + \frac{3}{2}p_Z^{(0)}A_y + \frac{1}{2}p_{ZZ}^{(0)}A_{yy}}, \quad (55)$$

whence detector efficiency factors are cancelled. Eq. (55) can be solved for $\frac{3}{2}A_y$, $\frac{1}{2}A_{yy}$:

$$\tfrac{3}{2}A_y = -\frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - b_1 a_2}, \qquad \tfrac{1}{2}A_{yy} = -\frac{a_1 c_2 - c_1 a_2}{a_1 b_2 - b_1 a_2}, \quad (56)$$

where

$$a_1 = p_Z^{(0)}l_{(1)} - p_Z^{(1)} \to p_Q(0.012\,l_{(1)} - 1),$$
$$a_2 = p_Z^{(0)}l_{(-1)} - p_Z^{(-1)} \to p_Q(0.012\,l_{(-1)} + 0.984),$$
$$b_1 = p_{ZZ}^{(0)}l_{(1)} - p_{ZZ}^{(1)} \to -p_Q(1.966\,l_{(1)} + 1),$$
$$b_2 = p_{ZZ}^{(0)}l_{(-1)} - p_{ZZ}^{(-1)} \to -p_Q(1.966\,l_{(-1)} + 0.952),$$
$$c_1 = l_{(1)} - 1, \qquad c_2 = l_{(-1)} - 1. \quad (57)$$

The expressions to the right of the arrows correspond to the values of p_Z and p_{ZZ} for the $m_I = 1, 0$ and -1 states. If this specialization is not made, however, the expressions are applicable for an arbitrary set of three (adequately separated) polarization values. An unpolarizated beam could, for instance, be used in place of the $m_I = 0$ beam, albeit with a considerable loss of sensitivity to A_{yy}.

Analogous quantities are then calculated from the right detector yields, and the mean values of the two determinations of $\frac{3}{2}A_y$ and $\frac{1}{2}A_{yy}$ are taken as the final result. As will be shown in sec. 4.4.4, the mean values so obtained are independent of all first-order alignment errors.

To determine A_{xx} by this method, a calculation exactly like the above one for A_{yy} is made, except one uses the up and down detectors rather than the left and right detectors. In this case the coefficient of p_Z is of no interest and is not calculated.

TABLE 2

Comparison of four methods for the measurement of spin-1 analyzing tensors.

Method	Tensor	First order spin errors	Dependence on current integration	Required # of intercalibrated polarization states
Rapid method	A_y	yes	no	1
	A_{xz}	yes	no	1
	$\frac{1}{2}(A_{xx}-A_{yy})$	yes	no	1
	A_{zz}	no	yes	2
Ratio method	A_y	no	yes	2
	A_{xx}	no	yes	2
	A_{yy}	no	yes	2
	A_{xz}	yes	yes	1
Monitor free method	A_y	yes	no	2
	A_{xx}	yes	no	2
	A_{yy}	yes	no	2
	$\frac{1}{2}(A_{xx}-A_{yy})$	yes	no	2
	A_{xz}	yes	no	2
Three spin state method	A_y	no	yes	3
	A_{xx}	no	yes	3
	A_{yy}	no	yes	3
	A_{xz}	no	yes	3

To determine A_{xz}, the up–down detectors with $\beta = 45°$ are used. In this case, for the up detector,

$$U_{(1)} = n^{(1)}N^{(1)}\Omega_U E_U I_0[1 + \tfrac{1}{2}p_{ZZ}^{(1)}A_{xz} - \tfrac{1}{4}p_{ZZ}^{(1)}A_{yy}],$$
$$U_{(0)} = n^{(0)}N^{(0)}\Omega_U E_U I_0[1 + \tfrac{1}{2}p_{ZZ}^{(0)}A_{xz} - \tfrac{1}{4}p_{ZZ}^{(0)}A_{yy}],$$
$$U_{(-1)} = n^{(-1)}N^{(-1)}\Omega_U E_U I_0[1 + \tfrac{1}{2}p_{ZZ}^{(-1)}A_{xz} - - \tfrac{1}{4}p_{ZZ}^{(-1)}A_{yy}]. \quad (58)$$

The down detector result is similar except the sign of the A_{xz} term is reversed, and A_{xz} is taken as the mean of the up and down determinations. This determination of A_{xz} is free of first-order spin angle errors. A second determination of A_{yy} is obtained as a byproduct, although this particular determination depends in first order on knowledge of the spin angle, β. The relative merits of the four methods presented are summarized in table 2.

4.4. Effect of misalignment on spin-1 analyzing power measurements

4.4.1. *Expression for the cross section in invariant form*

To discuss misalignment effects on the observable asymmetries, it is convenient to rewrite eq. (32) in terms of the unit vectors $\boldsymbol{k}$ (the beam direction), $\boldsymbol{n}$ (the normal to the scattering plane), $\boldsymbol{l}(=\boldsymbol{n}\times\boldsymbol{k})$, and $\boldsymbol{s}$ (the beam quantization axis). β is defined as the angle between the beam direction and the quantization axis, so that

$$\cos\beta = \boldsymbol{s}\cdot\boldsymbol{k}, \qquad \sin\beta = |\boldsymbol{s}\times\boldsymbol{k}|, \quad (59)$$

while ϕ is defined as the angle between the unit vector $\boldsymbol{s}\times\boldsymbol{k}/|\boldsymbol{s}\times\boldsymbol{k}|$ and $\boldsymbol{l}$, so that

$$\cos\phi = (\boldsymbol{s}\times\boldsymbol{k}\cdot\boldsymbol{l})/|\boldsymbol{s}\times\boldsymbol{k}|,$$
$$\sin\phi = (\boldsymbol{s}\times\boldsymbol{k}\cdot\boldsymbol{n})/|\boldsymbol{s}\times\boldsymbol{k}|. \quad (60)$$

From these equations we can write

$$\sin\beta\cos\phi = \boldsymbol{s}\times\boldsymbol{k}\cdot\boldsymbol{l} \equiv \boldsymbol{s}\cdot\boldsymbol{n},$$
$$\sin\beta\sin\phi = \boldsymbol{s}\times\boldsymbol{k}\cdot\boldsymbol{n} \equiv -\boldsymbol{s}\cdot\boldsymbol{l}. \quad (61)$$

Inserting eq. (61) into eq. (32), the cross section expression becomes

$$I(\theta,\phi) = I_0(\theta)\{1 + \tfrac{3}{2}p_Z A_y(\boldsymbol{s}\cdot\boldsymbol{n}) + p_{ZZ}A_{xz}(\boldsymbol{s}\cdot\boldsymbol{l}) \times \times(\boldsymbol{s}\cdot\boldsymbol{k}) + \tfrac{1}{4}p_{ZZ}(A_{xx}-A_{yy})[(\boldsymbol{s}\cdot\boldsymbol{l})^2 - -(\boldsymbol{s}\cdot\boldsymbol{n})^2] + \tfrac{1}{4}p_{ZZ}A_{zz}[3(\boldsymbol{s}\cdot\boldsymbol{k})^2 - 1]\}. \quad (62)$$

First-order expressions for the unit vectors $\boldsymbol{n}$ and $\boldsymbol{l}$ may be readily calculated. For this purpose we use a fixed coordinate system such that the centers of the four detectors lie in the coordinate planes, and with the x', y', z' axes as shown in fig. 8 (where only the left detector is shown). That is, we use a coordinate system such that the left detector is in the x', z' plane. The vector $\boldsymbol{R}$ describing the detector positions for the left, right, up and down detectors, respectively, has the components $(R\sin\theta, 0, R\cos\theta)$, $(-R\sin\theta, 0, R\cos\theta)$, $(0, R\sin\theta, R\cos\theta)$ and $(0, -R\sin\theta, R\cos\theta)$, where

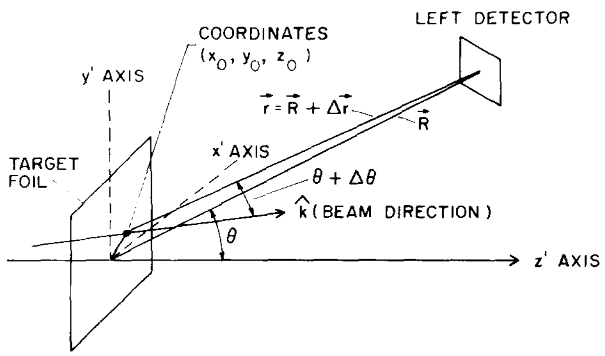

Fig. 8. Definitions of the appropriate vectors for an analysis of geometrical errors in spin-1 analyzing tensor measurements.

θ is the scattering angle. The source point is assumed to be at the position $\boldsymbol{r}_0$ [coordinates (x_0, y_0, z_0)], and the beam direction, $\boldsymbol{k}$ [coordinates (k_x, k_y, k_z)], is approximately along the z' axis. The z' axis is taken as a natural axis, such as a rotation axis of the apparatus. Note that we assume at present that the planes containing the left–right and up–down detectors are perpendicular. Allowance for this type of apparatus defect will be made in sec. 4.4.4 where an overall uncertainty in the spin direction is considered.

The unit vectors $\boldsymbol{n}$ and $\boldsymbol{l}$ are calculated for each position from the expressions

$$\boldsymbol{n} = \boldsymbol{k}\times\boldsymbol{r}/|\boldsymbol{k}\times\boldsymbol{r}|, \qquad \boldsymbol{l} = \boldsymbol{n}\times\boldsymbol{k}, \tag{63}$$

where

$$\boldsymbol{r} = \boldsymbol{R}+\Delta\boldsymbol{r} \equiv \boldsymbol{R}-\boldsymbol{r}_0 . \tag{64}$$

One obtains

$$\begin{aligned}
\boldsymbol{n} &= \alpha_y\boldsymbol{x}'+\boldsymbol{y}'-k_y\boldsymbol{z}', & \boldsymbol{l} &= \boldsymbol{x}'-\alpha_y\boldsymbol{y}'-k_x\boldsymbol{z}', \ (\text{left}),\\
\boldsymbol{n} &= \alpha_y\boldsymbol{x}'-\boldsymbol{y}'+k_y\boldsymbol{z}', & \boldsymbol{l} &= -\boldsymbol{x}'-\alpha_y\boldsymbol{y}'+k_x\boldsymbol{z}', \ (\text{right}),\\
\boldsymbol{n} &= -\boldsymbol{x}'-\alpha_x\boldsymbol{y}'+k_x\boldsymbol{z}', & \boldsymbol{l} &= -\alpha_x\boldsymbol{x}'+\boldsymbol{y}'-k_y\boldsymbol{z}', \ (\text{up}),\\
\boldsymbol{n} &= \boldsymbol{x}'-\alpha_x\boldsymbol{y}'-k_x\boldsymbol{z}', & \boldsymbol{l} &= -\alpha_x\boldsymbol{x}'-\boldsymbol{y}'+k_y\boldsymbol{z}', \ (\text{down}),
\end{aligned} \tag{65}$$

where

$$\alpha_x = k_x\cot\theta+x_0, \qquad \alpha_y = k_y\cot\theta+y_0 . \tag{66}$$

Assuming for the moment the detectors are identical, the error expressions for various sums and differences are

$$\begin{aligned}
\Delta\tfrac{1}{2}(L-R) &= -\tfrac{3}{2}p_Z A_y s_z k_y + p_{ZZ}A_{xz}(s_x s_z - s_z^2 k_x),\\
\Delta\tfrac{1}{2}(L+R) &= -p_{ZZ}A_{xz}s_y s_z k_y + \tfrac{1}{2}p_{ZZ}(A_{xx}-A_{yy})\times\\
&\quad\times s_y s_z k_y + \tfrac{3}{2}p_{ZZ}A_{zz}s_y s_z k_y,\\
\Delta\tfrac{1}{2}(U-D) &= \tfrac{3}{2}p_Z A_y(s_z k_x - s_x) + p_{ZZ}A_{xz}(s_y^2-s_z^2)\,k_y,\\
\Delta\tfrac{1}{2}(U+D) &= -\tfrac{3}{2}p_Z A_y s_y k_x - \tfrac{1}{2}p_{ZZ}(A_{xx}-A_{yy})\times\\
&\quad\times s_y s_z k_y + \tfrac{3}{2}p_{ZZ}A_{zz}s_y s_z k_y .
\end{aligned} \tag{67}$$

The nominal spin direction defines the y', z' plane in the present convention, so s_x is a first-order error quantity. Problems of the elimination of detector geometry effects and false asymmetries will be dealt with in the following section, as will methods for the elimination of the above polarization-dependent misalignment effects. In addition to the spatial misalignment included in eq. (67), we will later consider the effect of an overall uncertainty in the spin direction which is not cancelled by any flip procedure.

4.4.2. *Proper flips*

Beam misalignments in both the x, z and the y, z planes may give rise to false asymmetries unless a proper flip sequence is used to eliminate them. To first order, the error caused by displacements in the vertical and horizontal planes are independent, and we may generalize eq. (15) for solid target geometry:

$$\begin{aligned}
\Delta r_x &= -x_0\sin\theta, & \Delta r_y &= -y_0\sin\theta,\\
\Delta\theta_x &= -\frac{x_0}{R}\cos\theta-k_x, & \Delta\theta_y &= -\frac{y_0}{R}\cos\theta-k_y,
\end{aligned} \tag{68}$$

and eq. (16) for gas target geometry:

$$\begin{aligned}
\Delta r_x &= -\frac{x_0}{\sin\theta}, & \Delta r_y &= -\frac{y_0}{\sin\theta},\\
\Delta\theta_x &= -k_x, & \Delta\theta_y &= -k_y .
\end{aligned} \tag{69}$$

For a proper fourfold flip, the false asymmetry effects and detector geometry and efficiency effects are cancelled exactly. If a two-detector system is used, a proper flip is defined exactly as in our discussion of spin-$\frac{1}{2}$ analyzing power measurements.

The first-order polarization-correlated effects of eq. (67) are also cancelled by a proper flip. To see this cancellation, consider the geometric mean for the left detector:

$$L = (L_1 L_2 L_3 L_4)^{\frac{1}{4}}, \tag{70}$$

where

$$\begin{aligned}
L_1 &= n^{\mathrm{a}}N^{\mathrm{a}}E_1\Omega_1(\Delta r_1, \Delta\theta_1)\,I_{\mathrm{L}}(\theta+\Delta\theta_1)\times\\
&\quad\times[1 + p_Z\Delta_Z^{\mathrm{L}}(k_x, k_y, \alpha_x, \alpha_y) +\\
&\quad+ p_{ZZ}\Delta_{ZZ}^{\mathrm{L}}(k_x, k_y, \alpha_x, \alpha_y)],\\
L_2 &= n^{\mathrm{b}}N^{\mathrm{b}}E_2\Omega_2(\Delta r_2, \Delta\theta_2)\,I_{\mathrm{L}}(\theta+\Delta\theta_2)\times\\
&\quad\times[1 + p_Z\Delta_Z^{\mathrm{L}}(-k_y, k_x, -\alpha_y, \alpha_x) +\\
&\quad+ p_{ZZ}\Delta_{ZZ}^{\mathrm{L}}(-k_y, k_x, -\alpha_y, \alpha_x)],\\
L_3 &= n^{\mathrm{c}}N^{\mathrm{c}}E_3\Omega_3(\Delta r_3, \Delta\theta_3)\,I_{\mathrm{L}}(\theta+\Delta\theta_3)\times\\
&\quad\times[1 + p_Z\Delta_Z^{\mathrm{L}}(-k_x, -k_y, -\alpha_x, -\alpha_y) +\\
&\quad+ p_{ZZ}\Delta_{ZZ}^{\mathrm{L}}(-k_x, -k_y, -\alpha_x, -\alpha_y)],
\end{aligned}$$

$$L_4 = n^{\rm d} N^{\rm d} E_4 \Omega_4(\Delta r_4, \Delta\theta_4)\, I_{\rm L}(\theta+\Delta\theta_4) \times \times [1 + p_Z \Delta_Z^{\rm L}(k_y, -k_x, \alpha_y, -\alpha_x) + + p_{ZZ}\Delta_{ZZ}^{\rm L}(k_y, -k_x, \alpha_y, -\alpha_x)], \qquad (71)$$

with $I_{\rm L}(\theta)$ the cross section for scattering a polarized beam into the left detector with no misalignment present:

$$I_{\rm L}(\theta) = I_0(\theta)\,[1 + \tfrac{3}{2} p_Z A_y s_y + p_{ZZ} A_{xz} s_y s_z - - \tfrac{1}{4} p_{ZZ}(A_{xx}-A_{yy})\, s_y^2 + \tfrac{1}{4} p_{ZZ} A_{zz}(3 s_z^2 - 1)]. \qquad (72)$$

Eq. (72) is merely a restatement of eq. (62) for the left detector, i.e., for $\boldsymbol{n}$ along the y' axis. The error quantities of eq. (71) are obtained from eq. (67) by appropriate sums and differences:

$$\Delta\tfrac{1}{2}(L+R) + \Delta\tfrac{1}{2}(L-R) \equiv p_Z \Delta_Z^{\rm L} + p_{ZZ}\Delta_{ZZ}^{\rm L},$$
$$\Delta\tfrac{1}{2}(L+R) - \Delta\tfrac{1}{2}(L-R) \equiv p_Z \Delta_Z^{\rm R} + p_{ZZ}\Delta_{ZZ}^{\rm R},$$
$$\Delta\tfrac{1}{2}(U+D) + \Delta\tfrac{1}{2}(U-D) \equiv p_Z \Delta_Z^{\rm U} + p_{ZZ}\Delta_{ZZ}^{\rm U},$$
$$\Delta\tfrac{1}{2}(U+D) - \Delta\tfrac{1}{2}(U-D) \equiv p_Z \Delta_Z^{\rm D} + p_{ZZ}\Delta_{ZZ}^{\rm D}. \qquad (73)$$

L_1, L_2, L_3 and L_4 represent the yields obtained in the left detector with the detector array rotated about the beam through 0°, 90°, 180° and 270°, respectively; the subscript denotes the detector which is in the left position when the apparatus is in the particular configuration. If the misalignment parameters are $(k_x, k_y, \alpha_x, \alpha_y)$ in the initial configuration, for a proper rotation, the parameters become $(-k_y, k_x, -\alpha_y, \alpha_x)$, $(-k_x, -k_y, -\alpha_x, -\alpha_y)$ and $(k_y, -k_x, \alpha_y, -\alpha_x)$ for the remaining three configurations. Hence, all terms linear in these parameters are cancelled in first order by either a twofold or fourfold proper flip. (Here we need not distinguish between an arithmetic and geometric mean since they are identical for the first-order error terms considered here.)

4.4.3. *Nonproper flips*

We now consider the effect of a nonproper flip. In this case the beam is fixed in space, so that a given set of misalignment parameters $\Delta\theta$, Δr is associated with a position in space (e.g., "left"), rather than with a particular physical detector. For a four-detector system, the appropriate geometric mean for the left detector is

$$L = K^{\frac{1}{4}} \sqrt{[\Omega_1(\Delta\theta_{\rm L}, \Delta r_{\rm L})\, \Omega_2(\Delta\theta_{\rm L}, \Delta r_{\rm L}) \times \times \Omega_3(\Delta\theta_{\rm L}, \Delta r_{\rm L})\, \Omega_4(\Delta\theta_{\rm L}, \Delta r_{\rm L})]}\; I(\theta_{\rm L}, 0). \qquad (74)$$

For the right, up and down detectors, eq. (74) holds if all subscripts L are replaced by R, U, or D, respectively. $K^{\frac{1}{4}}$ is the factor $(n^{\rm a} n^{\rm b} n^{\rm c} n^{\rm d} N^{\rm a} N^{\rm b} N^{\rm c} N^{\rm d} E_1 E_2 E_3 E_4)^{\frac{1}{4}}$ in an obvious extension of the notation used in sec. 3.

As for the spin-$\frac{1}{2}$ case, it is necessary to consider the gas and solid target geometry separately. For solid targets, we may put

$$\Omega_{\rm L} \propto \frac{1}{(R+\Delta r_x)^2} \approx \frac{1}{R^2}\left(1 - \frac{2\Delta r_x}{R}\right),$$
$$\Omega_{\rm R} \propto \frac{1}{(R-\Delta r_x)^2} \approx \frac{1}{R^2}\left(1 + \frac{2\Delta r_x}{R}\right),$$
$$\Omega_{\rm U} \propto \frac{1}{(R+\Delta r_y)^2} \approx \frac{1}{R^2}\left(1 - \frac{2\Delta r_y}{R}\right),$$
$$\Omega_{\rm D} \propto \frac{1}{(R-\Delta r_y)^2} \approx \frac{1}{R^2}\left(1 + \frac{2\Delta r_y}{R}\right). \qquad (75)$$

Also, the cross sections may be expanded:

$$I(\theta_{\rm L}) = I_0(\theta)\,(1+G\Delta\theta_x),$$
$$I(\theta_{\rm R}) = I_0(\theta)\,(1-G\Delta\theta_x),$$
$$I(\theta_{\rm U}) = I_0(\theta)\,(1+G\Delta\theta_y),$$
$$I(\theta_{\rm D}) = I_0(\theta)\,(1-G\Delta\theta_y). \qquad (76)$$

Inserting eq. (68) into the definitions of the asymmetries ε_1–ε_5, we find for the first-order false asymmetries ε_1–ε_5, for solid target geometry,

$$\varepsilon_1' = \varepsilon_3' = (2\sin\theta_0 - G\cos\theta_0)\, x_0 - Gk_x,$$
$$\varepsilon_2' = \varepsilon_4' = (2\sin\theta_0 - G\cos\theta_0)\, y_0 - Gk_y,$$
$$\varepsilon_5' = 0. \qquad (77)$$

In the case of gas target geometry an analogous calculation making use of eq. (69) gives

$$\varepsilon_1' = \varepsilon_3' = \frac{x_0}{\sin\theta_0} + k_x \cot\theta_0 - Gk_x,$$
$$\varepsilon_2' = \varepsilon_4' = \frac{y_0}{\sin\theta_0} + k_y \cot\theta_0 - Gk_y,$$
$$\varepsilon_5' = 0. \qquad (78)$$

If the incident beam is now allowed to be polarized, the effect of the false asymmetries on the observed asymmetry parameters is as follows. Let ε_1–ε_5 be the asymmetries which would have been observed in the absence of misalignments, and $\varepsilon_1^{\rm obs}$–$\varepsilon_2^{\rm obs}$ those actually observed. For ε_1 and ε_2, the spin-$\frac{1}{2}$ result holds

$$\varepsilon_1^{\rm obs} = \frac{\varepsilon_1 + \varepsilon_1'}{1 + \varepsilon_1 \varepsilon_1'},$$

$$\varepsilon_2^{\rm obs} = \frac{\varepsilon_2 + \varepsilon_2'}{1 + \varepsilon_2 \varepsilon_2'}. \tag{79}$$

If a four-detector system is used to determine ε_3, ε_4 and ε_5, the situation is more complicated. We can write

$$L \propto K(1+\varepsilon_5)\left(1 + \frac{\varepsilon_3}{1+\varepsilon_5}\right)(1+\varepsilon_3'),$$

$$R \propto K(1+\varepsilon_5)\left(1 - \frac{\varepsilon_3}{1+\varepsilon_5}\right)(1-\varepsilon_3'),$$

$$U \propto K(1-\varepsilon_5)\left(1 + \frac{\varepsilon_4}{1-\varepsilon_5}\right)(1+\varepsilon_4'),$$

$$D \propto K(1-\varepsilon_5)\left(1 - \frac{\varepsilon_4}{1-\varepsilon_5}\right)(1-\varepsilon_4'), \tag{80}$$

from which we readily find

$$\varepsilon_3^{\rm obs} = \frac{[\varepsilon_3/(1+\varepsilon_5) + \varepsilon_3']\,(1+\varepsilon_5)}{1 + \frac{1}{2}(\varepsilon_3\varepsilon_3' + \varepsilon_4\varepsilon_4')},$$

$$\varepsilon_4^{\rm obs} = \frac{[\varepsilon_4/(1-\varepsilon_5) + \varepsilon_4']\,(1-\varepsilon_5)}{1 + \frac{1}{2}(\varepsilon_3\varepsilon_3' + \varepsilon_4\varepsilon_4')},$$

$$\varepsilon_5^{\rm obs} = \frac{\varepsilon_5 + \frac{1}{2}(\varepsilon_3\varepsilon_3' - \varepsilon_4\varepsilon_4')}{1 + \frac{1}{2}(\varepsilon_3\varepsilon_3' + \varepsilon_4\varepsilon_4')}. \tag{81}$$

It is noteworthy that even though ε_5 is free of first-order errors in the case of an unpolarized beam, such is not the case for a polarized beam. In this discussion of nonproper flips, we have ignored the polarization dependent errors of eq. (67) as these are generally much smaller than the false asymmetry errors which are present in this method. (We continue to restrict the term false asymmetry to describe only those asymmetries still present in the case of an unpolarized incident beam.)

4.4.4. *Residual errors in spin-1 asymmetry measurements*

Although the proper flip sequence eliminates all first-order effects associated with an imperfect apparatus*, there can still arise first-order errors from a lack of knowledge of the absolute direction of the spin quantization axis in space. For this discussion, let

$$\beta - \beta_0 = \Delta\beta, \qquad \phi - \phi_0 = \Delta\phi, \tag{82}$$

where β_0 and ϕ_0 are the intended spin angles, and where β and ϕ are the actual angles occurring at the target. To first order in $\Delta\beta$ and $\Delta\phi$, the error in various sums and difference of interest are

$$\Delta\tfrac{1}{2}(L-R) = I_0[\tfrac{3}{2}\cos\beta p_Z A_y \Delta\beta - \tfrac{1}{2}\sin 2\beta p_{ZZ} A_{xz}\Delta\phi],$$

$$\Delta\tfrac{1}{2}(U-D) = I_0[\cos 2\beta p_{ZZ} A_{xz}\Delta\beta + \tfrac{3}{2}\sin\beta p_Z A_y \Delta\phi],$$

$$\Delta\tfrac{1}{2}(L+R) = I_0[\tfrac{1}{2}\sin 2\beta p_{ZZ}(A_{yy} - A_{zz})\,\Delta\beta],$$

$$\Delta\tfrac{1}{2}(U+D) = I_0[\tfrac{1}{2}\sin 2\beta p_{ZZ}(A_{xx} - A_{zz})\,\Delta\beta]. \tag{83}$$

For the $\beta = 54.7°$ portion of the rapid method (sec. 4.3.1), one sees that a first-order $\Delta\beta$ error arises in each of the required quantities. The $\Delta\phi$ dependence is especially serious for A_{xz} measurements, unless a beam with $p_Z = 0$ is used, as previously noted. For the $\beta = 90°$ part of the methods described in secs. 4.3.2 and 4.3.3, one sees that A_y, A_{yy} and A_{xx} can be determined with no first-order dependence on $\Delta\beta$, $\Delta\phi$. In the method of sec. 4.3.2, where if $\beta = 45°$ and $p_Z = 0$ is used for the A_{xz} determination, there remains a first-order error in $\Delta\beta$ because of the denominator term $\frac{1}{2}(U+D)$. For the method of 4.3.3, a further first-order $\Delta\beta$ error arises since $\beta = 54.7°$. The method of sec. 4.3.4 is free of first-order spin angle effects for all four tensors, as will be elucidated in sec. 4.4.5.

If the left–right and up–down planes are not perpendicular, this can be accounted for by different values of $\Delta\phi$ in the first and second expressions of eq. (83). Thus this type of apparatus defect is of importance only for A_y and A_{xz} determinations by methods which depend on $\Delta\phi$ in first order. Of the methods described in secs. 4.3.1, 4.3.2, 4.3.3, and 4.3.4, only that described in sec. 4.3.1 is of this character.

As in the spin-$\frac{1}{2}$ case, even when a proper flip is made, a small component of nonproper flip may be present. Thus, the limit of error from this cause will depend on $k_x - k_{x'}$, $k_y - k_{y'}$, $x_0 - x_0'$ and $y_0 - y_0'$, where the primed quantities represent the parameter values for the "180° flipped" measurement.

4.4.5. *Errors in ratio measurements*

Ratio measurements are, of course, wholly dependent upon current integration or some other monitoring technique. Our chief concern here is with their dependence on misalignment effects. We continue to insist on methods which cancel detector efficiencies, solid angles, and false asymmetry effects exactly.

* z displacement effects on the effective scattering angle are assumed to be removed by the methods discussed in sec. 3.4.

In general, a ratio computed with a single detector will lead to a result containing first-order misalignment errors. To see this, we may combine the various error quantities derived in the previous sections to write the yields for left, right, up and down detectors, for a given beam charge (n), target thickness factor (N) and vector and tensor polarizations (p_Z, p_{ZZ}), as follows:

$$
\begin{aligned}
L_1 &= nN\Omega_1 E_1 I_1(\theta_1)\,[1 + p_Z(a_Z + b_Z) + \\
&\quad + p_{ZZ}(a_{ZZ} + b_{ZZ})]\,, \\
R_2 &= nN\Omega_2 E_2 I_0(\theta_2)\,[1 + p_Z(a_Z - b_Z) + \\
&\quad + p_{ZZ}(a_{ZZ} - b_{ZZ})]\,, \\
U_3 &= nN\Omega_3 E_3 I_0(\theta_3)\,[1 + p_Z(c_Z + d_Z) + \\
&\quad + p_{ZZ}(c_{ZZ} + d_{ZZ})]\,, \\
D_4 &= nN\Omega_4 E_4 I_0(\theta_4)\,[1 + p_Z(c_Z - d_Z) + \\
&\quad + p_{ZZ}(c_{ZZ} - d_{ZZ})]\,.
\end{aligned} \tag{84}
$$

The efficiency (E), solid angle factor (Ω) and scattering angle (θ) are assumed to be different for the four detectors; in the initial configuration of a flip sequence, the detectors numbered 1, 2, 3 and 4 are assumed to be in the left, right, up and down positions, respectively. The quantities a_Z, b_Z, a_{ZZ}, b_{ZZ}, etc., may be written as follows:

$$
\begin{aligned}
a_Z &= 0\,, \\
b_Z &= \underline{\underline{\tfrac{3}{2}\sin\beta A_y}} + \underline{\tfrac{3}{2}\cos\beta A_y \Delta\beta} - \tfrac{3}{2}\cos\beta A_y k_y\,, \\
a_{ZZ} &= \underline{\underline{\tfrac{1}{2}(\sin^2\beta A_{yy} + \cos^2\beta A_{zz})}} + \\
&\quad + \underline{\tfrac{1}{2}\sin 2\beta (A_{yy} - A_{zz})\,\Delta\beta} + \\
&\quad + \tfrac{1}{2}\sin 2\beta[-A_{xz}\alpha_y + \tfrac{1}{2}(A_{xx} - A_{yy})\,k_y + \tfrac{3}{2}A_{zz}k_y], \\
b_{ZZ} &= \underline{-\tfrac{1}{2}\sin 2\beta A_{xz}\Delta\phi} - A_{xz}\cos^2\beta k_x\,, \\
c_Z &= -\tfrac{3}{2}\sin\beta A_y \alpha_x\,, \\
d_Z &= \underline{\tfrac{3}{2}\sin\beta A_y \Delta\phi} + \tfrac{3}{2}\cos\beta A_y k_x\,, \\
c_{ZZ} &= \underline{\underline{\tfrac{1}{2}(\sin^2\beta A_{xx} + \cos^2\beta A_{zz})}} + \\
&\quad + \underline{\tfrac{1}{2}\sin 2\beta (A_{xx} - A_{zz})\,\Delta\beta} + \\
&\quad + \tfrac{1}{2}\sin 2\beta[-\tfrac{1}{2}(A_{xx} - A_{yy})\,k_y + \tfrac{3}{2}A_{zz}k_y]\,, \\
d_{ZZ} &= \underline{\underline{\tfrac{1}{2}\sin 2\beta A_{xz}}} + \underline{\cos 2\beta A_{xz}\Delta\beta} - \cos 2\beta A_{xz}k_y\,.
\end{aligned} \tag{85}
$$

TABLE 3

Asymmetry parameters.

	$\beta = 0°$	$\beta = 45°$	$\beta = 90°$
a_Z	0	[a]	0
b_Z	$\frac{3}{2}A_y(\Delta\beta - k_y)$	[a]	$\frac{3}{2}A_y$
a_{ZZ}	$\frac{1}{2}A_{zz}$	[a]	$\frac{1}{2}A_{yy}$
b_{ZZ}	$-A_{xz}k_x$	[a]	$-A_{xz}k_x$
c_Z	0	$-\frac{3}{\sqrt{2}}A_y\alpha_x$	$-\frac{3}{2}A_y\alpha_x$
d_Z	$\frac{3}{2}A_y k_x$	$\frac{3}{2\sqrt{2}}A_y(\Delta\phi + k_x)$	$\frac{3}{2}A_y\Delta\phi$
c_{ZZ}	$\frac{1}{2}A_{zz}$	$-\frac{1}{4}A_{yy} + \frac{1}{2}(A_{xx} - A_{zz})\Delta\beta$ $-\frac{1}{4}(A_{xx} - A_{yy})k_y + \frac{3}{4}A_{zz}k_y$	$\frac{1}{2}A_{xx}$
d_{ZZ}	$A_{xz}(\Delta\beta - k_y)$	$\frac{1}{2}A_{xz}$	$-A_{xz}(\Delta\beta - k_y)$

[a] These quantities may be easily calculated from eq. (85) but are not of interest for any of the methods discussed.

The zeroth-order terms in eq. (85) come from eq. (39), and are underlined twice. The first-order errors which come from an overall spin misalignment come from eq. (83) and are underlined once. The remaining first-order errors involve α_x, α_y, k_x and k_y; these are taken from eq. (67) with the substitution $s_z = \cos\beta$, $s_y = \sin\beta$. Terms involving s_x from eq. (67) have been dropped since these terms represent the same error as do the $\Delta\phi$ terms. a_Z is carried for the sake of symmetry.

One sees from the structure of eq. (84) that for a single detector there are, in general, four unknown quantities of interest; either a_Z, b_Z, a_{ZZ} and b_{ZZ} or c_Z, d_Z, c_{ZZ} and d_{ZZ}. Even in the special cases of interest, $\beta = 0°$ and $\beta = 90°$, at least three of these quantities is present for every case (see table 3).

We first consider $\beta = 0°$ from which we can determine A_{zz} by these methods. We use the left and right detector expressions, although for $\beta = 0°$ the up and down detectors lead to identical results. (In the $\beta = 0°$ error expressions of table 3, there is an apparent difference between the left–right and the up–down expressions because of our assumption that the spin lies in the y', z' plane.) Detector efficiencies may be eliminated by a 180° proper flip; this also removes in first order those misalignment terms proportional to α_x, α_y, k_x and k_y. Considering that $a_Z = 0$, we have for the geometric means

$$
\begin{aligned}
L &= \sqrt{(n^a n^b N^a N^b)}\,K[1 + p_Z b_Z + p_{ZZ}(a_{ZZ} - b_{ZZ})]\,, \\
R &= \sqrt{(n^a n^b N^a N^b)}\,K[1 - p_Z b_Z + p_{ZZ}(a_{ZZ} - b_{ZZ})]\,,
\end{aligned} \tag{86}
$$

where

$$K = \sqrt{[\Omega_1 \Omega_2 E_1 E_2 I(\theta_1) I(\theta_2)]}, \tag{87}$$

so that the quantity $\frac{1}{2}(L+R)$ is independent of b_Z and b_{ZZ}:

$$T_1 \equiv \tfrac{1}{2}(L+R) = \sqrt{(n^a n^b N^a N^b)}\, K(1+p'_{ZZ} a'_{ZZ}). \tag{88}$$

This quantity can then be measured with a different value of beam polarization p_Z, p_{ZZ}:

$$T'_1 = \tfrac{1}{2}(L'+R') = \sqrt{(n^c n^d N^c N^d)}\, K(1+p'_{ZZ} a_{ZZ}), \tag{89}$$

from which we obtain

$$a_{ZZ} \equiv \tfrac{1}{2} A_{zz} = \frac{T_1 - T'_1}{p_{ZZ} T'_1 - p'_{ZZ} T_1}, \tag{90}$$

provided that $\sqrt{(n^a n^b N^a N^b)} = \sqrt{(n^c n^d N^c N^d)}$.

Next consider the determination of A_{yy} and A_y. In this case we use $\beta = 90°$, and reference to table 3 shows that eq. (86) remains valid, and that a_{ZZ} $(\equiv \frac{1}{2} A_{yy})$ may be determined with no first-order errors. The difference $\frac{1}{2}(L-R)$ becomes

$$\tfrac{1}{2}(L-R) = \sqrt{(n^a n^b N^a N^b)}\, K(p_Z b_Z - p_{ZZ} b_{ZZ}). \tag{91}$$

Reference to table 3 shows that b_{ZZ} is zero if a proper flip is executed, so that the quantity

$$\frac{p_Z b_Z}{1+p_{ZZ} a_{ZZ}} \equiv \frac{\frac{3}{2} p_Z A_y}{1+\frac{1}{2} p_{ZZ} A_{yy}} \tag{92}$$

is also determined with no first-order error (and no dependence on current integration). Combination of this result with the result for a_{ZZ} obtained by eq. (90) results in an expression for b_Z $(\equiv \frac{3}{2} A_y)$ which is free of first-order errors but which now depends on current integration.

Finally, consider the determination of A_{xx}. Again we use $\beta = 90°$; in this case we have for the appropriate geometric means

$$U = \sqrt{(n^a n^b N^a N^b)}\, K'[1 + p_Z(c_Z + d_Z) + p_{ZZ}(c_{ZZ} + d_{ZZ})],$$
$$D = \sqrt{(n^a n^b N^a N^b)}\, K'[1 + p_Z(c_Z - d_Z) + p_{ZZ}(c_{ZZ} - d_{ZZ})], \tag{93}$$

where

$$K' = \sqrt{[\Omega_3 \Omega_4 E_3 E_4 I_0(\theta_3) I_0(\theta_4)]}, \tag{94}$$

so that the quantity $\frac{1}{2}(U+D)$ is

$$T_2 \equiv \tfrac{1}{2}(U+D) = \sqrt{(n^a n^b N^a N^b)}\, K'(1 + p_Z c_Z + p_{ZZ} c_{ZZ}). \tag{95}$$

Reference to table 2 shows that $c_Z = 0$ for a proper flip, so that the ratio method of eq. (90) may be applied to obtain a result for c_{ZZ} $(\equiv \frac{1}{2} A_{xx})$ free of first-order errors.

The method of sec. 4.3.4 may be examined in the light of eqs. (81) and (82). One sees that in this method the left detector ratios determine $a_Z + b_Z$ and $a_{ZZ} + b_{ZZ}$, the right detector ratios determine $a_Z - b_Z$ and $a_{ZZ} - b_{ZZ}$, etc. Thus, there are always enough equations, provided left–right or up–down pairs of detectors are used, to determine each of the quantities a_Z, b_Z, a_{ZZ} and b_{ZZ} or c_Z, d_Z, c_{ZZ} and d_{ZZ}. Analyzing powers free of first-order errors are available in each case. That is, for A_y, A_{yy}, one sees that for $\beta = 90°$, b_Z and a_{ZZ} each contain no error terms. For A_{xx}, again with $\beta = 90°$, c_{ZZ} contains no error terms. For A_{xz}, with $\beta = 45°$, d_{ZZ} contains no error terms. Thus this method can determine all four analyzing tensors with no first-order dependence on spin angle, as asserted in sec. 4.3.4.

5. Summary

In this paper we have sought to generalize the traditional concepts used in spin-$\frac{1}{2}$ analyzing power experiments to spin-1 experiments in the most direct way possible. In the spin-$\frac{1}{2}$ case, we spoke of proper flips where we meant that either the spin was reversed, or that the apparatus was rotated in such a way that an observer fastened to the apparatus would think that the spin was reversed. We showed that all first-order geometrical effects could then be removed.

In the spin-1 case, we found ourselves emphasizing the physical rotation of the apparatus since (1) reversing the quantization axis only reverses the vector part of the spin, so that such a reversal is not identical to an apparatus rotation, and (2) in some cases we wished to effectively rotate the spin 90° about the beam direction, which is difficult to do precisely in any system known to the authors. The choice of three spin states such as $m_I = 1$, $m_I = 0$ and $m_I = -1$ was shown to be analogous to the reversal of spin at the source in the spin-$\frac{1}{2}$ case.

In the spin-$\frac{1}{2}$ case, it was shown that it is possible to measure the analyzing power in a way simultaneously independent of first-order spin alignment errors and current integration, whereas in the spin-1 case it was found that both of these experimental features could not be simultaneously obtained.

It is a pleasure to thank D. D. Armstrong, J. L. McKibben and G. P. Lawrence, in collaboration with whom several experiments were performed which led to the development of much of the material presented in this paper. We would also like to thank L. L. Catlin, Jr., for his help in constructing an apparatus in which the present concepts are used. We also thank W. Gruebler for useful discussions about the various spin-1 methods, and C. K. Mitchell for help in demonstrating the usefulness of certain of the methods.

References

[1]) D. R. Waymire and N. Jarmie, Los Alamos Scientific Laboratory Report LA-4235 (1970).

[2]) R. C. Hanna, Proc. 2nd Intern. Symp. *Polarization phenomena of nucleons* (eds. P. Huber and H. Schopper; Birkhäuser Verlag, Basel, 1965) p. 280.

[3]) P. W. Keaton, Jr., D. D. Armstrong, L. L. Catlin and G. G. Ohlsen, Polarization measurement techniques, I and II, to be published as Los Alamos Scientific Laboratory Reports.

[4]) E. A. Silverstein, Nucl. Instr. and Meth. **4** (1959) 53.

[5]) P. W. Keaton, Jr. and G. G. Ohlsen, *Polarization phenomena in nuclear reactions* (University of Wisconsin Press, Madison, 1971) p. 852.

[6]) G. G. Ohlsen, Los Alamos Scientific Laboratory Report LA-4451 (1970).

[7]) G. G. Ohlsen, J. L. McKibben, G. P. Lawrence, P. W. Keaton, Jr. and D. D. Armstrong, Phys. Rev. Letters **27** (1971) 599.

[8]) G. P. Lawrence, G. G. Ohlsen, J. L. McKibben, P. W. Keaton, Jr. and D. D. Armstrong, *Polarization phenomena in nuclear reactions* (University of Wisconsin Press, Madison, 1971) p. 855.

[9]) R. R. Stevens, Jr. and G. G. Ohlsen, *Polarization phenomena in nuclear reactions* (University of Wisconsin Press, Madison, 1971) p. 858; and Los Alamos Scientific Laboratory Report LA-4465 (1970) p. 18-1.