Scale dependence of entrainment-mixing mechanisms in cumulus clouds

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Abstract

This work empirically examines the dependence of entrainment-mixing mechanisms on the averaging scale in cumulus clouds using in situ aircraft observations during the Routine Atmospheric Radiation Measurement Aerial Facility Clouds with Low Optical Water Depths Optical Radiative Observations (RACORO) field campaign. A new measure of homogeneous mixing degree is defined that can encompass all types of mixing mechanisms. Analysis of the dependence of the homogenous mixing degree on the averaging scale shows that, on average, the homogenous mixing degree decreases with increasing averaging scales, suggesting that apparent mixing mechanisms gradually approach from homogeneous mixing to extreme inhomogeneous mixing with increasing scales. The scale dependence can be well quantified by an exponential function, providing first attempt at developing a scale-dependent parameterization for the entrainment-mixing mechanism. The influences of three factors on the scale dependence are further examined: droplet-free filament properties (size and fraction), microphysical properties (mean volume radius and liquid water content of cloud droplet size distributions adjacent to droplet-free filaments), and relative humidity of entrained dry air. It is found that the decreasing rate of homogeneous mixing degree with increasing averaging scales becomes larger with larger droplet-free filament size and fraction, larger mean volume radius and liquid water content, or higher relative humidity. The results underscore the necessity and possibility of considering averaging scale in representation of entrainment-mixing processes in atmospheric models.

1. Introduction

Clouds have been considered as a major source of uncertainty in climate models because most cloud-related processes need to be represented with poorly understood parameterizations [Cess et al., 1988; Bony and Dufresne, 2005; Stephens, 2005; Wang et al., 2014; Zhang et al., 2014]. Among those processes that are parameterized, the poorest is the turbulent entrainment-mixing process [Liu et al., 2002; von Salzen and McFarlane, 2002; Zhang, 2009; Romps and Kuang, 2010; de Rooy et al., 2013], although it has long been recognized that entrainment-mixing processes affect warm rain initiation, aerosol indirect effect, cloud-climate feedback, and remote sensing of cloud microphysical properties [Paluch and Baumgardner, 1989; Blyth, 1993; Yum, 1998; Xue and Feingold, 2006; Kim et al., 2008; Del Genio and Wu, 2010; Ghan et al., 2011; Devenish et al., 2012; Kumar et al., 2012; Lu et al., 2013a].

Turbulent entrainment-mixing processes have been often studied with many conceptual models. The most used one is the homogeneous/inhomogeneous model [Baker et al., 1980, 1984; Freud et al., 2011]. In homogeneous entrainment-mixing process, all droplets are exposed to the same condition and simultaneously evaporate when dry air is entrained into cloud; in the extreme inhomogeneous entrainment-mixing process, some droplets completely evaporate, while other droplets are not affected. Homogeneous mixing scenario is found more common in shallow cumulus clouds [Jensen et al., 1985; Gerber et al., 2008; Lu et al., 2013c], whereas the inhomogeneous mixing scenario is more likely in stratocumulus clouds [Pawlowska et al., 2000; Burnett and Brenguier, 2007; Hannan et al., 2007; Lu et al., 2011]. Lehmann et al. [2009] pointed out that it was unclear whether the entrainment-mixing mechanism was predominantly homogeneous, inhomogeneous, or in between. Numerical simulations [Andrejczuk et al., 2009] and observations [Lehmann et al., 2009; Lu et al., 2011] showed that entrainment-mixing processes often fall between the above two extremes.

The unsettling situation is also reflected in various modeling studies that examine the impacts of entrainment-mixing processes on cloud microphysical and optical/radiative properties. Some studies found...
that assuming different entrainment-mixing mechanisms caused a significant impact on cloud albedo [Grabowski, 2006; Chosson et al., 2007; Slawinska et al., 2008], and formation of large drops [Lasher-Trap et al., 2005]. Morrison and Grabowski [2008] found that changing the entrainment-mixing mechanisms from the homogeneous to extreme inhomogeneous affected cloud microphysics and optical thickness, but such an impact was not as dramatic as in the simulations discussed by Chosson et al. [2007] and Grabowski [2006]. Hill et al. [2009] found that assuming different entrainment-mixing mechanisms caused a small difference in cloud microphysics and optical depth. However, both Morrison and Grabowski [2008] and Hill et al. [2009] pointed out that the effect of mixing mechanisms could be more significant for simulations over the entire cloud life cycle, especially during dissipation, when mixing processes are expected to dominate. Slawinska et al. [2012] found that the impact of the different entrainment-mixing mechanisms was significantly reduced, possibly due to the counteracting impacts of the subgrid-scale mixing and in-cloud activation, the mean characteristics of the entrained dry air, and numerical diffusion. In the above simulations, a given entrainment-mixing mechanism was assumed. Recently, Jarecka et al. [2013] explicitly treated mixing scenario in clouds. They found that the simulated homogeneity of mixing had a small impact on mean microphysical characteristics, which could be explained by the high humidity of the dry air involved in the subgrid-scale mixing processes.

A major challenge confronting the study of turbulent entrainment-mixing processes is that the related processes occur over a tremendous range of scales, from a cloud size down to the Kolmogorov microscale [Su et al., 1998]. In particular, Burnet and Brenguier [2007] found, using a stochastic model of entrainment mixing, that homogeneous entrainment-mixing mechanisms may appear to be extreme inhomogeneous due to the existence of droplet-free filaments and spatial averaging during measurements. Our recent observational study of the relationship between temperature and cloud droplet number concentration also suggested that the existence of droplet-free filament structure and spatial averaging during sampling partially contributed to the dominance of extreme inhomogeneous mixing in the stratocumulus clouds [Lu et al., 2011].

Further quantifying the scale dependence is obviously crucial for improving parameterization of entrainment-mixing processes in models of various resolutions. However, to the authors’ knowledge, there has been no systematic investigation on the scale dependence using observational data, especially on factors influencing the scale dependence.

The objective of this work is to fill this gap by analyzing the data collected during the Routine Atmospheric Radiation Measurement (ARM) Aerial Facility Clouds with Low Optical Water Depths Optical Radiative Observations (RACORO) field campaign over the ARM Southern Great Plains site near Lamont, Oklahoma, from 22 January to 30 June 2009 [Vogelmann et al., 2012].

2. RACORO Data and Analysis

The data set is the same as that used by Lu et al. [2012a], i.e., eight shallow cumulus flights (22 May, 23 May, 24 May, 11 June, 19 June, 23 June, 24 June, and 26 June 2009). The Twin Otter aircraft from the Center for Interdisciplinary Remotely Piloted Aircraft Studies made comprehensive observations. Cloud droplet size distributions (CDSDs) were measured by Cloud and Aerosol Spectrometer (CAS) with 10 Hz and Forward Scattering Spectrometer Probe (FSSP) with 1 Hz. The CAS probe measures aerosol particles and cloud droplets in 20 bins from 0.29 to 25 μm (radius), and the FSSP measures cloud droplets in 20 bins from 1.1 to 15.1 μm (radius). The calibrations of the instruments were carried out with spherical glass and polystyrene beads. The difference in optical properties of the glass and polystyrene beads as compared to water was taken into account in the calibration process. Figure 1 shows the comparison of liquid water content (LWC) from the two instruments at 1 Hz in the eight flights. The LWC from the CAS is calculated using droplets with bin average radius larger than 1 μm, and the LWC from the FSSP is calculated using all bins. Statistically, the LWC from the two independent measurements is consistent with each other, which gives confidence on the accuracy of the measurements. Since the CAS has a higher sampling rate than the FSSP, here only the results from the CAS will be used. Other cloud microphysical properties are also calculated using droplets with bin average radius larger than 1 μm from the CAS. The Cloud Imaging Probe (CIP) was used to measure droplets within 7.50–781 μm (radius) at a sampling rate of 1 Hz. A Rosemount probe and the Diode Laser Hygrometer (DLH) [Diskin et al., 2002; Podolske et al., 2003] were used to measure temperature and water vapor at a sampling rate of 10 Hz, respectively. Vertical velocity measurements were obtained with a five-hole gust probe on the nose of the Twin Otter.
The criteria for selecting cloud records are LWC > 0.001 g m$^{-3}$ and number concentration ($n$) > 10 cm$^{-3}$; the measured size distributions that are probably composed of large aerosols instead of cloud droplets can be eliminated by applying both criteria [Deng et al., 2009]. The sampling area of the CAS is 11.1 mm $\times$ 120 $\mu$m, and the true air speed is $\sim$50 m s$^{-1}$. So the sampling volume at 10 Hz is 11.1 mm $\times$ 120 $\mu$m $\times$ 50 m s$^{-1}$ $\times$ 0.1 s, i.e., 6.66 cm$^{-3}$. When the number concentration is 10 cm$^{-3}$, the number of droplets in the sampling volume is 66.6. Thus, the observations of each CDSD and microphysical properties should be reliable based on 66.6 droplets. The in-cloud mean CIP drizzle LWC (radius > 25 $\mu$m) over the observation period smaller than 0.005 g m$^{-3}$ is the criterion to identify nondrizzling clouds.

As stated in the paper by Lu et al. [2012a], only the data collected along horizontal legs are used. Nondrizzling growing cumulus clouds along a leg are selected with the following criteria: (1) CDSDs are thought to be in an individual cumulus cloud when the distance between them is less than 50 m; (2) 80% of vertical velocity in an individual cloud is positive [Gerber et al., 2008; Lu et al., 2012c]; (3) to select relatively large clouds, the number of CDSDs must be larger than 30; and (4) a cloud must be far enough from other clouds as determined with the following procedure. The temperature and water vapor mixing ratio in the environment are the mean values from the air that is $D$ to 2$D$ from the edge of the cloud core on both sides of the aircraft’s cloud penetration. $D$ can be thought of representing the grid size within a high-resolution model and is set to be 10, 20, 30, 40, 50, 100, 300, and 500 m. The edge of a cloud core is defined as the point where vertical velocity changes from negative to positive for the first time, going from the cloud edge toward the cloud interior (see Figure 1a in Lu et al. [2012a] for details). If the edge of a cloud core is within 3$D$ from the edge of another cloud core on both the left and right sides, then this cloud is discarded. The selected clouds must satisfy the fourth criterion for different $D$ values at the same time. See Lu et al. [2012a] for more explanations on $D$ and cloud selection. A total of 186 growing cumulus clouds satisfy all the four criteria. Cloud depths of the 186 cumulus clouds are typically ~200–500 m [Vogelmann et al., 2012]. The observation legs could be close to cloud top, in the middle of cloud, and close to cloud base. The mean droplet concentration and its standard deviation are 923 cm$^{-3}$ and 445 cm$^{-3}$, respectively; the mean liquid water content and its standard deviation are 0.2 g m$^{-3}$ and 0.2 g m$^{-3}$, respectively.

### 3. Definition of Homogeneous Mixing Degree and Its Calculation

As stated in the Introduction, entrainment-mixing mechanisms often fall between the two extremes—homogeneous mixing and extreme inhomogeneous mixing [Lehmann et al., 2009; Lu et al., 2011]. A continuous measure is desirable that can encompass all types of mixing mechanisms. Lu et al. [2013b] defined such a measure named as homogeneous mixing degree ($\psi$) based on the microphysical mixing diagram of $r_{cv}^3/n_{cv}^3$ versus $n_r/n_a$, where $r_{cv}$ and $r_{av}$ are the mean volume radius and adiabatic mean volume radius, respectively; $n_r$ and $n_a$ are the number concentration and adiabatic number concentration, respectively. Here a mixing diagram of $r_{cv}^3/n_{cv}^3$ versus LWC/LWC$_a$ instead of $n_r/n_a$ is used to define homogeneous mixing degree (Figure 2), where LWC$_a$ and LWC$_c$ are the liquid water content and adiabatic liquid water content, respectively. The reason for replacing $n_r/n_a$ with LWC/LWC$_a$ is to minimize influences of aerosol and vertical velocity, since number concentration is sensitive to them [Freud et al., 2008; Liu et al., 2008; Lu et al., 2012b; Mann et al., 2014] and to emphasize the effects of entrainment-mixing mechanisms. Figure 2 conceptually illustrates the main states involved in an entrainment-mixing event. The states are numbered from 1 to 3. State 1 is an adiabatic state with mean volume radius of $r_{av}$ and liquid water content of LWC$_a$. State 2 is just after entrainment but before mixing and evaporation, which has mean volume radius of $r_{vo}$ and liquid water content of LWC$_{vo}$, where $\chi$ is the...
mixing fraction of adiabatic cloud. State 3 is the state where new saturation is achieved after mixing and evaporation, with mean volume radius of \(r_{va}\) and liquid water content of \(LWC_c\). Homogeneous mixing degree is defined as

\[
\psi = \frac{m_1}{m_2} = \frac{1 - \frac{q_{vs} T_a}{L_v \chi(a_L \chi - q_{ve})}}{1 - \frac{L_w \chi}{L_v \chi}}. \tag{1}
\]

where “\(m_1\)” and “\(m_2\)” represent the length of two lines shown in Figure 2, respectively. This definition is similar to the inhomogeneous fraction defined based on effective radius versus liquid water content diagram in the paper by Gerber et al. [2008]. It is expected that \(\psi\) ranges from 0 to 1 for isobaric entrainment mixing; a larger value of \(\psi\) indicates a higher probability of homogeneous mixing. However, \(\psi\) could be smaller than 0 or larger than 1. For example, a cloud experiences inhomogeneous entrainment mixing below the aircraft horizontal leg; after inhomogeneous entrainment mixing, the diluted cloud is subject to an ascent and achieves the horizontal leg. The droplets in the diluted clouds have larger supersaturation and grow faster than those in adiabatic clouds because of smaller droplet number concentration and less competition for water vapor in diluted clouds. As a result, \(r_s\) is larger than \(r_{va}\) [Baker et al., 1980; Lasher-Trapp et al., 2005; Krueger, 2008; Lehmann et al., 2009; Lu et al., 2011], and \(\psi\) is smaller than 0. In addition, \(r_s\) larger than \(r_{va}\) could also be related to collision coalescence if droplets are large enough. Observation uncertainties of the properties that are needed in the calculation of homogeneous mixing degree may cause \(\psi\) smaller than 0 or larger than 1.

Similar to previous studies [Gerber et al., 2008; Lehmann et al., 2009; Lu et al., 2012c], \(\chi\) is calculated based on the conservation of total water and energy during the isobaric mixing at the aircraft observation level:

\[
q_L + q_{ve}(T) = \chi[q_{ve}(T_a) + q_{la}] + (1 - \chi)q_{ve}, \tag{2a}
\]

\[
c_p T = c_p T_a (1 - \chi) - L_v (q_{ve}(T_a - q_{ve})), \tag{2b}
\]

\[
q_{vs}(T) = 0.622 \frac{e_s(T)}{p - e_s(T)} \tag{2c},
\]

where \(q_{la}, T_a\) and \(q_{ve}(T_a)\) are, respectively, the liquid water mixing ratio, temperature, and saturation vapor mixing ratio in the adiabatic cloud parcel; \(q_{ve}\) and \(T_a\) are, respectively, the water vapor mixing ratio and temperature of the entrained dry air; \(q_s, T, \) and \(q_{ve}(T)\) are, respectively, the liquid water mixing ratio, temperature, and saturation vapor mixing ratio in cloud; and \(L_v, c_p, p, \) and \(e_s\) are, respectively, the latent heat, specific heat capacity at constant pressure, air pressure, and saturation vapor pressure at \(T\). The input quantities for these equations are \(q_L, q_{ve}, q_{ve}(T_a)\), and \(q_{ve}\); the output quantities are \(q_{ve}(T), T_a, \chi\), and \(T_{va}\).

The adiabatic water vapor mixing ratio \(q_{va}\) is derived from \(LWC_{va}\) that is assumed to be the maximum liquid water content within a cumulus cloud core. The water vapor mixing ratio corresponding to \(LWC_{va}\) is taken as the water vapor mixing ratio in the adiabatic cloud \(q_{va}(T_a)\), and the temperature \(T_{va}\) in the adiabatic cloud is calculated from \(q_{va}(T_a)\), assuming saturation in the adiabatic cloud. \(T_{va}\) and \(q_{ve}\) in entrained dry air are the mean values from the air that is \(D\) to \(2D\) from the edge of a cloud core.

In the calculation of \(\psi\), \(r_{va}\) is needed and obtained by

\[
r_{va} = \left(\frac{LWC_{va}}{4\pi n_a \rho} \right)^{1/3}, \tag{3}
\]

where \(\rho\) is the water density and \(n_a\) is assumed to be the maximum number concentration in an individual cloud. Note that there are uncertainties in the estimated values of \(LWC_{va}, n_a\), and \(r_{va}\) and the discussion about such uncertainty effects on \(\psi\) is deferred to section 4.1.
There exist two sources of uncertainty in the homogeneous mixing degree derived above: one from the parameter $c$ that remains in the range of 0.90 to 0.91; the parameter $a$ decreases only slightly from 0.36 to 0.33, and the parameter $b$ changes the most, decreasing from 0.56 to 0.40 when $D$ increases from 10 to 500 m. The variations of $a$ and $b$ cause a decrease in $\psi$ with increasing $D$, which is related to relative humidity in the dry air. As shown in Figure 3 of Lu et al. [2012a], the variation of $D$ manifests primarily in the variation of relative humidity, which significantly decreases as $D$ increases from 10 to 500 m. There is much less variation in temperature, only increasing ~0.7 K as $D$ increases from 10 to 500 m. Detailed analysis on the effect of relative humidity on homogeneous mixing degree is deferred to section 4.2.4. In addition, different clouds have different cloud sizes, horizontal penetration heights above cloud base, cloud dynamics, and different moist shells.

Considering that different clouds may entrain dry air at different values of $D$, we also examine the relationship between $\psi$ and $t$, assuming that the entrained dry air is from 10–1000 m away from the cloud core edge. This relationship exhibits similar scale dependence and is close to that for $D = 300$ m.

There exist two sources of uncertainty in the homogeneous mixing degree derived above: one from the measurement uncertainty of the variables needed as inputs in the calculation of homogeneous mixing degree. The standard error of the mean decreases as the averaging scale increases. Note that for different averaging scales, the sample number of the mean homogeneous mixing degree is the same, i.e., 186, because each cloud has one mean homogeneous mixing degree. When $\psi$ decreases as the averaging scale increases, the difference of $\psi$ among 186 clouds also becomes smaller for the larger averaging scale, correspondingly, the standard error of the mean decreases. The results for other $D$ values are similar and thus not shown.

To further quantify the scale dependence, the relationship between $\psi$ and $t$ in Figure 3 is fitted by the exponential function

$$\psi = a + b \times t^c,$$

where $a$, $b$, and $c$ are three fitting parameters and $t$ is the averaging time. The results for other $D$ values can also be fitted well by equation (4). It is interesting to note that when $D$ increases from 10 m to 500 m, the parameter $c$ remains in the range of 0.90 to 0.91; the parameter $a$ decreases only slightly from 0.36 to 0.33, and the parameter $b$ changes the most, decreasing from 0.56 to 0.40 when $D$ increases from 10 to 500 m. The variations of $a$ and $b$ cause a decrease in $\psi$ with increasing $D$, which is related to relative humidity in the dry air. As shown in Figure 3 of Lu et al. [2012a], the variation of $D$ manifests primarily in the variation of relative humidity, which significantly decreases as $D$ increases from 10 to 500 m. There is much less variation in temperature, only increasing ~0.7 K as $D$ increases from 10 to 500 m. Detailed analysis on the effect of relative humidity on homogeneous mixing degree is deferred to section 4.2.4. In addition, different clouds have different cloud sizes, horizontal penetration heights above cloud base, cloud dynamics, and different moist shells.

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The measurement errors of temperature, water vapor mixing ratio, and liquid water content are ±0.5°C [Podolske et al., 2003], ±3% [Podolske et al., 2003], and ±38% (Darrel Baumgardner, personal communications), respectively. The uncertainty in \( \psi \) is estimated using three values for each input variable. Taking temperature for example, the three values used are the observed temperature and observed ±0.5°C at a given level. The combination of the three variables produces 27 sets of input. The relationship between \( \psi \) and \( t \) for \( D = 50 \) m is plotted (not shown); there are 186 × 27 samples for each averaging time and distance window. The reason to use \( D = 50 \) m is that the relative humidity in the dry air for \( D = 50 \) m is in the middle among the relative humidity for all \( D \) values. The relative humidity for \( D = 10, 50, \) and \( 500 \) m are 91.2%, 84.3%, and 74.5%; the mean value of 91.2% and 74.5% is close to 84.3%. Thus, the result for \( D = 50 \) m should be representative and is used in the later analyses. The relationship between \( \psi \) and \( t \) considering the measurement errors of temperature, water vapor mixing ratio, and liquid water content can be fitted by

\[
\psi = 0.32 + 0.56 \times 0.90^t, \tag{5}
\]

which is quite close to the function in Figure 3

\[
\psi = 0.34 + 0.52 \times 0.90^t, \tag{6}
\]

The standard errors of the mean in the relationship between \( \psi \) and \( t \) considering the measurement errors are in the range of 0.0027 to 0.018, with the mean of 0.011. Therefore, the examination of scale dependence of homogeneous mixing in this study is not significantly affected by the measurement errors.

The adiabatic cloud core assumption could be another error source for homogeneous mixing degree. The assumed LWC_{\text{a}} along a horizontal leg might be less than the true LWC_{\text{a}} due to possible influence of...
entrainment-mixing processes. Other approaches for estimating a cloud base height and LWC have been reported in the literature, but they are not applicable here. For example, a cloud base height can be estimated using moisture and temperature from aircraft observations in the dry air below cloud, from surface stations, or obtained from some direct measurements [e.g., Clothiaux et al., 2000]. A cloud base height can also be estimated by fitting peak LWC values from different aircraft observation levels with a linear profile [e.g., Gerber et al., 2008]. LWC can be calculated with the cloud base height. Unfortunately, these approaches are not applicable in RACORO because cloud base heights varied significantly during a flight [Vogelmann et al., 2012], and it is not appropriate to assume a constant cloud base height for different cumulus clouds in a flight. In addition, the cumulus clouds analyzed here are shallow, and the properties of the shallow cumuli may change during the time when the aircraft changes its altitude for observations at different levels.

In addition to the uncertainty of LWC, the other two properties, rva and na, have similar problems due to possible effect of entrainment-mixing processes. Assume that the true adiabatic cloud has LWC, rva, and na; during the entrainment-mixing processes, LWC, rva, and na become LWC, rva, and na, respectively. To study the sensitivity of homogeneous mixing degree to the uncertainty of adiabatic cloud core assumption, LWC is assumed to be 1.25 times of LWC. Since LWC, rva, and na, are known from observation, rva and na can be calculated by

$$r_{va} = \frac{r_{va}}{1 - \frac{LWC}{LWC_{ad}}}^{1/3},$$

$$n_{aa} = \frac{LWC_{aa}}{4/3\pi r_{va}^2},$$

respectively, where Ψad and Ψaa are, respectively, the mixing fraction and homogeneous mixing degree in the entrainment-mixing process affecting assumed adiabatic cloud core. Equation (7) is derived from equation (1). Parameter Ψad can be calculated using equation (2) with some properties replaced. Parameters qL, T, and qvs(T) are replaced by qLaa, Taa, and qvs(Taa), i.e., the liquid water mixing ratio, temperature, and saturation vapor mixing ratio in the true adiabatic cloud core, respectively; qL, T, and qvs(T) are replaced by qLaa, Taa, and qvs(Taa), respectively.

To calculate rva and na, using equations (7) and (8), Ψaa is needed but unknown. Three assumptions of entrainment-mixing mechanism are made. Assumption 1: Ψaa for different t is, respectively, the same as Ψ shown in Figure 3. Assumption 2: Ψaa is equal to the mean Ψ for t = 0.1 s in Figure 3, i.e., the maximum value of the mean Ψ. Assumption 3: Ψaa is equal to the mean Ψ for t = 60 s in Figure 3, i.e., the minimum value of the mean Ψ. With Ψaa and Ψaa, rva and na can be calculated. Replacing rva and LWCC in equation (1) with rva and LWCC respectively, new homogeneous mixing degrees are calculated. Figure 4a shows homogeneous mixing degree as a function of t under different Ψaa assumptions for D = 50 m; Figure 4b enlarges the part of
Figure 4a for \( t < 10 \) s to show the results clearer. For assumption 1, the relationship between homogeneous mixing degree and \( t \) (the green line) is similar to the reference (the red line), where the LWC_{aa} is assumed to be the maximum liquid water content (Figure 3). The fitting equations indicate that the only difference between the two fitting lines is the intercept. The difference of the mean \( \psi \) with respect to the reference is in the range of 0.019 to 0.031 for different \( t \), and the mean difference is 0.026. For assumptions 2 and 3, the deviation of the mean \( \psi \) with respect to the reference has the mean values of 0.067 and 0.042, respectively. Furthermore, the fitting functions for different assumptions have similar shapes, because the parameter \( c \) in equation (4) is around ~0.90. Therefore, the adiabatic cloud core assumption could affect homogeneous mixing degree to some extent, but the homogeneous mixing degree calculated in this study is still reliable.

Another support for assuming adiabatic cloud core is that, Lu et al. [2014] also made the same assumption and found that homogeneous mixing degree is positively correlated with transition scale number,

\[ \psi_{1} - \psi_{0.1} \] as a function of (a) mean droplet-free filament size, (c) sum of droplet-free filament size, and (e) droplet-free filament fraction, respectively, in 186 growing cumulus clouds during RACORO. (b, d, and f) The same as in Figures 6a, 6c, and 6e but for \( \psi_{1}/\psi_{0.1} \). The dry air is assumed to be from \( D \) to \( 2D \) away from the edge of the cloud core, where \( D = 50 \) m. Parameters \( \psi_{0.1} \) and \( \psi_{1} \) represent homogeneous mixing degrees for the 10 Hz original data and for the data averaged every 1 s, respectively.
consistent with the theoretical expectation. The transition scale number, defined by Lu et al. [2011], theoretically represents the probability of homogeneous entrainment-mixing mechanisms.

4.2. Examination of Factors Affecting the Scale Dependence

4.2.1. Strength of Scale Dependence
To inspect the factors that affect the scale dependence, we use the difference \( \psi_1 - \psi_{0.1} \) and the ratio \( \psi_1/\psi_{0.1} \) for \( D = 50 \text{ m} \) to gauge the strength of the scale dependence. The homogeneous mixing degrees for 0.1 s (\( \psi_{0.1} \)) and for 1 s (\( \psi_1 \)) are used because 0.1 s and 1 s are two sampling rates that are commonly used in in situ aircraft measurements, and the CDSDs averaged over 0.1 s and 1 s have more samples than for other averaging time windows, e.g., 10 s. Figures 5a and 5b show the probability density functions of \( \psi_1 - \psi_{0.1} \) and \( \psi_1/\psi_{0.1} \), respectively. Generally speaking, the difference \( \psi_1 - \psi_{0.1} \) is negative and the ratio \( \psi_1/\psi_{0.1} \) is less than 1, confirming the previous results that entrainment-mixing mechanisms tend to be more inhomogeneous when the averaging scale is larger or the sampling rate is lower (Figure 3) [Burnet and Brenguier, 2007; Lu et al., 2011].

Figure 5 also indicates that both \( \psi_1 - \psi_{0.1} \) and \( \psi_1/\psi_{0.1} \) have wide ranges of values, suggesting that the scale dependence of entrainment-mixing mechanisms has different strength in different clouds. The effects of several plausible factors on the strength of the scale dependence of entrainment-mixing mechanisms are examined next.

4.2.2. Effect of Droplet-Free Filaments
One factor that may affect entrainment-mixing processes and their scale dependence is droplet-free filament, as measured by the mean droplet-free filament size, sum of droplet-free filament size, and droplet-free filament fraction (\( F \)). Figure 6 shows the relationships of \( \psi_1 - \psi_{0.1} \) and \( \psi_1/\psi_{0.1} \) with mean droplet-free filament size, sum of droplet-free filament size, and droplet-free filament fraction \( F \), respectively. In an individual cloud, there could be several droplet-free filaments. Each droplet-free filament size is calculated as follows. Since the sampling rate is 10 Hz, the sampling time difference (\( t_d \)) of two neighboring CDSDs is 0.1 s if there is no droplet-free filament between them. If filament exists, \( t_d \) should be larger than 0.1 s. The droplet-free filament size is estimated with the product of “\( t_d - 0.1 \)” and the aircraft speed (\( \sim 50 \text{ m s}^{-1} \)). The sum of the droplet-free filament size measures the total length of all the droplet-free filaments, and the droplet-free filament fraction in an individual cloud is calculated as the ratio of the sum of droplet-free filament size to cloud core width, which is estimated as the product of the aircraft observation time in an individual cloud and the aircraft speed. It is evident from Figure 6 that \( \psi_1 - \psi_{0.1} \) and \( \psi_1/\psi_{0.1} \) decrease with increasing mean droplet-free filament size, sum of droplet-free filament size, and droplet-free filament fraction. Moreover, the correlation coefficients with the droplet-free filament fraction (\( -0.36 \) for \( \psi_1 - \psi_{0.1} \) and \( -0.51 \) for \( \psi_1/\psi_{0.1} \)) are slightly larger than with the mean (\( -0.35 \) for \( \psi_1 - \psi_{0.1} \) and \( -0.48 \) for \( \psi_1/\psi_{0.1} \)) and sum (\( -0.30 \) for \( \psi_1 - \psi_{0.1} \) and \( -0.41 \) for \( \psi_1/\psi_{0.1} \)) of droplet-free filament sizes, suggesting that the droplet-free fraction captures the effect of droplet-free filaments better than the mean and sum of...
Note that majority of LWC analysis. Similar to Figure 7, Figure 8 shows the relationships of (Figure 8a) calculation of ψ versus LWCf/LWCc. As shown in Figure 9, the relationships of microphysical properties (Figure 8a) and LWCf/LWCc. According to Burnet and Brenguier [2007], in situ measurements of ψf/LWCf is generally smaller than rvf, because the entrainment-mixing mechanisms are close to homogeneous at high resolutions, as shown in Figure 3. Since ψf−ψf,0.1 and ψf/ψf,0.1 are negatively correlated with microphysics (rvf/LWCf and LWCf/LWCc) and F, respectively, it is important to check if microphysical properties (rvf/LWCf and LWCf/LWCc) and F are dependent. As shown in Figure 9, the relationships of microphysical properties (rvf/LWCf and LWCf/LWCc) with F are weak. So the microphysical properties and F are largely two independent factors. On the other hand, rvf/LWCf and LWCf/LWCc are related to each other with a strong positive correlation. So only one property (LWCf/LWCc)

Figure 8. The same as in Figure 7 but for (a) ψf−ψf,0.1 as a function of LWCf/LWCc and (b) ψf/ψf,0.1 as a function of LWCf/LWCc. LWCf and LWCc represent the mean values of liquid water content in cloud droplet size distributions adjacent to droplet-free filaments and within a whole cloud, respectively.

A few apparent exceptions are worth noting. Four cumulus clouds (the four overlapped dots in the upper left corner of each panel in Figure 6) have ψf−ψf,0.1 equal to 0 and ψf/ψf,0.1 equal to 1. Further analysis indicates that the mean droplet-free filament size, sum of droplet-free filament size, and droplet-free filament fraction of these clouds are all 0, suggesting that no droplet-free filaments exist in these clouds. Six clouds have ψf−ψf,0.1 larger than 0 and ψf/ψf,0.1 larger than 1. The behaviors of these clouds could be related to uncertainties in the estimation of LWCf, rvf, and nμ, and measurement errors of temperature, water vapor mixing ratio, and liquid water content. One cloud has ψf−ψf,0.1 larger than 0 and ψf/ψf,0.1 smaller than 1, which could also be caused by uncertainties as mentioned above; another possibility is inhomogeneous mixing with subsequent ascent, as discussed in section 3. Collision coalescence is not likely because the maximum mean volume radius is only 4.2 μm in this cloud.

4.2.3. Effect of Cloud Microphysics on Scale Dependence

To explore the sensitivity of scale dependence to cloud microphysics, Figure 7 shows the relationships of (Figure 7a) ψf−ψf,0.1 and (Figure 7b) ψf/ψf,0.1 versus rvf/LWCf, where rvf is the mean value of the mean volume radius of CDSDs adjacent to droplet-free filaments in a cloud; rvc is the mean value of the mean volume radius of all CDSDs in a cloud. One CDSD from each side (left and right) of a filament is taken in the calculation of rvf. So in total, two CDSDs are used. Four clouds without filament structure are excluded in the analysis. Similar to Figure 7, Figure 8 shows the relationships of (Figure 8a) ψf−ψf,0.1 and (Figure 7b) ψf/ψf,0.1 versus LWCf/LWCc, where LWCf is the mean value of liquid water content of CDSDs adjacent to droplet-free filaments in a cloud and LWCc is the mean value of liquid water content of all CDSDs in a cloud. The negative relationships indicate that the mean volume radius and liquid water content of CDSDs adjacent to droplet-free filaments play important roles in determining scale dependence of entrainment-mixing mechanisms.

To further examine the sensitivities of the relationships in Figures 7 and 8 to the rvf and LWCf uncertainties, we also use four CDSDs instead of two CDSDs for each filament to calculate rvf and LWCf, i.e., two CDSDs from each side (left and right) of a filament are used. The results are almost the same as those in Figures 7 and 8 (not shown).

Note that majority of LWCf is smaller than LWCc as expected from dilution and evaporation during entrainment-mixing processes. LWCf could also be partially reduced due to the droplet-free filaments smaller than ~5 m. According to Burnet and Brenguier [2007], in situ measurements of rvf tend to disguise the lowest rvf values in a spatially heterogeneous sample with the droplet-free filaments smaller than ~5 m. But still, rvf is generally smaller than rvc, because the entrainment-mixing mechanisms are close to homogeneous at high resolutions, as shown in Figure 3.
is used in the further analysis, because \( \text{LWC}_f/\text{LWC}_c \) has larger correlation coefficients than \( r_{rvf}^3/r_{rvc}^3 \) with \( \psi_1/\psi_{0.1} \) and \( \psi_1/\psi_{0.1} \) (Figures 7 and 8). Multivariable regression is then used to seek the combined effects of microphysics and \( F \)

\[
\psi_1 - \psi_{0.1} = -0.004960F - 0.09966 \frac{\text{LWC}_f}{\text{LWC}_c},
\]

where the coefficient of determination \( (R^2) \) is 0.42 with the \( P \) value smaller than 0.0001. Similarly, \( \psi_1/\psi_{0.1} \) can be expressed as

\[
\frac{\psi_1}{\psi_{0.1}} = 1 - 0.01727F - 0.7572 \frac{\text{LWC}_f}{\text{LWC}_c},
\]

where the \( R^2 \) is 0.39 with the \( P \) value smaller than 0.0001.

Because linear regressions are used in Figures 6e, 7a, and 8a, the \( R^2 \) in these figures are equal to the squared correlation coefficients, i.e., 0.13, 0.10, and 0.11, respectively. These squared correlation coefficients are much smaller than the \( R^2 \) in equation (9). The \( R^2 \) in Figures 6f, 7b, and 8b are 0.26, 0.12, and 0.15, respectively, much smaller than the \( R^2 \) in equation (10). A comparison of the coefficients of determination reveals that the two variable fittings are better than either of the single variable fitting. Therefore, the combined effects of microphysics and \( F \) are more significant on the scale dependence than the effect of each single factor (microphysics or \( F \)).

### 4.2.4. Effect of Relative Humidity

As discussed in section 4.1, another factor that affects the scale dependence of the entrainment-mixing processes is the relative humidity of the entrained air. Figure 10 shows that both \( \psi_1 \) and \( \psi_{0.1} \) increase with increasing relative humidity, which is assumed from \( D \) to \( 2D \) away from the edge of the cloud core. From left to right, the eight points in this figure correspond to \( D = 500, 300, 100, 50, 40, 30, 20, \) and \( 10 \) m, respectively. When the relative humidity is higher, evaporation is slower, and homogeneous mixing is more likely to occur, i.e., larger homogeneous mixing degree. Figure 11 further shows that \( \psi_1 - \psi_{0.1} \) and \( \psi_1/\psi_{0.1} \) decrease with an increase in relative humidity. The effect of relative humidity on the scale dependence can be explained as follows. If relative humidity is higher, a greater proportion of dry air is required (i.e., smaller \( \chi \)) in order to reduce liquid water content from the adiabatic value to the observed one. So for higher relative humidity, state 2 should move leftward, farther from state 1 in Figure 1. As a result, the length \( m_2 \) becomes smaller; homogeneous mixing degree and its variation (\( \psi_1 - \psi_{0.1} \) and \( \psi_1/\psi_{0.1} \)) increase, other conditions being equal. To more quantitatively examine the effect of relative humidity, equation (1) is differentiated

\[
\frac{\partial \psi}{\partial \chi} = \frac{(1 - C_1)C_2}{(\chi - C_2)^2}, \tag{11a}
\]

where

\[
C_1 = \frac{r_{rvf}^3}{r_{rvc}^3}, \tag{11b}
\]

\[
C_2 = \frac{\text{LWC}_f}{\text{LWC}_c}. \tag{11c}
\]
A smaller $\chi$ means a larger value of $\frac{1 - C_1 C_2}{C_1}$ and a larger absolute value of $\frac{\partial \psi}{\partial \chi}$, others being equal. Therefore, $\psi_1 - \psi_{0.1}$ and $\psi_{1/\psi_{0.1}}$ are larger for higher relative humidity, suggesting that high relative humidity of entrained air can enhance the scale dependence of entrainment-mixing mechanisms. Ideally, when the relative humidity is 100%, both the denominator and numerator in equation (11a) are equal to 0. In this case, there is no need to distinguish between entrainment-mixing mechanisms, as pointed out in previous studies [e.g., Lehmann et al., 2009].

5. Concluding Remarks

The scale dependence of entrainment-mixing mechanisms is examined using the data collected from shallow cumuli during the RACORO field campaign. A new measure of homogeneous mixing degree is defined based on the relationship between cubic mean volume radius and liquid water content, normalized by their own adiabatic values, respectively. Homogeneous mixing degree decreases significantly when the averaging time window increases from 0.1 s to 60 s, and such a variation can be well fitted by exponential functions. The base of the exponential function is close to a constant of 0.90 for different sources of entrained dry air. The adiabatic cloud core assumption and the measurement errors of temperature, water vapor mixing ratio, and liquid water content are examined, and the results indicate small effects on the calculated homogeneous mixing degree.

The strength of the scale dependence as measured by the difference $\psi_1 - \psi_{0.1}$ and ratio $\psi_{1/\psi_{0.1}}$ are further used to study factors influencing the scale dependence, where $\psi_1$ and $\psi_{0.1}$ are homogeneous mixing degrees for 1 Hz data and 10 Hz data, respectively. Three factors are found to be important in determining the strength of scale dependence. The first is droplet-free filament property. Parameters $\psi_1 - \psi_{0.1}$ and $\psi_{1/\psi_{0.1}}$ are both negatively correlated with mean droplet-free filament size, sum of droplet-free filament size, and droplet-free filament fraction, respectively. Among the three properties, droplet-free filament fraction captures the effect of droplet-free filaments the best. The strong influence of the droplet-free filaments is further reinforced by the fact that the four clouds that do not have filament structures are found to have no scale dependence. The second factor is mean volume radius or liquid water...
content of cloud droplet size distributions (CDSDs) adjacent to droplet-free filaments. Parameters $\psi_1-\psi_{0.1}$ and $\psi_1/\psi_{0.1}$ are, respectively, negatively correlated with $r_{av}^2/r_{av}^2$ and LWC/LWC, where $r_{av}$ and LWC are, respectively, the mean values of the mean volume radius and liquid water content of CDSDs adjacent to droplet-free filaments in a cloud and $r_{av}$ and LWC are, respectively, the mean values of the mean volume radius and liquid water content of all CDSDs in a cloud. The third is relative humidity in the entrained dry air. High relative humidity can enhance the scale dependence, consistent with theoretical analysis.

Several points are noteworthy. First, Lu et al. [2013b, 2014] explored parameterizations of entrainment-mixing mechanisms in cumulus and stratocumulus clouds with aircraft observations and numerical simulations. This study suggests that it is important to consider the scale dependence in the parameterizations of entrainment-mixing mechanisms. Second, the sampling rate is 10 Hz; data with a higher sampling rate could bring more insights on the scale dependence of entrainment-mixing mechanisms. Third, the droplet-free filament size used in this study is one-dimensional because the aircraft observation collects data along its own track, while the droplet-free filament in nature is three-dimensional. This could add noise to the relationships in Figure 6. Gerber et al. [2005] applied a statistical method to aircraft observational data to study hole size, which is conceptually similar to droplet-free filament, although there could be droplets in holes.

Except statistical methods, numerical simulation (e.g., direct numerical simulations) could be an important tool to study this topic. Finally, this study just scratches the surface of the scale dependence of entrainment-mixing processes, and more research is definitely needed. For example, on average, the homogeneous mixing degree in the cumulus clouds examined here appears to be larger than that in stratocumulus clouds collected at the same location [Lu et al., 2013b]. Future study will examine the difference in the scale dependence to improve our understanding of the effects of thermodynamics, dynamics, and microphysics.

References


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