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# A Lattice for the 50 GeV Muon Collider Ring

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## Abstract

A recent progress report on the lattice design of the 50-50 GeV muon collider is presented. The ring circumference needs to be as small as possible due to the short lifetime of the 50 GeV muons. The background at the detector is affected by the continuous decay of muons into electrons which requires a dipole between the high focusing quadrupoles and the detector. To obtain a luminosity on the order of  $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  it is required to have beam intensities on the order of  $1 \times 10^{12}$  particles per bunch. The rms momentum spread of the beam is equal to 0.12 % and the beta functions at the interaction point are equal to 4 cm. The maxima of the betatron functions at these quadrupoles are 1300 m, resulting in large chromaticities which must be corrected by local chromatic correction. Pairs of horizontal and vertical chromatic sextupoles are located at locations where the corresponding betatron functions are 100 m and the values of the horizontal dispersion functions are 3 and 2 m, respectively. They are carefully placed so that most of their nonlinear effects are canceled. The dynamic aperture is larger than 7 times the mean size of the beam for the momentum offsets larger than -6 and +10 sigmas.

## 1 INTRODUCTION

A Muon Collider research collaboration group has extensively been studying all aspects of the collider for the last several years. There are many advantages with respect to the electron or proton colliders. The size of the collider would be relatively small because of their rest energy being 0.1126 times smaller than protons (the circumference  $C$  for the 2 TeV  $\times$  2 TeV collider  $C \sim 6000$  m, while for the 50 TeV  $\times$  50 GeV collider is  $C \sim 350$  m). Muon  $\mu^+ \mu^-$  bunches will collide during 1000 turns (@ 2 TeV). This makes the number of collisions much larger than one as it is in the case of the linear electron colliders [3].

The muon collider would consist of many components like: the *Muon Production Section* (which includes a proton source, linac, proton driver, the liquid target and capture), the *Ionization Cooling Section*, the *Acceleration Section*—linacs, recirculating linacs, and at the end is the *Collider Ring*. The muon collider storage ring will have muon bunches  $\mu^+ \mu^-$  with energy in the center of mass of 2 TeV and luminosity of  $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ . For this luminosity the rms bunch length has to be very short (3 mm) which defines the betatron functions at the collision point of  $\beta^* = 0.003$  m. A proposal for the muon collider considers first a storage ring of 100 GeV center of mass  $\mu^+ \mu^-$  colliding energy. The parameters for the 50 GeV on 50 GeV collider, previously defined [1], are not as stringent as for the 4 TeV collider. The bunch length is estimated to be 4 cm with

a momentum spread of 0.12 %. Both the 4 TeV and 100 GeV muon collider rings have to be isochronous due to the short muon bunch lengths [2]. Because the muon decays very quickly, to allow the high values for luminosity, the circumference of the machine has to be as small as possible.

Unfortunately there are also some disadvantages with the muon collider. One of the limitations is the relatively short decay life time  $\tau = 2.2 \mu\text{s}$  which is partially overcome by rapidly increasing their energy (at 2 TeV  $\tau = 0.044$  s) [3]. The other consequence of the decay are the created electrons which heat the cryogenic magnets and make large background at the detectors. The background reduction at the detectors puts an additional constraint on the interaction telescope design pushing the quadrupoles away from the IP due to a necessary sweeping dipole, which removes the electrons. The quadrupole coil diameter has to be enlarged due to a shielding layer. The lattice design of the storage ring for the muon collider represents a real challenge in many respects:

- the  $\beta^*$  of 3 mm or 4 cm sets up the maximum beta functions at the 2 TeV storage ring up to  $\hat{\beta} \sim 600$  km, while at 50 GeV  $\hat{\beta} \sim 1300$ -1500 m.
- The chromaticity correction due to the triplet quadrupoles requires a separate local chromatic correction by pairs of sextupoles with phase advance of  $\pi$  between themselves and  $\pi$  away from the quadrupoles.
- The short bunches put a condition for an isochronous ring in the 2 TeV collider and approximately isochronous for the 50 GeV ring to avoid excessive RF. This is especially challenging for the small 350 m circumference muon collider because the *flexible momentum compaction* [4] module have to overcome a necessary positive dispersion through the dipoles within the local chromatic correction section around the IP.
- The muon ionization cooling studies had shown that the expected energy spread of the bunches would be of the order of 0.12%. In the electron storage rings for the synchrotron radiation or electron collisions, the high luminosity is obtained by reducing the emittance of the beams. The emittance depends on the values of the dispersion function ( $H = (D^2/\beta) + (D'\sqrt{\beta} + D\alpha/\sqrt{\beta})^2$ ). The limitations on the dispersion function are due to the limited aperture size. ( $\Delta x = D \cdot dp/p$  where  $dp/p \sim 0.12\%$ ).

More about the Muon Collider research, development and future plans can be found at other reports at this conference. The muon collider collaboration group has developed few

nice examples of the 50 GeV muon collider lattices and this represents a progress report on one of the designs. There are still possible improvements to the final design.

## 2 THE TELESCOPE WITH THE CHROMATIC CORRECTION

The sweeping superconducting dipole, with the magnetic of 8T and 2.5 m length, was found to be adequate, by a separate study, to reduce the background due the electrons from the muon decay.

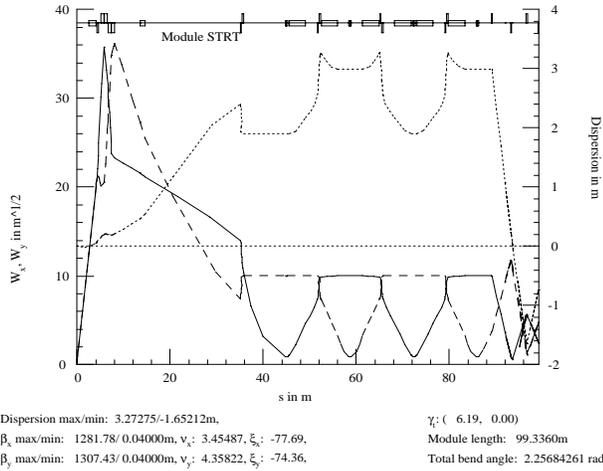


Figure 1: The interaction region with local chromatic correction.

The telescope at the interaction region is a major source of the chromaticity due to the large betatron functions at the high focusing quadrupoles. After a review of the positions and distances between the magnets within the superconducting cryostats in the present Relativistic Heavy Ion Collider it was found that the distance between the superconducting magnets could be shorter than previously considered. A distance of the first quadrupole from the interaction point IP as well as the separations between the quadrupoles became a little shorter than before. This small change lowered the maxima of the betatron functions from  $\hat{\beta} \sim 1560m$  to  $\hat{\beta}_x=1282m$   $\hat{\beta}_y=1307m$ . Fig. 1 represents the interaction region of the 50 GeV muon collider together with the chromatic correction part.

### 2.1 Chromaticity Correction of the High Focusing Triplets

The local chromatic correction of low-beta insertions have previously been studied and the similar solutions were developed [5] [6] [7]. There are two pairs of inter-lived [6] vertical and horizontal focusing sextupoles with a  $\pi$  phase advance with respect to the corresponding high beta triplets. The chromatic correction requires adequate values of the beta and dispersion functions at the sextupoles. A design of the chromaticity correction blocks was simplified

by setting a condition for the slope of the dispersion and betatron functions to be zero at the sextupoles (labeled in Fig. 2  $SX_2$  and  $SX_1$ ). It is enough to design a single block presented in the normalized dispersion space in Fig. 2 as a region from the top of the figure (location  $SX_2$ ) to the bottom (along the 4,5,6,7 to the last point labeled by  $SX_1$ ). The rest of three blocks are symmetric to the first one.

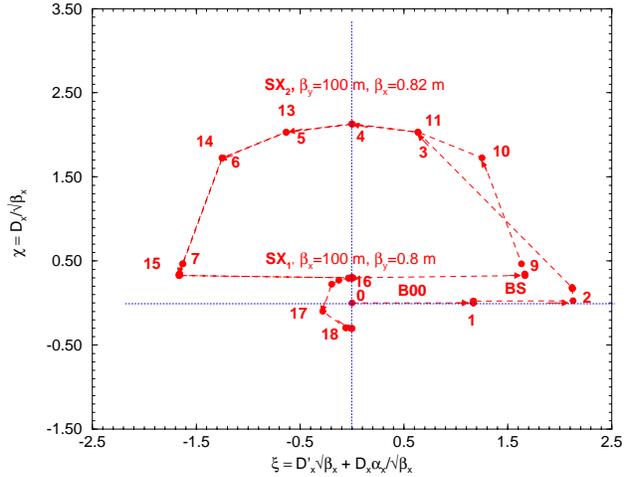


Figure 2: The local chromatic correction in normalized dispersion space.

## 3 FLEXIBLE MOMENTUM COMPACTION MODULE

The present designs of the lattice all have almost half of the ring with 350 m long circumference dedicated to the local chromatic correction where the dispersion has to be positive at the sextupoles.

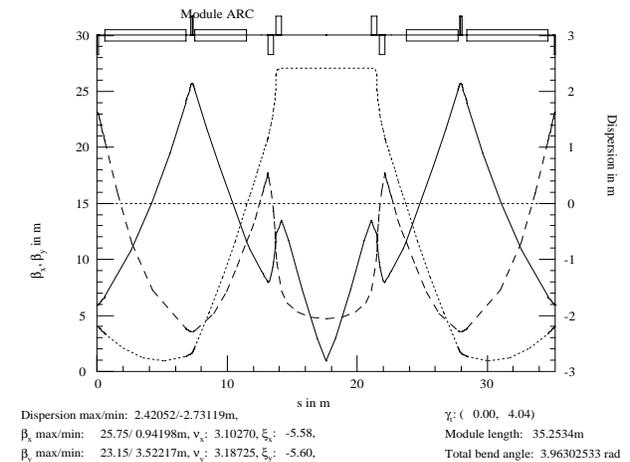


Figure 3: The interaction region with local chromatic correction.

The momentum compaction is defined as the integral of the dispersion function through the dipoles ( $\alpha = 1/C \int D(s) ds / \rho$ ). Due to a small positive value of the momentum compaction in half of the ring a request for the asynchronous condition becomes much harder to establish by the previously defined flexible momentum compaction *FMC* module. This is because in the regular *FMC* module dispersion function is partially positive through some dipoles. The major problem in adjusting the *FMC* module is to keep the dispersion function small. This is an important constraint because of the aperture limitations and a relatively large value of the momentum offset  $dp/p=0.12\%$ . Fig. 3 shows betatron functions inside of the modified *FMC* module. This module provides large enough negative value of the momentum compaction to cancel the influence of the local chromatic with the final focus telescope. This was accomplished by making negative dispersion function within the dipoles. At the same time maxima of the dispersion function are kept within small values ( $-2.7\text{ m} \leq D \leq 2.4\text{ m}$ ).

### 3.1 Tune and Chromaticity Dependence on Momentum

The global correction of the chromaticity can not remove the tune dependence on momentum due to the high focusing quadrupoles of the final focus telescope.

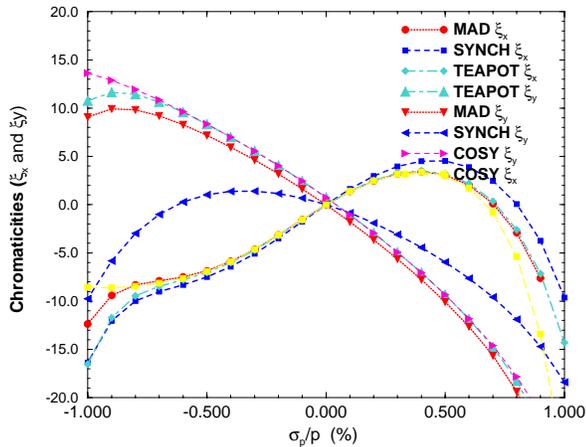


Figure 4: A Comparison of the Chromaticity Dependence on Momentum between four codes MAD, SYNCH, TEAPOT, and COSY.

It is very important to remove the second order tune shift on amplitude and the variations of the low beta  $\beta^*=0.04\text{ m}$  to allow high luminosity. Working conditions in the muon collider are not to be compared to the standard storage rings due to a small number of turns (less than 1000), but still during few hundred turns particles with the large amplitudes and with tunes close to a resonance are lost. An additional problem arises in the 50 GeV muon collider lattice design. Due to a small size and large dipole bend-

ing the existing lattice design and particle tracking codes have to be either modified or treated with additional caution. Fig. 4 represents a comparison of the four existing codes for the chromaticity dependence on momentum. A special attention should be given also when a dynamical aperture is explored by the particle tracking. For example, in the TEAPOT [8] thin lens element code, the dipoles had to be divided each into more than 64 parts until correct results were obtained. The higher orders in the momentum compaction factor are not correctly obtained with different codes (while the code COSY [9] compared very well to the analytical results).

## 4 CONCLUSIONS

We presented a progress report on a lattice design of the 50 GeV muon collider storage ring. The local chromaticity correction and modified *FMC* module are presented. Major problems in designing the muon collider lattice were emphasized. The large collaboration between different laboratories world wide, continuously helps in a process of solving all challenging aspects for the future muon collider.

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