EXAFS-50 Brookhaven National Laboratory Oct 30-31, 2023

Perfecting the Theory of XAFS

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University of Washington and SLAC

Office of

Science





Perfecting the Theory of XAFS

• TALK -

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- I. Challenges in XAFS Theory
- II. Theoretical advances over 50 years
- III. Beyond XAFS XANES, Multiplets, BSE & Corvus Workflows

I. Challenge of XAFS theory

X-ray Absorption Fine Structure



 $\chi = \frac{\mu - \mu_0}{\mu_0}$ Small correction to

atomic XAS - need high accuracy & very large energy range

Historical Interpretation ~ 50 yrs ago*

EXAFS

FT = Local structure



Many parameters - need to calibrate with "EXAFS standard"

Question: Can one calculate XAFS parameters?

Maybe or maybe not ?

"I always thought it was easier to

measure XAS than to calculate it."

Hans Bethe ca 1980

Standard electronic structure theory **FAILS**

XAS at K edge of Cu 1.2 Normalized Expl. FEFF8-final state, 300K **Ground state** FEFF8-ground state 1 DFT FEFF 0.8 0.6 Missing in DFT: Core-hole potential, mean free path & DW factors Fine structure too large 0.2 0 9000 9200 9400 9600 9800 10000

Energy (eV)

II. Theoretical Advances

Breakthrough: Spherical wave scattering theory FIXES Plane Wave Approximation

PHYSICAL REVIEW B

VOLUME 34, NUMBER 6

15 SEPTEMBER 1986

New high-energy approximation for x-ray-absorption near-edge structure

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Asymptotic high energy approximation: effective spherical wave scattering amplitudes f_{eff}

R_f for XAFS

Revised EXAFS Equation: Stern, Sayers, Lytle

$$\chi(k) = S_0^2 \sum_R \frac{|f_{\text{eff}}(k)|}{kR^2} \sin(2kR + \Phi_k) e^{-2R/\lambda_k} e^{-2\sigma^2 k^2}$$

Curved wave scattering theory $k \sim 25 \text{ Å}^{-1}$ $l \sim 25 f_{\text{eff}} \Phi_k$ Mean free paths $\lambda_k \sim 5 - 20 \text{ Å}$ and self energy $\Sigma(E)$ Vibrational damping $e^{-2\sigma^2 k^2}$

Multi-electron excitations $S_0^2 \sim 0.8$

- **Relativistic scattering potentials** *V(r)*
 - No codes in 1970's with all features !

Three key advances

Curved wave single-scattering amplitudes $f_{\rm eff}$ & phases φ Hedin-Lundqvist Self-energy $\Sigma(E)$

Relativistic Dirac-Fock scattering potentials *V(r)*



Approximations: Overlapped atom potentials, const S_0^2

Result: Theoretical XAFS Standards

Theoretical X-ray Absorption Fine Structure Standards JACS 113, 5136 (1991) J. J. Rehr.^{*,†} J. Mustre de Leon,^{†,‡} S. I. Zabinsky,[†] and R. C. Albers[§]

Contribution from the Department of Physics, FM-15, University of Washington, Seattle, Washington 98195, and Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545. Received November 13, 1990

Abstract: Theoretical X-ray absorption fine structure (XAFS) standards are developed for arbitrary pairs of atoms throughout the periodic table ($Z \le 94$). These standard XAFS spectra are obtained from *ab initio* single-scattering XAFS calculations, using an automated code, FEFF, which takes into account the most important features in current theories: (i) an exact treatment of curved-wave effects; (ii) approximate molecular potentials derived from relativistic atoms, (iii) a complex, energy-dependent self-energy; (iv) a well defined energy reference. FEFF also yields tables of XAFS phases and amplitudes as well as mean-free paths. Sample results are presented and compared with experimental results and with earlier work. We find that these theoretical standards are competitive with experimental standards, permitting XAFS analysis at lower wavenumbers and yielding distance determinations typically better than 0.02 Å and coordination numbers typically better than 20%. These standards also provide theoretical tests of chemical transferability in XAFS.

$$\chi_{l}^{(1)}(E) = -N A(E) \frac{|f_{\text{eff}}(\pi, k, R)|}{kR^{2}} \sin (2kR + 2\delta_{c} + \Phi) e^{-2R/\lambda} e^{-2\sigma^{2}k^{2}}$$

FEFF3 Accurate single-scattering standards

Challenge: Multiple-scattering in XAFS



Golden rule via Green's functions: Real space MS

$$\mathbf{G} = 1/(E - h' - \Sigma)$$

$$\mu(E) \sim -\frac{1}{\pi} \operatorname{Im} \langle i | \hat{\epsilon} \cdot \mathbf{r}' \operatorname{G}(\mathbf{r}', \mathbf{r}, E) \hat{\epsilon} \cdot \mathbf{r} | i \rangle$$

h' Final-state Hamiltonian with core-hole Σ Self-energy including mean-free-path

Multiple-scattering path expansion

$$\mu(E) \sim -\frac{1}{\pi} \operatorname{Im} \langle i | \hat{\epsilon} \cdot \mathbf{r'} \operatorname{G}(\mathbf{r'}, \mathbf{r}, E) \hat{\epsilon} \cdot \mathbf{r} | i \rangle$$

$$\begin{split} G &= G^{c} + \sum_{i \neq 0} G^{c} t_{i} G^{c} + \sum_{i \neq j \ i, j \neq 0} G^{c} t_{i} G^{0} t_{j} G^{c} + \cdots \\ &\equiv \sum_{\Gamma} G^{\Gamma}. \end{split}$$

Computational bottleneck $N \sim 100$, $lmax \sim 20$ Matrix products $Nlmax^2 x Nlmax^2 \sim 10^4 x 10^4$

Exact form useful only for low order 4-leg MS paths

Breakthrough: RA Separable Green's function

PHYSICAL REVIEW B

VOLUME 41, NUMBER 12

15 APRIL 1990-II

Scattering-matrix formulation of curved-wave multiple-scattering theory: Application to x-ray-absorption fine structure

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R. C. Albers

Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545 (Received 13 January 1989; revised manuscript received 4 January 1990)

$$G_{L,L'}(\rho) = \frac{e^{i\rho}}{\rho} \sum_{\lambda} \tilde{\Gamma}_{\lambda}^{L}(\rho) \Gamma_{\lambda}^{L'}(\rho)$$

$$\rho = kR_{\lambda_{1}}$$

$$\tilde{\Gamma}_{0}$$

$$\tilde{\Gamma}_{0}$$

$$\tilde{\Gamma}_{0}$$
Fast, accurate GF and high-order real-space
$$\tilde{\Gamma}_{3}$$
multiple-scattering path expansion

Cut diagrams

R

Breakthrough: Full multiple scattering - XANES

PHYSICAL REVIEW B

VOLUME 58, NUMBER 12

15 SEPTEMBER 1998-II

Real-space multiple-scattering calculation and interpretation of x-ray-absorption near-edge structure

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Full multiple scattering $G^{SC} = G^0 t G^0 + G^0 t G^0 t G^0 + \cdots$ by matrix inversion $= G^0 (1 - t G^0)^{-1}$ + SCF potentials



Implementation – FEFF6

PHYSICAL REVIEW B

VOLUME 52, NUMBER 4

15 JULY 1995-II

Multiple-scattering calculations of x-ray-absorption spectra

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Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

M. J. Eller Microsoft Corporation, Redmond, Washington 98052



Efficient MS path filters, algorithms and coding

Example: Accurate distances R_{nn} from XAFS





Partial answer to Hans Bethe: "XAFS can be easier to calculate than to measure - *IF* the structure is known" JJR

Review: 30 yrs of developments in XAFS





J. J. Rehr & R.C. Albers Rev. Mod. Phys. **72**, 621 (2000)

http://feff.phys.washington.edu

Validation with Expt: IFEFFIT & ARTEMIS

J. Synchrotron Rad. 8, 322 (2001)

IFEFFIT: interactive XAFS analysis and FEFF fitting

Matthew Newville*

Consortium for Advanced Radiation Sources, The University of Chicago, 5640 S Ellis Ave, Chicago, IL 60637, USA. E-mail: newville@cars.uchicago.edu

J. Synchrotron Rad. 12, 537 (2005)

ATHENA, ARTEMIS, HEPHAESTUS: data analysis for X-ray absorption spectroscopy using IFEFFIT

B. Ravel^a* and M. Newville^b

EXAFS vs XANES

PHYSICAL REVIEW B, VOLUME 65, 104107 (2002)

Parallel calculation of electron multiple scattering using Lanczos algorithms

A. L. Ankudinov,¹ C. E. Bouldin,² J. J. Rehr,¹ J. Sims,² and H. Hung²

¹Department of Physics, University of Washington, Seattle, Washington 98195 ²National Institute of Standards and Technology, Gaithersburg, Maryland 20899



Improved theory: Parameter free XAFS & XANES



Ab initio theory with NO adjustable parameters



Phys. Chem. Chem. Phys. 12, 5503-5513 (2010)

PERSPECTIVE

www.rsc.org/pccp | Physical Chemistry Chemical Physics

Parameter-free calculations of X-ray spectra with FEFF9

John J. Rehr,*^{*a*} Joshua J. Kas,^{*a*} Fernando D. Vila,^{*a*} Micah P. Prange^{*bc*} and Kevin Jorissen^{*a*}

Ingredients in *ab initio* XAS theory



- \checkmark Green's fn $G_{LR,L'R'}$
- \checkmark Atomic data Z < 138

 $\mu = \mu_0(1 + \chi)$

Separable propagators

Relativistic Dirac-Fock

- \checkmark Scattering potentials V(r) SCF, all-electron
- \checkmark RPA Core-hole V_c
- \checkmark HL Self-energy $\Sigma(E)$
- \checkmark Ab initio Debye-Waller σ^2
- ✓ Many body factor $S_0^2(E)$

Screening, excitonic effects

Mean-free path, energy shifts

Thermal vibrations

Multi-electron excitations

RPA Screened core-hole potential ✓



RPA improves on final state rule, half-core hole, etc

Many-pole GW Self-energy $\Sigma(E) * \checkmark$

Efficient ~ GW approximation for self-energy Σ & mean free path λ

Sum of HL plasmon-pole models matched to loss function

 $\Sigma(E) = iGW = \Sigma' - i\Gamma$

$$\varepsilon_2(\omega) = \frac{4\pi}{V} \operatorname{Im} \int d\mathbf{r} d\mathbf{r}' \operatorname{Tr}[d\chi(\mathbf{r},\mathbf{r}';\omega)d^{\dagger}],$$

***J.J. Kas** et. al, Phys Rev B **76**, 195116 (2007)



Ab initio Debye Waller factors ✓

An Initio Determination of Extended X-Ray Absorption Fine Structure Debye-Waller Factors

Fernando D. Vila, G. Shu, and John J. Rehr Department of Physics, University of Washington, Seattle, WA 98195

H. H. Rossner and H. J. Krappe Hahn-Meitner-Institut Berlin, Glienicker Strasse 100, D-14109 Berlin, Germany (Dated: August 23, 2005)

01



$$\sigma^{2} = \frac{\hbar}{\mu_{i}} \int_{0}^{\infty} \rho(\omega^{2}) \operatorname{coth} \frac{\beta \hbar \omega}{2} d\omega$$
$$\rho(\omega^{2}) = \langle Q_{i} | \delta(\omega^{2} - D) | Q_{i} \rangle$$
$$= \{6 - \text{step Lanczos recursion}\}$$

1

 $e^{-2\sigma^2k^2}$

Many pole model

for phonons



*F. Vila et al., Phys. Rev. B 76, 014301 (2007)

D dynamical matrix < DFT codes

ABINIT or Quantum Espresso

Anharmonic effects & cumulant expansion

PHYSICAL REVIEW B

VOLUME 48, NUMBER 1

1 JULY 1993-I

Thermal expansion and x-ray-absorption fine-structure cumulants

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J. J. Rehr Department of Physics, University of Washington, Seattle, Washington 98195

Effects of large vibrational disorder beyond MSRD

$$\langle e^{2ikR} \rangle = \exp[\sum_{n} \frac{C_n (2ik)^n}{n!}] \qquad C_n = \langle R^n \rangle_c$$

 C_1 Thermal expansion, C_2 MSRD, C_3 skew ... & relations between cumulants $C_1 C_2 \& C_3$ Q: How to treat S_0^2 theoretically?



Shake-up & shake-off theory of S_0^2

PHYSICAL REVIEW B

VOLUME 17, NUMBER 2

15 JANUARY 1978

Extended x-ray-absorption fine-structure amplitudes-Wave-function relaxation and chemical effects*

J. J. Rehr and E. A. Stern Department of Physics, University of Washington, Seattle, Washington 98195

R. L. Martin[†] and E. R. Davidson Department of Chemistry, University of Washington, Seattle, Washington 98195

Phasor sum of many-body overlap integrals

 $S_0^2 = \sum S_n^2 e^{2i(k_n - k)R}$ Huge computational bottleneck



Suggestion: Approximate S_n^2 with XPS

PHYSICAL REVIEW B 86, 165102 (2012)

K-edge x-ray absorption spectra in transition-metal oxides beyond the single-particle approximation: Shake-up many-body effects

M. Calandra,¹ J. P. Rueff,^{2,3} C. Gougoussis,¹ D. Céolin,² M. Gorgoi,⁴ S. Benedetti,⁵ P. Torelli,⁶ A. Shukla,¹ D. Chandesris,⁷ and Ch. Brouder¹



Theory: XPS from core hole Green's function G_c

Intrinsic multi-electron excitations & plasmons due to suddenly turned-on core-hole:



Core-hole Green's function $G_c(\omega)$



Validation: Satellites in XPS

 $A_c(\omega) = (1/\pi) |\text{Im } G_c(\omega)| \sim \text{XPS } J_k(E)$



J. J. Kas, F. D. Vila, J. J. Rehr, and S. A. Chambers, PRB **91**, 121112(R) (2015)

Application to S_0^2 in XAFS*

Cumulant approach for inelastic losses in x-ray spectra



*Ch. 27 in Sébilleau, Didier, Keisuke Hatada, and Ebert, "Multiple Scattering Theory for Spectroscopies." *Springer Proceedings in Physics* **204** (2018).

Improved theory: interference effects in S_0^2



Interference reduces losses near E_{θ}

All losses in convolution with particle-hole spectral function

$$\mu(\omega) = \int d\omega' \,\tilde{A}_K(\omega') \mu_K(\omega - \omega')$$

Example: Particle-hole cumulant for CeO₂*

 $Ce L_3 XAS of CeO_2$

Spectral function



Result: improves agreement in correlated systems

*J.J. Kas et al. Phys Rev B 94, 035156 (2016)

III. Beyond XAFS - XANES & Related spectra

RIXS and NRIXS

PHYSICAL REVIEW B 83, 235114 (2011)

Real-space Green's function approach to resonant inelastic x-ray scattering

J. J. Kas,¹ J. J. Rehr,^{1,*} J. A. Soininen,² and P. Glatzel³

¹Department of Physics, Box 351560, University of Washington, Seattle, Washington 98195-1560, USA ²Department of Physics, P.O. Box 64, University of Helsinki, FI-00014 Helsinki, Finland ³European Synchrotron Radiation Facility, B.P. 220, F-38043 Grenoble, France (Received 21 January 2011; revised manuscript received 7 April 2011; published 8 June 2011)

We present an *ab initio* theory of core and valence resonant inelastic x-ray scattering (RIXS) based on a real-space multiple-scattering Green's function formalism and a quasiboson model Hamiltonian. Simplifying assumptions are made that lead to an approximation of the RIXS spectrum in terms of a convolution of an effective x-ray absorption signal with the x-ray emission cross section. Additional many-body corrections are incorporated in terms of an effective energy-dependent spectral function. Example calculations of RIXS are found to give qualitative agreement with experimental data. Our approach also yields simulations of lifetime-broadening suppressed x-ray absorption, as observed in high-energy resolution fluorescence detection experiment. Finally, possible improvements to our approach are briefly discussed.



Compton, $S(q, \omega)$

PHYSICAL REVIEW B 85, 115135 (2012)

Real-space Green's function calculations of Compton profiles

Brian A. Mattern, Gerald T. Seidler,⁺ Joshua J. Kas, Joseph I. Pacold, and John J. Rehr Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA (Received 2 February 2012; revised manuscript received 16 March 2012; published 29 March 2012)

We report the development of a first-principles, real-space Green's function method for calculation of Compton profiles in the impulse approximation. For crystalline Be, we find excellent agreement with prior theoretical treatments requiring periodicity, with prior experimental measurements of the Compton profile, and with present measurements of the dynamical structure factor via nonresonant inelastic x-ray scattering (often also called x-ray Thomson scattering in the plasma physics community). We also find good agreement with prior experimental results for the Compton profile of Cu. This approach can be extended to disordered and very high-temperature systems, such as "warm dense matter," where theories presently used for the interpretation of inelastic x-ray scattering include condensed phase effects only at a perturbative level.



High temperature and extreme conditions

PRL 119, 176403 (2017) PHYSICAL REVIEW LETTERS

Finite Temperature Green's Function Approach for Excited State and Thermodynamic Properties of Cool to Warm Dense Matter

J. J. Kas and J. J. Rehr

PHYSICAL REVIEW B 104, 035144 (2021)

Real-space Green's function approach for x-ray spectra at high temperature

Tun S. Tan, J. J. Kas, and J. J. Rehr^{®*}

Finite *T* cumulant Green's function theory of XAS



Multiplet + Cumulant Approach

PHYSICAL REVIEW LETTERS 128, 216401 (2022)

Ab Initio Multiplet-Plus-Cumulant Approach for Correlation Effects in X-Ray Photoelectron Spectroscopy

J. J. Kas,^{1,*} J. J. Rehr⁽¹⁾,^{1,2} and T. P. Devereaux^{3,4}



Full spectrum optical constants: UV-X-ray



Full spectrum optical constant interface to the Materials Project

J.J. Kas^{a,b}, F.D. Vila^{a,b}, C.D. Pemmaraju^b, M.P. Prange^c, K.A. Persson^d, R.X. Yang^d, J.J. Rehr^{a,b,*}





100000

High-throughput calculations for ML and Al

All ~10⁵ materials & structures in MP Data base

SCIENTIFIC DATA

Including feff.inp

SCIENTIFIC DATA | 5:180151 | DOI: 10.1038/sdata.2018.151

Data Descriptor: High-throughput computational X-ray absorption

spectroscopy

Kiran Mathew^{1,*}, Chen Zheng^{2,*}, Donald Winston³, Chi Chen², Alan Dozier⁴, John J. Rehr⁵, Shyue Ping Ong² & Kristin A. Persson¹

npj Computational Materials

www.nature.com/npjcompumats

npj Computational Materials (2018) 4:12 ; doi:10.1038/s41524-018-0067-x Corrected: Author correction

ARTICLE OPEN

Automated generation and ensemble-learned matching of X-ray absorption spectra

Chen Zheng¹, Kiran Mathew², Chi Chen¹, Yiming Chen¹, Hanmei Tang¹, Alan Dozier³, Joshua J. Kas⁴, Fernando D. Vila⁴, John J. Rehr⁴, Louis F. J. Piper^{5,6}, Kristin A. Persson² and Shyue Ping Ong ¹

Improved XANES: Bethe-Salpeter Equation (BSE)

aka Particle-Hole Green's function



Efficient implementation of core-excitation Bethe-Salpeter equation calculations



K. Gilmore ^{a,b,*}, John Vinson ^c, E.L. Shirley ^c, D. Prendergast ^d, C.D. Pemmaraju ^d, J.J. Kas ^e, F.D. Vila ^e, J.J. Rehr ^e

IOP Publishing

J. Phys.: Condens. Matter 26 (2014) 363202 (24pp)

Journal of Physics: Condensed Matter doi:10.1088/0953-8984/26/36/363202

exciting: a full-potential all-electron package implementing density-functional theory and many-body perturbation theory

Andris Gulans¹, Stefan Kontur¹, Christian Meisenbichler¹, Dmitrii Nabok¹, Pasquale Pavone¹, Santiago Rigamonti¹, Stephan Sagmeister², Ute Werner¹ and Claudia Draxl^{1,3}

High accuracy XPS and XANES

Phys. Rev. B 95, 115112 (2017)

High-resolution valence and core excitation spectra of solid C60

via first-principles calculations and experiment

F. Fossard, K. Gilmore, G. Hug, J J. Kas, J J Rehr, E L Shirley and F D Vila

BSE * Particle-hole spectral function





Next generation XAS software

TIMES@SLAC SPEC@PNNL &UW FEFF10, Corvus, etc EOM-CC, Real-time, etc July 2017 U.S. Department of Energy: Office of Basic Energy Sciences Materials Sciences & Engineering Division Pacific Northwest National Laboratory Pacific Northwest ENERGY Street address: 902 Battelle Blvd., Richland WA 99354 ATIONAL LABORATOR' Postal Address: P.O. Box 999, Richland, WA 99352 Froudly Operated by Battelle Since 1965 **Theory Institute for Materials and Energy** Spectroscopies (TIMES) SPEC Scalable Predictive methods for **Excitations and Correlated phenomena** FWP # 100291

Division of Materials Science

SLAC National Accelerator Lab and Stanford University

Conclusions

Are we there yet after 50 years ?



XAFS – Fairly high accuracy now possible with *ab initio GF* theory & automated codes FEFF10

XANES, XES, RIXS, OPCONS etc.
High accuracy now possible with full potential BSE, DFT-MD and cumulant GF methods

NIRVANA: High accuracy Corvus workflows for full-spectrum XAS and related spectra may well be possible !

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