# FEEDBACK DESIGN METHOD REVIEW AND COMPARISON*

E. Onillon, BNL, Upton, NY

*Abstract*

Different methods for feedback designs are compared. These includes classical Proportional Integral Derivative (P. I. D.), state variable based methods like pole placement, Linear Quadratic Regulator (L. Q. R.), H_infinity and μ-analysis. These methods are then applied for the design and analysis of the RHIC phase and radial loop, yielding a performance, stability and robustness comparison.

## 1 INTRODUCTION

In the last two decades, new developments in control theory have been made, particularly in the field of state space based techniques like $H_2$ or $H_\infty$. The RHIC phase and radial loop have been designed using an $H_2$ approach (L. Q. R.), the state variables being beam phase, radius and the integral of the radius error. Studies, based on an $H_\infty$ approach, have been performed for a new design approach for those loops.

## 2 DESCRIPTION OF THE SYSTEM

The main variables used to describe the system are [1]: $\varphi$ the instantaneous phase deviation of the bunch from the synchronous phase, $\delta R$ the variations of the beam radius and $\delta\omega_{rf}$ the RF frequency deviation, b a scaling factor). The cavity transfer function is assumed to be the identity These variables are related by the two following transfer functions (Fig. 1) [1]: $\begin{matrix}\varphi = B_\varphi \delta\omega_{rf} \\ \delta R = B_R \delta\omega_{rf}\end{matrix}$ with $B_\varphi = \dfrac{s}{s^2+\omega_s^2}$ and $B_R = \dfrac{b}{s^2+\omega_s^2}$:

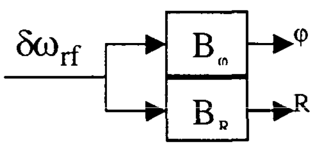


Figure 1: Block diagram

The system represented in Fig. 1 can be described using two state variables: $\begin{cases} x_1 = \dot{x}_2 = \varphi \\ x_2 = \dfrac{R}{b} = \dfrac{1}{s^2+\omega_s^2}\delta\omega_{rf} \end{cases}$

A third one, $x_3 = \int (R - R_{steer})dt$, is introduced to force the radius to follow its reference $R_{steer}$. These state variables, which are all observed, lead to the state space representation:

$$\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3\end{pmatrix} = \begin{pmatrix} 0 & -\omega_s^2 & 0 \\ 1 & 0 & 0 \\ 0 & b & 0 \end{pmatrix}\begin{pmatrix}x_1 \\ x_2 \\ x_3\end{pmatrix} + \begin{pmatrix}1 \\ 0 \\ 0\end{pmatrix}\delta\omega_{rf} + \begin{pmatrix}0 \\ 0 \\ -1\end{pmatrix} R_{steer} \quad \text{(eq 1)}$$

## 3 $H_\infty$ AND MIXED SENSITIVITY APPROACH

### *3.1 Sensitivity Functions and Loop Shaping*

If we consider the following block diagram [2] where K(s) is a feedback controller and G(s) the transfer matrix of the system.

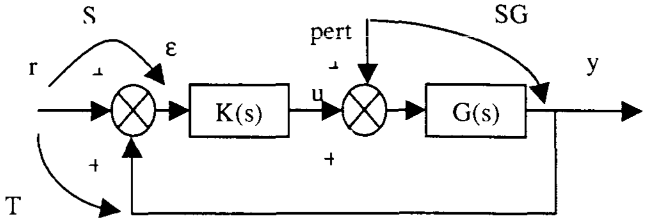


Figure 2: Sensitivity Function diagram

the transfer matrices relating the reference to the error ε and to the output y are respectively $S(s) = (I + G(s)K(s))^{-1}$ and $T(s) = (I + G(s)K(s))^{-1} G(s)K(s) = I - S(s)$

S(s) and T(s) are known respectively as the sensitivity function and the complementary sensitivity function.

From that diagram, one can see that

- a good reference tracking and a good rejection of the perturbation pert are obtained when S and SG are small
- the command effort is small when KS is small
- a good noise rejection is obtained when T is small

The gain of a transfer matrix, at a given frequency ω, will be characterized by its upper $\bar{\sigma}$ and lower $\underline{\sigma}$ singular values.

A transfer matrix G will be characterised by its $H_\infty$ norm defined as its biggest singular value: $\|G\|_\infty := \sup_\omega \bar{\sigma}(G(j\omega))$.

To design a feedback matrix K that matches the performance and robustness criteria, one will try to minimize S at low frequency (S behaves like the identity at high frequencies), and T at high frequency (T behaves like the identity at low frequencies), by choosing two

* Work performed under the auspices of the U.S. Department of Energy

weight matrices $W_1$ and $W_3$ that correspond to the shape of S and T or the open loop KL.
$\bar{\sigma}(S(j\omega)) \le |W_1^{-1}(j\omega)|$ and $\bar{\sigma}(T(j\omega)) \le |W_3^{-1}(j\omega)|$. These two requirements are combined into a single infinity norm specification of the form $\|T_{y_1u_1}\|_\infty \le 1$ where by definition

$T_{y_1u_1} = \begin{bmatrix} W_1S \\ W_3T \end{bmatrix}$, leading to the augmented plant:

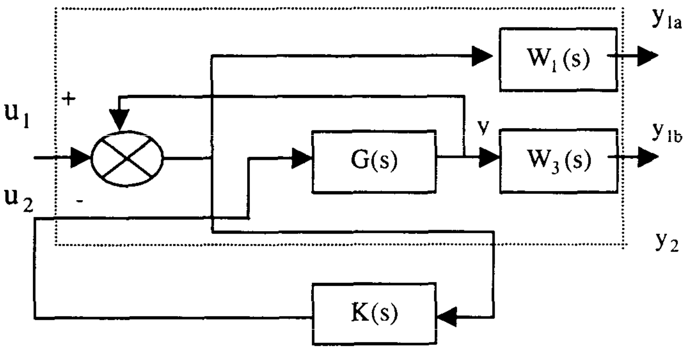


Figure 3: Augmented plant

### 3.2 Case of the phase and radial loop

$W_1^{-1}$ was chosen to be:

$$\begin{pmatrix} \frac{s^2+2.10^3s+1.410^6}{s^2+1.910^3s+10^6} & 0 & 0 \\ 0 & \frac{10^2s^2+810^4s+110^8}{s^2+2.310^3s+310^6} & 0 \\ 0 & 0 & \frac{10^2s^2+810^4s+110^8}{s^2+2.310^3s+310^6} \end{pmatrix}$$

and

$$W_3^{-1}: \begin{pmatrix} 8/(s+100) & 0 & 0 \\ 0 & 3010^6/s^2 & 0 \\ 0 & 0 & 10^2(s+9.310^3)/s^2 \end{pmatrix}$$

The system having a resonance at $\omega_s$, a bilinear transform has been performed to avoid a pole zero cancellation. A circle, which should contain the open loop poles, is defined [3]. The following results have been obtained:

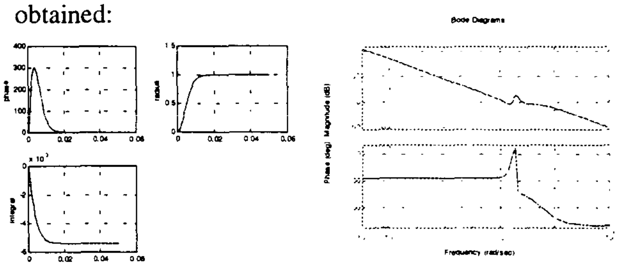

Figure 4: Step response and open loop Bode plot
The system settles in 10 ms. The phase and gain margins are respectively: 38 degrees and 9 dB.
The closed loop system is now:

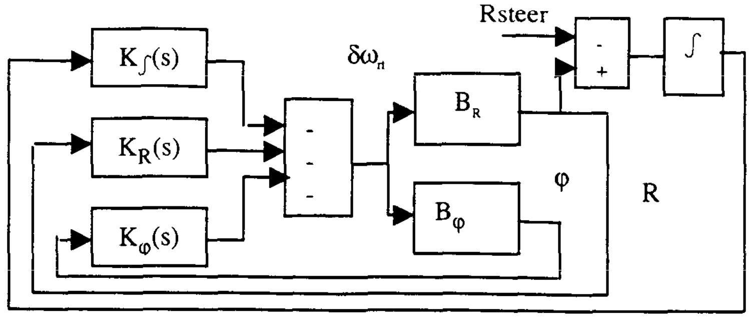


Figure 5: Closed loop system

$K_R(s)$, $K_\varphi$, $K_\int$ are the three feedback transfer functions:

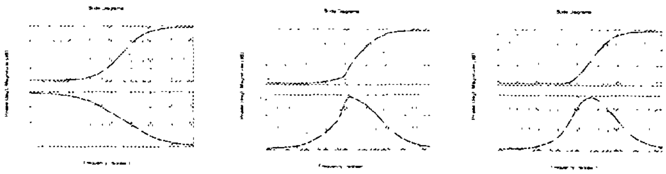

Figure 6: Controller Bode plot-

### 3.3 Robustness and μ analysis

One advantage of the previous approach is the ability to take into consideration the uncertainties on parameters or neglected dynamics. In the case of the phase and radial loop, the synchrotron frequency varies during acceleration: $\omega_s = \bar{\omega}_s\left(1 + \frac{\omega_0}{\bar{\omega}_s}\delta\right)$ which can be represented as follows:

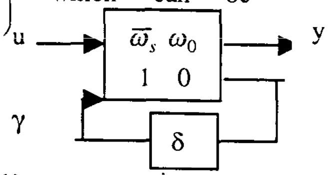


Figure 7: $\omega_s$ representation

The phase radial system, with no integral action, can now be represented as follows:

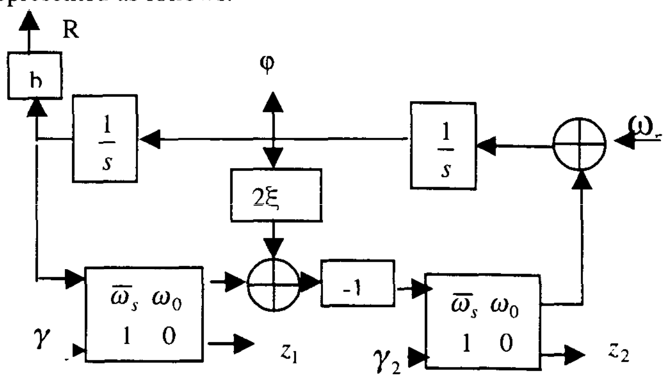


Figure 8: System representation

With $\Delta = \left\{ \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}, \delta_1 \in \Re, \delta_2 \in \Re \right\}$ and K the $H_\infty$ controller, one gets the generic M-Δ block diagram:

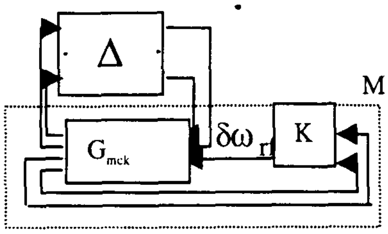


Figure 9: M Δ bloc diagram

The robustness is a measure of the size of the perturbation Δ that will make the system unstable. It requires the structured singular value μ of M with respect to the uncertainty Δ. The stability margin is defined as $\dfrac{1}{\max\limits_{\omega} \mu_\Delta(M(j\omega))}$ where

$\mu_\Delta(M) := \dfrac{1}{\min(\bar{\sigma}(\Delta) : \det(I - M\Delta = 0)}$. The following μ plot was obtained, where max(μ)=0.9 or $\delta_{\max} = 1.11$

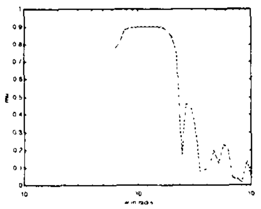

Figure 10: μ plot (μ as a function of ω in rad/s)

## 4 LQR APPROACH

Using the state variable representation defined in Eq 1, we can determine a Linear Quadratic Regulator (LQR), with the following quadratic performance index: $J = \frac{1}{2}\int_0^{+\infty}\left(X^T Q X + \omega_{rf}{}^T R \omega_{rf}\right)dt$, X being the state vector, Q minimising the deviation in states and R the input energy [3]. The Q and R matrices are chosen by the designer to obtain the desired system dynamic.

. With $Q = \begin{pmatrix} 600 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 10^6 \end{pmatrix}$ and $R = 10^{-6}$, one gets the following step radius response and open loop Bode plot:

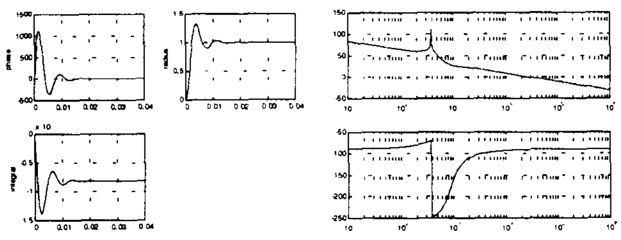

Figure 11: Step response and open loop Bode plot

This system settles also in 10 ms. The phase and gain margins are respectively: 90 degrees and infinity (propriety of LQR). The closed loop system is the same as in Fig.5 except that $K_R, K_\varphi, K_f$ are just gains.

## 5. CLASSICAL APPROACH

The phase and radial loop are two cascaded loops, the loop controllers being just classical filters.

With $K_\varphi(s) = 132\left(1 + \dfrac{1}{2.2\,10^{-3}s} + 5.5\,10^{-4}s\right)$ (PID) and $K_R(s) = 5\,10^3\,\dfrac{5\,10^3}{s + 5\,10^3}$ the following radius step response was obtained:

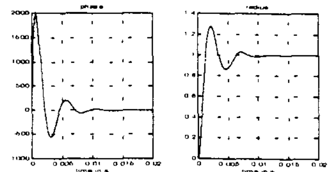

Figure 12: Step response (1 phase, 2 radius)

The system of Fig. 13 settles in 15 ms:

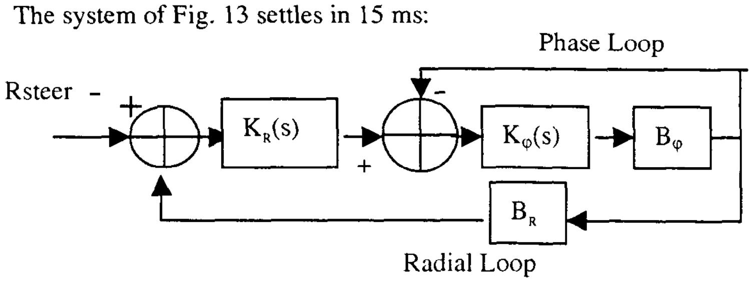


Figure 13: Closed loop system

## 6. CONCLUSION

The $H_\infty$ approach allows us to design a controller, by either shaping the open loop response or by defining a certain set of uncertainties and perturbations. Its realization will require the synthesis of three transfer functions. A robust analysis is then easy to perform.

The LQR approach will lead to a very simple realization: three gains and good stability margins. If the system is well known, it can lead to the programming of the feedback gains by switching to pole placement [4].

The traditional approach allows the decoupling between the phase and radial loop but the design of the controllers is more empirical.

## 7 REFERENCES


[1] RF Beam Control System for the AGS Booster, J. M. Brennan, BNL – 52438 Formal Report.

[2] Essentials of Robust Control, Kemin Zhou, Prenctice Hall.

[3] R. Y. Chiang, M. G. Safonov, Design of an $H_\infty$ Controller for a Lightly Damped System using a Bilinear Pole Shifting Transform, American Contr. Conf, May 90.

[4] The New BNL AGS Phase, Radial and Synchronization Loops, E. Onillon, J.M. Brennan, EPAC 1996.