Fermilab-TM-2202

Tevatron Separator Beam Impedance

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Introduction
This paper is a continuation of the work discussed in TM-2194. Measured longitudinal and transverse impedance of a Tevatron separator are presented.

Longitudinal impedance
The geometry of a separator is similar to an unterminated stripline which has a family of resonant frequencies. The effect of these modes on the beam (or stretched wire) may be negligible depending on the beam flight time and the period of the mode. The longitudinal fields will be localized to the plate ends and their combined effect on a particle that passes through will determine the beam impedance. This "transit time effect" suggests the integral of the kick can be small.

Stripline resonant modes:

$$\omega_n = n\pi\frac{c}{l} \qquad l = \textit{plate length},\ c = \textit{propagation velocity}$$

$$n \textit{ odd for odd modes}$$

$$n \textit{ even for even modes}$$

The beam flight time from one end of the plate to the other is:

$$\Delta t = \frac{l}{\upsilon} \qquad \upsilon = \textit{beam velocity}$$

The total longitudinal voltage is the sum from both ends:

$$\begin{aligned} V_{total} &= V\cos\omega\left(t+\frac{l}{2\upsilon}\right) \pm V\cos\omega\left(t-\frac{l}{2\upsilon}\right) \qquad (+ \textit{ for n odd}, \quad - \textit{ for n even}) \\ &= 2V\cos\omega t\cos\omega\frac{l}{2\upsilon} \qquad \textit{for n odd} \\ &= 2V\cos\omega t\sin\omega\frac{l}{2\upsilon} \qquad \textit{for n even} \\ &= 0 \qquad \textit{for } \omega = n\pi\frac{c}{l} \textit{ and } \upsilon = c \end{aligned}$$

Provided the resonant frequencies of the modes are given by $\omega=n\pi\upsilon/l$ and the velocity of the beam is υ, the separator will have no longitudinal impedance.

Effect of power cables
The separator power cables attach about 12 ½" from one end of the plates through a 50Ω resistor. Typically the cables are a few meters long and terminate into a 1MΩ impedance. The presence of

the cables extend the electrical length of the plates and shifts the frequency of the modes. The 101" plates are ½ wavelength long at 58MHz. The plate and 80" power supply cable (v/c=.48) are ½ wavelength at about 23MHz.

$$\omega_n = \frac{n\pi}{\frac{l_{plate}}{\upsilon_{plate}} + \frac{l_{cable}}{\upsilon_{cable}}}$$

This shift in frequency results in a measurable longitudinal and transverse impedance for the separators. However, the presence of the series resistor substantially reduces this impedance.

Measured Longitudinal impedance

A .010" diameter tin plated copper wire was stretched through the separator to form a 330Ω TEM coaxial transmission line. Each end was matched to 50Ω with resistive L pads. A network analyzer was used to measure the transmission (S21) through the wire from 30KHz to 500MHz under 4 conditions:

- with the power supply cables and series resistor
- with the power supply cables but without the 50Ω series resistor
- without the power supply cables

The longitudinal impedance is determined from the measurement of S21.

$$Z_L = 2Z_0 \ln(S_{21}) = 2Z_0 \frac{S_{21}[db]}{20\log(e)}$$

$$\frac{Z_L}{n} = Z_L \frac{\omega_0}{\omega} \qquad \omega_0 = 2\pi \ rotation \ frequency$$

The transverse impedance can be estimated from the longitudinal impedance.

$$Z_T = \frac{2c}{\omega b^2} Z_L \qquad where\ b\ is\ the\ radius\ of\ the\ structure$$

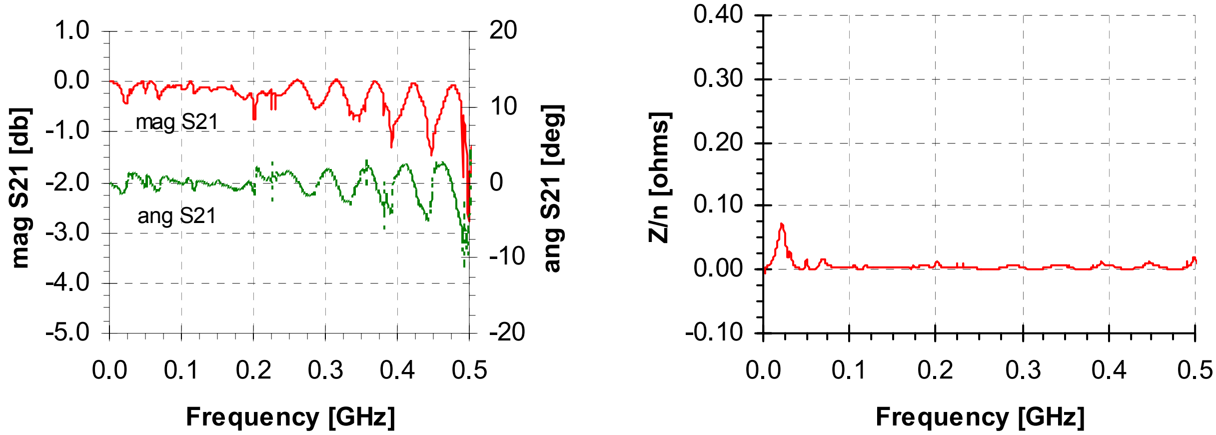
$$Z_T = \frac{2c}{\omega_0 b^2} \frac{Z_L}{n} \qquad \frac{2c}{\omega_0 b^2} = 3.100x10^6 \quad for\ b = 1'', \frac{\omega_0}{2\pi} = 47713Hz$$

The measurements have been corrected for the skin effect losses on the wire (10.6 db/100ft at 1GHz) and the wire delay (121" at v=c). This was done by measuring a 2 ⅜" ID beam pipe and using that data to normalize separator measurements.

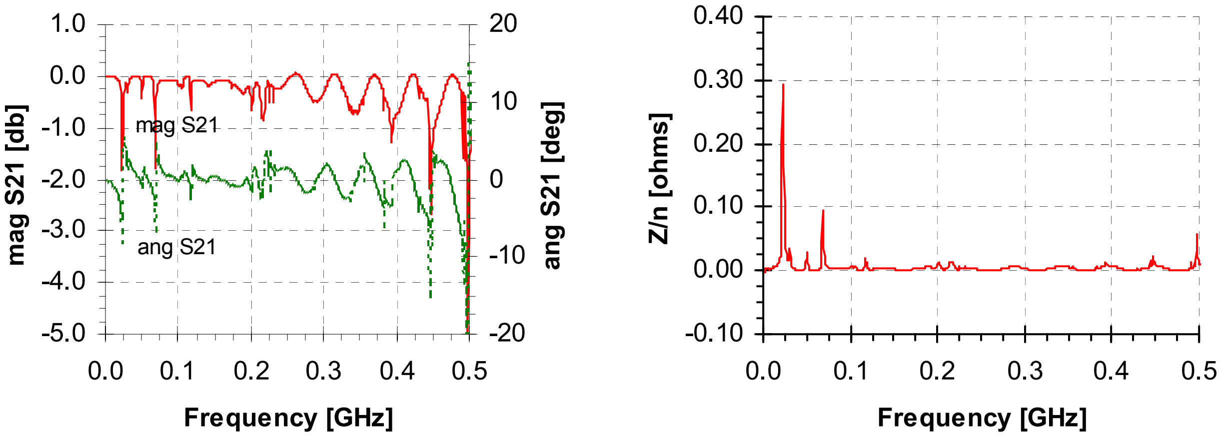
Without the power supply cables attached, no impedance is seen below 200 MHz. The mode at 203.8 MHz is not understood but is not present with the power supply cables. The distance from the end of the plate to the cable is about ¼ wavelength at this frequency.

With the cables attached, but without the damping resistor, impedance is seen at 22.5 and 67.8MHz. These frequencies correspond to where the cable and the plate are ½ and 1 ½ wavelengths long. These modes are substantially reduced with the series 50Ω damping resistor in the cable.

With PS cable and 50Ω resistor



PS cable without resistor



no PS cable

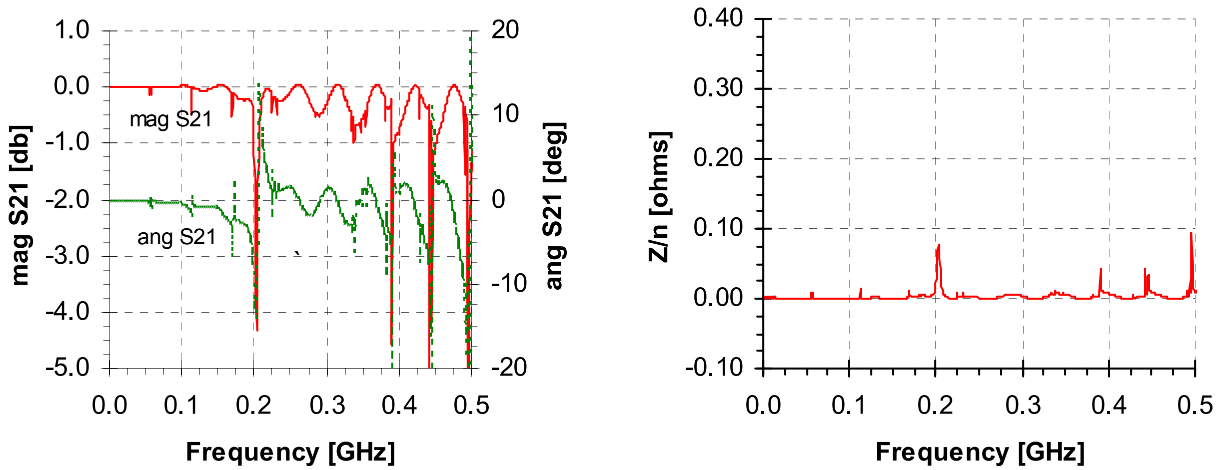


Figure 1. Longitudinal impedance.

With PS cable and 50Ω resistor

PS cable without resistor

no PS cable

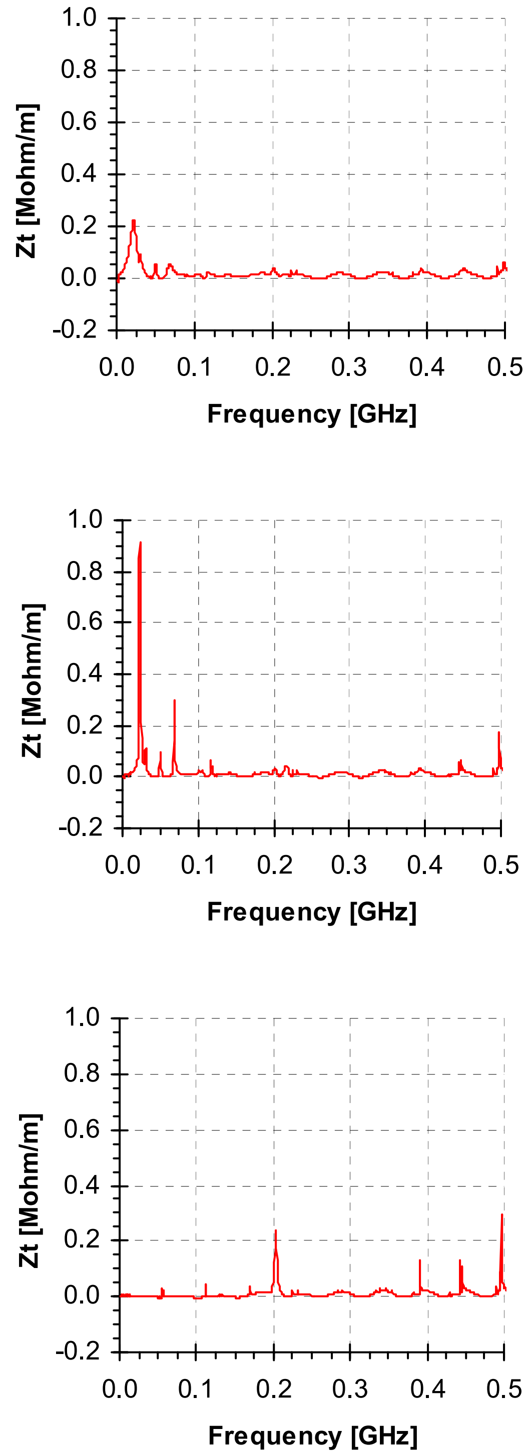


Figure 2. Transverse impedance estimated from measured longitudinal impedance.

Transverse impedance
Electric and magnetic fields can be induced between the plates referred to as differential modes. The modes occur at the same frequencies as the longitudinal modes, or when the plates are multiples of ½ wavelength long.

The total deflecting kick on the beam from the electric and magnetic fields is:

$$kick = P_x = \int_{t=t_0}^{t_0+\frac{l}{\upsilon}} F\, dt$$

$$F = q(E + \upsilon \times B) \qquad \upsilon = beam\ velocity \quad l = separator\ length$$

Let the electric and magnetic fields alternate along the plate by n half periods. The magnetic field lags behind the electric field by 90°. The fields reverse direction along the plates according to the mode number.

$$\omega_n = n\pi \frac{c}{l}$$

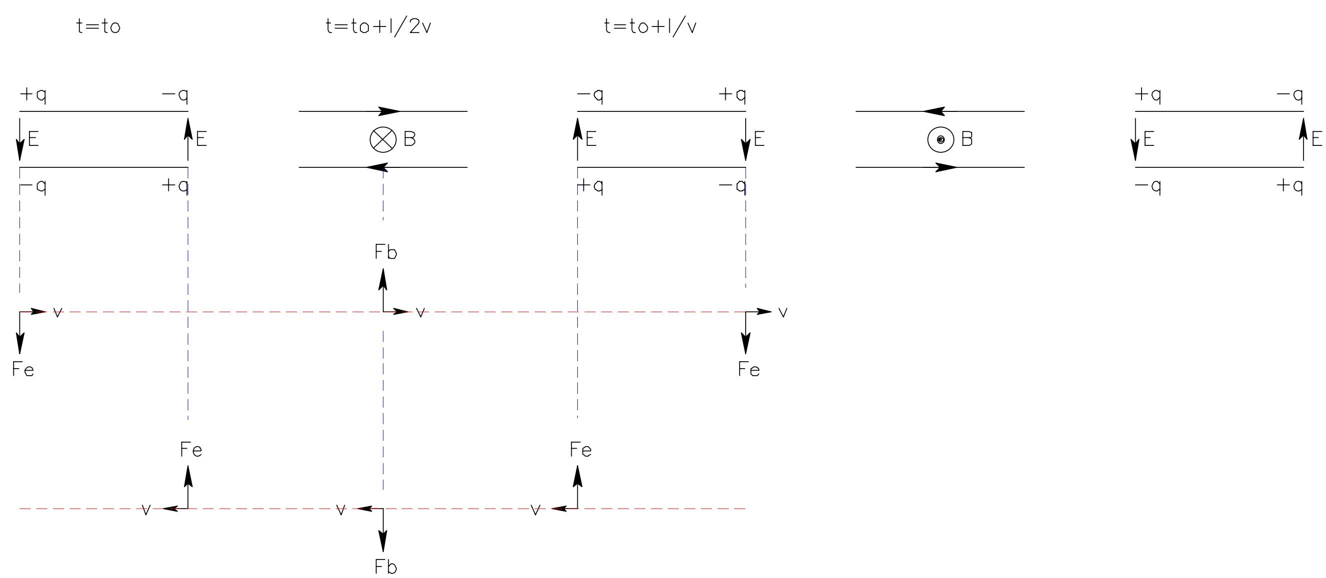


Figure 3. The electric and magnetic forces are in opposition for either direction of particle travel. The lowest frequency mode (n=1) is shown above. The particle travels through the plates in ½ period. The electric field has opposite direction at the ends. The charge moving from one end to the other generates the maximum magnetic field ¼ period after the peak voltage occurs.

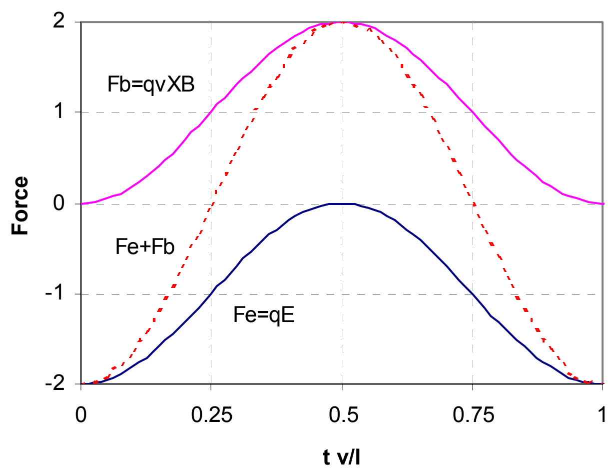


Figure 4. Forces exerted on a charged particle traveling from left to right for mode 1. The velocity is assumed to be the speed of light making Emax = υBmax.

For a charge traveling left to right:

$$E(x) = E_{\max} \cos \omega t \cos\left(n\pi \frac{z}{l} \right)$$

$$B(x) = B_{\max} \sin \omega t \sin\left(n\pi \frac{z}{l} \right)$$

$$\textit{fields seen by a particle moving at velocity } \upsilon \textit{ entering at } t = t_0 \quad (z = \upsilon(t - t_0))$$

$$E(t) = E_{\max} \cos \omega t \cos\left(n\pi \frac{\upsilon}{l}(t - t_0) \right) = \frac{E_{\max}}{2} \left\{ \cos\left(\omega t - n\pi \frac{\upsilon}{l}(t - t_0) \right) + \cos\left(\omega t + n\pi \frac{\upsilon}{l}(t - t_0) \right) \right\}$$

$$B(t) = B_{\max} \sin \omega t \sin\left(n\pi \frac{\upsilon}{l}(t - t_0) \right) = \frac{B_{\max}}{2} \left\{ \cos\left(\omega t - n\pi \frac{\upsilon}{l}(t - t_0) \right) - \cos\left(\omega t + n\pi \frac{\upsilon}{l}(t - t_0) \right) \right\}$$

$$F(t) = \frac{qE_{\max}}{2} \left\{ \cos\left(\omega t - n\pi \frac{\upsilon}{l}(t - t_0) \right) + \cos\left(\omega t + n\pi \frac{\upsilon}{l}(t - t_0) \right) \right\}$$
$$- \frac{q\upsilon B_{\max}}{2} \left\{ \cos\left(\omega t - n\pi \frac{\upsilon}{l}(t - t_0) \right) - \cos\left(\omega t + n\pi \frac{\upsilon}{l}(t - t_0) \right) \right\}$$

$$\textit{for } \omega = \omega_n = n\pi \frac{\upsilon}{l}, \upsilon = c, \textit{ and } E_{\max} = \upsilon B_{\max}$$

$$F(t) = qE_{\max} \cos\left(2n\pi \frac{\upsilon}{l} t - n\pi \frac{\upsilon}{l} t_0 \right)$$

$$\int_{t=t_0}^{t_0+\frac{l}{\upsilon}} F\,dt = qE_{\max}\left\{\frac{\sin\left(2n\pi\frac{\upsilon}{l}t - n\pi\frac{\upsilon}{l}t_0\right)}{2n\pi\frac{\upsilon}{l}}\right\}\Bigg|_{t=t_0}^{t_0+\frac{l}{\upsilon}}$$

$$= 0$$

Provided the resonant frequency of the modes is given by ω=nπυ/l, the velocity of the beam is υ, and that Emax = υBmax, the separator will have no transverse impedance. As can be seen in Figure 2, the average force as the particle travels through the separator will be zero.

Measured Transverse impedance

The beam impedance can be determined from the attenuation measured along two stretched wires placed in the separator and driven differentially. Two parallel .010" diameter tin plated copper wires separated by 1cm were stretched through the separator to form a 530Ω TEM balanced transmission line. Each end was matched to 100Ω with resistive L pads and combined with a 100Ω broad band 180 degree hybrid. A network analyzer was used to measure the transmission (S21) through the wire from 30KHz to 500MHz under 4 conditions:

With the wires in the plane of the plates
- with the power supply cables and resistor
- with the power supply cables without the 50Ω series resistor
- without the power supply cables

With the wire plane perpendicular to the plane of the plates
- with the power supply cables and resistor

The transverse impedance is determined from the S21 measurement.

$$Z_T = \frac{c}{\omega\Delta^2}2Z_0\ln(S_{21}) = \frac{c}{\omega\Delta^2}2Z_0\frac{S_{21}[db]}{20\log(e)} \qquad \Delta \textit{ is the wire separation}$$

An attempt was made to normalize the measurements using a similar length beam pipe. However, the characteristic impedance was slightly different, resulting in significant error at frequencies where the wires were multiple ½ wavelengths long. The best results were found by using the "off plane" data to normalize the measurements. (wire plane perpendicular to the plane of the plates).

Without the power supply cables attached, no impedance is seen below 200 MHz. Again, the mode at 199.4MHz is not understood but is not present with the power supply cables.

With the cables attached but without the damping resistor, impedance is seen at 22.5 and 67.8MHz. These frequencies correspond to where the cable and the plate are ½ and 1 ½ wavelengths long. The modes are substantially reduced with the series 50Ω damping resistor.

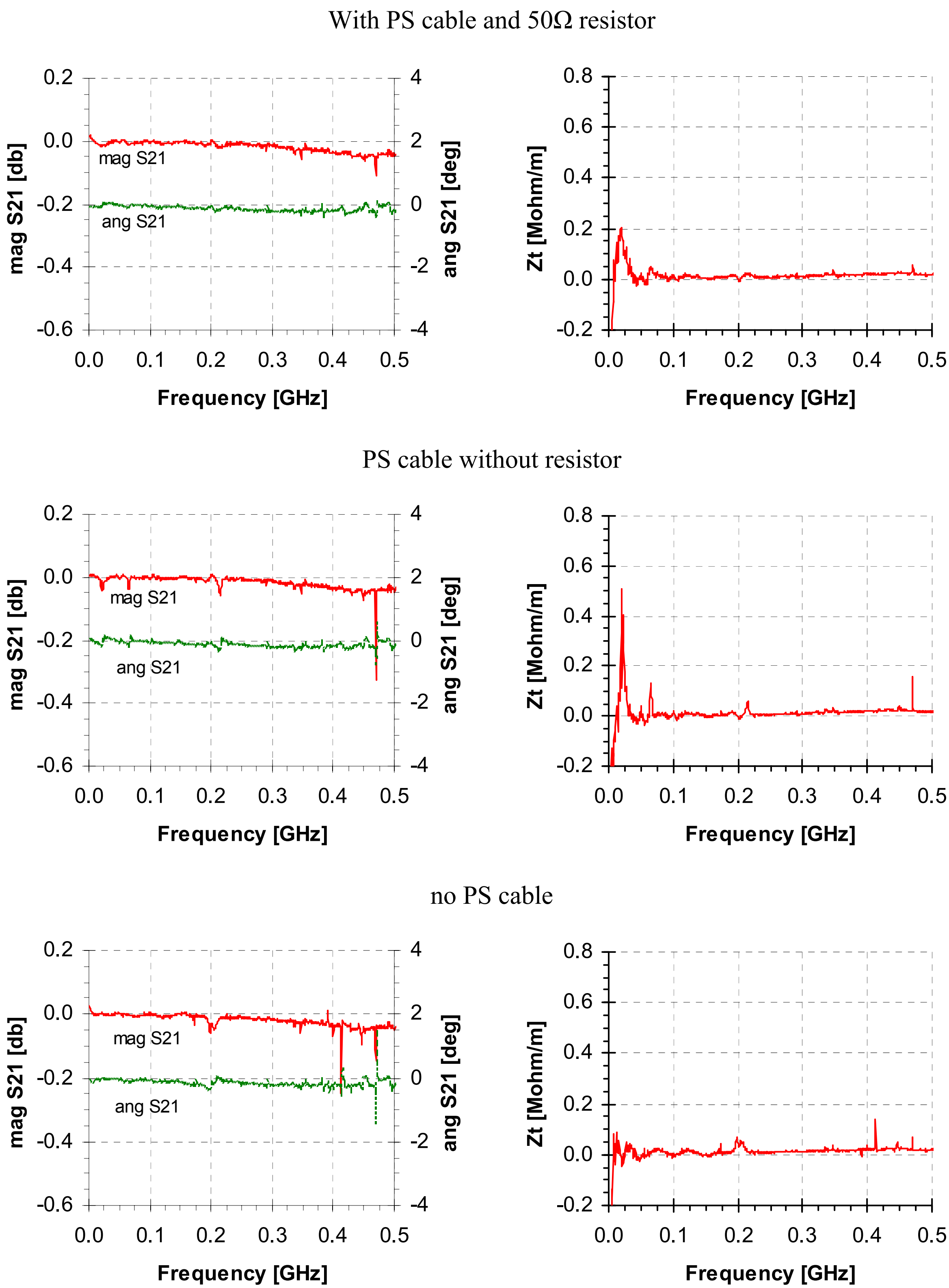


Figure 5. Transverse impedance as measured with a balanced or differential transmission line.

Discussion

Discontinuities along the device may introduce modes that are not harmonically related to the beam flight time and thus produce beam impedance. Such discontinuities can be caused by ceramic standoffs, cable connection points, or changes in the vacuum enclosure such as pump out or access ports and bellows. The geometry of the plate ends can also shift the frequency of the modes resulting in impedance. Such behavior may explain the dominant mode at 200 MHz seen without the cables attached.

The transverse impedance at 22.5MHz is small, about 0.2 MΩ/m. However, there are 24 separators installed in the tevatron. Provided the power supply cable lengths are the same on all separators, the total impedance would be 4.8 MΩ/m.

The measured attenuation on the stretched wire is small and the calculated beam impedance depends heavily on how the data is normalized. However, similar results were found on several independent measurements using different wire spacing. In addition, the agreement between the direct transverse measurements and the estimated transverse impedance from the longitudinal measurement is reassuring. I estimate these impedances are good to about ±25%.

It would be interesting to place a probe near the series 1 MΩ resistor to measure the amplitude of beam induced signals on the cables of one of the separators.

References:

1) Sacherer, F., Nassibian, G.; "Methods for Measuring Transverse Coupling Impedances in Circular Accelerators"; Nuc. Inst. And Meth., Vol. 159, pgs 21-27, 1979.
2) Panofsky, W.K.H., Wenzel, W.A.; "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", Rev. Sci. Instrum., Vol 27, pg. 967, 1956.
3) Crisp, J., Fellenz, B.; "Measured Longitudinal Beam Impedance of a Tevatron Separator"; Fermilab TM-2194; November 25, 2002.
4) Chao, A., Tigner, M. ; "Handbook of Accelerator Physics and Engineering"; World Scientific Publishing; 1999; pg. 570.
5) "Reference Data for Radio Engineers"; sixth edition; Howard W. Sams; 1984.

Drawings:

Electrode Support Assembly	2214-ME-261599
Overall Layout of Mounting Stands	2214-ME-276699

Miscellaneous Data:

Plate length	101"	
Plate gap	1.969"	
Plate width	7.8"	
ID of vacuum tank	13.5"	
Flange to flange length	120"	
Conflat vacuum flanges	6"	
Differential plate impedance	81Ω	estimated
Common mode plate impedance	42Ω	estimated
High Voltage cable	Dielectric Sciences 2134, 50Ω, 80" long, v/c=0.48 attaches 12.3" from one end of the plate	

Stretched Wire Data:

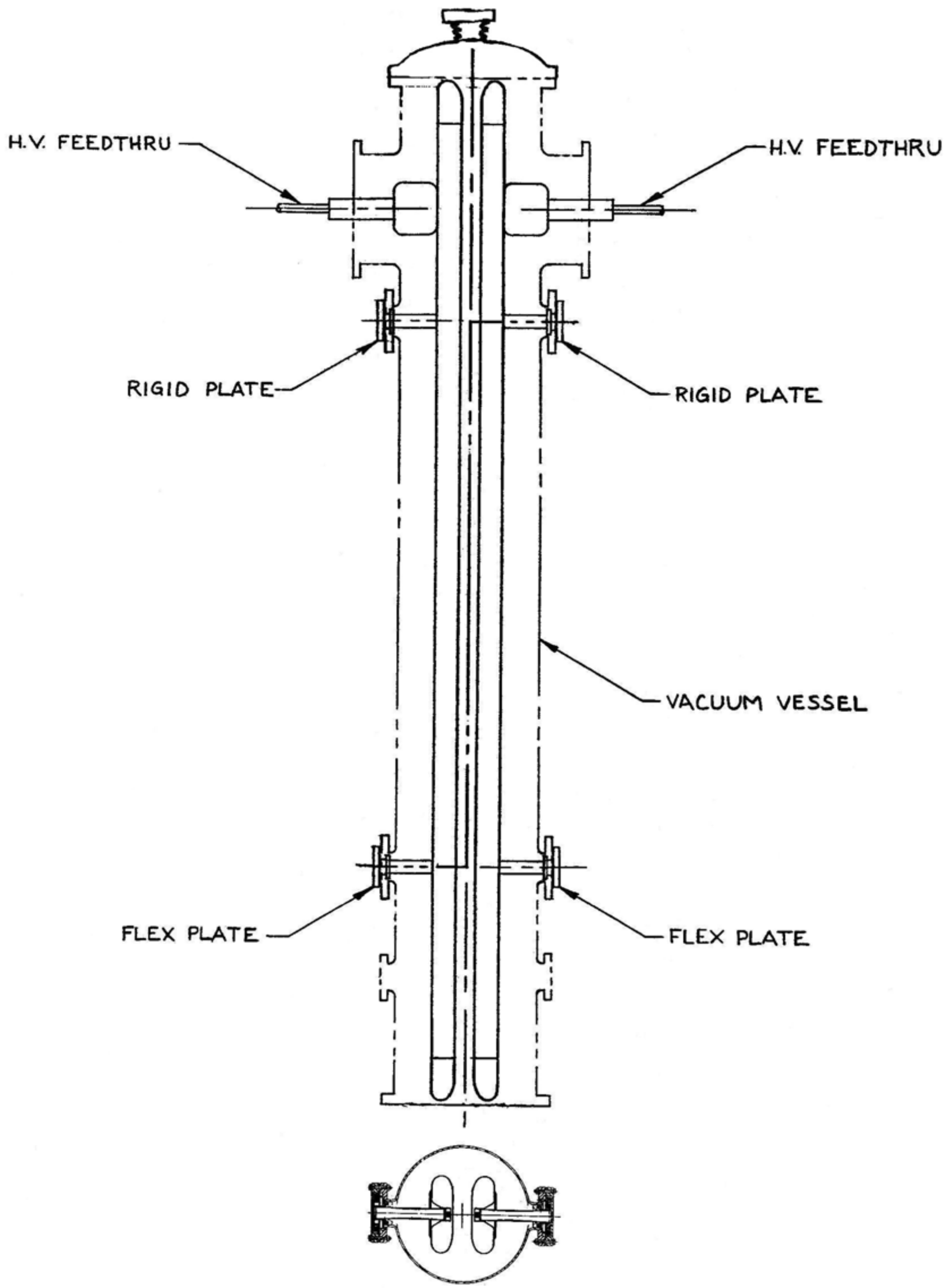
<u>Coaxial line</u> (longitudinal measurement)

Wire diameter	.010	inch (#30 tin plated copper wire)
Wire length	120	inch
Characteristic impedance	331	ohms
Wire loss	10.6	db/100ft @1GHz
Matching L pad	-27.8	db (300/55 ohms)

<u>Balanced line</u> (transverse measurement)

Wire diameter	.010	inch (#30 tin plated copper wire)
Wire separation	1	cm
Wire length	120	inch
Characteristic impedance	518	ohms
Wire loss	13.6	db/100ft @1GHz
Matching L pad	-18.3	db (200/130 ohms)

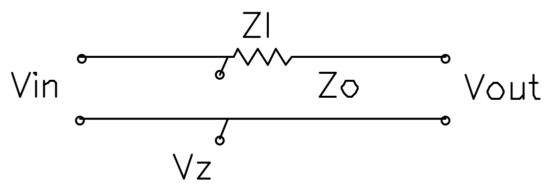
Sketch of Tevatron separator:



Appendix: (useful equations for beam impedance measurements)

Longitudinal Beam Impedance

lumped impedance:



$$Vz = Vinc + Vref = Vin\left(1 + \frac{(Z_L + Z_0) - Z_0}{(Z_L + Z_0) + Z_0}\right)$$

$$Vout = Vz\frac{Z_0}{Z_L + Z_0} = Vin\frac{Z_0}{Z_L + Z_0}\frac{2(Z_L + Z_0)}{Z_L + 2Z_0}$$

$$\frac{Vout}{Vin} = S_{21} = \frac{2Z_0}{Z_L + 2Z_0}$$

$$Z_L = 2Z_0\frac{1 - S_{21}}{S_{21}}$$

high frequency approximation:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$at\ high\ frequency \quad R \ll \omega L\ and\ G \ll \omega C$$

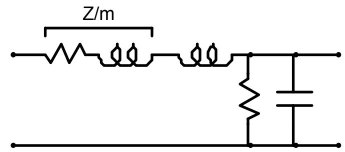
$$Z_0 \approx \sqrt{\frac{L}{C}} \qquad \alpha \approx \frac{R}{2Z_0} + \frac{G}{2}Z_0 \qquad \beta \approx \omega\sqrt{LC} = \frac{\omega}{\upsilon_p}$$

$$for\ G\ small \quad \alpha \approx \frac{R}{2Z_0}$$

$$\frac{Vout}{Vin} = S_{21} = e^{-\alpha l}$$

$$R = -2Z_0 \ln S_{21}$$

distributed impedance:



$$\gamma = \sqrt{(R + j\omega L)\, j\omega C} = \alpha + j\beta$$

$$\gamma^2 = (R + j\omega L)\, j\omega C$$

$$R = \frac{\gamma^2}{j\omega C} - j\omega L = \frac{\alpha^2 - \beta^2 + j2\alpha\beta}{j\omega C} - j\omega L$$

$$\frac{Z}{m} = 2\alpha \frac{\beta}{\omega C} - j\left(\omega L + \frac{\alpha^2 - \beta^2}{\omega C}\right)$$

$$assume L = \frac{Z_o}{\upsilon} C = \frac{1}{Z_o \upsilon} \beta = \frac{\omega}{\upsilon}$$

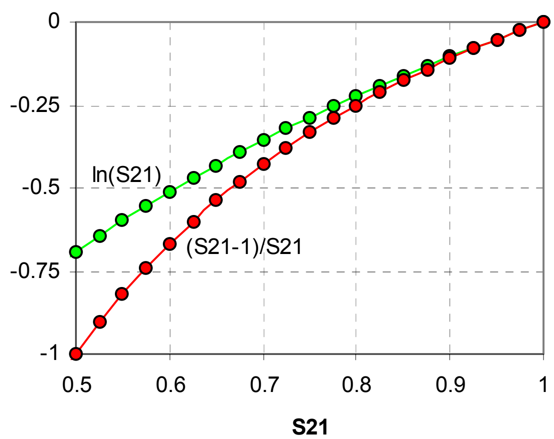
$$\frac{Z}{m} = 2\alpha Z_o - j\frac{\alpha^2}{\beta} Z_o$$

$$Z_{tot} = 2Z_o \alpha l \left(1 - j\frac{\alpha l}{2\beta l}\right) = 2Z_o \ln S_{21} \qquad \alpha l = \ln S_{21} \quad \beta l = angle\ of\ S_{21}$$

$$Z_L = 2Z_0 \frac{1 - S_{21}}{S_{21}} \qquad for\ lumped\ impedance$$

$$= -2Z_0 \ln S_{21} \qquad for\ distributed\ impedance$$

$$= 2\ Z_0 \frac{S_{21}[db]}{20 \log(e)}$$



Transverse Impedance

Panofsky equation: (estimate transverse impedance from longitudinal measurement)

$$Z_T = \frac{2c}{\omega b^2} Z_L \qquad \textit{for pipe radius} = b$$

$$Z_T = \frac{2c}{\omega b^2} 2 Z_0 \frac{S_{21}[db]}{20 \log(e)} \qquad \textit{for distributed impedance}$$

Transverse impedance: (use a balanced or differential transmission line)

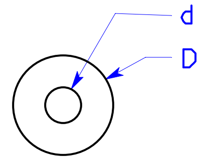
$$Z_T = \frac{c}{\omega \Delta^2} 2Z_0 \left(\frac{1 - S_{21}}{S_{21}} \right) \qquad \textit{for lumped impedance} \quad (\Delta = \textit{wire spacing})$$

$$= \frac{c}{\omega \Delta^2} 2Z_0 \ln S_{21} \qquad \textit{for distributed impedance}$$

$$Z_T = \frac{c}{\omega \Delta^2} 2Z_0 \frac{S_{21}[db]}{20 \log(e)}$$

Characteristic Impedance

coaxial line:

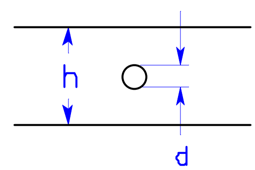


$$Zo = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{D}{d} = 60\ln\frac{D}{d}$$

Zo=331Ω For 0.010" diameter tin plated copper wire in a 2.5" pipe:
Tin plated 30 gauge (.010") wire is 26.6 ohms/m at 1 GHz (rho = 1.14e-7 ohms/m)

$$\alpha \approx \frac{R}{2Z_0} = \frac{26.6}{2\,x\,331}\frac{Np}{m}\quad 20\log(e)(.0254)(12)(100) = 10.6\frac{db}{100\,ft}@1GHz$$

parallel plate line:



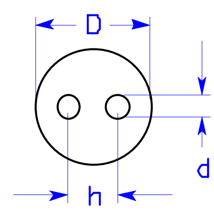
$$Zo = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{4h}{\pi d} = 60\ln\frac{4h}{\pi d}$$

Zo=291Ω For 0.010" diameter tin plated copper wire between 1" plates:
Tin plated 30 gauge (.010") wire is 26.6 ohms/m at 1 GHz (rho = 1.14e-7 ohms/m)

$$\alpha \approx \frac{R}{2Z_0} = \frac{26.6}{2\,x\,290}\frac{Np}{m}\quad 20\log(e)(.0254)(12)(100) = 12.1\frac{db}{100\,ft}@1GHz$$

balanced line:

$$Zdiff = \frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{2h}{d}\frac{1-\left(\frac{h}{D}\right)^2}{1+\left(\frac{h}{D}\right)^2}\right) \approx 120\ln\frac{2h}{d} \qquad (d << h, D)$$



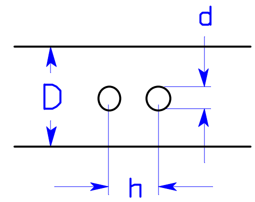
$$Zcom = \frac{1}{4\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{h}{2d}\frac{1-\left(\frac{h}{D}\right)^4}{\left(\frac{h}{D}\right)^2}\right) \approx 30\ln\frac{D^2}{2hd}$$

Zo=518Ω For 0.010" diameter tin plated copper wires in a 2.5" pipe:
Tin plated .010" wire is 26.6 ohms/m at 1 GHz (rho = 1.14e-7 ohms/m)

$$\alpha \approx \frac{R}{2Z_0} = \frac{2\,x\,26.6}{2\,x\,518}\frac{Np}{m}\quad 20\log(e)(.0254)(12)(100) = 13.6\frac{db}{100\,ft}@1GHz$$

balanced line between parallel plates:

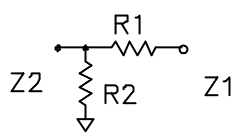
$$Zdiff = 120\ln\left(\frac{4D}{\pi d} TanH \frac{\pi h}{2D}\right) \approx 120\ln\frac{2h}{d} \qquad (d << h, D)$$

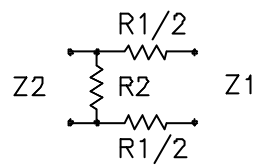


Zo=437Ω For 0.010" diameter tin plated copper wire between 1" plates:
Tin plated .010" wire is 26.6 ohms/m at 1 GHz (rho = 1.14e-7 ohms/m)

$$\alpha \approx \frac{R}{2Z_0} = \frac{2x26.6}{2x437}\frac{Np}{m} \quad 20\log(e)(.0254)(12)(100) = 16.1\frac{db}{100\,ft}@1GHz$$

impedance matching:





$$R1 = \sqrt{Z1\,(Z1 - Z2)}$$

$$R2 = \frac{Z1\,Z2}{R1}$$

For Z1=331Ω, Z2=50Ω:	R1=305Ω, R2=54.3Ω-27.76db xmsn	
For Z1=518Ω, Z2=200Ω:	R1=406Ω, R2=255Ω	-18.32db xmsn
	R1/2=203Ω, R2/2=128Ω	
For Z1=285Ω, Z2=50Ω:	R1=259Ω, R2=55Ω	-26.40db xmsn
For Z1=441Ω, Z2=200Ω:	R1=326Ω, R2=271Ω	-19.93db xmsn
	R1/2=163Ω, R2/2=136Ω	