Parameterizations of Entrainment-Mixing Mechanisms and their Effects on Cloud Droplet Spectral Width Based on Numerical Simulations

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**Key points:**

- Turbulent entrainment-mixing mechanisms are examined and parameterized.
- Relationship between cloud droplet spectral width and entrainment-mixing mechanisms is quantified.
- Factors and physical mechanisms are dissected by two newly defined quantities measuring the competition of complete/partial evaporation.
Abstract

Entrainment-mixing mechanisms significantly affect cloud droplet number concentration, radius, and spectral shape. Quantitative examination of entrainment-mixing effects on cloud droplet spectral width is lacking. This study examines the effects of entrainment-mixing processes on cloud microphysics by 12,218 different setups, each simulated 10 times using the Explicit Mixing Parcel Model (EMPM) driven by the observational data from the Third Tibetan Plateau Atmospheric Scientific Experiment (TIPEX-III) campaign. Parameterizations of entrainment-mixing mechanisms are developed by relating homogeneous mixing degree to transition scale number that depends on the dissipation rate and droplet evaporation time scale. The correlation between relative dispersion of cloud droplet size distribution and homogeneous mixing degree changes from negative to positive with the decreasing homogeneous mixing degree. The different relationships are closely related to the competition between complete and partial droplet evaporation and the number concentration of small droplets, which are quantitatively described by two newly introduced dimensionless numbers. The competition is significantly affected by relative humidity and mixing fraction of entrained air as well as turbulence dissipation rate, but not much by cloud droplet number concentration. Especially, when relative humidity and dissipation rate are high, there is only a negative correlation. This study sheds new light on generalizing the homogeneous/inhomogeneous concept by considering relative dispersion, and also provides parameterizations of entrainment-mixing processes and relative dispersion for atmospheric models.
1. Introduction

As “the world water tower”, the Tibetan Plateau (TP) tremendously affects water budget in this region, as well as the radiation balance, weather and climate of East Asia (Chen et al., 2020; Fu et al., 2020; Wu et al., 2015; Xu et al., 2008, 2014) and global stratosphere (Bian et al., 2020). In the boreal summer, convection is easy to be triggered and clouds prevail over TP (Chen et al., 2019; Li and Zhang, 2016; Li et al., 2020; Liu et al., 2015; Wang et al., 2014; Yu et al., 2004). Clouds significantly affect the energy balance (Grabowski and Morrison, 2011; Ramanathan et al., 2001; Stevens and Feingold, 2009; Wang, 2015; Yang et al., 2015; Yu et al., 2007; Zelinka et al., 2017), atmospheric motion at all scales (Lee, 2012), and water distribution (Moncrieff et al., 1997; Ramanathan et al., 2001). Cloud liquid-phase processes over TP play important roles in the development of cloud and precipitation (Li et al., 2008; Li and Zhang, 2017; Qiu et al., 2019; Xu et al., 2020; Zhao et al., 2017a; Zhao et al., 2017b), while parameterizations of liquid-phase processes remain uncertain in microphysics schemes of general circulation models or even cloud-resolving models (e.g., Gao et al., 2016; Grabowski, 2006; Gettelman and Morrison, 2015).

In particular, the turbulent entrainment-mixing process is still poorly understood, although the process has been studied for decades since Stommel (1947). The entrainment-mixing processes influence not only cloud properties (e.g., droplet size, number concentration, and spectral width) by dilution and evaporation, but also precipitation formation, radiative transfer, cloud life cycle and aerosol indirect effect.
Several conceptual models have been proposed for entrainment-mixing mechanisms (Baker and Latham, 1979; Gao et al., 2020; Telford, 1996; Wang et al., 2009; Yang et al., 2016; Yeom et al., 2017; Yum et al., 2015). The most popular one is based on the concepts of homogeneous/inhomogeneous mixing mechanisms (e.g., Baker et al., 1980; Tölle and Krueger, 2014). During homogeneous entrainment-mixing, all droplets are exposed to the same environment of supersaturation or undersaturation and evaporate simultaneously. Both number concentration and droplet size decrease. In the extreme inhomogeneous entrainment-mixing, some droplets near undersaturated air are completely evaporated while the rest of droplets remain unaffected. Number concentration decreases but droplet size does not change. Which mixing mechanism dominates at a particular location and time is still a matter of great debate. Some studies (Burnet and Brenguier, 2007; Jensen et al., 1985; Lu et al., 2013b; Yum and Hudson, 2001) stated that the dominant entrainment-mixing mechanisms was homogeneous, while others (Burnet and Brenguier, 2007; Freud et al., 2011; Gerber et al., 2008; Haman et al., 2007; Pawlowska and Brenguier, 2000) supported the dominance of extreme inhomogeneous mixing. The effects of different entrainment-mixing mechanisms on cloud microphysical and radiative properties have been found significant in some studies (Chosson et al., 2007; Grabowski, 2006; Lasher-trapp et al., 2005; Slawinska et al., 2008) and weak in others (Morrison and Grabowski, 2008; Slawinska et al., 2012). However, the results of Schmeissner et al. (2015) and Morrison...
and Grabowski (2008) suggested that the effects could be related to the stages of the cloud life cycle. Recently, Hoffmann and Feingold (2019) found that extreme inhomogeneous mixing increased droplet growth significantly. Therefore, our understanding on entrainment-mixing mechanisms and their effects on cloud microphysics are still in debate.

The entrainment-mixing processes occur on small-scales that cannot be resolved in most numerical atmospheric models including large-scale models, and even cloud-resolving and large-eddy simulation models; parameterizations of entrainment-mixing mechanisms are thus needed. Several studies have attempted to develop parameterizations (Andrejczuk et al., 2009; Jarecka et al., 2013; Lu et al., 2013a, 2014a; Gao et al., 2018). These parameterizations, based on the limited observational data or a limited number of simulations, are still full of uncertainties, for example, the scatter of the parameterizations. Developing parameterizations based on a large number of simulations driven by different observational data is desirable.

Furthermore, entrainment-mixing processes have important effects on shaping cloud droplet size distributions (CDSDs) by affecting the local environments and trajectories of droplets (Cooper, 1989; Kumar et al., 2012). The relative width of CDSD is often represented by relative dispersion ($d$), defined as the ratio of standard deviation ($\sigma$) to mean radius of cloud droplets ($r_m$), and its importance has been emphasized in the parameterizations of effective radius, autoconversion process (Beard and Ochs, 1993; Cooper et al., 2013; Chandrakar et al., 2016; Desai et al., 2019; Liu et al., 2002;
Lu et al. (2020; Xie et al., 2013; Wang et al., 2011; Zhao et al., 2006; Zhao et al., 2019) and cloud water sedimentation flux (Ackerman et al., 2009). Numerical simulations (Andrejczuk et al., 2004; Jensen and Baker, 1989; Lasher-trapp et al., 2005; Su et al., 1998; Tölle and Krueger, 2014) and observations (Bera et al., 2016a; Bera et al., 2016b; Burnet and Brenguier, 2007; Haman et al., 2007; Jensen et al., 1985; Lu et al. 2013b) found that $d$ and/or $\sigma$ in diluted cloud was larger than that in near-adiabatic cloud. Lu et al. (2013b) showed that the entrainment-mixing processes led to an increase of $d$ due to the decrease of mean radius. Tölle and Krueger (2014) indicated that CDSDs could broaden during entrainment-mixing, and the effects of different factors (e.g., dissipation rate, relative humidity) were discussed. Besides the importance of entrainment-mixing to $d$, $d$ is also important to entrainment-mixing parameterizations. As pointed out by Tölle and Krueger (2014), the homogeneous entrainment-mixing lines in the mixing diagram of droplet size vs number concentration (Burnet and Brenguier, 2007; Lu et al., 2013a; Yum et al., 2015) only require that no droplets completely evaporate; however, different droplets may experience different subsaturation histories, and $d$ is expected to change along these homogeneous entrainment-mixing lines. Kumar et al. (2018) also emphasized the evolution of $d$ during entrainment-mixing. Therefore, the quantitative relationship between $d$ and entrainment-mixing mechanisms is critical to understand the broadening of CDSDs and the parameterizations of both $d$ and entrainment-mixing process. However, such a quantitative relationship is lacking.

To fill these gaps, we apply the Explicit Mixing Parcel Model (EMPM) driven by observational data of meteorological information and aerosols over TP. To minimize
the uncertainties in simulations, tens of thousands of cases are simulated with different combinations of model inputs. We compare the differences between the parameterizations of entrainment-mixing mechanisms at the final and instantaneous states. Moreover, we quantify the relationship between $d$ and entrainment-mixing mechanisms, and further examine the effects of the key factors (e.g., kinetic energy dissipation rate ($\epsilon$), relative humidity of entrained air ($RH_e$), fraction of entrained air ($f$), and initial cloud droplet number concentration ($n_d$)) on the relationship.

The rest of the paper is organized as follows: Section 2 describes the EMPM and observational data. Theoretical framework is presented in section 3. The characteristics and differences of entrainment-mixing parameterizations are shown in section 4. Section 5 presents the relationship between $d$ and homogeneous mixing degree ($\psi$), and the impacting factors. The conclusions and discussions are given in section 6.

2. Model and Observational Data

EMPM was developed by Krueger et al. (1997), Su et al. (1998) and Tölle and Krueger (2014), based on Kerstein's linear eddy model (Kerstein, 1988, 1992). This model can simulate the interaction between cloudy air and its ambient air; the history of each droplet is tracked during the isobaric mixing process. As stated in Su et al. (1998), EMPM calculates the condensation/evaporation of individual droplets depending on their local environment (temperature, water vapor mixing ratio and pressure). The local environmental information is predicted by a linear eddy model.
The simulated domain is 20 m (length) * 1 mm (width) * 1 mm (height), which consists of 12,000 grid cells. The same setting has been used in other studies (e.g., Lu et al., 2013a; Tölle and Krueger, 2014). The basic concept of the EMPM is as follows (Krueger et al., 2008): First, a cloud adiabatically ascends and stops after reaching the entrainment level. Second, at the entrainment level, isobaric mixing process occurs between the cloudy parcel and entrained air with a segment of the cloudy parcel replaced by the same-sized entrained air. Third, the unsaturated entrained air is torn into many parts with different sizes and randomly arranged in the cloudy parcel at a finite turbulent rate; the droplets can grow or evaporate depending on their local environment.

Table 1 shows the setting of model simulations. Meteorological information is from the observations at the Naqu station (92.1°E, 31.5°N) during the Third Tibetan Plateau Atmospheric Scientific Experiment (TIPEX-III). The cloud base heights were measured by the Vaisala CL31 ceilometer (made by Vaisala, Finland). The cloud base pressure, temperature and water vapor were measured by soundings (see Zhao et al., 2017b for details) at 05:00, 12:00, and 17:00 Local Time (Universal Time Coordinated + 6 h) in July and August 2014. As mentioned above, liquid-phase processes play important roles in the development of cloud and precipitation over TP. To focus on cloud liquid-phase processes, only the soundings of 23 days with the cloud base temperature larger than 0 °C are used. Different from previous studies focusing on only one height or assuming one initial value of liquid water content (LWC) (Andrejczuk et al., 2009; Gao et al., 2018; Lu et al., 2013a), the entrainment height in
this study is set to be 200, 300 and 400 m above cloud base in different cases, respectively, to be more realistic. The temperature at these heights could be lower than 0 °C, but still higher than -5 °C, as shown in Figure 1a; ice/mixed-phase processes are expected to be weak.

RH_e at these three heights is taken from the soundings at the same heights (Figure 1b), assuming that the entrained air is from the environment outside the humid shells. In reality, air entrained in cumulus clouds comes from the surrounding shell, which is slightly moister and cooler than the environment outside the shell (Heus and Jonker, 2008). The entrainment induced changes in observed cloud droplet size distributions in Gerber et al. (2008) also supports this conclusion. As discussed in Lu et al. (2012), for the three-layer model proposed by Heus and Jonker (2008), cloud, environment and the surrounding shell are considered, so RH_e from the shell should be used; for the two-layer entraining plume parameterizations, there are only cloud and environment (e.g., Zhang and Mcfarlane, 1995). In this case, RH_e is often assumed from the environment outside the shells. Since there is no observation of aerosol number concentration at Naqu, the initial aerosol information measured by the Printed Optical Particle Spectrometer at Lhasa (Cui et al., 2018) is employed in the model. The average aerosol number concentration at all cloud base heights is 191.5 cm^{-3}. In the EMPM, aerosol particles are prescribed and assumed to be at their equilibrium states without considering the detailed deliquescence process (Krueger et al., 1997; Su et al., 1998; Tölle and Krueger, 2014). The curvature and solution effects of droplet are included (Su et al., 1998). Vertical velocity (w) is set to be 2 m s^{-1} before reaching the entrainment
level, which is reasonable according to the cloud radar observations at Naqu (Liu et al., 2015). At the entrainment level, $w$ is equal to 0 m s$^{-1}$ for isobaric mixing. Sensitivity tests are carried out under different conditions. $n_d$ is set to be $1/3$, $2/3$, 1, $4/3$, $5/3$ and 2 times of 191.5 cm$^3$. As to the kinetic energy dissipation rate, $\varepsilon$ is set to be in the range of $10^{-5} \sim 5 \times 10^{-2}$ m$^2$s$^{-3}$, which is reasonable according to observations (Meischner et al., 2001). Entrained blob size is 2 m, so the variation of $f$ from 0.1 to 0.7 is realized through changing the blob number. The cases with $f = 0.1$ are not analyzed because the effect of residual supersaturation in most of these cases contributes to the distinct increase in mean volume radius ($r_v$), and further significantly becomes greater than the effect of the entrainment-mixing process. The reason for the existence of residual supersaturation is that some parts of domain not affected by entrained dry air still experience the residual supersaturation produced during the vertical movement before reaching the entrainment level. In total, more than 23,000 cases are performed, and 12,218 cases that do not dissipate completely are analyzed here. For each of the 12,218 cases, 10 realizations are performed to have a good statistic, each with a different random number seed. The selecting criteria are that LWC is larger than 0.001 g m$^{-3}$ and number concentration is larger than 10 cm$^{-3}$ (Lu et al., 2014b).

3. Theoretical Framework

Four measures of $\psi$, i.e., homogeneous mixing degree, have been proposed and used to quantify the various types of mixing process (Gao et al., 2018; Lu et al., 2013a,
226 2014a, 2014b, 2018). Because the results from the four $\psi$ are similar, this paper focuses
on the one defined as:

$$
\psi = \frac{(\ln n_i - \ln n_0)}{(\ln n_i - \ln n_f)}, \quad (1a)
$$

$$
n_i = n_0 - \frac{LWC_0 - LWC_f}{\frac{4}{3} \pi r_{va}^3 \rho_w}, \quad (1b)
$$

228 where $n_0$ and LWC$_0$ are the number concentration and liquid water content after
entrainment but before evaporation, respectively, and $n_0$ can also be thought of as the
number concentration assuming homogeneous mixing mechanism; $n_i$ is the number
concentration assuming extreme inhomogeneous mixing mechanism; $n_c$ is the number
concentration during entrainment-mixing; $r_{va}$ is the adiabatic radius; $\rho_w$ is the water
density. LWC$_i$ is the liquid water content after mixing and evaporation at the new
equilibrium state, which is defined as the first time when the domain-averaged relative
humidity is larger than 99.5% (Lehmann et al., 2009) and the difference between LWC$_0$
and instantaneous LWC during mixing and evaporation is larger than 98% of the
difference between LWC$_0$ and minimum liquid water content during the mixing process.

238 For homogeneous mixing, $\psi = 1$, and for extreme inhomogeneous mixing, $\psi = 0$.

239 Meanwhile, the transition scale number ($N_L$) introduced by Lu et al. (2011) based
on the transition length ($l^*$) (Lehmann et al., 2009) is used to parameterize $\psi$:

$$
N_L = \frac{l^*}{\eta} = \frac{e^{1/2} \tau_{evap}^{3/2}}{\eta}, \quad (2a)
$$

$$
l^* = e^{1/2} \tau_{evap}^{3/2}, \quad (2b)
$$
\[ \tau_{\text{evap}} = \frac{r_{\text{va}}^2}{2A S_e}, \quad (2c) \]

\[ \eta = (\nu^3/\varepsilon)^{1/4}, \quad (2d) \]

where \( A \) is a function of pressure and temperature (Rogers and Yau, 1989), \( S_e \) supersaturation of the dry air, \( \eta \) Kolmogorov microscale, \( \nu \) kinematic viscosity, and \( \tau_{\text{evap}} \) evaporation time for a single droplet. Lu et al. (2018) compared \( \tau_{\text{evap}} \) and several other time scales, including the phase relaxation time and reaction time scale (e.g., Lehmann et al., 2009; Lu et al., 2011), and pointed out that \( \tau_{\text{evap}} \) was the most appropriate when the scientific focus was on the variations of droplet size and number concentration in entrainment-mixing processes. The parameterization of the effect of entrainment-mixing processes on microphysics can be realized by building a relationship between \( \psi \) and \( N_L \) such that

\[ \psi = c \exp(a N_L^b) \quad (3) \]

where \( a, b, \) and \( c \) are the three fitting parameters.

One can apply the parameterization to the numerical models that do not resolve the entrainment-mixing process. The number concentration after mixing and evaporation, \( n_c \), is needed in the double-moment bulk microphysics schemes. According to equation (1a), the relationship between \( n_c \) and \( \psi \) can be expressed as:

\[ n_c = n_i^{1-\psi} n_0^\psi. \quad (4) \]

There are three steps to obtain \( n_c \): first, calculate \( N_L \) by equations (2a) - (2d), second, calculate \( \psi \) through \( N_L \) by equation (3), and third, calculate \( n_c \) by equations (4) and (1b).

The input quantities are \( \varepsilon, A, S_e, \eta, r_{\text{va}}, n_0, \text{LWC}_0 \text{ and LWC}_l \). \( \varepsilon \) can be calculated using...
the method in Deardorff (1980). The calculation of $A$ can be found in Rogers and Yau (1989). $S_e$ is equal to $\text{RH}_e - 1$. The mean volume radius, number concentration and liquid water content before evaporation can be taken as $r_{va}$, $n_0$ and $\text{LWC}_0$ for the current entrainment-mixing event, respectively. $\text{LWC}_f$ can be calculated with $\text{LWC}_0$, relative humidity, air pressure and temperature.

4. Parameterization of Entrainment-Mixing Processes

As in Lu et al. (2013a, 2014a), parameterizations of entrainment-mixing mechanisms at the three entrainment heights are developed (Figures 2a, b, c) with the microphysical properties corresponding to $\text{LWC}_f$ at the equilibrium state. The results in Figure 2 suggest that $\psi$ and $N_L$ have good positive correlations at all entrainment heights. Interestingly, both parameter $a$ and $c$ decrease with the increasing entrainment height, whereas the variation of $b$ is opposite. Because $\psi$ is smaller at the lower entrainment height, the entrainment-mixing mechanisms gradually shifts from inhomogeneous to homogeneous with the increasing entrainment height, which is consistent with the results obtained in trade-wind shallow cumuli simulated by a large-eddy simulation (Jarecka et al., 2013). Jarecka et al. (2013) argued that the reason was greater turbulence intensities and larger droplet sizes at a higher height. Since turbulence dissipation rate does not change with height in the EMPM, the variation of $\psi$ is mainly caused by the variation of droplet size through affecting $\tau_{\text{evap}}$. The larger $r_{va}$ at a higher height causes larger $\tau_{\text{evap}}$ and $N_L$ (equation (2)), which further increases $\psi$ (equation (3)).

Besides, the parameterization of entrainment-mixing based on the final state results is
developed by integrating all the cases that satisfy the criteria of cloud after entrainment-mixing at all entrainment heights (Figure 2d):

$$\psi = 113.39 \exp(-0.90N_L^{-0.26}). \quad (5)$$

Natural clouds cannot be always at new equilibrium states after entrainment-mixing, or more often, many clouds are still on the way towards new equilibrium states. For example, Figure 3 shows the variations of $\psi$ and corresponding microphysical properties in the microphysical mixing diagram (Burnet and Brenguier, 2007) of a typical case with $\varepsilon = 1 \times 10^{-4}$ m$^2$ s$^{-3}$, RH$_e = 82.3\%$, $f = 0.2$, and $n_d = 127.6$ cm$^{-3}$ at the entrainment height of 200 m. The temporal evolution of $\psi$ is not monotonic; the reason is that $r_v$ generally decreases but also increases in a short period during the entrainment-mixing process. Detailed analysis is deferred to Section 5. Therefore, it is interesting to include instantaneous microphysics to develop entrainment-mixing parameterizations.

Note that in the beginning of the entrainment-mixing, some droplets keep growing by the condensation process under the effect of residual supersaturation (Kumar et al., 2017, 2018), while the other droplets in contact with the entrained dry air evaporate. When condensation is stronger than evaporation, the cubic normalized mean volume radius $(r_v/r_{va})^3$ is greater than 1.0 and LWC$_t$ is even larger than LWC$_0$. In this case, $n_i$ is larger than $n_0$ (equation (1b)). Since some droplets could completely evaporate, $n_0 > n_c$. Therefore, $n_i > n_0 > n_c$. Equation (1a) shows that the absolute value of $\ln n_c - \ln n_i$ is larger than that of $\ln n_0 - \ln n_i$ and $\psi > 1$. This study only focuses on the part with $(r_v/r_{va})^3 \leq 1.0$ and $\psi$ in the range of 0–100\%.
Similar to Andrejczuk et al. (2009) and Gao et al. (2018), the instantaneous parameterization of entrainment-mixing mechanisms is developed (Figure 4), such that

\[ \psi = 107.96 \exp(-0.95 N_L^{-0.35}) \]  

(6)

The coefficients in equations (5) and (6) are close to each other. To further explore the reasons responsible for the two similar parameterizations, each simulation case is divided into 60 time intervals since the time needed to reach an equilibrium state varies significantly in different cases. With the data in all cases at each time interval, 60 instantaneous parameterizations are developed. As shown in Figure 5, the parameter \( a \) first decreases and then increases whereas \( b \) always increases. As a result, \( \psi \) first decreases and then increases; the parameter \( c \) only affects the value of \( \psi \), but not its trend. Entrainment-mixing mechanisms transform from homogeneous to inhomogeneous, and then evolves towards homogeneous, which also can be found in Krueger et al. (2008). The evolution of entrainment-mixing mechanisms during the mixing process corroborates the envisions that view the mixing process as the initial filamentation between cloudy and entrained dry air by large-scale eddies and the following molecular homogenization of Kolmogorov microscale (Broadwell and Breidenthal, 1982; Jensen and Baker, 1989). In the first 10 out of 60 time intervals, both \( a \) and \( b \) are quite different from those in equation (5), respectively. Simulation in Kumar et al. (2017) also showed significant variations of microphysics in the first dozen seconds after mixing started. The parameterizations in the remaining 50 time intervals have \( a \) and \( b \) close to those in equations (5), respectively, because relative humidity in the model domain is close to 100% and evaporation is weak. This is why
equations (5) and (6) are similar. Note that the pentagram and triangle in Figure 5 might
or might not be on the curve. The reason is that some entrainment-mixing processes are
short, and interpolation is taken to produce data in 60 intervals. These data from the
interpolation are additional to the original output of the EMPM, and can affect $a$ and $b$
during fitting.

Two steps are taken to evaluate how the parameterizations work. First, $n_c$ is
calculated using equations (4) and (6) with the input quantities $\varepsilon, A, S_e, \eta, r_{va}, n_0, LWC_0$
and $LWC_f$. Second, compare the $n_c$ from the first step with the real $n_c$ calculated from
cloud droplet size distributions in the EMPM. Figure 6 shows that the two $n_c$ values
agree with each other well.

5. Effects of Entrainment-Mixing on Cloud Droplet Spectral Width

As mentioned above, it is critical to include $d$ in study of the entrainment-mixing
parameterizations, and the first step is to quantitatively examine the relationship
between $d$ and $\psi$, which is generally lacking in the literature. Tölle and Krueger (2014)
defined standard deviation of squared droplet radius difference at two time points
during entrainment-mixing to represent spectral width, and discussed the effects of
different factors (e.g., dissipation rate, relative humidity) on spectral width.
5.1 Relationship between Relative Dispersion and Homogeneous Mixing Degree

Figure 7 shows that on average, $d$ first increases with increasing $\psi$, peaks at some typical value of $\psi$ (~55%), and then decreases with further increasing $\psi$. The relationship from the 12,218 EMPM simulations can be well fitted with:

$$d = 0.03 + 0.008 \psi - 7.17 \times 10^{-5} \psi^2$$

(7)

Equation (7) can serve as an expression to calculate $d$ from $\psi$, after $\psi$ is obtained from equations (6). Figure S3 in the Supporting Information shows the joint probability density function of relative dispersion ($d$) parameterized with equation (7) vs $d$ calculated from the cloud droplet size distributions in the EMPM simulations.

Performance of equation (7) is also tested with the mean $d$ in each $\psi$ bin in Figure 7, with the dot size representing the weight, i.e., the number of data points in each $\psi$ bin. Figure S3 shows that equation (7) can well describe the mean $d$ in each bin of $\psi$ (the correlation coefficient is 0.99 with $p$ less than 0.01), but does not well describe the individual cases with notable scatter (the correlation coefficient is 0.45 with $p$ less than 0.01). This conclusion is expected because Figure 7 shows that $d$ has a wide range for each bin of $\psi$. Possible physical reasons are explained as follows. $\psi$ reflects how $n_c$ and $r_v$ vary during entrainment-mixing. Both of the two quantities are calculated based on the integral of CDSD. However, $d$ is related to not only the integral of CDSD, but also the different sizes of droplets in each bin of CDSD. Therefore, $\psi$ can capture the main trend of the relationship between $d$ and $\psi$, but does not well represent the fluctuation of $d$ in each bin of $\psi$, which depends on stochastic processes. As discussed in Desai et al. (2019), stochastic processes contribute significantly to the increase of $d$. Tölle and
Krueger (2014) also pointed out that droplets of different sizes experienced different supersaturation histories. In the following subsections, the input variables that may influence the relationship between $d$ and $\psi$ are examined, including $\varepsilon$, RH$_e$, $f$ and $n_d$. Representative simulation cases are selected from the 12,218 EMPM simulations and their parameters are shown in Table 2. The ensemble mean results of each simulation in the 10 realizations are analyzed. Since the duration time of mixing and evaporation in each realization can be different (71.5~83.75 s for the control run; 35.75~38.75 s for the $\varepsilon$ effect; 233~263.75 s for the RH$_e$ effect; 116~138.5 s for the $f$ effect; 61.25~62 s for the $n_d$ effect), time in each realization is normalized by its own duration time, to combine the simulations in the 10 realizations.

5.2 Control Run

The control run is conducted with $\varepsilon$ of $1 \times 10^{-4}$ m$^2$s$^{-3}$, RH$_e$ of 85%, $f$ of 0.2, and $n_d$ of 127.6 cm$^{-3}$. Figure 8 shows the relationships between $d$ and $\psi$, between $\psi$ and small droplets concentration ($n_s$) defined as the number concentration of droplets with radii smaller than half of the smallest peak radius in each simulation, and between $d$ and $n_s$. Guo et al. (2018) found that number concentration of small droplets was critical to determine $d$ based on in situ observations of deep convection. Tölle and Krueger (2014) also emphasized the role of small droplets in the broadening of CDSD. It is interesting to find that the correlation between $d$ and $\psi$ is negative in the beginning of entrainment-mixing process (Stage 1; before the pentagram) and then becomes positive during the further development of entrainment-mixing (Stages 2 and 3; after the pentagram). Such
a relationship is consistent with that in Figure 7. The relationship between $n_s$ and $\psi$ is negative during Stage 1 and positive during Stages 2 and 3, similar to the relationship between $d$ and $\psi$. The relationship between $d$ and $n_s$ is positive for all the three stages, which is consistent with observational results (Guo et al., 2018). Therefore, the effects of entrainment-mixing mechanisms on $d$ can be largely explained by the effects on $n_s$.

It is expected that $n_s$ is determined by the balance between a decrease in $n_s$ resulting from complete evaporation of droplets and an increase in $n_s$ due to partial evaporation of larger droplets. The decrease in $n_s$ within one time step due to complete evaporation can be approximately represented by $-\Delta n_c = n_{c,j+1} - n_{c,j}$ since most completely evaporated droplets are small droplets; the subscripts $j$ and $j+1$ are the time $j$ and $j+1$, respectively. The increase in $n_s$ due to partial evaporation can be represented by $-\frac{\Delta n_s - \Delta n_c}{\Delta n_c}$, where $\Delta n_s = n_{s,j+1} - n_{s,j}$ is the difference of $n_s$ within one time step. Therefore, $-\Delta n_c$ and $-\frac{\Delta n_s - \Delta n_c}{\Delta n_c}$ can represent the degrees of complete and partial evaporation, respectively. Two non-dimensional quantities, the ratio of partial to complete evaporation $\left(\frac{\Delta n_s - \Delta n_c}{\Delta n_c}\right)$ and the percentage of small droplets $\frac{n_s}{n_c}$, are introduced to measure the relative strength between partial and complete evaporation, and the effects of small droplets, respectively.

To further examine the physical processes, Figure 9 shows the temporal evolutions of CDSDs and some key quantities including $d$, $\psi$, $-\Delta n_c$, $-\frac{\Delta n_s - \Delta n_c}{\Delta n_c}$, and $\frac{n_s}{n_c}$. In the very beginning of Stage 1, droplets only partially evaporate and CDSDs slightly broaden toward small droplets. The smallest droplet size is still larger than the radius criterion of $n_s$, so $\Delta n_c = 0$, $\psi = 100\%$, and $d$ only slightly increases. The mixing
following the homogeneous mixing line within the first few seconds is consistent with
Kumar et al. (2017). During the further mixing and evaporation, complete evaporation
increases with increasing $-\Delta n_c$, while partial evaporation is more profound than
complete evaporation because the ratio of partial to complete evaporation is greater
than 1. The small droplets accumulate as shown by the increasing percentage of small
droplets, and thus $d$ increases. Although the partial evaporation dominates, complete
evaporation is also strong as indicated by the high value of $-\Delta n_c$, which is induced by
the large entrained air size and low relative humidity of entrained dry air at the
beginning of entrainment-mixing. As a result, $\psi$ decreases and is negatively correlated
with $d$ during Stage 1.

During Stage 2, complete evaporation is stronger than partial evaporation with the
ratio of partial to complete evaporation less than 1. As a result, the percentage of small
droplets and $\psi$ decrease. The CDSDs are narrowed due to the decreasing small droplets,
so $d$ also decreases and is positively correlated with $\psi$. During Stage 3, complete
evaporation is weak, because of the increasing relative humidity and the decreasing
entrained air size. Partial evaporation becomes more effective than complete
evaporation since the ratio of partial to complete evaporation is generally greater than
1. This is similar to Stage 1, but the relationship between $d$ and $\psi$ is opposite between
Stages 1 and 3. The reason is that complete evaporation during Stage 3 is weaker than
that during Stage 1, i.e., smaller $-\Delta n_c$ during Stage 3, which results in a decrease and an
increase in $\psi$ during Stages 1 and 3, respectively. The increase of $\psi$ during Stage 3 is
consistent with the conclusion that mixing becomes more homogeneous as it
approaches saturation in previous studies \cite{Freud2011,Jensen1989,Krueger2008}.

5.3 Effects of Turbulence Dissipation Rate

To study the effects of $\varepsilon$, we vary it from $1\times10^{-5}$ to $1\times10^{-2}$ m$^2$s$^{-3}$ with the other parameters being the same as those in the control run. Figure 10 shows the evolutions of CDSDs, $d, \psi, -\Delta n_c, (\Delta n_s - \Delta n_c) / \psi$, and “$n_s / n_c$” for $\varepsilon = 1\times10^{-3}$ m$^2$s$^{-3}$. The faster mixing between cloudy air and entrained dry air causes the less time to reach final new saturation, compared with the control run (Figure 9). During the whole period, the ratio of partial to complete evaporation is always greater than 1 and the percentage of small droplets increases, which means that partial evaporation is stronger than complete evaporation. There is no Stage 2 where complete evaporation is stronger than partial evaporation, because the existence of Stage 2 needs enough small droplets available for complete evaporation. In this sensitivity test, the stronger turbulence homogenizes the water and temperature fields and the percentage of small droplets at the end of Stage 1 (5.8\%) is much smaller than that in the control run (9.5\%). There are less small droplets available for complete evaporation than in the control run, and therefore, Stage 2 does not exist.

Besides the results with $\varepsilon = 1\times10^{-4}$ and $1\times10^{-3}$ m$^2$s$^{-3}$ discussed above, it is also interesting to examine the relationships for even weaker and stronger turbulence in Figure 8. When turbulence dissipation rate is really low ($\varepsilon = 1\times10^{-5}$ m$^2$s$^{-3}$), some droplets near the entrained dry air evaporate significantly or even completely.
Nevertheless, the rest of droplets far away from the entrained dry air experience condensational growth due to supersaturation. These results are not included in Figure 8 and there is a positive correlation between $d$ and $\psi$ during Stage 3. In contrast, when turbulence is quite strong ($\varepsilon = 1 \times 10^{-2} \text{ m}^2\text{s}^{-3}$), partial evaporation dominates with few droplets completely evaporated. $\psi$ slightly decreases or even barely changes (close to 100%) and CDSDs slightly broaden toward small droplets. The slight shift from homogeneous mixing towards inhomogeneous mixing has been reported in direct numerical simulation for high turbulent kinetic energy (Andrejczuk et al., 2006). The increase in $d$ is also reported in Pinsky et al. (2016) when mixing is homogeneous. Because of the fast mixing process, new saturation can be realized in a short time within Stage 1. Therefore, there are no Stages 2 and 3, and $d$ is negatively correlated with $\psi$ (Figure 8).

5.4 Effects of Relative Humidity of Entrained Environmental Air

Figure 11 shows the relationships between $d$, $\psi$ and $n_s$ with RH$_e$ varying from 55% to 90.2%; the other parameters are the same as those in the control run. When RH$_e$ is 90.2%, Stage 1 is long, and there is even no Stages 2 and 3. Therefore, $d$ and $\psi$ are negatively correlated for RH$_e = 90.2\%$. The correlation between $d$ and $\psi$ is negative during Stage 1 and positive during Stages 2 and/or 3 for RH$_e = 55\%$.

To further demonstrate the RH$_e$ effects, the case with RH$_e = 55\%$ is shown in Figure 12. The variations of complete evaporation and the ratio of partial to complete evaporation are similar to those in the control run. Because of the more rapid
evaporation, the accumulation and subsequent decrease of small droplets are much faster during Stages 1 and 2 than in the control run. As a result, the duration for Stages 1 and 2 in the 55% RH_e case is notably shorter than that in the control run, as mentioned above. Oppositely, the stage after Stage 2 is much longer until a new saturation and this stage is actually a mixture of Stages 2 and 3, because of the ratio of partial to complete evaporation fluctuates around 1. Generally speaking, during this stage, the entrainment-mixing mechanism shifts from inhomogeneous towards homogeneous mixing with fluctuation of $\psi$. This transition is also found in the mixing diagram trajectories by Kumar et al. (2014, 2018). The physical processes during the three stages are generally the same as those in the control run, except for the circling period highlighted by the vertical dotted lines. The circling phenomenon occurs because the phase of $\psi$ is ahead of $d$ (Figure 12b). In addition, since Stage 3 is much longer for RH_e = 55% than for higher RH_e, it is expected that the positive correlation will become stronger if more low relative humidity cases are included in Figure 7. More studies are needed to substantiate this conclusion.

5.5 Effects of Entrained Environmental Air Fraction

Figure 13 shows the relationships between $d$ and $\psi$, between $n_s$ and $\psi$, and between $d$ and $n_s$. The results with $f$ from 0.2 (the control run) to 0.4 are compared. The cases with the fraction from 0.5 to 0.7 are not included since the clouds are totally dissipated under the meteorological conditions in Table 2. There is a negative correlation during Stage 1 and a positive correlation during Stages 2 and 3 between $d$ and $\psi$ when $f = 0.2$. 
When $f = 0.3$, the relationships basically can be divided into Stages 1 and 3, similar to the case with $\varepsilon = 1 \times 10^{-3}$ m$^2$s$^{-3}$ (Figure 10). When $f = 0.4$, $d$ is generally negatively correlated with $\psi$ during Stage 1 and weakly positively correlated during Stages 2 and 3. Figure 14 further shows the timeseries of $d$, $\psi$ and other related quantities for $f = 0.4$. The larger $f$ contributes to the more pronounced broadening feature, as pointed out by Jensen et al. (1985), Lasher-trapp et al. (2005) and Tölle and Krueger (2014). Partial evaporation is greater than complete evaporation since the ratio of partial to complete evaporation is greater than 1 for normalized time < 0.46 (Stage 1), while the high value of $-\Delta n_c$ indicates that complete evaporation is also strong. As a result, the percentage of small droplets increases and $\psi$ decreases; there is a negative correlation between $d$ and $\psi$ during Stage 1. After that, the ratio of partial to complete evaporation fluctuates around 1, thus Stages 2 and 3 cannot be distinguished. The percentage of small droplets slightly increases, which causes a weak positive correlation between $d$ and $\psi$.

5.6 Effects of Cloud Droplet Number Concentration

To explore the $n_d$ effects, $n_d$ is set from 63.8 to 957.5 cm$^{-3}$, and the other settings are the same as in the control run. Figure 15 shows that the relationships are similar to the results in the control run. Besides, there are several points showing a positive correlation in the beginning of Stage 1 in Figure 15a (brown points) because complete evaporation of some droplets near entrained dry air causes $\psi$ to decrease drastically in the beginning of the mixing process. To further illustrate the effects of $n_d$, the case with $n_d = 957.5$ cm$^{-3}$ is shown in Figure 16. Again, the evolutions of the ratio of partial to
complete evaporation, the percentage of small droplets and other quantities are similar to those in the control run. One difference is that $d$ and $\psi$ increase more significantly than in the control run during Stage 3, which is related to the faster increasing rate of the percentage of small droplets.

5.7 Effects of Initial Cloud Droplet Spectral Width

To study the effects of the CDSD width on the above conclusions, initial CDSD for the EMPM is produced as follows. The shape of the initial CDSD is the same as the CDSD at the beginning of Stage 2 in the control run (ensemble mean based on the 10 realizations). To produce even wider CDSDs, the droplet number concentrations of the bins < 7.5 μm radius are multiplied by 2, 4, 6, 8, and 10 times, respectively. These five CDSDs are adjusted to be consistent with all other simulations, having the same liquid water content of 0.4 g m$^{-3}$ (Figure S1 in the Supporting Information). Figure S2 shows that Stage 1 still exists but is weakened for the wider initial CDSDs. Therefore, Stage 1 is more typical for a previously unaffected cloud.

In addition, CDSDs in adiabatic parcels are intrinsically narrow. If the initial CDSD is wide, it is expected to have experienced at least one entrainment-mixing event (labeled as A). When the initial CDSD is further affected by another entrainment-mixing event (labeled as B), the CDSD width varies accordingly. In the above discussions with Figures S1 and S2, we only focus on event B with initial wide CDSD after event A but before event B. Since the topic is how entrainment-mixing affects the width of CDSD, it may be also needed to include event A which starts with narrow
CDSD. The combination of events A and B indicates the whole effects of entrainment-mixing on the CDSD width.

5.8 Synthesis of Different Effects

As mentioned above, the whole temporal evolutions of microphysics during entrainment-mixing can be generally divided into three stages based on the characteristics of partial and complete evaporation. As summarized in the schematic diagram (Figure 17), the correlation between $d$ and $\psi$ generally changes from negative during Stage 1 to positive during Stages 2 and 3. In Stage 1, although complete evaporation is strong, partial evaporation is stronger than complete evaporation and small droplets increase, leading to a negative $d$-$\psi$ correlation. In Stage 3, complete evaporation is weak and partial evaporation is stronger, $d$ and $\psi$ are positively correlated. When complete evaporation is more significant than partial evaporation, small droplets drastically decrease. $d$ and $\psi$ are positively correlated (Stage 2).

It is noteworthy that not all the three stages always occur in a simulation, and which one occurs depending on the strength of the dominant effects. These results are consistent with theoretical expectation. The results of Kumar et al. (2014) and Tölle and Krueger (2014) suggested that homogeneous mixing contributed to the broadening of CDSDs, but the increase in $d$ was relatively smaller than that for inhomogeneous mixing. When mixing changes from homogeneous to inhomogeneous, $d$ significantly increases. It is expected that there is a negative correlation between $d$ and $\psi$ (Stage 1). When the entrainment-mixing mechanism is extremely inhomogeneous, the cloud
droplet size distribution after entrainment-mixing is as narrow as the adiabatic size
distribution, i.e., small $d$. Therefore, when $\psi$ decreases, there should be a maximum $d$
and a positive correlation between $d$ and $\psi$ (Stages 2 and 3).

6. Conclusions and Discussions

Liquid phase processes play important roles in cloud physics and precipitation over
the Tibetan Plateau (TP), but parameterizations of liquid phase processes are full of
uncertainties, including entrainment-mixing. Especially, quantitative examinations of
entrainment-mixing effects on cloud droplet spectral width are lacking.

To overcome this limitation, entrainment-mixing processes are simulated with the
Explicit Mixing Parcel Model (EMPM) driven by the observational data from the Third
Tibetan Plateau Atmospheric Scientific Experiment (TIPEX-III). Nearly 23,000 cases
are simulated at three entrainment heights (200 m, 300 m, 400 m) above cloud base
with varying parameters, including kinetic energy dissipation rate ($\varepsilon$), relative humidity
of entrained environmental air ($\text{RH}_e$), fraction of entrained environmental air ($f$), and
initial cloud droplet number concentration ($n_d$). A total of 12,218 cases that satisfy the
criteria of cloud after entrainment-mixing are analyzed. The parameterizations of
entrainment-mixing mechanisms are developed with microphysical properties at final
equilibrium states and/or instantaneous states. Meanwhile, the effects of mixing
mechanisms on cloud droplet spectral width are quantitatively examined by analyzing
the relationship between relative dispersion of cloud droplet size distribution ($d$) and
homogeneous mixing degree ($\psi$). Different factors affecting the relationship are also
examined. In models, $d$ can be calculated from $\psi$ that is obtained from the parameterizations of entrainment-mixing mechanisms.

The parameterizations of entrainment-mixing mechanisms with final state microphysics are developed for different entrainment heights, while previous studies focused on only one height or assumed one initial value of liquid water content (Andrejczuk et al., 2009; Gao et al., 2018; Lu et al., 2013a). Entrainment-mixing regimes shift from inhomogeneous mixing to homogeneous mixing with the increasing height. The lower entrainment height means the shorter condensation time and the smaller droplet sizes. The smaller droplets are more easily to experience complete evaporation and mixing is more inhomogeneous.

A parameterization is also developed for instantaneous state microphysics, which is similar to that for the final state microphysics. The mixing process of each case is divided into 60 time intervals to explore the temporal evolution. Entrainment-mixing mechanisms transform from homogeneous to inhomogeneous, and then to homogeneous with increasing time intervals. The difference between parameterizations from the instantaneous states and from the final states is largest in the first 10 time intervals. In the remaining 50 time intervals, the difference is relatively small. Therefore, the parameterization based on all the instantaneous state microphysics is close to that based on the final state microphysics. The application of the parameterization based on instantaneous state microphysics shows that the parameterized number concentration agrees with the EMPM results well.
As mentioned above, it is critical to include $d$ in the entrainment-mixing parameterizations and also to consider entrainment-mixing in the $d$ parameterizations. The first step is to examine the quantitative relationship between $d$ and $\psi$, which is lacking in the literature. Intriguingly, when the data in all cases are included, $d$ and $\psi$ are negatively correlated, and then positively correlated with decreasing $\psi$. Sensitivity tests are carried out to examine the effects of different factors on the relationship during entrainment-mixing events. Two non-dimensional quantities are introduced to measure the strength of partial and complete evaporation and the effects of small droplets. The results indicate that the number concentration of small droplets is a key quantity to represent the competition of partial and complete evaporation, and then determines the relationship between $d$ and $\psi$. The correlation between $d$ and $\psi$ and the competition between partial and complete evaporation of droplets are affected by different factors.

An additional quantity seems needed to depict small droplets in bulk microphysical parameterizations in future studies.

To examine the effects of different factors, an entrainment-mixing event is divided into three stages where the relationships are different. The negative correlation exists under all conditions during Stage 1, except when $\epsilon$ is really low, condensation due to residual supersaturation significantly becomes greater than evaporation due to entrainment-mixing. When RH_e and $\epsilon$ are high, there is only negative correlation, because partial evaporation is good enough to obtain new saturation. Except for high RH_e and $\epsilon$, the positive correlation during Stage 3 exists under all conditions. Stage 3 is often at the end of entrainment-mixing processes; complete evaporation is not easy
to occur and partial evaporation is greater than complete evaporation. The positive correlation during Stage 2 may or may not exist. Stage 2 needs greater complete evaporation than partial evaporation for the significant evaporation of small droplets and occurs at moderate $\varepsilon$ and RH$_e$. This is determined by whether accumulation of small droplets during Stage 1 is significant to cause complete evaporation of a large number of small droplets during Stage 2 to satisfy the condition of stronger complete evaporation than partial evaporation. As to $f$, both Stages 2 and 3 exist for $f = 0.2$ and 0.4, but there is only Stages 1 and 3 for $f = 0.3$. It is interesting that $n_d$ does not affect the relationship between $d$ and $\psi$, although $n_d$ affects the two quantities themselves.

Several points are noteworthy. First, the parameterizations are expected to be general. One reason is that the EMPM is designed for studying entrainment-mixing processes, not limited to specific locations. All factors affecting entrainment-mixing mechanisms are considered in the model. Another reason is that the quantities used for developing the parameterizations well connect the dynamical and microphysical aspects of entrainment-mixing processes. The parameterizations combine all the important factors. Second, more studies with low relative humidity are needed, such as 22% used in Indian monsoon cloud simulations (Kumar et al., 2017). It is expected that the positive correlation between $d$ and $\psi$ will be stronger with more low relative humidity cases. Third, only number concentration and droplet size are included in the concept of homogeneous and inhomogeneous entrainment-mixing. The results from this study indicate that relative dispersion should be incorporated to generalize the concept, for example, including relative dispersion in the definition of homogeneous...
mixing degree and entrainment-mixing parameterizations. Fourth, vertical velocity is an important factor governing the relative dispersion of droplet size distribution (Liu et al., 2006; Lu et al., 2012; Yum and Hudson, 2005). The cases presented here represent a limited scenario where dilution makes buoyancy of cloud parcel to zero. Assuming isobaric mixing can be treated as the first step to understand entrainment-mixing mechanisms and their effects on relative dispersion, following many previous studies (Andrejczuk et al., 2009; Gao et al., 2018; Kumar et al., 2018; Lu et al., 2020; Tölle and Krueger, 2014). Also following these studies, it is assumed that the entrained air does not contain aerosol. It is needed in future studies to consider the subsequent ascent process after entrainment and activation during entrainment. Fifth, this study only focuses on a single entrainment-mixing event. Qualitatively, Lu et al. (2014a) found that homogeneous mixing degree fluctuated with increasing number of multiple entrainment events and time interval between two events. Also qualitatively, Tölle and Krueger (2014) pointed out that multiple entrainment events led to a broader droplet spectrum at the high entrained air fraction. Besides, Jensen et al. (1985) stated that secondary mixing events often occurred with the broadening of CDSD and possibly the production of bimodal CDSD. It is interesting to quantitatively study the variations of microphysics during multiple entrainment-mixing events.
Symbol list

A: function of pressure and temperature

\( a \): parameter of parameterization in equation (3)

\( b \): parameter of parameterization in equation (3)

\( c \): parameter of parameterization in equation (3)

\( D \): domain size

\( d \): scale of entrained air

\( f \): fraction of entrained dry air

\( l' \): transition length

LWC\(_{0}\): liquid water content after entrainment but before evaporation

LWC\(_{f}\): liquid water content after mixing and evaporation at the new equilibrium state

\( n_{a} \): adiabatic cloud droplet number concentration

\( n_{c} \): cloud droplet number concentration during entrainment-mixing

\( n_{d} \): initial cloud droplet number concentration

\( n_{o} \): number concentration after the homogeneous mixing

\( n_{s} \): number concentration after the extreme inhomogeneous mixing

\( n_{s} \): number concentration of small droplets

\( N_{L} \): transition scale number

\( \psi' \): homogeneous mixing degree

\( r_{va} \): adiabatic mean volume radius

\( r_{v} \): mean volume radius

\( r_{mi} \): mean radius
$R^2$: coefficient of determination

RH$_e$: relative humidity of entrained air

$S_e$: supersaturation of the dry air

$\varepsilon$: kinetic energy dissipation rate

$\tau_{\text{evap}}$: evaporation time scale

$d$: relative dispersion of cloud droplet size distribution

$\sigma$: standard deviation of cloud droplet size distribution

$\eta$: Kolmogorov microscale

$v$: kinematic viscosity

$w$: vertical velocity
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Caption List

Figure 1. The averaged vertical profiles of (a) temperature and (b) relative humidity (RH\textsubscript{e}) from soundings over the Tibetan Plateau during the 23 selected days of July and August, 2014. The error bars show the average, minimum, and maximum of the quantities. The red and blue dash lines correspond to the entrainment heights at 400 m above the maximum and the minimum cloud base heights, respectively. The vertical yellow dash line is the threshold of -5 °C.

Figure 2. Parameterizations of cloud entrainment-mixing mechanisms over the Tibetan Plateau by relating homogeneous mixing degree (ψ) to transition scale number (N\textsubscript{L}) at final states at (a) 200 m, (b) 300 m, (c) 400 m, and (d) 200 m, 300 m, 400 m above cloud bases. The contours represent the joint probability density function (PDF) of ψ vs N\textsubscript{L}. The magenta dots and error bars are mean values and standard deviations of ψ in each N\textsubscript{L} bin, respectively. The mean values are fitted using a weighted least squares method with the number of data points in each N\textsubscript{L} bin as the weight. The fitting equations, coefficients of determination (R\textsuperscript{2}) and p value are also given.

Figure 3. (a) Temporal evolution of the ratio of cubic mean volume radius (r\textsubscript{v}\textsuperscript{3}) to its adiabatic value (r\textsubscript{va}\textsuperscript{3}) as a function of the ratio of number concentration (n\textsubscript{c}) to its adiabatic value (n\textsubscript{a}), with the arrows to indicate the direction of time, and (b) temporal evolution of homogeneous mixing degree (ψ) in a representative case at the entrainment height of 200 m, with turbulence dissipation rate of 1\times10\textsuperscript{-4} m\textsuperscript{2}s\textsuperscript{-3}, entrained air relative
humidity of 82.3%, entrained air fraction of 0.2, and initial droplet number concentration of 127.6 cm$^{-3}$.

Figure 4. Parameterization of cloud entrainment-mixing mechanisms over the Tibetan Plateau by relating homogeneous mixing degree ($\psi$) to transition scale number ($N_L$) at instantaneous states at 200 m, 300 m and 400 m above cloud bases. The contours represent the joint probability density function (PDF) of $\psi$ vs $N_L$. The magenta dots and error bars are mean values and standard deviations of $\psi$ in each $N_L$ bin, respectively. The mean values are fitted using a weighted least squares method with the number of data points in each $N_L$ bin as the weight. The fitting equation, coefficient of determination ($R^2$) and $p$ value are also given.

Figure 5. Temporal evolutions of the parameters $a$ and $b$ in the parameterization of equation (3) developed with cloud microphysics in each time interval (from 1 to 60), with the arrow to indicate the direction of temporal development. The pentagram and triangle represent the parameterizations developed with microphysical properties at the final states and all the instantaneous states, respectively.

Figure 6. The cloud droplet number concentration ($n_c$) during entrainment-mixing obtained by the parameterization of $n_c$ (equation (6)) vs. $n_c$ calculated from cloud droplet size distributions in the EMPM with 12,218 cases for 10 realizations of each simulation.
Figure 7. Joint probability density function (PDF) of relative dispersion ($d$) vs homogeneous mixing degree ($\psi$) based on 12,218 simulations for 10 realizations of each simulation. The magenta dots and error bars are mean values and standard deviations of $d$ in each $\psi$ bin, respectively. The mean values are fitted using a weighted least squares method with the number of data points in each $\psi$ bin as the weight. The fitting equation, coefficient of determination ($R^2$) and $p$ value are also given.

Figure 8. Relationships (a) between relative dispersion ($d$) and homogeneous mixing degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$ and $n_s$ in four simulations with different turbulence dissipation rate ($\varepsilon$). The big data points represent the beginning of entrainment-mixing processes. The pentagram and triangle are the turning points where Stage 1 shifts to Stage 2 or 3, and Stage 2 shifts to Stage 3, respectively.

Figure 9. The control run (Table 2). Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud droplet size distributions (CDSDs), complete evaporation ($-\Delta n_c$), the ratio of partial to complete evaporation “$(\Delta n_s - \Delta n_c) / (-\Delta n_c)$” and the percentage of small droplets “$n_s / n_c$”. See text for more explanations on $-\Delta n_c$, “$(\Delta n_s - \Delta n_c) / (-\Delta n_c)$”, and “$n_s / n_c$”. The vertical dash lines represent the moments when Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 3, respectively. The red dash line represents the radius criterion of $n_s$. The black dot line represents $(\Delta n_s - \Delta n_c) / (-\Delta n_c) =$
Figure 10. Different dissipation rate ($\varepsilon = 1 \times 10^{-3} \text{ m}^2\text{s}^{-3}$) compared to the control run. Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud droplet size distributions (CDSDs), complete evaporation ($-\Delta n_c$), the ratio of partial to complete evaporation “($\Delta n_s - \Delta n_c$) / ($-\Delta n_c$)”, and the percentage of small droplets “$n_s / n_c$”. See text for more explanations on $-\Delta n_c$, “($\Delta n_s - \Delta n_c$) / ($-\Delta n_c$)”, and “$n_s / n_c$”. The vertical dash lines represent the moments when Stage 1 shifts to Stage 3. The red dash line represents the radius criterion of $n_s$. The black dot line represents ($\Delta n_s - \Delta n_c$) / ($-\Delta n_c$) = 1.

Figure 11. Relationships (a) between relative dispersion ($d$) and homogeneous mixing degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$ and $n_s$ in three simulations with different relative humidity of entrained environment air ($R_{He}$). The big data points represent the beginning of entrainment-mixing processes. The pentagram and triangle are the turning points where Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 2/3, respectively.

Figure 12. Different relative humidity of entrained environmental air ($R_{He} = 55\%$) compared to the control run. Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud droplet size distributions (CDSDs), complete evaporation ($-\Delta n_c$), the ratio of partial to complete evaporation “($\Delta n_s - \Delta n_c$) / ($-\Delta n_c$)”,...
and the percentage of small droplets “$n_s / n_c$”. See text for more explanations on $-\Delta n_c$,

$((\Delta n_s - \Delta n_c) / (\Delta n_c))$, and “$n_s / n_c$”. The vertical dash lines represent the moments when

Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 2/3, respectively. The vertical dot

lines correspond to the circling period. The red dash line represents the radius criterion

of $n_s$. The black dot line represents $(\Delta n_s - \Delta n_c) / (-\Delta n_c) = 1$.

Figure 13. Relationships (a) between relative dispersion ($d$) and homogeneous mixing
degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$
and $n_s$ in three simulations with different mixing fraction of entrained air ($f$). The big

data points represent the beginning of entrainment-mixing processes. The pentagram

and triangle are the turning points where Stage 1 shifts to Stage 2 and/or 3, and Stage 2

shifts to Stage 3, respectively.

Figure 14. Different entrained air fraction ($f = 0.4$) compared to the control run.

Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud
droplet size distributions (CDSDs), complete evaporation ($-\Delta n_c$), the ratio of partial to

complete evaporation “$((\Delta n_s - \Delta n_c) / (-\Delta n_c))$”, and the percentage of small droplets “$n_s / n_c$”. See text for more explanations on $-\Delta n_c$, “$((\Delta n_s - \Delta n_c) / (-\Delta n_c))$”, and “$n_s / n_c$”. The

vertical dash line represents the moment when Stage 1 shifts to Stages 2 and 3. The red
dash line represents the radius criterion of $n_s$. The black dot line represents $(\Delta n_s - \Delta n_c)$

/ $(-\Delta n_c) = 1$.  

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Figure 15. Relationships (a) between relative dispersion ($d$) and homogeneous mixing degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$ and $n_s$ in four simulations with different initial droplet number concentration ($n_d$). The big data points represent the beginning of entrainment-mixing processes. The pentagram and triangle are the turning points where Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 3, respectively.

Figure 16. Different initial droplet number concentration ($n_d = 957.5 \text{ cm}^{-3}$) compared to the control run. Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud droplet size distributions (CDSDs), complete evaporation (-$\Delta n_c$), the ratio of partial to complete evaporation “($\Delta n_s - \Delta n_c$) / (-$\Delta n_c$)”, and the percentage of small droplets “$n_s / n_c$”. See text for more explanations on -$\Delta n_c$, “($\Delta n_s - \Delta n_c$) / (-$\Delta n_c$)”, and “$n_s / n_c$”. The vertical dash lines represent the moments when Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 3, respectively. The red dash line represents the radius criterion of $n_s$. The black dot line represents ($\Delta n_s - \Delta n_c$) / (-$\Delta n_c$) = 1.

Figure 17. Schematic diagram of the evolutions of relative dispersion of cloud droplet size distribution ($d$), homogeneous mixing degree ($\psi$), small droplets ($n_s$), and their relationships and reasons.
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### Table 1. Parameters for EMPM simulations of entrainment-mixing processes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain size, $D$</td>
<td>20 m</td>
</tr>
<tr>
<td>Vertical velocity, $w$</td>
<td>2 m s$^{-1}$</td>
</tr>
<tr>
<td>Entrained blob size, $l$</td>
<td>2 m</td>
</tr>
<tr>
<td>Initial droplet number concentration, $n_d$</td>
<td>63.8, 127.6, 191.5, 255.3, 319.2, 383.1 cm$^{-3}$</td>
</tr>
<tr>
<td>Entrained air fraction, $f$</td>
<td>0.1~0.7</td>
</tr>
<tr>
<td>Dissipation rate, $\varepsilon$</td>
<td>$10^{-5}$, $10^{-4}$, $10^{-3}$, $10^{-2}$, $5\times10^{-5}$, $5\times10^{-4}$, $5\times10^{-3}$, $5\times10^{-2}$ m$^2$s$^{-3}$</td>
</tr>
<tr>
<td>Entrainment heights (above cloud base)</td>
<td>200, 300, 400 m</td>
</tr>
<tr>
<td>Sounding data (July and August, 2014)</td>
<td>23 days</td>
</tr>
</tbody>
</table>
Table 2. Parameters in the control run and sensitivity tests.

<table>
<thead>
<tr>
<th>Category</th>
<th>Dissipation rate, $\varepsilon$ (m$^2$s$^{-3}$)</th>
<th>Entrained air relative humidity, RH$_e$ (%)</th>
<th>Entrained air fraction, $f$</th>
<th>Initial droplet number, $n_d$ (cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control run</td>
<td>$10^{-4}$</td>
<td>85</td>
<td>0.2</td>
<td>127.6</td>
</tr>
<tr>
<td>Case 1: $\varepsilon$ effect</td>
<td>$10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$</td>
<td>85</td>
<td>0.2</td>
<td>127.6</td>
</tr>
<tr>
<td>Case 2: RH$_e$ effect</td>
<td>$10^{-4}$</td>
<td>55, 85, 90.2</td>
<td>0.2</td>
<td>127.6</td>
</tr>
<tr>
<td>Case 3: $f$ effect</td>
<td>$10^{-4}$</td>
<td>85</td>
<td>0.2, 0.3, 0.4</td>
<td>127.6</td>
</tr>
<tr>
<td>Case 4: $n_d$ effect</td>
<td>$10^{-4}$</td>
<td>85</td>
<td>0.2</td>
<td>63.8, 127.6, 957.5</td>
</tr>
</tbody>
</table>
Figure 1. The averaged vertical profiles of (a) temperature and (b) relative humidity (RH$_e$) from soundings over the Tibetan Plateau during the 23 selected days of July and August, 2014. The error bars show the average, minimum, and maximum of the quantities. The red and blue dash lines correspond to the entrainment heights at 400 m above the maximum and the minimum cloud base heights, respectively. The vertical yellow dash line is the threshold of -5 °C.
Figure 2. Parameterizations of cloud entrainment-mixing mechanisms over the Tibetan Plateau by relating homogeneous mixing degree ($\psi$) to transition scale number ($N_L$) at final states at (a) 200 m, (b) 300 m, (c) 400 m, and (d) 200 m, 300 m, 400 m above cloud bases. The contours represent the joint probability density function (PDF) of $\psi$ vs $N_L$. The magenta dots and error bars are mean values and standard deviations of $\psi$ in each $N_L$ bin, respectively. The mean values are fitted using a weighted least squares method with the number of data points in each $N_L$ bin as the weight. The fitting equations, coefficients of determination ($R^2$) and $p$ value are also given.
Figure 3. (a) Temporal evolution of the ratio of cubic mean volume radius \( (r_v^3) \) to its adiabatic value \( (r_{va}^3) \) as a function of the ratio of number concentration \( (n_c) \) to its adiabatic value \( (n_a) \), with the arrows to indicate the direction of time, and (b) temporal evolution of homogeneous mixing degree \( (\psi) \) in a representative case at the entrainment height of 200 m, with turbulence dissipation rate of \( 1 \times 10^{-4} \text{ m}^2\text{s}^{-3} \), entrained air relative humidity of 82.3%, entrained air fraction of 0.2, and initial droplet number concentration of 127.6 cm\(^{-3}\).
Figure 4. Parameterization of cloud entrainment-mixing mechanisms over the Tibetan Plateau by relating homogeneous mixing degree ($\psi$) to transition scale number ($N_L$) at instantaneous states at 200 m, 300 m and 400 m above cloud bases. The contours represent the joint probability density function (PDF) of $\psi$ vs $N_L$. The magenta dots and error bars are mean values and standard deviations of $\psi$ in each $N_L$ bin, respectively. The mean values are fitted using a weighted least squares method with the number of data points in each $N_L$ bin as the weight. The fitting equation, coefficient of determination ($R^2$) and $p$ value are also given.
Figure 5. Temporal evolutions of the parameters $a$ and $b$ in the parameterization of equation (3) developed with cloud microphysics in each time interval (from 1 to 60), with the arrow to indicate the direction of temporal development. The pentagram and triangle represent the parameterizations developed with microphysical properties at the final states and all the instantaneous states, respectively.
Figure 6. The cloud droplet number concentration \( (n_c) \) during entrainment-mixing obtained by the parameterization of \( n_c \) (equation (6)) vs. \( n_c \) calculated from cloud droplet size distributions in the EMPM with 12,218 cases for 10 realizations of each simulation.
Figure 7. Joint probability density function (PDF) of relative dispersion ($d$) vs homogeneous mixing degree ($\psi$) based on 12,218 simulations for 10 realizations of each simulation. The magenta dots and error bars are mean values and standard deviations of $d$ in each $\psi$ bin, respectively. The mean values are fitted using a weighted least squares method with the number of data points in each $\psi$ bin as the weight. The fitting equation, coefficient of determination ($R^2$) and $p$ value are also given.
Figure 8. Relationships (a) between relative dispersion ($d$) and homogeneous mixing degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$ and $n_s$ in four simulations with different turbulence dissipation rate ($\varepsilon$). The big data points represent the beginning of entrainment-mixing processes. The pentagram and triangle are the turning points where Stage 1 shifts to Stage 2 or 3, and Stage 2 shifts to Stage 3, respectively.
Figure 9. The control run (Table 2). Temporal evolutions of relative dispersion \( (d) \), homogeneous mixing degree \( (\psi) \), cloud droplet size distributions (CDSDs), complete evaporation \( (-\Delta n_c) \), the ratio of partial to complete evaporation \( \left(\frac{\Delta n_s - \Delta n_c}{-\Delta n_c}\right) \) and the percentage of small droplets \( \frac{n_s}{n_c} \). See text for more explanations on \( -\Delta n_c \), \( \left(\frac{\Delta n_s - \Delta n_c}{-\Delta n_c}\right) \), and \( \frac{n_s}{n_c} \). The vertical dash lines represent the moments when Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 3, respectively. The red dash line represents the radius criterion of \( n_s \). The black dot line represents \( \frac{\Delta n_s - \Delta n_c}{-\Delta n_c} = 1 \).
Figure 10. Different dissipation rate ($\varepsilon = 1 \times 10^{-3}$ m$^2$s$^{-3}$) compared to the control run. Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud droplet size distributions (CDSDs), complete evaporation ($-\Delta n_c$), the ratio of partial to complete evaporation "$(\Delta n_s - \Delta n_c) / (-\Delta n_c)$", and the percentage of small droplets "$n_s / n_c$". See text for more explanations on $-\Delta n_c$, "$(\Delta n_s - \Delta n_c) / (-\Delta n_c)$", and "$n_s / n_c$". The vertical dash lines represent the moments when Stage 1 shifts to Stage 3. The red dash line represents the radius criterion of $n_s$. The black dot line represents $(\Delta n_s - \Delta n_c) / (-\Delta n_c) = 1$. 
Figure 11. Relationships (a) between relative dispersion ($d$) and homogeneous mixing degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$ and $n_s$ in three simulations with different relative humidity of entrained environment air (RH$_e$). The big data points represent the beginning of entrainment-mixing processes. The pentagram and triangle are the turning points where Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 2/3, respectively.
Figure 12. Different relative humidity of entrained environmental air (RH$_e$ = 55%) compared to the control run. Temporal evolutions of relative dispersion ($d$), homogeneous mixing degree ($\psi$), cloud droplet size distributions (CDSDs), complete evaporation ($-\Delta n_c$), the ratio of partial to complete evaporation “$(\Delta n_s - \Delta n_c) / (-\Delta n_c)$”, and the percentage of small droplets “$n_s / n_c$”. See text for more explanations on $-\Delta n_c$, “$(\Delta n_s - \Delta n_c) / (-\Delta n_c)$”, and “$n_s / n_c$”. The vertical dash lines represent the moments when Stage 1 shifts to Stage 2, and Stage 2 shifts to Stage 2/3, respectively. The vertical dot lines correspond to the circling period. The red dash line represents the radius criterion of $n_c$. The black dot line represents $(\Delta n_s - \Delta n_c) / (-\Delta n_c) = 1$. 
Figure 13. Relationships (a) between relative dispersion ($d$) and homogeneous mixing degree ($\psi$), (b) between small droplets number concentration ($n_s$) and $\psi$, (c) between $d$ and $n_s$ in three simulations with different mixing fraction of entrained air ($f$). The big data points represent the beginning of entrainment-mixing processes. The pentagram and triangle are the turning points where Stage 1 shifts to Stage 2 and/or 3, and Stage 2 shifts to Stage 3, respectively.
Figure 14. Different entrained air fraction \( f = 0.4 \) compared to the control run. Temporal evolutions of relative dispersion \( d \), homogeneous mixing degree \( \psi \), cloud droplet size distributions (CDSDs), complete evaporation \( -\Delta n_c \), the ratio of partial to complete evaporation \( \frac{(\Delta n_s - \Delta n_c)}{(-\Delta n_c)} \), and the percentage of small droplets \( \frac{n_s}{n_c} \). See text for more explanations on \( -\Delta n_c \), \( \frac{(\Delta n_s - \Delta n_c)}{(-\Delta n_c)} \), and \( \frac{n_s}{n_c} \). The vertical dash line represents the moment when Stage 1 shifts to Stages 2 and 3. The red dash line represents the radius criterion of \( n_s \). The black dot line represents \( \frac{(\Delta n_s - \Delta n_c)}{(-\Delta n_c)} =1 \).
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Figure 17. Schematic diagram of the evolutions of relative dispersion of cloud droplet size distribution ($d$), homogeneous mixing degree ($\psi$), small droplets ($n_s$), and their relationships and reasons.

<table>
<thead>
<tr>
<th>Stage</th>
<th>$\psi$ VS. $d$</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Negative</td>
<td>$P &gt; C$, strong $C$</td>
</tr>
<tr>
<td>2</td>
<td>Positive</td>
<td>$C &gt; P$</td>
</tr>
<tr>
<td>3</td>
<td>Positive</td>
<td>$P &gt; C$, weak $C$</td>
</tr>
</tbody>
</table>