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Colliding Crystalline Beams

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COLLIDING CRYSTALLINE BEAMS *

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Abstract

The understanding of crystalline beams has advanced to the point where one can now, with reasonable confidence, undertake an analysis of the luminosity of colliding crystalline beams. Such a study is reported here. It is necessary to observe the criteria, previously stated, for the creation and stability of crystalline beams. This requires, firstly, the proper design of a lattice. Secondly, a crystal must be formed, and this can usually be done at various densities. Thirdly, the crystals in a colliding-beam machine are brought into collision. We study all of these processes using the molecular dynamics (MD) method. The work parallels what was done previously, but the new part is to study the crystal-crystal interaction in collision. We initially study the zero-temperature situation. If the beam-beam force (or equivalent tune shift) is too large then overlapping crystals can not be created (rather two spatially separated crystals are formed). However, if the beam-beam force is less than but comparable to that of the space-charge forces between the particles, we find that overlapping crystals can be formed and the beam-beam tune shift can be of the order of unity. Operating at low but non-zero temperature can increase the luminosity by several orders of magnitude over that of a usual collider. The construction of an appropriate lattice, and the development of adequately strong cooling, although theoretically achievable, is a challenge in practice.

1 INTRODUCTION

For the last decade there has been interest in, and experimental effort to achieve crystalline beams. The interest, besides being intrinsic for this new state of matter, is primarily due to the possibility of studying the physics of completely space-charge dominated beams, the possibility of studying Wigner crystal, and the possibility of using crystalline beams to obtain very high luminosity colliders. It is the later possibility that we study in this work.

The ground state of a crystalline beam was proposed by Dikanskiǐ and Pestrikov[1] based on an experimental anomaly observed on an electron-cooled proton beam at NAP-M, and was first studied using the MD method by Schiffer and co-workers[2]. At the same time, experimental efforts have succeeded in achieving very low beam temperatures, but not yet a crystalline state[3].

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2 THE GROUND STATE

Particle motion can be described by a Hamiltonian[4]-[7] in the rest frame (x, y, z, t) of a circulating reference particle in which the orientation of the axes is rotating so that the axes are constantly aligned to the radial (x), vertical (y), and tangential (z) direction. Consider a system of ions with electric charge Z_0e and atomic mass M_0 under Coulomb interaction and external fields. Measure dimensions in units of the characteristic distance ξ_0 with $\xi_0^3 = r_0\rho^2/\beta^2\gamma^2$, time in units of $\rho/\beta\gamma c$, and energy in units of $\beta^2\gamma^2 Z_0^2 e^2/\xi_0$, where $r_0 = Z_0^2e^2/M_0c^2$ is the classical radius, βc and $\gamma M_0 c^2$ are the velocity and energy of the reference particle, and ρ is the radius of curvature in bending regions of magnetic field B_0. In a bending region with pure dipole magnetic field, the Hamiltonian is

$$H_i \;=\; \frac{1}{2}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{1}{2}x^2 - \gamma x P_z + V_{Ci} \qquad (1)$$

where $V_{Ci} = \sum_j [(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2]^{-1/2}$ is the Coulomb potential, $P_{x,y,z}$ are the canonical momenta, and the summation, j, is over all the other particles and their image charges[4] in the same beam. In a non-bending region with longitudinal electric field and non-dipole magnetic fields, the Hamiltonian is

$$\begin{aligned} H_i \;=\; & \frac{1}{2}\left(P_x^2 + P_y^2 + P_z^2\right) - \frac{n_1}{2}(x^2 - y^2) \\ & -n_{1s}xy - \frac{n_2\xi_0}{6}(x^3 - 3xy^2) + V_{Ci} + U_s, \end{aligned} \qquad (2)$$

where the normal quadrupole, skew quadrupole, and sextupole strengths are represented by $n_1 = -\frac{\rho}{B_0}\frac{\partial B_y}{\partial x}$, $n_{1s} = -\frac{\rho}{B_0}\frac{\partial B_y}{\partial y}$, $n_2 = -\frac{\rho}{B_0}\frac{\partial^2 B_y}{\partial x^2}$, respectively. U_s can be expressed in terms of electric field E_s measured in the laboratory frame, $\frac{\partial U_s}{\partial z} = -\frac{Z_0eE_s\xi_0}{M_0c^2}\left(\frac{\rho}{\xi_0\beta\gamma}\right)^2$.

We have done both analytic and numerical calculations using the equations derived from these Hamiltonians and the molecular dynamics (MD) methods. The details of the numerical methods have been provided in Ref.[4].

We start with a study of the ground state, as previously reported in Ref. [5]. It has been shown that there are two necessary conditions for the formation and maintenance of a crystalline beam. They are as follows:

1. The storage ring must be alternating-gradient (AG) focusing and the energy of the beam must be less than the transition energy of the ring; i.e., $\gamma < \gamma_T$.
2. The ring lattice periodicity is at least $2\sqrt{2}$ as high as the maximum betatron tune.

Condition (1) arises from the criterion of stable kinematic motion under Coulomb interaction when particles are subject to bending in a storage ring. Condition (2) arises from the criterion that there is no linear resonance between the phonon modes of the crystalline structure and the machine lattice periodicity.

Existing storage rings upon which attempts have been made to obtain crystalline beams do not satisfy the conditions just stated, although with minor modifications they would. The requisite cooling in order to obtain a crystalline state is delineated in Ref. [7]. What is required is reasonably powerful and preferably "tapered" cooling; i.e., cooling to the same average angular velocity. These requirements on a cooling system would seem to be achievable in practice.

3 BEAM-BEAM MODELLING

Our numerical study of colliding crystals is done using the MD code SOLID.[4] The newly added element is that now there are two interacting crystals moving in opposite directions. The interaction occurs once a period in a very short time, so it is treated as a lumped kick in momentum. The kick on particle i can be represented by a Hamiltonian

$$H_i = \sum_j \frac{(1+\beta^2)\gamma\xi_0}{\rho\sqrt{b_{min}^2 + b_{ij}^2}} \tag{3}$$

where $b_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2$ is the square of the transverse separation and $b_{min} = (1+\beta^2)r_0/(4\beta^2\gamma^2\xi_0)$ is the minimum separation in the beam rest frame, and the summation, j, is over all the particles in the opposite beam. We find that if the kick is large comparing with that of the crystalline space charge, then the ground state is two crystals separated in space at the crossing point; i.e. there are no overlapping (Fig.1d). If, however, the beam-beam effect is not too large then the two crystals do overlap and beam-beam nuclear interactions can occur.

A convenient measure of both the beam-beam and the space-charge forces is given by assuming a uniform charge distribution within the beam. This is, of course, an under-estimate of the actual space-charge and beam-beam forces when the beam is crystallized, since the crystalline beam has ordered structures (e.g. for Fig. 1a the actual space-charge force is under-estimated by this tune-shift formula by about a factor of 4). Let R be the radius of the machine, β_{xy}^* be the β values at the crossing point, N_B be the number of crossing per revolution, N_0 be the number of ions per bunch, λ_0 be the peak number of ions per unit length, and a be the full transverse radius of the bunch, we have:

$$\Delta\nu_{sc} = \frac{-\lambda_0 R r_0 \beta_{xy}}{\beta^2\gamma^3 a^2}, \quad \Delta\nu_{bb} = \frac{-N_B N_0 (1+\beta^2) r_0 \beta_{xy}^*}{4\pi\beta^2\gamma a^2} \tag{4}$$

With these expressions, the ratio of beam-beam force to space-charge force is independent of the transverse beam size (or temperature) as the beams are cooled down.

It is necessary, in order to have significant beam-beam nuclear interactions, to form crystals with many shells. Unfortunately, crystals with many shells (beyond three or four) requires excessive computer time. We have, therefore, modelled a crystal by representing N_{MP} ions by a macro particle of charge $N_{MP}Z_0$ and mass $N_{MP}M_0$. Thus we replace ξ_0 by $\xi = N_{MP}^{1/3}\xi_0$. That is, of course, not the same as a many shell crystal with $N_{MP} = 1$ but it does replicate some of the phenomena. Numerical studies, albeit only with a few shells, indicates minimal sensitivity of our results to shell number.

4 RESULTS

We first study the formation of crystals with different values of the beam-beam force in comparison with the space-charge force. The results are shown in Fig 1. Beam and machine parameters are listed in Table 1 ($\xi_0 = 0.68\,\mu$m). It can be seen that crystalline ground states (zero-temperature state) can be found with any value of beam-beam force. At the ground states, particles of the two intersecting beams do not collide at the crossing point. Collision only occurs at non-zero temperature when the amplitude of the particle transverse thermal motion is larger than the minimum transverse separation of the particles of the opposite beam.

We then study the heating rates of the crystals as functions of crystal temperature. The study is done in the absence of external cooling and, therefore, tells us the amount of cooling required to form and maintain the crystal. The normalized crystal temperature $T = T_x + T_y + T_z$ is defined with $T_{x,y,z}$ the deviation of P_x, P_y and P_z from their ground-state values, squared and averaged over particles. T is related to the conventional beam temperature T_B at high temperature by

$$T_B \approx \frac{\beta^2\gamma^2 M_0 c^2 \xi^2}{2k_B\rho^2} T = 12.3\ N_{MP}^{2/3} T \ \ [\mathrm{K}] \tag{5}$$

with k_B the Boltzmann constant. Starting with finite-temperature states and evaluating the rate at which the beams absorb energy from the lattice, we present in Fig. 2 the relative increase of temperature per lattice period for the four cases displayed in Fig. 1. Comparing with the case of no beam-beam collision (long dash line), cases with $\Delta\nu_{bb} = 0.08$ and 0.27 have similar heating rate down to normalized temperature of about 0.01.

Finally, in Fig 3, we show the values of $\Delta\nu_{sc}$ and $\Delta\nu_{bb}$ as a function of beam temperature for the last three cases. These curves show that as the temperature is reduced the

Table 1: Beam and machine parameters.

Quantity	Value
Ion species	proton
Ring circumference, $2\pi R$	251.3 [m]
Number of lattice periods per turn	100
Energy (γ)	22
Horizontal & vertical tunes, ν_x, ν_y	30.99, 30.89
Transition energy, γ_T	29.4
Dipole bending radius, ρ	10 [m]
Maximum $\beta_{x,y}$	4.1 [m]
Minimum $\beta_{x,y}$	0.6 [m]
Average $\beta_{x,y}^*$	2.4 m
RF voltage, V per period	1 [MV]
RF harmonic number, h	10^5
Synchrotron tune, ν_s	0.26

beam becomes smaller and hence there is an increase in both of these quantities. Most important, however, is the increase in $\Delta\nu_{bb}$ which shows that a larger value of the beam-beam effect can be tolerated at low temperatures than at high temperatures. Of course, in order to have nuclear luminosity, ions in the two beams must overlap on a scale much smaller than that which is shown in the figure. Such overlap does not occur when $T = 0$, but will occur at sufficiently high temperature as indicated by the solid lines in Fig. 3. Operating at temperatures just above these values increase the luminosity by about two orders of magnitude above that allowed in normal colliders!

The "bottom line", then, is that our work suggests (only "suggests" as a number of possibly important phenomena are only studied by extrapolation) that combining cooling with colliding beams is a useful thing to do. Perhaps the

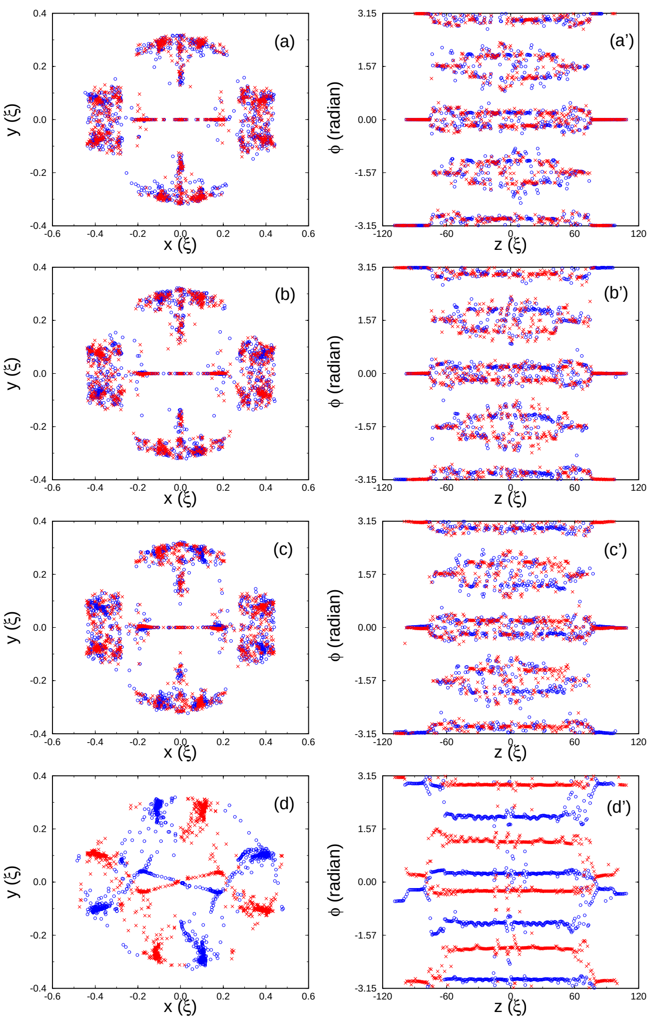

Figure 1: A series of figures showing the formation of crystalline ground states with 1000 macro particles in each beam. In each case the space charge tune shift $\Delta\nu_{sc} = -3.8$ and the beam-beam tune shift $\Delta\nu_{bb} = 0, 0.08, 0.27$, and 2.7 ($N_{MP}$=1, 1, 40, and 40, 000). The crosses correspond to one beam while the circles correspond to the other. ϕ is the polar angle. The actual temperature is near $T \approx 10^{-6}$.

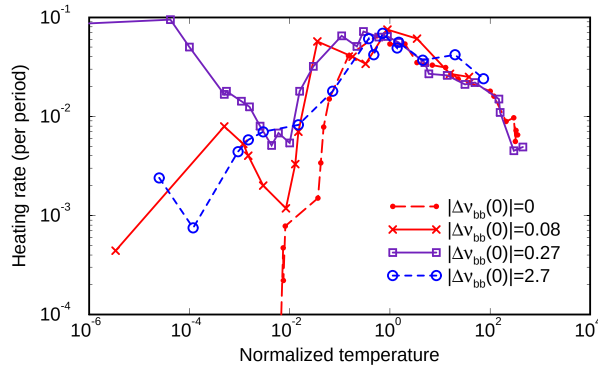

Figure 2: The heating rate for the crystals shown in Fig. 1. Each crystal has 1000 macro particles and $\Delta\nu_{bb} = 0, 0.08$, 0.27, and 2.7.

gain in beam-beam collision rate, while maintaining beam stability, can be as much as two orders of magnitude. Further study, and especially experimental study, would appear to be called upon.

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5 REFERENCES

[1] N.S. Dikanskiĭ, D.V. Pestrikov, Workshop on Electron Cooling and Related Applications, KfK 3846, ed. H. Poth, (1984).

[2] J.P. Schiffer, P. Kienle, Z. Phys. **A 321**, 181 (1985); A. Rahman, J.P. Schiffer, Phys. Rev. Lett. **57**, 1133 (1986); J.P. Schiffer et al, Z. Phys.A-Atomic Nuclei 331, **71** (1988).

[3] S. Schroder et al, Phys. Rev. Lett. 64, 2901 (1990); J.S. Hangst, et al, Phys. Rev. Lett. **67**, 1238 (1991).

[4] J. Wei, X.-P. Li, A.M. Sessler, PAC, p. 3527 (1993), and more extensively as BNL Report 52381 (1993).

[5] J. Wei, X.-P. Li, A.M. Sessler, Phys. Rev. Lett. **73**, 3089 (1994).

[6] J. Wei, et al, *Crystalline Beams and Related Issues*, World Scientific, Singapore p.229 (1996).

[7] J. Wei, H. Okamoto A.M. Sessler, Phys. Rev. Lett. **80**, 2606 (1998).

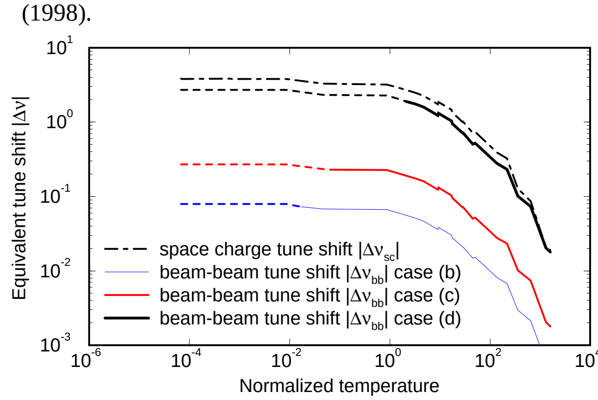

Figure 3: The beam-beam tune shift $\Delta\nu_{bb}$ and the space-charge tune shift $\Delta\nu_{sc}$ as functions of temperature. The solid lines indicate regions where particles of opposite beams will overlap so that $|\Delta\nu_{bb}|$ is a proper measure of luminosity. Structures of cases (b), (c), and (d) of Fig. 1 correspond to the respective curves at about $T \approx 10^{-6}$.