Beam Position Monitors for the pEDM Experiment

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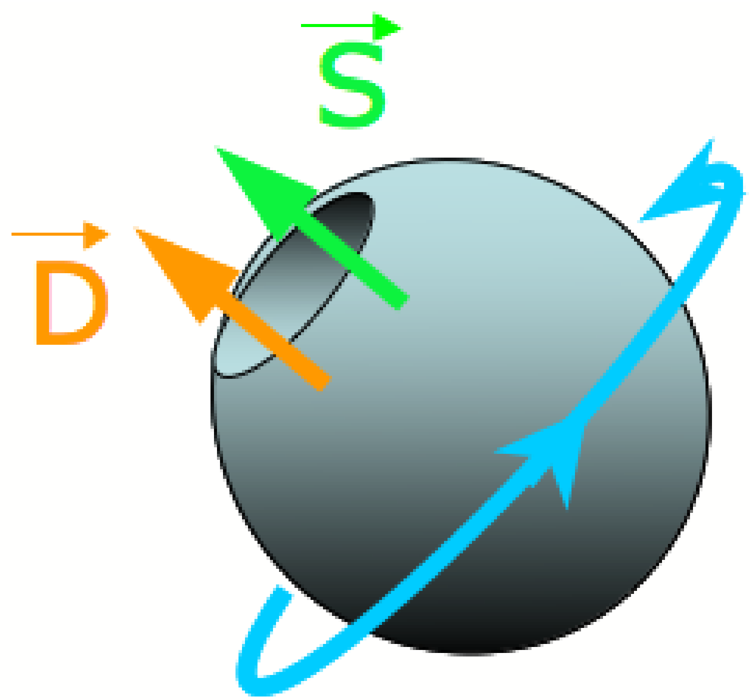


Limitation on the pEDM experiment due to radial magnetic fields

- The total spin precession is given by :

$$\frac{d\boldsymbol{s}}{dt} = \boldsymbol{\mu} \times \boldsymbol{B} + \boldsymbol{d} \times \boldsymbol{E}, \quad \text{where } |\boldsymbol{s}| = \hbar/2$$

$$\Rightarrow \omega_v = \frac{2\left(\mu B_r + dE_r\right)}{\hbar}$$

- Here ω_v is the angular precession frequency of the spin out of the plane.
- The precession due to an EDM at the level of 10^{-29} e·cm given by :

$$\omega_v^{\mathrm{EDM}} = \frac{2dE}{\hbar} = \frac{2dEc}{\hbar c} = \frac{2 \times 1 \times 10^{-31}\ \mathrm{e \cdot m} \times 10.5\ \mathrm{MV/m} \times 0.95 \times 3 \times 10^{8}\ \mathrm{m/s}}{197\ \mathrm{MeV \cdot fm}}$$

$$\omega_v^{\mathrm{EDM}} = 3\ \mathrm{nrad/s}$$

- Precession into vertical also caused by a radial magnetic field $\boldsymbol{B}_r$
- Effect on precession is indistinguishable from an EDM
- What is the maximum $\boldsymbol{B}_r$ allowed?
- How will we measure $\boldsymbol{B}_r$?

Limitation on the pEDM experiment due to radial magnetic fields

- Non-zero $\boldsymbol{B}_r$ results in vertical Lorentz force
- Lorentz force in opposite directions for CW and CCW beams
- Compensated by net vertical electric field : $\boldsymbol{E}_v = -\boldsymbol{\beta} \times \boldsymbol{B}_r$
- Spin precession in vertical due to $\boldsymbol{B}_r$ using lab-frame quantities (see Jackson, in cgs) :

$$\begin{aligned}
\frac{d\boldsymbol{s}}{dt} &= \frac{e}{mc}\boldsymbol{s}\times\left[\left(\frac{g}{2} - 1 + \frac{1}{\gamma}\right)\boldsymbol{B}_r - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1}\right)\boldsymbol{\beta}\times\boldsymbol{E_v}\right] \\
&= g\frac{e}{2mc}\frac{1}{\gamma^2}\boldsymbol{s}\times\boldsymbol{B}_r \\
&= \frac{1}{\gamma^2}\boldsymbol{\mu}\times\boldsymbol{B}_r \quad \text{(normal relation modified by focusing E field)}
\end{aligned}$$

$\Rightarrow$ The EDM precession into the vertical at ω_v could be caused by $\boldsymbol{B}_r$ of magnitude :

$$\begin{aligned}
\hbar\omega_v &= 2\mu B_r/\gamma^2 \Rightarrow \\
B_r &= \frac{\hbar\omega_v}{2\mu}\gamma^2 = \frac{1.05\times10^{-34}\ \mathrm{J\cdot s}\times 3\times10^{-9}\ \mathrm{rad/s}\times 1.25^2}{2\times1.41\times10^{-26}\ \mathrm{J/T}} = 2.2\times10^{-17}\ \mathrm{T}
\end{aligned}$$

$\Rightarrow$ Net radial magnetic field of 0.22 pG would causes precession equivalent to pEDM of $d_p = 10^{-29}$ e·cm

Radial $\boldsymbol{B}$ field splits CW and CCW beam in vertical direction

- Lorentz force from B_r of opposite sign for CW and CCW beams $\Rightarrow$ they split vertically
- Expanding $\boldsymbol{B}_r$ in multipoles, write the equation of motion in vertical y :

$$\frac{d^2y}{d\theta^2} + Q_y^2 y \;=\; \frac{\beta c R_0}{E_r} \sum_{N=0}^{\infty} B_{rN} \cos\left(N\theta + \phi_N\right)$$

- This has solutions :

$$\delta y(\theta) \;=\; \pm \sum_{N=0}^{\infty} \frac{\beta c R_0 B_{rN}}{E_r} \left[\frac{1}{Q_y^2 - N^2}\right] \cos(N\theta + \phi_N) + y_0 \cos(Q_y\theta + \phi_Q),$$

- Q_y is vertical betatron tune, last term is vertical betatron oscillation
- Distortion of equilibrium orbit of opposite sign for the CW and CCW beams
- Only N=0 term, B_{r0}, leads to $\langle \delta y_{CW} - \delta y_{CCW} \rangle \neq 0$
- With vertical tune $Q_y \approx 0.1$, average vertical displacement of each beam :

$$\delta y \;=\; \pm\frac{\beta c R_0 B_r}{E_r Q_y^2} = \pm\frac{0.6 \times 3 \times 10^8 \text{ m/s} \times 40 \text{ m} \times 2.2 \times 10^{-17} \text{ T}}{10.5 \times 10^6 \text{ V/m} \times 0.1^2} \;=\; \pm 1.5 \times 10^{-12} \text{ m}.$$

$\Rightarrow$ Net radial magnetic field B_r of 2.2×10^{-17} T splits the CW and CCW beams vertically by $\approx$ 3.0 pm

Separated counter-circulating beams produce a magnetic dipole

- To detect splitting, consider $\boldsymbol{B}$ fields created by beams
- For displacements from origin by δx and δy, $\boldsymbol{B}$ from single beam :

$$\boldsymbol{B}(r,\phi) = \frac{\mu_0}{4\pi}\frac{2I}{r}\left\{\left[-\sin\phi + \left(-\frac{\delta x}{r}\sin 2\phi + \frac{\delta y}{r}\cos 2\phi\right)\right]\hat{\boldsymbol{x}} + \left[+\cos\phi + \left(-\frac{\delta x}{r}\cos 2\phi + \frac{\delta y}{r}\sin 2\phi\right)\right]\hat{\boldsymbol{y}}\right\}$$

- Using 2×10^{10} protons/beam (I=2.2 mA), consider $\boldsymbol{B}$ field at r=2 cm away :

$$\boldsymbol{B}(r = 2\ \mathrm{cm}, \phi) \ = \ \frac{4\pi \times 10^{-7}}{4\pi}\frac{2 \times 2.2 \times 10^{-3}\ \mathrm{A}}{2 \times 10^{-2}\ \mathrm{m}}\hat{\boldsymbol{\phi}} \ \approx \ 220\ \mu\mathrm{G}$$

- If counter-circulating beam has same charge within a ppm, $\boldsymbol{B} \approx 220$ pG $\hat{\boldsymbol{\phi}}$ at r=2 cm
- If beams split by $\pm\delta y$, can detect at $\phi = \{0, \pi\}$ looking at $\boldsymbol{B} \cdot \hat{\boldsymbol{x}}$
- To move signal off of DC, modulate the vertical tune at ω_m between 20 Hz and 1 kHz
- Set $Q_y \Rightarrow Q_y \times (1 - m\cos(\omega_m t))$ where modulation depth $m \approx 0.1$

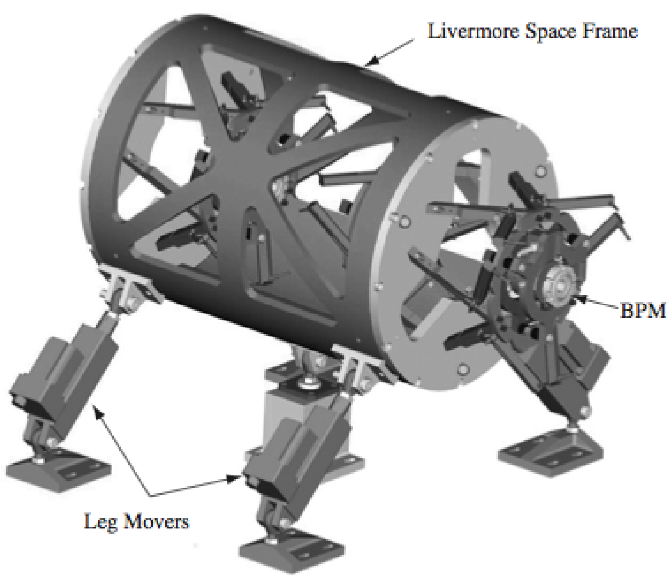
$$\Rightarrow \boldsymbol{B}(r, \phi = (0,\pi), \omega_m) \ = \ \frac{\mu_0}{4\pi}\frac{2I}{r}\left[\frac{\delta y \times 4m\cos\omega_m t}{r}\right]\hat{\boldsymbol{x}}.$$

Magnetic field sensitivity required to measure beam splitting

- Modulating 3 pm splitting of beams leads to peak field of 0.6×10^{-3} fT at ω_m
- Have roughly 10^4 stores of 10^3 seconds to measure this field (for run duration 10^7 s)
- Need to determine $\boldsymbol{B}$ from beams to 0.6×10^{-1} fT per store of 1000 s
- Need sensitivity of $\approx\ 1.9$ fT$/\sqrt{\mathrm{Hz}}$ at ω_m for $S/N\ \approx\ 1$

$\Rightarrow$ Is equivalent to determining splitting of beams $2\delta y \leq 0.3$ nm per store

- ILC final focus requires BPMs with nm level resolution for single shots of 10^{10} electrons
- Single shot resolution of 16 nm has been demonstrated with
  TM_{110} dipole-mode RF cavity BPMs
  (S. Walston *et al.*, Nucl. Instrum. Methods A **578**, 1 (2007))
- We just need to measure *relative* splittings of beams
- Position and tilt of our BPMs not nearly as critical as ILC application

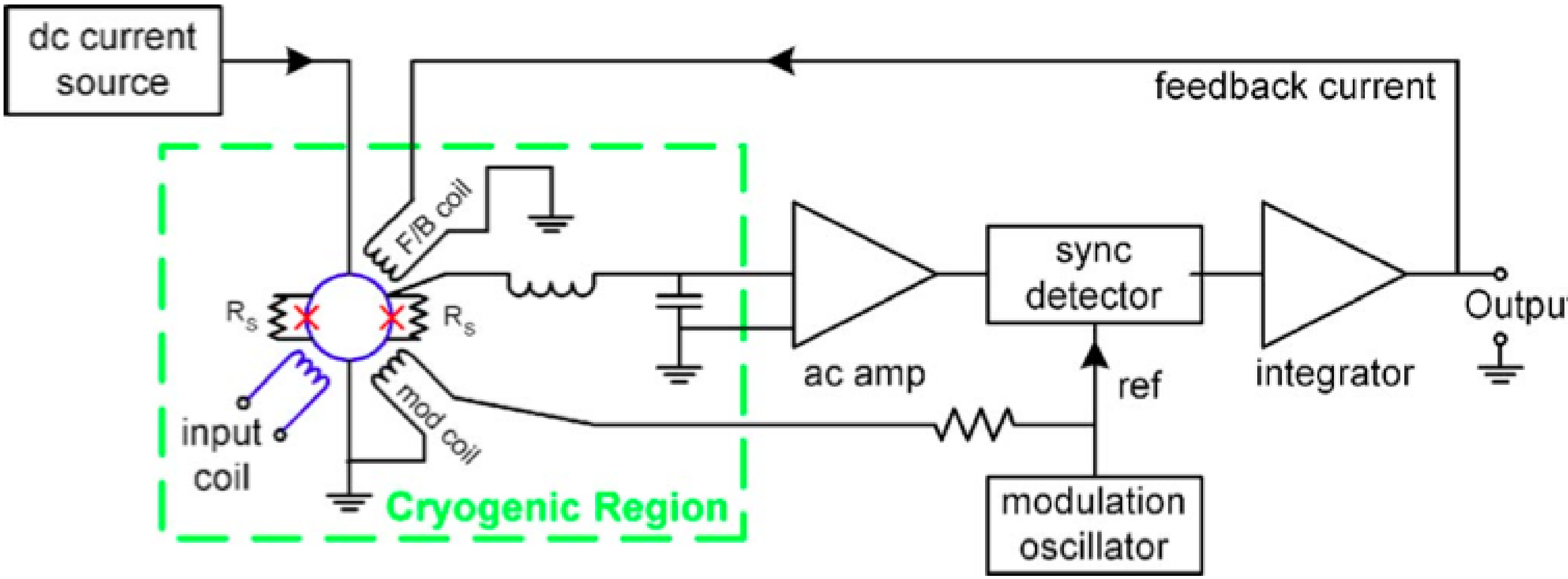


- At least two approaches have demonstrated ability to detect such fields
- (i) K SERF magnetometer developed by M. Romalis' group at Princeton
  (J.C. Allred, R.N. Lyman, T.W. Kornack, and M.V. Romalis, Phys. Rev. Lett. **89**, 130801 (2002))
  - Have demonstrated sensitivity of ≈ 1 fT/$\sqrt{\mathrm{Hz}}$ at $\omega \approx 2\pi\times$ 50 Hz
    (T.W. Kornack, S.J. Smullin, S.-K. Lee, and M.V. Romalis, Appl. Phys. Lett. **90**, 223501 (2007))
- (ii) Commercially available low temperature superconductor DC SQUIDs (LTS dc SQUIDs)
  - Systems from Tristan Technologies have demonstrated $\delta B \leq 1$ fT/$\sqrt{\mathrm{Hz}}$
  - http://www.tristantech.com
  - Many examples in literature of non-commercial devices with similar sensitivity
    (0.7 fT/$\sqrt{\mathrm{Hz}}$ by W. Vodel and K. Mäkiniemi, Meas. Sci. Technol. **3**, 1155 (1992))
  - Systems primarily developed for study of heart and brain biomagnetic fields
- Will focus on solution using SQUIDs
  - Commercially available
  - Implementation and operation might be simpler than SERF magnetometers

$\Rightarrow$ System performance often limited by magnetic field noise - not the magnetometer

$\Rightarrow$ Need to reduce magnetic field noise at ω_m below sensitivity of magnetometer

Block diagram of a SQUID BPM system



(From R.L. Fagaly, Rev. Sci. Instrum. **77**, 101101 (2006))

- Position sense coils adjacent to beams, orient to detect $\hat{x}$ (radial) component of $\boldsymbol{B}$
- Attach sense coil leads to SQUID input coil
- Connect SQUID output to signal input of lock-in amplifier
- Modulate vertical tune at ω_m, connect ω_m oscillator to reference input of lock-in
- Output of lock-in amplifier has component of SQUID output at modulation frequency
- Digitize, record, and possibly use for feedback to beams

Parameters of a SQUID BPM system

- $\boldsymbol{B}$ field sensitivity depends on input current noise of SQUID and coil inductance

- For maximum sensitivity, need to match inductance of sense coil to input coil of SQUID
- For LSQ/20 LTS dc SQUID of Tristan Tech., input coil inductance $L \approx 1.8\mu$H
- A 4 turn coil, 4 cm long $\times$ 1.5 cm high (area of 6 cm^2) has $L \approx 1.6\mu$H

- LSQ/20 + flux locked loop and iMAG SQUID controller has $\delta I_{\text{noise}} \leq$ 0.7 pA/$\sqrt{\text{Hz}}$
- Magnetic field sensitivity extracted from flux sensitivity :

$$\begin{aligned}
\delta\Phi_{\text{noise}} = NA\delta\boldsymbol{B}_{\text{noise}} &= \delta I_{\text{noise}} \times (L_{\text{input}} + L_{\text{sense}}) \\
\delta\boldsymbol{B}_{\text{noise}} &= \frac{(0.7 \times 10^{-12}\ \text{A}/\sqrt{\text{Hz}}) \times (3.54 \times 10^{-6}\ \text{H})}{(4\ \text{turns}) \times (6 \times 10^{-4}\ \text{m}^2/\text{turn})} \\
&= 1.0\ \text{fT}/\sqrt{\text{Hz}}.
\end{aligned}$$

$\Rightarrow$ If ambient field noise at ω_m is $\leq$ 1 fT/$\sqrt{\text{Hz}}$, combined noise $\leq$ 2 fT/$\sqrt{\text{Hz}}$
$\Rightarrow$ A single system is sensitive enough to measure $\boldsymbol{B}_r$ to the required level

- Of course, would never rely on a single system
- Also want to improve $S/N \gg$1

Parameters of a SQUID BPM system

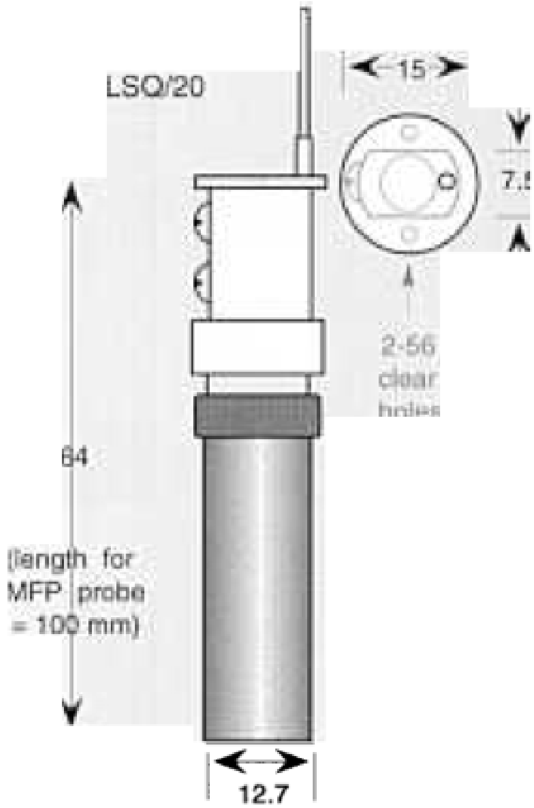
- There are 6 straight sections, each $\approx$ 24 cm long, in which SQUID BPMs could be used
- Use 20 SQUIDs in each straight section, 10 sense coils on either side of beam
- Coils would overlap : coil pairs would see the same signal and local magnetic field noise
- However, input current noise of each SQUID is not common
- As long as ambient field noise at or below 1 fT$\sqrt{\mathrm{Hz}}$ not hurt much by this scheme

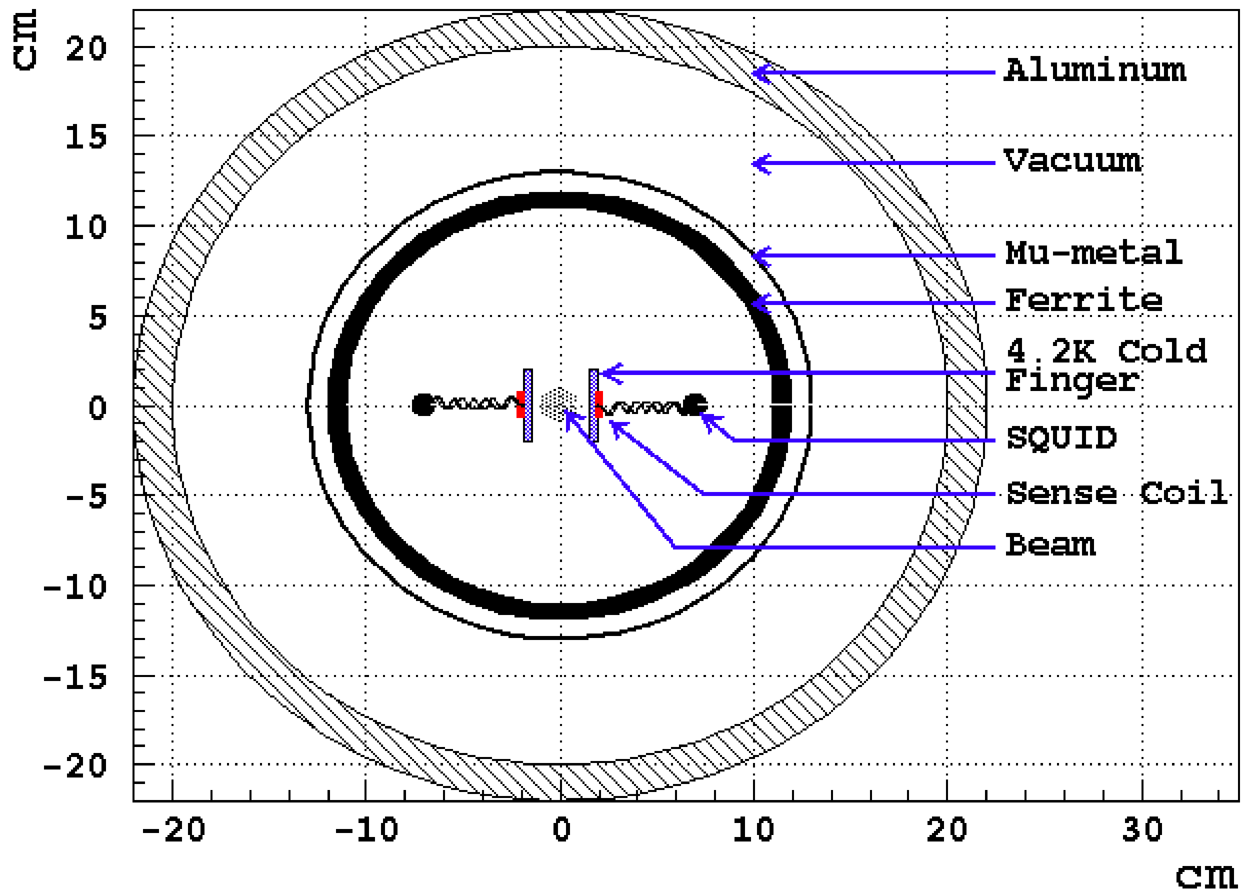
- Assume we can achieve 1 fT$\sqrt{\mathrm{Hz}}$ ambient field noise at ω_m
- Each coil pair has sensitivity of $\approx$ 1.3 fT$\sqrt{\mathrm{Hz}}$
- Average of 60 independent pairs has 0.17 fT$\sqrt{\mathrm{Hz}}$ at ω_m
- Recall we want to limit $\boldsymbol{B}_r \leq 2.2 \times 10^{-17}$ T, which causes 3 pm splitting of beams
- Splitting of $2\delta y \approx 3$ pm $\Leftrightarrow$ $\boldsymbol{B}$ from beams of 0.6$\times 10^{-3}$ fT
- In 1 store, determine $\boldsymbol{B}$ from beams to 5.3$\times 10^{-3}$ fT, splitting of beams to $2\delta y \approx 27$ pm
- In 1 store determine $\boldsymbol{B}_r$ to $\approx\ 22 \times 10^{-17}$ T

- In 100 stores, we should be able to measure $\boldsymbol{B}_r$ to required limit

$\Rightarrow$ In 10^4 stores, should be able to measure $\boldsymbol{B}_r$ to required sensitivity with $S/N \gg 5$

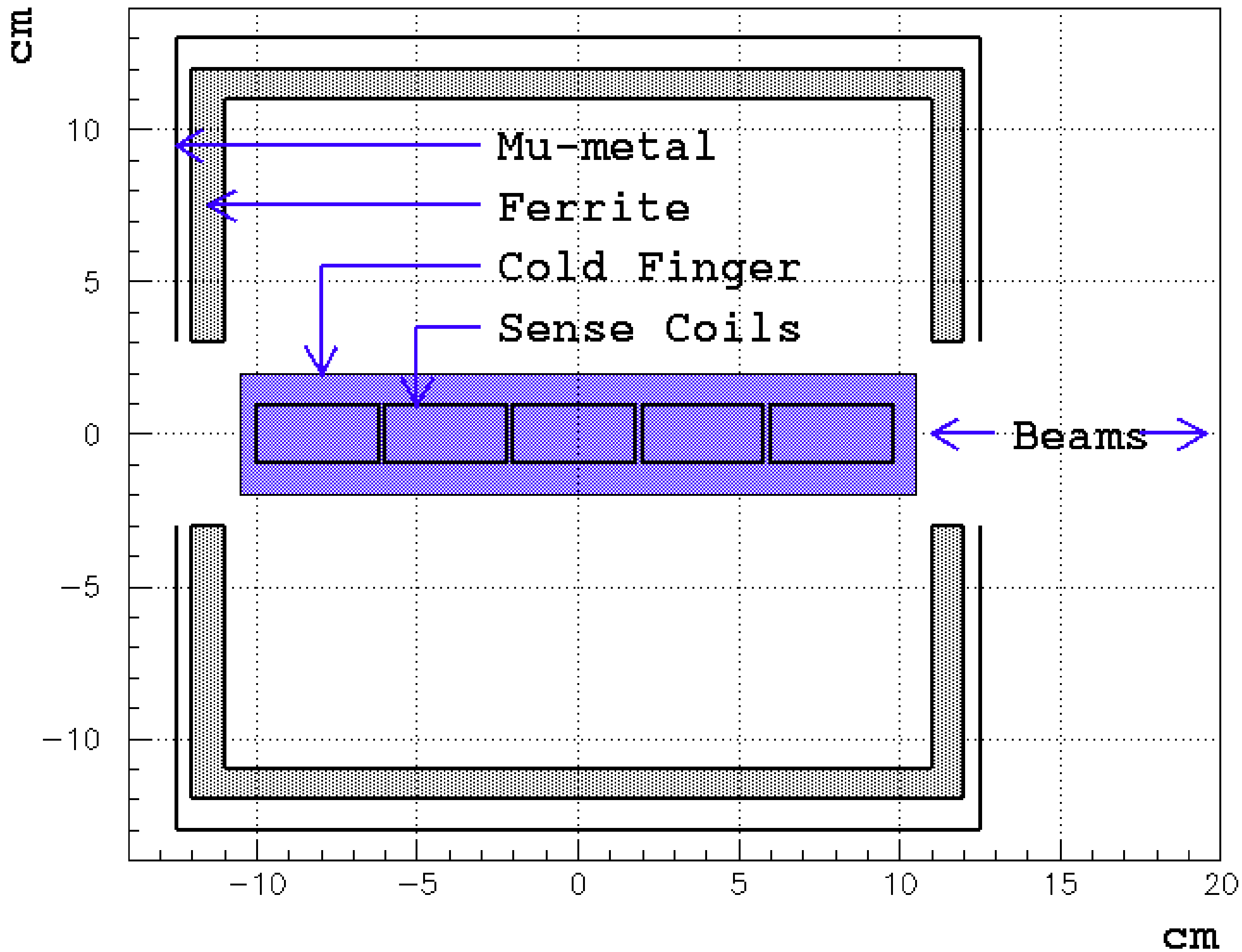
Schematic of a SQUID BPM system





- Tristan Technology LSQ/20 SQUID
- 64 mm long, 12.7 mm diameter
- ≤ 1 fT$/\sqrt{\mathrm{Hz}}$

- Beam's eye view schematic of a SQUID BPM system
- Sense coils, leads, SQUIDs at 4.2K; leads and SQUIDs in superconducting shields
- Ferrite and μ-metal at room temp.
- More magnetic shielding outside Al vacuum chamber



- Side view schematic of a SQUID BPM system
- Sense coils, leads, SQUIDs at 4.2K; leads and SQUIDs in superconducting shields
- Ferrite and μ-metal at room temp, more magnetic shielding outside vacuum chamber

Magnetic shielding

- Would like to achieve 1 fT$\sqrt{\mathrm{Hz}}$ ambient field noise at ω_m
- Sense coils will pick up magnetic noise from :

  (i) Sources outside the BPM shields

  (ii) Magnetic noise from the shields themselves

  (iii) Magnetic noise from beam at $\omega \neq \omega_m$

- Sense coils leads and SQUIDs have additional superconducting shield

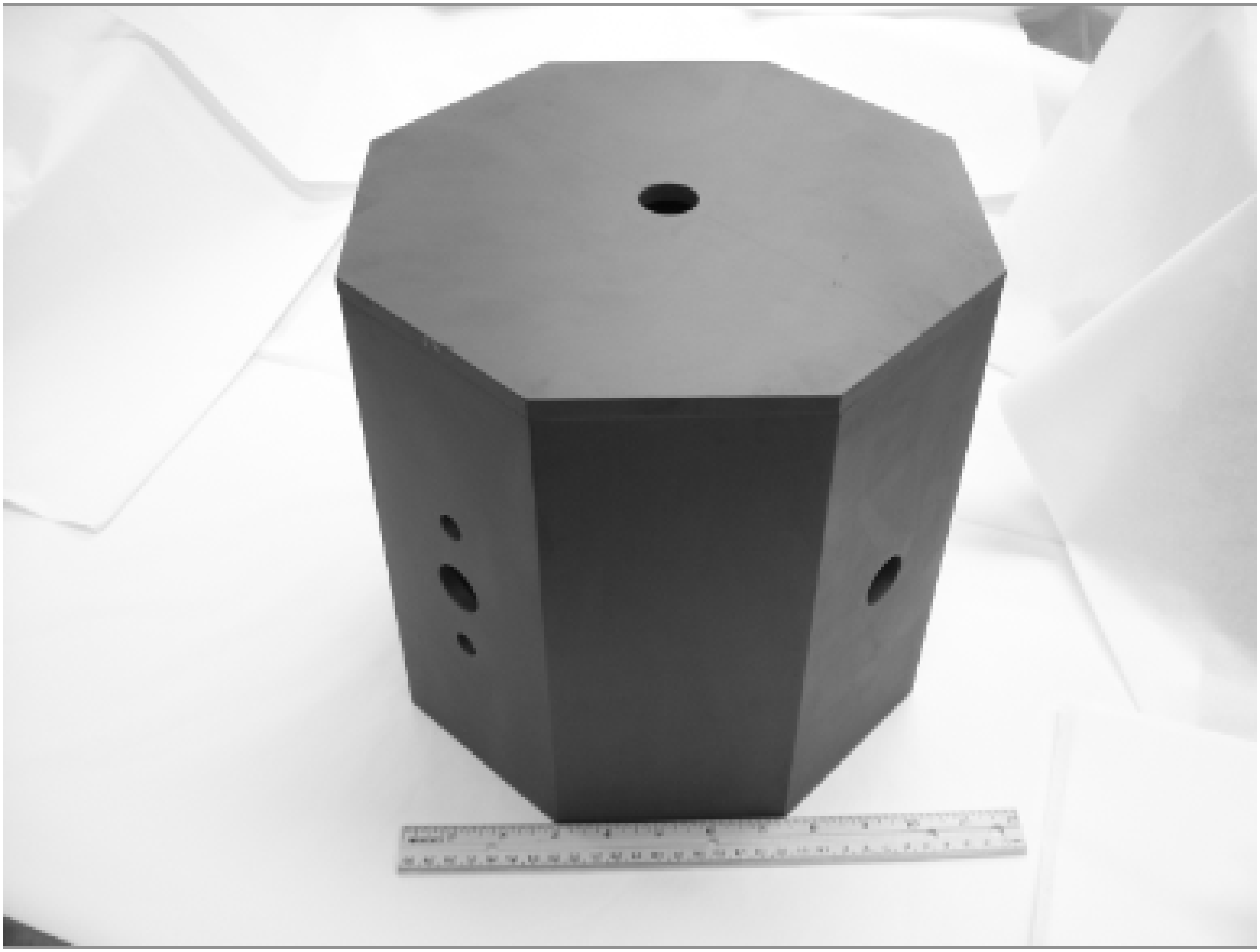
$\Rightarrow$ System noise dominated by noise picked up by sense coils

- 0.75 fT$/\sqrt{\mathrm{Hz}}$ at $\omega > 2\pi\times$ 35 Hz achieved with 3 layers of μ-metal and ferrite shield (T.W. Kornack, S.J. Smullin, S.-K. Lee, and M.V. Romalis, Appl. Phys. Lett. **90**, 223501 (2007))
- Shielded a 10 cm diameter cylindrical volume, residual field $<$ 2 nT

- We will have 3 layers of μ-metal outside of the beam pipe
- In addition : beam pipe will act as eddy current shield, shielding factor $H_{\mathrm{int}}/H_{\mathrm{ext}} \approx 1/(1+i\omega\tau)$ should help above 60 Hz
- Inside beam pipe : additional μ-metal and 1 cm thick MnZn ferrite shield

$\Rightarrow$ Shielding of environment and noise from shields should be comparable to Romalis

Magnetic Shielding from CMI-Ferrite



Magnetic shielding

- Our dimensions slightly different
- Along axis of long ferrite, shield noise expected

$$\delta B_{\mathrm{magn}} \approx \frac{0.26\mu_0}{r\sqrt{t}}\sqrt{\frac{4kT\mu''}{\omega\mu'^2}}, \tag{1}$$

- Here r and t are inner radius and thickness, loss factor $\mu''\mu_0/\mu'^2 = 1.46 \times 10^{-6}$
- Given our finite shield length, expect $\delta B_{\mathrm{magn}} \leq 3.2$ fT/$\sqrt{\mathrm{Hz}}$ at 1 Hz

⇒For $\omega_m > 10$ Hz, expect noise from shields $\delta B_{\mathrm{magn}} < 1$ fT/$\sqrt{\mathrm{Hz}}$

- Noise from beam will be at 1-70 MHz, depending on bunching
- Frequencies far outside the SQUID flux-locked loop bandwidth of a few kHz
- This noise from beam can perturb operation of SQUID
- Can be shielded using layer of indium-tin-oxide (ITO) on inner surface of sense-coil cold finger
- Image currents in ITO will reduce signal a bit, but high frequency attenuated significantly

Magnetic shielding

- Using conductivity $\sigma(\mathrm{ITO}) \approx 10^4\ \Omega^{-1}$m^{-1} ITO skin depth δ:

$$\delta = 1/\sqrt{2f\mu\sigma}$$

- 2.5 mm of ITO would reduce 70 MHz noise by factor 20. Noise from ITO itself :

$$\delta B_{\mathrm{curr}} \approx \frac{1}{\sqrt{8\pi}}\frac{\mu_0\sqrt{kT\sigma t}}{a} \approx 10\ \mathrm{fT}/\sqrt{\mathrm{Hz}},$$

- where $a = 2$ mm is distance from ITO to coils, and noise drops with frequency as $1/f$

$\Rightarrow$ For ω_m above 20 Hz magnetic noise from ITO would not limit measurement

- If move to long bunch scheme, to shield 1 MHz noise requires few hundred μm of Al
- Conductivity of Al 3500 times that of ITO, $\delta B_{\mathrm{curr}} \approx 80$ fT$/\sqrt{\mathrm{Hz}}$ at 1 Hz

$\Rightarrow$ For ω_m above 150 Hz, magnetic noise from Al would not limit measurement

$\Rightarrow$ For $\omega_m > 150$ Hz, expect noise from shields $\delta B_{\mathrm{magn}} < 1$ fT$/\sqrt{\mathrm{Hz}}$

- What about ground motion at ω_m?

(i) Beam samples $\approx \pm 3$ cm $\times \pm 1.5$ cm $\times 260$ m

- Small motions won't change $\langle \boldsymbol{B}_r \rangle$ significantly

(ii) If sense coils move up/down wrt beams at ω_m ...

- See part of $\boldsymbol{B} \approx 200$ pG $\hat{\boldsymbol{\phi}}$ from δI, that couples in $\hat{\boldsymbol{x}}$
- Estimate this fraction for 1 nm motion as $\approx 10^{-9}$ m$/10^{-2}$ m
- Could anticipate field of 200 pG $\times 10^{-7} \approx 0.2$ fT at ω_m

(iii) Local gradients should be less than 2 nT/cm

- For 1 nm motion, see field 2 nT/cm $\times$ 1 nm $=$ 0.2 fT

$\Rightarrow$ Will need to keep motion at ω_m at nm level

Options to reduce requirements on BPMs

- What other options do we have if 2 fT/$\sqrt{\mathrm{Hz}}$ at ω_m not possible?

- Stochastic cooling may allow Q_y to go from 0.1 $\Rightarrow$ 0.03
- Beam splitting goes up by factor 10, required BPM resolution drops by factor 10

- Increase Q_y modulation depth m from 0.1 to 0.15-0.2 : becomes factor 2$+$ easier

- May just have to run longer to get required limits

Summary and Conclusions

- Net radial magnetic field of 0.22 pG would causes precession equivalent to pEDM of $d_p = 10^{-29}$ $e\cdot$cm
- This field would split the CW and CCW beams by 3 pm
- Magnetic field from beams split in vertical has radial component
- By modulating vertical tune, can look for this field using SQUIDs and lock-in amplifier
- Require sensitivity ≤ 1 fT$/\sqrt{\mathrm{Hz}}$ at ω_m
- A single SQUID magnetometer has this sensitivity
- Magnetic shielding with noise < 1 fT$/\sqrt{\mathrm{Hz}}$ above 35 Hz has been demonstrated
- Large effort required :
  - Design cold finger/cryostat, integrate with other elements of experiment
  - Integration of SQUID controller output with lock in, DAQ, many parameters to be determined

  $\Rightarrow$ Demonstrating that this works in storage ring environment will be necessary

  $\Rightarrow$ Systematics : thermal, dimensional stability, ground motion, slow changes in B, ...

  $\Rightarrow$ Great challenge and a great opportunity