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Electric dipole moments as probes of new physics

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Abstract

We review several aspects of flavour-diagonal *CP*-violation, focussing on the role played by the electric dipole moments (EDMs) of leptons, nucleons, atoms, and molecules, which constitute the source of several stringent constraints on new *CP*-violating physics. We dwell specifically on the calculational aspects of applying the hadronic EDM constraints, reviewing in detail the application of QCD sum-rules to the calculation of nucleon EDMs and *CP*-odd pion–nucleon couplings. We also consider the current status of EDMs in the Standard Model, and on the ensuing constraints on the underlying sources of *CP*-violation in physics beyond the Standard Model, focussing on weak-scale supersymmetry.
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1. Introduction

The search for violations of fundamental symmetries has played a central role in the development of particle physics in the 20th century. In particular, tests of the discrete symmetries, charge conjugation *C*, parity *P*, and time-reversal *T*, have been of

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paramount importance in establishing the structure of the Standard Model (SM). Perhaps the most famous example was the discovery of parity violation in the weak interactions [1], which led to the realization that matter fields should be combined into asymmetric left- and right-handed chiral multiplets, one of the cornerstones of the Standard Model. The observation of *CP*-violation, via the mixing of Kaons [2], also subsequently provided strong evidence for the presence of three quark and lepton generations, via the Kobayashi–Maskawa mechanism [3], prior to direct experimental evidence for the third family.

It is interesting to recall that one of the first tests of this kind, actually pre-dating the discovery of parity violation in the weak interactions, was a probe of parity invariance within the—at the time unknown—theory of the strong interactions. In 1949, Purcell and Ramsey argued, in a way that at the time was not fully appreciated that, lacking a theory of the strong interactions, there was no way to "derive" parity invariance and thus one must confirm parity conservation by experimental tests, or discover the lack thereof, rather than rely on a belief that nature "must be symmetric." As a probe of parity violation, Purcell and Ramsey proposed that one consider an intrinsic electric dipole moment of the neutron. Placed in a magnetic and electric field, a neutral non-relativistic particle of spin S can be described by the following Hamiltonian:

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d\mathbf{E} \cdot \frac{\mathbf{S}}{S}. \tag{1.1}$$

Under the reflection of spatial coordinates, $P(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}$, whereas $P(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$. The presence of a non-zero d would therefore signify the existence of parity violation. It was soon realized that d also breaks time-reversal invariance. Indeed, under time reflection, $T(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}$ and $T(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$. Therefore a non-zero d may exist if and only if both parity and time-reversal invariance are broken. Analysis of the existing experimental data on neutron scattering from spin zero nuclei led to the conclusion, $|d_n| < 3 \times 10^{-18} e$ cm [4]. Such a result probes physics at distances much shorter than the typical scale of nuclear forces ~ 1 fm, or the Compton wavelength of the neutron. This initial limit on the neutron EDM implied that P and T were good symmetries of the strong interactions at percent-level precision.

It was only some 25 years later with the emergence of QCD that the possibility of *T*-violation (or *CP*-violation, on assuming the *CPT* theorem) in the strong interactions had some theoretical underpinning. Indeed, QCD allows for the addition of a dimension-four term, known as the θ-term, with a dimensionless coefficient θ which, if non-zero, would signify the violation of both P and T. This term is somewhat unusual, being a purely topological boundary term, but its value determines a superselection sector in QCD [6] and its presence is intrinsically tied to an elegant feature of the theory, namely the mechanism via which the mass of the η' meson is lifted well-above the scale one might naturally expect given its apparent status as a Goldstone boson [5]. However, were $\theta \sim \mathcal{O}(1)$, one would predict a neutron EDM of sufficient size to ensure that the original analysis of Purcell and Ramsey would have detected it. In fact θ is now known to be tuned to better than one part in 10^9! This tuning is the well-known *strong CP problem* of the Standard Model, which

has been with us for more than 25 years, and has led to interesting dynamical mechanisms for its resolution; some of these have important consequences and predictions for other aspects of particle physics and cosmology.

The required tuning of this *CP*-odd parameter in QCD comes into sharp focus when we put QCD into its rightful place within the Standard Model, which necessarily means coupling it to the electroweak sector and massive quarks in particular. In this case the physical value of θ acquires a contribution from the overall phase of the quark mass matrix. In this sense the strong *CP* problem can be phrased as the absence, to high precision, of *flavour-diagonal CP*-violation within the Standard Model. This situation could not contrast more strongly with the situation in the flavour-changing sector, which is where all currently observed *CP*-violating effects reside. Indeed, the original discovery of *CP*-violation in the system of neutral Kaons [2] can be explained within this sector through the elegant and indeed rather minimal model of Kobayashi and Maskawa, which links *CP*-violation to the single physical phase in the unitary CKM mixing matrix describing transitions between the three generations of quarks [3]. This picture has recently received significant support—indeed essential confirmation—through experiments using neutral *B* mesons [7]. In contrast to θ, the phase in the CKM mixing matrix requires no tuning at all—its effects are nicely masked in the appropriate channels by the flavour structure of the Standard Model. Indeed, it turns out that the predictions for any *CP*-violating effect in the flavour-conserving channel induced by CKM mixing are minuscule, thus denying any hopes of detecting the experimental manifestation of CKM physics in these channels in the foreseeable future.

Searches for flavour-diagonal *CP*-violation, while insensitive to the CKM phase, thus inherit on the flip-side the status as one of the unique, essentially "background" free, probes of new physics. Electric dipole moments, through continuous experimental development since the work of Purcell and Ramsey, remain our most sensitive probes of this sector. All existing searches have failed to detect any intrinsic EDM, and indeed the precision to which EDMs are now known to vanish is remarkable, and sufficient to render them some of the most important precision tests of the Standard Model. In this more general context, the strong *CP* problem, associated with the tuning of θ, becomes just the most highly tuned example among many possible *CP*-odd operators which could contribute to the observable EDMs of nucleons, leptons, atoms, and molecules. Anticipating the presence of such *CP*-odd sources is not without motivation. Indeed, one of the strongest motivations comes from cosmology, where the success of the inflationary scenario, together with the observed cosmological dominance of baryons over antibaryons, suggests that a non-zero baryon number was generated dynamically in the early Universe. According to the Sakharov criteria [8], this requires a source of *CP*-violation, and within the Standard Model, neither the Kobayashi–Maskawa phase nor the θ-term can create conditions that would lead to the generation of an appreciable net baryon number. This strongly suggests the presence of another, yet to be discovered, source of *CP*-violation in nature. Although exceptions exist, e.g., the leptogenesis scenario, EDMs generally provide a highly sensitive diagnostic for these new *CP*-odd sources.

The second prominent motivation arises from theoretical prejudices about the physics of the Fermi scale, i.e., the mechanism for electroweak symmetry breaking, currently the focus of intense theoretical and experimental work. There are several theoretical motivations to believe that new physics, beyond the SM Higgs boson, should become apparent at, or just above, this scale, with weak-scale supersymmetry (SUSY) being a prominent example. Flavour-diagonal *CP*-violation constitutes a powerful probe of these scales, since any new physics need not provide the same flavour-dependent suppression factors as does the SM, while the SM itself constitutes a negligible background. These precision tests are thus highly complementary to direct searches at colliders. A rough estimate, based on the decoupling of new physics as the inverse square of its characteristic energy scale Λ, currently gives us the possibility to probe an order one *CP*-violating source at up to $\Lambda \sim 10^6$ GeV. In many weakly coupled theories, such as SUSY, this scale is somewhat lower, but often is still beyond the reach of existing and/or projected colliders. As with the link between the Kobayashi–Maskawa mechanism and the three-generation structure, one might hope that flavour-diagonal *CP*-violation, or perhaps the lack thereof, will tell us something profound about the matter sector.

The level of experimental precision achieved in EDM searches has improved dramatically since the early work of Purcell and Ramsey, and has been broadened to many atomic and nuclear quantities. Indeed, following significant progress throughout the past decade, the EDMs of the neutron [9], and of several heavy atoms and molecules [10–15], have been measured to vanish to remarkably high precision. From the present standpoint, it is convenient to classify the EDM searches into three main categories, distinguished by the dominant physics which would induce the EDM, at least within a generic class of models. These categories are: the EDMs of paramagnetic atoms and molecules; the EDMs of diamagnetic atoms; and the EDMs of hadrons, and nucleons in particular. For these three categories, the experiments that currently champion the best bounds on *CP*-violating parameters are the atomic EDMs of thallium and mercury and that of the neutron, as listed in Table 1.

The upper limits on EDMs obtained in these experiments can be translated into tight constraints on the *CP*-violating physics at and above the electroweak scale, with each category of EDM primarily sensitive to different *CP*-odd sources. For example, the neutron EDM can be induced by *CP*-violation in the quark sector, while paramagnetic EDMs generally result from *CP*-violating sources that induce the electron EDM. Despite the apparent difference in the actual numbers in Table 1, all three limits on d_n, d_{Tl}, and d_{Hg} actually have comparable sensitivity to fundamental *CP*-violation, e.g., superpartner masses and *CP*-violating phases, and thus

Table 1
Current constraints within three representative classes of EDMs

Class	EDM	Current bound
Paramagnetic	^{205}Tl	$|d_{Tl}| < 9 \times 10^{-25} e$ cm (90% C.L.) [10]
Diamagnetic	^{199}Hg	$|d_{Hg}| < 2 \times 10^{-28} e$ cm (95% C.L.) [11]
Nucleon	n	$|d_n| < 6 \times 10^{-26} e$ cm (90% C.L.) [9]

play complementary roles in constraining fundamental *CP*-odd sources. This fact can be explained by the way the so-called Schiff screening theorem [16] is violated in paramagnetic and diamagnetic atoms. The Schiff theorem essentially amounts to the statement that, in the non-relativistic limit and treating the nucleus as point-like, the atomic EDMs will vanish due to screening of the applied electric field within a neutral atom. The paramagnetic and diamagnetic EDMs result from violations of this theorem due, respectively, to relativistic and finite-size effects, and in heavy atoms such violation is maximized. For heavy paramagnetic atoms, i.e., atoms with non-zero electron angular momentum, relativistic effects actually result in a net enhancement of the atomic EDM over the electron EDM. For diamagnetic species, the Schiff screening is violated due to the finite size of the nucleus, but this is a weaker effect and the induced EDM of the atom is suppressed relative to the EDM of the nucleus itself. These factors equilibrate the sensitivities of the various experimental constraints in Table 1 to more fundamental sources of *CP*-violation.

In this paper, we will review in detail the calculational aspects of applying the current bounds on EDMs to constrain new *CP*-violating sources. In order to make the discussion as systematic as possible, we will proceed by working our way upwards in energy scale, using several effective *CP*-odd Lagrangians at the relevant thresholds. In the next section, we begin by discussing the current status of the experimental constraints within three generic classes, namely the neutron EDM, and the EDMs of paramagnetic and diamagnetic atoms, and describing the contributions to these EDMs at the nuclear scale. We then move to the QCD scale and introduce an effective *CP*-odd effective quark–gluon Lagrangian which plays an important role in the subsequent analysis. The leading term in this effective theory, of dimension four, is the θ-term and we briefly review the strong *CP*-problem and some of its proposed resolutions. We then turn in Section 3 to QCD computations of the EDMs, and dwell on some of the calculational aspects which are currently some of the major sources of uncertainty in the application of EDM constraints (for a detailed discussion of many other aspects we refer the reader to [17]). In Section 4, we turn to the generation of these observables within specific models of *CP*-violation, reviewing first the significant sources of suppression within the Standard Model, and then focussing on weak-scale supersymmetry, and the MSSM in particular, as the source of new physics at the electroweak scale. We discuss the generic constraints that EDMs impose on combinations of *CP*-violating parameters in the SUSY-breaking sector, and also explore some additional effects which may arise in special parameter regimes. We also emphasize the stringent EDM constraints on combined sources of *CP*- and flavour-violation in more general models. Finally, we conclude in Section 5 with an outlook on future experimental and theoretical developments.

2. EDMs as probes of *CP*-violation

The majority of EDM experiments are performed with matter as opposed to anti-matter. Therefore, the conclusion about the relation between d and *CP*-viola-

tion relies on the validity of the *CPT* theorem. The interaction $d\mathbf{E} \cdot \mathbf{S}$ for a spin 1/2 particle then has the following relativistic generalization:

$$H_{T,P\text{-odd}} = -d\mathbf{E} \cdot \frac{\mathbf{S}}{S} \quad \rightarrow \quad \mathcal{L} = -d\frac{\mathrm{i}}{2}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}. \tag{2.2}$$

Parenthetically, it is worth remarking that the precision of EDM experiments has now reached a level sufficient to provide competitive tests of *CPT* invariance, since one can also consider a *CP*-even, but *CPT*-odd, relativistic form of $d\mathbf{E} \cdot \mathbf{S}$, namely $\mathcal{L} = d\overline{\psi}\gamma^{\mu}\gamma_5\psi F_{\mu\nu}n^{\nu}$, with a preferred frame $n^{\nu} = (1,0,0,0)$, which spontaneously breaks Lorentz invariance and *CPT*.

The problem of calculating an observable EDM from the underlying *CP*-violation in a given particle physics model can be conveniently separated into different stages, depending on the characteristic energy/momentum scales. At each step the result can be expressed as an effective Lagrangian in terms of the light degrees of freedom with Wilson coefficients that encode information about *CP*-violation at higher-energy scales. As usual in effective field theory, it is convenient to classify all possible effective *CP*-violating operators in terms of their dimension, with the operators of lowest dimension usually leading to the largest contributions. This logic may need to be refined if symmetry requirements imply that certain operators are effectively of higher dimension than naive counting would suggest. This is actually the case for certain EDM operators due to gauge invariance, as discussed in more detail below.

We will present this analysis systematically in order of increasing energy scale, working our way upwards in the dependency tree outlined in Fig. 1, which allows us to remain entirely model-independent until the final step where some high-scale model of *CP*-violation can be imposed and then subjected to EDM constraints.
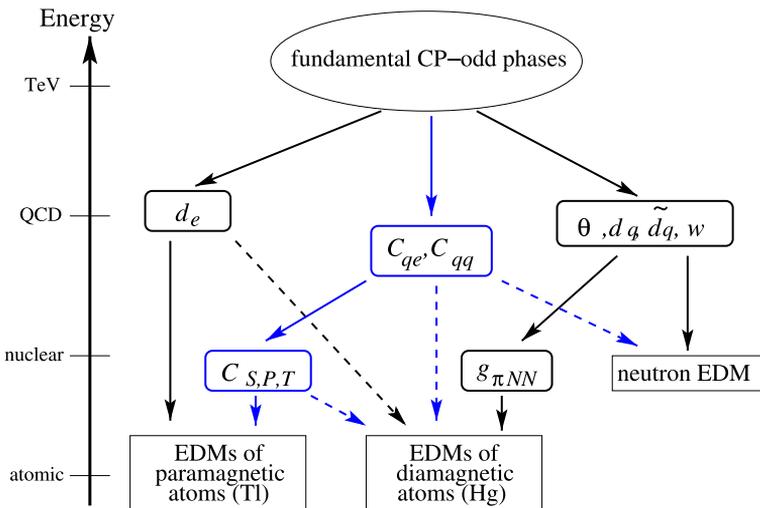

Fig. 1. A schematic plot of the hierarchy of scales between the *CP*-odd sources and three generic classes of observable EDMs. The dashed lines indicate generically weaker dependencies.

2.1. Observable EDMs

Let us begin by reviewing the lowest level in this construction, namely the precise relations between observable EDMs and the relevant *CP*-odd operators at the nuclear scale. At leading order, such effects may be quantified in terms of EDMs of the constituent nucleons, d_n and d_p (where the neutron EDM is already an observable), the EDM of the electron d_e, and *CP*-odd electron–nucleon and nucleon–nucleon interactions. In the relevant channels these latter interactions are dominated by pion exchange, and thus we must also consider the *CP*-odd pion–nucleon couplings $\bar{g}_{\pi NN}$ which can be induced by *CP*-odd interactions between quarks and gluons. To be more explicit, we write down the relevant *CP*-odd terms at the nuclear scale

$$\mathscr{L}_{\text{eff}}^{\text{nuclear}} = \mathscr{L}_{\text{edm}} + \mathscr{L}_{\pi NN} + \mathscr{L}_{eN}, \tag{2.3}$$

which can be split into terms for the nucleon (and electron) EDMs

$$\mathscr{L}_{\text{edm}} = -\frac{i}{2} \sum_{i=e,p,n} d_i \overline{\psi}_i (F\sigma) \gamma_5 \psi, \tag{2.4}$$

the *CP*-odd pion–nucleon interactions

$$\mathscr{L}_{\pi NN} = \bar{g}_{\pi NN}^{(0)} \bar{N}\tau^a N \pi^a + \bar{g}_{\pi NN}^{(1)} \bar{N} N \pi^0 + \bar{g}_{\pi NN}^{(2)} (\bar{N}\tau^a N \pi^a - 3\bar{N}\tau^3 N \pi^0), \tag{2.5}$$

and finally *CP*-odd electron–nucleon couplings

$$\begin{aligned} \mathscr{L}_{eN} = &\; C_S^{(0)} \bar{e} i\gamma_5 e \bar{N} N + C_P^{(0)} \bar{e} e \bar{N} i\gamma_5 N + C_T^{(0)} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{N} \sigma^{\alpha\beta} N \\ &+ C_S^{(1)} \bar{e} i\gamma_5 e \bar{N} \tau^3 N + C_P^{(1)} \bar{e} e \bar{N} i\gamma_5 \tau^3 N + C_T^{(1)} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{N} \sigma^{\alpha\beta} \tau^3 N. \end{aligned} \tag{2.6}$$

In certain rare cases, *CP*-odd nucleon–nucleon forces are not mediated by pions, in which case the effective Lagrangian must be extended by a variety of *contact* terms, e.g., $\bar{N}N\bar{N}i\gamma_5 N$, and the like.

The dependence of the observable EDMs on the corresponding Wilson coefficients relies on atomic and nuclear many-body calculations which would go beyond the scope of this review to cover here (see the reviews [17,18] for further details). However, we will briefly summarize the current status of these calculations, before turning to our major focus which is the calculation of these coefficients in terms of higher scale *CP*-odd sources.

As alluded to earlier on, it is convenient to split the discussion into three parts, corresponding roughly to the three classes of observable EDMs which currently provide constraints at a similar level of precision; namely: EDMs of paramagnetic atoms and molecules, EDMs of diamagnetic atoms, and the neutron EDM.

2.1.1. EDMs of paramagnetic atoms—thallium EDM

Paramagnetic systems, namely those with one unpaired electron, are primarily sensitive to the EDM of this electron. At the non-relativistic level, this is far from obvious due to the Schiff shielding theorem which implies, since the atom is neutral, that any applied electric field will be shielded and so an EDM of the unpaired electron will not induce an atomic EDM. Fortunately, this theorem is violated by rela-

tivistic effects. In fact, it is violated strongly for atoms with a large atomic number, and even more strongly in molecules which can be polarized by the applied field. For atoms, the parametric enhancement of the electron EDM is given by [19,20,18]

$$d_{\text{para}}(d_e) \sim 10 \frac{Z^3 \alpha^2}{J(J+1/2)(J+1)^2} d_e, \tag{2.7}$$

up to numerical $\mathcal{O}(1)$ factors, with J the angular momentum and Z the atomic number. This enhancement is significant, and for large Z, the applied field can be enhanced be a factor of a few hundred within the atom. This feature explains why atomic systems provide such a powerful probe of the electron EDM, since the "effective" electric field can be much larger than one could actually produce in the lab.

Although the electron EDM is the predominant contributor to any paramagnetic EDM is most models, one should bear in mind that other contributions may also be significant in certain regimes. In particular, significant CP-odd electron–nucleon couplings may also be generated, due for example to CP-violation in the Higgs sector. Among these couplings, C_S plays by far the most important role for paramagnetic EDMs because it couples to the spin of the electron and is enhanced by the large nucleon number in heavy atoms.

Among various paramagnetic systems, the EDM of the thallium atom currently provides the best constraints on fundamental CP-violation. A number of atomic calculations [20–22] (see also [17] for a more complete list) have established the relation between the EDM of thallium, d_e, and the CP-odd electron–nucleon interactions C_S:

$$d_{\text{Tl}} = -585 d_e - e\ 43\ \text{GeV} \times (C_S^{(0)} - 0.2 C_S^{(1)}) \tag{2.8}$$

with C_S expressed in isospin components. The relevant atomic matrix elements are known to within 10–20% [18].

As we discuss later on, current experimental work is focussing on the use of paramagnetic molecules, e.g., YbF and PbO [15,23], which can provide an even larger enhancement of the applied field due to polarization effects, have better systematics, and may bring significant progress in measuring/constraining d_e and C_S.

2.1.2. EDMs of diamagnetic atoms—mercury EDM

EDMs of diamagnetic atoms, i.e., atoms with total electron angular momentum equal to zero, also provide an important test of CP-violation [17]. In such systems the Schiff shielding argument again holds to leading order. However, in this case it is violated not by relativistic effects but by finite-size effects, namely a net misalignment between the distribution of charge and EDM (i.e., first and second moments) in the nucleus of a large atom (see e.g. [18] for a review). However, in contrast to the paramagnetic case, this is a rather subtle effect and the induced atomic EDM is considerably suppressed relative to the underlying EDM of the nucleus.

To leading order in an expansion around the point-like approximation for the nucleus, the contributions arise from an octopole moment (which is only relevant for states with large spin, and will not be relevant for the cases considered here), and the Schiff moment $\vec{S}$, which contributes to the electrostatic potential

$$V_E = 4\pi \vec{S} \cdot \vec{\nabla}\delta(\vec{r}). \tag{2.9}$$

CP-odd nuclear moments, such as $\vec{S}$, can arise from intrinsic EDMs of the constituent nucleons and also *CP*-odd nucleon interactions. It turns out that the latter source tends to dominate in diamagnetic atoms and thus, since such interactions are predominantly due to pion exchange, we can ascribe the leading contribution to *CP*-odd pion–nucleon couplings $\bar{g}_{\pi NN}^{(i)}$ for $i = 0, 1, 2$ corresponding to the isospin.

There are of course various additional contributions, which are generically subleading, but may become important in certain models. Schematically, we can represent the EDM in the form

$$d_{\text{dia}} = d_{\text{dia}}(S[\bar{g}_{\pi NN}, d_N], C_S, C_P, C_T, d_e), \tag{2.10}$$

where we note that electron–nucleon interactions may also be significant, as is the electron EDM itself [17] (although in practice the electron EDM tends to be more strongly constrained by limits from paramagnetic systems and thus is often neglected).

Currently, the strongest constraint in the diamagnetic sector comes from the bound on the EDM of mercury—at the atomic level, this is in fact the most precise EDM bound in existence. As should be apparent from the above discussion, computing the dependence of d_{Hg} on the underlying *CP*-odd sources is a non-trivial problem requiring input from QCD and nuclear and atomic physics. In particular, the computation of $S(\bar{g}_{\pi NN})$ is a non-trivial nuclear many-body problem, and has recently been reanalysed with the result [24]

$$S(^{199}\text{Hg}) = -0.0004 g\bar{g}^{(0)} - 0.055 g\bar{g}^{(1)} + 0.009 g\bar{g}^{(2)} \ e \ \text{fm}^3, \tag{2.11}$$

where $g = g_{\pi NN}$ is the *CP*-even pion–nucleon coupling, and $\bar{g}^{(i)} = \bar{g}_{\pi NN}^{(i)}$ denote the *CP*-odd couplings. The isoscalar and isotensor couplings have been significantly reduced relative to previous estimates, while the isovector coupling—which generically turns out to be most important—has been less affected (within a factor 2). This nonetheless provides some indication of the difficulties inherent in the calculation, and makes precision estimates more difficult. Moreover, it is worth noting that the suppression of the overall coefficient in front of $g\bar{g}^{(0)}$ below O(0.01) is the result of mutual cancellation between several contributions of comparable size, and therefore is in some sense accidental and may not hold in future refinements of these nuclear calculations.

Putting the pieces together, we can write the mercury EDM in the form

$$d_{\text{Hg}} = -(1.8 \times 10^{-4} \ \text{GeV}^{-1})e\bar{g}_{\pi NN}^{(1)} + 10^{-2}d_e + (3.5 \times 10^{-3} \ \text{GeV})eC_S^{(0)}, \tag{2.12}$$

where we have limited attention to the isovector pion–nucleon coupling and C_S which turns out to be the most important for *CP*-violation in supersymmetric models.

2.1.3. Neutron EDM

The final class to consider is that of the neutron itself, whose EDM can be searched for directly with ultracold neutron technology, and currently provides one of the strongest constraints on new *CP*-violating physics. In this case, there is clearly no additional atomic or nuclear physics to deal with, and we must turn di-

rectly to the next level in energy scale, namely the use of QCD to compute the dependence of d_n on *CP*-odd sources at the quark–gluon level. This statement also applies to many of the other quantities we have introduced thus far, including in particular the *CP*-odd pion–nucleon coupling. Indeed, it is only paramagnetic systems that are partially immune to QCD effects, although even there we have noted the possible relevance of electron–nucleon interactions.

2.2. The structure of the low-energy Lagrangian at 1 GeV

The effective *CP*-odd flavour-diagonal Lagrangian normalized at 1 GeV, which is taken to be the lowest perturbative quark–gluon scale, plays a special role in EDM calculations. At this scale, all particles other than the *u*, *d*, and *s* quark fields, gluons, photons, muons, and electrons can be considered heavy, and thus integrated out. As a result, one can construct an effective Lagrangian by listing all possible *CP*-odd operators in order of increasing dimension

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{dim}=4} + \mathscr{L}_{\text{dim}=5} + \mathscr{L}_{\text{dim}=6} + \cdots \tag{2.13}$$

There is only one operator at dimension four, the QCD θ-term

$$\mathscr{L}_{\text{dim}=4} = \frac{g_s^2}{32\pi^2} \bar{\theta} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}, \tag{2.14}$$

where on account of the axial $U(1)$ anomaly, the physical value of θ—denoted $\bar{\theta}$—also includes the overall phase of the quark mass matrix

$$\bar{\theta} = \theta + \text{Arg Det} M_q. \tag{2.15}$$

The anomaly can be used to shuffle contributions between the θ-term and imaginary quark masses, but only the combination $\bar{\theta}$ is physical and we choose to place it in front of $G\tilde{G}$ taking $\text{Det } M_q$ to be real. It should be apparent that if any of the quarks were massless, we could then rotate θ away and it would have no physical consequences.

At the dimension-five level, there are (naively) several operators: EDMs of light quarks and leptons and colour electric dipole moments of the light quarks

$$\mathscr{L}_{\text{dim}=5} = -\frac{i}{2} \sum_{i=u,d,s,e,\mu} d_i \overline{\psi}_i (F\sigma)\gamma_5 \psi_i - \frac{i}{2} \sum_{i=u,d,s} \tilde{d}_i \overline{\psi}_i g_s (G\sigma)\gamma_5 \psi_i, \tag{2.16}$$

where $(F\sigma)$ and $(G\sigma)$ are a shorthand notation for $F_{\mu\nu}\sigma^{\mu\nu}$ and $G_{\mu\nu}^a t^a \sigma^{\mu\nu}$.

In fact, in most models these operators are really dimension-six operators in disguise. The reason is that, if we proceed in energy above the electroweak scale and assume the system restores $SU(2) \times U(1)$ as in the Standard Model, gauge invariance ensures that these operators must include a Higgs field insertion [25]. Indeed, were we to write the basis of down quark EDMs and CEDMs above the electroweak scale, we should specify the following list of dimension-six operators [25]

$$\mathscr{L}_{\text{"dim}=5\text{"}}^{\text{EW}} = \frac{i}{2\sqrt{2}} \bar{Q}_L \big[2d_1^{\text{EW}}(B\sigma) + d_2^{\text{EW}}\tau^i(W^i\sigma) + d_2^{\text{EW}}\lambda^a(G^a\sigma) \big](\Phi/v)D_R + \text{h.c.}, \tag{2.17}$$

which are defined in terms of left-handed doublets $Q_L = (U, D)_L$ and right-handed singlets D_R and the Higgs doublet Φ, and in terms of the $U(1)$, $SU(2)$, and $SU(3)$ field strengths $B_{\mu\nu}$, $W^i_{\mu\nu}$, and $G^a_{\mu\nu}$.

The lesson we draw from (2.17) with regard to EDMs is that, if generated, these operators must be proportional to the Higgs v.e.v. below the electroweak scale, and consequently must scale at least as $1/M^2$ for $M \gg M_W$. In practice, this feature can also be understood in most models by going to a chiral basis, where we see that these operators connect left- and right-handed fermions, and thus require a chirality flip. This is usually supplied by an insertion of the fermion mass, i.e., $d_f \sim m_f/M^2$, again implying that the operators are effectively of dimension six.

Consequently, for consistency we should also proceed at least to dimension six where we encounter the *CP*-odd three-gluon Weinberg operator and a host of possible four-fermion interactions, $(\bar{\psi}_i \Gamma \psi_i)(\bar{\psi}_j \Gamma \gamma_5 \psi_j)$, where Γ denotes several possible scalar or tensor Lorentz structures and/or gauge structures, which are contracted between the two bilinears. We limit our attention to a small subset of the latter that will be relevant later on

$$\mathcal{L}_{\dim=6} = \frac{1}{3} w \, f^{abc} G^a_{\mu\nu} \tilde{G}^{\nu\beta,b} G^{\mu,c}_\beta + \sum_{i,j} C_{ij} (\bar{\psi}_i \psi_i)(\bar{\psi}_j i\gamma_5 \psi_j) + \cdots \tag{2.18}$$

In this formula, the operators with C_{ij} are summed over all light fermions. Going once again to a chiral basis, we can argue as above that the four-fermion operators, which require two chirality flips, are in most models effectively of dimension eight. Nonetheless, in certain cases they may be non-negligible.

2.3. The strong CP problem

The leading dimension-four term in the *CP*-odd Lagrangian given in Eq. (2.14) has a special status, in that it is a marginal operator, unsuppressed by any heavy scale. It is also a total derivative—we can write $G\tilde{G} = \partial_\mu \mathcal{K}^\mu$ with $\mathcal{K}^\mu$ the Chern–Simons current—and thus plays no role in perturbation theory. However, $\mathcal{K}^\mu$ is not invariant under so-called *large* gauge transformations and thus one may expect that the θ-term becomes relevant at the non-perturbative level. That it does so can be argued at the semi-classical level using instanton methods, and more generally can be understood within QCD via this relation to the $U(1)$ problem. In particular, we note that the same operator $G\tilde{G}$ arises as the θ-term in the Lagrangian, and also as an anomaly for the axial $U(1)$ current $\mathcal{J}^\mu_A$, i.e., for massless quarks

$$\partial_\mu \mathcal{J}^\mu_A = \frac{\alpha_s}{2\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}. \tag{2.19}$$

This leads to an intrinsic link between two physical phenomena: namely the θ-dependence of physical quantities and the absence of a light pseudo-Goldstone boson associated with spontaneous breaking of the axial current $\mathcal{J}^\mu_A$ [5] (the corresponding state, the η' is instead rather heavy, $m_{\eta'} \gg m_\pi$). Although it would take us too far afield to review the story of this link in detail (see e.g. [5,26–30]), let us note that

in the large N limit, as discussed by Witten and Veneziano [27,28], use of the anomaly equation leads to a simple relation that exemplifies this connection

$$m_{\eta'}^2 = \frac{4N_\mathrm{f}}{f_\pi^2} \left(\frac{\mathrm{d}^2 E}{\mathrm{d}\theta^2}\right)_{\theta=0}^{\mathrm{YM}}, \tag{2.20}$$

where N_f is the number of flavours. This relation expresses the η' mass in terms of the θ-dependence of the vacuum energy in a theory with no light quarks.

In turn, if we now take for granted that $m_{\eta'} \gg m_\pi$, use of the anomaly relation allows precise calculations of the θ-dependence of physical observables [26,29]. In particular, one can obtain an expression for the θ-dependence of the vacuum energy $E(\theta)$. At leading order in $\bar\theta$,

$$E(\bar\theta) = \frac{1}{2}\bar\theta^2\chi(0) = -\frac{\mathrm{i}}{8\pi^2}\bar\theta^2 \lim_{k\to0} \int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}px} \left\langle \frac{\alpha_s}{2\pi}G\tilde{G}(x), \frac{\alpha_s}{2\pi}G\tilde{G}(0)\right\rangle, \tag{2.21}$$

where $\chi(0)$ is known as the topological susceptibility. Making use of the anomaly relation and assuming $m_{\eta'} \gg m_\pi$, this may be evaluated as [29]

$$E(\bar\theta) = -\frac{1}{2}\bar\theta^2 m_*\langle\bar{q}q\rangle + \mathcal{O}(\bar\theta^4, m_*^2), \tag{2.22}$$

where $\langle\bar{q}q\rangle$ is the quark vacuum condensate and m_* is the reduced quark mass, given by

$$m_* = \frac{m_u m_d}{m_u + m_d}, \tag{2.23}$$

in two-flavour QCD. This dependence on the reduced quark mass can be straightforwardly understood on recalling that $\bar\theta$ becomes unphysical as soon as any quark eigenstate becomes massless. Indeed, we see that the result is essentially fixed up to an order one coefficient by the dictates of the anomalous Ward identity. On general grounds, we would expect $E(\bar\theta) \sim \bar\theta^2 m_* \Lambda_\mathrm{had}^3$, where Λ_had is the characteristic hadronic scale required on dimensional grounds, which the calculation above identifies with the quark condensate.

The quadratic dependence of the vacuum energy on $\bar\theta$, since it is determined effectively by a two-point function, implies that generic CP-odd observables will inherit a leading linear dependence on $\bar\theta$. In particular, although we will discuss a more detailed calculation in the next section, we can obtain a similar order of magnitude estimate for the neutron EDM

$$d_n \sim e\,\frac{\bar\theta m_*}{\Lambda_\mathrm{had}^2} \sim \bar\theta \cdot (6 \times 10^{-17})\,e\,\mathrm{cm}, \tag{2.24}$$

where we identified $\Lambda_\mathrm{had} = m_n$ and used conventional values for the light quark masses. The experimental bound then translates into the limit

$$|\bar\theta| < 10^{-9}. \tag{2.25}$$

This remarkable degree of tuning in the value of $\bar\theta$ then constitutes the *strong CP problem*. It is aggravated by the fact that $\bar\theta$ is a dimensionless parameter, and thus

can receive corrections from unspecified sources of *CP*-violation at an arbitrarily high scale.

If we discard the possibility that this tuning is simply accidental, and search for a theoretical explanation for why $\bar{\theta}$ is very small, then we find that the existing theoretical attempts to solve the strong *CP* problem can be divided into those that are based either on continuous symmetries or on spontaneously broken discrete symmetries. To some extent, these two possibilities can also be motivated by two extreme reference points, namely when $\bar{\theta}$ is either fully rotated to sit in front of $G\tilde{G}$, or to manifest itself as an overall phase of the quark mass matrix. Although inherently basis-dependent, the former viewpoint suggests that θ is essentially tied to the gluonic structure of QCD, while the latter emphasizes instead its links to the flavour sector.

2.3.1. Dynamical relaxation of $\bar{\theta}$

The energy of the QCD vacuum as a function of $\bar{\theta}$ (2.22) has a minimum at $\bar{\theta} = 0$. Thus the relaxation of the θ-parameter to zero is possible if one promotes it to a dynamical field, called the *axion* [31–33]. This is motivated by the assumption that the Standard Model, augmented by appropriate additional fields, admits a chiral symmetry $U(1)_{PQ}$, acting on states charged under $SU(3)_c$. When this symmetry is spontaneously broken at a necessarily high-scale f_a, a pseudo-scalar Goldstone boson—the axion—survives as the only low-energy manifestation. Symmetry dictates that the essential components of the axion Lagrangian are very simple

$$\mathscr{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a(x)}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}, \tag{2.26}$$

leading to a field-dependent shift of θ

$$\bar{\theta} \rightarrow \bar{\theta} + \frac{a}{f_a}. \tag{2.27}$$

If the effects of non-perturbative QCD are ignored, this Lagrangian possesses a symmetry, $a \rightarrow a + \text{const.}$, and a is a massless field with derivative couplings to the SM fields, i.e., $\partial_\mu a \bar{\psi}\gamma_\mu\gamma_5\psi$, that are not important for the solution of the strong *CP* problem.

Below the QCD scale, one finds that $U(1)_{PQ}$ is explicitly broken by the chiral anomaly, and thus the axion is in reality a *pseudo*-Goldstone boson and acquires a potential. The form of this potential can be read directly from our earlier discussion of the θ-dependence of the vacuum energy, namely $E(\theta) \sim \chi(0)\theta^2/2 + \cdots$ Accounting for the shift in (2.27), the effective axion Lagrangian becomes

$$\mathscr{L}_a^{\text{eff}} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}\chi(0)\left(\bar{\theta} + \frac{a}{f_a}\right)^2 + \cdots \tag{2.28}$$

We see from (2.27) that the vacuum expectation value of the axion field $\langle a \rangle$ renormalizes the value of $\bar{\theta}$ so that all observables depend on the $(\bar{\theta} + \langle a \rangle / f_a)$ combination. At the same time, such a combination must vanish in the vacuum as it minimizes the value of the axion potential in (2.28). This dynamical relaxation then solves the strong *CP* problem. This cancellation mechanism works independently of the "initial" value of $\bar{\theta}$, which is why it is very appealing. However, the excitations around $\langle a \rangle$ correspond to a massive axion particle with

$$m_a \sim \frac{1}{f_a}|\chi(0)|^{1/2}, \tag{2.29}$$

a formula analogous to that discussed earlier for η' (2.20). For large f_a the axion is very light and thus has significant phenomenological consequences. Indeed, the negative results of direct and indirect searches for "invisible axions" [34,35], where f_a is a free scale as we have been discussing here, have now imposed a rather large bound, $f_a > 10^{10}$ GeV, while cosmological constraints imply, on the contrary, that it cannot be too much larger than this (see e.g. [36]).

An aspect of the axion mechanism that is perhaps not stressed as often as it should be is that there can be other contributions to the axion potential which shift its minimum away from $(\bar{\theta} + a/f_a) = 0$ [37]. In particular, in the analysis above, we included only the leading term corresponding to the vacuum energy. However, if there are other CP-odd operators $\mathcal{O}_{CP}$ present at low scales, QCD effects may also generate terms linear in θ via non-zero mixed correlators of the form

$$\chi_{\mathcal{O}_{CP}}(0) = -i \lim_{k \to 0} \int d^4x \, e^{ik \cdot x} \langle 0|T(G\tilde{G}(x), \mathcal{O}_{CP}(0))|0\rangle. \tag{2.30}$$

An example of this type is the quark chromoelectric dipole moment, $\mathcal{O}_{CP} = \tilde{d}_q \bar{q} G \sigma \gamma_5 q$, appearing in (2.16). The axion potential is then modified

$$\mathcal{L}_a^{\text{eff}} = \frac{1}{2} \partial_\mu a \partial^\mu a - \chi_{\mathcal{O}_{CP}}(0)\left(\bar{\theta} - \frac{a}{f_a}\right) - \frac{1}{2}\chi(0)\left(\bar{\theta} + \frac{a}{f_a}\right)^2 + \cdots \tag{2.31}$$

and exhibits a minimum shifted from zero. The size of this *induced* contribution to θ, i.e., $\theta_{\text{ind}} = -\chi_{\mathcal{O}_{CP}}(0)/\chi(0)$, is linearly related to the coefficient of the CP-odd operator $\mathcal{O}_{CP}$ generating $\chi_{\mathcal{O}_{CP}}(0)$. These effects therefore need to be taken into account in computing the observable consequences of CP-odd sources in axion scenarios, and will be important for us later on.

Before moving on, it is worth recalling that, were it realized, the simplest solution to the strong CP problem would fall into the class we are discussing, namely the possibility that $m_u = 0$ in the Standard Model Lagrangian normalized at a high-scale M, or more generically, $\det(Y_u(M)) = 0$. In this situation, the Lagrangian already possesses the appropriate chiral symmetry without the addition of extra fields and, as we have discussed, $\theta(M)$ then becomes unphysical. Since the identification of light quark masses is indirect, using meson and baryon spectra and chiral perturbation theory, the possibility that $m_u = 0$ has been debated at length in the literature [38,39], but is strongly disfavoured by conventional chiral perturbation theory analysis, with recent results implying $m_u/m_d = 0.553 \pm 0.043$ [40], and this conclusion is beginning to be backed up by unquenched (but chirally extrapolated) lattice simulations which suggest similar values, $m_u/m_d = 0.43 \pm 0.1$ [41].

2.3.2. Engineering $\bar{\theta} \simeq 0$

Another way to approach the strong CP problem is to assume that either P or CP or both are exact symmetries of nature at some high-energy scale. Then one can declare that $\theta G\tilde{G}$ be zero at this high scale as a result of symmetry. Of course, to

account for the parity- and *CP*-violation observed in the SM, one has to assume that these symmetries are spontaneously broken at a particular scale $\Lambda_{P(CP)}$.

The model building problem that this sets up—one which has been made particularly manifest by the consistency of the recently observed *CP*-violation in *B*-meson decays with the KM mechanism—is that one needs to ensure that the subsequent corrections to θ are small, while still allowing for an order one KM phase. Symmetry breaking at $\Lambda_{P(CP)}$ may generate the θ-term at tree level through, e.g., imaginary corrections to the quark mass matrices M_u and M_d

$$\bar{\theta} \sim \text{Arg Det}\,(M_u M_d) + \cdots \tag{2.32}$$

where such corrections could affect either the Yukawa couplings $Y_{u(d)}$ or the Higgs vacuum expectation values, $v_{u(d)}$ (in the SM $v_u = v_d^*$), while the ellipsis denotes the phases of other coloured fermions. For comparison, the SM CKM-type phase (in basis-invariant form) is [42]

$$\theta_{\text{KM}} \sim \text{Arg Det}\big[M_u M_u^\dagger, M_d M_d^\dagger\big] \tag{2.33}$$

and one is then led to consider models for flavour in which the second phase (2.33) can be large, as is required, while the first (2.32) vanishes, or is at least highly suppressed.

One class of models uses exact parity symmetry at some high-energy scale which implies $L \leftrightarrow R$ reflection symmetry in the Yukawa sector and thus hermitian Yukawa matrices which do not contribute to $\bar{\theta}$ [43]. However, this necessitates the extension of the SM gauge group to incorporate $SU(2)_R$, and the reality of $v_{u(d)}$ comes as an additional constraint on the model which can be achieved, e.g., in its supersymmetric versions [44,45]. One can instead just demand that *CP* be an exact symmetry at high scales, which is then broken spontaneously and *CP*-violation enters via complex vacuum expectation values of additional scalar fields. Models of this type can be constructed in which the mass matrices are complex, but have a real determinant [46,47], although often it is difficult to obtain a sufficiently large CKM phase. An interesting recent suggestion for getting round this problem is to use low-scale supersymmetry breaking [48] (see also earlier ideas [49,50]), while *CP* is broken spontaneously at a much higher scale where SUSY is still exact. Strong interactions in the *CP*-breaking sector can then generate a large CKM phase, while a SUSY non-renormalization theorem ensures that $\bar{\theta}$ is not generated until the much lower scale where SUSY is broken.

All models of the type discussed above, that attempt to solve the strong *CP* problem by postulating exact parity or *CP* at high scales, have to cope with the very tight bound on $\bar{\theta}$. Indeed, it is not enough to obtain $\bar{\theta} = 0$ at tree level, as loop effects at and below $\Lambda_{P(CP)}$ can lead to a substantial renormalization of the θ-term (see e.g. [51,52]). If the effective theory reduces to the SM below the scale $\Lambda_{P(CP)}$, the residual *low-scale* corrections to the θ-term can only come via the Kobayashi–Maskawa phase and the resulting value for $\bar{\theta}(\delta_{KM})$ is small. However, this does not guarantee that the threshold corrections at $\Lambda_{P(CP)}$ are also small, as they will depend on different sources of *CP*-violation and do not have to decouple in the limit of large $\Lambda_{P(CP)}$. Such corrections are necessarily model-dependent. However, if the underlying theory

is supersymmetric at the scale $\Lambda_{P(CP)}$ and the breaking of supersymmetry occurs at a lower-scale Λ_{SUSY}, one expects the corrections to $\bar{\theta}$ to be suppressed by power(s) of the small ratio $\Lambda_{\text{SUSY}}/\Lambda_{P(CP)}$ [48].

To summarize this section, we comment that the way the strong *CP* problem is resolved affects the issue of how large additional non-CKM *CP*-violating sources can be. The axion solution, as well as $m_u = 0$, generically allows for the presence of *arbitrarily* large *CP*-violating sources above a certain energy scale. This scale is determined by comparison of higher-dimension *CP*-odd operators (i.e., dim $\geqslant 5$) induced by these sources with the current EDM constraints. On the contrary, models using a discrete symmetry solution to the strong *CP* problem usually have tight restrictions on the amount of additional *CP*-violation even at higher scales in order to avoid potentially dangerous contributions to the θ-term.

3. QCD calculation of EDMs

Having discussed the θ-term in detail, we now take a more general approach and consider all the relevant operators up to dimension six in the *CP*-odd Lagrangian (2.13), and move to the next level in energy scale in Fig. 1. To proceed, we need to determine the dependence of the nucleon EDMs, pion–nucleon couplings, etc., on these quark–gluon Wilson coefficients normalized at 1 GeV, i.e.

$$\begin{aligned} d_n &= d_n(\bar{\theta}, d_i, \tilde{d}_i, w, C_{ij}), \\ \bar{g}_{\pi NN} &= \bar{g}_{\pi NN}(\bar{\theta}, \tilde{d}_q, w, C_{ij}). \end{aligned} \tag{3.34}$$

The systematic project of deducing this dependence was first initiated some 20 years ago by Khriplovich and his collaborators, and is clearly a non-trivial task as it involves non-perturbative QCD physics. It is nonetheless crucial in terms of extracting constraints, and in particular one would like to do much better than order of magnitude estimates so that the different dependencies of the observable EDMs may best be utilized in constraining models for new physics.

It is this problem that we will turn to next. In order to be concrete, we will limit our discussion to the nucleon EDMs and pion–nucleon couplings. The electron–nucleon couplings, of which C_S plays the most important role for the EDM of paramagnetic atoms, receive contributions from the semi-leptonic four-fermion couplings C_{qe} in (2.18), which may be determined straightforwardly using low-energy theorems for the matrix elements of quark bilinears in the nucleon (see e.g. [53]).

Before we delve into some of the details of these calculations, it is worth outlining a checklist of attributes against which we can compare the various techniques available for these calculations. We list below several features that such techniques would *ideally* possess:

- *Chiral invariance*, including the relevant anomalous Ward identities, provides a very strong constraint on the manner in which *CP*-odd sources may lead to physical observables in the QCD sector. As an example, distributing $\bar{\theta}$ arbitrarily between $G\tilde{G}$ and $\bar{q}i\gamma_5 q$ cannot alter the prediction for $d_n(\bar{\theta})$, and the answer must

also depend on the correct combination of quark masses, namely the reduced mass m_*. These symmetry constraints are therefore very powerful, and allow a consistency check of the QCD estimates. Calculations are therefore more transparent if these constraints can be "built in."

- In addition to these chiral properties, the need to deal first of all with the tuning of $\bar{\theta}$ means that ideally the procedure should also correctly account for additional contributions generated under PQ relaxation. As argued in the previous section, the presence of *CP*-odd operators can shift the position of the axion expectation value, leading to a new class of contributions, $d_n(\theta_{\text{ind}})$. More specifically, this is the case for the CEDM sources, the presence of which implies that $\bar{\theta}$ must be substituted not by zero but by θ_{ind} given by [54]:

$$\theta_{\text{ind}} = -\frac{m_0^2}{2} \sum_{q=u,d,s} \frac{\tilde{d}_q}{m_q}, \tag{3.35}$$

independently of the specific details of the axion mechanism. Here m_0^2 determines the strength of the following mixed quark–gluon condensate

$$g_s \langle \bar{q} G \sigma q \rangle \equiv m_0^2 \langle \bar{q} q \rangle. \tag{3.36}$$

Thus PQ symmetry may lead to additional vacuum contributions to the EDM.

- In order to make consistent use of the EDM constraints on fundamental *CP*-odd phases, it would be desirable to have the *same method* available for obtaining estimates of d_n and $\bar{g}_{\pi NN}$ in terms of the relevant Wilson coefficients. Since different techniques have different sources of errors, use of the same method may allow a reduction in the uncertainty between the relative coefficients which, given a suite of different constraints, is ultimately more important than the overall uncertainty.
- Ideally, the method used should allow for a systematic *estimate of the precision* of the relevant QCD matrix elements, i.e., within a framework allowing for a treatment of higher-order subleading corrections.
- The generic dependence of d_q on m_q poses an additional challenge for obtaining precise results, through the poorly known values of the light quark masses, and their strong dependence on the QCD normalization scale. This uncertainty can be ameliorated given a method which generates answers which depend only on scale-invariant combinations such as, i.e., $(m_u + m_d)\langle \bar{q} q \rangle = -f_\pi^2 m_\pi^2$ or m_u/m_d.
- For EDMs, the contribution of operators that are not suppressed by the light quark masses, m_u and m_d, is of considerable phenomenological interest. An ideal method would lead to a quantitative prediction for whether $\tilde{d}_s$ or w can compete with the contributions of light quark EDMs and CEDMs. A well-known example in the 0^+ channel suggests that this may indeed happen: i.e., $\langle N|m_s \bar{s} s|N \rangle > \langle N|m_q \bar{q} q|N \rangle$, where $q = u, d$.

Since this is a non-perturbative QCD problem, the tools at our disposal are limited. Ultimately, the lattice may provide the most systematic treatment, but for the moment we are limited to various approximate methods and none that are currently

available can satisfy all of the demands listed above, although we will argue that combining QCD sum-rules with chiral techniques can satisfy most of them. While one can make use of various models of the infrared regime of QCD, we prefer here to limit our discussion to three (essentially) model-independent approaches, which vary both in their level of QCD input, and in genericity as regards the calculations to which they may be applied.

However, we will first recall what is perhaps the most widely used approach for estimating the contribution of quark EDMs to the EDM of the neutron. This is the use of the $SU(6)$ quark model, wherein one associates a non-relativistic wavefunction to the neutron which includes three constituent quarks and allows for the two spin states of each. Obtaining the contribution of quark EDMs to d_n then amounts to evaluating the relevant Clebsch–Gordan coefficients and one finds

$$d_n(d_q)^{\mathrm{QM}} = \tfrac{1}{3}(4d_d - d_u). \tag{3.37}$$

Although one may raise many questions regarding the reliability, and expected precision, of this result, we will emphasize here only the significant disadvantage that this approach cannot be used for a wider class of CP-odd sources, relevant to the generation of d_n and $\bar{g}_{\pi NN}$.

3.1. Naive dimensional analysis

Although historically not the first, conceptually the simplest approach is a form of QCD power-counting which goes under the rather unassuming name of "naive dimensional analysis" (NDA) [55]. This is a scheme for estimating the size of some induced operator by matching loop corrections to the tree-level term at the specific scale where the interactions become strong. In practice, one uses a dimensionful scale $\Lambda_{\mathrm{had}} \sim 4\pi f_\pi$ characteristic of chiral symmetry breaking, and a dimensionless coupling $\Lambda_{\mathrm{had}}/f_\pi$ to parametrize the coefficients. The claim is that, to within an order of magnitude, the dimensionless "reduced coupling" of an operator below the scale Λ_{had} is given by the product of the reduced couplings of the operators in the effective Lagrangian above Λ_{had} which are required to generate it. The reduced couplings are determined by demanding that loop corrections match the tree-level terms, and for the coefficient $c_{\mathcal{O}}$ of an operator $\mathcal{O}$ of dimension D, containing N fields, is given by $(4\pi)^{2-N} \Lambda_{\mathrm{had}}^{D-4} c_{\mathcal{O}}$. A crucial, and often rather delicate, point is the precise scale at which one should perform this matching. Within the quark sector, the identification of this scale with $4\pi f_\pi$ often seems to work quite well. However, for gluonic operators, the implied matching occurs at a very low scale where g_s is very large, up to $g_s \sim 4\pi$, and NDA has proved more problematic in this sector.

To illustrate this approach, let us consider the neutron EDM induced by θ, in this case realized as an overall phase θ_q of the quark mass matrix, and also the EDM and CEDM of a light quark. The dimension five neutron EDM operator has reduced coupling $d_n \Lambda_{\mathrm{had}}/(4\pi)$. Above the scale Λ_{had} we need the reduced couplings of the electromagnetic coupling of the quark, $e/(4\pi)$, and the CP-odd quark mass term, $\theta_q m_q/\Lambda_{\mathrm{had}}$. Thus we find

$$d_n(\theta_q, \mu) \sim e\theta_q(\mu)\frac{m_q(\mu)}{\Lambda_{\mathrm{had}}^2}, \tag{3.38}$$

where the μ-dependence reflects the choice of matching scale. To obtain a similar estimate for the contribution of a light quark EDM, we note simply that it has a reduced coupling given by $d_d\Lambda_{\mathrm{had}}/(4\pi)$ and thus

$$d_n(d_q, \mu) \sim d_q(\mu), \tag{3.39}$$

which can be contrasted with the quark model estimate above. The contribution of the quark CEDM is similar, but one needs in addition the reduced electromagnetic coupling of the quark, $e/(4\pi)$, so that

$$d_n(\tilde{d}_q, \mu) = \frac{eg_s(\mu)}{4\pi}\tilde{d}'_q(\mu), \tag{3.40}$$

where we have redefined the CEDM operator so that $\tilde{d}_q = g_s\tilde{d}'_q$. This makes the factor g_s explicit, which seems crucial to the success of NDA for gluonic operators as the matching needs to be performed at a large value of g_s, e.g., $g_s \sim 4\pi$ as noted above.

These examples indicate on one hand the simplicity of this approach and also its general applicability, but also the fact that it does not easily allow one to combine different contributions into a single result for the neutron EDM. In particular, these estimates have uncertain signs and thus can only be used independently with an assumption that the physics which generates them does not introduce any correlations. This will not generically be the case.

3.2. Chiral techniques

Historically, the first model-independent calculation of the neutron EDM [56] made use of chiral techniques to isolate an infrared log-divergent contribution in the chiral limit (for an earlier bag model estimate, see [57]). This was one of the landmark calculations which made the strong *CP* problem, and indeed the magnitude of the required tuning of θ, quite manifest.

The basic observation was that, given a *CP*-odd pion–nucleon coupling $\bar{g}_{\pi NN}$, one could generate a contribution to the neutron EDM via a π^--loop (see Fig. 2) which was infrared divergent in the chiral limit. In reality this log-divergence is cut off by the finite pion mass, and one obtains
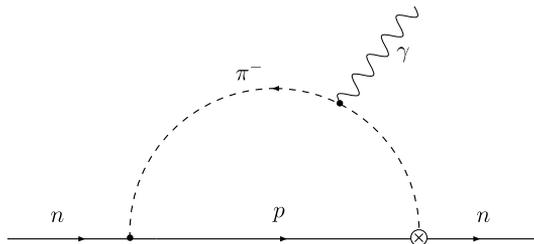

Fig. 2. Chirally enhanced contribution to the neutron EDM.

$$d_n^{\chi\log} = \frac{e}{4\pi^2 M_n} g_{\pi NN} \bar{g}_{\pi NN}^{(0)} \ln\frac{\Lambda}{m_\pi}, \tag{3.41}$$

where Λ is the relevant UV cutoff, i.e., $\Lambda = m_\rho$ or M_n. One can argue that such a contribution cannot be systematically cancelled by other, infrared finite, pieces and thus the bound one obtains on $\bar{g}_{\pi NN}^{(0)}$ in this way is reliable in real-world QCD.

This reduces the problem to one of computing the relevant *CP*-odd pion–nucleon couplings. For a given *CP*-odd source $\mathcal{O}_{CP}$, we have

$$\langle N\pi^a|\mathcal{O}_{CP}|N'\rangle = \frac{\mathrm{i}}{f_\pi}\langle N|[\mathcal{O}_{CP}, J_{05}^a]|N'\rangle + \text{rescattering}, \tag{3.42}$$

justified by the small *t*-channel pion momentum. The possible rescattering corrections will be discussed below. If we now specialize to the θ-term, as in [56], with $\mathcal{O}_{CP} = -\theta_q m_* \sum_f \bar{q}_f i\gamma_5 q_f$ then the commutator reduces to the triplet nucleon sigma term, and we find

$$\bar{g}_{\pi NN}^{(0)}(\theta_q) = \frac{\theta_q m_*}{f_\pi}\langle p|\bar{q}\tau^3 q|p\rangle\left(1 - \frac{m_\pi^2}{m_\eta^2}\right). \tag{3.43}$$

One can then determine $\langle N|\bar{q}\tau^a q|N\rangle$ from lattice calculations or, as was done in [56], by using global $SU(3)$ symmetry to relate it to measured splittings in the baryon octet.

The final factor on the right-hand side of (3.43) reflects the vanishing of the result in the limit that the chiral anomaly switches off and η (or η' in the three-flavour case) is a genuine Goldtone mode. This factor is numerically close to one and was ignored in [56]. It arises because in (3.42) we should also take into account the fact that the *CP*-odd mass term can produce η from the vacuum and thus, in addition to the PCAC commutator, there are rescattering graphs with η produced from the vacuum and then coupling to the nucleon, and the soft pion radiated via the *CP*-even pion–nucleon coupling [58].

Although this technique is not universally applicable, one can also contemplate computing the contribution of certain other sources, e.g., the quark CEDMs. Using the same PCAC-type reduction of the pion in $\langle N\pi^a|\mathcal{O}_{CP}|N'\rangle$ as in (3.42), one can reduce the calculation of $\bar{g}_{\pi NN}$ to the matrix elements of dimension five *CP*-even operators. In doing this, one has to take into account a subtlety for CEDM sources, first pointed out in [59,58,60], namely that a second class of contributions, the pion-pole diagrams (Fig. 3B), now contribute at the same order in chiral perturbation theory. In an alternative but physically equivalent approach, one can perform a chiral rotation in the Lagrangian to set $\langle 0|\mathcal{L}_{\slashed{C}P}|\pi^0\rangle = 0$, thus making this additional source of *CP*-violation explicit at the level of the Lagrangian [61].

After this consistent PCAC reduction of the pion, the intermediate result for $\bar{g}_{\pi NN}$ takes the following form:

$$\frac{1}{2f_\pi}\langle N|\tilde{d}_u(\bar{u}g_s G\sigma u - m_0^2\bar{u}u) - \tilde{d}_d(\bar{d}g_s(G\sigma)d - m_0^2\bar{d}d)|N\rangle + \frac{m_*}{f_\pi}(\bar{\theta} - \theta_{\text{ind}})\langle N|\bar{q}\tau^3 q|N\rangle. \tag{3.44}$$
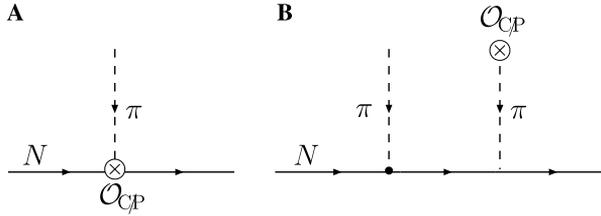
Fig. 3. Two classes of diagrams contributing to the *CP*-odd pion–nucleon coupling constant.

The second line in this expression contains the same matrix element as (3.43), and vanishes when PQ symmetry sets the axion minimum to θ_{ind} (3.35). The remaining terms are proportional to the following combination

$$\langle N|\overline{q}g_s G\sigma q - m_0^2\overline{q}q|N\rangle, \tag{3.45}$$

which unfortunately *cannot* be estimated using chiral techniques, and requires genuine QCD input. A naive vacuum saturation hypothesis in (3.45) leads to the vanishing of this expression. This is a rather fundamental problem which limits the precision of various approaches, e.g., those based on the use of low-energy theorems to estimate (3.45) [62,63], to obtain the dependence of $\overline{g}_{\pi NN}$ on the CEDMs.

This limited applicability is one problem that currently afflicts the chiral approach. A more profound issue is that the terms enhanced by the chiral log, while conceptually distinct, are not necessarily numerically dominant. Indeed, there are infrared finite corrections to (3.41) which, while clearly subleading for $m_\pi \to 0$, are not obviously so in the physical regime. This dependence on threshold corrections has been observed to provide a considerable source of uncertainty [64] (see also [65]).

3.3. QCD sum-rules techniques

An alternative to considering the chiral regime directly is to first start at high energies, making use of the operator product expansion, and attempt to construct QCD sum-rules [66] for the nucleon EDMs, or the *CP*-odd pion–nucleon couplings. This approach in principle allows for a systematic treatment of all the sources, and is motivated in part by the success of such approaches to the calculation of baryon masses [67] and magnetic moments [68]. For a recent review of some aspects of the application of QCD sum-rules to nucleons, see e.g. [69].

The basic idea is familiar from other sum-rules applications. One considers the two-point correlator of currents, $\eta_N(x)$, with quantum numbers of the nucleon in question in a background with non-zero *CP*-odd sources, an electromagnetic field $F_{\mu\nu}$, and also a soft pion field π^a

$$\Pi(Q^2) = i \int d^4x\, e^{ip\cdot x}\langle 0|T\{\eta_N(x)\overline{\eta}_N(0)\}|0\rangle_{\cancel{C}P,F,\pi}, \tag{3.46}$$

where $Q^2 = -p^2$, with p the current momentum. It is implicit here that the soft pion field admits PCAC reduction, and then in the case of CEDM sources corresponds to an external field coupled to the operator $\overline{q}g_s G\sigma q - m_0^2\overline{q}q$, as in (3.44) and (3.45).

One then computes the correlator at large Q^2 using the operator product expansion (OPE), generalized to incorporate condensates of the fields, and then matches this to a phenomenological parametrization corresponding to an expansion of the nucleon propagator to linear order in the background field and *CP*-odd sources, and corresponding higher excited states in the relevant channel. In practice, one makes use of a Borel transform to suppress the contribution of excited states, and then checks for a stability domain in Q^2, or rather the corresponding Borel mass M, where the two asymptotics may be matched.

Let us discuss this procedure in a little more detail. For the neutron, there are two currents with lowest dimension that are commonly used in QCD sum-rules and lattice calculations

$$
\begin{aligned}
\eta_1 &= 2\epsilon_{abc}(d_a^T C\gamma_5 u_b)d_c, \\
\eta_2 &= 2\epsilon_{abc}(d_a^T C u_b)\gamma_5 d_c,
\end{aligned}
\tag{3.47}
$$

of which the second, η_2, vanishes in the non-relativistic limit, and lattice simulations have shown that η_1 indeed provides the dominant projection onto the neutron state [69,70]. In the truncated OPE expansion, an admixture of η_2 can nonetheless be used to optimize convergence, and thus it is natural to parametrize the current in the form $\eta_n = \eta_1 + \beta\eta_2$ introducing an unphysical parameter β. The truncated OPE will then inherit a dependence on this parameter, which can then be fixed to improve convergence once the sum-rule has been constructed for a given physical quantity.

The correlator (3.46) exhibits various Lorentz structures (LS) in its OPE and, in selecting one to consider, one needs to be aware that in a *CP*-violating background the coupling of the current to the neutron state, described by a spinor v, is not invariant under chiral rotations, i.e., $\langle 0|\eta_n|0\rangle = \lambda e^{i\alpha\gamma_5/2}v$. It turns out that of the terms contributing to the neutron EDM, there is a unique structure which is invariant under chiral rotations, namely $\mathrm{LS} = \{F\sigma\gamma_5, \not{p}\}$, thus this is the natural quantity on which to focus in constructing a sum-rule for the EDM (for alternatives, see [71,72]). Correspondingly, $\mathrm{LS} = \not{p}$ is the relevant chiral-invariant structure for the $\bar{g}_{\pi NN}$ sum-rule.

In constructing a phenomenological model for the current correlator, it is apparent that in expanding to linear order in the external field we are effectively considering a three-point function. It is then not particularly useful to work with the spectral density, as is standard for two-point functions, and the conventional approach is to parametrize the correlator itself, i.e., $\Pi^{\mathrm{phen}} = \mathrm{LS}f(p^2) + \cdots$ The function $f(p^2)$ will in general have an expansion in double and single-pole terms, and then a continuum modelling the transitions between excited states. A Borel transform can be applied to suppress the contribution of excited states. However, a well-known complication [68] of baryon sum-rules in external fields is that the single-pole terms, corresponding to transitions between the neutron and excited states, are not exponentially suppressed by the Borel transform and thus provide the leading contribution from the excited states, with a coefficient which is not sign definite. This must then be treated as a phenomenological parameter to be determined from the sum-rules themselves. In this approximation, we then find [73,58,74]

$$\Pi_{(d)}^{\text{phen}} = \frac{\mathrm{i}}{2} \{F\sigma\gamma_5, \not{p}\} \left(\frac{\lambda^2 d_n m_n}{(p^2 - m_n^2)^2} + \frac{A}{p^2 - m_n^2} + \cdots \right), \tag{3.48}$$

$$\Pi_{(\bar{g})}^{\text{phen}} = 2\not{p} \left(\frac{\lambda^2 \bar{g}_{\pi NN} m_n}{(p^2 - m_n^2)^2} + \frac{A'}{p^2 - m_n^2} + \cdots \right), \tag{3.49}$$

where the constants A, A' parametrize the single-pole contributions. One can then go further and construct a full continuum model to match the high-Q^2 asymptotics, but as discussed below this refinement has minimal impact in comparison to the single-pole terms A and A'. We now turn to the calculation of the OPE for d_n and $\bar{g}_{\pi NN}$.

3.3.1. Nucleon EDM calculations

The OPE for d_n is conveniently constructed in practice by first computing the generalized quark propagator, expanded in the presence of the background field, the CP-odd sources, and also the vacuum condensates. One then computes the relevant contractions in (3.46) to obtain the OPE to the appropriate order. Although it would take us too far afield to describe this procedure in detail, we can exhibit some of the dominant physics by looking at just one class of diagrams which arise in evaluating the OPE for (3.46). In particular, in Fig. 4 two of the quarks in the nucleon current propagate without interference, carrying the large current momentum, while the third is taken to be soft and so induces a dependence on the appropriate chiral quark condensates. We may then make use of standard arguments [29] used to determine $E(\theta)$ (2.22), which utilize the anomaly in the axial current and the fact that $m_\eta \gg m_\pi$, to determine the dependence of these condensates on the CP-odd source.

Let us consider the contribution of $\bar{\theta}$. For additional control over the chiral transformation properties of the answer, we split $\bar{\theta}$ into several terms in the Lagrangian

$$\mathcal{L}_{\text{dim}=4} = \frac{g_s^2}{32\pi^2} \theta_G G_{\mu\nu}^a \widetilde{G}^{\mu\nu,a} - \sum_{i=u,d,s} m_i \theta_i \overline{\psi}_i i\gamma_5 \psi_i. \tag{3.50}$$
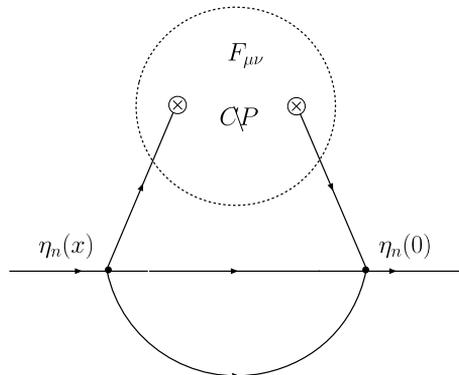

Fig. 4. A leading contribution to the neutron EDM within QCD sum-rules. Sensitivity to the CP-violating source enters through the two soft quark lines which lead to a dependence on the chiral condensate.

The diagram in Fig. 4 then leads to a dependence on

$$m_q \langle 0|\bar{q}\sigma_{\mu\nu}\gamma_5 q|0\rangle_{\theta,F} = \mathrm{i}m_* \theta_G \langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F + \mathcal{O}(m_*^2)$$
$$= \mathrm{i}\chi_q \theta_G m_* F_{\mu\nu}\langle\bar{q}q\rangle + \mathcal{O}(m_*^2). \tag{3.51}$$

In the first equality the dependence on θ_G has been determined as in the standard computation of $E(\theta)$ in (2.22) [29], where the terms of $\mathcal{O}(m_*^2)$ are subleading only because there is no $U(1)$-axial Goldstone mode, i.e., $m_\eta \gg m_\pi$. The dependence of (3.51) on θ_G rather than $\bar{\theta}$ means that there are additional contributions from θ_q that have to be taken into account, at this order in the OPE, in order to restore the dependence on the physical $\bar{\theta}$ combination. In fact, it turns out that the procedure is generally more complicated and one must also incorporate the the mixing of the currents η_1 and η_2 (3.47) with their chirally rotated (or CP-conjugate) counterparts [73,58] in order to restore the dependence on $\bar{\theta}$ and exclude unphysical parameters such as $\theta_G - \theta_q$.

In the second line of (3.51) we have introduced the so-called electromagnetic susceptibility of the quark vacuum, χ_q, given by

$$\langle \bar{q}\sigma_{\mu\nu}q \rangle_F = \chi_q F_{\mu\nu}\langle\bar{q}q\rangle \tag{3.52}$$

and chosen for simplicity to be favour independent, $\chi_q = \chi e_q$ [68]. In fact, χ_q is one among a whole series of mixed quark–gluon condensates that have to be taken into account in the calculation of the OPE for d_n. However, it turns out that numerically χ is rather large, $\chi = -(5\text{–}9)\,\mathrm{GeV}^{-2}$ [75,76], and results in the diagram of Fig. 4 being numerically very important. The remaining condensates are numerically quite small in comparison, and we will neglect them in what follows, referring the reader to [77,73,58,60,74] for more of the details involved in these calculations.

The leading-order and next-to-leading-order contributions to the OPE induced by the CP-odd operators of dimension four and five are given by

$$\Pi^{\mathrm{OPE}}(Q^2) = -\frac{\mathrm{i}}{64\pi^2}\langle\bar{q}q\rangle\{F\sigma\gamma_5,\slashed{p}\}\left(\ln(-p^2)[\pi_{\mathrm{LO}}^{(\chi)} + \pi_{\mathrm{LO}}^{(q)}] - \frac{4}{p^2}\ln\left(-\frac{\Lambda_{\mathrm{IR}}^2}{p^2}\right)[\pi_{\mathrm{NLO}}]\right). \tag{3.53}$$

At leading order the quark EDMs induce $\pi_{\mathrm{LO}}^{(q)}$

$$\pi_{\mathrm{LO}}^{(q)} = d_d[10 + 6\beta^2] - d_u[3 + 2\beta - \beta^2], \tag{3.54}$$

while the θ-term, and the CEDMs, are responsible for $\pi_{\mathrm{LO}}^{(\chi)}$

$$\pi_{\mathrm{LO}}^{(\chi)} = 4(1+\beta)^2\chi_d m_d P_d - (1+\beta)^2\chi_u m_u P_u + 2(1-\beta^2)m_*(\chi_u + \chi_d)(P_u - P_d), \tag{3.55}$$

where $P_q = \theta_q + (\langle\bar{q}\gamma_5 q\rangle_{\slashed{C}P}/\mathrm{i}\langle\bar{q}q\rangle)$ contains the dependence on the CP-odd sources.

The next-to-leading-order terms in (3.53), π_{NLO}, are associated with a class of diagrams in which one of the propagators exhibits a logarithmic infrared divergence as can be seen in (3.53). The magnitude of these terms is clearly ambiguous through the logarithmic dependence on the cutoff Λ_{IR}, although in practice with a cutoff $\Lambda_{\mathrm{IR}} \sim \Lambda_{\mathrm{QCD}}$ the logarithms are not particularly large in the momentum regime for

which the resulting sum-rules are optimal. We will not need to exhibit them explicitly here, but it is important that for all of these terms $\pi_{\text{NLO}} \propto (1 - \beta)$, which differs from the $(1 + \beta)^2$ dependence of many of the dominant leading-order terms.

The truncated OPE in (3.53) necessarily depends on the unphysical parameter β, which can therefore be chosen in such a way as to optimize the convergence of the expansion. Such optimization problems arise in many areas of physics [78], and in the present case with only two terms of the series in hand, the most practical approach is to use "fastest apparent convergence" (FAC), which involves choosing β to set the highest known term in the series to zero. Historically, Ioffe [79] introduced an FAC-like criterion in analysis of the mass-sum rule, which has proved quite successful. For the CP-even sector this involves the choice $\beta = -1$ to cancel the subleading terms. We shall also follow this approach as it has the added advantage of cancelling the ambiguous infrared logarithmic terms. As first discussed in [73,58], for the CP-odd sector this involves the choice $\beta = 1$. The difference in β, as compared with the choice in the CP-even sector, is not surprising as β is unphysical and there is then no reason to expect optimal choices to be the same for different physical observables.

A remarkable feature of the choice $\beta = 1$ becomes apparent if we now rewrite the OPE expression for $\beta = 1$. All the subleading logarithmic terms in (3.53) are now cancelled, while the leading-order terms adopt the elegant form:

$$
\begin{aligned}
\pi_{\beta=1}^{(\chi)} &= 4[4\chi_d m_d P_d - \chi_u m_u P_u], \\
\pi_{\beta=1}^{q} &= 4[4d_d - d_u].
\end{aligned}
\tag{3.56}
$$

It is remarkable that the contribution of the u quark in each term is precisely $-1/4$ that of the d quark, which is the combination suggested by the $SU(6)$ quark model! One might suspect that this is due to the minimal valence quark content of the current, but the fact that this structure arises only for the choice $\beta = 1$ is rather surprising given that only the η_1 current survives in the non-relativistic limit.

One of the most important aspects of the whole calculation is the consistent treatment of the CP-odd vacuum condensates entering via the CP-odd sources P_q. This calculation can be done relatively easily with the use of chiral techniques, and at leading order in $m_{\pi(\eta)}^2/m_{\eta'}^2$. A useful constraint on this calculation is that the anomalous chiral Ward identity must be respected for each quark flavour. This provides a useful check if we consider artificially decoupling the s-quark, while at the same time sending either m_u or m_d to zero. In this regime, the dependence on $\bar{\theta}$ in particular must vanish, which fixes the remaining quark mass dependence in terms of m_*. The final result for the P_u and P_d sources, in the limit of $m_{u(d)} \ll m_s$, reads

$$
m_{u(d)} P_{u(d)} = m_* \bar{\theta} - \frac{m_* m_0^2}{2} \left(\frac{\tilde{d}_{u(d)} - \tilde{d}_{d(u)}}{m_{d(u)}} - \frac{\tilde{d}_s}{m_s} \right),
\tag{3.57}
$$

which respects the anomalous chiral Ward identity. If $\bar{\theta}$ is removed by PQ symmetry, i.e., substituted by θ_{ind} (3.35), Eq. (3.57) simplifies even further

$$
m_q P_q^{\text{PQ}} = -\frac{m_0^2 \tilde{d}_q}{2}.
\tag{3.58}
$$

Putting all the ingredients together, after a Borel transform of (3.53) and (3.48), and using $\beta = 1$ as discussed earlier, we obtain the sum-rule

$$\lambda^2 m_n d_n + AM^2 = -\frac{M^4}{32\pi^2} e^{m_n^2/M^2} \langle \bar{q}q \rangle \left[\pi_{\beta=1}^{(\chi)} + \pi_{\beta=1}^q \right] + O(M^0), \qquad (3.59)$$

where M is the Borel mass, and the relevant contributions are given in (3.56). Clearly, the presence of three parameters d_n, λ, and A in (3.59) necessitates the use of additional sum-rules, and the coupling λ is conveniently obtained from the well-known sum-rules for the two-point correlation function of the nucleon currents in the CP even sector (see e.g. [69] for a review).

Rather than reviewing the full analysis, let us consider a simple estimate obtained from the leading-order terms in the OPE of (3.59) a la Ioffe's derivation of the nucleon mass formula [67]. We set $A = 0$, and taking $M = m_n \sim 1$ GeV, we divide the sum-rule (3.59) by the standard CP-even sum-rule for λ obtained for the Lorentz structure $\not{p}$ and $\beta = 1$ (the choice $\beta = -1$ in the latter sum-rule leads to a similar result). The resulting estimate takes the following form:

$$d_n^{\text{est}} = \frac{8\pi^2 |\langle \bar{q}q \rangle|}{m_n^3} \left[\frac{2\chi m_*}{3} e(\bar{\theta} - \theta_{\text{ind}}) + \frac{1}{3}(4d_d - d_u) + \frac{\chi m_0^2}{6}(4e_d \tilde{d}_d - e_u \tilde{d}_u) \right], \quad (3.60)$$

where θ_{ind} again is a linear combination of $\tilde{d}_q/m_q$ (3.35). The coefficient in front of the square brackets in (3.60) is very close to 1, given Ioffe's estimate for m_n, $m_n^3 \simeq 8\pi^2 |\langle \bar{q}q \rangle|$ [67]. Indeed, this estimate shows no deviation at all from the naive quark model result for $d_n(d_q)$! Using Vainshtein's value for χ, $\chi = -N_c/(4\pi^2 f_\pi^2) \sim -9$ GeV^{-2} [76], obtained using pion dominance for the longitudinal part of certain anomalous triangle diagrams, along with the Ioffe formula for m_n, the estimate for $d_n(\bar{\theta})$ becomes

$$d_n^{\text{est}} = -\frac{em_* \bar{\theta}}{2\pi^2 f_\pi^2}, \qquad (3.61)$$

which coincides with the chiral estimate (3.41) if $g_A \langle p|\bar{q}\tau^3 q|p \rangle \ln(\Lambda/m_\pi)$ is of order 2, where $g_A \simeq g_{\pi NN} f_\pi/m_n$. Needless to say that within the accuracy of both methods the two estimates coincide. If $\bar{\theta}$ is removed by PQ symmetry, then within the same approximation the resulting estimate reads

$$d_n^{\text{est}} = \frac{4}{3}d_d - \frac{1}{3}d_u - \frac{2m_\pi^2 e}{m_n(m_u + m_d)} \left(\frac{2}{3}\tilde{d}_d + \frac{1}{3}\tilde{d}_u \right), \qquad (3.62)$$

where the approximate relation $m_0^2 \simeq -m_n^2$ has been used, along with $(m_u + m_d)|\langle \bar{q}q \rangle| = f_\pi^2 m_\pi^2$. One immediately sees that the CEDM contributions are in fact significant and comparable in magnitude to the effects induced by quark EDMs.

We can give a more precise numerical treatment by making use of the following parameter values: For the quark condensate, we take a central value of

$$\langle 0|\bar{q}q|0 \rangle = -(0.225 \text{ GeV})^3, \qquad (3.63)$$

while for the condensate susceptibilities, we have [75]

$$\chi \simeq -5.7 \pm 0.6 \text{ GeV}^{-2}, \quad m_0^2 \simeq -0.8 \text{ GeV}^2. \qquad (3.64)$$

This determination of χ is based on spectral sum-rules [75] and is slightly lower than the value obtained in [76].

A more systematic treatment of the sum-rule [73,58,74] indicates that a stability domain exists for relatively low Borel mass scales, of $M \sim \mathcal{O}(0.8 \text{ GeV})$. Convergence of the OPE is apparently not in danger as this low scale arises via the two-step procedure used, in which the OPE is naturally formulated around the neutron scale, $M \sim 1 \text{ GeV}$, while the chiral techniques used to extract the dependence of the condensates on the *CP*-violating sources lower the effective scale, but also introduce additional combinatoric suppression factors as the dimension of the condensates increases. In order to test the stability of the sum-rule, and obtain an estimate for the uncertainty due to the handling of excited states, one can generalize the expression for (3.59), by including a more systematic parametrization of the continuum, and also by including one-loop anomalous dimensions for the currents and condensates entering the sum-rule. However, as alluded to earlier, these refinements have a rather minimal impact, moving the stability domain by no more than 10–15%. This is relatively small compared to the primary sources of error, namely the saturation hypothesis for the condensates, the need to extract the single-pole term from the sum-rules and, perhaps most significantly, the dependence on β.

Extracting a numerical central value from the sum-rule, employing numerical estimates for the condensates (3.64), and estimating the precision through consideration of the sources of error listed above, we find the results first presented in [73,74]

$$
\begin{aligned}
d_n(\bar{\theta}) &= (1 \pm 0.5)\frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3}\bar{\theta} \times 2.5 \times 10^{-16} e\,\text{cm}, \\
d_n^{\text{PQ}}(d_q, \tilde{d}_q) &= (1 \pm 0.5)\frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3}[1.1e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.4(d_d - 0.25d_u)],
\end{aligned}
\tag{3.65}
$$

where we intentionally split the formula into two parts, $d_n(\bar{\theta})$ and $d_n(d_q, \tilde{d}_q)$, in the presence of PQ symmetry. In the generic case, the two lines in (3.65) must be added together and $\bar{\theta}$ substituted by $\bar{\theta} - \theta_{\text{ind}}$.

The result (3.65) offers several interesting consequences. Note that the overall factor of $\langle \bar{q}q \rangle$ combines with the light quark masses from short-distance expressions for d_i and $\tilde{d}_i$ to give a result $\sim f_\pi^2 m_\pi^2 (1 + \text{O}(m_u/m_d))$ thus reducing the uncertainty due to poor knowledge of the absolute values of quark masses and condensates. The contribution from the strange quark CEDM is suppressed by the m_*/m_s ratio, and is completely removed at this order once the PQ symmetry is switched on. However, the use of nucleon currents with only valence quarks leads one to suspect that gluon and sea-quark contributions may be under-estimated, since they enter only at higher orders. It is possible that the question of $d_n(\tilde{d}_s)$ may be resolved in future lattice simulations, given an appropriate lattice implementation of chiral symmetry.

Compared to the techniques outlined previously, this approach has the significant advantage that all of the sources up to dimension five can be handled systematically and thus relative signs and magnitudes can be consistently tracked. As indicated in (3.65), one can also make a systematic estimate of the precision of the result, where the errors are due to the contribution of excited states, neglected higher dimensional

operators in the OPE, and also an ambiguity in the nucleon current. Comparing the numerical result with those obtained using NDA and chiral techniques one finds, as is to be expected, that the results agree in terms of order of magnitude. In particular, our result for d_n implies a bound $|\bar{\theta}| < 3 \times 10^{-10}$.

In progressing to consider the contribution from sources of higher dimension, problems arise through the appearance of certain infrared divergences at low orders in the OPE, while a number of unknown condensates also enter and render a corresponding calculation for dimension six sources intractable. One can nonetheless estimate the contribution of these operators by utilizing a trick which involves relating the EDM contribution to the measured anomalous magnetic moment μ_n via the γ_5-rotation of the nucleon wavefunction induced by the *CP*-odd source [37],

$$d_n \sim \mu_n \frac{\langle N|\delta\mathscr{L}_{CP}|N\rangle}{m_n \bar{N} i\gamma_5 N}. \tag{3.66}$$

One may analyse the γ_5-rotation using conventional sum-rules techniques, and for the Weinberg operator, one can obtain the following estimate [80]

$$|d_n(w)| \sim |\mu_n| \frac{3 g_s m_0^2}{32\pi^2} w \ln(M^2/\mu_{\mathrm{IR}}^2) \simeq e \ 22 \ \mathrm{MeV} \ w(1 \ \mathrm{GeV}), \tag{3.67}$$

taking $M/\mu_{\mathrm{IR}} = 2$, where M is the Borel mass and μ_{IR} is an infrared cutoff, and $g_s = 2.1$.

We can also apply this technique for the contribution of four-fermion operators. For SUSY models with generic parameters *CP*-odd four-fermion operators are negligible due to the double helicity-flip requiring an m_q^2 dependence and rendering these operators effectively of dimension eight. However, for large $\tan\beta$, there are enhancements for operators proportional to C_{ij} with $i, j = d, s, b$ which can partially overcome this suppression thus altering the conventional picture of EDM sources (see Fig. 1). An important class of contributions in this case involves the four-fermion operators with a *b*-quark. The contribution of these sources to d_n can again be estimated using the same technique as above [81]

$$|d_n(C_{ij})| \sim e \ 2.6 \times 10^{-3} \ \mathrm{GeV}^2 \left(\frac{C_{bd}(m_b)}{m_b} + 0.75 \frac{C_{db}(m_b)}{m_b} \right). \tag{3.68}$$

We should emphasize that both the dimension six estimates above necessarily have a precision not better than $\mathcal{O}\,(100\%)$, and one cannot reliably extract the sign. Fortunately, the numerical size of these dimension six contributions is often negligible, and thus does not significantly impact the phenomenological application of EDM constraints.

3.3.2. Calculation of $\bar{g}_{\pi NN}$

The other primary source of nuclear EDMs, leading to the observable EDMs in diamagnetic atoms, arises through nucleon interactions mediated by pion exchange with *CP*-violation in the pion–nucleon vertex. As discussed in the preceding subsection, the calculation of these couplings involves essentially two steps: the first is a PCAC-type reduction of the pion in $\langle N\pi^a|\mathcal{O}_{CP}|N'\rangle$ as in (3.42), and the second is

an evaluation of the resulting matrix elements over the nucleon. It is this second part for which QCD sum-rules may usefully be employed, and here we sketch its application to the computation of the dependence of $\bar{g}_{\pi NN}$ on dimension five CP-odd sources in (2.16) [61].

The main difficulty, as alluded to in the previous section, is the partial cancellation between the $m_0^2 \bar{q}q$ and $\bar{q}g_s G\sigma q$ sources in (3.44). Within the sum-rule analysis, one can nonetheless trace this cancellation up to the next-to-next-to-leading order [61]. Indeed, following the approach outlined earlier of fixing the unphysical parameter β to suppress the highest calculated order in the OPE leads again to $\beta = 1$. However, in this case, the choice $\beta = 1$ sets all the LO, NLO, and NNLO terms to zero! This cancellation is, however, seemingly an artefact of the purely valence-quark content of the current, and is not dictated by symmetry. Thus, although the result will necessarily have a strong dependence on β. one can obtain a numerical estimate by varying β through an appropriate range. To get some idea of the size of the result, let us work only with the current η_1 which survives in the non-relativistic limit and has the dominant overlap with the neutron state, i.e., set $\beta = 0$. Assuming also the dominance of the nucleon double-pole term and the leading-order term in the OPE, and separating different isospin structures, we find [61]

$$\bar{g}_{\pi NN}^{(1)} \sim \frac{3}{4} \frac{m_0^2}{f_\pi}(\tilde{d}_u - \tilde{d}_d), \quad \bar{g}_{\pi NN}^{(0)} \sim \frac{3}{20} \frac{m_0^2}{f_\pi}(\tilde{d}_u + \tilde{d}_d). \tag{3.69}$$

Unless $\tilde{d}_u - \tilde{d}_d \simeq 0$, the CP-odd coupling $\bar{g}_{\pi NN}^{(1)}$ is several times larger than $\bar{g}_{\pi NN}^{(0)}$. Concentrating therefore on $\bar{g}_{\pi NN}^{(1)}$, we have

$$\bar{g}_{\pi NN}^{(1)} = 3 \times 10^{-12} \frac{\tilde{d}_u - \tilde{d}_d}{10^{-26} \text{ cm}} \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} \frac{|m_0^2|}{0.8 \text{ GeV}^2}, \tag{3.70}$$

where we re-instate the dependence on the relevant condensates, normalized to their central values. The estimate (3.70) is half the size of the value for $\bar{g}_{\pi NN}^{(1)}$ used in [59].

The full numerical treatment of the sum-rule for the isospin-one coupling produces the following result at next-to-next-to-leading order [61]

$$\bar{g}_{\pi NN}^{(1)} = 2_{-1}^{+4} \times 10^{-12} \frac{\tilde{d}_u - \tilde{d}_d}{10^{-26} \text{ cm}} \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3}, \tag{3.71}$$

where the poor precision is essentially due to the cancellation of the leading terms as described above. We emphasize that a more precise calculation of the matrix element (3.45) would significantly enhance the quality of constraints one could draw from the experimental bounds on diamagnetic EDMs, and thus constitutes a significant outstanding problem. Again, it seems progress may have to wait for further developments in lattice QCD.

In considering the contribution of dimension six sources to $\bar{g}_{\pi NN}$, we note first that the three-gluon Weinberg operator $GG\tilde{G}$ is additionally suppressed by m_q and can be neglected. However, as for d_n, for SUSY models with large $\tan\beta$, certain four-fermion operators $C_{q_1 q2}$ may be relevant, and can be obtained via vacuum factorization, as the two diagrams in Fig. 3 now fail to cancel

$$\bar{g}_{\pi NN}^{(1)} \sim -8 \times 10^{-2} \text{ GeV}^2 \left(\frac{0.5C_{dd}}{m_d} + 3.3\kappa \frac{C_{sd}}{m_s} + \frac{C_{bd}}{m_b}(1 - 0.25\kappa) \right), \tag{3.72}$$

where $\kappa \equiv \langle N|m_s\bar{s}s|N\rangle/220$ MeV, with the preferred value $\kappa \simeq 0.5$ [53].

4. EDMs in models of CP-violation

We have now moved to the highest level in Fig. 1, which is where the EDM constraints can be applied to directly constrain new sources of CP-violation. In this section, we will briefly discuss these constraints, first looking at why the Standard Model itself provides such a small background, and then why most models of new physics, and supersymmetry in particular, tend to overproduce EDMs and are thus subject to stringent constraints.

4.1. EDMs in the Standard Model

The recent discovery and exploration of CP-violation in the neutral B-meson system [7] is, along with existing data from CP-violation observed in K-mesons, (within current precision) in accord with the minimal model of CP-violation known as the Kobayashi–Maskawa (KM) mechanism [3]. This introduces a 3×3 unitary quark mixing matrix V in the charged current sector of up and down-type quarks taken in the mass eigenstate basis

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\bar{U}_L W^+ V D_L + (\text{h.c.}) \right). \tag{4.73}$$

This model possesses a single CP-violating invariant in the quark sector, $J_{CP} = \text{Im}(V_{tb}V_{td}^*V_{cd}V_{cb}^*) \simeq 3 \times 10^{-5}$. This combination, along with θ_{QCD}, is the only allowed source of CP-violation in the Standard Model (treating "Standard Model neutrinos" as massless). In addition to this, CP-violation in the SM vanishes in the limit of an equal mass for any pair of quarks of the same isospin, e.g., d and s, u and c, etc. These two conditions are extremely powerful in suppressing any KM-induced CP-odd flavour-conserving amplitude.

4.1.1. Quark and nucleon EDMs
The necessity of four electroweak vertices requires that any diagram capable of inducing a quark EDM have at least two loops. Moreover, it turns out that all EDMs and colour EDMs of quarks vanish exactly at the two-loop level [83], and only three-loop diagrams survive [84,82], as in Fig. 5. A leading-log calculation of the three-loop amplitude for the EDM of the d-quark produces the following result [82]:

$$d_d = e \frac{m_d m_c^2 \alpha_s G_F^2 J_{CP}}{108\pi^5} \ln^2(m_b^2/m_c^2) \ln(M_W^2/m_b^2). \tag{4.74}$$

Upon the inclusion of the other contributions, it produces a numerical estimate

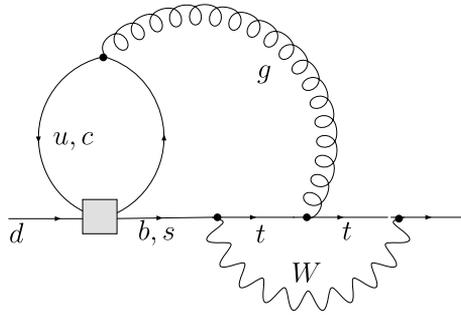$$d_d^{\text{KM}} \simeq 10^{-34} e \text{ cm}. \tag{4.75}$$

Fig. 5. A particular three-loop contribution [82] to the *d*-quark EDM induced by the KM phase in the standard model. The box vertex denotes a contacted *W*-boson line connected to the light quarks, while it is implicit that the external photon line is to be attached as appropriate to any charged lines.
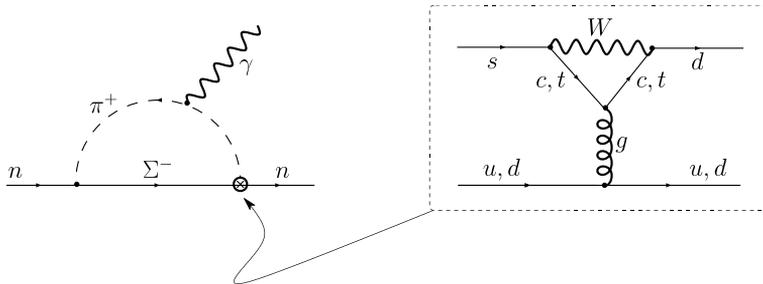

Fig. 6. A leading contribution to the neutron EDM in the Standard Model, arising via a four-quark operator generated by a strong penguin, and then a subsequent enhancement via a chiral π^+ loop.

The only relevant operator that is not zero at two-loop order is the Weinberg operator [85], but its numerical value also turns out to be extremely small. Indeed, the largest Standard Model contribution to d_n comes not from quark EDMs and CEDMs, but instead from a four-quark operator generated by a so-called "strong penguin" diagram shown in Fig. 6. This is enhanced by long-distance effects, namely the pion loop, and it has been estimated that this mechanism could lead to a KM-generated EDM of the neutron of order [86]

$$d_n^{\text{KM}} \simeq 10^{-32} e \, \text{cm}. \tag{4.76}$$

However, this is still six to seven orders of magnitude smaller than the current experimental limit.

4.1.2. Lepton EDMs

The KM phase in the quark sector can induce a lepton EDM via a diagram with a closed quark loop, but a non-vanishing result appears first at the four-loop level [87] and therefore is even more suppressed, below the level of

$$d_e^{\text{KM}} \leqslant 10^{-38} e \, \text{cm}, \tag{4.77}$$

and so small that the EDMs of paramagnetic atoms and molecules would be induced more efficiently by, e.g., Schiff moments and other *CP*-odd nuclear momenta.

In this regard, we note that recent data on neutrino oscillations point toward the existence of neutrino masses, mixing angles, and possibly of new CKM-like phase(s) in the lepton sector. Under the assumption that neutrinos are Majorana particles, the presence of these new *CP*-odd phases in the lepton sector allows for a non-vanishing two-loop contributions to d_e [88], without any further additions to the Standard Model. However, recent calculations [89] show that a typical seesaw pattern for neutrino masses and mixings only induces a tiny contribution to the EDMs in this way, of $O(m_e m_\nu^2 G_F^2)$, unless a fine-tuning of the light neutrino masses is tolerated in which case d_e could reach $10^{-33} e$ cm. Therefore, within this minimal extension of the Standard Model allowing for massive neutrinos, the electron EDM is not the best way to probe *CP*-violation in the lepton sector.

4.1.3. Probing the scale of new physics

The Standard Model predictions for EDMs described above are well beyond the reach of even the most daring experimental proposals. This implies in turn that the Kobayashi–Maskawa phase provides a negligible background and thus any positive detection of an EDM would necessarily imply the presence of a non-KM *CP*-violating source. Before we consider some of the models which provide motivations for anticipating such a discovery, it will first be useful to consider in more general terms how high an energy scale one could indirectly probe with EDM measurements. Indeed, we are led to ask first of all, what energy scale of new *CP*-violating physics is probed with the current experimental sensitivity to EDMs? Second, given the small KM background, we might also ask for the largest energy scale that could be probed *in principle* before reaching the level where the Standard Model KM contributions would become significant.

To try and answer these questions in a systematic way, let us consider a toy model containing a scalar field ϕ (which is Higgs-like, but need not be the SM Higgs) coupled to the SM fermions with scalar and pseudo-scalar couplings, $\mathcal{L}_{int} = -\sum_i \phi \bar{\psi}_i (y_i + iz_i \gamma_5) \psi_i$. The presence of both scalar and pseudo-scalar couplings y_i and z_i breaks *CP* invariance.

Assuming that the scalar mass M is large, we integrate this field out and match the resulting coefficients with the Wilson coefficients listed in (2.16) and (2.18). In particular, at tree level, we obtain the following contribution to (2.6)

$$C_S^{(0)} \simeq \frac{1}{M^2} z_e (3(y_d + y_u) + \kappa y_s + \cdots), \tag{4.78}$$

where the ellipsis stands for the contribution of heavy quark flavours, and the couplings are normalized at 1 GeV. If we now make the assumption that there is no correlation with other sources of *CP*-violation, e.g., the electron EDM d_e, then with the use of the experimental constraint on d_{Tl} and the results of atomic calculations (2.8), we arrive at the following limit on the *CP*-odd combination of couplings and the mass M

$$\frac{1}{M^2} z_e (y_d + y_u + 0.3\kappa y_s) \leqslant \frac{1}{(1.5 \times 10^6 \text{ GeV})^2}. \tag{4.79}$$

Given the most optimistic assumption about the possible size of these couplings, i.e., $z_e y_q \sim \mathrm{O}(1)$, we can conclude that the current experimental EDM sensitivity translates to a bound on M as high as $M_{CP} \sim 10^6$ GeV. If instead we insert the largest Kobayashi–Maskawa prediction for the Tl EDM of order $\sim 10^{-35} e$ cm in place of the current sensitivity, we obtain $M_{CP}^{\max} \sim 10^{11}$ GeV, as the ultimate scale which can be probed via these dimension-six operators before the onset of the "KM background."

For comparison, allowing for arbitrarily large flavour-violating couplings of ϕ to fermions, we can also deduce the sensitivity level to new flavour physics (NFP) in a similar way. For example, requiring that four-fermion operators that change flavour by two units, e.g., $\bar{s}\gamma_5 d \bar{s}\gamma_5 d$ and the like, do not introduce new contributions to Δm_B, Δm_K, and ϵ_K that are larger than the SM contribution, one typically finds that $M_{\mathrm{NFP}} > 10^7 – 10^8$ GeV in this scenario. Thus, we see that the sensitivity of EDMs is already approaching this benchmark and, unlike the constraints from the $\Delta F = 2$ sector, can be significantly improved in the near future.

At this point, we should emphasize that in this example, we are relaxing all constraints on the flavour structure by allowing order one couplings of the scalar field ϕ to the light fermions. These couplings violate chirality maximally, and if they were part of a more realistic construction, for example a two-Higgs doublet model, one would expect that y_i and z_i will have to scale according to the fermion mass m_i. In this case, the sensitivity to M clearly drops dramatically, and the tree-level interactions (4.78) are not necessarily the dominant contributions to EDMs, as heavy flavours may contribute in a more substantial way via loop effects [90,91]. Indeed, if new physics above the electroweak scale preserves chirality, as is often assumed, one expects that for the light flavours $d_i \sim e \times (1–10)$ MeV$/M^2$. Taking the electron EDM, and the Tl EDM bound, as a concrete example we find under this more restrictive assumption that

$$d_e \sim e \times \frac{m_e}{M^2} = 10^{-23} e \text{ cm} \times \left(\frac{1 \text{ TeV}}{M} \right)^2 \Rightarrow M_{CP} \sim 70 \text{ TeV}, \tag{4.80}$$

and consequently the current level of sensitivity to new CP-violating chirality-preserving physics drops somewhat, but for reference this scale is still well beyond the centre-of-mass energy of the LHC.

If we put the current EDM bounds into the broader context of precision tests of the Standard Model, we see that the present bounds in Table 1 imply that EDMs occupy an intermediate position in sensitivity to mass scales for new physics, between the electroweak precision tests (EWPTs) and flavour violation in $\Delta F = 2$ processes noted above. The EWPTs from LEP impose a bound, $M_{\mathrm{EWPT}} > $ few TeV, through constraints on various dimension-six operators, e.g., oblique corrections to gauge boson propagators. Since $M_{\mathrm{EWPT}} \gg M_Z$, this has been dubbed the "little hierarchy problem" [92]. Indeed, while there are general expectations that the

Standard Model is an effective theory, and will be corrected at scales of about a TeV, it is clear from this discussion that precision constraints in many sectors do not contain any hints of new physics beyond the Higgs at the weak scale, and in this sense EDMs are no exception. The remarkably large-scale M_{CP} implied by EDM limits requires, at least within our current level of understanding, a tuning in the CP-odd sector of physics beyond the standard model that we lack a coherent explanation for. The recent data from BaBar and Belle on CP-violation in the neutral B-meson sector, which thus far is consistent with the KM model, within which CP-violation is maximal within the confines of the flavour structure, only makes this tuning more pronounced, since we lack a strong motivation to enforce any additional CP-violating phases to be small. Moreover, further experimental progress in the near future could, given null results, push the value of M_{CP} closer to $M_{\rm NFP}$. From this viewpoint EDMs provide our most powerful tool in probing the question of whether CP-violation and flavour physics are intrinsically linked, as indeed they are within the electroweak Standard Model. This issue stands out as one of the most important ways in which EDMs may assist in demystifying some of the less constrained parts of the Standard Model.

4.2. EDMs in supersymmetric models

Having demonstrated the generic importance of EDM constraints for TeV-scale physics, we would now like to make this analysis more concrete by focussing on models with electroweak-scale supersymmetry and reviewing their predictions for EDMs.

Supersymmetric extensions of the Standard Model provide perhaps the most natural solution to the gauge hierarchy problem by automatically cancelling the quadratically divergent contributions to the Higgs mass. Supersymmetry is thought of here as a symmetry of nature at high energies, whereas at the electroweak scale and below it is obviously broken. Ensuring that supersymmetry breaking does not re-introduce quadratic divergences, and is compatible with the observed low-energy spectrum, still allows for a large number of new dimensionful parameters, unfixed by any symmetry, that are usually called the soft-breaking parameters. The minimal realization, known as the Minimal Supersymmetric Standard Model (MSSM), has been the subject of numerous theoretical studies, and also experimental searches, for over two decades. While no experimental evidence for SUSY exists, the MSSM retains a pre-eminent status among models of TeV-scale physics in part through several indirect virtues, e.g., gauge-coupling unification and a "natural" dark matter candidate. For full details of the MSSM spectrum and the parametrization of the soft-breaking terms, we address the reader to any of the comprehensive reviews on MSSM phenomenology [93].

The unbroken sector of the MSSM contains, besides the gauge interactions, the Yukawa couplings parametrized by 3×3 Yukawa matrices in flavour space, $\mathbf{Y}_u$, $\mathbf{Y}_d$, and $\mathbf{Y}_e$. These matrices source the tree-level masses of matter fermions

$$\mathbf{M}_u = \mathbf{Y}_u \langle H_2 \rangle, \quad \mathbf{M}_d = \mathbf{Y}_d \langle H_1 \rangle, \quad \mathbf{M}_e = \mathbf{Y}_e \langle H_1 \rangle, \tag{4.81}$$

where $\langle H_1 \rangle$ and $\langle H_2 \rangle$ are two Higgs vacuum expectation values related to the SM Higgs v.e.v. via $\langle H_2 \rangle^2 + \langle H_1 \rangle^2 = v^2/2$. In SUSY models, anomaly cancellation in the Higgsino sector requires the introduction of at least two Higgs superfields as above. In addition to Yukawa couplings, the supersymmetry-preserving sector contains the so-called μ-term that provides a Dirac mass to the higgsinos (the superpartners of the Higgs bosons) and contributes to the mass term of the Higgs potential

$$V_{\text{Higgs}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_{12}^2 H_1 H_2 + |\mu|^2 (|H_1|^2 + |H_2|^2) + \cdots \tag{4.82}$$

where the ellipses denote quartic terms fixed by supersymmetry and gauge invariance [93]. m_1^2, m_2^2, and m_{12}^2 are soft-breaking parameters that may attain negative values thus driving electroweak symmetry breaking. By suitable phase redefinitions of H_1 and H_2, one can restrict to real Higgs v.e.v.s and introduce the parameter, $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$.

Among the remaining soft-breaking parameters one has gaugino mass terms and squark and slepton masses,

$$-\mathscr{L}_{\text{mass}} = \frac{1}{2} \sum_{i=1,2,3} M_i \bar{\lambda}_i \lambda_i + \sum_{S=Q,U,D,L,E} \tilde{S}^\dagger \mathbf{M}_{\tilde{S}}^2 \tilde{S}, \tag{4.83}$$

where λ_i are the gaugino (Majorana) spinors, with i labelling the corresponding gauge group, $U(1)$, $SU(2)$ or $SU(3)$. Each gaugino mass M_i can be complex. The second sum spans all the squarks and sleptons and contains five Hermitian 3×3 mass matrices in flavour space. Finally, the soft-breaking terms also include three-boson couplings allowed by gauge invariance, such as $\tilde{Q} H_2 \mathbf{A}_u \tilde{U}$, that are called A-terms and are parametrized by three arbitrary complex matrices $\mathbf{A}_u$, $\mathbf{A}_d$, and $\mathbf{A}_e$. In the construction above we have limited the discussion to the R-parity conserving case, which only allows an *even* number of superpartners in each physical vertex, and is imposed to reduce problems with baryon number violation. Even with this restriction, if we count all the free parameters in this model we find a huge number, of $\mathcal{O}(100)$, with a few dozen new CP-violating phases! Truncating this number is fully justified only within the context of a fully specified supersymmetry-breaking mechanism, which may then enforce additional symmetries and relations among parameters.

Without going into the details of the dynamics behind SUSY breaking, it will be enough for our purposes to simply assume that the following, very restrictive, conditions are fulfiled:

$$\begin{aligned} \mathbf{M}_S^2 &= m_S^2 \mathbf{1}; \quad \text{for } S = Q, U, D, L, E, \quad \text{``degeneracy''} \\ \mathbf{A}_i &= A_i \mathbf{Y}_i; \quad \text{for } i = u, d, e, \quad \text{``proportionality.''} \end{aligned} \tag{4.84}$$

Strictly speaking, such conditions can only be imposed at a specific normalization point above the weak scale, as the renormalization group evolution of the MSSM parameters will modify these relations. Moreover, these conditions can only be imposed with limited precision at this scale due to threshold effects. Nonetheless, such a restrictive flavour universality ansatz in the scalar mass sector, and proportionality of the trilinear soft-breaking terms to the Yukawa matrices has the utility that it greatly reduces the number of independent soft-breaking parameters. Even so, a significant number of CP-violating phases remain

$$\mathrm{Arg}\,(\mu M_i m_{12}^{2*}); \quad \mathrm{Arg}\,(A_f M_i^*); \quad \mathrm{Arg}\,(M_i M_j^*), \quad \mathrm{Arg}\,(A_f A_{f'}^*). \tag{4.85}$$

Going to an even more restrictive framework, by assuming a common phase for the gaugino masses and another common phase for A_i reduces the number of independent *CP*-violating parameters to two. Using phase redefinitions, one can choose the phase of the gaugino mass to be zero, and use $\theta_A = \mathrm{Arg}(A)$ and $\theta_\mu = \mathrm{Arg}(\mu)$ as the basis for parametrizing *CP*-violation.

It has been known for over 20 years that even in the absence of new flavour physics, large EDMs can be induced at the one-loop level within a single generation [94,95]. Indeed, one would anticipate large EDMs as both of the reasons that rendered $d_i(\delta_{KM})$ very small, namely high loop order and also mixing angle/Yukawa coupling suppression, are not present for EDMs induced by the phases of the soft-breaking parameters.

Fig. 7 exhibits examples of one-loop diagrams at the supersymmetric threshold that generate non-zero contributions to the *CP*-odd Lagrangians (2.14) and (2.16). If we leave aside the problematic *s*-quark CEDM, then at one loop we can concentrate on diagrams involving just the first generation of quarks and leptons. Within the parametrization described above, the phases residing in μ and A permeate the squark, selectron, chargino, and neutralino spectrum, which in the mass eigenstate basis translates into complex phases in the quark–squark–gluino and fermion–sfermion–chargino(neutralino) vertices. To make this explicit, for a moment let us truncate the flavour space to one generation and write down the expression for the 2×2 *d*-squark mass matrix at the electroweak scale in the basis of $\tilde{d}_L$ and $\tilde{d}_R$

$$M_{\tilde{d}}^2 = \begin{pmatrix} m_Q^2 + \mathrm{O}(v^2) & -m_d(\mu \tan\beta + A_d^*) \\ -m_d(\mu^* \tan\beta + A_d) & m_D^2 + \mathrm{O}(v^2) \end{pmatrix}, \tag{4.86}$$

where we further assume that the soft masses m_Q^2 and m_D^2 are large relative to the weak scale, and thus we can ignore subleading $\mathrm{O}(v^2)$ corrections to the diagonal entries. Similar expressions can be written for the selectron mass matrix with the obvious substitutions in (4.86), and for the *u* squark, where in addition one has to exchange $\tan\beta$ by $\cot\beta$. In the generic case of three generations, M^2 becomes a 6×6 matrix with 3×3 blocks which are traditionally called M_{LL}^2, M_{LR}^2, M_{RL}^2, and M_{RR}^2. For our purposes, the crucial terms in (4.86) are the off-diagonal components
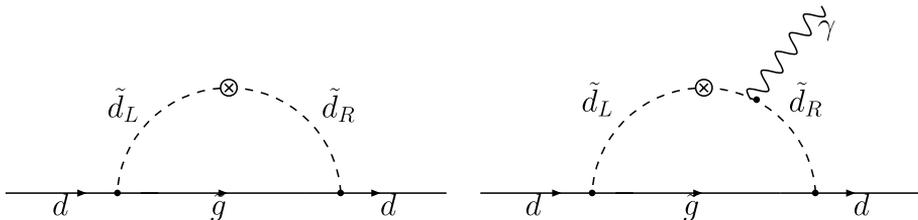

Fig. 7. One-loop SUSY threshold corrections in the down quark sector induced by a gluino–squark loop. On the left, a threshold correction generating $\mathrm{Im}(m_d)$, while on the right the analogous diagram for the EDM. The *CP*-violating source enters via the highlighted vertex, squark-mixing in the present case.

$$(M_{\tilde{d}}^2)_{LR} = -m_d(\mu \tan \beta + A_d^*) \tag{4.87}$$

which contain the *CP*-odd phases. By virtue of being proportional to the small mass m_d, such a term can be treated as a perturbation and accounted for by an explicit mass insertion on the squark line, as in Fig. 7. Note that the natural range for $\tan \beta$, in the interval between 1 and 60, allows for a significant enhancement of the μ-dependent term in (4.86). The phase of μ also modifies the spectrum of charginos and neutralinos, the mass eigenstates of the superpartners of the Z, W, γ and Higgs bosons.

With this notation in hand we see that, for example, the squark–gluino loop diagram generates an imaginary d-quark mass correction that contributes to $\Delta\bar{\theta}_{\text{rad}}$,

$$\text{Im}\, m_d = -m_d \frac{\alpha_s}{3\pi} \frac{M_3(\mu \tan \beta \sin \theta_\mu - A_d \sin \theta_A)}{M_Q^2} I(M_3, m_Q, m_D). \tag{4.88}$$

The loop function I is normalized in such a way that $I(m,m,m) = 1$; its exact form (see e.g. [81,96]) is not important for our discussion. The ratio $\text{Im}(m_d)/m_d$, along with contributions from other quark flavours, represents a one-loop renormalization of the θ-term. It is important to observe that it only depends on the SUSY mass ratio and thus does not decouple if A, μ, M_3, and $m_{Q(D)}$ are pushed far above the electroweak scale. Applying the bound on the θ-term to the combined tree level and one-loop results (4.88), with degenerate SUSY mass parameters as above, we find $|\bar{\theta}_{\text{tree}} + 10^{-2}\delta_{CP}| < 10^{-9}$, where δ_{CP} is a linear combination of $\sin \theta_\mu$ and $\sin \theta_A$ with $\mathcal{O}(1)$ coefficients. If there is no axion and $\bar{\theta}_{\text{tree}}$ vanishes instead by symmetry arguments, it follows that the phases of the soft-breaking parameters must be tuned to within a factor of 10^{-7} in order to satisfy the EDM bounds. Therefore, an incredibly tight constraint on the phases of the SUSY soft-breaking parameters can be obtained in models which invoke high-scale symmetries to resolve the strong *CP* problem.

However, if the PQ symmetry removes the θ-term, such radiative corrections to $\bar{\theta}$ have no physical consequences, and the residual EDMs are determined by higher-dimensional operators. The relevant expressions for the one-loop-induced d_i and $\tilde{d}_i$ contributions can be found in, e.g., [96]. Here we would just like to demonstrate the main point implied by these SUSY EDM calculations in a simplistic model in which *all* soft-breaking parameters are taken equal to a unique scale M_{SUSY} at the electroweak scale, i.e., $M_i = m_Q = m_D = \cdots = |\mu| = |A_i| = M_{\text{SUSY}}$. Working at leading order in v^2/M_{SUSY}^2, we can then present the following compact results for all dimension-five operators (with $q = d,u$),

$$\frac{d_e}{e\kappa_e} = \frac{g_1^2}{12} \sin \theta_A + \left(\frac{5g_2^2}{24} + \frac{g_1^2}{24}\right) \sin \theta_\mu \tan \beta,$$

$$\frac{d_q}{e_q\kappa_q} = \frac{2g_3^2}{9}\left(\sin \theta_\mu [\tan \beta]^{\pm 1} - \sin \theta_A\right) + \text{O}(g_2^2, g_1^2), \tag{4.89}$$

$$\frac{\tilde{d}_q}{\kappa_q} = \frac{5g_3^2}{18}\left(\sin \theta_\mu [\tan \beta]^{\pm 1} - \sin \theta_A\right) + \text{O}(g_2^2, g_1^2).$$

The notation $[\tan \beta]^{\pm 1}$ implies that one uses the plus(minus) sign for $d(u)$ quarks, g_i are the gauge couplings, and $e_u = 2e/3$, $e_d = -e/3$. For the quarks we quoted the ex-

plicit result only for the gluino–squark diagram that dominates in this limit. All these contributions to d_i are proportional to κ_i, a universal combination corresponding to the generic dipole size

$$\kappa_i = \frac{m_i}{16\pi^2 M_{\text{SUSY}}^2} = 1.3 \times 10^{-25}\ \text{cm} \times \frac{m_i}{1}\ \text{MeV} \left(\frac{1\ \text{TeV}}{M_{\text{SUSY}}}\right)^2, \qquad (4.90)$$

which varies by a factor of a few for $i = e, d, u$ depending on the value of the fermion mass. The perturbative nature of the MSSM provides a loop suppression factor in (4.90) so that κ_i is about two orders of magnitude smaller than the estimate (4.80). Correspondingly, the reach of the current EDM constraints in SUSY models cannot exceed the scale of a few TeV.

In (4.90) the quark masses should be normalized at the high scale, M_{SUSY}. To make the explicit connection with the dipole operators in (2.16), the results of Eq. (4.89) should be evolved down to the low-energy normalization point of 1 GeV using the relevant anomalous dimensions (see e.g. [81]). Plugging these results into the expressions for d_n, d_{Tl}, and d_{Hg} and comparing them to the current experimental bounds, we arrive at a set of constraints on θ_A and θ_μ depending on M_{SUSY} and $\tan\beta$. In Fig. 8, we plot these constraints in the (θ_μ, θ_A)-plane for $M_{\text{SUSY}} = 500$ GeV and $\tan\beta = 3$. The region allowed by the EDM constraints is at the intersection of all three bands around $\theta_A = \theta_\mu = 0$. One can observe that the combination of all three constraints strengthens the bounds on the phases, and protects against the accidental cancellation of large phases that can occur within one particular observable. The uncertainty in the QCD calculations of $\bar{g}_{\pi NN}^{(1)}$ and the nuclear calculation of $S(g_{\pi NN}^{(1)})$ discussed earlier may affect the width of the d_{Hg} constraint band, but do not change its slope on the (θ_μ, θ_A) plane.

Before we review the most common approaches to address the "overproduction" of EDMs in supersymmetric models, for completeness, we will briefly discuss some of the additional contributions which become important when $\tan\beta$ is large, a regime favoured for consistency of the MSSM Higgs sector with the final LEP results [97].
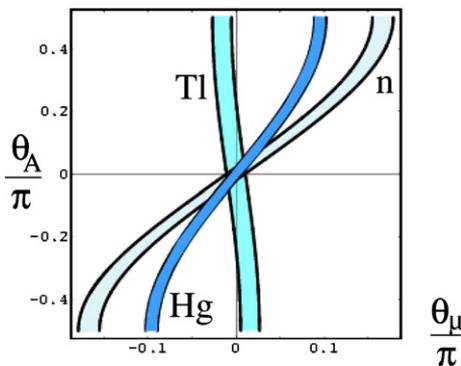

Fig. 8. The combination of the three most sensitive EDM constraints, d_n, d_{Tl}, and d_{Hg}, for $M_{\text{SUSY}} = 500$ GeV, and $\tan\beta = 3$. The region allowed by EDM constraints is at the intersection of all three bands around $\theta_A = \theta_\mu = 0$.

One simple observation is that the EDMs of down quarks and electrons, induced by θ_μ at one loop, grow linearly with $\tan\beta$ (4.89). However, at the two-loop level, there are additional contributions from the phase of the A-parameter which may also be $\tan\beta$-enhanced [98]. A typical representative of the two-loop family is presented in Fig. 9. At large $\tan\beta$ the additional loop factor can be overcome, and these two-loop effects have to be taken into account alongside the one-loop contributions in (4.89). For example, the stop-loop contribution to the electron EDM in the same limit of a large universal SUSY mass is given by

$$d_e^{\text{two loop}} = -e\kappa_e \frac{\alpha Y_t^2}{9\pi} \ln\left[\frac{M_{\text{SUSY}}^2}{m_A^2}\right] \sin(\theta_A + \theta_\mu)\tan\beta, \tag{4.91}$$

where m_A is the mass of the pseudo-scalar Higgs boson, that we took to be smaller than M_{SUSY}, $Y_t \simeq 1$ is the top quark Yukawa coupling in the SM, and $\kappa_e \simeq 0.6 \times 10^{-25}$ cm. For very large values of $\tan\beta$ additional contributions from sbottom and stau loops, which are enhanced by higher powers of $\tan\beta$, also have to be taken into account [81,98].

Finally, the second, and in some sense more profound change, is that at large $\tan\beta$, the observable EDMs of neutrons and heavy atoms receive significant contributions not only from the EDMs of the constituent particles, e.g., d_e and d_q, but also from CP-odd four-fermion operators [99]. The relevant Higgs-exchange diagram is shown in Fig. 9. The CP-violation in the Higgs-fermion vertex originates from the CP-odd correction to the fermion mass operator in Fig. 7. These diagrams, since they are induced by Higgs exchange, receive an even more significant enhancement by $(\tan\beta)^3$. In the same approximation as before, the value of the thallium EDM induced by this Higgs-exchange mechanism, and normalized to the current experimental limit, is given by

$$\frac{d_{\text{Tl}}}{[d_{\text{Tl}}]_{\text{exp}}} \simeq \frac{\tan^3\beta}{330}\left(\frac{100\text{ GeV}}{m_A}\right)^2\left[\sin\theta_\mu + 0.04\sin(\theta_\mu + \theta_A)\right]. \tag{4.92}$$
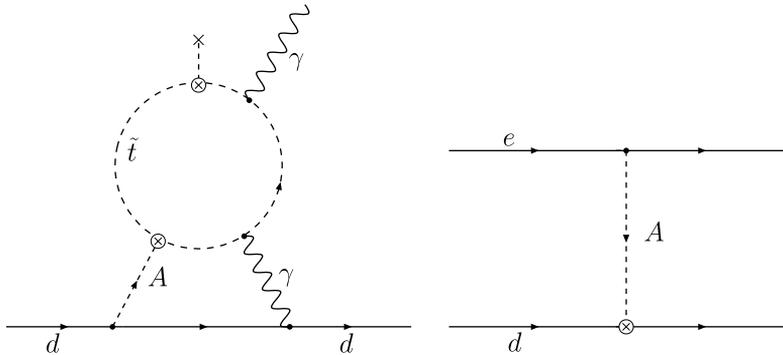

Fig. 9. Additional corrections to the EDMs. On the left two-loop Barr–Zee-type graphs mediated by a stop-loop and a pseudo-scalar Higgs, while on the right we have a Higgs-mediated electron–quark interaction C_{de} with CP-violation at the Higgs-quark vertex. There is a second diagram with CP-violation at the Higgs-electron vertex mediated by H.

Notice that this result *does not* scale to zero as $M_{\text{SUSY}} \to \infty$. Although just an $O(10^{-3}–10^{-2})$ correction for $\tan\beta \sim O(1)$, these Higgs-exchange contributions become very large for $\tan\beta \sim O(50)$ [99,100,81] (see also [101]).

4.2.1. The SUSY CP problem

Fig. 8 exemplifies the so-called SUSY *CP* problem: either the *CP*-violating phases are small, or the scale of the soft-breaking masses is significantly larger than 1 TeV, or schematically

$$\delta_{CP} \times \left(\frac{1\text{ TeV}}{M_{\text{SUSY}}}\right)^2 < 1. \tag{4.93}$$

The need to provide a plausible explanation to the SUSY *CP* problem has spawned a sizable literature, and the following modifications to the SUSY spectrum have been discussed.

- *Heavy superpartners*. If the masses of the supersymmetric partners exhibit certain hierarchy patterns the SUSY *CP* problem can be alleviated. One of the more actively discussed possibilities is an inverted hierarchy among the slepton and squark masses, i.e., with the squarks of the first two generations being much heavier than the stops, sbottoms, and staus, i.e., $(\mathbf{M}_S^2)_{ij} \gg (\mathbf{M}_S^2)_{i3}, (\mathbf{M}_S^2)_{33}$, where $i,j = 1,2$ is the generation index [102]. It is preferable to have masses of the third-generation sfermions under the TeV scale because they enter into radiative corrections to the Higgs potential, and making them too heavy would re-introduce the fine-tuning of the Higgs mass whose resolution was one of the primary motivations for weak-scale SUSY. Such a framework suppresses the one-loop EDMs which become immeasurably small if the scale of the u and d squarks is pushed all the way to ~ 50 TeV, as suggested by the absence of SUSY contributions in $\Delta m_{K(B)}$. This does not mean, however, that the EDMs in such models become comparable to $d_i(\delta_{\text{KM}})$. Indeed, the two-loop contributions to d_i and w involving the third-generation sfermions are not small in this framework, and indeed are at (or sometimes above) the level of current experimental sensitivity. Also, this means of suppressing the EDMs would not necessarily work in the large $\tan\beta$ regime where Higgs exchange may induce a large value for C_S that is not as sensitive to M_{SUSY} as the EDM operators. We note that future improvements in experimental precision will allow a stringent probe of such scenarios.
- *Small phases*. A rather obvious possibility for suppressing EDMs is the assumption of an exact (or approximate) *CP* symmetry of the soft-breaking sector. This is essentially a "model-building" option, and various ways of avoiding the SUSY flavour and *CP* problem in this way have been suggested in the past 15 years [103–105]. The idea of using low-energy supersymmetry breaking looks especially appealing, as it can also help in constructing an axion-less solution to the strong *CP* problem [48]. If the *CP*-odd phases in the soft-breaking sector are exactly zero and the conditions (4.84) are imposed *exactly* at the unification scale as a constraint on the high-scale model, we can ask about the scale of EDMs induced

by SUSY diagrams due purely to δ_{KM}. Since such an MSSM framework would possess the same flavour properties as the SM, one expects proportionality to the same *CP*-odd invariant combination of mixing angles, namely J_{CP}, and suppression by differences of Yukawa couplings [95]. Then it is easy to understand that the superpartner contributions to the down quark (C)EDM will necessarily be suppressed by the equivalent of $\delta_{CP} \sim JY_c^2 \sim 10^{-9}$, which is again six to seven orders of magnitude below current experimental capabilities, and thus not significantly larger than the EDMs induced in the SM [106].

- *Accidental cancellations*. Another possibility entertained in recent years [107] is the partial or complete cancellation between the contributions of several *CP*-odd sources to physical observables, thus allowing for $\delta_{CP} \sim O(1)$ with $M_{SUSY} < 1$ TeV. Since the number of potential *CP*-odd phases is large, and the superpartner mass spectrum is clearly unknown, one cannot exclude this possibility in principle. However, as we illustrated in Fig. 8, d_n, d_{Tl}, and d_{Hg} depend on different combinations of phases, and the possibility of such a cancellation looks improbable. A more thorough exploration of the MSSM parameter space in search of acceptable solutions that pass the EDM constraints was performed in [108,109], and did not identify any significant regions of cancellation.

- *No electroweak-scale supersymmetry*. Of course, there is always the possibility that other mechanisms (or no easily identifiable mechanism at all) lie behind the gauge hierarchy problem and the SM is a good effective theory valid up to energy scales *much larger* than 1 TeV. In this case there is no SUSY *CP* problem by definition. One of the recently suggested scenarios [110] exploits the possibility of a large number of electroweak vacua to invoke anthropic reasoning for selecting the "right" vacuum, thus side-stepping naturalness arguments for expecting new physics at the weak scale. Ref. [110] assumes that all the scalar superpartners are very heavy, but leaves gauginos and Higgsinos under a TeV, in order to preserve gauge-coupling unification and a dark matter candidate. This eliminates the one-loop induced EDMs, but leaves room for two-loop contributions [98,111] generated by chargino loops via a diagram similar to that shown in Fig. 9 with *A* replaced by the light Higgs. This scenario can also be probed with the predicted sensitivity of future EDM experiments.

4.3. SUSY EDMs from flavour physics

EDMs can also serve as a sensitive probe of non-minimal flavour physics. Indeed, the assumptions of proportionality and universality in the soft-breaking sector (4.84) at a given high-energy scale are highly idealized, and are not expected to hold with arbitrary precision. In this subsection, we would like to show that EDMs are sensitive to flavour-changing terms in the soft-breaking sector, and provide significant constraints on SUSY models with non-minimal flavour structure.

For concreteness, let us assume that (4.84) holds approximately, and the perturbations are small. Around the electroweak scale, and in a basis with diagonal quark mass matrices, the soft-breaking mass matrices can be approximated as

$$\mathbf{M}_S^2 = \text{diag}\left(m_{S11}^2, m_{S22}^2, m_{S33}^2\right) + \delta M_{Sij}^2, \tag{4.94}$$

where, as before, S labels the different squarks and sleptons, and $i \neq j$. Using this approximation, we can calculate the contributions to the relevant observables using δM_{Sij}^2 as a perturbation via insertions along the squark line, as in Fig. 10.

Calculating the gluino one-loop diagram in the approximation of equal SUSY mass scales, $(\mathbf{M}_S^2)_{ii} = M_i^2 = |\mu|^2 = M_{\text{SUSY}}^2$, we arrive at the following result for the d-quark EDM, and the imaginary correction to the d quark mass

$$\begin{aligned}
\text{Im}\, m_d &= -\delta_{131}^d \times m_b \frac{\alpha_s \tan \beta}{18\pi}, \\
d_d &= \delta_{131}^d \times e_d m_b \frac{\alpha_s \tan \beta}{45\pi M_{\text{SUSY}}^2},
\end{aligned} \tag{4.95}$$

where δ_{131}^d denotes the following CP-odd dimensionless combination

$$\delta_{131}^d = \frac{\text{Im}\left(\delta M_{Q13}^2 e^{i\theta_\mu} \delta M_{D31}^2\right)}{M_{\text{SUSY}}^4}. \tag{4.96}$$

In (4.95), for simplicity, we neglected the contributions from the A parameters, and retained only the mixing coefficients between the first and the third generations. There are two important points about Eq. (4.95) that we should emphasize here: δ_{131} can be non-zero even if $\theta_\mu = 0$, and both $\text{Im}(m_d)$ and d_d are enhanced relative to (4.88) and (4.89) by the large ratio $(m_b/m_d) \sim 10^3$, which can compensate the suppression associated with flavour violation. In the case of u quark operators, this enhancement factor is even larger, $m_t/m_u \sim 10^5$.

As we have seen in the previous subsection, renormalization of $\bar{\theta} \sim \text{Im}(m_q)/m_q$ can be very large, capable of producing bounds on δ_{131}^d and δ_{131}^u at the 10^{-9} level or better unless $\bar{\theta}$ is removed via PQ symmetry. In the latter case, using (4.95) and similar results in the lepton sector, one obtains the following sensitivity of EDMs to the above combination of flavour-changing transitions on electron, u, and d quark lines for $M_{\text{SUSY}} = 1$ TeV

$$\delta_{131}^e \sim 10^{-4}\text{–}10^{-3}; \quad \delta_{131}^u \sim 10^{-6}\text{–}10^{-5}; \quad \delta_{131}^d \sim 10^{-4}\text{–}10^{-3}. \tag{4.97}$$

Thus, EDMs independently provide very stringent constraints on the combined sources of flavour- and CP-violation in the soft-breaking sector. These constraints
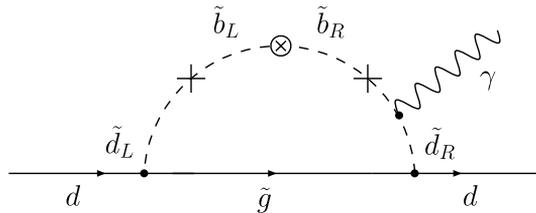

Fig. 10. Contribution of flavour-changing processes to the d-quark EDM. The middle insertion on the sfermion line corresponds to LR mixing proportional to m_b; the insertions on the left and on the right correspond to flavour transitions in the LL and RR squark mass sector.

are complementary to those coming from K and B meson physics and searches for lepton flavour violation. Note also that these calculations need to be modified in the large $\tan\beta$ regime to include Higgs-mediated contributions [112,99] which may dominate over the one-loop results for d_i (4.95).

It is important to realize that the apparent enhancement of EDMs in (4.95) by the ratios of heavy to light quark and lepton masses occurred because of the presence of flavour-changing terms in *both LL* and *RR* sectors of the squark–slepton mass matrices. Indeed, to make this point transparent we can write

$$\delta^d_{131} = \mathrm{Arg}\left[(\delta^d_{13})_{LL}(\delta^d_{33})_{LR}(\delta^d_{31})_{RR}\right], \tag{4.98}$$

in terms of "mass insertions" [113] $\delta^f_{ij} = M^2_{\tilde f ij}/m^2_{\tilde f}$, which, although the distinction is not crucial here, are usually defined in a slightly different basis to the one we have been using. $m^2_{\tilde f}$ denotes here the average sfermion mass-squared. The status of the LL and RR insertions is in general rather different, and particularly so within the MSSM where the latter are essentially absent.

To see this in more detail, we recall that flavour-changing terms in the LL sector are natural, as they are induced by renormalization group evolution of the soft-breaking parameters (see e.g. [114]) even if one assumes the conditions (4.84) at the unification scale. Starting from the universal boundary conditions (4.84) for all scalar masses, equal to m^2_0, and A parameters at some high-energy scale Λ_{UV}, one can obtain an expression for $\mathbf{M}^2_Q$ at a lower-energy scale Λ, which at one loop is given by

$$\mathbf{M}^2_Q = m^2_0 \mathbf{1} - \frac{3m^2_0 + A^2}{16\pi^2} \ln\left(\frac{\Lambda^2_{\mathrm{UV}}}{\Lambda^2}\right)\left(\mathbf{Y}^\dagger_u \mathbf{Y}_u + \mathbf{Y}^\dagger_d \mathbf{Y}_d\right) + \cdots \tag{4.99}$$

The ellipsis denotes "flavour-blind" contributions and also higher-order terms. Depending on the particular model of SUSY breaking Λ_{UV} can be anywhere between a few tens of TeV and the Planck scale. The presence of *both* up and down Yukawa matrices in (4.99) guarantees the appearance of flavour-changing contributions in the LL entries of the squark mass matrices. At the superpartner threshold, $\Lambda = M_{\mathrm{SUSY}}$, the flavour-changing terms in the down squark sector will evidently be

$$\delta M^2_{\tilde d ij} \simeq -\frac{3m^2_0 + A^2}{16\pi^2} \ln\left(\frac{\Lambda^2_{\mathrm{UV}}}{M^2_{\mathrm{SUSY}}}\right) Y^2_t V^*_{ti} V_{tj}, \tag{4.100}$$

where V is the CKM matrix, and Y_t is the top quark Yukawa coupling. If the scale Λ_{UV} is very high, i.e., comparable to the Planck or GUT scale, the logarithm is large and can entirely offset the loop factor. Therefore, the natural size of the 13 entry in the down squark LL sector is $\sim M^2_{\mathrm{SUSY}} V_{td} \simeq 0.01 M^2_{\mathrm{SUSY}}$. The situation in the RR sector is completely different. There the absence of any $\mathbf{Y}_u$-dependence in the RG equations for $\mathbf{M}^2_D$ forbids the generation of substantial flavour-changing transitions, unless the MSSM spectrum is modified above certain energies so that the RG equations for the right-handed squark masses acquire flavour dependence.

A number of SUSY scenarios have been proposed which describe plausible patterns of small deviations from (4.84), allowing for significant RR contributions. In models with $SO(10)$ unification and $\Lambda_{\mathrm{UV}} > \Lambda_{\mathrm{GUT}}$ the running of the soft-breaking

parameters extends *above* the unification scale, where the RG equations are modified by the presence of new field degrees of freedom. For $SO(10)$ GUTs this modification introduces significant flavour dependence in the RR sector of squark and slepton mass matrices [115], even if the restrictions (4.84) are imposed at the Planck scale. The resulting flavour-changing terms for down squarks δM_{Dij}^2 are of the same order of magnitude as in the LL sector, leading to the prediction $\delta_{131}^d \sim 10^{-4}$, which is right at the border-line of current experimental sensitivity (4.97) [116,117]. Similar effects may be generated by heavy sterile neutrinos. The light neutrino mass scale might, via the seesaw mechanism, be pointing to the existence of a new energy scale, $M_R \sim Y_\nu^2 v^2/(0.1$–$0.001\ \mathrm{eV})$ related to heavy sterile (or "right-handed") neutrinos. If Λ_{UV} is larger than M_R, the RG equations for sleptons will be modified above M_R with an effect similar to that above, namely a non-trivial flavour dependence will be imprinted on the slepton mass matrices. The importance of such an effect will depend on the size of the neutrino Yukawa couplings Y_ν and, with certain Yukawa patterns, an observable or nearly observable electron EDM might be induced [118]. Of course, if the scale of SUSY breaking is lower than M_R (or Λ_{GUT}) there are no significant consequences for EDMs unless one allows for other "diagonal" phases in this sector.

5. Conclusions and future directions

Recent years have seen significant progress in the experimental tests of *CP*-violation in the Standard Model. Experimental verification of direct *CP*-violation in Kaon decay, and in particular the spectacular measurements of *CP* asymmetries for neutral *B* meson decays at BaBar and Belle have provided solid confirmation of the overall validity of the Kobayashi–Maskawa mechanism. The current status of *CP*-violation in flavour-changing processes is such that (within errors) it does not necessitate the introduction of any additional *CP*-violating sources. At the same time, there is ample (experimental) room for the existence of new *CP*-violating physics to which the *K* and *B* meson data are not sensitive. This concerns, primarily, *CP*-violation in flavour-conserving channels. The existence of such new sources is hinted at, albeit indirectly, by the baryon asymmetry in the Universe. The search for *CP*-violation in flavour-conserving channels, and the search for EDMs in particular, should thus remain high on the priority list for particle physics. The strong suppression of EDMs induced purely by the Kobayashi–Maskawa phase, combined with prospects for improving the experimental sensitivity, places EDM searches at the forefront in probing *CP*-violating physics beyond the Standard Model.

Moreover, beyond their direct sensitivity, current (and future) null results for EDM searches also provide very powerful constraints on models for new physics. Indeed, as we have discussed, the sensitivity for example to *CP*-violation in the soft-breaking sector of SUSY models allows us to probe soft-breaking masses as large as a few TeV. In this indirect sense, EDMs are often sensitive to energy scales beyond the reach of future collider experiments, and play a central role in the full suite of precision tests of the Standard Model. The scales probed by EDMs and by the

constraints on flavour-changing neutral currents are not too dis-similar, and the gap may continue to narrow with future progress in EDM searches. This only heightens the tension between the observed *CP*-violation in the flavour-changing sector and the lack thereof in flavour-diagonal channels, of which the strong *CP* problem is the most manifest example. We seem compelled to question whether *CP* and flavour are as intrinsically linked in general as they are within the Kobayashi–Maskawa model? This is one aspect of what one might hope would be answered by a general "theory of flavour." EDMs will clearly continue to provide a crucial probe in tackling this question.

In this concluding section we would like to emphasize some directions on the experimental and theoretical front that are likely to bring future progress in establishing the nature of flavourless *CP*-violation at and above the electroweak scale.

5.1. Experimental developments

There are a number of developments in experimental techniques to search for EDMs which promise to narrow the gap between the current limits and the KM background in all of the EDM classes discussed in this review. In particular, novel techniques for storing neutrons in liquid helium, in progress at LANSCE in Los Alamos [119] and under investigation for an experiment at PSI, will help to improve the measurement of many fundamental parameters in neutron physics, including d_n. On another front, the suggestion that *CP*-violating effects can be significantly enhanced in exotic nuclei possessing an octopole moment [120] drives several experiments searching for EDMs of isotopes of Ra and Rn [121]. We note that such experiments should be pursued primarily because of their potential to discover *CP*-violation, as null results will not be as constraining as those from d_n or d_{Hg} due to large uncertainties in the calculations of nuclear matrix elements. In future measurements of paramagnetic EDMs, the resulting sensitivity to d_e and C_S will be significantly improved via the use of paramagnetic molecules, such as YbF and PbO, that can be polarized and thus allow a huge enhancement of the applied field [15,23]. The anticipated precision will allow probes of the electron EDM down to $10^{-30}e$ cm or below [15,23]. In this regard, we should mention an interesting alternative approach, which involves the measurement of the tiny electron EDM-induced magnetic flux when an electric field is applied to a particular garnet crystal. This project is also already in development at LANSCE [122].

Perhaps the most interesting proposal of recent years is the new approach to measure the EDMs of *charged* particles in storage rings. An initial proposal to measure the muon EDM at the $10^{-24}e$ cm level [123] has since evolved into the idea to measure the EDMs of ions and nuclei [124], and in particular the deuteron EDM at the $10^{-27}e$ cm level. Although clearly not a diamagnetic system, the deuteron EDM can be placed in the same category as it is primarily sensitive to the same nuclear scale *CP*-violating source, namely the *CP*-odd pion–nucleon couplings $\bar{g}_{\pi NN}$. More precisely, one finds [125–127],

$$d_D = (d_n + d_p) - (1.3 \pm 0.3 \text{ GeV}^{-1})e\bar{g}_{\pi NN}^{(1)}, \tag{5.101}$$

where the second term generically dominates, leading to a similar dependence on *CP*-odd sources as for mercury. However, in comparison to mercury, the deuteron EDM proposal has the significant advantage of requiring only straightforward nuclear calculations, since it is a weakly bound state, which leads to the relatively precise dependence on $\bar{g}_{\pi NN}^{(1)}$ in (5.101). Using the result (3.71) for $\bar{g}_{\pi NN}^{(1)}$, and accounting for the subleading corrections, one obtains [126]

$$d_D \simeq 6_{-3}^{+11} e(\tilde{d}_d - \tilde{d}_d) + \mathcal{O}(\tilde{d}_d + \tilde{d}_u, d_u, d_d). \tag{5.102}$$

Thus a measurement of d_D at the $\sim 10^{-27} e$ cm level would correspond to a sensitivity to light quark CEDMs at the level of 10^{-28} cm [126], which is at least two orders of magnitude better than the current limits from $d_{\rm Hg}$. Above all else, these proposals to measure EDMs in storage rings deserve special considerations as they depart from the dominant philosophy that EDM measurements demand the study of neutral objects in parallel electric and magnetic fields.

To place this activity in context, we should bear in mind that probably the most important single question for particle physics—the origin of electroweak symmetry breaking—will be subjected to serious experimental scrutiny with the Large Hadron Collider coming on-line within a few years. Besides the discovery of the Higgs boson(s), it may provide an answer to the gauge hierarchy problem, and indeed uncover a plethora of new particles or resonances above the electroweak scale. EDM experiments, which might of course discover new physics before the LHC switches on, can subsequently play a complementary role in providing constraints on (or signatures of) *CP*-violating couplings (e.g., in the Higgs sector of the MSSM). The projected level of sensitivity in coming years will be more than competitive in this regard with collider probes. Moreover, strangely enough, the absence of new physics (beyond the Higgs—or whatever might play this role) at the TeV scale would not remove motivations for EDM searches. Indeed, as we argued in this review, EDMs are sensitive to *CP*-violation at multi-TeV scales, and thus represent one of the few classes of low-energy precision measurements that are sensitive to such high-energy scales.

Another important experimental direction relevant to *CP*-violating physics is the search for axions. As we reviewed, one of the more natural resolutions of the strong *CP* problem predicts the existence of a light pseudo-scalar particle, the axion. The developments of recent years in cosmology have lent considerable weight to the presence of a non-baryonic cold dark matter component of the energy density in the Universe. Although the popularity of supersymmetric models continues to focus attention on the lightest supersymmetric particle (or LSP) as a natural dark matter candidate at the weak scale, axions with a coupling f_a^{-1} below its astrophysical bound in fact still represent a viable alternative, thus providing additional motivation for the continuation of axion searches.

5.2. Theoretical developments

On the theoretical side, beyond questions of the precise generation mechanisms of *CP*-odd sources in specific new physics models, it is clear that the primary limitation on the full application of the observational bounds arises through the limited preci-

sion of QCD and nuclear calculations. Perhaps the most afflicted quantity at present is the *CP*-odd pion–nucleon constant, as induced in particular by the CEDMs of light quarks. As we have discussed, this is a fundamental parameter controlling the level of the constraints imposed by diamagnetic atoms, which can currently be calculated only to limited precision due to large cancellations in the relevant nucleon matrix elements. Another important issue concerns the strange quark CEDM contribution both to $\bar{g}_{\pi NN}$ and the neutron EDM d_n, and whether or not it is underestimated in the leading-order sum-rules analysis [59,128]. It would clearly be worthwhile to revisit these aspects. However, it seems likely that significant quantitative progress will come only from ab initio lattice calculations. This is a very challenging task, since a viable lattice calculation would necessarily have to respect chiral symmetry both at the level of quarks and gluons and also among the observable matrix elements between the hadronic states, since this is the underlying reason for the suppression of $d_n(\bar{\theta})$ by m_* and the partial suppression of $d_n(\tilde{d}_q)$. To that end, it will be important to implement a calculation displaying all the required symmetries, and in this sense $d_n(\bar{\theta})$ would be a good place to start, as many features of the answer, such as the dependence on m_* and on $\bar{\theta} = \theta + \arg \det M_q$, are enforced by symmetry allowing for independent checks of the calculation. On the nuclear side, we noted that recent reanalyses of the Schiff moment indicate that various many-body effects, e.g., polarization, can be significant and thus further progress in this area would assist significantly in improving the quality of constraints on $\bar{g}_{\pi NN}$ in different isospin channels. It will also be important, in guiding future experimental ideas, to clarify the size of the enhancement of *CP*-violation in exotic nuclei with octupole deformations.

In conclusion, the limits on flavour-diagonal *CP*-violation produced by the null results of existing EDM searches already provide strong constraints on new physics at and above the electroweak scale. Developments in coming years promise to provide us with a wealth of new information about the nature of *CP*-violation and TeV-scale physics, complementary to studies of electroweak symmetry breaking at colliders and flavour studies with K and B mesons.

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