YN (and YY) interactions from hypernuclei

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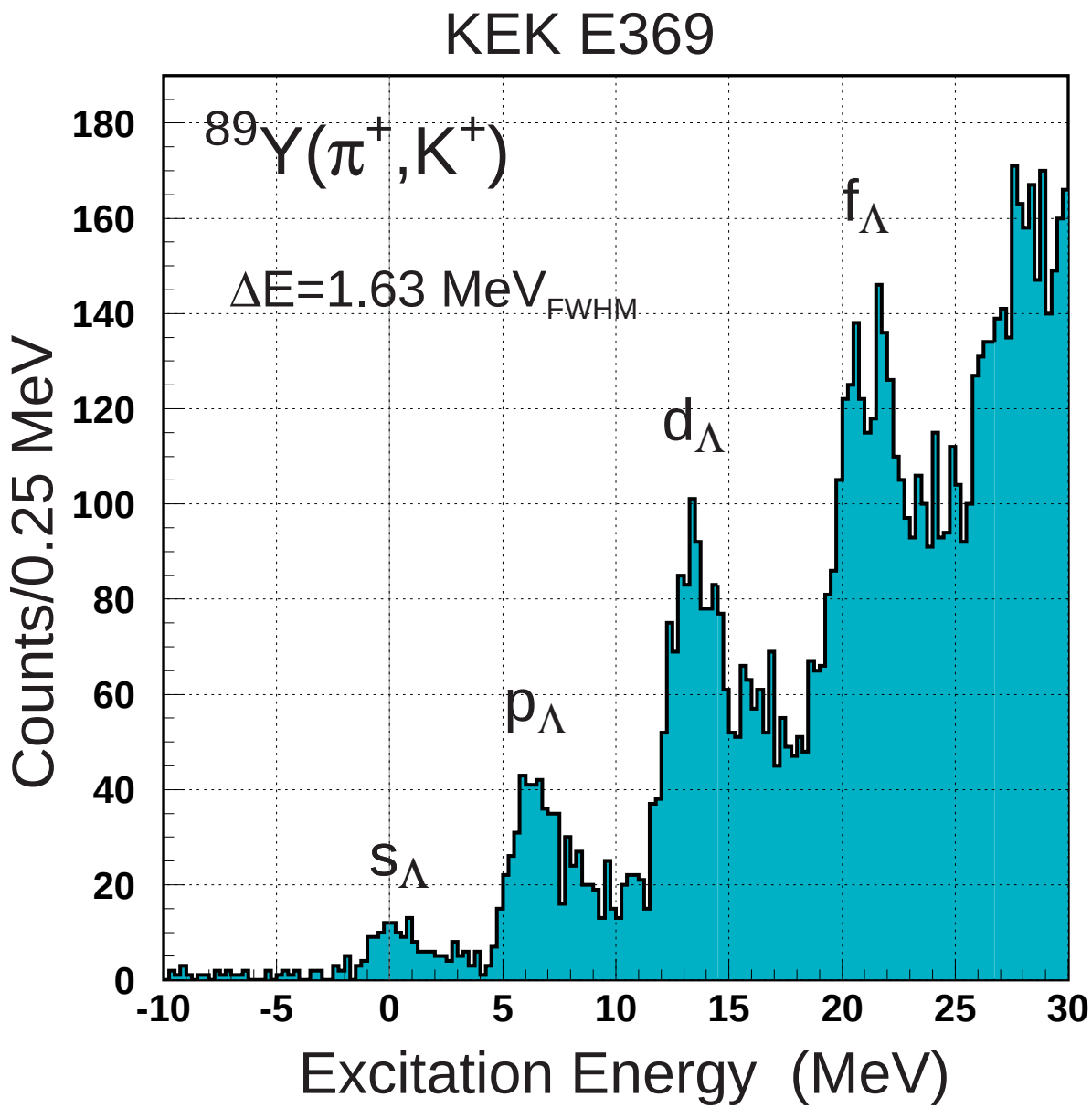
$m_\Lambda = 1115.68 \sim m_n + 176$ MeV $\qquad \tau_\Lambda \sim 260$ ps $\qquad \tau_{HY} \sim 200$ ps

$m_{\Sigma^-} = 1197.45$ MeV $\qquad m_{\Sigma^0} = 1192.64$ MeV $\qquad m_{\Sigma^+} = 1189.37$ MeV

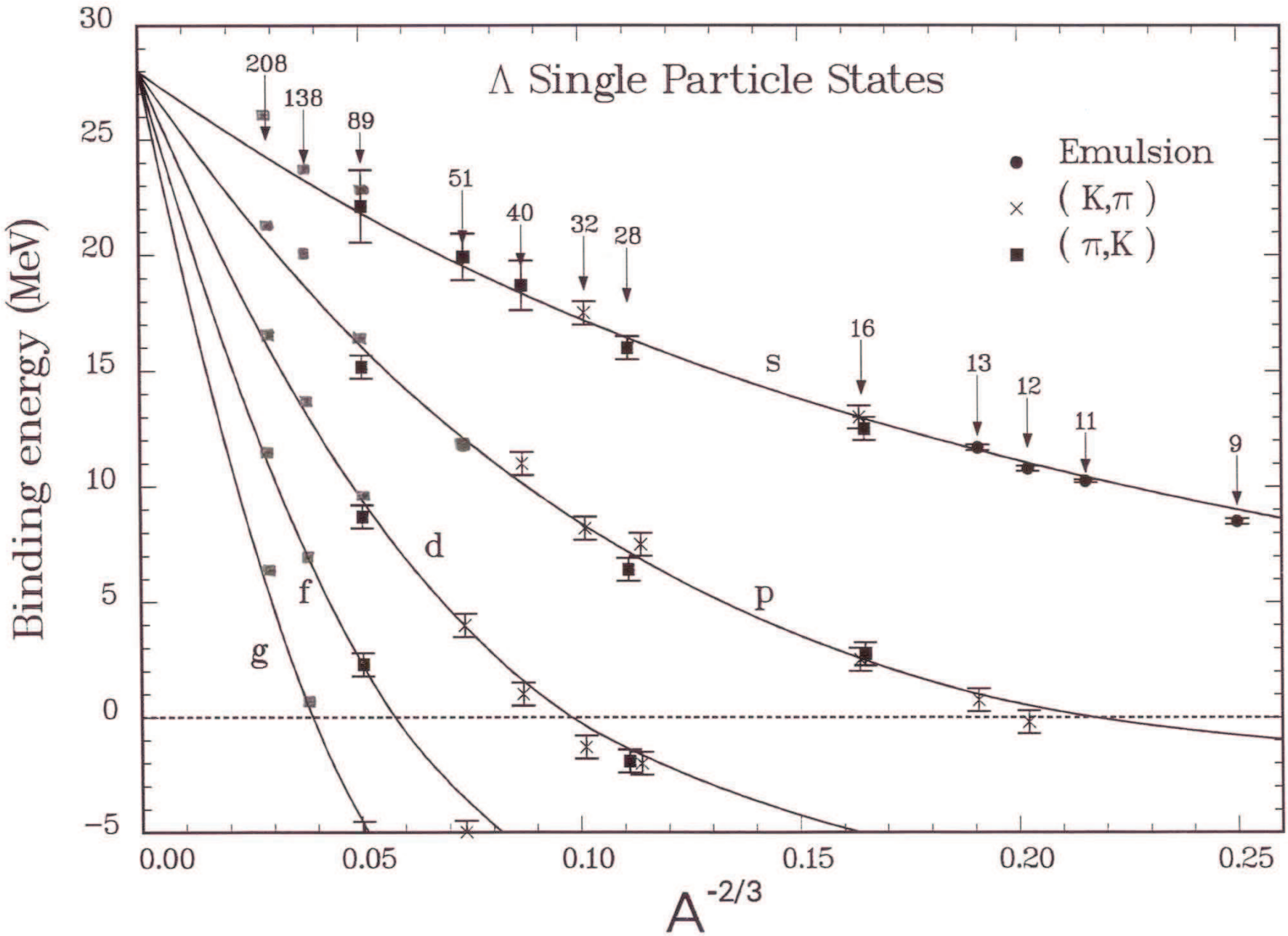
Studies of Λ hypernuclei

- $n(K^-, \pi^-)\Lambda$ – stopped or in-flight; first emulsions $\Rightarrow B_\Lambda$, then CERN, BNL, KEK, Frascati
- $n(\pi^+, K^+)\Lambda$ – BNL, KEK
- Hypernuclear γ-ray spectroscopy with Hyperball
  $(\pi^+, K^+\gamma)$ at KEK and $(K^-, \pi^-\gamma)$ at BNL $\qquad$ 1998 – 2005
- $p(e, e'K^+)\Lambda$ – JLab, Hall A and Hall C $\qquad$ Ongoing
- $p(K^-, K^+)\Xi^-$; $\qquad \Xi^- + p \rightarrow \Lambda + \Lambda$

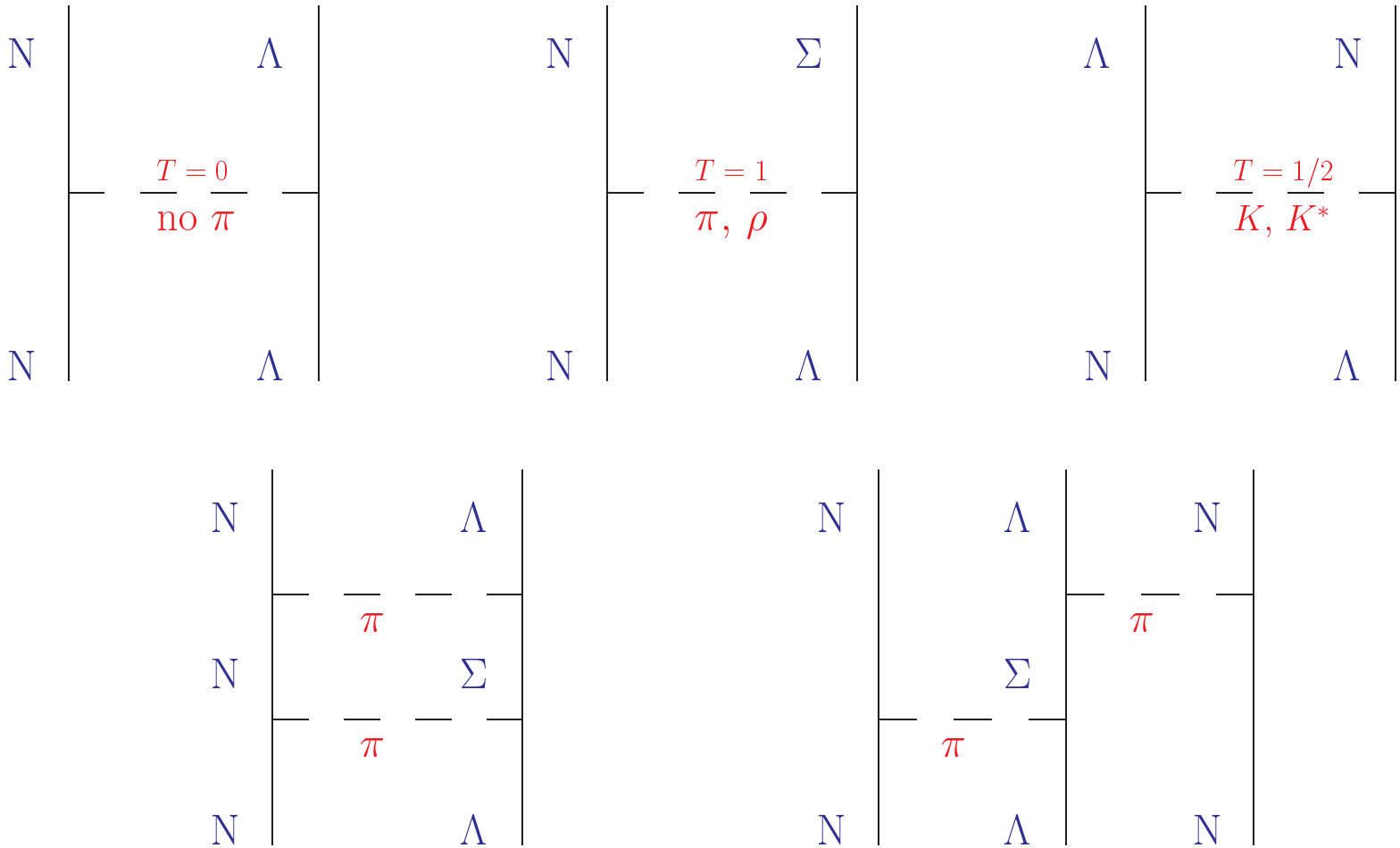


Hotchi et al., Phys. Rev. C 64 (2001) 044302 $B_\Lambda = 23.11 \pm 0.10$ MeV

Update: Millener, Dover, Gal PRC 38, 2700 (1988)



$S = -1 \quad T = 1/2 \qquad \Lambda N - \Sigma N$



$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\; s_N \cdot s_\Lambda + V_{LS}(r)\; l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r)\; l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r)\; S_{12}$$

$$V_0 = 1/4\,{}^1V_C + 3/4\,{}^3V_C \qquad V_\sigma = {}^3V_C - {}^1V_C$$

For $p_N s_Y$ $\quad V_{\Lambda N} = \bar{V} + \Delta\, s_N \cdot s_\Lambda + S_\Lambda\, l_N \cdot s_\Lambda + S_N\, l_N \cdot s_N + T\, S_{12}$

Parameters in MeV

	$\bar{V}$	Δ	S_Λ	S_N	T
$N\Lambda$-$N\Lambda$ $\;A = 7-$?		0.430	-0.015	-0.390	0.030
$A = 11 - 16$		0.330	-0.015	-0.350	0.024
$N\Lambda$-$N\Sigma$	1.45	3.04	-0.085	-0.085	0.157

Can write the central Λ-Σ coupling interaction as

$$\sqrt{4/3}\; t_N \cdot t_Y\; \overline{V}' + \sqrt{4/3}\; s_N \cdot s_Y\; t_N \cdot t_Y\; \Delta'$$

The factor $\sqrt{4/3}$ arises from defining t_Y as an operator that changes a Λ into a Σ

Diagonal matrix element $\sqrt{4/3}\sqrt{T(T+1)}\overline{V}' + a(J)\langle J_c || \sum_i s_i t_i || J_c \rangle \Delta'$

Off-diagonal matrix element $b(J)\langle J_c' || \sum_i s_i t_i || J_c \rangle \Delta'$

The important Λ-Σ coupling matrix elements involve a Σ coupled to the same core state as the Λ and states connected to this by a large Gamow-Teller matrix element.

S-Shell Λ Hypernuclei

Hypernucleus	$J^\pi(gs)$	B_Λ MeV	J^π	E_x MeV
${}^3_\Lambda$H	$1/2^+$	0.13(5)		
${}^4_\Lambda$H	0^+	2.04(4)	1^+	1.04(5)
${}^4_\Lambda$He	0^+	2.39(3)	1^+	1.15(4)
${}^5_\Lambda$He	$1/2^+$	3.12(2)		

Ab Initio Calculations

- $A = 3, 4$ A. Nogga et al., PRL 88 (2002) 172501
  Faddeev and Faddeev-Yakubovsky
- $A = 4$ E. Hiyama et al., PRC 65 (2002) 011301(R)
  Jacobi-coordinate Gaussian basis
- $A = 3, 4, 5$ H. Nemura et al., PRL 89 (2002) 142504
  Stochastic variation with correlated Gaussians

$\Lambda - \Sigma$ coupling for ${}^4_\Lambda$H and ${}^4_\Lambda$He

Y. Akaishi et al., PRL 84 (2000) 3539

$$|{}^4_\Lambda\mathrm{He}(\mathrm{T}=1/2)\rangle = \alpha s^3 s_\Lambda + \beta s^3 s_\Sigma$$

From ΛN$-\Sigma$N g matrix for $0s$ orbits

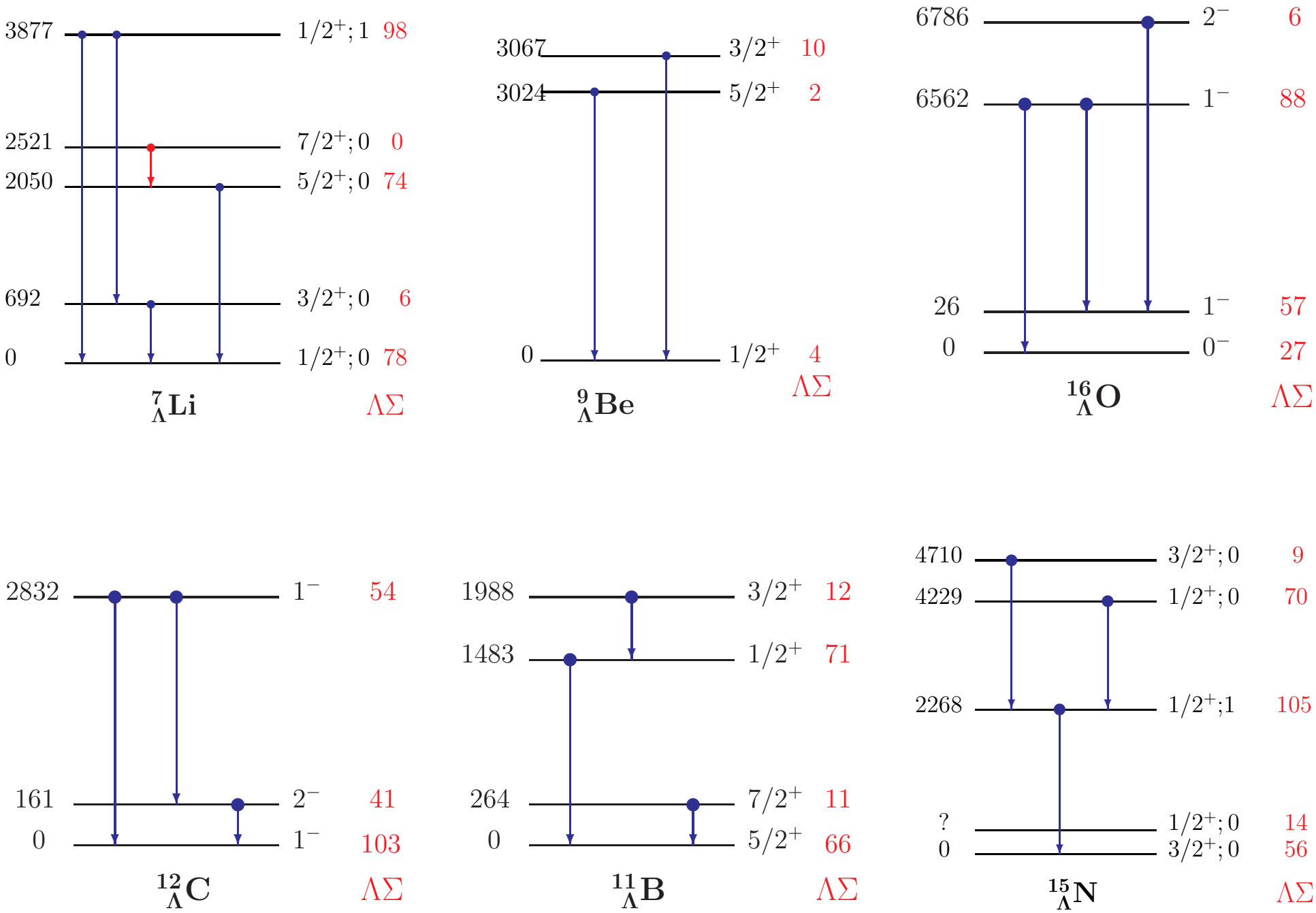
$v = \langle s^3 s_\Lambda | g | s^3 s_\Sigma \rangle$, $\quad \Delta E \sim 80$ MeV $\qquad {}^3g_{ss} = 4.8 \qquad {}^1g_{ss} = -1.0$

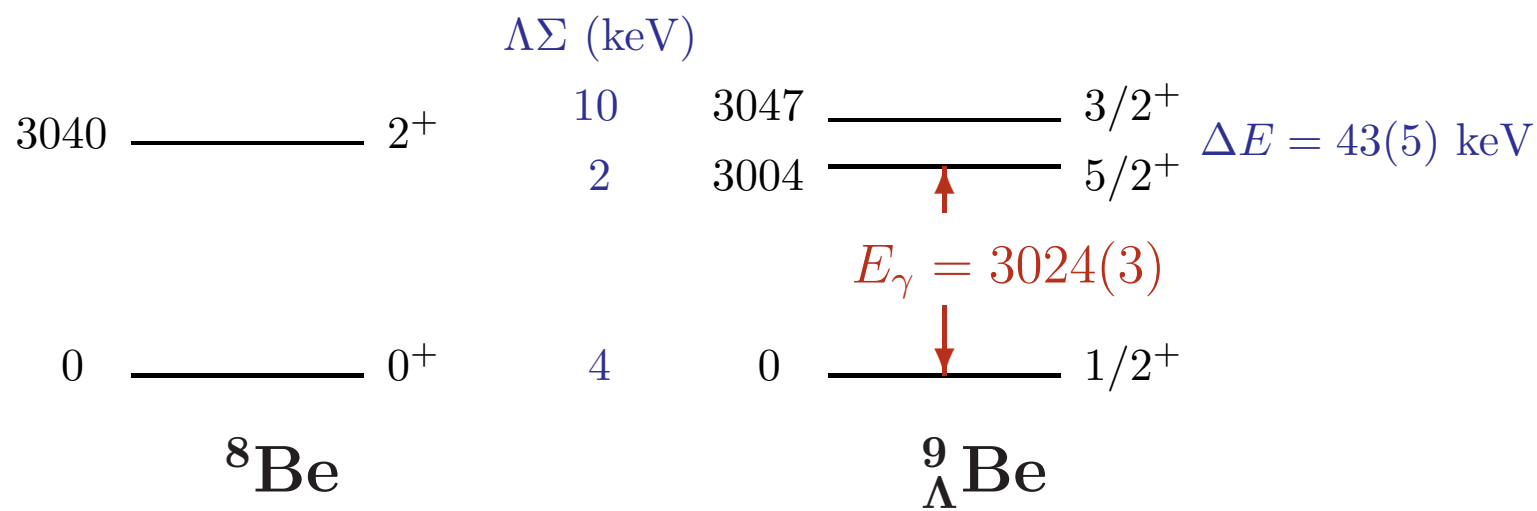
$\bar{V} = 3.35 \qquad \Delta = 5.8$

$0^+ \qquad v = \frac{3}{2}\,{}^3g_{ss} - \frac{1}{2}\,{}^1g_{ss} = \bar{V} + \frac{3}{4}\Delta \qquad$ Admixture $\sim -v/\Delta E$

$1^+ \qquad v = \frac{1}{2}\,{}^3g_{ss} + \frac{1}{2}\,{}^1g_{ss} = \bar{V} - \frac{1}{4}\Delta \qquad E^{shift} \sim v^2/\Delta E$

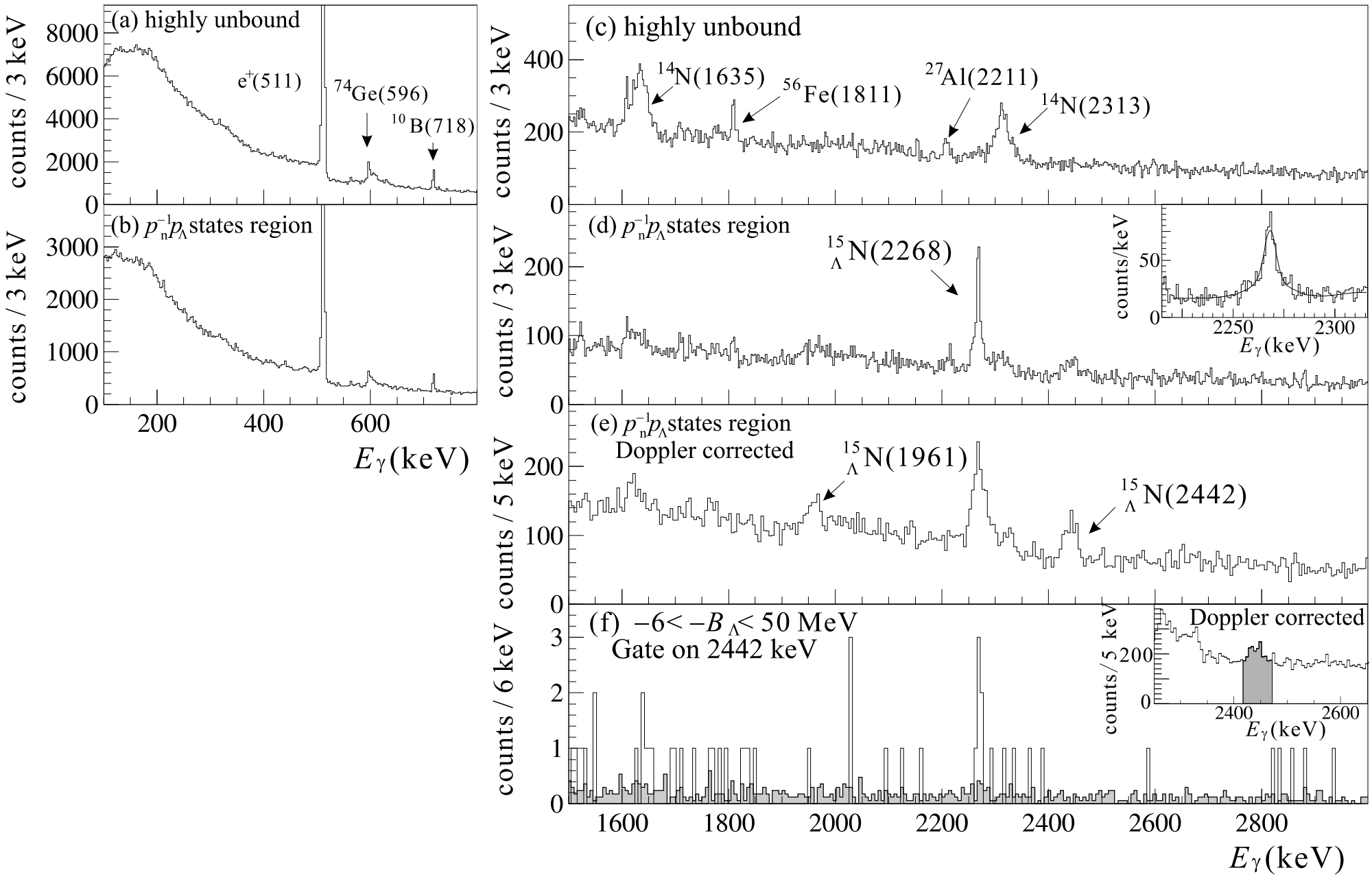
NSC97f: for $0^+ \quad v \sim 7.7$ MeV $\Rightarrow E^{shift} \sim 0.74$ MeV


3877
1/2+; 1
98
2521
7/2+; 0
0
2050
5/2+; 0
74
692
3/2+; 0
6
0
1/2+; 0
78
7ΛLi
ΛΣ
3067
3/2+
10
3024
5/2+
2
0
1/2+
4
9ΛBe
ΛΣ
6786
2−
6
6562
1−
88
26
1−
57
0
0−
27
16ΛO
ΛΣ
2832
1−
54
161
2−
41
0
1−
103
12ΛC
ΛΣ
1988
3/2+
12
1483
1/2+
71
264
7/2+
11
0
5/2+
66
11ΛB
ΛΣ
4710
3/2+; 0
9
4229
1/2+; 0
70
2268
1/2+;1
105
?
1/2+; 0
14
0
3/2+; 0
56
15ΛN
ΛΣ

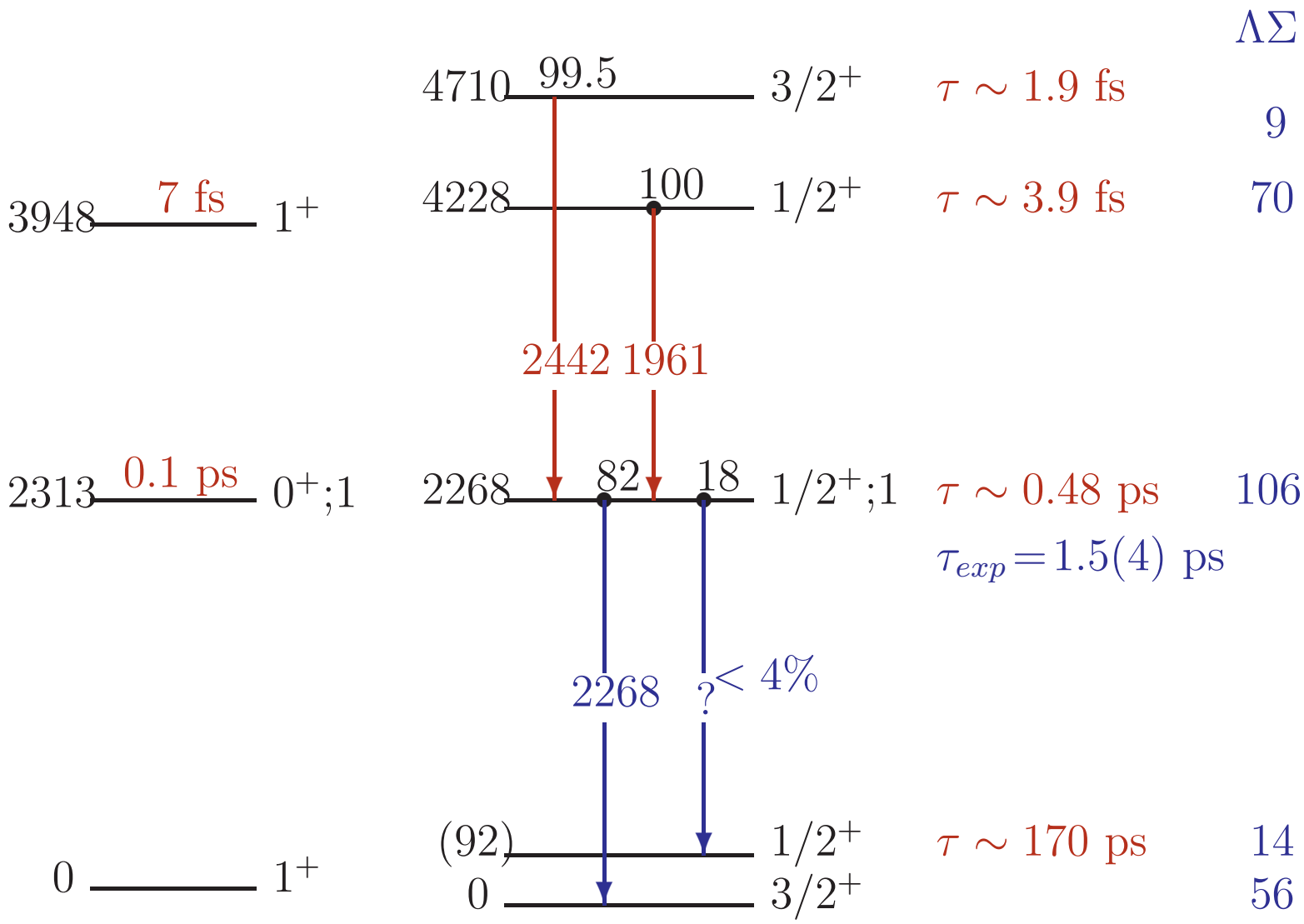


$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE
	-0.033	-2.467	0.000	0.939	
-8	-14	37	0	28	44 keV

- Order of $3/2^+$, $5/2^+$ not determined from this experiment.
- In 2001 run on ^{10}B target, the upper level is seen following $^{10}_{\Lambda}\mathrm{B} \rightarrow {}^{9}_{\Lambda}\mathrm{Be} + p$ enabling us to deduce $J^\pi = 3/2^+$ for the upper member of the doublet.


(a) highly unbound
e+(511)
74Ge(596)
10B(718)
counts / 3 keV
(b) p-n-1pΛ states region
counts / 3 keV
Eγ(keV)
(c) highly unbound
14N(1635)
56Fe(1811)
27Al(2211)
14N(2313)
counts / 3 keV
(d) p-n-1pΛ states region
15ΛN(2268)
counts/keV
Eγ(keV)
counts / 3 keV
(e) p-n-1pΛ states region
Doppler corrected
15ΛN(1961)
15ΛN(2442)
counts / 5 keV
(f) −6< −BΛ< 50 MeV
Gate on 2442 keV
Doppler corrected
counts/5 keV
Eγ(keV)
counts / 6 keV
Eγ(keV)

$^{15}_{\Lambda}$N γ rays – Hyperball, BNL E930 ('01)



Excitation energies and weak-coupling wave functions for ${}^{15}_{\Lambda}\mathrm{N}$.

$J_n^\pi;T$	E_x (keV)	Wave function
$3/2_1^+;0$	0	$0.9985\,1_1^+;0\times s_\Lambda + 0.0318\,1_2^+;0\times s_\Lambda + 0.0378\,2_1^+;0\times s_\Lambda$
$1/2_1^+;0$	96	$0.9986\,1_1^+;0\times s_\Lambda + 0.0503\,1_2^+;0\times s_\Lambda$
$1/2_1^+;1$	2282	$0.9990\,0_1^+;1\times s_\Lambda + 0.0231\,1_1^+;1\times s_\Lambda + 0.0206\,0_2^+;1\times s_\Lambda$
		$-\,0.0261\,0_1^+;1\times s_\Sigma$
$1/2_2^+;0$	4122	$-0.0502\,1_1^+;0\times s_\Lambda + 0.9984\,1_2^+;0\times s_\Lambda$
$3/2_2^+;0$	4624	$-0.0333\,1_1^+;0\times s_\Lambda + 0.9984\,1_2^+;0\times s_\Lambda + 0.0363\,2_1^+;0\times s_\Lambda$

M1 decays from $1/2^+;1$ in ${}^{15}_{\Lambda}\mathrm{N}$

$1/2^+;1 \to 3/2^+;0$	a_f	a_i	core M1	
large component	$0.9979 \times$	$0.9988 \times$	(-0.251)	-0.250
$1_2^+, s_\Lambda$ admixture	$0.0318 \times$	$0.9988 \times$	$(\ \ 2.957)$	$+0.095$
Σ admixture				$+0.011$
				-0.137

$1/2^+;1 \to 1/2^+;0$	a_f	a_i	core M1	
large component	$0.9983 \times$	$0.9988 \times$	(-0.251)	-0.250
$1_2^+, s_\Lambda$ admixture	$0.0545 \times$	$0.9988 \times$	$(\ \ 2.957)$	$+0.161$
Σ admixture				$+0.008$
				-0.081

$g_\Lambda = -1.226\ \mu_N \qquad g_{\Lambda\Sigma} = 3.22\ \mu_N$

Doublet spacings in p-shell hypernuclei

	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
${}^{7}_{\Lambda}$Li	$3/2^+$	$1/2^+$	72	628	−1	−4	−9	693	692
${}^{7}_{\Lambda}$Li	$7/2^+$	$5/2^+$	74	557	−32	−8	−71	494	471
${}^{8}_{\Lambda}$Li	2^-	1^-	151	396	−14	−16	−24	450	(442)
${}^{9}_{\Lambda}$Li	$5/2^+$	$3/2^+$	116	530	−17	−18	−1	589	
${}^{9}_{\Lambda}$Li	$3/2^+_2$	$1/2^+$	−80	231	−13	−13	−93	−9	
${}^{9}_{\Lambda}$Be	$3/2^+$	$5/2^+$	−8	−14	37	0	28	44	43
${}^{11}_{\Lambda}$B	$7/2^+$	$5/2^+$	56	339	−37	−10	−80	267	264
${}^{11}_{\Lambda}$B	$3/2^+$	$1/2^+$	61	424	−3	−44	−10	475	505
${}^{12}_{\Lambda}$C	2^-	1^-	61	175	−12	−13	−42	153	161
${}^{15}_{\Lambda}$N	$3/2^+_2$	$1/2^+_2$	65	451	−2	−16	−10	507	481
${}^{16}_{\Lambda}$O	1^-	0^-	−33	−123	−20	1	188	23	26
${}^{16}_{\Lambda}$O	2^-	1^-_2	92	207	−21	1	−41	248	224

Λ-Σ and spin-dependent contributions to ground-state binding energies

	${}^{7}_{\Lambda}$Li	${}^{8}_{\Lambda}$Li	${}^{9}_{\Lambda}$Li	${}^{9}_{\Lambda}$Be	${}^{10}_{\Lambda}$B	${}^{11}_{\Lambda}$B	${}^{12}_{\Lambda}$B	${}^{13}_{\Lambda}$C	${}^{15}_{\Lambda}$N	${}^{16}_{\Lambda}$N
	$1/2^+$	1^-	$3/2^+$	$1/2^+$	1^-	$5/2^+$	1^-	$1/2^+$	$3/2^+$	1^-
Λ-Σ	78	160	183	4	35	66	103	28	59	62
Δ	419	288	350	0	125	203	108	-4	40	94
S_Λ	0	-6	-10	0	-13	-20	-14	0	12	6
S_N	94	192	434	207	386	652	704	841	630	349
T	-2	-9	-6	0	-15	-43	-29	-1	-69	-45
Sum	589	625	952	211	518	858	869	864	726	412
Expt	5.58	6.80	8.50	6.71	8.89	10.24	11.37	11.69		13.76
		6.84	8.29		9.11					$*$
$\overline{V}$	-0.94	-1.02	-1.06		-1.05	-1.04	-1.05	-0.96		-0.93

$*$ $B_\Lambda = 13.76(16)$ MeV: F. Cusanno et al. PRL 103, 202501 (2009)

To get a rough $\overline{V}$, take $B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) = 3.12$ MeV as s_Λ single-particle energy, and subtract sum from experimental B_Λ value.

Double one-pion exchange ΛNN interaction

Gal, Soper, and Dalitz: Ann. Phys. (N.Y.) 63, 53 (1971)

Independent of Λ spin. Averaged over s_Λ wave function gives

$$V_{NN}^{eff} = \sum_{klm} Q_{lm}^{k}(r_1, r_2) \left[\sigma_1, \sigma_2\right]^k \cdot \left[C_l(\hat{r_1}), C_m(\hat{r_2})\right]^k \ \tau_1 \cdot \tau_2$$

Parameters in MeV

Q_{00}^0	Q_{22}^0	Q_{22}^1	$Q_{02}^2 = Q_{20}^2$	Q_{22}^2
0.026	1.037	−0.531	−0.049	0.245

- Q_{00}^0 and Q_{22}^0 give repulsive contributions to B_Λ that depend quadratically on the number of p-shell nucleons in the core.
- Q_{22}^1 represents an anti-symmetric spin-orbit interaction that behaves rather like S_N

Future Hypernuclear γ-ray Spectroscopy

- Hyperball-J at J-PARC (Tamura)
- $(K^-, \pi^-\gamma)$ $\quad p_K = 1.1$ or 1.5 GeV/c
- Larger spin-flip amplitudes - test case ${}^4_\Lambda$He $1^+ \rightarrow 0^+$
- p-shell and light sd-shell nuclei as targets. First ^{19}F
- $(K^-, \pi^0\gamma)$ also possible