

## Exploring parameterization for turbulent entrainment-mixing processes in clouds

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[1] Different turbulent entrainment-mixing processes (e.g., homogeneous and inhomogeneous) occur in clouds; accurate representation of these processes is critical for improving cloud-related parameterizations in large-scale models, but poorly understood and quantified. Using in situ aircraft observations over the U. S. Department of Energy's Atmospheric Radiation Measurement Southern Great Plains site during the March 2000 Cloud Intensive Observation Period and numerical simulations with the Explicit Mixing Parcel Model (EMPM), here we explore the potential of using degree of homogeneous mixing as a measure to quantify these different mechanisms and examine various microphysical measures of homogeneous mixing degree and their relationships to entrainment-mixing dynamics as measured by transition scale numbers. Three different microphysical measures for the homogeneous mixing degree are newly defined and each is coupled with one of two different transition scale numbers. Both observations and simulations show that all the combinations have positive correlated relationships; simulations further show that the tightest relationship is between the measure of homogeneous mixing degree considering adiabatic number concentration and the transition scale number accounting for mixing fraction of dry air. A parameterization of the entrainment-mixing processes is advanced according to the relationships of homogeneous mixing degree measures to transition scale numbers.

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### 1. Introduction

[2] Turbulent entrainment-mixing processes are critical to many outstanding issues related to clouds, including aerosol indirect effects, cloud-climate feedbacks, warm-rain initiation and remote sensing retrieval of cloud microphysical properties [Paluch and Baumgardner, 1989; Yum, 1998; Ackerman et al., 2004; Kim et al., 2008; Del Genio and Wu, 2010; Lu et al., 2011; Kumar et al., 2012; Lu et al., 2013]. Several different entrainment-mixing mechanisms have been proposed that may result in different cloud microphysical properties. Consider the homogeneous/inhomogeneous mixing model pioneered by Baker and Latham [1979] and Baker et al. [1980] for example. In the homogeneous mixing scenario, all droplets evaporate simultaneously; in the

extreme inhomogeneous mixing scenario, some droplets evaporate completely while the rest of droplets do not evaporate at all. Although the conceptual model is well established, our understanding is still far from complete. Some observations suggested that the entrainment-mixing process was close to homogeneous [e.g., Jensen et al., 1985; Burnet and Brenguier, 2007; Lehmann et al., 2009]; others pointed to an inhomogeneous scenario [Pawłowska et al., 2000; Burnet and Brenguier, 2007; Haman et al., 2007; Freud et al., 2008, 2011; Gerber et al., 2008; Lehmann et al., 2009].

[3] The existence and our poor understanding of different types present a great challenge to accurate representation of entrainment-mixing processes and their various effects in atmospheric models. For example, Lasher-Trapp et al. [2005] found more large droplets were produced in a three-dimensional cloud model assuming inhomogeneous mixing than assuming homogeneous mixing. Using large eddy simulation, Chosson et al. [2007] found cloud albedo bias changed from  $-3\%$  to  $-31\%$  when assuming both mixing mechanisms alternatively in a fragmented and thin stratocumulus cloud. Similarly, with a cloud-resolving model, Grabowski [2006] found that the amount of solar energy reaching the surface was the same in the pristine case assuming the homogeneous mixing scenario as in the polluted case with the extreme inhomogeneous mixing; the same conclusion was also reported by Slawinska et al. [2008] using large eddy simulations with a one-moment microphysics scheme. Although later Morrison and Grabowski [2008], Hill et al. [2009], and

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Slawinska *et al.* [2012] found significantly reduced effect of mixing mechanisms in simulations, Morrison and Grabowski [2008] and Hill *et al.* [2009] pointed out the effect of mixing mechanisms could be more significant over the entire cloud life cycle, especially during dissipation of clouds. Therefore, it is critical to distinguish homogeneous and inhomogeneous mixing. Furthermore, in real clouds, entrainment-mixing processes often fall between the above two extremes [Andrejczuk *et al.*, 2009; Lu *et al.*, 2011], posing an additional need for a quantitative measure that spans continuously over the two extreme types. This paper is motivated by such needs and focuses on examining the measure of homogeneous mixing degree, transition scale number [Lu *et al.*, 2011], and their relationships. Both in situ observations and numerical simulations are utilized in our analysis. A parameterization of turbulent entrainment-mixing process is advanced from the results.

[4] The rest of the paper is organized as follows. Section 2 defines three new measures of homogeneous mixing degree and describes the calculations of two transition scale numbers. Section 3 presents the results from in situ observations and numerical simulations. A summary is presented in section 4.

## 2. Homogeneous Mixing Degree and Transition Scale Number

### 2.1. Microphysical Measures of Homogeneous Mixing Degree

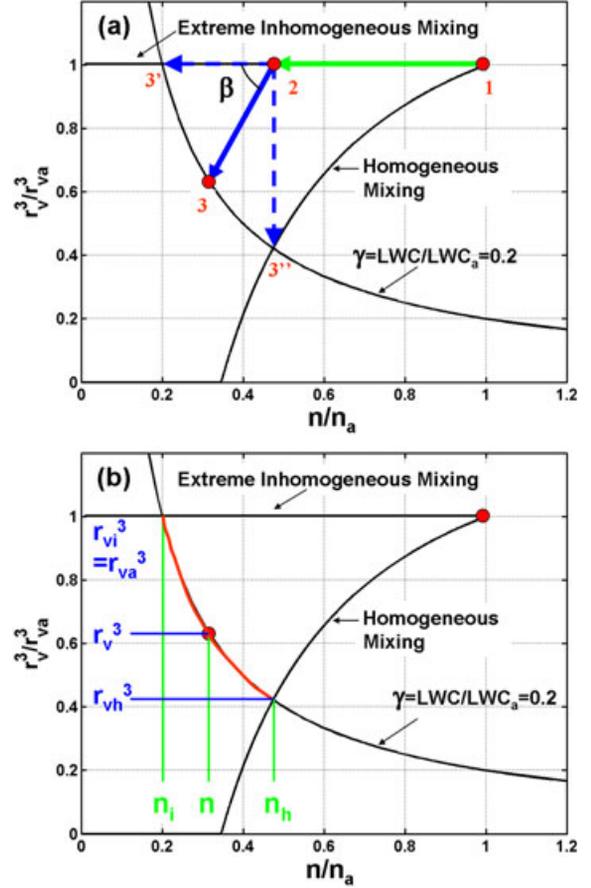
[5] The diagram of volume mean radius ( $r_v$ ) versus number concentration ( $n$ ), or its normalized version,  $r_v^3/r_{va}^3$  versus  $n/n_a$  ( $r_{va}$  and  $n_a$  are adiabatic volume mean radius and number concentration, respectively) has been widely used to study homogeneous/inhomogeneous entrainment-mixing mechanisms [Burnet and Brenguier, 2007; Lehmann *et al.*, 2009; Lu *et al.*, 2011]. Based on the diagram of  $r_v^3/r_{va}^3$  versus  $n/n_a$ , we can define three measures of homogeneous mixing degree.

[6] The first measure of homogeneous mixing degree ( $\psi_1$ ) is defined with the help of Figure 1a. Similar to Figure 5 in Krueger [2008], Figure 1a conceptually illustrates the sequence of states involved in an entrainment and isobaric mixing event. The states are numbered from 1 to 3. State 1 is an adiabatic cloud, which has adiabatic number concentration of  $n_a$  and volume mean radius of  $r_{va}$ . State 2 is just after entrainment but before evaporation, which has  $n_h$  and  $r_{va}$ . From States 2 to 3, mixing and evaporation occur; the number concentration and volume mean radius become  $n$  and  $r_v$ , respectively, after mixing and evaporation. The extreme inhomogeneous and homogeneous mixing scenarios are denoted by 3' and 3'', respectively. The angle between the line linking States 2 and 3'' and the extreme inhomogeneous mixing line is  $\pi/2$ ; the angle between the line linking States 2 and 3 and the extreme inhomogeneous mixing line is  $\beta$ . As an example, liquid water content is assumed to decrease to 0.2 of the adiabatic value.

[7] A trigonometrical analysis relates  $\beta$  to the slope of the line linking States 2 and 3:

$$\beta = \tan^{-1} \left( \frac{\frac{r_v^3}{r_{va}^3} - 1}{\frac{n}{n_a} - \frac{n_h}{n_a}} \right), \quad (1)$$

[8] Normalizing  $\beta$  by  $\pi/2$  gives the first measure of homogeneous mixing degree:



**Figure 1.** (a) Mixing diagram illustrating the definition of the first homogeneous mixing degree. The three black solid lines correspond to extreme inhomogeneous mixing, homogeneous mixing (relative humidity of the dry air is 66%), and contour of  $\gamma=0.2$ ;  $\gamma$  is the ratio of liquid water content (LWC) to its adiabatic value ( $LWC_a$ ). See text for the meanings of the other lines and symbols. (b) Same as Figure 1a but for the definition of the second and third homogeneous mixing degree.

$$\psi_1 = \frac{\beta}{\pi/2} \quad (2)$$

[9] Obviously,  $\psi_1$  is between 0 and 1 and measures the degree of homogeneous mixing: a larger  $\psi_1$  indicates a higher degree of homogeneous mixing, with  $\psi_1=1$  for homogeneous mixing and  $\psi_1=0$  for extreme inhomogeneous mixing.

[10] The second measure of homogeneous mixing degree ( $\psi_2$ ) is illustrated with Figure 1b. When a mixing process is homogeneous, the number concentration and volume mean radius are  $n_h$  and  $r_{vh}$ , respectively; when a mixing process is extreme inhomogeneous, the number concentration and volume mean radius are  $n_i$  and  $r_{va}$ , respectively. In reality, the mixing scenario is between the two extremes with number concentration of  $n$  and volume mean radius of  $r_v$ . The second measure of homogeneous mixing degree is expressed as

$$\psi_2 = \frac{1}{2} \left( \frac{n-n_i}{n_h-n_i} + \frac{r_v^3-r_{va}^3}{r_{vh}^3-r_{va}^3} \right), \quad (3) \quad \frac{dr}{dt} = A \frac{s}{r}, \quad (11a)$$

where

$$r_{vh}^3 = \frac{n}{n_h} r_v^3, \quad (4)$$

$$n_i = \frac{r_v^3}{r_{va}^3} n. \quad (5)$$

[11] The third measure of homogeneous mixing degree ( $\psi_3$ ) can be defined by recognizing that the logarithm operation linearizes the nonlinear  $r_v^3/r_{va}^3 - n/n_a$  relation such that

$$\psi_3 = \frac{\ln n - \ln n_i}{\ln n_h - \ln n_i} = \frac{\ln r_v^3 - \ln r_{va}^3}{\ln r_{vh}^3 - \ln r_{va}^3}. \quad (6)$$

[12] In a numerical study, *Morrison and Grabowski* [2008] introduced an empirical parameter to quantify the homogeneous mixing degree such that

$$n = n_0 \left( \frac{q_L}{q_{L0}} \right)^\alpha, \quad (7)$$

where  $n_0$  and  $q_{L0}$  are number concentration and liquid water mixing ratio after entrainment but before evaporation, respectively;  $n_0$  is identical to  $n_h$  in equation (3);  $n$  and  $q_L$  are final number concentration and liquid water mixing ratio, respectively, after mixing and evaporation. These two properties are taken as the values after new saturation is achieved. It can be shown that  $\psi_3$  is uniquely related to  $\alpha$  by

$$\psi_3 = 1 - \alpha, \quad (8)$$

[13] Note that *Morrison and Grabowski* [2008] prescribed three  $\alpha$  values (=0, 0.5, and 1) in their simulations, instead of examining the link between  $\alpha$  and microphysical relationships.

## 2.2. Transition Scale Numbers

[14] The transition scale number ( $N_L$ ) is a dynamical measure of the occurrence probability of homogeneous or inhomogeneous entrainment-mixing process [*Lu et al.*, 2011]; the larger the  $N_L$ , the stronger the homogeneous entrainment-mixing process and the weaker the inhomogeneous entrainment-mixing process.  $N_L$  is calculated as the ratio of transition length ( $L^*$ ) introduced by *Lehmann et al.* [2009] to the Kolmogorov microscale ( $\eta$ ):

$$N_L = \frac{L^*}{\eta} = \frac{\varepsilon^{1/2} \tau_{\text{react}}^{3/2}}{\eta}, \quad (9)$$

where  $\varepsilon$  is eddy dissipation rate.  $\eta$  is given by

$$\eta = \left( \frac{v^3}{\varepsilon} \right)^{1/4} \quad (10)$$

where  $v$  is the kinematic viscosity [*Wyngaard*, 2010]. The reaction time  $\tau_{\text{react}}$  is the time when droplets have completely evaporated or relative humidity has reached 99.5% ( $s > -0.005$ ) [*Lehmann et al.*, 2009] whichever is first satisfied; it is calculated by the following equations:

$$\frac{ds}{dt} = -Brs, \quad (11b)$$

where  $r$  is cloud droplet radius,  $s$  supersaturation,  $A$  a function of pressure and temperature, and  $B$  a function of pressure and temperature and proportional to cloud droplet number concentration [*Rogers and Yau*, 1989]. If  $n_a$  is used in  $B$ , scale number is denoted by  $N_{La}$ ; while if  $n_0$  is used, scale number is denoted by  $N_{L0}$ . For a group of droplets,  $r$  in the above equations is replaced with  $r_{va}$ .

## 3. Relationship between Homogeneous Mixing Degree and Transition Scale Number

[15] Although a positive relationship between the microphysical measures of homogeneous mixing degree and transition scale number is expected from the fact that the two measures quantify the probability of the homogeneous mixing process from different perspectives, the details of the relationship remain elusive. Next we explore this relationship by examining both in situ cloud observations and numerical simulations.

### 3.1. Observational Results

[16] The stratocumulus clouds were collected over the U. S. Department of Energy's Atmospheric Radiation Measurement Southern Great Plains site during the March 2000 Cloud Intensive Observation Period (IOP). Cloud droplet and drizzle size distributions were measured with a Particle Measuring Systems (PMS) Forward Scattering Spectrometer Probe (FSSP-100) and an optical array probe (1D-C), respectively. The FSSP probe sizes and counts cloud droplets in 15 bins, with bin centers from 2.7 to 30  $\mu\text{m}$  radius; the 1D-C probe has 30 bins with bin centers from 12.1 to 300  $\mu\text{m}$  radius. Measurements of both instruments are corrected with standard procedures [*Dye and Baumgardner*, 1984; *Baumgardner et al.*, 1985; *Baumgardner*, 1987; *Baumgardner and Spowart*, 1990]. Air temperature, air pressure, and dew point were measured with Rosemount Model 102, Rosemount Model 1201F1, and EG&G Model 137 probes, respectively. Relative humidity is calculated based on the dew point, air temperature, and air pressure. True air speed was measured by Rosemount 1221F. The turbulence dissipation rate is derived from the true airspeed using a structure function (see Appendix A in *Lu et al.* [2011] for details). The 1Hz data are used in this study.

[17] Sixteen non-drizzling horizontal flight legs in five flights were used to study homogeneous/inhomogeneous mixing with the relationship of  $r_v-n$  by *Lu et al.* [2011]. One leg was affected by homogeneous mixing, 13 were affected by extreme inhomogeneous mixing, and two were affected by inhomogeneous mixing with subsequent ascent. In the definitions of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$ , subsequent ascent after the mixing process is not considered, so the legs, except the two legs affected by inhomogeneous mixing with subsequent ascent, are used here.

[18] To obtain the properties used in the calculations of  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $N_{La}$ , and  $N_{L0}$ , mixing fraction of dry air ( $f$ ) for each of the 14 legs is needed and can be calculated based

on the equations [Burnet and Brenguier, 2007; Gerber et al., 2008; Lehmann et al., 2009]:

$$q_L + q_{vs}(T) = [q_{vs}(T_a) + q_{La}](1-f) + q_{ve}f, \quad (12a)$$

$$c_p T = c_p T_a(1-f) + c_p T_e f - L_v [q_{La}(1-f) - q_L], \quad (12b)$$

$$q_{vs}(T) = 0.622 \frac{e_s(T)}{p - e_s(T)}, \quad (12c)$$

where  $T$ ,  $q_{vs}(T)$ , and  $q_L$  are, respectively, the average temperature, saturation vapor mixing ratio, and liquid water mixing ratio along each leg;  $T_e$  and  $q_{ve}$  are, respectively, temperature and water vapor mixing ratio of the entrained dry air from above the cloud tops;  $e_s$  is saturation vapor pressure;  $c_p$  is specific heat capacity at constant pressure;  $p$  is air pressure;  $L_v$  is latent heat;  $T_a$ ,  $q_{vs}(T_a)$ , and  $q_{La}$  are, respectively, the temperature, saturation vapor mixing ratio, and liquid water mixing ratio in the adiabatic cloud parcel. The maximum liquid water mixing ratio along a leg is assumed to be  $q_{La}$ ; the temperature corresponding to  $q_{La}$  is assumed to be  $T_a$ . Then  $f$  along the horizontal legs is calculated with the input parameters  $q_{La}$ ,  $T_a$ ,  $q_L$ ,  $T_e$ , and  $q_{ve}$  (Table 1). With  $f$  and assuming the maximum number concentration along a leg is  $n_a$ ,  $n_h$  can be calculated with  $n_h = n_a(1-f)$ . The average number concentration along a leg is taken to be  $n$ . Thus,  $r_v$  is given by

$$r_v = \left( \frac{q_L \rho_{\text{air}}}{4/3\pi n \rho} \right)^{1/3} \quad (13)$$

where  $\rho$  and  $\rho_{\text{air}}$  are water density and air density, respectively. Similarly,  $r_{va}$  is calculated with  $q_{La}$  and  $n_a$ . Then  $r_{vh}$  and  $n_i$  are calculated by equations (4) and (5), respectively.

[19] With all the above properties,  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $N_{La}$ , and  $N_{L0}$  can be calculated and the results are shown in Figure 2. The positive correlations between  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$  and  $N_{La}$ ,  $N_{L0}$  are evident for 12 out of 14 legs, but with large scatter. One reason for the scatter of data points is the influence of dissipation rate, relative humidity, adiabatic number concentration, and mixing fraction of dry air, which will be discussed later; another reason could be the uncertainties of obtaining the quantities needed in equations (1)–(11b) from observations. Also note that  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  along the other two legs are negative,

which cannot be shown in the logarithmic space. The reason for the negative values is that  $n$  along each of these two legs is smaller than  $n_i$ , which may be related to the uncertainties of the cloud properties determined from observations, such as  $n_a$ . Freud et al. [2011] pointed out that  $n_a$  determined by observed maximum number concentration may be sensitive to small-scale processes (e.g., local strong updraft near cloud base, since  $n_a$  is sensitive to updraft [e.g., Lu et al., 2012a]) and be affected by the extent of dilution that the measured cloud has experienced; they developed a new approach to estimate  $n_a$  but this approach is not applicable to the stratocumulus clouds in this study, because applying this approach requires penetrations at different levels in convective clouds. Fortunately, these two legs with negative homogeneous mixing degrees have small  $N_{La}$  and  $N_{L0}$  ( $<10$ ) and the data points for these legs would be located in the bottom left corner if Figure 2 was plotted in the linear space, still supporting the positive relationships of  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$  to  $N_{La}$ ,  $N_{L0}$ . Also note that the values of neither  $N_{La}$  nor  $N_{L0}$  are equal to the scale numbers used in Lu et al. [2011]; the reason is that the number concentration and mean radius used in Lu et al. [2011] were instantaneous values observed by aircraft at 1 Hz; after instantaneous scale number was calculated, the values were averaged to the leg-averaged scale number. This is different from the calculations of  $N_{La}$  and  $N_{L0}$ . For example, the number concentration and droplet size used to calculate  $N_{La}$  are adiabatic number concentration and adiabatic volume mean radius, respectively.

### 3.2. Numerical Results

[20] As discussed above, some properties (e.g.,  $n_a$ ) determined from observations have uncertainties that affect the calculations of  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $N_{La}$ , and  $N_{L0}$ , and thus their relationships. To further examine the relationships in detail, the Explicit Mixing Parcel Model (EMPM) is employed.

#### 3.2.1. Model Description and Simulation Parameters

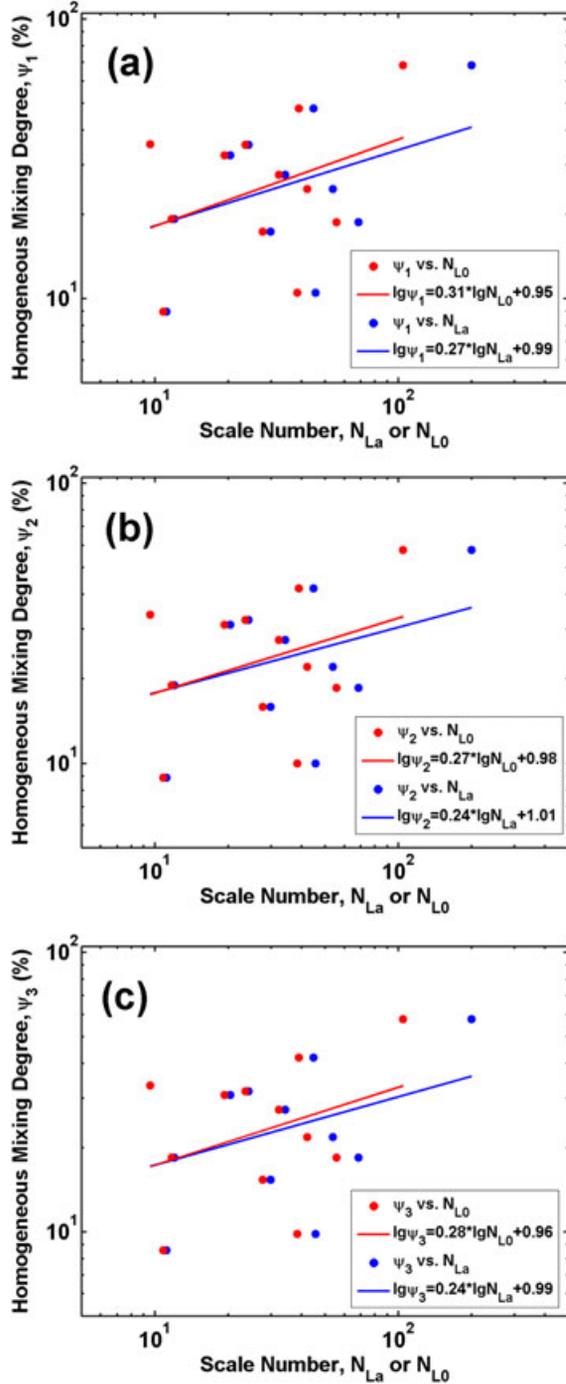
[21] The EMPM was developed by Krueger et al. [1997]; Su et al. [1998] further included individual droplet growth in the model. The model depicts the fine-scale internal structure of a rising parcel using a 1D domain. The internal

**Table 1.** General Characteristics Above Cloud Tops and Along Horizontal Aircraft Legs in Stratocumulus During the 2000 Cloud Intensive Observation Period

Case	Leg	$T_e$ (K)	$RH_e$ (%)	$p$ (hPa)	$q_{La}$ (g kg <sup>-1</sup> )	$T_a$ (K)	$n_a$ (cm <sup>-3</sup> )	$r_{va}$ (μm)	$q_L$ (g kg <sup>-1</sup> )	$n$ (cm <sup>-3</sup> )	$r_v$ (μm)	$f$
03 March 2000	1	273.6	0.61	879.2	0.61	270.8	470.6	7.1	0.28	240.9	7.5	0.21
03 March 2000	2	273.6	0.61	882.8	0.39	271.0	501.3	6.0	0.23	330.9	6.2	0.12
03 March 2000	3	273.6	0.61	889.8	0.48	271.5	491.1	6.4	0.20	270.4	6.2	0.19
03 March 2000	5	273.6	0.61	849.1	0.62	269.6	403.0	7.4	0.35	245.6	7.4	0.16
17 March 2000	1	268.1	0.62	850.2	0.19	277.0	89.4	8.2	0.13	55.3	8.4	0.14
17 March 2000	2	268.1	0.62	913.9	0.61	274.2	331.0	8.0	0.30	205.3	7.6	0.27
19 March 2000	1	275.4	0.42	877.6	0.56	271.0	443.9	7.0	0.21	190.4	7.3	0.16
19 March 2000	2	275.4	0.42	893.9	0.36	272.0	489.1	5.9	0.15	215.8	5.8	0.11
19 March 2000	3	275.4	0.42	900.2	0.41	272.1	640.8	5.6	0.16	240.1	5.6	0.12
19 March 2000	4	275.4	0.42	893.3	0.39	273.5	531.5	5.8	0.14	250.5	5.5	0.13
19 March 2000	5	275.4	0.42	886.7	0.40	273.3	489.0	6.0	0.19	278.8	5.7	0.11
19 March 2000	6	275.4	0.42	896.5	0.34	273.0	545.5	5.6	0.15	270.0	5.4	0.10
19 March 2000	7	275.4	0.42	906.7	0.31	273.5	537.9	5.4	0.13	287.5	5.0	0.10
21 March 2000	1	272.2	0.27	728.2	1.03	275.8	204.5	10.3	0.49	102.7	10.6	0.20

<sup>a</sup>With air temperature  $T_e$  and relative humidity  $RH_e$  above the cloud tops, air pressure  $p$ , adiabatic liquid water mixing ratio  $q_{La}$ , temperature  $T_a$  corresponding to  $q_{La}$ , adiabatic droplet number concentration  $n_a$ , adiabatic volume mean radius  $r_{va}$ , average liquid water mixing ratio  $q_L$ , average droplet number concentration  $n$ , average volume mean radius  $r_v$ , and mixing fraction of dry air  $f$  along different horizontal legs.

<sup>b</sup>The leg numbers are the same as those in Figure 1 in Lu et al. [2011].



**Figure 2.** Observational relationships between the (a–c) three measures of homogeneous mixing degree ( $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ) and the two scale numbers ( $N_{La}$ ,  $N_{L0}$ ), respectively, along 12 horizontal aircraft legs in stratocumulus clouds observed during the March 2000 Cloud Intensive Observation Period. Two legs with negative homogeneous mixing degree are not shown in the logarithmic space. See text for the details.

structure evolves in the model as a consequence of discrete entrainment events and explicit turbulent mixing based on the linear eddy model developed by Kerstein [1988; 1992]. As summarized by Krueger [2008], the model works as follows. First, the parcel ascends adiabatically above cloud base, while the droplets grow by condensation. Second,

when entrainment occurs, the subsaturated entrained air replaces a same-sized segment of the cloudy parcel. Third, the cloudy air and the newly entrained air undergo a finite rate turbulent isobaric mixing process, during which many droplets encounter the entrained subsaturated air, resulting in partial or even total evaporation of some droplets.

[22] Table 2 summarizes the input parameters of the model. In this study, the cloud base and environmental information are taken from Hawaiian trade cumulus cloud measurements [Raga *et al.*, 1990]. The cloud base pressure, temperature, and water vapor mixing ratio are 964.0 hPa, 293.6 K, and  $15.7 \text{ g kg}^{-1}$ , respectively. The  $n_a$  is  $102.7 \text{ cm}^{-3}$ ; these droplets are randomly assigned to the 20 m (width)  $\times$  0.001 m (height)  $\times$  0.001 m (depth) model domain. The entrainment level is set at the pressure of 888.9 hPa where the temperature and relative humidity in the entrained air are 289.3 K and 88%, respectively. Updraft is set to be a constant,  $2 \text{ m s}^{-1}$ , before the entrainment level; after that the parcel stops rising and isobaric mixing occurs. The grid size is set to be  $0.0017 \text{ m} \times 0.001 \text{ m} \times 0.001 \text{ m}$ . The time step of model output is 0.75 s.

[23] To explore the relationship between the homogeneous mixing degree and the transition scale number, we perform a suite of simulations with different combinations of model parameters:  $n_a$  is set to be 102.7, 205.4, 308.1, 410.8, and  $513.5 \text{ cm}^{-3}$ ; relative humidity (RH) is set to be 11%, 22%, 44%, 66%, and 88%;  $\varepsilon$  is set to be  $1 \times 10^{-5}$ ,  $5 \times 10^{-4}$ ,  $1 \times 10^{-3}$ ,  $5 \times 10^{-3}$ ,  $1 \times 10^{-2}$ , and  $5 \times 10^{-2} \text{ m}^2 \text{ s}^{-3}$ ; blob number is set to be 2–9 and the entrained blob size is  $2 \text{ m} \times 0.001 \text{ m} \times 0.001 \text{ m}$ ; so mixing fraction of dry air is 0.2–0.9. When the blob number is 1, the mixing fraction of dry air is small and most of the domain is occupied by adiabatic cloud. In the adiabatic cloud, droplet size increases due to residual supersaturation. As a result,  $r_v$  over the domain increases, especially for higher RH. To minimize this effect and focus on the entrainment-mixing processes, the blob number starts from 2.

### 3.2.2. Results from EMPM

[24] Figure 3 compares the relationships of homogeneous mixing degree measures ( $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ) to the transition scale numbers ( $N_{La}$ ,  $N_{L0}$ ). In every entrainment and isobaric mixing process, liquid water mixing ratio decreases sharply. When the liquid water mixing ratio stops decreasing for a 5 s period, it is assumed that a new saturation level is achieved; the  $n$  and  $r_v$  in the domain are considered to be the final values used in the calculations of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$ . The values of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  are not sensitive to the waiting period of “5 s.” Investigations of  $\psi_1$  with a waiting period of 3 s and 10 s have relative differences of 0.5–1% when compared to the control of 5 s. In some sensitivity tests, clouds completely evaporate due to the large mixing fraction of dry air and/or low relative humidity (e.g., mixing fraction of dry air from 0.3 to 0.9, RH=11%); these cases are not included in Figure 3. Positive relationships between the three measures of homogeneous mixing degree and the scale numbers are found in two regimes: RH=11%, 22%, 44%, and 66% and RH=88%. The reason for the two regimes is that different conditions are satisfied in solving equations (11a) and (11b) to obtain  $\tau_{\text{react}}$  and then  $N_L$ ; for RH=11%, 22%, 44%, and 66% and for RH=88%, the first condition “droplets have completely evaporated” and the second condition “relative humidity has reached 99.5%” are satisfied, respectively. For RH=88%,

**Table 2.** Input Parameters for the EMPM

	Parameters	Values
Cloud base conditions	Cloud base pressure (hPa)	964.0
	Cloud base temperature (K)	293.6
	Cloud base water vapor mixing ratio ( $\text{gkg}^{-3}$ )	15.7
	Adiabatic droplet number concentration, $n_a$ ( $\text{cm}^{-3}$ )	102.7, 205.4, 308.1, 410.8, 513.5
Entrainment conditions	Domain size (m)	$20 \times 0.001 \times 0.001$
	Entrainment level pressure (hPa)	888.9
	Entrained air temperature (K)	289.3
	Entrained air relative humidity, RH (%)	11, 22, 44, 66, 88
	Vertical velocity	Before mixing $2 \text{ms}^{-1}$ , after mixing $0 \text{ms}^{-1}$ .
	Grid size (m)	0.0017
	Dissipation rate, $\epsilon$ ( $\text{m}^2 \text{s}^{-3}$ )	$1 \times 10^{-5}$ , $5 \times 10^{-4}$ , $1 \times 10^{-3}$ , $5 \times 10^{-3}$ , $1 \times 10^{-2}$ , $5 \times 10^{-2}$
	Entrained air blob size ( $\text{m}^3$ )	$2 \times 0.001 \times 0.001$
	Entrained air blob number	2, 3, 4, 5, 6, 7, 8, 9
Mixing fraction of dry air, $f$	0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9	

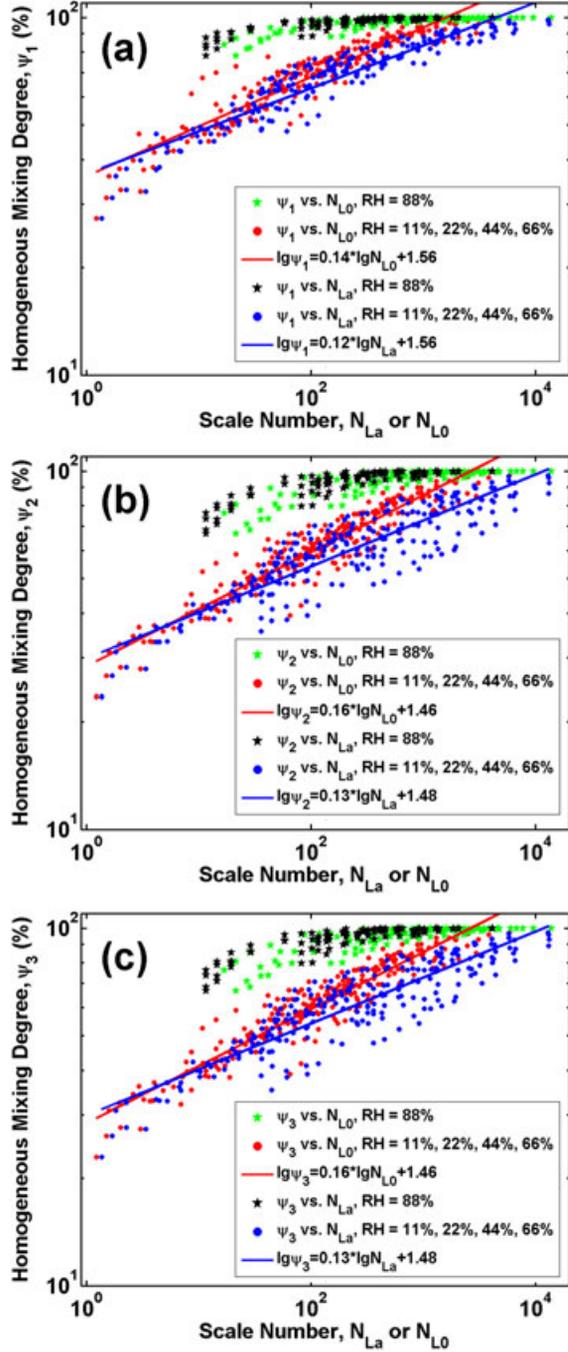
most of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  are close to 100%; it is less important to distinguish homogeneous and inhomogeneous mixing when entrained dry air has a high relative humidity since evaporation is relatively small [Lehmann *et al.*, 2009; Devenish *et al.*, 2012; Slawinska *et al.*, 2012]. Only when mixing fraction of dry air is large, it becomes important to distinguish mixing types; for example, as shown in Figure 3b, some green and black points (RH=88%) with  $f=0.5$  have  $\psi_2$  around 70%. This is supported by Figure 1 in Freud *et al.* [2011], where the homogeneous mixing line for RH=90% is close to the extreme inhomogeneous mixing line when adiabatic fraction (the ratio of liquid water content to its adiabatic value) is larger than 0.4; only when adiabatic fraction becomes smaller (i.e., mixing fraction of dry air becomes larger), e.g., 0.2, the homogeneous mixing line deviates quite remarkably from the extreme inhomogeneous mixing line. Here we will focus on the regime with RH=11%, 22%, 44%, and 66%; the best fitting lines are also shown in Figure 3.

[25] As shown in Figure 3,  $\psi_2$  versus  $N_L$  and  $\psi_3$  versus  $N_L$  are close to each other, while  $\psi_1$  versus  $N_L$  has the tightest relationship. To be clearer, Figure 4 shows that  $\psi_2$  and  $\psi_3$  are close to each other, and  $\psi_1$  has larger values than  $\psi_2$  (and  $\psi_3$ ). Since the upper limit of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  is 100%,  $\psi_1$  has a smaller range than  $\psi_2$  and  $\psi_3$ , and  $\psi_1$  versus  $N_L$  is the tightest. The physical reason for the tightest  $\psi_1$  versus  $N_L$  may be that the calculation of  $\psi_1$  needs one more constraint (i.e.,  $n_a$ ) than  $\psi_2$  and  $\psi_3$  [equations (1)–(8)]. A tight relationship is important for a parameterization; thus it is suggested to use  $\psi_1$  if  $n_a$  is available [ $n_a$  is necessary in equation (1)], otherwise, use  $\psi_2$  or  $\psi_3$ . Figure 3 also shows that the relationships of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  to  $N_{L0}$  are tighter than those of  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  to  $N_{La}$ , which is especially true for  $\psi_2$  and  $\psi_3$ . To figure out the reasons, Figure 5 compares  $\psi_3$  versus  $N_{La}$  and  $\psi_3$  versus  $N_{L0}$  with a mixing fraction of dry air changing from 0.2 to 0.5; others being equal, the  $\psi_3$  is smaller with a larger mixing fraction of dry air; for example,  $\psi_3$  decreases from “1” to “2” or from “I” to “II.” This is consistent with observations in cumuli analyzed by Burnet and Brenguier [2007]; they found that mixing approached the homogeneous type when mixing fraction of dry air was small and approached the inhomogeneous type when mixing fraction of dry air was large. Similar conclusion can also be drawn from Figures 5a and 5b in Lehmann *et al.* [2009]. Other studies have also proven that mixing fraction of dry

air is critical for mixing processes [Jensen and Baker, 1989; Hicks *et al.*, 1990; Schlüter, 2006; Jeffery, 2007].  $N_{La}$  cannot reflect the role of dry air mixing fraction but  $N_{L0}$  can. The reason is in the calculation of  $N_{La}$ , the number concentration used in the calculation of  $B$  in equations (11a) and (11b) is the adiabatic number concentration which is independent of mixing fraction; as for  $N_{L0}$ , the number concentration used is the number concentration just after entrainment, which considers the mixing fraction. As a result, the change from “I” to “II” is along a vertical line while that from “1” to “2” is toward the bottom left corner of Figure 5; thus the relationship of  $\psi_3$  versus  $N_{L0}$  (the red points and squares) is tighter than that of  $\psi_3$  versus  $N_{La}$  (the green points and squares). Thus it is suggested to use  $N_{L0}$  instead of  $N_{La}$  in the future studies.

[26] Although the relationships are tighter with  $N_{L0}$ , dispersion of the data points in Figure 3 still exists. To further determine the influencing factors, the effects of dissipation rate, relative humidity, adiabatic number concentration, and mixing fraction of dry air are examined. As exemplified in Figure 6, in every group (e.g., “ $\psi_1$  versus  $N_{L0}$ , RH=11%” in Figure 6a), the dissipation rates of the six data points from left to right are  $1 \times 10^{-5}$ ,  $5 \times 10^{-4}$ ,  $1 \times 10^{-3}$ ,  $5 \times 10^{-3}$ ,  $1 \times 10^{-2}$ , and  $5 \times 10^{-2} \text{m}^2 \text{s}^{-3}$ , respectively. Smaller dissipation rates correspond to smaller homogeneous mixing degree values, because a smaller dissipation rate means a slower mixing process, which is favorable for inhomogeneous mixing process [e.g., Baker *et al.*, 1984]. Comparison of the results with RH=66% and 11% shows that lower RH causes smaller  $\psi$  and a larger slope of  $\psi$  versus  $N_{L0}$  (Figure 6a); lower RH means faster evaporation of droplets, making mixing more likely inhomogeneous [e.g., Siebert *et al.*, 2006]. Figure 6b shows that  $\psi$  is smaller for a larger adiabatic droplet number concentration, because a larger adiabatic number concentration causes smaller droplets, increasing the likelihoods of complete evaporation and extreme inhomogeneous mixing [e.g., Hill *et al.*, 2009]. As discussed above, larger mixing fraction of dry air causes more inhomogeneous mixing, i.e., smaller  $\psi$  (Figures 5 and 6c). These factors altogether affect the relationships between the homogeneous mixing degree and the scale number.

[27] In addition, the combination of  $N_{L0}$  and a new approach for estimating entrainment rate ( $\lambda$ ) [Lu *et al.*, 2012b, 2012c] presents a potential to link the two sides of entrainment-mixing processes, i.e., entrainment rate and

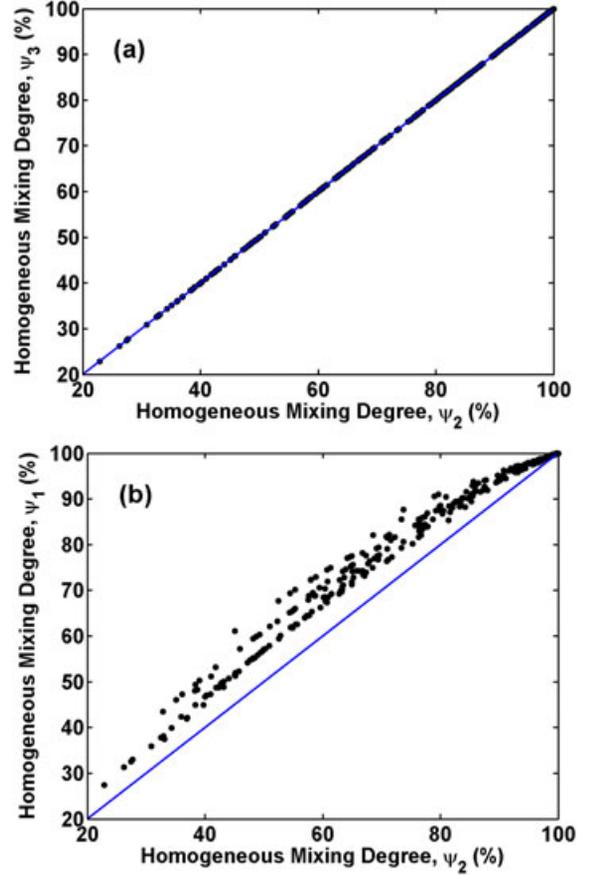


**Figure 3.** The relationships between the (a–c) three measures of homogeneous mixing degree ( $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ) and the two scale numbers ( $N_{La}$ ,  $N_{L0}$ ), respectively, under all kinds of conditions as listed in Table 2 except the cases where clouds completely dissipate.

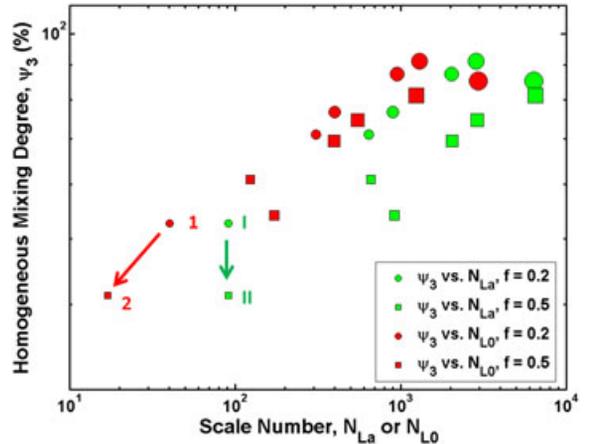
the entrainment-mixing mechanisms. This new approach is directly derived from the definition of  $\lambda$  and relates it to  $f$  and the height above cloud base ( $h$ ).

$$\lambda = -\frac{\ln(1-f)}{h} \quad (14)$$

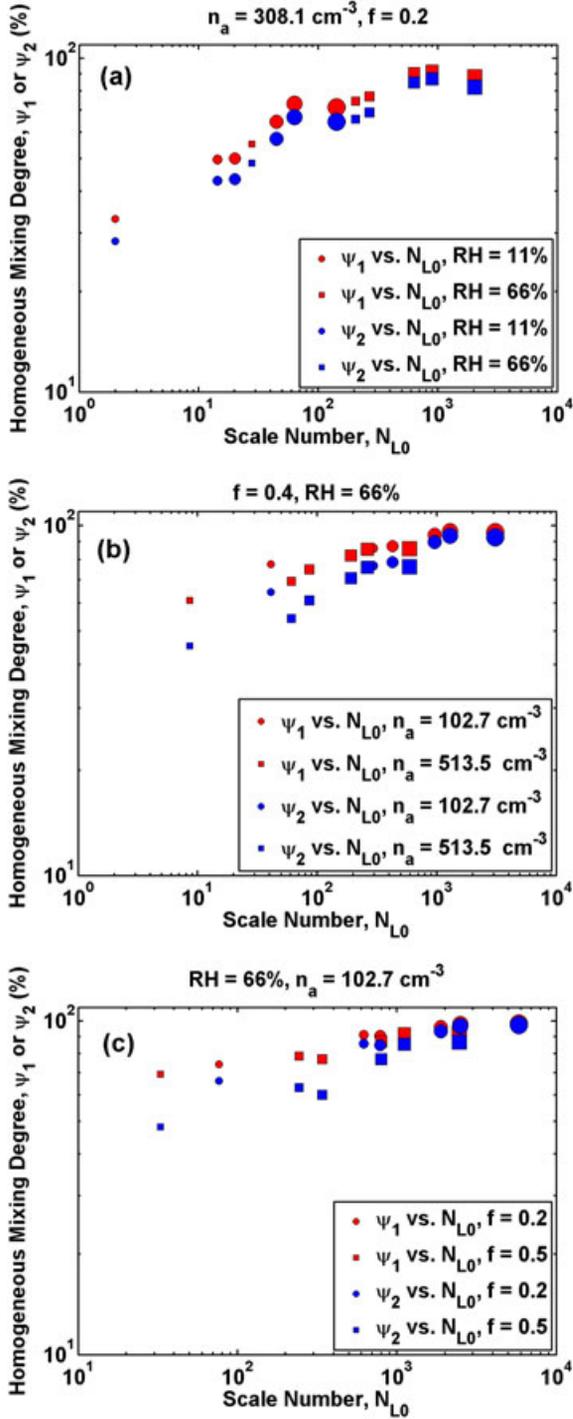
[28] Since in the current simulation, the model domain does not move upward during mixing, so  $h$  is fixed for



**Figure 4.** (a) The relationship between the third homogeneous mixing degree ( $\psi_3$ ) and the second homogeneous mixing degree ( $\psi_2$ ); (b) the relationship between the first homogeneous mixing degree ( $\psi_1$ ) and  $\psi_2$ .



**Figure 5.** The relationships between the homogeneous mixing degree ( $\psi_3$ ) and the two scale numbers ( $N_{La}$ ,  $N_{L0}$ ), respectively, with two mixing fractions of dry air ( $f$ ) being 0.2 and 0.5. The relative humidity (RH) and adiabatic number concentration ( $n_a$ ) are 66% and  $205.4\text{cm}^{-3}$ , respectively; in every group (e.g.,  $\psi_3$  versus  $N_{La}$ ,  $f=0.2$ ), dissipation rates are  $1 \times 10^{-5}$ ,  $5 \times 10^{-4}$ ,  $1 \times 10^{-3}$ ,  $5 \times 10^{-3}$ ,  $1 \times 10^{-2}$ , and  $5 \times 10^{-2}\text{m}^2\text{s}^{-3}$  for the six data points (from left to right and from small to big size), respectively.



**Figure 6.** (a) Relationships between two measures of homogeneous mixing degree ( $\psi_1$ ,  $\psi_2$ ) and the scale number ( $N_{L0}$ ), respectively, with relative humidity (RH) being 11% and 66%; adiabatic number concentration ( $n_a$ ) and mixing fraction of dry air ( $f$ ) are  $308.1 \text{ cm}^{-3}$  and 0.2, respectively. (b) Relationships between  $\psi_1$ ,  $\psi_2$ , and  $N_{L0}$ , respectively, with  $n_a$  being 102.7 and  $513.5 \text{ cm}^{-3}$ ;  $f$  and RH are 0.4 and 66%, respectively. (c) Relationships between  $\psi_1$ ,  $\psi_2$ , and  $N_{L0}$ , respectively, with  $f$  being 0.2 and 0.5; RH and  $n_a$  are 66% and  $102.7 \text{ cm}^{-3}$ , respectively. In every group (e.g.,  $\psi_1$  versus  $N_{L0}$ , RH=11%), dissipation rates are  $1 \times 10^{-5}$ ,  $5 \times 10^{-4}$ ,  $1 \times 10^{-3}$ ,  $5 \times 10^{-3}$ ,  $1 \times 10^{-2}$ , and  $5 \times 10^{-2} \text{ m}^2 \text{ s}^{-3}$  for the six data points (from left to right and from small to big size), respectively.

different simulations and  $\lambda$  is determined only by  $f$ . Since  $\lambda$  increases with increasing  $f$ , the effect of  $\lambda$  on the relationship of  $\psi$  versus  $N_L$  (not shown) is similar to the effect of  $f$  as shown in Figure 6c. Future study will go beyond single isobaric mixing event since the EMPM and the new approach are applicable for multiple mixing events at different heights, which is believed to provide more insights on the link between the two sides of entrainment-mixing processes.

#### 4. Summary

[29] The relationship between the homogeneous mixing degree and the transition scale number is examined with in situ cloud observations over the U.S. Department of Energy's Atmospheric Radiation Measurement Southern Great Plains site during the March 2000 Cloud Intensive Observation Period and numerical simulations using the Explicit Mixing Parcel Model. Three measures of homogeneous mixing degree are newly defined and two transition scale numbers are used. As theoretically expected, the three homogeneous mixing degrees are all positively related to the two scale numbers based on both the observations and simulations. Numerical simulations further show that among the three measures of homogeneous mixing degree, it is suggested to use the first homogeneous mixing degree ( $\psi_1$ ) if adiabatic number concentration ( $n_a$ ) is available, because  $\psi_1$  has the tightest relationship with the two scale numbers; otherwise, use the second or third homogeneous mixing degree ( $\psi_2$  or  $\psi_3$ ) instead. As to transition scale numbers, it is suggested to use  $N_{L0}$  instead of  $N_{La}$  in the future studies of entrainment-mixing processes because the relationships of  $N_{L0}$  to the three homogeneous mixing degree measures are tighter than the relationships of  $N_{La}$  to the three measures. The reason is that  $N_{L0}$  considers mixing fraction of dry air while  $N_{La}$  does not. Besides this advantage of  $N_{L0}$ , the combination of  $N_{L0}$  and a new entrainment rate estimation approach [Lu *et al.*, 2012c] presents a potential to link the two sides of entrainment-mixing processes (homogeneous/inhomogeneous entrainment-mixing mechanisms and entrainment rate).

[30] The relationship between the homogeneous mixing degree and transition scale number also suggests a new parameterization for entrainment-mixing processes in models with two-moment microphysical schemes. The transition scale number can be calculated from the microphysical properties [volume mean radius ( $r_{va}$ ) and adiabatic number concentration ( $n_a$ ) or number concentration just after dilution ( $n_0$ )] and meteorological elements before mixing and evaporation for every cloud grid at every time step in models; the degree of homogeneous mixing can be estimated from the transition scale number based on the best fitting lines in Figure 3; with the homogeneous mixing degree and equations (2), (3), or (6), the values of number concentration ( $n$ ) and volume mean radius ( $r_v$ ) after the mixing and evaporation can be calculated.

[31] Three points are noteworthy. First, the relationship between the microphysical homogeneous mixing degree and transition scale number is obtained from in situ observations and numerical simulations in this study. Further theoretical exploration is needed to improve our understanding and quantification of the relationship. Second, all isobaric mixing events in this study are assumed to be between an adiabatic parcel and dry air. This assumption may make the mixing look more homogeneous than reality as secondary mixing events

may start with droplets smaller than the droplets in an adiabatic parcel. In future work, we will decrease the size of one dry blob and assume that the number of entraining dry air blobs during multiple successive entrainment events can be described by a stochastic function, such as a Gaussian distribution, since entrainment mixing is a stochastic process [Romps and Kuang, 2010]. Thus the effect of secondary mixing may be examined in detail. Third, the evolution of droplet size distributions during entrainment mixing processes should be further studied, because the  $r_v$ - $n$  relationship has its own limitation. Even in fully homogeneous mixing, the smallest droplets may fully evaporate and  $n$  may be smaller than the number concentration during homogeneous mixing ( $n_h$ ); as a result,  $r_v$  may be slightly larger than volume mean radius during homogeneous mixing ( $r_{vh}$ ) [Freud et al., 2011].

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