A new approach for simultaneous estimation of entrainment and
detrainment rates in non-precipitating shallow cumulus

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Key points:

● A new approach for simultaneously estimating entrainment and detrainment rates in cumulus clouds is presented.

● Estimated entrainment and detrainment rates can reproduce cloud thermodynamic variables.

● Effects of environmental relative humidity on entrainment rate and detrainment rate are consistent with buoyancy sorting.
Abstract.

A new approach is developed for estimating entrainment and detrainment rates in cumulus clouds based on aircraft observations. Equations relating entrainment and detrainment rates to gross entrainment and detrainment are derived. This approach is applied to the Holistic Interactions of Shallow Clouds, Aerosols, and Land-Ecosystems (HI-SCALE) field campaign, supported by the U.S. Department of Energy’s Atmospheric Radiation Measurement (ARM) program. The results show that both entrainment and detrainment rates decrease with increasing height. Sensitivity tests with different detrained air assumptions yield similar results. The entrainment and detrainment rates can reproduce the cloud thermodynamic variables. Partial correlation analysis indicates that entrainment rate is positively correlated with environmental relative humidity, and detrainment rate is negatively correlated with environmental relative humidity and positively correlated with entrainment rate. This new approach can be applied to other cloud observations to obtain a dataset of entrainment and detrainment rates in cumulus clouds.

Key words: Entrainment rate; detrainment rate; new approach; aircraft observation; cumulus
Entrainment and detrainment rates are key properties in the simulation of cumulus clouds. Previous studies have diagnosed entrainment and detrainment rates in simulation experiments. However, there is no published approach for simultaneous estimation of entrainment and detrainment rates using aircraft observations. A new approach is presented to fill this gap. This approach is applied to aircraft observations of shallow cumulus clouds. Both entrainment and detrainment rates decrease with increasing height. The results are verified by reproducing the variables in the cloud. Using the new approach to obtain a dataset of entrainment and detrainment rates could help improve shallow convection parameterizations in models.
1. Introduction

Cumulus convection is an important component in the atmospheric energy and hydrologic cycles (Arakawa, 2004). The vertical transport of heat, moisture, and momentum generated on or near the Earth's surface to high levels are significantly affected by cumulus clouds (Donner et al., 2011; Guo et al., 2014; Nie & Kuang, 2012; Wang et al., 2018). Accurate parameterizations of cumulus clouds in large-scale models are critical to the simulations of precipitation (Lin et al., 2013; Wang et al., 2017), the Madden-Julian oscillation (Del Genio et al., 2012; Zhang & Song, 2009), monsoons (Zou & Zhou, 2011), and tropical cyclones (Zhao et al., 2018a).

The entrainment and detrainment are key processes that affect cumulus convection, representing the inflow of environmental air into the cloud and the outflow of cloudy air into the environment, respectively (Blyth, 1993; de Rooy et al., 2013; Houze, 1993; Simpson & Wiggert, 1969; Wang, 2020; Xue & Feingold, 2006). In mass flux convection schemes, which are widely used, entrainment and detrainment have become increasingly crucial (de Rooy et al., 2013; Murata & Ueno, 2005; Nie & Kuang, 2012). For example, climate sensitivity (Klocke et al., 2011; Zhao, 2014), precipitation (Cooper et al., 2013), and monsoons (Yang et al., 2015) in models are sensitive to entrainment and/or detrainment. Many studies have developed parameterizations of entrainment rate ($\varepsilon$) by relating $\varepsilon$ to cloud radius, vertical velocity, buoyancy, vertical gradient of buoyancy, and/or environmental relative humidity (RH) (Bera & Prabha, 2019; Lu et al., 2018; Simpson & Wiggert, 1969; Stirling & Stratton, 2012; von Salzen & McFarlane, 2002; Zhang et al., 2016). Compared with $\varepsilon$, detrainment rate ($\delta$) has
been relatively understudied. However, $\delta$ is a key factor that determines the vertical variation of mass flux (Böing et al., 2012; de Rooy & Siebesma, 2008). In some Eddy-Diffusivity Mass-Flux (EDMF) schemes (e.g., Neggers et al. (2009)), $\epsilon$ is parameterized and cloud mass flux is obtained by an equation for updraft vertical velocity and assumptions about updraft cloud fraction. Therefore, $\delta$ can be deduced subject to assumptions of cumulus updraft fraction. In many other cumulus parameterization schemes, $\epsilon$ and $\delta$ are parameterized to obtain the vertical variation of cloud mass flux, since $\epsilon$, $\delta$, and cloud mass flux are constrained by mass continuity. For example, some studies have parameterized $\delta$ as a function of cloud radius, vertical velocity, and critical mixing fraction between cloudy and environmental air (Böing et al., 2012; Bretherton et al., 2004; Dawe & Austin, 2013; de Rooy & Siebesma, 2008; Tiedtke, 1989). The buoyancy sorting schemes have been introduced and widely applied (Bretherton et al., 2004; Kain & Fritsch, 1990; Raymond & Blyth, 1986), which also considers both $\epsilon$ and $\delta$ (Bretherton et al., 2004).

To understand entrainment and detrainment processes and improve their parameterizations, it is critical to estimate $\epsilon$ and $\delta$ accurately from observations and high-resolution models. Specifically, $\epsilon$ is often estimated based on conserved properties (e.g., total water mixing ratio ($q$), liquid water potential temperature, moist static energy ($h$)) in cloudy and environmental air with the bulk plume approach (Bera & Prabha, 2019; Betts, 1975; Esbensen, 1978; Gerber et al., 2008; Neggers et al., 2003; Nie et al., 2016; Yanai et al., 1973; Zhang et al., 2016). Lu et al. (2012b) proposed an approach for estimating $\epsilon$ with the mixing fraction between cloudy and environmental air, which had less uncertainty than the
above traditional plume approach. Fewer studies have been devoted to the evaluation of $\delta$, and most of them are based on numerical simulations through the equation (Bera & Prabha, 2019; Böing et al., 2012; de Rooy et al., 2013; Siebesma, 1998):

$$\delta = \epsilon - \frac{1}{M_c} \frac{dM_c}{dz},$$

(1)

where $M_c$ is cloud mass, $z$ is height. Although it is straightforward to calculate $dM_c / dz$ in simulations, it is quite challenging to obtain $dM_c / dz$ from aircraft observations of cumulus clouds that are not very deep. Romps (2010), Dawe & Austin (2011), and Wang (2020) diagnosed $\epsilon$ and $\delta$ at each grid cell of large-eddy simulations (LES) by directly calculating the entrainment and detrainment of air into and out of clouds. While these approaches are direct, they are only limited to high-resolution numerical simulations. In contrast, Norgren et al. (2016) presented an approach for estimating the amount of gross entrainment ($m_e$) and detrainment ($m_d$) based on aircraft observations. The gross entrainment and detrainment are defined as the integrated fractional mass entrained and detrained, respectively, from cloud base to a given height level (see Section 3 for details). However, they cannot be directly used in cumulus parameterizations where $\epsilon$ and $\delta$ are needed (Arakawa & Schubert, 1974), which are defined as the fractional mass entrained and detrained per unit height, respectively (equations (6-7)). Currently, to the best of our knowledge, there is no published approach for simultaneously estimating $\epsilon$ and $\delta$ using aircraft in situ observations.

To fill this gap, we present a new approach for estimating $\epsilon$ and $\delta$. The new approach is then applied to aircraft observations of shallow cumulus clouds during the Holistic Interactions of Shallow Clouds,
Aerosols, and Land-Ecosystems (HI-SCALE) field campaign (Fast et al., 2019). Note that we focus on non-precipitating shallow cumulus clouds. One reason is that they are globally prevalent over oceans and land (Norris, 1998) and significantly affect the thermodynamic structure (Neggers et al., 2007) and large-scale circulation (Nie, 2013; Siebesma, 1998; Tiedtke, 1989); the other reason is that these clouds are one of the leading factors influencing the cloud-climate feedbacks (Bony et al., 2004; Bony & Dufresne, 2005; Nie, 2013). The conclusions from this study may be qualitatively applicable to deep cumulus clouds, but special attention should be paid to quantitative application.

2. Data

During the HI-SCALE field campaign, two Intensive Observational Periods (IOPs) were conducted by the Department of Energy’s Gulfstream-1 (G-1) aircraft over the Southern Great Plains (SGP) site in north-central Oklahoma, one in the spring between 24 April and 21 May and the other in late summer between 28 August and 24 September 2016. Details of aircraft payload in the HI-SCALE field campaign (38 flights) were documented in Fast et al. (2019). Flight parameters and meteorological conditions (including ambient temperature, static pressure, humidity, and wind speed) were measured by Aircraft-Integrated Meteorological Measurement System (AIMMS-20) at 20 Hz. Cloud droplet size distributions (CDSDs) were measured by the Fast-Cloud Droplet Probe (FCDP) at 10 Hz. FCDP can size particles from 0.5 to 25 μm (radius) in 20 bins. Here, only the droplets with bin-average radii larger than 1 μm and smaller than 25 μm are used to calculate cloud microphysical properties, including cloud number concentration ($N_c$), liquid water content (LWC), and volume-mean radius ($r_v$). The High
Volume Precipitation Spectrometer-3 (HVPS) measured particles from 37.5 to 4537.5 μm (radius) at 1 Hz. Temperature measured by AIMMS is corrected for wet cooling by the empirical equation based on Fig. 8b in Lawson & Cooper (1990). In total, 477 non-precipitating shallow cumulus clouds along horizontal penetrations are identified during 13 flights (25 April, 27 April, 1 May, 2 May, 11 May, 18 May, 20 May, 1 September, 4 September with 2 flights, 15 September, 17 September, and 20 September 2016). The criteria used to identify the cases include: CDSDs with $N_c$ larger than 10 cm$^{-3}$ and LWC larger than 0.001 g m$^{-3}$ are considered to be cloud records (Deng et al., 2009; Lu et al., 2012a; Lu et al., 2013); the mean drizzle LWC in a cloud calculated from HVPS during each penetration is required to be smaller than 0.005 g m$^{-3}$ (Lu et al., 2013); one individual cloud penetration width must exceed 150 m.

3. New Approach for Estimating Entrainment and Detrainment Rates

Norgren et al. (2016) presented an approach for estimating gross entrainment and detrainment based on the conservation of three variables: $M_c$, $q$, and $h$.

\[ m_a + m_c - m_d = 1, \]  
\[ q_a m_a + q_c m_c - q_d m_d = q_c, \]  
\[ h_a m_a + h_c m_c - h_d m_d = h_c, \]

where mass fractions $m_a = M_a / M_c$, $m_c = M_c / M_c$, and $m_d = M_d / M_c$; $M_a$, $M_c$, $M_d$, and $M_c$ are cloud masses at the aircraft observation level; the subscripts a, e, d, and c represent adiabatic cloudy air,
laterally entrained air, laterally detrained air, and aircraft-sampled cloudy air, respectively. In equations (2 - 4), $m_a$, $m_c$, and $m_d$ are the three unknowns. The rest of the variables are measured from aircraft observations (see Section 4 for details): $q$ is the sum of water vapor mixing ratio and liquid water mixing ratio and $h$ is the sum of the enthalpy per unit mass of air, the potential energy, and the latent heat content (Wallace & Hobbs, 2006). The approach of Norgren et al. (2016) can deduce $m_a$, $m_c$, and $m_d$, which are very useful in the investigation of vertical transport of mass and energy. However, one needs $\varepsilon$ and $\delta$ to make a connection to cumulus parameterizations.

Rearrangement of equation (1) and integration from cloud base ($z_0$) to a certain level in cloud (e.g., aircraft observation level) above the sea level yield

$$
\int_{z_0}^{z} (\varepsilon - \delta) dz = \int_{M_c(z_0)}^{M_c(z)} \frac{dM_c}{M_c}. 
$$

(5)

Using subscripts $e$ and $d$ to represent the contributions of entrainment and detrainment to $M_c$, respectively, $\varepsilon$ and $\delta$ can be respectively represented by

$$
\varepsilon = \frac{1}{M_c} \frac{dM_c}{dz},
$$

(6)

$$
\delta = \frac{1}{M_c} \frac{dM_c}{dz}.
$$

(7)

Rearranging equations (6) and (7) and integrating from $z_0$ to $z$ yield
To obtain the layer-averaged entrainment and detrainment (for simplicity of use of symbols, from now on we use \( \varepsilon \) and \( \delta \) to denote layer-averaged entrainment and detrainment) for the depth from \( z_0 \) to \( z \) (see more details in the supporting information), equations (5) and (8) become:

\[
\varepsilon - \delta = \frac{1}{H} \ln \frac{M_c(z)}{M_c(z_0)},
\]

\[
\frac{M_e(z) - M_e(z_0)}{\varepsilon} = \frac{M_d(z) - M_d(z_0)}{\delta},
\]

respectively, where \( H = z - z_0 \) is the height above the cloud base; \( M_c(z) \) and \( M_c(z_0) \) represent the mass of cloud parcel at \( z \) and \( z_0 \), respectively; \( M_e(z) \) and \( M_e(z_0) \) represent the mass of entrained air at \( z \) and \( z_0 \), respectively; \( M_d(z) \) and \( M_d(z_0) \) represent the mass of detrained air at \( z \) and \( z_0 \), respectively. Since \( M_e(z_0) \) and \( M_d(z_0) \) are both zero, i.e., there is no entrained or detrained mass flux at cloud base \( z_0 \), equation (10) becomes

\[
\frac{M_e(z)}{\varepsilon} = \frac{M_d(z)}{\delta}.
\]

Combination of (9) with (11) yields

\[
\varepsilon = \frac{M_e(z)}{H(M_e(z) - M_d(z))} \ln \frac{M_c(z)}{M_c(z_0)},
\]

\[
\delta = \frac{M_d(z)}{H(M_e(z) - M_d(z))} \ln \frac{M_c(z)}{M_c(z_0)}.
\]

Since \( \frac{m_d}{M_c} = \frac{M_d}{M_c} \), \( \frac{m_e}{M_c} = \frac{M_e}{M_c} \), and \( \frac{m_a}{M_c} = \frac{M_a}{M_c} \) (Norgren et al., 2016), the expressions for \( \varepsilon \) and \( \delta \)
are

\[ \varepsilon = \frac{m_e}{H(m_e - m_a)} \ln \frac{1}{m_a}, \]

(14)

\[ \delta = \frac{m_d}{H(m_e - m_d)} \ln \frac{1}{m_a}, \]

(15)

respectively.

Note that due to the limitation of observational data that each cloud can only be detected once, we can only calculate the \( \varepsilon \) and \( \delta \) once for each cloud. \( H_m = (z - z_0)/2 \) is used as a height marker for the calculated \( \varepsilon \) and \( \delta \), similar to Lu et al. (2012a).

The above derivation indicates that our new approach is a two-step procedure: first obtain \( m_a, m_e, \) and \( m_d \) with the approach of Norgren et al. (2016), and then obtain \( \varepsilon \) and \( \delta \) with equations (14) and (15), respectively. If there are multiple-level observations in the clouds, the vertical distributions of \( \varepsilon \) and \( \delta \) will be more accurate with a higher vertical resolution by repeating the above procedure and obtain vertical profiles of \( \varepsilon \) and \( \delta \). For example, if there are three horizontal penetrations (P1, P2, and P3 from low to high), we can calculate \( \varepsilon \) and \( \delta \) between P1 and P2, and also between P2 and P3, respectively.
4. Calculation of Gross Entrainment and Detrainment

The mass fractions $m_a$, $m_e$, and $m_d$ are calculated using equations (2 - 4) following Norgren et al. (2016). The air 1 km away from the cumulus cloud edges is assumed to be entrained into clouds, and the meteorological properties in the air during a horizontal penetration are averaged and taken as the meteorological properties of the entrained dry air for that penetration ($q_e$ and $h_e$). Lu et al. (2012a) found that the properties in the air 0.5 km away from the cumulus cloud edges are close to the environmental air represented by the aircraft vertical sounding. The reason for choosing “1 km” is to make sure that the environmental air used is reasonably far away from the cloud. The cloud sample with maximum LWC in each cloud is assumed to be the adiabatic cloudy air to calculate $q_a$ and $h_a$ (Lu et al., 2012a; Lu et al., 2020). The in-cloud properties ($q_c$ and $h_c$) in each cloud are the average of all cloud samples. Laterally detrained air is assumed to be composed of 90% of the air with saturation moist static energy at the environmental temperature and 10% of mean aircraft-sampled cloudy air; $q_d$ and $h_d$ are calculated under this assumption.

Similar to Figure 3 in Norgren et al. (2016), Figure 1 shows that $m_e$ and $m_d$ tend to increase with increasing altitude in the flight on 1 May 2016. The results of other flights are similar. These vertical distributions are primarily consistent with the results of the Gulf of Mexico Atmospheric Composition and Climate Study (GoMACCS) field campaign in Norgren et al. (2016), indicating that our results are reasonable. Similar to the results of Norgren et al. (2016), $m_d$ is much smaller than $m_e$. Generally, $m_d$ is smaller than 50%, while $m_e$ varies from ~0% to ~80%. Barnes et al. (1996) found in observations
that net entrainment was predominant in the early stages of growing clouds, while net detrainment
often occurred during the dissipation phase. In HI-SCALE, the observation period was between 0900
and 1400 local time (Coordinated Universal Time (UTC) + 6 h). Sampling the clouds during the
morning and early afternoon and the cloud selecting criteria may limit the number of dissipating clouds.
Therefore, $m_d$ in the analyzed clouds is small. The increasing $m_e$ with increasing height is consistent
with many previous observational studies as well. For example, researchers found that the adiabatic
fraction (the ratio of the observed LWC to the adiabatic LWC) decreases with the increasing height
(Ackerman, 1963; Cotton, 1975; Houze, 1993; Lu et al., 2008; Warner, 1970).

To test if the vertical trends of $m_e$ and $m_d$ depend on the mixture assumption of 90/10 the air with
saturation moist static energy at the environmental temperature and the cloudy air, sensitivity tests are
conducted. Note that detrained air is often assumed to be neutrally buoyant and saturated at the
environmental mean temperature in convection parameterization schemes (Arakawa & Schubert,
1974). We have tested this assumption, i.e., the mixture assumption of 100/0, and also other mixture
assumptions of 80/20, 70/30, 60/40, and 50/50. Results show that the vertical distributions of $m_e$ and
$m_d$ remain consistent with each other (Figure S1).

5. Vertical Distributions and Verifications of Entrainment and Detrainment Rates

After analyzing $m_e$ and $m_d$, Figure 2 is plotted to show the vertical variations of $\varepsilon$ and $\delta$ with the
increasing $H_m$. It is critical to estimate the cloud base. As mentioned previously, the HI-Scale campaign was carried out over the SGP site, the same location as the Routine AAF (ARM Aerial Facility) Clouds with Low Optical Water Depths (CLOWD) Optical Radiative Observations (RACORO) field campaign. The cumulus clouds during the HI-Scale were expected to have different cloud base heights even during the same flight, as pointed out in Vogelmann et al. (2012) about RACORO. The cloud base for each cloud is estimated by extrapolating the maximum LWC in each cloud to the height with LWC = 0 (Lu et al., 2012a). There are other approaches for estimating cloud base heights using aircraft observed LWC vertical profiles (Gerber et al., 2008; Lu et al., 2012b), lidar observations (Clothiaux et al., 2000), meteorological information below cloud base (Wallace & Hobbs, 2006), etc. However, these approaches are not applicable to this study (Lu et al., 2012a). The cloud base estimation uncertainties are discussed in Lu et al. (2012a).

As shown in Figure 2, both $\varepsilon$ and $\delta$ generally decrease with the increasing $H_m$, consistent with previous studies (Bera & Prabha, 2019; Böing et al., 2012; Dawe & Austin, 2011, 2013; Romps, 2010). Results from many LES simulations showed that both $\varepsilon$ and $\delta$ generally decrease first with the increasing height, and then gradually increase when approaching the cloud tops (Böing et al., 2012; Dawe & Austin, 2011; Romps, 2010). Dawe & Austin (2013) used LES to estimate the probability density functions of $\varepsilon$ and $\delta$ in a large number of cumulus clouds and obtained similar vertical distributions of $\varepsilon$ and $\delta$. For the lower layers of the cloud, our results are similar to the results in the literature; it should be noted that the thickness of the shallow cumulus clouds in HI-Scale is generally small. The $\varepsilon$ and
δ measured by Bera & Prabha (2019) using the bulk plume approach and equation (1) also showed a decreasing trend with increasing height, indicating that our calculation results are credible.

Similar to Section 4, sensitivity tests are also conducted to analyze the impacts of different mixture fraction assumptions on $\varepsilon$ and $\delta$. Results show that changing the assumption does not affect the vertical trends of entrainment and detrainment rates (Figure S2). Figure S3 and Table S1 further quantitatively compare the results assuming different mixing fractions to those assuming 90/10. Assuming different mixing fractions has negligible effects on entrainment rate. Although the data points of $\delta$ become scattered when the mixing fraction gradually changes from 90/10 to 50/50, the data points still generally distribute along the 1:1 line. The relative change is $+2.8\%$, $+2.9\%$, $+7.8\%$, $+12.7\%$, $+21.6\%$, and the correlation coefficients are 0.93, 0.98, 0.94, 0.92, 0.88 for the mixing fractions of 100/0, 80/20, 70/30, 60/40, 50/50, respectively.

After showing the results from the new approach, the accuracy of the results is further verified. Houze (1993) presented an idealization of a rising cloud parcel interacting with its environment and the conservation equation for a variable $A$ can be derived as

$$
\frac{dA_c}{dz} = \left(\frac{dA_e}{dz}\right)_a + \frac{1}{M_e} \frac{dM_e}{dz} (A_e - A_c) + \frac{1}{M_c} \frac{dM_d}{dz} (A_c - A_d),
$$

(16)

where $A_c$ is the variable in a cloud, $(d A_c / d z)_a$ is the rate of change of $A_c$ in adiabatic clouds, $A_e$ is the variable entrained from the environment, $A_d$ is the variable detrained from the cloud, $M_c^{-1} d M_c / d z$ is...
is the entrainment rate, \( \frac{M_c^{-1} d M_d}{dz} \) is the detrainment rate; see the supporting information for detailed derivation. Houze (1993) assumed that \( A_d \) in the detrained air was equal to \( A_c \) in the cloud and removed the last term of equation (16), i.e., detrainment had no effect on \( A_c \). However, this term is retained here, because the detrained air is assumed to be the mixture of the cloudy air and the air with saturation moist static energy at the environmental temperature.

Based on equation (16) and the estimated \( \varepsilon \) and \( \delta \), as well as the observations from the aircraft vertical sounding at the cloud base heights, we can calculate the thermodynamic variables like \( h_c \) and total water mixing ratio (\( q_c \)) at the horizontal penetration heights and compare the calculated values with the observations. Figure 3 shows that the correlations are good for both \( h_c \) and \( q_c \), indicating that the calculated \( \varepsilon \) and \( \delta \) can reproduce the thermodynamic variables, which is a good verification of the accuracy of the results.

6. Effects of Environmental Relative Humidity on Entrainment and Detrainment Rates

Given the important role of environmental RH in affecting convection (Abbott & Cronin, 2021; Axelsen, 2005; Lu et al., 2018; Stirling & Stratton, 2012), it is interesting to study the relationships between \( \varepsilon \), \( \delta \), and environmental RH. Compared with the correlation coefficient, the partial correlation coefficient is a better choice for analyzing the correlations when three or more variables are not independent. The partial correlation measures the linear correlation between two variables with the linear influence of other variables removed (Freund & Wilson, 2003). Table S2 shows the partial
correlation coefficients between $\varepsilon$, $\delta$, and environmental RH. These correlations can be explained by
the buoyancy sorting concept (Bretherton et al., 2004; de Rooy et al., 2013; de Rooy & Siebesma, 2008; Emanuel, 1991; Kain & Fritsch, 1990). According to buoyancy sorting, cloud and environmental air are mixed together with different fractions near cloud edges. The positively buoyant mixtures with a mixing fraction of environmental air ($\chi$) smaller than the critical mixing fraction ($\chi_c$) are entrained into the cloud. Oppositely, the negatively buoyant mixtures with $\chi$ larger than $\chi_c$ are detrained.

Bretherton et al. (2004) derived that

$$\varepsilon = \varepsilon_0 \chi_c^2,$$

(17)

$$\delta = \varepsilon_0 (1 - \chi_c)^2,$$

(18)

where $\varepsilon_0$ is a parameter related to height above the ground. Furthermore, de Rooy & Siebesma (2008) derived the expression that relates $\chi_c$ to RH given by

$$\chi_c = \frac{A}{B - RH},$$

(19)

where $A$ and $B$ are two parameters. Hence, $\varepsilon$ is expected to be high and $\delta$ is low when environmental RH is high. Table S2 shows that $\varepsilon$ is positively correlated with environmental RH with the partial correlation coefficient of 0.66, as expected in equation (17). Similar conclusions were reached in many studies (Axelsen, 2005; Bera & Prabha, 2019; Lu et al., 2018; Stanfield et al., 2019; Stirling & Stratton, 2012), while many others found a weak or even opposite correlation (Bechtold et al., 2008; Böing et al., 2012; Derbyshire et al., 2011; Stanfield et al., 2019; Zhao et al., 2018b).
The relationship between $\delta$ and RH is relatively weak $(-0.28)$, which is due to two opposite factors. On the one hand, equations (17 and 18) can be rewritten as

$$\delta = \varepsilon \left( \frac{1}{\chi} - 1 \right)^2,$$

(20)

which indicates that $\delta$ and $\varepsilon$ tend to be positively correlated; a similar conclusion was also reached in Stirling & Stratton (2012). The positive correlations between $\delta$ and $\varepsilon$ and between $\varepsilon$ and RH cause a positive correlation between $\delta$ and RH. On the other hand, equations (18, 19, and 20) show that buoyancy sorting leads to a negative correlation between $\delta$ and RH. The combined results of the two factors weaken the relationship between $\delta$ and RH, and also the relationship between $\varepsilon$ and $\delta$.

Note that $\varepsilon$ and/or RH are often used to parameterize $\delta$ (Böing et al., 2012; Bretherton et al., 2004; Bush et al., 2015; Siebesma, 1998; Stirling & Stratton, 2012). Therefore, equation (21) is fitted to connect $\delta$ with $\varepsilon$ and RH, trying to determine $\delta$ as a function of $\varepsilon$ and RH.

$$\delta = \varepsilon^{3.42} (0.23 - \text{RH} / 4.06)^{0.81}.$$ 

(21)

The fitting has a good correlation coefficient of 0.69, but the root mean squared error is 0.28, which is also due to two opposite factors mentioned above. When multiple-level observations are available, it will be interesting to analyze the effects of the vertical gradient of moist static energy on $\varepsilon$ and $\delta$. 


A new approach is presented for estimating entrainment rate \( (\varepsilon) \) and detrainment rate \( (\delta) \) in cumulus clouds. The two-step approach first estimates the mass fractions with the approach of Norgren et al. (2016) and then relates the mass fractions to \( \varepsilon \) and \( \delta \). The new approach is applied to the aircraft observation of shallow cumulus clouds during the HI-SCALE field campaign, to estimate \( \varepsilon \) and \( \delta \). The mass fractions of entrained air \( (m_e) \) and detrained air \( (m_d) \) gradually increase with increasing altitude, and this distribution is similar to the results of Norgren et al. (2016). The distribution of \( m_e \) is consistent with previous studies (Ackerman, 1963; Cotton, 1975; Houze, 1993; Lu et al., 2008; Warner, 1970). Sensitivity experiments of different assumptions of detrained air show similar vertical distributions.

After analyzing the calculations of \( m_e \) and \( m_d \), the vertical distributions of \( \varepsilon \) and \( \delta \) are analyzed. Both \( \varepsilon \) and \( \delta \) generally decrease with increasing height above the cloud base, consistent with previous studies. Sensitivity tests are also conducted to analyze the impact of different assumptions on \( \varepsilon \) and \( \delta \). Results show that the data points of \( \delta \) become relatively scattered when the mixing fraction of detrained air gradually changes from 90/10 to 50/50, the data points still generally distribute along the 1:1 line. An equation is derived to relate \( \varepsilon \) and \( \delta \) to the thermodynamic variables, with which the calculated \( \varepsilon \) and \( \delta \) can reproduce the change of the thermodynamic variables. Partial correlation analysis of \( \varepsilon \), \( \delta \), and environmental RH confirms the combined influence of the two opposite factors on the relationship between \( \delta \) and RH. One is that the positive correlations between \( \delta \) and \( \varepsilon \) and between \( \varepsilon \) and RH could cause a positive correlation between \( \delta \) and RH, and the other is that a negative correlation between \( \delta \)
and RH is supported by buoyancy sorting.

Note that the interaction between cloud and environment over some small time change is a very important question, because environmental variables affect the entrained air properties and further affect cloud properties, and cloud properties affect detrained air properties and thus environmental variables. However, it is really challenging to answer this question from aircraft observations, because the observations only provide cloud snapshots. This question could be addressed in future studies with high-resolution numerical simulations, for example, large-eddy simulations. From the viewpoint of macroscale behavior of the entrainment processes related to cumulus parameterization, cloud properties would follow moist adiabatic processes without entrainment. Thus, entrainment rates are inferred from how much the cloud properties deviate from their adiabatic values. This line of thinking inevitably involves some time-averaging implicitly. This is consistent with the fact that most existing cumulus parameterization schemes and associated representation of entrainment and detrainment rates assume that cumulus updrafts are in steady-state.

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Figure 1. Mass fractions of entrained air ($m_e$) (a) and detrained air ($m_d$) (b) as a function of altitude in the flight on 1 May 2016 in the Holistic Interactions of Shallow Clouds, Aerosols, and Land-Ecosystems (HI-SCALE) field campaign. Here the altitude on the y-axis is the height of horizontal penetrations above sea level.
Figure 2. Entrainment rate ($\varepsilon$) (a) and detrainment rate ($\delta$) (b) as a function of $H_m = (z - z_0)/2$ in the non-precipitating cumuli in the HI-SCALE field campaign, where $z$ is the height of aircraft horizontal penetration above sea level and $z_0$ is the cloud base height.
Figure 3. Calculated moist static energy ($h_c$) versus observed $h_c$ (a) and calculated total water mixing ratio ($q_c$) versus observed $q_c$ (b) in the 477 non-precipitating cumuli in the HI-SCALE field campaign. $R$ and $p$ are correlation coefficient and significant level, respectively.