Ultradense Quark Stars from Perturbative QCD*

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The equation of state for cold quark matter is studied in perturbative quantum chromodynamics up to second order in the strong coupling constant α_s. The equation of state allows for a new class of solution at high density besides the one for ordinary neutron stars which is formed by deconfined matter. The resulting mass-radius relation exhibits extremely dense stars with maximum masses of about $0.3 M_\odot$ and radii below 2 km.

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I. INTRODUCTION AND MOTIVATION

The history of neutron stars dates back to the year of 1932, when Chadwick discovered the neutron [1]. During the very same year, Landau predicted the existence of neutron stars. A couple of years later, Baade and Zwicky suggested the connection between neutron stars and the phenomenon of supernovae explosions. However, it was not until 1939 that the first neutron star theoretical calculations were performed by Tolman and Oppenheimer, and by Volkoff [2]. Nowadays, the equations derived in this calculation are known as the Tolman-Oppenheimer-Volkoff (TOV) equations, and are at the basis of stellar structure calculations. The hypothesis of neutron stars formation in supernovae explosions was strengthened by the discovery of radio pulsars by Hewish *et al.* [3]. Nevertheless, new possibilities for compact stars came also from the quark model Gell-Mann and Zweig proposed for hadrons and its future consequences [4]. Perhaps the first proposal of superdense quark stars appeared in the work of Ivanenko and Kurdgelaidze [5], in 1965. But it was the discovery of asymptotic freedom that opened the way for the hypothesis that matter at the high densities found in neutron star cores could be a quark soup due to hadrons overlap. This ponderation, together with a discussion on the possibility of superfluidity and superconductivity effects, was done in a remarkable paper by Collins and Perry [6] ten years after the idea of quark stars appeared for the first time. At this time, the subject attracted some attention and motivated many papers on the quark-hadron phase transition, the high-density regime of QCD, and the study of neutron stars, culminating with the first systematic quark star phenomenology based on high-density perturbative QCD results presented by Freedman and McLerran (see [7] and references therein). The next major development in this field came when Witten proposed the idea of *strange matter*, i.e., that quark matter rather than nuclear matter might be the ground state of QCD at finite baryon number [8]. This assumption lead Farhi and Jaffe to the study of the stability of strange matter [9] and stimulated the investigation of self-bound strange stars [10]. From this point on, an entire zoo of possibilities arose: strange stars, different families of neutron stars, hybrid stars, etc [1].

In order to test all those possibilities, one has to compare theoretical predictions to actual astronomical observables. For the sake of simplicity we will focus on two of them: the total mass and the total radius of the star. The way to calculate these quantities in a given model is by solving the TOV equations, which are derived from Einstein's field equations assuming a static and spherically symmetric star (see, *e.g.*, Ref. [11]). The TOV equations have the following form:

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]\left[1-\frac{2G\mathcal{M}(r)}{r}\right]^{-1} , \tag{1}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2\epsilon(r) \ ; \ \ \mathcal{M}(R) = M , \tag{2}$$

where p is the pressure, ϵ is the energy density, r is the radial coordinate, and G is Newton's gravitational constant. The total radius is represented by R and the total mass by M. The quantity $\mathcal{M}(r)$ gives the mass of the star up

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to a radius r. Given $\mathcal{M}(0) = 0$, $\epsilon(0) = \epsilon_c$, the energy density at the center of the star, and the equation of state (EoS) $p = p(\epsilon)$, one can integrate the TOV equations from the origin until the pressure $p(r)$ becomes zero at $r = R$. Given $p = p(\epsilon)$, $M(\epsilon_c)$ defines a family of stars. Extrema in $M(\epsilon_c)$ signal gravitational instability and this defines a maximum mass (see [11] for details). Different types of stars have different EoS and, therefore, different astronomical output.

The usual approach to quark stars relies on the MIT bag model for the EoS and provides results that depend strongly on the bag parameter B. In this work, we present calculations of the equation of state of cold and dense quark matter using the perturbative expansion up to second order in the strong coupling constant $\alpha_s = g^2/(4\pi)$, which is allowed to run according to the renormalization group equation. The typical densities found inside quark stars allow for a sensible use of perturbation theory [12]. Following this procedure, we find significant deviations from the equation of state obtained from the MIT bag model. Nevertheless, we can also reproduce the results obtained from the usual approach in a particular limit of this more fundamental model.

It is not our aim to provide a realistic and accurate description of the phenomenology related to quark stars by studying higher order corrections to the thermodynamic potential. We intend to highlight the essential difference between the usual approach, which uses the MIT bag model to obtain the EoS, and one which is solely based on perturbative QCD as a guideline to what might happen at very high densities. The perturbative approach provides phenomenological results that depend on fundamental quantities, the beta function, instead of some phenomenological bag parameter [13].

II. USUAL APPROACH TO STRANGE STARS

The basic physical picture adopted in the usual approach [1,10] to strange stars is that of strange matter described by a Fermi gas of up, down and strange quarks, and electrons, where the region the quarks live in is characterized by a constant energy density B, the bag parameter. Since the stellar temperature, in the case of neutron stars and quark stars, is much smaller than the typical chemical potentials, one can assume zero temperature from the beginning. Moreover, one assumes chemical equilibrium, so that

$$\begin{aligned} d &\longrightarrow u + e^- + \overline{\nu_e} \quad , \\ u + e^- &\longrightarrow d + \nu_e \quad , \\ s &\longrightarrow u + e^- + \overline{\nu_e} \quad , \\ u + e^- &\longrightarrow s + \nu_e \quad , \\ s + u &\longrightarrow d + u \quad , \end{aligned} \tag{3}$$

which implies $\mu_d = \mu_s \equiv \mu$ and $\mu_u + \mu_e = \mu$, as $\mu_\nu = 0$. Overall charge neutrality implies

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0\,, \tag{4}$$

where the particle density is $n_i = -(\partial\Omega_i/\partial\mu_i)$, so that there is only one independent chemical potential. Here, Ω_i is the thermodynamic potential, usually the one for a free gas plus eventual $O(\alpha_s)$ corrections with α_s taken to be constant.

For the simple case $m_u = m_d = m_s = 0$, $\alpha_s = 0$, and $\mu_e = 0$ ($n_e/n_u \approx 0$), the EoS simplifies to

$$p(\epsilon) = \frac{1}{3}(\epsilon - 4B) \quad . \tag{5}$$

It is remarkable that, for intermediate values of m_s, the correction to the result above is less than 4%. The EoS is dominated essentially by B. The global properties of strange stars one obtains with such an EoS are the following (for $B^{1/4} = 145\ MeV$) [10]:

$$M_{max} = \frac{0.0258}{G^{3/2}B^{1/2}} \approx 2M_\odot \quad , \tag{6}$$

$$R_{max} = \frac{0.095}{G^{1/2}B^{1/2}} \approx 11\ km \quad , \tag{7}$$

$$\epsilon_c^{max} = 19.2B \approx 2 \times 10^{15}\ g/cm^3 \approx 8\epsilon_0 \quad . \tag{8}$$

$$\epsilon_{surf} = 4B \approx 4 \times 10^{14}\ g/cm^3 \approx 2\epsilon_0 \quad , \tag{9}$$

$$\epsilon_{surf}^{NS} = \epsilon_{solidFe} \approx 7.8\ g/cm^3 \quad . \tag{10}$$

Here $\epsilon_0 \approx 2.5 \times 10^{14} g/cm^3$ is the energy density of normal nuclear matter and $\epsilon_{solidFe}$ is the density of solid iron, found in the surface of neutron stars. One can see the strong dependence of the results on B and the very different patterns that arise for strange stars as compared to neutron stars. However, for $M \approx 1.4 M_{\odot}$ the range of possibilities for the total mass and the total radius in each case is almost the same.

III. QUARK STARS FROM PERTURBATIVE QCD

We consider the case of three-flavor massless quarks at zero temperature. Matter in compact stars is in β-equilibrium and depends on the quark (or baryon) chemical potential as well as the electrochemical potential. The electrochemical potential vanishes for massless three-flavor quark matter as it is charge neutral by itself. Then, the chemical potentials of the up, down, and strange quarks must be equal so that one has equal Fermi momenta and equal abundances of all three light quarks in matter. So, we have only one independent chemical potential as before.

The thermodynamic potential of a plasma of massless quarks and gluons was calculated perturbatively up to $O(\alpha_s^2)$, in a momentum-space subtraction scheme (MOM) with a dimension dependent Landau gauge by Freedman and McLerran [7]. Baluni [14] did a similar calculation using the MOM scheme in Feynman gauge. The results of these works are consistent with each other and can be transformed into the $\overline{\text{MS}}$ subtraction scheme [15], resulting in the following transformation of the coupling constant:

$$\frac{\alpha_s^{\text{MOM}}}{\pi} = \frac{\alpha_s^{\overline{\text{MS}}}}{\pi} \left[1 + \mathcal{A}\ \frac{\alpha_s^{\overline{\text{MS}}}}{\pi} \right] \quad , \tag{11}$$

where $\mathcal{A} = 151/48 - (5/18)N_f$ with N_f being the number of flavors. The translation between schemes up to this order corresponds to a shift in the constant of the second-order term of the original (MOM) potential. Then, the final form for the thermodynamic potential is given by

$$\begin{aligned} \Omega(\mu) = &\frac{3\mu^4}{4\pi^2} \left\{ 1 - 2\left(\frac{\alpha_s}{\pi}\right) \right. \\ &\left. - \left[G + N_f \ln \frac{\alpha_s}{\pi} + \left(11 - \frac{2}{3}N_f\right) \ln \frac{\bar{\mu}}{\mu} \right] \left(\frac{\alpha_s}{\pi}\right)^2 \right\} , \end{aligned} \tag{12}$$

where in the $\overline{\text{MS}}$ scheme $G = 10.374 - 0.536 N_f + N_f \ln N_f$ and $\bar{\mu}$ is the renormalization subtraction point. The scale dependence of the strong coupling constant $\alpha_s(\bar{\mu})$ obeys [16]:

$$\begin{aligned} \alpha(\bar{\mu}) = &\frac{4\pi}{\beta_0 \ln\left(\bar{\mu}^2/\Lambda_{\overline{\text{MS}}}^2\right)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln\left[\ln\left(\bar{\mu}^2/\Lambda_{\overline{\text{MS}}}^2\right)\right]}{\ln\left(\bar{\mu}^2/\Lambda_{\overline{\text{MS}}}^2\right)} \right. \\ &+ \frac{4\beta_1^2}{\beta_0^4 \ln^2\left(\bar{\mu}^2/\Lambda_{\overline{\text{MS}}}^2\right)} \left(\left(\ln\left[\ln\left(\bar{\mu}^2/\Lambda_{\overline{\text{MS}}}^2\right)\right] - \frac{1}{2} \right)^2 \right. \\ &\left.\left. + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4} \right) \right] \quad , \end{aligned} \tag{13}$$

where $\beta_0 = 11 - 2N_f/3$, $\beta_1 = 51 - 19N_f/3$, and $\beta_2 = 2857 - 5033N_f/9 + 325N_f^2/27$.

There is some freedom in the actual choice of $\bar{\mu}$, provided that it minimizes the contribution generated by logarithmic corrections to the thermodynamic potential of the form $(\ln(\bar{\mu}^2/\mu^2))^n$ [7,14]. The only scale in the problem is the Fermi momentum of the quarks which equals the quark chemical potential for massless particles. A natural choice is to set $\bar{\mu} = \mu$. Later, we will discuss the sensitivity of our result with respect to the choice of μ. We fix the scale $\Lambda_{\overline{\text{MS}}}$ by requiring that $\alpha_s = 0.3089$ at $\bar{\mu} = 2$ GeV as taken from the fit of the Particle Data Group to the experimental data [16]. This gives $\Lambda_{\overline{\text{MS}}} = 0.365$ GeV for $N_f = 3$.

From the knowledge of the thermodynamic potential we can immediately obtain the pressure, $p(\mu) = -\Omega(\mu)$, the quark number density $n = (\partial p/\partial \mu)$, and the energy density $\epsilon = -p + \mu n$. Therefore, we have all the ingredients that are necessary to solve the TOV equations with this new EoS and, then, obtain the relevant astrophysical features solely in terms of α_s and the beta function.

The pressure, in units of the pressure of a free gas, is shown in Figure 1 as a function of the quark chemical potential. There, we show the results up to first order in α_s and the one which includes the $O(\alpha_s^2)$ contribution. It is clear from this plot that the interactions between quarks can not be ignored, even in this large-μ region. Moreover, contrary to the case of finite-temperature perturbation theory [17], the series is reasonably well-behaved. Then, perturbation theory seems to be applicable for the pressure in the range of μ considered, but it does not imply that it works well for other observables as pointed out recently by Rajagopal and Shuster [18]. The remarkable feature in this figure is the fact that the pressure vanishes for $\mu = 0.767\ GeV$, so that the star is self-bound and represents a new class of solution at ultrahigh densities. This fact is emphasized in Figure 2. There one can see that the result for the case $\bar{\mu} = \mu$ represents a new branch in the EoS and can not be matched with hadronic equations of state [19]. In fact, there is a big gap separating the two regions.

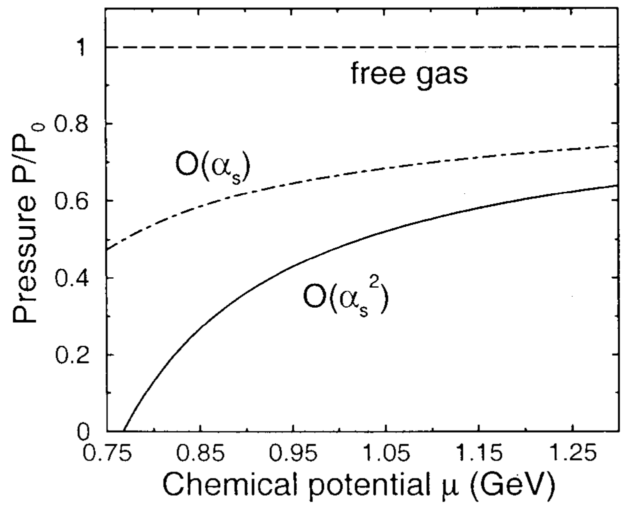

FIG. 1. Total pressure (in units of the free gas pressure) as a function of the chemical potential for the case $\bar{\mu} = \mu$. We show the result up to first order in α_s and the one which includes the second-order contribution.

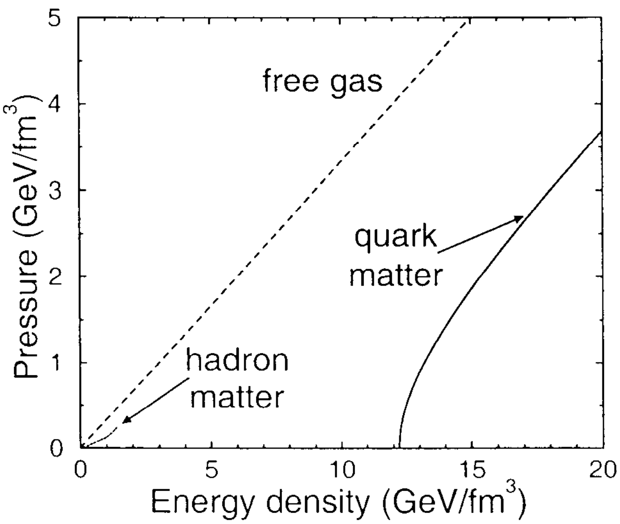

FIG. 2. Equation of state for the case $\bar{\mu} = \mu$. The free gas and a hadronic equation of state are displayed for comparison.

Figure 3 shows the total mass as a function of the central energy density for the $\bar{\mu} = \mu$ case. As mentioned before, extrema in such a plot signal gravitational instability and define a maximum allowed total mass for a stable star. For the case under consideration, the maximum mass is of the order of $0.35 M_\odot$, and the central energy density can reach values almost a hundred times greater than ϵ_0. These results should be compared to the ones obtained by using the

MIT bag model equation of state, namely $M_{max} \approx 2M_{\odot}$ and $\epsilon_c^{max} \approx 8\epsilon_0$.

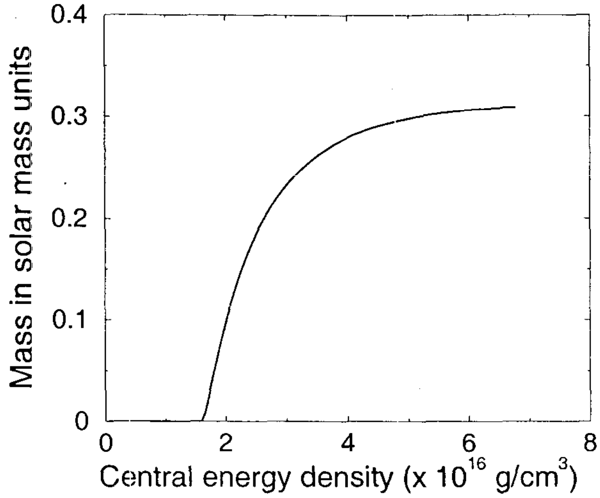

FIG. 3. Total mass (in solar mass units) as a function of the central energy density, for the case $\bar{\mu} = \mu$.

Figure 4 displays the mass-radius relation for the cases $\bar{\mu} = \mu$, $\bar{\mu} = 2\mu$ and $\bar{\mu} = 3\mu$. They should be compared to the usual results for the maximum mass, quoted above, and for the maximum radius, $R_{max} \approx 11km$. One can see the strong dependence on the choice of the scale $\bar{\mu}$. In fact, for higher values of $\bar{\mu}$, one can reach the region that reproduces the results obtained by using the usual approach.

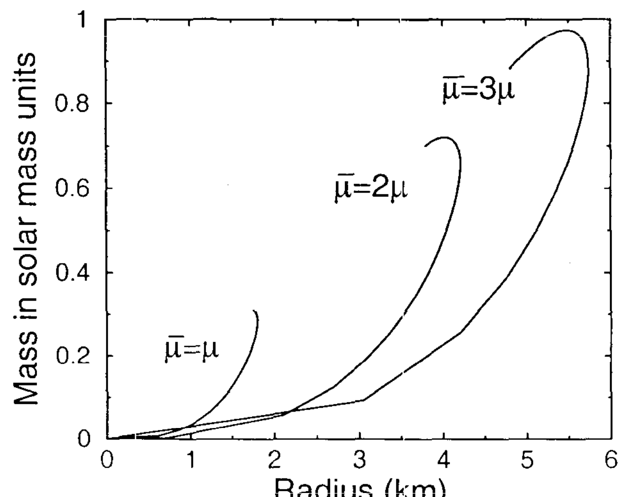

FIG. 4. Mass-radius relation for the cases $\bar{\mu} = \mu$, $\bar{\mu} = 2\mu$ and $\bar{\mu} = 3\mu$.

IV. CONCLUSION AND FUTURE WORK

Although the patterns of the curves for the mass-radius relation presented in Figure 4 resemble those obtained for strange stars within the MIT bag model approach, the phenomenology encountered in our treatment is very different. The quark stars which result from perturbative QCD are roughly five times smaller, they are less massive ($M \approx 0.3 - 0.9\ M_{\odot}$ as compared to $M \approx 2\ M_{\odot}$ for strange stars), and much denser. They represent an entirely new class of ultradense stars. The strong dependence on the choice of the scale $\bar{\mu}$, which must be chosen in a way that minimizes the contribution of higher-order logarithmic corrections [7], is of course unpleasant and is under current investigation [13]. However, it does not diminish the spectrum of exciting possibilities openened by the results provided by perturbative QCD. The possible existence of quark stars could also have considerable impact on the study of cold quark matter in connection with color superconductivity [20]. They might be detected in micro-lensing experiments.

In fact, the MACHO project has reported micro-lensing events for the Large Magellanic Cloud [21], interestingly with mass ranges close to our calculated quark star masses of $M = 0.15$–$0.9 M_{\odot}$.

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