Factors Governing Cloud Growth and Entrainment Rates in Shallow Cumulus and Cumulus Congestus During GoAmazon2014/5

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Key Points:

- Entrainment rates of Amazon cumulus and congestus from surface-based remote sensing and radiosondes are $0.58 \pm 0.10$ km$^{-1}$.
- Estimated entrainment rates are most dependent on cloud thickness and buoyancy, according to linear regression and random forest analyses.
- Correlations between entrainment and environmental variables vary across dry, wet, and transition seasons.
Abstract

Shallow cumulus and cumulus congestus clouds play an important role in the large-scale tropical circulation by mixing heat and moisture vertically and preconditioning the environment for deeper convection. Different representations of these shallow clouds account for much of the spread in General Circulation Model (GCMs) climate sensitivity, potentially because of how entrainment is represented in GCM parameterizations. This study uses observations from the Department of Energy's Atmospheric Radiation Measurement (ARM) mobile facility deployed at Manacapuru, Brazil, during the Green Ocean Amazon (GoAmazon2014/5) Campaign. Environmental thermodynamic profiles and observations of cloud-top height are used to constrain an entraining plume model to estimate bulk entrainment rates. Estimates of cloud-top height are obtained from a combination of vertically-pointing W-band ARM cloud radar (WACR) and 1290-MHz Radar Wind Profiler (RWP) observations. A combination of radiosonde, microwave radiometer profiler (MWRP) and microwave radiometer (MWR) observations provides new best estimates of the environmental thermodynamic state. We quantify uncertainty in entrainment rates considering uncertainties in estimated cloud-top height, environmental thermodynamic properties, and assumed initial parcel characteristics. We find entrainment rates ranging from 0.16 to 2.8 km$^{-1}$ with an average of 0.58 ± 0.10 km$^{-1}$ over a selected population of 469 shallow cumulus and cumulus congestus clouds. Using the retrieved estimates of entrainment rate, we evaluate several entrainment closures that are currently used in atmospheric models or have been proposed based on theory or large-eddy simulation. Entrainment rates in cumulus clouds are weakly correlated with low-level buoyancy, cloud depth, and cloud size.
1 Introduction

Convective clouds are formed from rising air parcels, to first order driven by buoyancy and modulated by environmental stability and mixing with environmental air. Both shallow cumulus and congestus cloud types play an important role in tropical climate dynamics by distributing heat and moisture vertically (Riehl et al., 1951), and in the case of congestus providing a contribution to the net diabatic heating term proportional to their precipitation (Schumacher et al., 2008; Stachnik et al., 2013). They also can act as a precursor for deeper convection (Neggers et al., 2007) by moistening and destabilizing the stable layers that inhibit them (Mechem & Oberthaler, 2013; Riehl et al., 1951; Stevens, 2007). Despite their importance, some aspects of these clouds are not fully understood, and their contribution to tropical dynamics is not correctly represented in global climate models (GCMs) (Nam et al., 2012; Williams & Tselioudis, 2007). GCMs underestimate the sensitivity of convection to the tropospheric humidity (Derbyshire et al., 2004), including over the Amazon region (Lintner et al., 2017). As a result, shallow cumulus and congestus cloud fractions tend to be underestimated (Nam et al., 2012; Williams & Tselioudis, 2007). While there have been recent improvements (Xie et al., 2019), GCMs struggle to represent the diurnal cycle of convective precipitation correctly, by initializing and deepening convection too quickly (Del Genio & Wu, 2010; Stirling & Stratton, 2012).

One reason for these shortcomings may be how entrainment is represented in GCM convective parameterizations (Del Genio, 2012). Entrainment is the rate at which environmental air is mixed into a cloudy updraft and is a first-order mechanism that governs the depth to which clouds penetrate (Bretherton et al., 2004). Entrainment affects the vertical transport of heat, humidity, and momentum (Brast et al., 2016), and has also been found to broaden the cloud
droplet size distribution, enhancing precipitation production (Cooper et al., 2013). Lateral entrainment, occurring at cloud edges, was first introduced by Stommel (1947). However, historically, entrainment was thought to occur at cloud top and mix into the cloud through penetrative downdrafts created by evaporative cooling from the entrained environmental air (Paluch, 1979; Squires, 1958). Other studies using large eddy simulations (LES) (Heus et al., 2008) and observations (Lin & Arakawa, 1997) cast doubt on the cloud top entrainment theory and support Stommel’s (1947) idea of lateral entrainment. In this study, we work from the premise that environmental air is continuously entrained at lateral cloud edges into a cloudy parcel as it rises, reducing its buoyancy.

Because entrainment is not resolved in GCMs, assumptions must be made about its behavior. Most current convective parameterizations are based on an entraining-plume framework, with an entrainment rate formulated as a function of various environmental or cloud properties, which are identified using LES. These entrainment relationships are speculative and remain underexplored, especially regarding the complicated couplings between variables.

Entrainment has been extensively studied in LES, but observational techniques to estimate entrainment are limited (e.g., Drueke et al., 2019; Jensen & Del Genio, 2006; Masunaga & Luo, 2016; Takahashi et al., 2017; Wagner et al., 2013). Estimating entrainment observationally is difficult because of the challenge of deploying an instrument suite capable of jointly sampling environmental thermodynamics and cloud properties with high spatial and temporal resolution. We use a combination of several surface-based remote-sensing instruments deployed during the Green Ocean Amazon (GoAmazon2014/5) field campaign to develop a method to estimate entrainment rates and their observational uncertainties in shallow cumulus and cumulus congestus clouds. We use these retrieved entrainment rates to evaluate common
entrainment closures used in GCM convective parameterizations, and we quantify any correlations with environmental variables.

2 Data

Data for this project are from the GoAmazon2014/5 field campaign, which took place in Manacapuru, Manaus, Brazil from January 2014 through December 2015 (Martin et al., 2016, 2017; see also Giangrande et al., 2017). One component of this field campaign was the deployment of the Department of Energy’s Atmospheric Radiation Measurement (ARM) Mobile Facility (AMF, Miller et al., 2016), which includes the W-band ARM cloud radar (WACR), radar wind profiler (RWP), radiosondes, microwave radiometer profiler (MWRP), microwave radiometer (MWR), and surface meteorology systems (MET). Properties of these instruments are summarized in Table 1. More information about the instruments during GoAmazon2014/5 can be found in the citations listed in the table.

We use the Active Remote Sensing of Clouds (ARSCL) product (ARM, 2014a; Clothiaux et al., 2000; Kollias et al., 2005), which combines WACR reflectivities, ceilometer, and micropulse lidar data to provide estimates of cloud properties, including cloud-base height (CBH), cloud-top height (CTH), and cloud thickness. The CBH estimate is generally taken from the laser-based ceilometer. The WACR is highly sensitive to small cloud droplets, but also severely attenuates in rain. To overcome this problem, we employ the RWP, which yields a more accurate cloud echo or top boundary estimates in the presence of precipitation. In the absence of large scatterers, the RWP may detect Bragg scattering resulting from sharp density gradients, such as those associated with inversions. Additional echo-classification using reflectivity, vertical velocity, and spectrum width was performed to distinguish between non-meteorological
scattering and relevant cloud echo conditions (e.g., Geerts & Dawei, 2004; Giangrande et al., 2013; Steiner et al., 1995).

To supplement the relatively infrequent radiosonde launches, we use the MWRP and MWR to develop best estimates of thermodynamic profiles for periods when precipitation is not present. We note that the MWRP did not become available for this field campaign until November 2014.

**Table 1**: Properties of the instruments deployed during GoAmazon2014/5 and used in this study.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Resolution</th>
<th>Purpose</th>
<th>Citation</th>
</tr>
</thead>
<tbody>
<tr>
<td>WACR (95 GHz)</td>
<td>42.86-m range gate, 2.048-s temporal spacing</td>
<td>Reflectivity, vertical velocity, spectrum width</td>
<td>Kollias et al., 2016</td>
</tr>
<tr>
<td>RWP (1290 MHz)</td>
<td>200-m range gate, 14-s temporal spacing</td>
<td>Reflectivity, vertical velocity, spectrum width, echo classification and boundaries</td>
<td>Giangrande et al., 2013, 2016; Feng &amp; Giangrande, 2018</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>4-5 launches daily</td>
<td>Thermodynamic profile</td>
<td>Giangrande et al., 2017; ARM, 2014b</td>
</tr>
<tr>
<td>MWRP</td>
<td>100-250 m vertical resolution, 1-min temporal spacing</td>
<td>Thermodynamic profile</td>
<td>ARM, 2014c</td>
</tr>
<tr>
<td>MWR</td>
<td>1-min temporal spacing</td>
<td>Precipitable water vapor (PWV), liquid water path (LWP)</td>
<td>Turner et al., 2007; ARM, 2014d</td>
</tr>
<tr>
<td>VARANAL</td>
<td>25 hPa vertical resolution, 3-hr intervals, 110-km circular domain</td>
<td>Large-scale forcings, specifically vertical velocity and moisture convergence profiles</td>
<td>Tang et al., 2016; Giangrande et al., 2017; ARM, 2014e</td>
</tr>
<tr>
<td>MET</td>
<td>1-min means</td>
<td>Surface temperature and equivalent potential temperature</td>
<td>ARM, 2014f</td>
</tr>
</tbody>
</table>

3 Methods

3.1 Observational estimates of entrainment rate

The entrainment rate (ER) is estimated with an idealized entraining-plume model, based on the classic entraining plume model, given by equation (1) (Betts 1975).

\[
\frac{\partial \theta_{ep}}{\partial z} = -\varepsilon(\theta_{ep} - \theta_e),
\]  

(1)
where $\theta_{ep}$ is the equivalent potential temperature of the parcel, $\theta_e$ is the equivalent potential temperature of the environment, and $\varepsilon$ is the entrainment rate. We use newly developed best estimates of maximum CTH and the thermodynamic environment to infer entrainment rates for each cumulus and congestus cloud sampled during the field campaign. The convective parcel is assumed to undergo linear mixing of environmental air as it ascends according to

$$
\theta_{ep} (z + \Delta z) = \frac{\theta_{ep}(z) + \varepsilon \Delta z \theta_e}{1 + \varepsilon \Delta z},
$$

(Jensen & Del Genio, 2006, hereafter JD06). This method assumes $\theta_e$ is conserved in adiabatic motions and that environmental air is mixed into the parcel continuously at a constant rate of $\varepsilon$. First, the observational $\theta_e$ profile is smoothed in the vertical using a moving average with a window width of 150 vertical grid points (~1.5 km) to minimize fine-scale variations. Entrainment rate is found iteratively, starting with an initial value of $\varepsilon = 0.01$ km$^{-1}$, and then increasing $\varepsilon$ in increments of 0.001 km$^{-1}$ until the (entraining) level of neutral buoyancy (ELNB) coincides with the CTH (Figure 1). Thus, these methods require an accurate measurement of the CTH, a representative thermodynamic environment, and an assumption about initial parcel thermodynamic properties and parcel ascent.

Entrainment estimates obtained from JD06 are closely tied to theoretical assumptions of the method; specifically, that convection can be represented by an idealized plume undergoing continuous, lateral entrainment with height. Historically, while the plume approach has been used in models, convection may be better represented by a series of transient thermals (Morrison et al., 2020; Peters et al., 2020). In the case of transient thermals, each thermal would grow in a subsequently moister environment, entraining higher $\theta_e$ air, and growing taller than the previous thermal. We anticipate that ER estimates obtained from the JD06 bulk plume model are smaller than those found using the thermal approach, because of the assumption of entraining pristine
environmental air with lower $\theta_e$, as opposed to an environment that might be moistened from previous thermals. Furthermore, the nature of the method used to calculate entrainment will impact the ER. Romps (2010) found that the “direct” method yields a higher ER compared to the bulk method, because air around the cloud (the cloud shell) is more humid than the environment. Therefore, more entrainment is required in the direct method to produce a given amount of dilution.

This entraining-plume method (and any method relying solely on profiling instruments) can provide ER estimates only for clouds passing directly over the radar, and therefore constitutes only a small sample of the total cloud population. Also, the possibility exists that the clouds being sampled are transient, meaning that some congestus observed could be at an intermediate point of their lifecycle and later go on to become deep cumulonimbus (Luo et al., 2009; Mechem & Oberthaler, 2013). In those cases, the observations of cloud top underrepresent the final CTH, and the ER will be overestimated. This method further assumes the maximum cloud height for any given cloud sampled by the radar is associated with the strongest updraft and therefore smallest value of entrainment. In reality, we are not always sampling the strongest updraft or center of every cloud that passes over the radar. Assuming sampling along a random chord of a circular cloud, we could be underestimating the cloud size by 22% (Jorgensen et al., 1985). If the cloud shape is ellipsoidal rather than circular, the bias in cloud size could be as high as 32% (Borque et al., 2014). How such ‘random radius’ chording biases predict associated biases in CTH, updraft intensity, or other cloud properties is not obvious and is beyond solely observational efforts to address (e.g., Wang et al., 2020a). The next two subsections address newly formulated best estimates of CTH and thermodynamic profiles.
Figure 1. The entraining-plume model used to find the entrainment rate. The blue lines represent the environmental $\theta_e$ and $\theta_{es}$. The black line is the undiluted parcel $\theta_e$, and the red lines are the $\theta_e$ of parcels experiencing different amounts of entrainment. Here, the entrainment rate is 0.15 km$^{-1}$ since that is the rate at which the CTH equals the ELNB.

3.2 Best estimate of CTH

A cloud is defined as 1 minute of contiguous (neighboring) profiles where the ARSCL reflectivity and mean Doppler velocity (MDV) are both defined. This study focuses on active cumulus and congestus clouds; we define these clouds as those having a maximum height between 1 and 9 km (JD06; Johnson et al., 1999), cloud base below 1 km, thickness greater than 300 m (Zhang & Klein, 2013), and positive buoyancy below cloud base to ensure clouds are surface-based and not forced (Stull, 1985). It is important to note that the echo top and cloud top are not necessarily the same, and therefore the echo height is an estimate of CTH.

As mentioned previously, the WACR strongly attenuates in the presence of precipitation (Giangrande et al., 2010; Haynes et al., 2009), resulting in an underestimate of CTH that leads to an overestimate of the entrainment rate. The lower-frequency RWP is unaffected by attenuation in rain, which may yield a more accurate estimate of CTH in precipitating cloud/congestus cloud.
contexts. For this reason, we use the RWP in combination with the ARSCL product to form a best estimate of CTH for all cloud cases. As above, an RWP-based echo classification product (Feng & Giangrande, 2018; Giangrande et al., 2016, 2017) is used to distinguish echoes from Bragg and Rayleigh (cloud, meteorological) scattering. The RWP CTH is taken to be the maximum height of the echo classified as either “convection,” “weak convection,” or “cloud.” The multi-instrument best estimate of the CTH is the maximum value of the ARSCL and RWP CTH values (Figure 2). The RWP CTH is most representative in precipitating clouds where the WACR beam is strongly attenuated. The ARSCL CTH is the best estimate of CTH in the cases where the cloud does not contain large enough hydrometeors and the RWP is not sensitive enough to observe the total cloud or any cloud. Approximately 70% of our CTH values are from the RWP.

Figure 2. The WACR reflectivity and RWP echo classification for a congestus cloud on April 4th, 2014. The WACR is clearly attenuating because of the precipitation and does not sample the entire cloud. Therefore, we use the RWP-sampled CTH.

A total of 893 shallow and congestus clouds were identified during the GoAmazon2014/5 deployment. However, when considering only the clouds for which the MWRP was operational and those for which the iterative ER estimation algorithm converges, the number of cases reduces to 469. Figure 3 shows a histogram of the CTH of all clouds used in this study. Although
Giangrande et al. (2020) indicates a maximum frequency of occurrence at 2 km associated with shallow cumulus, our methods filter out most of these clouds because of the requirement of positive buoyancy and thickness greater than 300 m.

![Histogram of cloud top heights](image)

**Figure 3.** A histogram of cloud top heights of all the clouds used in this study.

### 3.3 Best-estimate thermodynamic profile

Because radiosondes are only launched every six hours, ER calculations based on the sounding profiles alone may be up to six hours older than the cloud observation (soundings used from periods after the cloud may be influenced by convection), in which case the sounding may not be representative of the thermodynamic environment in which the cloud grew. We employ the temperature and moisture profiles from the MWRP to produce more representative thermodynamic profiles than the soundings alone. The MWRP has improved temporal resolution relative to the 6-hourly soundings, but the retrievals exhibit positive temperature and negative moisture biases at the surface and a positive moisture bias in the upper levels, so the MWRP profiles cannot be used directly. To overcome these shortcomings, we use the relative differences of the MWRP temperature and moisture profiles between the time of the cloud and the time of the most recent radiosonde profile to adjust the radiosonde temperature and moisture profiles. We further constrain the moisture profile using the MWR PWV retrieval (Turner et al., 1998).
First, the moisture profile is linearly scaled such that its PWV equals the MWR PWV. Next, any supersaturated layers are reduced to 100% RH, in which case the final adjusted sounding PWV will be less than the MWR PWV. Above 10 km altitude, where MWRP profiles are unavailable, the most recent sounding is assumed to be representative. Evaluation of the best-estimate methods can be found in the Supporting Information. Though we have done our best to create the most representative thermodynamic profile, a possibility exists that some of cases are contaminated with cold pools from prior convection, and the environment which was sampled is not representative of the environment in which the cloud developed. In an attempt to crudely identify the influence of strong cold pools generated from precipitation, we use the MET to identify cold pools directly, using a surface $\theta_e$ threshold of -4 K, a value consistent with the mean cold-pool $\theta_e$ perturbation from Zuidema et al. (2017, their Fig. 5). Of the 91 cold pools identified this way, 52 were associated with precipitation passing over the site and a wetting of the MWRP instrument and therefore not included in the “more stringent” category discussed below in Sec. 4.1.1. When the additional 39 cold-pool cases were removed, ER decreased by only 0.02 km$^{-1}$, which is within the uncertainty bounds.

3.4 Identifying environmental controls on entrainment rates

Environmental controls on ER are explored to evaluate common entrainment closures and identify other variables not commonly used in closures that potentially impact ER. A linear regression is performed of the entrainment rate and each environmental variable, including surface-based (most unstable, MU) and mixed-layer (ML, calculated over the lowest 100 hPa) convective available potential energy (CAPE), CAPE in the lowest 5 km, convective inhibition (CIN), RH in various layers, horizontal wind shear, environmental lapse rate (ELR , over 1-3 km or 3-6 km layers, depending on CTH), maximum buoyancy in the lowest 5 km,
large-scale vertical velocity at 700 mb, and the vertical integral of moisture convergence. We focus on the ER relationship with CAPE and buoyancy in the lowest 5 km to be more representative of shallow and congestus cloud growth environments.

Correlations between entrainment and cloud properties, including cloud depth and cloud size are also found. The cloud depth is the best estimate CTH minus the ARSCL CBH. The cloud size is calculated using the length of time the cloud is observed by the WACR multiplied by the average wind speed (from the radiosonde) in the cloud layer. Uncertainties in estimating cloud size with this method assume that the advection of the cloud is captured by the wind speed from the soundings and that the cloud evolves minimally as it passes over the radar. Overestimating the advection speed may therefore result in an overestimation of cloud size. Our cloud size estimate is also likely not the size of a single updraft but contains detrained condensate and elements from other clouds that are overlapped in the vertical.

Although simple linear regression provides a basic measure of how individual variables influence ER, the relationships among the different variables are likely highly covarying, and the influence of the variables on the ER is nonlinear (sum of the $R^2$ is not 1). For this reason, a broader array of statistical methods is needed to identify and explore possible relationships between variables. We use a random forest (RF) approach to evaluate the controls on ER.

RF is an ensemble learning method of nonlinear regression and an effective predictive analytical approach (Breiman, 2001). This method tends to have robustness against overfitting, as well as enhanced predictability, because it uses an ensemble of random, diverse trees which are likely to have offsetting biases (Breiman, 2001). We use the ‘Rpart’ R package based on the methods in Breiman et al. (1984) to create the forest. All the environmental and cloud variables are input into a RF to identify the variables with the largest effect on ER. The model tuning
parameters can be controlled with cross validation techniques to avoid overfitting the data (Kuhn & Johnson, 2013) in which case the model would not be a good predictor of new data. In the cross validation, we use a subset of the full dataset to train the model, a random 80% sample of the data. Then the model is tested on the remaining 20% subset of data. 10-fold cross validation with 1000 repeats is used. The model with the smallest root-mean-square-error (RMSE) between the training observations and test model output is chosen as the final model.

3.5 Sensitivity tests

Uncertainty in our estimates of ER arises from three different sources that can be broadly categorized as: (1) measurement uncertainty contributions to the retrieval of CTH and thermodynamic profiles, which are used as input to the entrainment algorithm; (2) sampling issues; and (3) uncertainty associated with certain assumptions in the bulk-plume method. Details of how we quantify entrainment uncertainty from instrumental and retrieval uncertainties are presented in Appendix A. We acknowledge the sampling uncertainties associated with profiling instruments (e.g., the chording discussion in Sec. 3.1), but the uncertainty analysis does not explicitly take this into account.

Uncertainties associated with the bulk-plume method arise from the choices made in the ER calculation, including how the initial parcel properties are formulated and how the parcel ascends. Parcels that ultimately form cumulus clouds generally originate at the surface (Lin, 1999a); however, using surface properties for the initial parcel may yield a parcel that is too buoyant. The actual parcel thermodynamics may be better represented by mixed-layer properties. The entrainment sensitivity to different assumptions about the initial parcel are quantified, including using surface-based and parcels originating from the lowest 100 m, 500 m, and 1 km mixed layer.
Pseudoadiabatic ascent assumes all of the water that is condensed is immediately rained out, whereas moist adiabatic ascent preserves the total water in the parcel, leaving it available for evaporation if the parcel descends. Pseudoadiabatic ascent does not consider hydrometeor loading, so the parcel will be more buoyant relative to moist adiabatic parcel ascent. In reality, neither of these extremes occurs and the actual parcel ascent is somewhere in between. Neither pseudoadiabatic nor moist adiabatic ascent includes the latent heat of fusion, which is reasonable for these clouds given the low ice content. The entrainment sensitivity to the ascent condition is quantified.

4 Best-Estimate Observational Entrainment Rates

Our best-estimate entrainment rates in warm, low clouds range from 0.16 to 2.8 km\(^{-1}\), with a mean of 0.58 km\(^{-1}\) and standard deviation of 0.39 km\(^{-1}\). These values are similar but exhibit a wider variation than JD06, who found values that ranged from 0.1 to 0.68 km\(^{-1}\) in 67 congestus clouds from Nauru Island in the Tropical Western Pacific. Lu et al. (2018) found entrainment rates measured from a bulk-plume model ranging from 1-3 km\(^{-1}\) in continental shallow cumulus.

4.1 Observational evaluation of entrainment closures in atmospheric models

4.1.1 Linear regression

The results of the linear regression analysis between the ER and environmental and cloud variables for warm, low clouds are shown in Table 2. We include only clouds for which our iterative ER estimation algorithm converges, and which occurred during the period the RWP was operational. A more stringent set of criteria excludes the cases where the difference between the radar CTH and ELNB is greater than 400 m to ensure that the entrainment and cloud top proxy (ELNB) are representative of the actual observed cloud. The more stringent criteria also include
only the cases using the MWRP and MWR best estimate of thermodynamics. We find 469 cases that meet the basic criteria and 221 cases that meet the more stringent criteria. The ER and explanatory variable correlations in the cases that meet the more stringent criteria are slightly improved from the basic criteria, as expected. Unless otherwise indicated, the results below are of the cases with the more stringent criteria.

Table 2: Linear correlation coefficients of entrainment rate and environmental and cloud variables for the cases which meet the basic criteria and more stringent criteria are presented and compared to JD06.

<table>
<thead>
<tr>
<th>ER vs…</th>
<th>R² in basic criteria cases</th>
<th>R² in more stringent criteria cases</th>
<th>JD06 R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>RH in the cloud layer</td>
<td>0.08</td>
<td>0.01</td>
<td>0.18</td>
</tr>
<tr>
<td>RH (0-2 km)</td>
<td>0.09</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>RH (2-4 km)</td>
<td>0.08</td>
<td>0.005</td>
<td>0.20</td>
</tr>
<tr>
<td>MU-CAPE in the lowest 5 km</td>
<td>0.007</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>ML-CAPE in the lowest 5 km</td>
<td>0.02</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>0.001</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>Max buoyancy</td>
<td>0.21</td>
<td>0.27</td>
<td>0.19</td>
</tr>
<tr>
<td>CIN</td>
<td>0.004</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>ELR</td>
<td>0.01</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>Vertical Velocity</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Moisture Convergence</td>
<td>0.002</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Cloud size</td>
<td>0.13</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>Cloud thickness</td>
<td>0.20</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Average ER (km⁻¹)</td>
<td>0.50</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>

The most common entrainment dependencies in the literature include inverse relationships between entrainment rates and updraft velocity (Neggers et al., 2002), cloud size
(Bechtold et al., 2001; Khairoutdinov & Randall, 2006; Stirling & Stratton, 2012), cloud thickness (Bretherton et al., 2004; Siebesma et al., 2003), and buoyancy (Lin, 1999b), and a positive relationship with environmental RH (Drueke et al., 2020; Lu et al., 2018; Stirling & Stratton, 2012), all supported by findings from LES. ER in GoAmazon2014/5 clouds have the strongest relationship with cloud thickness, which explains about 28% of the variance (Figure 4a). This relationship has a negative correlation, in agreement with the convective parameterizations which prescribe entrainment as inversely proportional to the depth, i.e., $\sim 1/H$ (Siebesma et al., 2003). A physical interpretation for this relationship is not apparent but could be explained by the covarying relationships between cloud depth and cloud size, for which the interpretation is clear (explained below).

Entrainment is moderately correlated with the maximum buoyancy in the lowest 5 km ($R^2 = 0.27$, Figure 4b). JD06 also found a correlation with $R^2 = 0.19$ in maritime congestus clouds. Lin (1999b) found an inverse relationship between cumulus entrainment and buoyancy in a cloud-resolving model (CRM) simulation. The negative relationship between ER and buoyancy in our results is consistent with Lin (1999b) and the idea that a cloud with larger buoyancy will have a larger vertical velocity, less time to entrain, and a smaller resulting ER (Neggers et al., 2002).

Early fluid tank experiments (Morton et al., 1956; Turner, 1969) found an inverse relationship between entrainment and plume radius leading to parameterizations of entrainment with the same relationship (Bechtold et al., 2001; Kain & Fritsch, 1990). The lateral cloud size explains about 21% of the variance in ER and is negatively correlated with ER (Figure 4c), in agreement with LES (Khairoutdinov & Randall, 2006; Stirling & Stratton, 2012). A wider cloud should be able to shield its updraft core from entraining environmental air, so the buoyant core is
not subjected to as much dilution by environmental air and therefore retains a stronger vertical velocity, relative to a narrower cloud (Bechtold et al., 2001; Kain & Fritsch, 1990).

Because of chording, we do not sample every cloud’s updraft, nor the center of the updrafts we do sample. We are able to observe vertically coherent updrafts in only a small number (37) of the congestus clouds. Vertical air motions are identified by the techniques described in Giangrande et al. (2016). The width of the updraft is found by visually identifying the updraft core ($w > 0$) and then transforming it to a physical width by multiplying the updraft time by the average wind speed in the cloud layer. In these clouds, entrainment is weakly negatively correlated with the updraft lateral size ($R^2 = 0.12$, Figure 4d), which is consistent with the lab findings of Morton et al. (1956) and Turner (1969) mentioned above. However, in clouds where a coherent updraft could be identified, the median of the updraft velocity is not correlated with entrainment (Figure 4e). This result contradicts the negative correlation between ER and buoyancy, which would naturally suggest a negative relationship between ER and updraft speed (Lin, 1999b; Neggers et al., 2002). Given the small number of cases, our confidence in the ER-$w$ relationship is low. We are more confident in the retrievals of buoyancy and cloud size, both of which show a negative relationship with ER.

We do not find a strong correlation with RH in the cloud layer ($R^2 = 0.01$) or with low-level (0-2 km) RH ($R^2 = 0.04$). However, our results support the qualitative (visual) results of these past observational studies of a positive correlation between ER and RH (Figure 4f). Both observational and LES studies support a positive relationship, with JD06 finding a relatively strong relationship between congestus ER and low-level (2-4 km) RH ($R^2 = 0.2$) and mid-level (5-7 km) RH ($R^2 = 0.18$), Lu et al. (2018) reporting a strong positive relationship with RH in 8 shallow clouds ($R = 0.8$), and Drueke et al. (2020) finding a robust positive relationship using
LES. The physical interpretation for the positive relationship is that in moister environments, the cloud retains more entrained air (less detrainment due to negative buoyancy), continues to rise and entrain, and then ultimately experiences more dilution (Lu et al., 2018; see also Drueke et al., 2020).

We also evaluate relationships with environmental variables explored by JD06 that have not been tested with LES nor are typically used in convective parameterizations, specifically the relationships between ER and CAPE, shear, and CIN. Entrainment is not correlated with either the MU- or ML-CAPE in the lowest 5 km ($R^2 = 0.03, 0.02$, respectively, Figure 4g), whereas JD06 find a slightly larger correlation ($R^2 = 0.12$). Given that ER is strongly anticorrelated with buoyancy, one might also expect a similar relationship with CAPE. However, our data show little correlation between maximum buoyancy and CAPE. Differences between our study and JD06 could be attributed to the geographic differences between the maritime tropical congestus observed by JD06 and the continental tropical clouds of the Amazon, a difference also recently found by Wang et al. (2020b). Sea breezes and other mesoscale circulations (Burleyson et al., 2016) unique to the Amazon may contribute to the differences, along with possible aerosol-mediated feedbacks on the local humidity (Abbot & Cronin, 2021). However, establishing mechanistic cause-and-effect relationships is beyond the scope of this study.

In agreement with JD06, we find negligible relationships ($R^2 < 0.1$) with the average shear from the surface to 700 hPa and CIN. We also observe no relationships with the vertical integral of moisture convergence (Figure 4h), large-scale vertical velocity at 700 hPa (Figure 4i), or ELR in the cloud layer.

The results were further broken down between the wet and dry seasons to assess seasonal variability of these dependencies. For simplicity, we define the wet season as December,
January, February, March, and April (Giangrande et al., 2017). The dry season is taken to be June, July, August, and September, and the transition seasons are May and October-November. The variables exhibiting the largest difference across the three regimes are shown in Table 3. Low mid-level RH, enhanced buoyancy and shear, and fewer, less organized cases characterize the dry season.

**Figure 4.** Entrainment rate versus several environmental and cloud variables. The error bars represent the standard deviation of entrainment in appropriately sized bins.
Figure 5. Entrainment versus cloud size (a), maximum buoyancy in the lowest 5 km (b), and cloud thickness (c) during the wet (green), dry (orange), and transition (blue) seasons.

The average ER is nearly the same in the dry (0.54 km$^{-1}$), wet (0.53 km$^{-1}$), and transition seasons (0.46 km$^{-1}$) (Table 3). The shallower CTH and lower mid-level RH in the dry season have offsetting effects on the ER resulting in small changes between the seasons. Correlations between ER and environmental variables are largely similar across the different seasons. Cloud size and maximum low-level buoyancy are more strongly correlated with ER in the dry season (Figure 5a,b). Cloud thickness is slightly more correlated with ER in the wet season (Figure 5c). Low-level CAPE and moisture convergence also have slightly larger R$^2$ in the wet season (not
shown). The stronger correlations in the dry season may be explained by a better estimation of ER in the dry season. The dry season has a lower cloud frequency and therefore the thermodynamic profile has less contamination because of other cloud or deep convective events.

**Table 3:** Mean values of several variables in each of the dry, wet, and transition seasons. These are the variables with the largest seasonal differences.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dry (n=126)</th>
<th>Wet (n=158)</th>
<th>Transition (n=119)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ER (km⁻¹)</td>
<td>0.526</td>
<td>0.538</td>
<td>0.459</td>
</tr>
<tr>
<td>RH (4-7 km) (%)</td>
<td>50.8</td>
<td>76.6</td>
<td>66.5</td>
</tr>
<tr>
<td>CIN (J kg⁻¹)</td>
<td>-76.8</td>
<td>-50.8</td>
<td>-68.9</td>
</tr>
<tr>
<td>Shear (cm s⁻¹)</td>
<td>0.35</td>
<td>0.19</td>
<td>0.23</td>
</tr>
<tr>
<td>Low-level CAPE (J kg⁻¹)</td>
<td>216</td>
<td>201</td>
<td>228</td>
</tr>
<tr>
<td>Low-level buoyancy (N kg⁻¹)</td>
<td>0.097</td>
<td>0.074</td>
<td>0.081</td>
</tr>
<tr>
<td>Cloud size (km)</td>
<td>11.4</td>
<td>9.2</td>
<td>8.9</td>
</tr>
</tbody>
</table>

4.1.2 Random Forests

The explanatory variables in an RF are analyzed by their “importance” to the dependent variable (ER in this case), which is defined as the reduction of the sum of squared error when that variable is used in a split, averaged over all trees in the forest. The results of the RF model show that the cloud size, maximum low-level buoyancy, and thickness have the most importance in determining entrainment (Figure 6). These variables also had the highest linear R² value of the ones we tested. The other environmental variables have small impact on ER. The RF model explains 54% of the variance in ER. The RMSE between the observed (testing data) and model predicted ER is 0.21 km⁻¹, so this model is able to predict ER with some accuracy. However, since the RF results explain just over half of the variance in ER, other factors are likely contributing to ER variability besides those used in this study (discussion on potential sources of variability in section 5).
We also use RF to evaluate the seasonal differences of the entrainment relationships in low clouds. The relative importance of each variable in each season is shown in Figure 6. In agreement with the linear analysis, cloud thickness is more important in the wet and transition seasons. Cloud size is somewhat more important in the dry season, in agreement with the linear analysis, but not as apparent, and much more important in the transition season. The linear analysis also found a stronger relationship between entrainment and buoyancy in the dry season, and the RF confirms this. The linear analysis did not suggest a large seasonal difference in the relationship with CAPE, but the RF shows CAPE with larger importance during the wet season. RH has consistent importance in the wet and dry seasons but is slightly more important in the transition season. These results, along with the linear regression results, suggest that entrainment could have different dependencies in different seasons, but the reasons for these differences are unknown. We speculate that these differences could be caused by more organized convection in the wet season (Wang et al., 2019) than the dry, or by differences in the convective forcing mechanisms (i.e., mesoscale variability).
We test the sensitivity of our ER estimates against the assumptions we use in the bulk-plume model and how we determine the environmental correlations. We have assumed that parcels originated at the surface and ascended pseudoadiabatically, and that the environmental profiles were representative of the environments in which the clouds grew.

Instead of assuming that cumulus are formed from surface-based parcels (Lin, 1999a), we assume the parcel initial conditions are representative of a mixed layer of varying depths. Figure 7 shows the differences of ER between a surface-based parcel and a mixed-layer average over the lowest 1000, 500, and 100 m. Surface-based ER estimates are larger than the 1000 m and 500
m ML ER estimates for the majority of cases (68% and 60% of the cases, respectively). For a well-mixed boundary layer, we would not expect any difference between surface-based and ML parcels, and therefore no difference in ER. We find the largest ER changes are from the 1000 m and 500 m ML parcels, which have an average difference of 0.048 (10%) and 0.050 km\(^{-1}\) (10.5%), respectively. The 100 m ML difference is much smaller (5.1%). Our choice of initial parcel properties yields a magnitude of uncertainty similar to the CTH and thermodynamic profile uncertainties (see Appendix for CTH and thermodynamic profile uncertainty details). However, regardless of the choice of initial parcel origin, the variables with the largest correlations with entrainment do not change (not shown).

Figure 7. Mixed-layer parcel entrainment rates versus the surface-based parcel entrainment rates for a 1000 m mixed layer (a), 500 m mixed layer (b), and 100 m mixed layer (c).

Pseudoadiabatic ascent assumes all of the water that is condensed is immediately rained out, whereas moist adiabatic ascent preserves the total water in the parcel, yielding a lower buoyancy because of hydrometeor drag and evaporative cooling during mixing. The assumption of moist adiabatic ascent represents the lower limit of the calculated ER with the true ER somewhere in between the pseudoadiabatic and moist adiabatic limits. We observe that ERs calculated using pseudoadiabatic ascent are an approximate factor of 3 larger than those calculated using moist adiabatic ascent (Figure 8). JD06 finds that pseudoadiabatic ascent ER
estimates are a factor of 2 larger than moist adiabatic ascent ER estimates. We speculate that this is caused by the larger difference in LNB between the pseudoadiabatic and moist ascent observed during GoAmazon2014/5 compared to those on Nauru Island, leading to a larger ER difference. Nonetheless, the ER calculated from moist adiabatic ascent exhibits similar correlations with the explanatory variables, except that buoyancy is less correlated with entrainment and low-level CAPE is more correlated (not shown).

**Figure 8:** Entrainment rates calculated from moist adiabatic ascent versus pseudoadiabatic ascent. The solid black line is the 1:1 line.

To evaluate the representativeness of our best-estimate thermodynamic profiles, following JD06 we explore whether the correlations between ER and environmental variables differ when calculating ER using more recent soundings, specifically, uncorrected soundings within 1 or 3 hours of the cloud observations. We find that using more recent soundings does not change the correlations, strongly suggesting that our best-estimate profiles are credible representations of the environment associated with the cloud observations.
5 Conclusions

This study estimates entrainment rates in cumulus and congestus clouds using observations collected during the GoAmazon2014/5 field campaign. We expand on the work of JD06 by: (1) including measurements from additional sensors (RWP, MWRP, MWR) to better constrain the estimates of entrainment rate; (2) applying these techniques to a much greater number of cases; (3) considering a different meteorological environment (tropical continental); and (4) including a particular focus on uncertainty quantification. We evaluate common entrainment closures supported by LES using standard linear correlation analysis and random forest regression methods that embrace the nonlinearity and covariability inherent in the relationships among the different variables.

The main findings of our analysis are summarized below:

- We estimate entrainment rates ranging from 0.16 to 2.8 km\(^{-1}\) with an average of 0.58 ± 0.10 km\(^{-1}\) in 469 clouds.

- Entrainment rate is best correlated with cloud thickness (R\(^2\)=0.28), maximum buoyancy in the lowest 5 km (R\(^2\)=0.27), and cloud size (R\(^2\)=0.21). These variables exhibiting the strongest correlations are also dominant factors in the random forest analysis.

- Entrainment rate tends to be best correlated with cloud thickness and low-level CAPE in the wet season, buoyancy in the dry season, and RH during the transition season.

Our analysis gives some mixed observational support for several of the entrainment closures supported by LES, but the correlations between entrainment and the environmental and cloud variables are fairly weak and do not fully explain the variability in entrainment rate. The small spatial and temporal scales used to represent the thermodynamic profile, observation uncertainties, or limitations of the bulk-plume method could explain the remaining variance.
Though we have improved the thermodynamic profile and CTH estimates, observational limitations and uncertainties from the retrievals remain, which we have worked to quantify. The Amazon region has considerable mesoscale variability not represented in vertical profiles, including the continental effects mentioned above as well as cold pools or dry/moist layers resulting from prior convection, which may all impact the entrainment rates.

Limitations of the theoretical assumptions of the bulk-plume method include representing convection by an idealized plume undergoing continuous, linear entrainment with height. Convection that is more transient than steady may be better represented by a bubble entraining at cloud top (Yano, 2014). Entrainment may vary with height. Several studies have estimated the vertical dependence of entrainment and found that entrainment is maximum at cloud base and decreases above (de Rooy et al., 2013; Lin, 1999b; Lu et al., 2012). However, JD06 tested the sensitivity of their results to height-varying entrainment rates and found little change in their results.

Our results strongly suggest that future field campaigns use the RWP in conjunction with cloud radars to obtain accurate macrophysical cloud properties, especially when sampling clouds with larger hydrometeors. Supplementing relatively infrequent soundings with high-temporal-resolution remote-sensing retrievals to improve observational estimates of the thermodynamic profiles is highly desirable. Future research should further explore seasonal entrainment dependencies and the nature of those dependencies but would require additional field deployments or long-term LES of the Amazon environment. In addition, it would be beneficial to compare the observations with LES simulations of GoAmazon2014/5 clouds and entrainment rates, and further explore the role of nonlinear relationships and interactions between variables in predicting entrainment rates and the mechanisms behind the relationships.
Appendix A Quantifying Entrainment Rate Uncertainty

We quantify the uncertainty in the estimated entrainment based on instrumental and retrieval uncertainties that are well characterized using standard error propagation techniques (Taylor, 1982). Some of the following choices may seem arbitrary yet are necessary given the number of degrees of freedom especially in the height-dependent uncertainties of the retrieved thermodynamic profiles.

Uncertainty in the CTH measurements is attributable to the minimum detectable signal (Rayleigh scattering in cloud/precipitation) for the RWP. The RWP range gate spacing (200 m) is used as the uncertainty in the CTH, a value that is both a convenient choice and in reasonable agreement with calculations of radar reflectivity from LES with size-resolving microphysics done by Mechem et al. (2015). Using output from their control simulation and assuming a radar sensitivity of 5 dBZ, the median underestimate of echo-top height is 278 m. Although derived from a single simulation of continental precipitating congestus, this calculation nevertheless provides confidence in the reasonableness of our 200-m value. This uncertainty is then propagated through to ER by increasing and decreasing all of the CTH values by 200 m, computing the ER, and comparing those rates to the original rates. Although the WACR range gate spacing is smaller (42.86 m), to be conservative and avoid confusion we use 200 m as an uncertainty for all CTH measurements.

Underestimating CTH by 200 m overestimates the average ER of the campaign by 0.075 km\(^{-1}\), an 11.9% change. Similarly, overestimating CTH by 200 m underestimates the average ER by 0.082 km\(^{-1}\), a 17.6% change. The changes in ER are not symmetric when the uncertainty is added and subtracted, so we use the average magnitude change of all ER changes as the uncertainty in entrainment because of CTH uncertainty, 0.079 km\(^{-1}\) (15%). When all cases are
considered, the average ER uncertainty for both adding and subtracting the CTH uncertainty is 0.06 km$^{-1}$ (11.3%).

Assessing the uncertainty in the thermodynamic profile is less straightforward, since we use a combination of several instruments and retrievals with different uncertainty characteristics. We estimate the uncertainty in the temperature profile as the sum of squares of the MWRP (Cadeddu & Liljegren, 2018) and sounding (Holdridge et al., 2011) uncertainties in three layers. The profile is then smoothed to remove sudden changes in the vertical temperature.

The moisture uncertainty profile is found in a similar manner to the temperature uncertainty profile. The profile is broken down into 4 layers, and the uncertainty is calculated in each of the layers as the sum of squares between the sounding RH (Holdridge et al., 2011), MWRP total vapor density (Cadeddu & Liljegren, 2018), and MWR PWV (Gaustad & Turner, 2007) uncertainties. All of the instrumental uncertainty measurements are converted to mixing ratio units (g kg$^{-1}$) using a representative temperature and pressure in the layer.

A representative case having CTH of 6 km and an ER of 0.28 km$^{-1}$ is used as a baseline to propagate the uncertainty in temperature and moisture and estimate the total uncertainty in these thermodynamic parameters. The temperature and moisture uncertainty (±1σ) profiles are added to and subtracted from the baseline case temperature and moisture profiles, in all possible combinations. These new uncertainty profiles are substituted into the ER calculation to find the changes in ER from the baseline case.

The ER uncertainty caused by sounding, MWRP, and MWR measurement uncertainties is found by averaging the magnitudes of the four uncertainties, resulting from the four possible combinations of adding and subtracting the temperature and moisture uncertainty profiles, 0.065 km$^{-1}$. The entrainment uncertainty caused by cloud top uncertainty and by thermodynamic
retrieval uncertainty are of similar magnitude. We combine the ER uncertainties caused by both sources of uncertainty using the sum of squares to find the final ER uncertainty. This value is 0.102 km$^{-1}$ for the more stringent cases and 0.088 km$^{-1}$ for all cases.

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Data availability: All ARM data sets from the GoAmazon2014/5 campaign used for this study are available through the ARM discovery website: https://adc.arm.gov/discovery/#/results/site_code::mao

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