A High Power RF Coupler Design for Muon Cooling
RF Cavities

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Abstract

We present a high power RF coupler design for an interleaved 5/8 805 MHz standing wave accelerating structure proposed for an muon cooling experiment at FNAL. The coupler, in its simplest form, is a rectangular waveguide directly connected to an accelerating cell through an open slot on the cavity side-wall or end-plates. Two of such couplers are needed to feed the interleaved cavities. Current high power RF test requires the coupler to be at critical coupling. Numerical simulations on the coupler designs using MAFIA will be presented.

1 INTRODUCTION

One of the most critical issues towards the feasibility of building an \( \mu^+ - \mu^- \) collider is how to effectively cool the six-dimensional emittance of the muon beams down by a factor of \( 10^9 \) to \( 10^6 \). Substantial progress has been made recently on the cooling studies both theoretically and experimentally. Among many possible cooling schemes, ionization cooling has been chosen to be a candidate to cool the \( \mu \) beams at early stage. An muon cooling collaboration, headed by FNAL, was established a few years ago to concentrate on the ionization cooling studies [1]. An experiment to demonstrate ionization cooling was proposed and has been developed since 1997. The cooling channel consists of about 20 cooling sections and each section has a 1.3 meters long linac and two absorbers surrounded by a super-conducting solenoid. A highest possible gradient accelerating structure is required to compensate the longitudinal beam energy losses in each section. An \( 5/8 \) interleaved structure has been developed between LBNL, FNAL, BNL and University of Mississippi [2, 3], and will be employed for this purpose. The structure closely resembles a pillbox cavity, but has the conventional beam irises covered by thin beryllium foils to enhance the on-axis accelerating fields. This brings the ratio of the surface field to accelerating field to one while for conventional structures it is typically between 2 to 5. The structure may be cooled down to liquid nitrogen temperature to further reduce the RF power losses on cavity wall which increases the shunt impedance by nearly another factor of two. Table 1 lists the relevant RF parameters of the cavity.

A three-cell low power test model, as shown in Figure 1, has been built and is currently under test at LBNL. A high power test on the structure will be conducted at FNAL after learning all the necessary information from the test model. A 5 Tesla super-conducting solenoid has also been designed and built at LBNL, and will be installed at FNAL for the high power test to study the performance of the structure under strong magnetic fields.

Table 1: RF parameters of the \( 5/8 \) interleaved cavity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (MHz)</td>
<td>805</td>
</tr>
<tr>
<td>Speed of muon ((\beta = v/c))</td>
<td>0.87</td>
</tr>
<tr>
<td>Shunt impedance ((Z_T^2)) (MO/m)</td>
<td>36.2</td>
</tr>
<tr>
<td>Average gradient (MV/m)</td>
<td>34</td>
</tr>
<tr>
<td>Peak power (MW/m)</td>
<td>32</td>
</tr>
<tr>
<td>Quality factor (Q_0)</td>
<td>18,700</td>
</tr>
<tr>
<td>Cell length (cm)</td>
<td>8.1</td>
</tr>
<tr>
<td>Beryllium thickness/radius (mm)</td>
<td>0.127/80</td>
</tr>
</tbody>
</table>

Figure 1: The three-cell copper test model of the \( 5/8 \) interleaved RF structure. The open irises will be covered by two beryllium windows of 127 \( \mu \)m of thickness and 16 cm of diameter.

2 THE COUPLER

The coupler, in its simplest form is a rectangular open slot either on the side wall or on the end-plate of the cavity and connected directly to a waveguide. For the high power test, the RF power has to be coupled through the end-plate since there will be no access available to the side wall once the super-conducting solenoid is in place. The rectangular open slot has been chosen to have a cross section of \( 12 \text{ cm} \times 2 \text{ cm} \). The coupler and waveguide need to be fitted within the bore size of the super-conducting solenoid of 440 cm of...
ID, and leave a clear passage for the $\mu$ beams at region of $R \leq 5$ cm, and transition to a standard WR975 waveguide. Critical coupling is required at resonant frequency of 805 MHz.

2.1 The Simulation Model

MAFIA (T3 module) time domain simulations were used to determine the coupling between the coupler and the cavity. A simplified 3D model was used to simulate the coupler + cavity system. The model consists of a pill-box cavity with one of the end-plates having a rectangular open slot coupled to a WR975 waveguide, shown in Figure 2. The area of the slot was kept unchanged while the depth of the slot was varied to adjust the coupling strength. The radius of the cavity was tuned to get the resonant frequency of 805 MHz. It is worth pointing out that only one single pill-box cell is needed to simulate the coupling of the system if each accelerating cell is identical, the coupling constant for the multi-cell case is then determined by,

$$J_{e}(N) = N \cdot J_{e}(1) = N \cdot \frac{Q_{0}}{Q_{ext}}, \quad Q_{ext} = \omega \frac{U}{P_{ext}}, \quad (1)$$

where $J_{e}(1)$, $J_{e}(N)$ are coupling constants for single and $N$ cells; $U$ is the stored energy in the single cavity; $\omega$ is the angular resonant frequency and $P_{ext}$ is the power loss to a matched load; $N$ is the number of total cells. For the three-cell interleaved 5 structure (only two accelerating cells) we design $J_{e}(1) = 2$ with the one cell model.

![Figure 2: A 3D solid MAFIA model used for the coupling simulations](image)

2.2 Simulation Procedures

The waveguide boundary condition at the waveguide port was used. the electro-magnetic field in the cavity was excited by a broad band dipole pulse with its frequency centered around 805 MHz (fundamental TM010 mode). To simulate the waveguide boundary conditions a two-dimension frequency domain run is necessary in order to obtain the propagation modes in the waveguide. These waveguide modes are loaded into the waveguide port when the time domain simulation starts. To avoid exciting unwanted higher order modes, a filament (a line dipole) was defined on the axis with a length of about half the cavity gap and its polarization pointed to $z$. The band-width of the pulse was chosen to be wide enough (short pulse), but not too wide to drive the nearest higher order modes. For a high $Q$ cavity, the induced $em$ fields will be dominated by the fundamental mode quickly once the excitation pulse is gone. The power loss from the cavity + coupler system (perfect conductor was assumed) is the one exiting from the waveguide port, namely $P_{ext}$. The $em$ fields in the cavity are then oscillating at its natural resonant frequency, and its amplitude (either $E$ or $B$ field) exponentially decays with a time constant of $\tau_{ext}$ given by,

$$\tau_{ext} = \frac{2Q_{ext}}{\omega} = \frac{2U}{P_{ext}}. \quad (2)$$

For a weakly coupled system ($J_{e} < 1$), $\tau_{ext}$ could be as long as $10^4$ times of one RF period. Measuring $\tau_{ext}$ of the $em$ amplitudes from a short time domain simulation, for instance 100 RF periods, becomes difficult and inaccurate. Nevertheless, the induced time varying $em$ fields in the cavity does provide enough information to allow for an accurate measurement of the resonant frequency of the system. The resonant frequency can simply be calculated by counting the time of the last 10-20 RF periods. The time step of the simulations is of the order of $10^{-12}$ to $10^{-13}$ seconds, which gives at least one order of magnitude better accuracy of the frequency than the one obtained from FFT when approximately total 100 RF periods ($\sim 10^{-7}$ seconds) are used. Two methods (we call energy and decay methods) have been used for $Q_{ext}$ calculation. These methods have been discussed in detail in [4]. At most two MAFIA time domain runs are needed for each configuration. Both $Q_{0}$ and $Q_{ext}$ can be calculated from the time domain simulation results. After obtaining the resonant frequency $\omega$ (discussed above), we excite the system with a $sin(\omega t)$ line dipole. The $em$ fields in the cavity should build up coherently. The fields are recorded at a few carefully chosen time steps during the simulation, and used later in the MAFIA post-processor (T3) to reconstruct quasi-time harmonic $em$ fields. Taking the advantage of many built-in functions in p3, both $Q_{0}$ and $Q_{ext}$ can be readily calculated [4]. This method (energy method) works very well for high $Q$ systems with weak coupling where the decay method becomes impractical because a long time record is needed. For a strongly coupled system, both methods work equally well, but the decay method is more efficient if only $Q_{ext}$ is measured. To save cpu time, typically 50 ~ 80 RF periods are long enough for each simulation. A model with about 11,000 mesh points takes approximately 20 minutes of cpu time on an Ultra 60 Unix Sun-Workstation.

It is important to point out that when using the energy method, the continuous dipole excitation fields contribute to both the stored energy in the cavity and $P_{ext}$. In general one should wait until the $em$ fields of the cavity mode dominate before recording the fields where the contribu-
tions from the dipole become negligible. However, for a strongly coupled system (or a complex system), the time that it takes for the cavity mode to dominate could be quite long, this contribution can not be neglected in a short time-domain run. However the dipole fields can be subtracted in the post-processor because their contributions are steady over the time. By recording the fields at two time steps, \( Q_0 \) and \( Q_{ext} \) can be computed by,

\[
Q_0 = \omega \frac{U(t_2) - U(t_1)}{P_w(t_2) - P_w(t_1)}; \quad Q_{ext} = \omega \frac{U(t_2) - U(t_1)}{P_{ext}(t_2) - P_{ext}(t_1)},
\]

where \( U(t_1), P_w(t_1) \) and \( P_{ext}(t_1) \) are stored energy in the cavity, power loss on cavity wall and power loss in the matched load at time \( t_1 \), respectively. Equation 3, in fact, can be used even at times when the dipole contributions become negligible. Another alternative solution is to compute for longer time (more cpu time of course!) until the dipole fields becomes negligible. The \( Q_0 \) and \( Q_{ext} \) are simply given by,

\[
Q_0 \approx \omega \frac{U(t_1)}{P_w(t_1)}; \quad Q_{ext} \approx \omega \frac{U(t_1)}{P_{ext}(t_1)}.
\]

Calculations of \( U(t_1), P_w(t_1) \) and \( P_{ext}(t_1) \) using time-domain simulation results are also presented in details in [4].

### 2.3 Simulation Results

Based on the methods discussed above, MAFIA simulations were performed on the model shown in Figure 2. As an example for the energy method, Figure 3 shows a snap shot of electric fields in the cavity + coupler system driven by a continuous line dipole resonating at \( \omega \). The fundamental TM\(_{010}\) mode is clearly dominant in the system. Fields at the coupling slot can barely be seen. Figure 4 gives a time record of a magnetic component of \( B_z(t) \) monitored at a large radius. It is apparent that the field is oscillating at a single mode, namely the fundamental mode, after the excitation pulse. In this example, the \( Q \) is fairly high and the expected exponential decay can hardly be observed at this time scale. Nevertheless zooming into the last 10-20 RF periods, the resonant frequency of the system can be measured.

![Figure 3: A snap shot of electric fields in the cavity and the coupler](image)

For a strongly coupled system, the \( \epsilon \mu \) fields exhibit an exponential decay pattern (it looks like a linear pattern if time record is less than \( \tau_{ext} \)). Calculation of \( Q_{ext} \) is straightforward, and given by,

\[
Q_{ext} = -\left(\frac{\omega}{2}\right) \ln B_z(t_2) - \ln B_z(t_1),
\]

where \( B_z(t_1) \) is the \( B_z \) measured at \( t_1 \).

### 3 CONCLUSION

The coupler has been designed using the MAFIA time domain simulations. The coupling slot dimension was determined to be 12 cm \( \times \) 2 cm \( \times \) 8 cm (width \( \times \) height \( \times \) depth) to give critical coupling (\( Q_0 = 2 \)) at the frequency of 805 MHz for a three-cell \( \frac{3}{2} \) interleaved structure. The MAFIA time domain simulations and the methods we developed have been proved to be a powerful tool for our coupler designs, and can be easily applied to other similar applications.

### 4 REFERENCES


